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THE FEASIBILITY OF A LOW-ORDER MODEL OF A MOIST GENERAL CIRCULATION

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A low-order two-level quasi-geostrophic model of a moist general circulation is formulated. The model consists of a system of 27 ordinary differential equations, or, in one variant, 20 equations. Modeled processes include evaporation from the ocean, condensation and evaporation within the atmosphere, precipitation reflection of solar radiation by clouds, and absorption and emission of long-wave radiation by (Over)

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clouds and water vapor. A portion of the region represented by a single grid point may be subsaturated while another portion contains clouds.

A wide variety of numerical experiments may be performed with the model. When the prescribed solar heating is horizontally uniform, there are sometimes two distinct states of stable equilibrium - a very cold state and a very warm state.

FOREWORD

The principal aim of Contract F-19628-78-C-0032 is to elucidate the influence of water as an atmospheric constituent on the over-all behavior of the atmosphere. The processes involving water which will particularly concern us are evaporation from the earth's surface with its accompanying cooling of the surface, condensation or evaporation within the atmosphere with its accompanying warming or cooling, precipitation, reflection of short-wave radiation by clouds, and absorption and emission of long-wave radiation by clouds and water vapor. We are interested in how the atmospheric wind and temperature fields differ from what they would be if water were not present, and in how the various processes which we have mentioned contribute to the differences. We are also interested in such matters as the contributions of these processes to the changes in the wind and temperature fields which would accompany a change in the intensity of the solar radiation.

Our specific goal has been the construction of a low-order numerical model of the general atmospheric circulation which incorporates all of the processes involving water which we have mentioned. By a low-order model we mean one defined by a minimal number of prognostic ordinary differential equations which can yield a qualitatively valid description of the processes involved. We do not expect a low-order model to give quantitatively correct results, and we certainly do not expect it to produce good day-to-day weather forecasts.

The most obvious advantage of a low-order model is the speed with which numerical integrations can be performed. This allows us to examine many more cases than could economically be treated with a large model. Perhaps equally important is the fact that the paucity of dependent variables greatly facilitates the analysis of the results.

The principal section of this report, entitled "The feasibility of a low-order model of a moist general circulation", is devoted to a description of the low-order moist model which we have formulated. We also describe some preliminary numerical experiments in which the solar heating is taken to be horizontally uniform. We intend to submit this section to a journal for publication, after we have performed and analyzed a few numerical experiments with horizontally variable solar heating.

Under a previous contract (F 19628-77-C-0026) we introduced the concept of moist available energy (MAE). This is identical to available potential energy in a dry atmosphere, but in a moist atmosphere it is defined in such a manner that both dry-adiabatic and moist-adiabatic processes preserve the sum of MAE and kinetic energy. MAE may be generated and destroyed by diabatic heating and cooling. It may also be generated by evaporation from the earth's surface and destroyed by precipitation, but it is not altered by condensation and evaporation within the atmosphere. We devised a graphical procedure for evaluating the MAE present in a given atmospheric state.

One of the first tasks under the present contract was to devise a numerical procedure for evaluating MAE. We felt at that time that this task should precede construction of our low-order model, and that the low-order model, when formulated, should treat MAE in a qualitatively correct manner. As our low-order model stands at present, we have not identified any quantity which possesses the properties of MAE.

Having developed a rapid numerical procedure for evaluating MAE, we felt that it would be desirable to apply it to a sequence of real weather situations. In order to perform the evaluations, we had to introduce assumptions regarding the amount of liquid water at each point in the atmosphere, since liquid water is not one of the quantities which is observed on a routine basis. We soon discovered that our results were highly sensitive to the manner in which we estimated the liquid water content. We therefore decided to abandon our attempts to study the day-to-day variations of MAE, at least until such time as suitable methods of estimating liquid water content became available, possibly through new satellite observations.

Another of our aims was to develop a low-order moist model spanning many scales of motion, possibly from the global scale to the cumulus scale, in contrast to the model described in this report, which covers only the largest scales. It became evident that such a model could not easily be formulated until the present moist model was in working order. As a result we have not progressed with this task.

As described in the main section of this report, we have already uncovered one surprising result, namely that with certain intensities of horizontally uniform solar heating, there are two markedly different equilibrium distributions of temperature and moisture. We believe that further experiments with the model will uncover some equally interesting results.

THE FEASIBILITY OF A LOW-ORDER MODEL OF A MOIST GENERAL CIRCULATION

1. Introduction

The past two decades have seen the ever-increasing application of highly simplified systems of equations to various problems involving the circulation of the atmosphere or the ocean or some other fluid system. Such systems of equations have come to be known as "low-order models." The construction of a low-order model of the atmosphere usually begins with the selection of a set of M prognostic partial differential equations, and some auxiliary diagnostic equations, which approximate the equations governing the atmosphere; the M dependent variables usually include the wind components and the temperature, and sometimes the water content. The vertically continuous atmosphere is first approximated by an array of N layers; the three-dimensional field of each dependent variable is replaced by N two-dimensional fields, and the equations are modified accordingly. Often $N = 2$; sometimes $N = 1$. Each two-dimensional field is then expanded in a series of orthogonal functions - generally a double Fourier series if the earth is approximated by an infinite plane or channel, or a series of spherical harmonics if spherical geometry is retained - and the coefficients of the orthogonal functions become the dependent variables in the new equations. Finally the system is truncated by omitting reference to all but a small number K of terms in each series. The resulting KMN prognostic ordinary differential equations can retain some of the essential nonlinearity of the advective process even when $K = 3$.

A similar procedure may be used when the fluid system is a portion of the atmosphere, all or a portion of the ocean, or a laboratory model of the atmosphere or the ocean. Variations are possible; for example, the value of N or K may be different for different variables. Ordinarily, the advective terms in a low-order model will preserve total energy if those in the original equations do so also, provided that the total energy is a quadratic function of the dependent variables.

One does not ordinarily expect quantitatively correct results from a low-order model when the truncation of the series is severe. It is widely assumed, however, perhaps mainly as an article of faith, that a carefully constructed low-order model can give results which are qualitatively realistic, at least over certain ranges of the adjustable parameters. The advantages of low-order models are that in some instances analytic solutions may be found, while, when numerical methods must be used, the computations needed to simulate a given span of real time may be fewer by several orders of magnitude than those required when a large global circulation model or something equally detailed is used. Also, the paucity of variables can facilitate the subsequent interpretation of the numerical results. Thus, with a model consisting of only eight prognostic ordinary differential equations, we were able, using analytic procedures, to reproduce the qualitative arrangement in parameter space of the transition between the Hadley and Rossby regimes, as observed in laboratory experiments (Lorenz, 1962). When we increased the number of equations to fourteen, we were able, using numerical procedures, to reproduce the transition from steady Rossby flow to vacillation, with relatively little computational effort (Lorenz, 1963).

