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JAN 81 W P D'AMICO, M D FULLER
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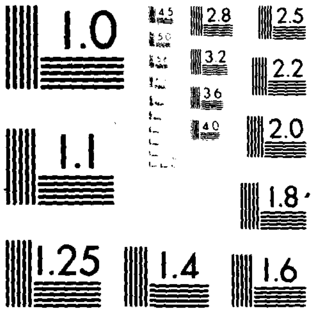

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MEMORANDUM REPORT ARBRL-MR-03070

(Supersedes IMR No. 682)

EXPERIMENTAL STUDY OF A LIQUID-FILLED CYLINDER WITH UNEQUAL INTERNAL DIAMETERS

William P. D'Amico, Jr.
Michael D. Fuller

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A series of tests was conducted using a liquid-filled gyroscope to study the behavior of cylinders with dissimilar internal diameters. The discontinuity in the internal diameter of the container produced a lip that modified the characteristic liquid eigenfrequency of the cylinder and the amplitude behavior of the gyroscope. A simple correction for this unusual geometry can be formulated by using an average aspect ratio, which is based upon the volume and height of the container. The use of an average aspect ratio only grossly defines the differences between the gyroscope data and the available predictive models.

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I. INTRODUCTION

A series of gyroscope tests was conducted to study the behavior of fluid-filled cavities comprised of two cylindrical sections with dissimilar internal diameters. A typical configuration is shown in Figure 1. The discontinuity produced by the different internal diameters presents a geometry that is not presently amenable to analysis. However, as a first approximation an average radius, $\bar{a}$, can be obtained from the total internal volume, V_T, and the overall height, 2c*. An average aspect ratio, $c/\bar{a}$, can be used to empirically examine this type of unusual geometry and for a limited number of tests with the liquid in a state of rigid body rotation, it was determined that the average aspect ratio could be used to assess the behavior of a liquid-filled gyroscope. Significant uncertainty will exist if engineering designs are based only upon this parameter.

II. BACKGROUND

A cut-away view of the 155mm, M687 binary projectile is shown in Figure 2. In the binary concept, two non-toxic chemicals are carried in tandem fashion within the projectile. Upon launch, the burst discs are severed and the two chemicals mix to form a toxic agent. Often, the individual liquid components must be stored in plastic liners within the payload canisters. Once the burst discs have been ruptured, a cylindrical internal geometry is produced. The canister design for the M687 was selected using an analysis by Stewartson.[1] This analysis applies only to a liquid-filled projectile with a cylindrical payload compartment and identifies the natural frequencies of oscillation of the liquid in a totally or partially filled cylinder. Furthermore, it is assumed that the liquid is spinning as a rigid body and that the centrifugal forces upon the liquid are much larger than body forces such as gravity or drag. Under these circumstances, a linear theory for an inviscid liquid predicts that the motion of the projectile will be unstable if certain of the natural frequencies of oscillation of the liquid are close to the fast precessional frequency of the projectile.** Key results from this theory indicate that the natural frequencies (eigenfrequencies) of an inviscid liquid depend only upon the aspect ratio of the cylinder (c/a, the ratio of the height to diameter) and the fill ratio $(1-b^2/a^2)$. In functional form,

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[1] *K. Stewartson, "On the Stability of a Spinning Top Containing Liquid," J. Fluid Mech., Vol. 5, Part 4, September 1959, pp. 577-592.*

* $\bar{a} = (V_T/2c\pi)^{1/2}$

***In aeroballistic terminology, the fast precessional frequency is called the nutational frequency.*

the eigenfrequencies are then

$$\tau_{nj} = F_n\left(\frac{c/a}{2j+1},\ 1-b^2/a^2\right), \tag{1}$$

where n and j are the radial and longitudinal mode numbers, respectively. The mechanism for projectile instability remains as a resonance between a liquid eigenfrequency, τ_{nj}, and the nondimensional fast frequency motion of the projectile ($\tau_{NU}=\dot{\phi}_1/p$, where $\dot{\phi}_1$ is the coning frequency while p is the spin.[2] Wedemeyer used a boundary layer correction technique to account for the effects of viscosity[3] and an averaging procedure to account for ogival-shaped internal geometries.[4] The techniques of Reference 4 can not be applied to geometries like those of Figure 1.

Consider the case where one of the plastic liners in Figure 2 is eliminated. The resulting internal geometry is not cylindrical. Casually, one may suppose that the discontinuity in the diameter is not severe, but experience with laboratory gyroscopes indicates that subtle changes in internal geometry produce dramatic changes in the predicted eigenfrequencies and in the stability of the gyroscope. Actually, the original canister design for the M678 utilized two canisters of dissimilar internal diameters. When the projectile developers realized the restrictions of the available theory, the design was modified to yield a geometry more suitable to the Stewartson analysis. This geometry restriction along with other considerations resulted in a projectile weight approximately 10% lower than that of the parent projectile, the M483A1. This weight difference is not desirable for ballistic similitude; it would be advantageous for newer designs to achieve a closer approximation to the inertial characteristics of the parent projectile.