Many of the recent large global circulation models treat the atmosphere as a mixture of dry air, water vapor, liquid water, and perhaps ice. Low-order models of small-scale systems also have frequently been moist models. Low-order general-circulation models, however, have generally treated the atmosphere as an ideal gas. Reasons for this state of affairs are not hard to identify. First, although simple products, such as those appearing in advective terms, assume a reasonably simple form when transformed into Fourier space, some of the nonlinear functions arising in moist-atmosphere thermodynamics, such as those relating saturation mixing ratio to temperature and pressure, transform into very complicated expressions.

With large models this difficulty may be circumvented by transforming the variables from Fourier space to grid-point space, performing the thermodynamic computations at each grid point, and then transforming back to Fourier space, at every time step, thanks to the Fast Fourier Transform. With low-order models this procedure introduces another difficulty. The mixing ratio, which is commonly used as a measure of water-vapor content, is always positive, but on a global scale it may vary by one or even two orders of magnitude in the horizontal direction. A highly truncated Fourier series which best approximates the mixing ratio over the bulk of the atmosphere is likely to indicate negative values at some points and values representing supersaturation at others.

The purpose of this work is to demonstrate that the formulation of a low-order model of a moist general circulation is nevertheless feasible. We shall construct a model in which the basic variables are Fourier coefficients, and we shall transform the variables to and from grid-point space. We shall include the thermodynamic effects of phase changes of water, and also the effects of water vapor and liquid water on short-wave and long-wave radiation, which we shall formulate in a simplified manner compatible with the low vertical resolution of the model. We shall not include the possible presence of ice. We shall allow liquid water to remain for a while in the form of clouds before returning to the earth as rain.

Since the atmosphere receives much of its energy in the form of latent heat through evaporation from the ocean, we shall find it convenient to let the earth's surface consist entirely of ocean. The simplest arrangement, in the sense of entailing the fewest dependent variables, would be to let the ocean have zero heat capacity, and to determine its temperature diagnostically by requiring that it neither gain nor lose energy. Such an ocean might more properly be thought of as wet land. It is more realistic, and just as simple computationally, to give the ocean a finite heat capacity, in which case the ocean temperature becomes another dependent variable. We shall not consider an ocean of infinite heat capacity, which would require prespecifying the ocean temperature.

To avoid the particular difficulties which we have noted, we shall make a few departures from the most commonly used procedures. First, we shall use a quantity representing total water content rather than water-vapor content as a basic variable. We shall use an auxiliary diagnostic equation to specify how much of the total water is vapor and how much is liquid; if this equation is judiciously formulated the problem of supersaturation will never arise. In formulating the equation we shall recognize that each grid point actually represents a large area, and that a portion of the area may be subsaturated while another portion contains clouds.

Next, our variable representing total water content will be the "total dew point" rather than the total-water mixing ratio. By the total dew point we mean the value which the ordinary dew point would acquire if all the liquid water were converted to vapor, at constant pressure. Like the temperature, the total dew point should vary by a factor of less than two in the horizontal direction, and even a highly truncated Fourier series representing it should not produce negative values.

In the following sections we shall formulate first the equations of a continuous atmosphere, then the equations of a layered atmosphere, and finally the equations of a low-order model. Since we shall be concerned mainly with feasibility, we shall at times suggest alternative formulations, and our numerical values, when not dictated by the physics, will simply be suggested values. We shall present the results of a few numerical integrations, which will constitute an initial test of the model.

2. The continuous equations

Our independent variables will be time t , pressure p , and horizontal coordinates x and y . Our basic dependent variables will be height z , horizontal velocity derivable from a stream function ψ and a velocity potential χ , individual pressure change ω , total dew point W , air temperature T , and, in our principal version, sea-surface temperature S . From these we shall derive the auxiliary variables total-water mixing ratio w , water-vapor mixing ratio r , saturation mixing ratio m corresponding to air temperature and pressure, and saturation mixing ratio s corresponding to sea-surface temperature and sea-level pressure. We shall let the atmosphere occupy a channel of infinite west-east extent, bounded laterally by frictionless walls at $y = 0$ and $y = \pi D$, and below by the surface $p = p_4$, whose height will be allowed to vary. Suggested values for D and p_4 are 2000 km, if the channel is supposed to simulate the extra-tropical latitudes of one hemisphere, and 1000 mb.

Our basic diagnostic equations will be the hydrostatic, geostrophic, and continuity equations

$$\partial z/\partial p = -RT/(gp) \quad , \tag{1}$$

$$f_0 \psi = gz \quad , \tag{2}$$

$$\nabla^2 \chi + \partial \omega/\partial p = 0 \quad , \tag{3}$$

where R is the gas constant for air, g is the acceleration of gravity, and f_0 is the constant average value of the variable Coriolis parameter f. We shall let ω vanish at the top and bottom of our model atmosphere.

Our prognostic equations will be the vorticity equation

$$\partial \nabla^2 \psi/\partial t = -J(\psi, \nabla^2 \psi + f) - f_0 \nabla^2 \chi + \nabla^2 F \, , \tag{4}$$

the equation for continuity of water content

$$dw/dt = G \, , \tag{5}$$

and the first law of thermodynamics applied separately to the atmosphere and the ocean,

$$d(c_p T + L w)/dt = RT\omega/p + H \quad , \tag{6}$$

$$d(cS)/dt = E \quad , \tag{7}$$

where L is the latent heat of condensation of water vapor, assumed constant, c_p is the specific heat of air at constant pressure, c is the specific heat of liquid water, and J denotes a Jacobian with respect to x and y. Here $\nabla^2 F$, G, H, and E are source and sink terms for vorticity, atmospheric water content, and atmospheric and oceanic enthalpy; F includes friction, G includes evaporation and precipitation, and H and E include diabatic heating and the effects of evaporation from the ocean surface (but not the effects of condensation within the atmosphere). By writing (6) as a prognostic equation for specific enthalpy $c_p T + L w$, we automatically include the thermodynamic effects of water. The omission from (4) of the nonlinear terms containing χ or ω is consistent with the geostrophic approximation. Suitable numerical values for the constants are $f_0 = 10^{-4} s^{-1}$, $g = 9.8 \ ms^{-2}$, $c_p = 1000 \ m^2 s^{-2} K^{-1}$, $c = 4185 \ m^2 s^{-2} K^{-1}$, $R = (2/7) c_p$, and $L = 2.5 \times 10^6 m^2 s^{-2}$.

We shall relate the mixing ratios to the temperatures by the approximate formulas

$$w = c' W^{\mu}/p \quad , \tag{8}$$

$$\mu = c' T^{\mu}/p \quad , \tag{9}$$

$$s = c' S^{\mu}/p_4 \quad , \tag{10}$$

where C' and μ are constants for which appropriate values will presently be introduced. To close the system we need an equation relating ν to the other variables. The standard assumption is that there is no liquid water if the total water is insufficient for saturation, i.e., $\nu = \omega$ if $\omega < \mu$, and no supersaturation if the total water is sufficient, i.e., $\nu = \mu$ is $\omega \geq \mu$. Sometimes condensation is assumed to occur when the relative humidity $\eta = \nu/\mu$ reaches some value below saturation, perhaps 80 per cent. As we have noted, we prefer a formulation where a portion of the region represented by a single grid point may be subsaturated, while another portion may contain clouds. We shall choose a formula which makes the liquid water content $\omega - \nu$ small when the degree of subsaturation $\mu - \nu$ is large, and vice versa. A simple formula of this sort, which makes $\nu/\omega \to 1$ as $\omega \to 0$, and $\nu/\mu \to 1$ as $\omega \to \infty$, is

$$(\omega - \nu)(\mu - \nu) = \gamma^2 \nu^2 \tag{11}$$

where γ is a small constant. Eqs. (1)-(11) form our closed system in the 11 basic and auxiliary dependent variables.

Choosing $\gamma = 1/4$ makes $\eta = 0.47$ when $\omega = \mu/2$, $\eta = 0.80$ when $\omega = \mu$, and $\eta = 0.95$ when $\omega = 2\mu$. We also note that choosing $\gamma = 0$ reduces (11) to the standard assumption.

To arrive at formulas (8)-(10) we write the Clausius-Clapeyron equation in the approximate form

$$dq_s(T)/dT = \varepsilon L q_s(T)/(RT^*T) \quad , \tag{12}$$

where $e_s(T)$ is the saturation vapor pressure of water at temperature T, $\varepsilon = 0.622$ is the ratio of the molecular weights of water and air, and T^* is a constant temperature typical of the atmosphere, say 273 K. This modification from the more exact form, in which T^2 instead of $T^* T$ would appear in the denominator, and L would vary with T, makes $e_s(T)$ proportional to T^{μ}, where $\mu = \varepsilon L/(RT^*)$ is a constant whose appropriate value is about 20.0. Eq. (8) then follows from the approximate relation $M = \varepsilon\, e_s(T)/p$; the constant c' is to be chosen so that M = 0.0038 when T = 273 K and p = 1000 mb. Eqs. (9) and (10) are arrived at similarly.

From (1) and (2) we obtain the thermal wind equation

$$\partial\psi/\partial p = -RT/(f_0 p) \quad , \tag{13}$$

after which further reference to z is superfluous. Our closed system is now (3)-(11) and (13).

We would prefer a prognostic equation for W to one for w. From (5) and (8) we find that

$$dW/dt = \nu W \omega/p + \nu W G/w \quad , \tag{14}$$

where $\nu = 1/\mu$. Likewise, we would prefer a prognostic equation for T to one for $c_p T + Lw$. We find from (11) that

$$dv/dt = v_w\, dw/dt + v_u\, du/dt \quad , \tag{15}$$

where

$$v_w = v(u - v)/[w(u - v) + u(w - v)] \,, \tag{16}$$

$$v_u = v(w - v)/[w(u - v) + u(w - v)] \,. \tag{17}$$

With the aid of (9) and (15), (6) becomes

$$(c_p + c_q)\, dT/dt = (\kappa c_p + \nu c_q)\, T\omega/p + (H - L v_w G) \,, \tag{18}$$

where $\kappa = R/c_p$ and $c_q = \mu L u v_u / T$.

Finally, we need to rewrite (17) and (18) with local time derivatives. We obtain

$$\begin{aligned} \partial W/\partial t = & -J(\psi, W) - \nabla\chi \cdot \nabla W - \omega\, \partial W/\partial p \\ & + \nu W \omega/p + \nu W G/w \end{aligned} \tag{19}$$

$$\begin{aligned} \partial T/\partial t = & -J(\psi, T) - \nabla\chi \cdot \nabla T - \omega\, \partial T/\partial p \\ & + [(\kappa c_p + \nu c_q)/(c_p + c_q)]\, T\omega/p + (H - L v_w G)/(c_p + c_q) . \end{aligned} \tag{20}$$

To interpret the coefficient of $T\omega/p$ in (20) we may picture an adiabatic chart, with pressure and temperature as coordinates, drawn so that the slope of the dry adiabats is κ. The slope of the lines of constant saturation mixing ratio is then ν, while the ratio $(\kappa c_p + \nu c_q)/(c_p + c_q)$, which lies between κ and ν, is the effective slope of the moist adiabats. It would equal the standard slope of the moist adiabats if $r_m = 1$, and that of the dry adiabats if $r_m = 0$. The factor r_m in c_q is defined by (17), and actually $r_m = 1$ if $\Lambda = 1$ and $r_m = 0$ if $\Lambda = 0$, and $0 < r_m < 1$ otherwise. Eq. (20) thus recognizes that under adiabatic lifting Λ increases, so that there is somewhat less condensation, and the rate of cooling is somewhat closer to the dry-adiabatic, then if Λ remained fixed at 100 per cent.

3. The layered model

For our layered model we shall choose a simple form of the two-layer model, in which the basic dependent variables include ψ at each of two levels, but T at only one level. Such models have been called "2-1/2-dimensional", since the wind field is 3-dimensional while the temperature field is effectively 2-dimensional. It is consistent with this formulation to let the basic variables include W at only one level.

We shall let the atmosphere consist of an active troposphere extending upward from 1000 to 200 mb, and an isothermal dry stratosphere extending upward from 200 mb. The stratosphere will play no role except to exert the 200-mb pressure at its base. We shall let the ocean consist of an isothermal mixed layer extending downward to 8000 mb (about 70 m), and a deeper ocean which will play no role. We shall use subscripts 0,1,2,3 and

4 to denote the value of an atmospheric quantity at 200, 400, 600, 800, and 1000 mb respectively, and subscripts 5 and 6 to denote an oceanic quantity at 4500 and 8000 mb. We shall sometimes choose other numerical values for p_5 and p_6.