Recently, a small group of projectiles which had canisters with dissimilar internal diameters and with inertial properties that matched the M483A1 was tested by the Chemical Systems Laboratory (CSL). Two geometries were tested, and observations of projectile range indicated one design to be stable and one unstable. Test geometries are provided on the next page in a tabular form. The discontinuity in the diameters is identified as a "lip" and from the notation of Figure 1 the lip is defined as a-d.

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[2] *Engineering Design Handbook, Liquid-Filled Projectile Design, AMC Pamphlet No. 706-165, U.S. Army Materiel Development and Readiness Command, Washington, D.C., April 1969. AD 853719.*

[3] *E.H. Wedemeyer, "Viscous Corrections to Stewartson's Stability Criterion," BRL Report No. 1325, Aberdeen Proving Ground, Maryland, June 1966. AD 489687.*

[4] *E.H. Wedemeyer, "Dynamics of Liquid-Filled Shell: Noncylindrical Cavity," BRL Report 1326, Aberdeen Proving Ground, MD., August 1966. AD 823441.*

PROJECTILE DESIGN*	AVERAGE ASPECT RATIO	HEIGHT OVERALL	FORWARD CANISTER		REAR CANISTER		
			HEIGHT	DIAM	HEIGHT	DIAM	LIP
A	4.469	50.31	15.85	10.80	34.46	11.11	0.32
B	4.438	49.79	16.84	10.80	32.95	11.43	0.64

**All dimensions are in cm.*

Design A was unstable, while B was stable. Yawsondes were not employed, so details of the flight histories are not available. The stability of a liquid-filled projectile cannot be predicted by the Stewartson theory except for very long flight times where the liquid is in a quasi-rigid body rotation with the projectile. Substantial experimental and numerical work has been accomplished for the portion of the trajectory where the liquid is not yet in rigid body rotation. Wedemeyer[5] provided the rationale for a model of spin-up from rest, and this model has been improved by the inclusion of viscous diffusion terms by Sedney and Gerber[6]. Kitchens and Gerber[7] developed analyses for the prediction of spin decay for liquid-filled shell, and their results were consistent with yawsonde data. Another experimental technique was also utilized: using yawsonde and radar data, the angular momentum histories for liquid payloads were computed by Mark.[8] The prediction of time to spin-up from References 7 and 8 were consistent when the angular motion of the projectile was small enough to produce spin histories that were smooth.[9,10]

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5. *E.H. Wedemeyer, "The Unsteady Flow within a Spinning Cylinder," Journal of Fluid Mechanics, Vol. 20, Part 3, 1964, pp. 383-399. Also see BRL Report No. 1252, Aberdeen Proving Ground, MD., October 1963. AD 771919.*
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8. *A. Mark, "Measurements of Angular Momentum Transfer in Liquid-Filled Projectiles," Ballistic Research Laboratory Technical Report No. ARBRL-TR-2029, November 1977. AD A051056.*
9. *W.P. D'Amico, W.H. Clay, and A. Mark, "Yawsonde Data for M687-Type Projectiles with Application to Rapid Spin Decay and Stewartson-Type Spin-Up Instabilities," Ballistic Research Laboratory Report in preparation.*
10. *C.W. Kitchens, and R. Sedney, "Conjecture for Anomalous Spin Decay of the 155mm Binary Shell (XM687)," Ballistic Research Laboratory Report No. 2026, October 1977. AD A050311.*

The time to spin-up can be estimated using the Ekman number ($E=\nu/c^2p$), where ν is the kinematic viscosity of the liquid, c is the half height of the container, and p is the spin of the projectile. The characteristic time to spin-up is

$$t_c \text{ (spin-up)} = E^{-1/2}p^{-1} \qquad (2)$$

References 7 and 8 indicate that for M687-type shell with simulant payloads, t_c is often as large as 10 seconds for a Charge 4 launch. It is not known whether the A design was unstable during the early or later portions of the trajectory, i.e., it is not known if the instability was produced by a Stewartson eigenfrequency. Hence, gyroscope tests which only address the fully spun-up case may not provide needed information to explain the observed projectile instabilities.

III. TESTS WITH A VARIABLE DIAMETER CONTAINER

A model was constructed with several inserts to produce three values of 2d (see Figure 1). Table 1 lists the pertinent dimensions and the average aspect ratios, $c/\bar{a}$. During these experiments, the fill ratio was varied while the fast precessional frequency of the empty gyroscope, τ_{NU}, was held constant. The actual coning frequency of the liquid plus solid parts of the gyroscope will vary and this frequency, when scaled by the spin, is defined as TAU R. The amplitude growth rate is also scaled by the spin and is defined as TAU I. This procedure will identify the fill ratio at which maximum amplitude growth of the gyroscope occurs. The geometry that was selected produced a Stewartson resonance for mode numbers n=1 and j=0. The annulus with a lip depth of zero was inserted and then the fill ratio range was traversed. This procedure was then repeated for annuli with successively smaller annulus diameters (larger lips). Figure 3 shows a comparison between the predicted and measured behavior of the gyroscope for Annulus #1. The data appear to be shifted slightly in fill ratio. On the other hand, the geometry produced by Annulus #1 was not exactly that of a cylinder (see note 1 within Table 1). Also, the growth rates were very rapid and amplitude histories did not provide a large number of oscillations for the determination of the growth rate. Figure 4 compares predicted and measured precessional frequencies for Annulus #1. A shift in fill ratio of approximately 2% is evident. This large an error could not have occurred due to filling procedures. The difference between the predicted and measured values of TAU R and TAU I are most likely caused by the cavity not being exactly cylindrical, as previously noted.