Our basic dependent variables will then be ψ_1, ψ_3, χ_1, χ_3, ω_2, W_2, T_2, and S_5. We must, however, recognize the existence of the variables at other levels, and, in formulating expressions for radiative heating, we must let W_2 and T_2 determine W and T at other levels in a reasonable manner. We shall first require that in each vertical column, T be proportional to p^λ, where λ is a prespecified constant. Noting that $\lambda = 0$ or $\lambda = \kappa = 2/7$ would imply respectively an isothermal or dry-adiabatic lapse rate, we find that a reasonable value of λ would be about 0.175. We next specify that the relative humidity r be constant within a column. It then follows that w/μ, and hence W/T is constant, so that W is also proportional to p^λ, while w, v, and μ are proportional to $p^{(\lambda\mu - 1)}$.

From (3) and (13) we find, since $\omega_0 = \omega_4 = 0$, that

$$\nabla^2 \chi_1 = -\omega_2 / \Delta p \quad , \tag{21}$$

$$\nabla^2 \chi_3 = \omega_2 / \Delta p \quad , \tag{22}$$

$$\psi_1 - \psi_3 = h T_2 \quad , \tag{23}$$

where $\Delta p = (p_4 - p_0)/2$ and $h = R\Delta p/(f_0 p_2)$. In view of (21) and (22), the vorticity equation (4) yields

$$\partial \nabla^2 \psi_1/\partial t = -J(\psi_1, \nabla^2\psi_1 + f) + f_0 \omega_2/\Delta p + \nabla^2 F_1 \quad , \tag{24}$$

$$\partial \nabla^2 \psi_3/\partial t = -J(\psi_3, \nabla^2\psi_3 + f) - f_0 \omega_2/\Delta p + \nabla^2 F_3 \quad . \tag{25}$$

Letting $\psi_2 = (\psi_1 + \psi_3)/2$, we find from (23)-(25) that

$$\partial \nabla^2 T_2/\partial t = -J(\psi_2, \nabla^2 T_2) - J(T_2, \nabla^2 \psi_2 + f) + 2R\omega_2/(h^2 p_2) + \nabla^2(F_1 - F_3)/h \quad , \tag{26}$$

while

$$\partial \nabla^2 \psi_2/\partial t = -J(\psi_2, \nabla^2\psi_2 + f) - h^2 J(T_2, \nabla^2 T_2)/4 + \nabla^2(F_1 + F_3)/2 \; . \tag{27}$$

Because W and T are proportional to p^λ , the terms $\omega\, \partial W/\partial p$ and $\omega\, \partial T/\partial p$ in (19) and (20) may be replaced by $\lambda W \omega/p$ and $\lambda T \omega/p$, and, since we may assume that $\chi_2 = (\chi_1 + \chi_3)/2 = 0$, (19) and (20) yield

$$\partial W_2/\partial t = -J(\psi_2, W_2) + (\nu - \lambda) W_2 \omega_2/p_2 + \nu W_2 G_2/\varpi_2 \quad , \tag{28}$$

$$\partial T_2/\partial t = -J(\psi_2, T_2) + \left[(\chi C_p + \nu C_{q2})/(C_p + C_{q2}) - \lambda\right] T_2 \omega_2/p_2 + (H_2 - L \varpi_w G_2)/(C_p + C_{q2}) \quad , \tag{29}$$

while (7) becomes

$$\partial S_5/\partial t = E_5/c \quad . \tag{30}$$

The diagnostic equations (8)-(12), with suitable subscripts added, complete the system.

We may now eliminate the time derivatives from (26) and (29), and obtain the ω -equation

$$\begin{aligned}&[(\kappa c_p + \nu c_{q2})/(c_p + c_{q2}) - \lambda] T_2 \omega_2/p_2 - 2R \nabla^{-2}\omega_2/(h^2 p_2) \\ &\quad = J(\psi_2, T_2) - \nabla^{-2} J(\psi_2, \nabla^2 T_2) - \nabla^{-2} J(T_2, \nabla^2\psi_2 + f) \\ &\quad - (H_2 - L w_w G_2)/(c_p + c_{q2}) + (F_1 - F_3)/h \quad ,\end{aligned} \tag{31}$$

where ∇^{-2} is the inverse of ∇^2. With ω given by the diagnostic equation (31), we end up with a system of four prognostic equations (27)-(30) governing the dependent variables ψ_2, W_2, T_2, and S_5.

4. Sources and sinks

Although the new dependent variables are ostensibly the values of quantities at specific levels, they are supposed to represent entire layers, and their changes due to external influences should represent the effect of the external influences upon entire layers. For example, we *wish* T_2 *to be affected* by the total absorption and emission of radiation by the atmosphere, rather than just the convergence of radiative flux at level 2, which would be difficult to estimate.

The quantities $\nabla^2 F_1$, $\nabla^2 F_3$, G_2 , H_2 , and E_5 represent sources of vorticity of momentum, water, and enthalpy per unit mass. On the other hand, such quantities as radiative flux and exchanges across the ocean-atmosphere interface are generally expressed in amounts per unit horizontal area. Accordingly, we shall denote the respective sources per unit area by $\nabla^2 F_1'$, $\nabla^2 F_3'$, G_2' , H_2', and E_5'. We find that

$$F_1 = \Delta p^{-1} g \, F_1' , \tag{32}$$

$$F_3 = \Delta p^{-1} g \, F_3' , \tag{33}$$

and, in view of the assumed vertical variations of w and T ,

$$G_2 = \lambda\mu \, p_2^{\lambda\mu-1} \left(p_4^{\lambda\mu} - p_0^{\lambda\mu} \right)^{-1} g \, G_2' , \tag{34}$$

$$H_2 = (1+\lambda) \, p_2^{\lambda} \left(p_4^{1+\lambda} - p_0^{1+\lambda} \right)^{-1} g \, H_2' , \tag{35}$$

while

$$E_5 = \left(p_6 - p_4 \right)^{-1} g \, E_5' . \tag{36}$$

For such a crude model a nonlinear formulation of friction seems unwarranted, and we shall let

$$F_1' = -k'(\psi_1 - \psi_3) \quad , \tag{37}$$

$$F_3' = -k\psi_4 + k'(\psi_1 - \psi_3) \quad , \tag{38}$$

where $\psi_4 = (3\psi_3 - \psi_1)/2$. An appropriate value for k would be $C_D \rho_4^* U_4^*$, where C_D is a surface drag coefficient, say 2.0×10^{-3}, and ρ_4^* and U_4^* are a typical density and wind speed at level 4, say 1.2 kg m^{-3} and 10 m s^{-1}. Probably all that can be said about k' is that is should be smaller than k , and we shall let $k' = k/4$.

The processes entering G_2 are evaporation and precipitation. We shall let

$$G_2' = k(s_5 - \sigma_4) - k''(w_2 - \sigma_2) \quad , \tag{39}$$

where $\sigma_4 = \sigma_2 (p_4/p_2)^{\lambda\mu - 1}$, and k is the same as in (38). In parameterizing precipitation we have assumed a "half life" for clouds. We observe that when (39) is substituted into (34), the coefficient of $s_5 - \sigma_4$, with the suggested numerical values, becomes about (5 days)$^{-1}$. If the half life for clouds is about 16 hours, the e-folding time is about 1 day, and $k'' = 5k$.

Exchanges of sensible and latent heat across the atmosphere-ocean interface may be handled similarly. We find that

$$H_2' = k c_p (S_5 - T_4) + k L (s_5 - r_4) + R_2' , \tag{40}$$

$$E_5' = -k c_p (S_5 - T_4) - k L (s_5 - r_4) + R_5' , \tag{41}$$

where $T_4 = T_2 (p_4/p_2)^\lambda$, and R_2' and R_5' denote the effects of radiation.

We shall assume that all short-wave radiation which is not reflected by clouds is absorbed by the ocean. Short-wave radiation then does not enter R_2' , and its contribution to R_5' is $Q_0(1-a)$, where Q_0 is the intensity of the solar beam and a is the albedo. We shall let clouds be perfect reflectors of short-wave radiation, whence the albedo equals the fraction of the sky covered by clouds. This fraction may be parameterized in a number of ways; the formula $a = r^4$ seems to yield reasonable values.

We shall let the ocean surface be a black-body absorber and emitter of long-wave radiation. Absorption and emission by the atmosphere is more complicated. We need to know the atmosphere's fractional absorptivity and emissivity, and the effective temperatures at which it radiates upward and downward. It would be pointless to introduce all of the intracacies of the appropriate radiation formulas in a model which is so crude in other respects. Instead we shall use a modification of Simpson's method.

In his computations of the atmosphere's radiation balance, Simpson (1928) treated the atmosphere as being completely transparent in the 8.5-11 micron band, so that the earth's surface rather than the atmosphere would radiate to space in these wave lengths. He assumed that a layer of air containing 0.3 mm of precipitable water, i.e., 0.3 kg of water vapor per m^2 of cross section, would be completely opaque below 7 and above 14 microns, so that the uppermost such layer, generally in the upper troposphere, would radiate to space in these wave lengths. In the intermediate bands the atmosphere would radiate to space, at an intermediate temperature.

We shall simplify Simpson's treatment by assuming as a first approximation that the cloud-free fraction of the atmosphere radiates upward and downward with a certain fraction a', say 0.7, of the intensity of black-body radiation, at temperatures T_0' and T_4' respectively. We shall let the cloud-free atmosphere absorb the fraction a' of the radiation from the earth's surface, while the fraction $1 - a'$ passes through. We shall neglect any variations of a' which ought to occur because the wave length of maximum absorption and emission shifts with temperature. We shall let the clouds radiate upward and downward as black bodies, also at temperatures T_0' and T_4'. These temperatures will occur at pressures p_0' and p_4', which will be the levels above which and below which, respectively, the amount of water vapor is $V^*/2$, where $V^* = 0.3 \text{ kg m}^{-2}$.

Letting v^* be the value which v_2 would possess if the total water vapor in a column were V^*, we find, since v is proportioned to $p^{\lambda\mu-1}$, that

$$(p_0')^{\lambda\mu} = p_0^{\lambda\mu} + \tfrac{1}{2}(v^*/v_2)(p_4^{\lambda\mu} - p_0^{\lambda\mu}) \quad , \tag{42}$$

$$(p_4')^{\lambda\mu} = p_4^{\lambda\mu} - \tfrac{1}{2}(v^*/v_2)(p_4^{\lambda\mu} - p_0^{\lambda\mu}) \quad . \tag{43}$$

With the numerical values we have been using, v^* is about 3×10^{-5} kg m^{-3}.

Eqs. (42) and (43) appear satisfactory when v_2 is large, but they become unreasonable when v_2 is very small, since $p_0' > p_4'$ when $v_2 < v^*$, and p_0' and p_4' move right out of the troposphere when $v_2 < v^*/2$. It would be more reasonable to have p_0' and p_4' approach an intermediate value p_2' as $v_2 \to 0$. Formulas which fulfill this condition, but differ only slightly from (42) and (43) when v_2 is large, are

$$(p_0')^{\lambda\mu} = p_0^{\lambda\mu} + \tfrac{1}{2} v^* \left(p_4^{\lambda\mu} - p_0^{\lambda\mu} \right) / (v_2 + v^*) , \tag{44}$$

$$(p_4')^{\lambda\mu} = p_4^{\lambda\mu} - \tfrac{1}{2} v^* \left(p_4^{\lambda\mu} - p_0^{\lambda\mu} \right) / (v_2 + v^*) . \tag{45}$$

With our chosen numerical values the exponent $\lambda\mu$ is 3.5 .

In addition, in the limit as $v_2 \to 0$ there should be no absorption or emission by water vapor. We shall fulfill this condition by modifying the factor a' by the additional factor $v_2/(v_2 + v^*)$. We find, then, that

$$R_3' = \left[a + (1-a)\, a'\, v_2/(v_2 + v^*) \right] \sigma \left(S_5^4 - T_4'^4 - T_0'^4 \right) , \tag{46}$$

$$R_5' = -\sigma S_5^4 + \left[a + (1-a)\, a'\, v_2/(v_2 + v^*) \right] \sigma T_4'^4 + Q_0 (1-a) , \tag{47}$$

where $T_0' = T_2(p_0'/p_2)^\lambda$, $T_4' = T_2(p_4'/p_2)^\lambda$, and $\sigma = 5.67 \times 10^{-8} kg\ s^{-3} K^{-4}$ is the Stefan-Boltzman constant.