TABLE 1. DIMENSIONS FOR THE VARIABLE DIAMETER MODEL

Annulus Number	Depth of Lip (cm)	Interior Diameter of Annulus[2] 2d (cm)	Average Diameter[3] $2\bar{a}$ (cm)	Fill Ratio for Maximum Amplitude Growth (%)		Average Aspect Ratio $c/\bar{a}$	Relative Decrease in Volume[4] (percent)
				Measured	Predicted		
1	0.00[1]	14.09	14.09	73	72	0.998	----[1]
2	0.32	13.51	13.87	77.5	75	1.014	3.8
3	0.64	12.87	13.61	81.5	79	1.033	7.4
4	0.95	12.24	13.36	82.5	83	1.053	10.8

---

1. *An annulus with "no lip" was used, but actual measurements indicated a 0.4% diametrical difference between this annulus and the lower portion of the model.*

2. *All annuli were 6.045 cm in height and were fitted into a modified cylinder to produce an overall interior height of 14.069 cm.*

3. *Calculated from the total available volume.*

4. *A cylinder with 2a and 2c was used as the maximum volume.*

Figure 5 shows the data for the model with the variable diameters. Table I lists comparisons between the measured and predicted fill ratios for maximum amplitude growth. The cases of smaller values of 2d seem to be better approximated by the use of the average aspect ratio within the Stewartson/Wedemeyer model, but at best only a crude description of the complete gyroscope motion can be obtained. The data indicate that the presence of a lip increases the resonant fill ratio. For a Stewartson analysis, this would imply that the fundamental eigenfrequency was reduced.* The data for the smallest annulus seems to correlate well with the average aspect ratio, but this is probably fortuitous. These experiments have indicated that the presence of a lip can be accounted for by the use of an average aspect ratio and that smaller values of 2d can substantially decrease the growth rate of the gyroscope.

IV. TESTS WITH SCALE MODELS OF THE FLIGHT HARDWARE

A second series of tests were conducted with 3/4 scale cavities of the original flight hardware. The fast precessional frequency of the gyroscope, τ_{NU}, was held constant at approximately 0.078, while the fill ratio was varied. The aspect ratios for the two models indicate that the most probable mode numbers for instability were n=1 and j=2. This frequency, 0.078, is very close to that which would be expected for an M687-type projectile. Model A, the unstable flight vehicle case, showed no measureable amplitude growth for any fill ratio. The air core of the model did become disturbed at a fill ratio of 60.7%. This observation indicates that the natural damping of the gyroscope was suppressing a potential instability. The predicted fill ratio for maximum amplitude growth using the average aspect ratio was 60.5%.

Model B did provide a well defined picture for amplitude growth versus fill ratio. Figure 6 gives a comparison between the data and the predicted growth rates. Sensitivity calculations for the behavior of the gyroscope were made to determine the effects of small changes in the aspect ratio. The error bar located at the peak of the predicted growth rate curve (Figure 6) indicates a shift of ±1% of $c/\bar{a}$ (= 4.365). The experimental data are beyond the error bars and indicate a real shift in the fill ratio for maximum growth. The shift of the maximum growth rate was to a higher fill ratio and was similar to the tests conducted with the variable annulus model. The difference between the predicted and observed maximum growth rates for Model B was approximately 20%.

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* The curves on pp. 8 - 23 (Figures 8-9) of Reference 2 furnish the basis for this conclusion.

V. DISCUSSION

The gyroscope experiments have shown that for geometries similar to the Models A and B the fill ratio for maximum growth rate will shift towards a larger value (see Figures 8-9 of Reference 2) and that the maximum amplitude growth rate will decrease. The gyroscope experiments gave no indication as to why one version should have been stable and the other not; in fact, Model A which had unstable flight histories had no measureable yaw growth rate on the gyroscope. Experience with liquid-filled shell has indicated that flight instabilities normally occur prior to liquid spin-up. Eigen-oscillations can occur during spin-up and will result in a projectile instability whose mechanism is identical to that of a Stewartson type. During spin-up, however, the eigenfrequency spectrum will change as the liquid changes its state of rotation. A numerical procedure has been developed to determine the time-dependent eigenfrequency history for a completely filled cylinder.[11] The applicability of these computational procedures has not been verified for projectiles as yet, although some correlation was achieved from flight data reported in Reference 9. The analysis for the time-dependent eigenfrequencies is linear, while laboratory experimental data suggests that nonlinear effects in eigenfrequency, gyroscope amplitude growth rate and internal liquid pressures are present.[12,13] In spite of all of these uncertainties, the average aspect ratios of the flight hardware were used to compute time-dependent eigenfrequency histories. For these calculations, a launch spin of 628.3 rad/s and a kinematic viscosity of 0.01 cm^2/s yielded an Ekman number of 5.29×10^{-8} . The Ekman spin-up time was 6.93 s.