5. The truncated equations

To convert our system into a low-order model, we shall approximate each dependent variable by a truncated double Fourier series in x and y. We shall use the same truncation which we used previously in a study of vacillation (Lorenz, 1963), to which the reader is referred for details not included here.

Recalling that the domain of the variables is an infinite strip with boundaries at $y = 0$ and $y = \pi D$, we let $x_0 = x/D$ and $y_0 = y/D$. If X then represents any one of the variables ω, ψ, W, T, or S, we let

$$X = \sum_{i=0}^{6} X_i \Phi_i , \tag{48}$$

where

$$\left.\begin{aligned}
\Phi_0 &= 1 \\
\Phi_1 &= 2 \sin y_0 \cos n x_0 \\
\Phi_2 &= 2 \sin y_0 \sin n x_0 \\
\Phi_3 &= \sqrt{2} \cos y_0 \\
\Phi_4 &= 2 \sin 2y_0 \cos n x_0 \\
\Phi_5 &= 2 \sin 2y_0 \sin n x_0 \\
\Phi_6 &= \sqrt{2} \cos 2y_0
\end{aligned}\right\} \tag{49}$$

for some positive constant n. In our numerical work we shall let $n = 2$.

We observe that $\Phi_0, \cdots, \Phi_6$ satisfy the relations

$$\overline{\Phi_i \Phi_j} = \delta_{ij} \quad , \tag{50}$$

$$D^2 \nabla^2 \Phi_i = -a_i \Phi_i \quad , \tag{51}$$

where the bar denotes an average over the whole domain, δ_{ij} is the Kronecker delta, and $a_0 = 0$, $a_1 = a_2 = 1 + n^2$, $a_3 = 1$, $a_4 = a_5 = 4 + n^2$, and $a_6 = 4$. Each dependent-variable field thus consists of an overall average, a superposed zonally symmetric portion with two north-south modes, and a single superposed wave with two north-south modes and an arbitrary longitudinal phase. We have ordered the variables so that subscripts divisible by three refer to the zonally symmetric field. We include the effect of a variable Coriolis parameter f by letting $f_3 = -f_0/4$.

In general an arbitrary nonlinear function Z of $\Phi_0, \cdots, \Phi_6$, and hence such a function of the dependent variables, cannot be expressed exactly as a linear combination of $\Phi_0, \cdots, \Phi_6$, and further approximation is needed. In view of the orthogonality relation (50), the appropriate value of Z_i is $\overline{Z \Phi_i}$. If Z is a simple product, say $Z = XY$ and X and Y have the form of (48), we readily find that

$$
\left.\begin{aligned}
Z_0 &= \sum_{i=0}^{6} X_i Y_i \\
Z_1 &= (XY)_{01} - \left[(XY)_{16} - (XY)_{34}\right]/\sqrt{2} \\
Z_2 &= (XY)_{02} - \left[(XY)_{26} - (XY)_{35}\right]/\sqrt{2} \\
Z_3 &= (XY)_{03} + \left[(XY)_{14} + (XY)_{25} + (XY)_{36}\right]/\sqrt{2} \\
Z_4 &= (XY)_{04} + (XY)_{13}/\sqrt{2} \\
Z_5 &= (XY)_{05} + (XY)_{23}/\sqrt{2} \\
Z_6 &= (XY)_{06} - (X_1 Y_1 + X_2 Y_2 - X_3 Y_3)/\sqrt{2}
\end{aligned}\right\} \qquad (52)
$$

where $(XY)_{ij}$ stands for $X_i Y_j + X_j Y_i$. Likewise, if $Z = -J(X,Y)$, we find that

$$
\left.\begin{aligned}
Z_0 &= 0 \\
Z_1 &= 5b\,[XY]_{23} + 8b\,[XY]_{56} \\
Z_2 &= 5b\,[XY]_{31} + 8b\,[XY]_{64} \\
Z_3 &= 5b\,[XY]_{12} + 4b\,[XY]_{45} \\
Z_4 &= 8b\,[XY]_{26} + 4b\,[XY]_{53} \\
Z_5 &= 8b\,[XY]_{61} + 4b\,[XY]_{34} \\
Z_6 &= 8b\,[XY]_{15} + 8b\,[XY]_{42}
\end{aligned}\right\} \qquad (53)
$$

where $[XY]_{ij}$ stands for $X_i Y_j - X_j Y_i$ and $b = 8n\sqrt{2}/(15\pi D^2)$.

When Z represents a more complicated function of the dependent variables, such as one of the high powers or fractional powers appearing in the source-sink terms in (27)-(31), an analytic expression for Z_i would be prohibitively complicated. To approximate Z_i it would be preferable to evaluate Z and Φ_i at each of a well-chosen set of points spanning the domain, and then to calculate the average product of Z and Φ_i.

Accordingly, we shall introduce a grid of points into the domain. At each time step we shall evaluate each dependent variable (except ψ_2) at each point, and then compute the source-sink terms in (27)-(31) separately at each point, finally multiplying each value so obtained by the value of Φ_i at the same point, and averaging. Since the functions Φ_i are trigonometric, this procedure is nothing more than transforming from Fourier space to grid-point space and back again.

The estimates of $\overline{Z\Phi_i}$, where Z is a source-sink term, will be more accurate the larger the number of points chosen, but since the model as a whole is rather crude regardless of how many points are chosen, there is little to gain by choosing too many. We shall settle for a grid of 16 points, which will be located at the intersections of the lines $y_0 = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8$ with the lines $n x_0 = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

To solve the ω-equation (31) we must either evaluate a product implicitly in Fourier space or invert ∇^2 implicitly in grid-point space. We shall choose the former procedure. We first evaluate the coefficient of ω_2/p_2 in the first term of (31) (call it B) in grid-point space, and then transform B into Fourier space. Since $(\omega_2)_0 = 0$, the coefficients of $\Phi_1, \cdots, \Phi_6$ in the Fourier transform of (31), with $(B\omega_2)_i$ as in (52), form a set of six algebraic equations to be solved for $(\omega_2)_1, \cdots, (\omega_2)_6$. The terms $(W_2\omega_2)_i$ in the Fourier transforms of (28) may then be evaluated, using (52).

6. Testing the model

With a typical low-order dry model of the general circulation the determination of an equilibrium solution, when the external heating is horizontally uniform, is a trivial matter, but, with our moist model it is not at all obvious what such a solution should look like. The motion should vanish, but the equilibrium values of W_2, T_2, and S_5 are solutions of rather complicated nonlinear equations, obtained by equating G_2', H_2', and E_5' to zero.