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[11] *C.W. Kitchens, Jr., N. Gerber, and R. Sedney, "Oscillations of a Liquid in a Rotating Cylinder: Part I. Solid-Body Rotation," Ballistic Research Laboratories Technical Report ARBRL-TR-02081, June 1978. AD A057759.*

[12] *W.E. Scott and W.P. D'Amico, "Amplitude-dependent behavior of a liquid-filled gyroscope," J. Fluid Mech., Vol. 60, Part 4, pp. 751-758, 1973.*

[13] *R.D. Whiting, "An Experimental Study of Forced Asymmetric Oscillations in a Rotating Liquid-Filled Cylinder," Ballistic Research Laboratories Report in publication.*

The time-dependent eigenfrequency predictions are given in Figure 7.

This technique presently can only identify the eigenfrequency history and cannot define the magnitude of the overturning moment associated with a particular eigenfrequency. It is obvious, however, that if an eigenfrequency changes rapidly, resonance with projectile will not be long in duration and, therefore, the time of action or application of the overturning moment during resonance will be less. Hence, the slope of the eigenfrequency history in the vicinity of $\tau_{nj} = \tau_{NU} = 0.078$ is important for an M687-type projectile. The histories shown in Figure 7 are very similar and none of the small differences between them suggest that dramatic differences in flight behavior should exist. It is interesting to note that the eigenoscillation is prograde, i.e., positive, and therefore in the direction of spin, during the early portions of the flight history and then becomes retrograde during the later stages. The asymptotes of the eigenfrequency histories at large times would be analagous to the Stewartson eigenfrequency. For the cases considered, these frequencies would be approximately -0.065 and -0.09. The smallest Stewartson eigenfrequency is zero and retrograde eigenfrequencies have no moment associated with them, since resonance is only possible for $\tau_{NU} > 0$. Hence, for the mode numbers considered, a Stewartson instability is not possible.

It is apparent that important and essential features of the fluid mechanics are not included in the use of the average aspect ratio for computing the Stewartson model or time-dependent eigenfrequency histories. This approximate method does not provide sufficient information for a confident engineering design of a canister with dissimilar internal diameters. It is possible that turbulence is generated by the lip during launch from a gun, and this turbulence could have a dramatic effect on the spin-up history of the liquid and the eigenfrequencies associated with the resulting flow.

VI. CONCLUSIONS

A series of gyroscope tests were conducted to investigate the effect of dissimilar diameters within a cylindrical cavity. The use of an average aspect ratio was not sufficient to accurately model the frequency and amplitude response of a liquid-filled gyroscope. Also, these data do not explain instabilities observed in field tests.

VII. ACKNOWLEDGEMENTS

The authors performed this work in support of Mr. John Ferriter of the CSL. Funds were provided through the Independent Laboratory In-House Research program of CSL.

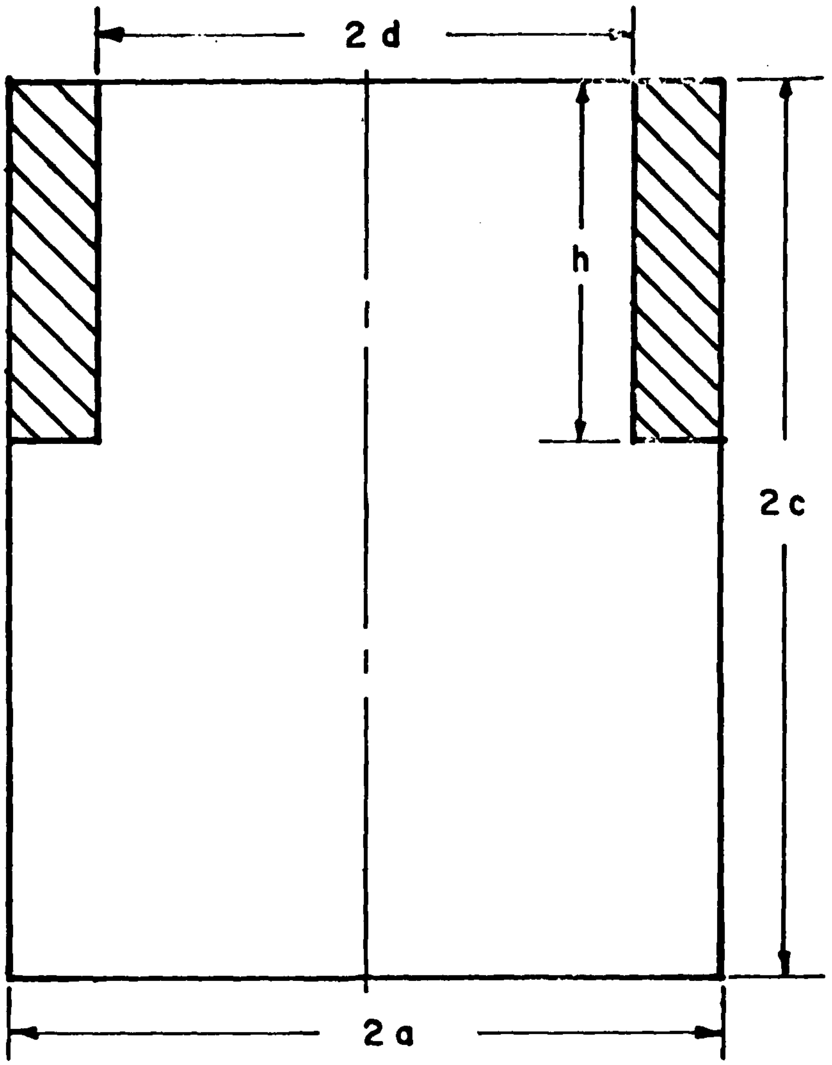

Figure 1. Cavity with two cylindrical sections of dissimilar internal diameters.

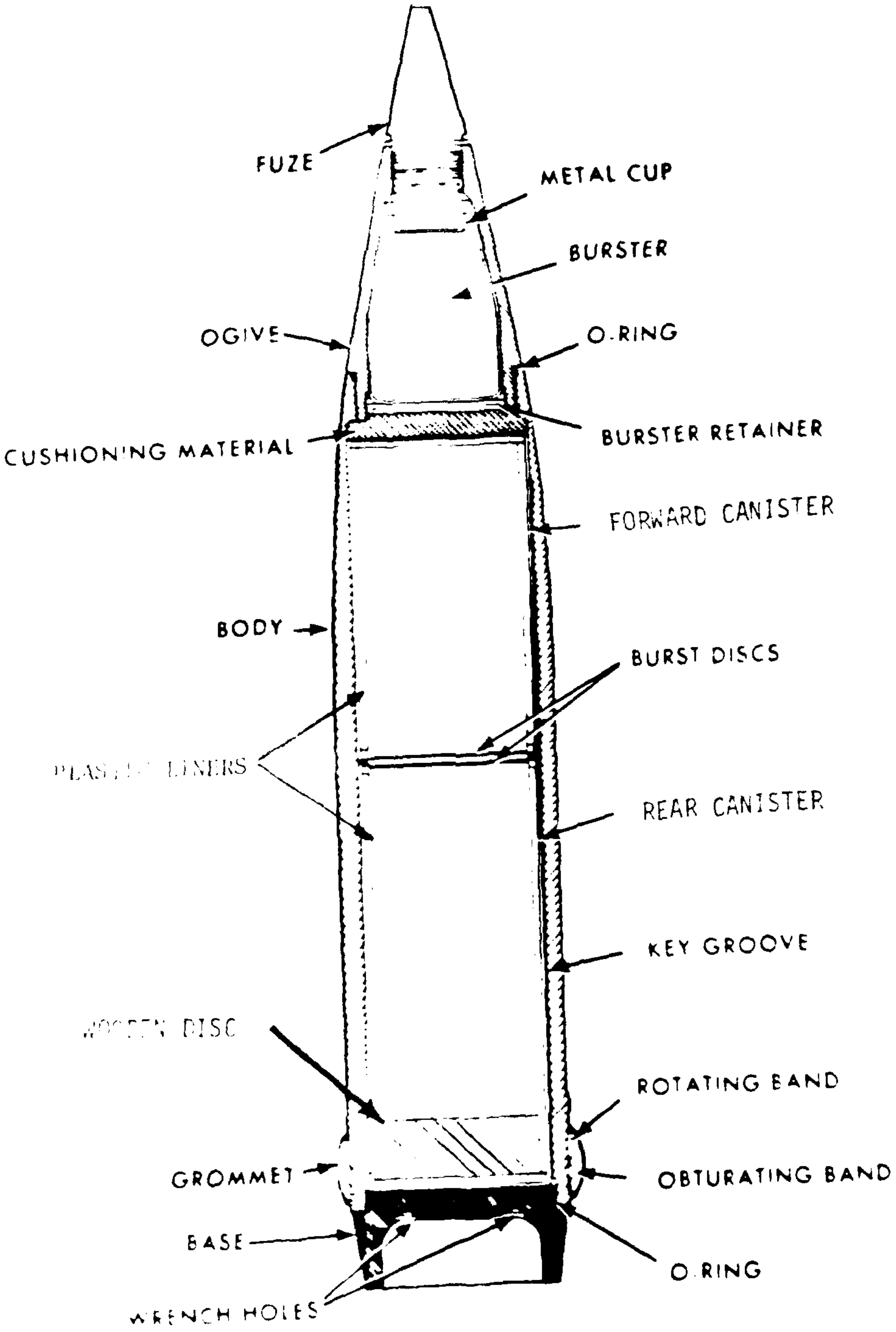

Figure 2. Cut-away view of the 155mm M687 binary projectile.