As an initial test of the model we shall determinethese equilibrium values for various values of $Q_{\cdot 1}$. More definitive tests with horizontally varying $Q_{\cdot 1}$ have yet to be performed. Assuming that a computer program to integrate the equations of the model has been written in any case, the way to determine the equilibrium solution which requires the least additional programming, although not the least computing, is to run the model from arbitrary initial conditions until a steady state is approached. Since there are no horizontal variations, transformations from Fourier space to grid-point space and back again are unnecessary, and the computation may be performed at a single grid point. Because of the large heat capacity of the ocean, the approach to equilibrium may require many years, and we should note that the final state, as opposed to the process of approaching it, is independent of the ocean's heat capacity. We shall therefore use an ocean with a 2-m instead of a 70-m mixed layer, whose heat capacity is comparable to that of the atmosphere.

We shall express the solar heating Q_0 in terms of a planetary temperature T_Q, where $Q_0 = \sigma T_Q^4$. We do not include the albedo a in the definition of T_Q, since a is not prespecified.

We have obtained several numerical solutions using the suggested numerical values of the constants, with values of T_Q varying from 258 K to 288 K. We note that $Q_0 = 340$ W m^{-2}, the approximate average value for the earth, when $T_Q = 278$ K. We have found it convenient to use the surface values W_4 and T_4 instead of W_2 and T_2 as output, although W_2 and T_2 remain the working variables. In each run W_4, T_4, and S_5 all equal T_Q initially; T_Q would be the equilibrium value for S_5 if the greenhouse effect exactly canceled effect of the albedo. We have used 1.5-hour time steps. After two years, in each run, except one where $T_Q = 273$ K, when five years are needed, the approach to equilibrium becomes so nearly exponential that the final state can be easily extrapolated by hand computation.

Table 1 shows the results. The most outstanding feature is the sensitivity of the equilibrium solution to T_Q; when T_Q is low, W_4, T_4, and S_5 are very low, and when T_Q is high, W_4, T_4, and S_5 are very high. The change as T_Q changes from 268 K to 273 K is especially abrupt. It is also noteworthy that T_4 is below W_4 and S_5 when T_Q is low, and above W_4 and S_5 when T_Q is high, while W_4 is always slightly below S_5.

Table 1. Equilibrium values of surface total dew point W_4, surface temperature T_4, sea-surface temperature S_5, and relative humidity r, for various values of planetary temperature T_Q, as determined in first set of numerical runs.

T_Q	258	263	268	273	278	283	288
W_4	224.1	229.0	234.5	306.4	315.9	322.9	329.1
T_4	223.1	227.6	232.8	312.4	324.7	334.2	342.9
S_5	225.4	230.1	235.7	306.9	316.3	323.3	329.4
r	0.84	0.85	0.85	0.62	0.53	0.47	0.42

These features, once observed, are readily accounted for. We note first that with our chosen values for the constants, w_4 and v_4 are about 3.6 times w_2 and v_2. We thus find from (39) that when G_2' vanishes,

$$w_4 = (1 - 3.6k/k'')\, v_4 + (3.6\,k/k'')\, S_5 \qquad (54)$$

It follows that if we had chosen $k'' = 3.6\,k$, which seems just as reasonable as our choice of $5\,k$, w_4 and S_5 would be equal. With $k'' = 5\,k$, w_4 becomes a weighted average of v_4 and S_5, and, since $v_4 < w_4$, $S_5 > w_4$, whence $S_5 > W_4$.

Suppose next that we have located a value of T_Q for which $T_4 = S_5$, and that we then increase T_Q. If W_4, T_4, and S_5 were to increase in proportion, the relative humidity, and hence the cloud albedo, would remain fixed. The latent heat transferred from the ocean to the atmosphere would then increase as T_Q^{20}, while the heat returned to the ocean through radiation or sensible-heat exchange would increase only as T_Q^4 or T_Q. Equilibrium would then no longer exist, and the atmosphere would warm up while the ocean cooled. With equilibrium reestablished, $T_4 > S_5$. Likewise, if T_Q should decrease, $T_4 < S_5$.

It follows, since W_4 and S_5 vary similarly, that the relative humidity, and hence the cloud cover and the cooling effect of its albedo, must decrease as T_Q increases. On the other hand, the greenhouse effect, which depends upon the total water-vapor content, increases when T_Q increases. The equilibrium values of W_4, T_4, and S_5 should then increase of decrease very rapidly as T_Q increases or decreases, and should be below T_Q when T_Q is low and above T_Q when T_Q is high.

The suddeness of the increase in T_4 as T_Q reaches 273 K was not anticipated, and it suggests that there may actually be two stable equilibria, with an unstable euilibrium in between. This idea is supported by the observation that for $T_Q = 273$ K, the rate at which T_4 departs from its initial value continually increases for the first three years. To test our hypothesis we have performed some additional 2-year runs, increasing T_Q in steps from 258 K to 288 K, and then decreasing T_Q in steps back to 258 K. In each run after the first one the initial conditions are the final conditions of the previous run. We find indeed that between $T_Q = 268$ K and $T_Q = 278$ K there are multiple equilibria, comprising a cold regime which is approached when the initial temperatures are low and a warm one which is approached when they are high. Fig. 1 shows the equilibrium values of T_4 plotted against T_Q . The large dots indicate the results of particular runs. The dots fit two smooth curves.

Presumably these curves are portions of a single smooth S-shaped curve; the intermediate portion passes through the unstable equilibria, and in Fig. 1 it has been sketched by eye. It could be located by a successive-approximation procedure beginning with a good initial guess, such as the guess shown in Fig. 1.

We cannot say at this point whether the model has passed or flunked its first test. The cold and warm regimes seem unreasonably cold and warm by comparison with the real atmosphere, and indeed the oceans would be thoroughly frozen in the oold regime while the implicit assumption that the mixing ratios are fairly small would not hold in the warm regime. However, in reality no column of air with its underlying ocean is isolated, since there is always a circulation in the atmosphere and in the ocean between the more strongly and less strongly heated regions. Also, we do not know how the real atmosphere would behave if the earth's surface were all ocean.