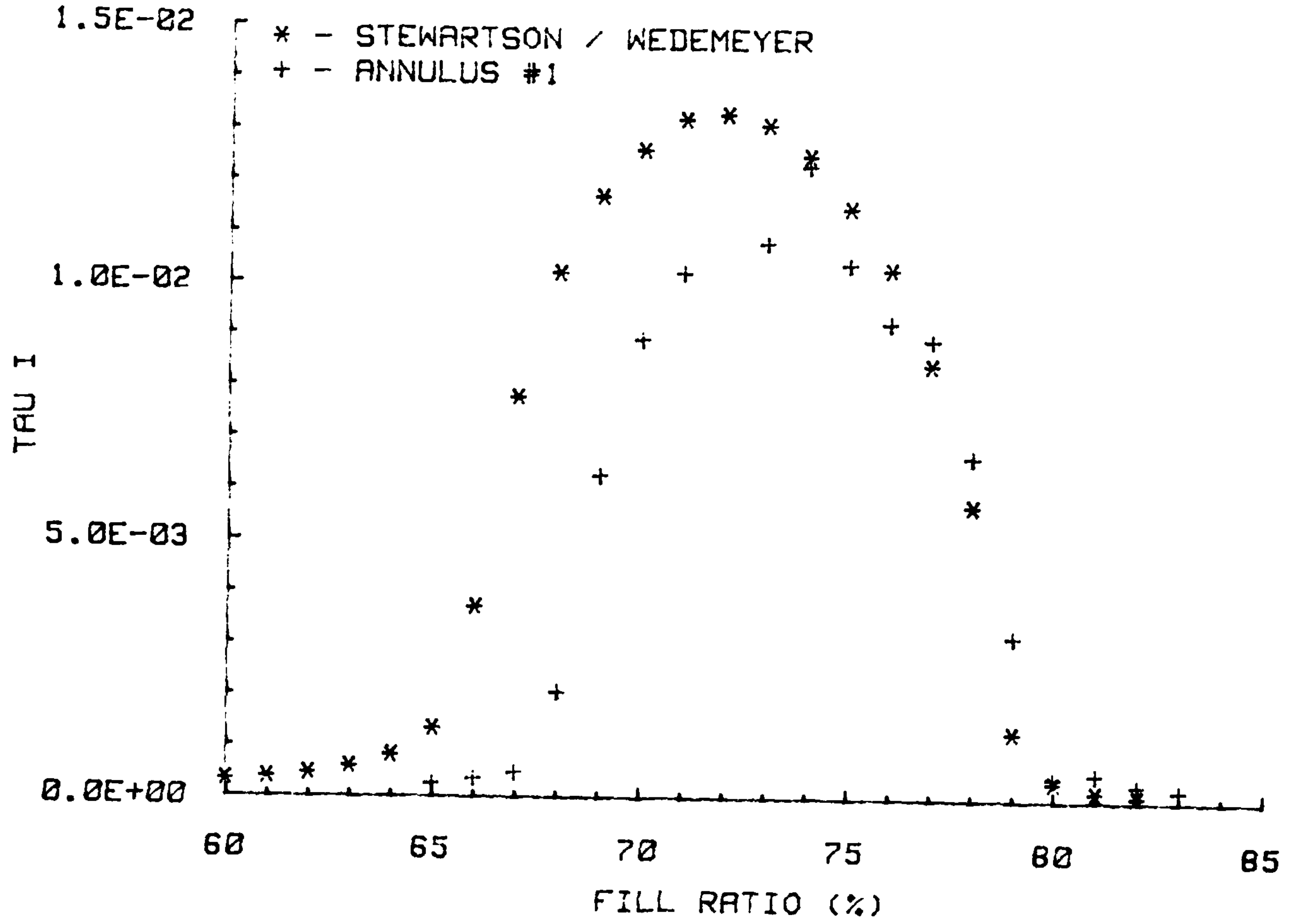

Figure 3. Comparison of predicted and measured amplitude growth rates (τ_I) for the variable diameter model with annulus #1.