The absence of a stable equilibrium value of T_4 between 260 K and 290 K is not necessarily serious. The approach to a cold or a warm stable equilibrium is so slow that when T_Q varies *horizontally*, the resulting circulations will have ample time to prevent either equilibrium from being approached. We might also observe that a curve of T_4 against S_5 would have a nearly constant slope, so that if the

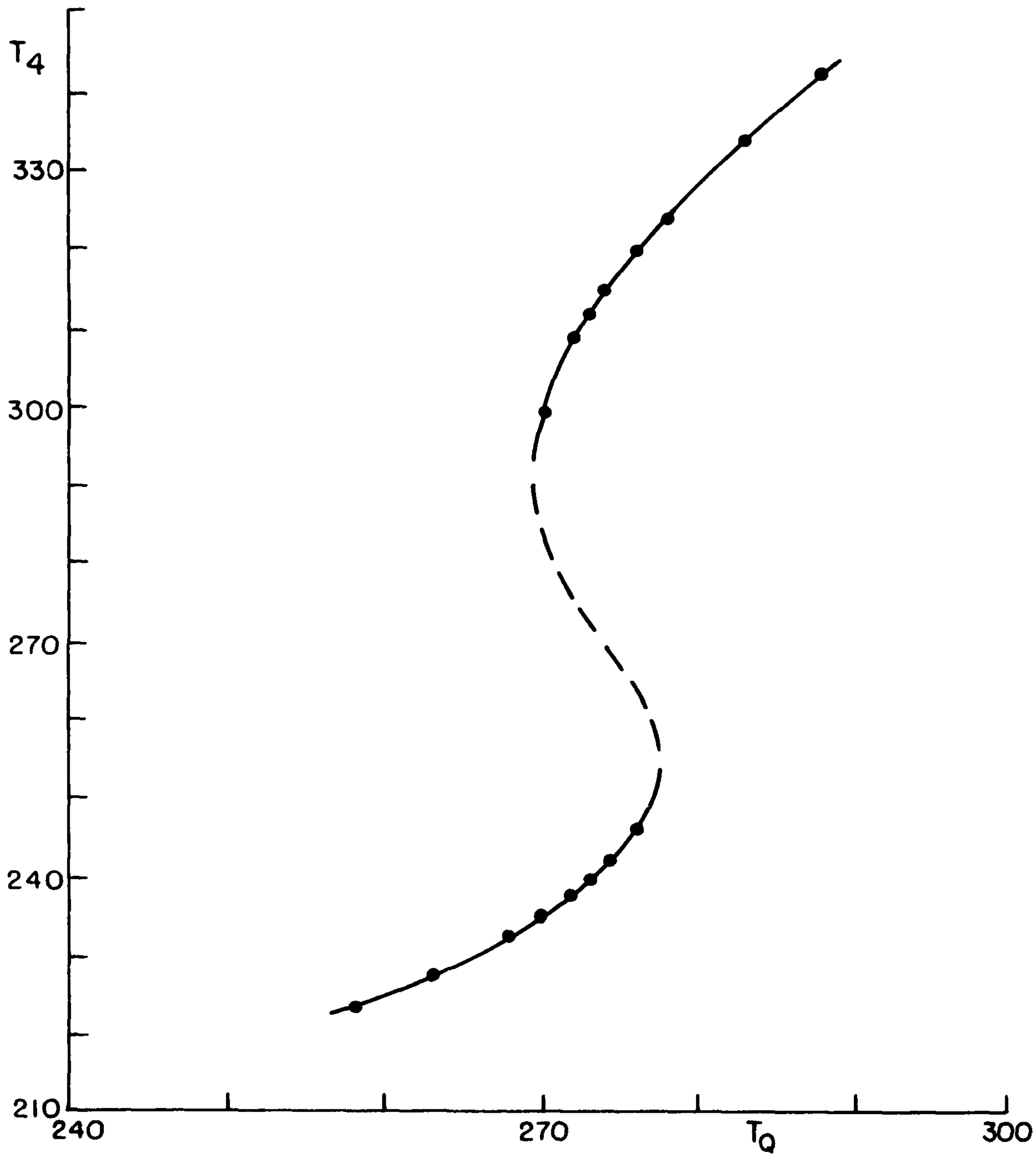

Fig. 1. Equilibrium surface temperatures T_4 corresponding to planetary temperatures T_Q. Large dots indicate computation points. Dashed portion of curve indicates unstable equilibria.

sea-surface temperature had been prespecified, this phenomenon would not have occurred.

7. Concluding remarks

We have constructed a low-order moist general circulation model which contains 20 prognostic ordinary differential equations if the ocean is assumed to have zero heat capacity, and 27 equations if it has finite heat capacity. The model atmosphere is composed of dry air, water vapor, and liquid water, and the thermodynamic and radiative effects of water vapor and clouds are included.

We have run the model for the special case when the solar heating is horizontally uniform, and have obtained an unexpected result, namely, that for certain intensities of solar heating there are two decidedly different equilibrium solutions. We have not yet performed any runs with horizontally variable solar heating, which should prove to be of greater interest.

We have constructed the model in such a way that modifications may easily be made, and we anticipate that, in subsequent development of the model, modifications will be made. These might include changes in the numerical values of constants, alterations of specific formulas such as the one relating cloud amount to relative humidity, or explicit recognition of the partially absorbing water-vapor bands. It is easy to add to the model, say by giving the ocean surface a prespecified albedo, including the absorption and emission of long-wave radiation by a prespecified concentration of carbon dioxide, or including some absorption of short-wave radiation by the atmosphere.

We must recognize, however that the low horizontal and vertical resolution preclude from the start the possibility of closely reproducing the atmospheric circulation. The purpose of the model is to test qualitatively the influence of various moist processes over a far wider range of conditions than is economically feasible with a large model. Inclusion of such processes as reflection by the ocean is desirable only if it contributes to the main purpose.

We believe that the weakest features of the model are the assumptions of a uniform lapse rate of temperature and a uniform tropopause pressure. Together these imply a perfect relation between surface temperature and tropopause temperature, and hence, except when very little moisture and cloudiness is present, a close relation between surface temperature and the temperature at which the bulk of the outgoing radiation takes place. There should therefore be large variations of outgoing radiation with latitude. In the real atmosphere these variations are not so great, because in the tropics the uppermost layers of water vapor are very high, and are as cold as those in the polar regions, which are not so high. Hence the real atmosphere should require a greater cross-latitude heat transport than the model. Also, the stabilization of the lapse rate when kinetic energy is released would be a desirable feature.

It is our ultimate intention to attempt to eliminate these weaknesses. We should be able to allow the vertically uniform lapse rate to vary horizontally, thereby adding seven more prognostic equations to the model. We prefer to find some means for representing the tropopause height in terms of the surface temperature and the lapse rate, and perhaps some other quantities, instead of letting the tropopause height become still another prognostic variable.

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