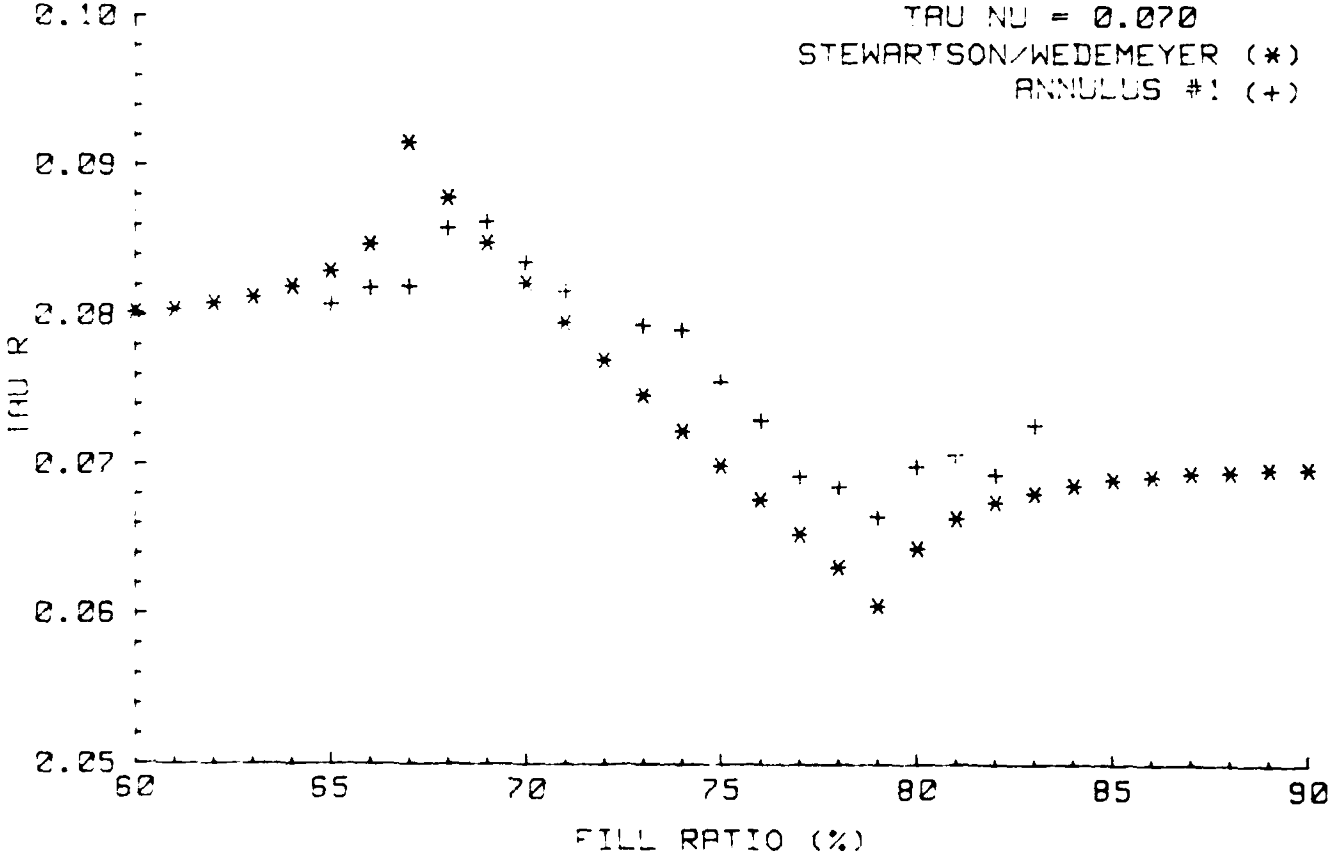

Figure 4. Comparison of predicted and measured coning frequencies (τ_R) rates for the variable diameter model with annulus #1.

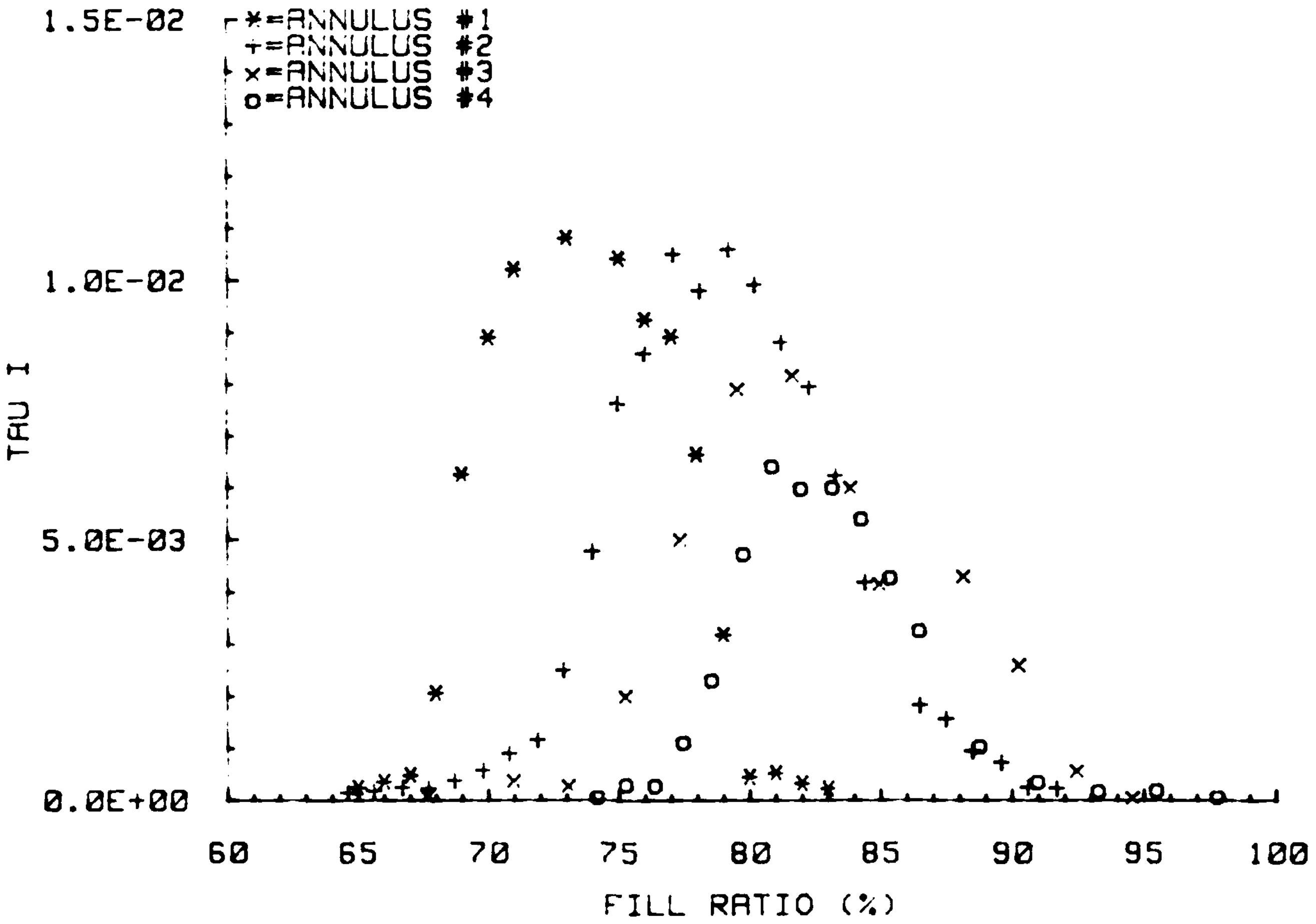

Figure 5. Fill ratio versus amplitude growth rate for the variable diameter model.

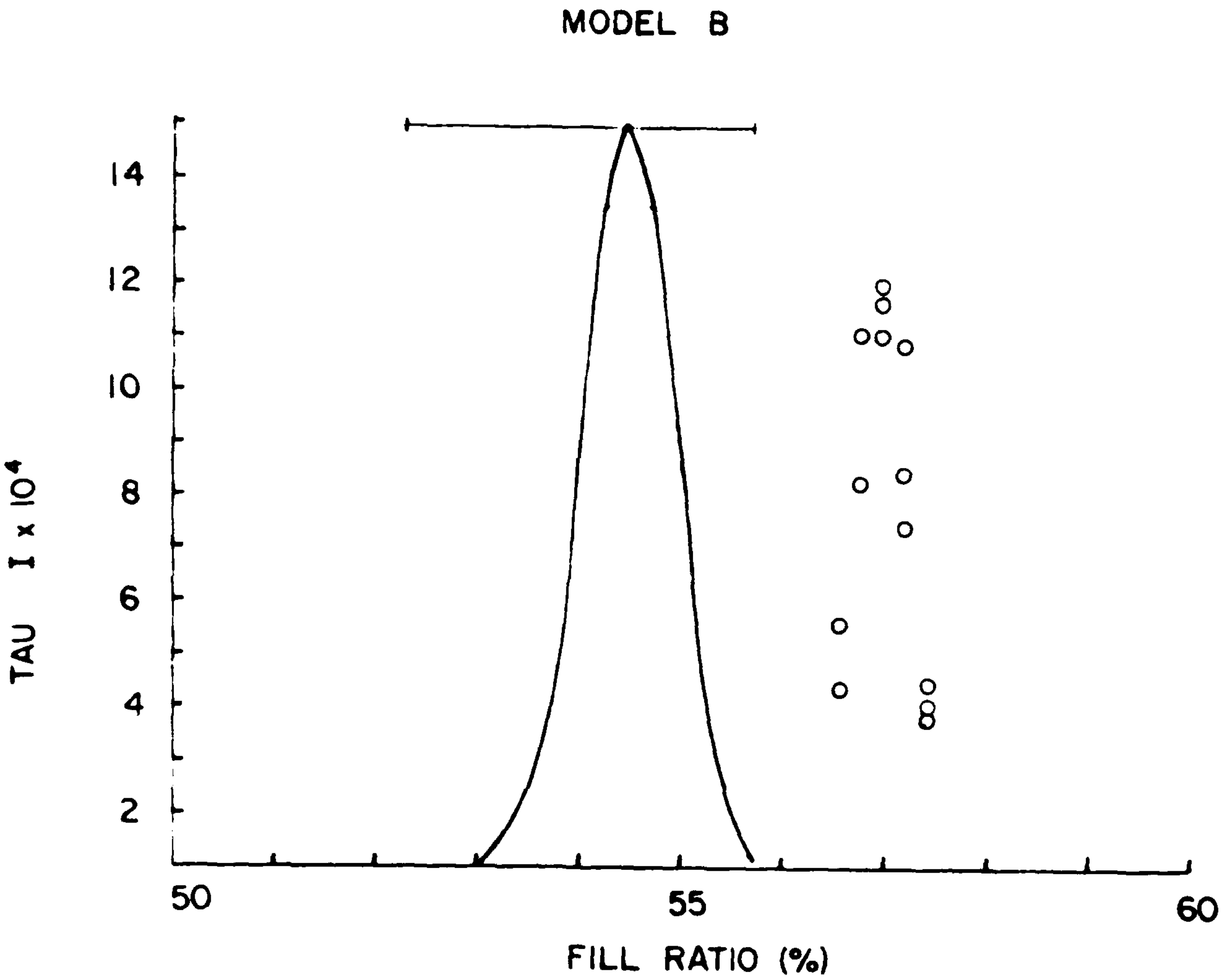

Figure 6. Comparison of predicted and measured amplitude growth rates (τ_I) for Test Hardware B.

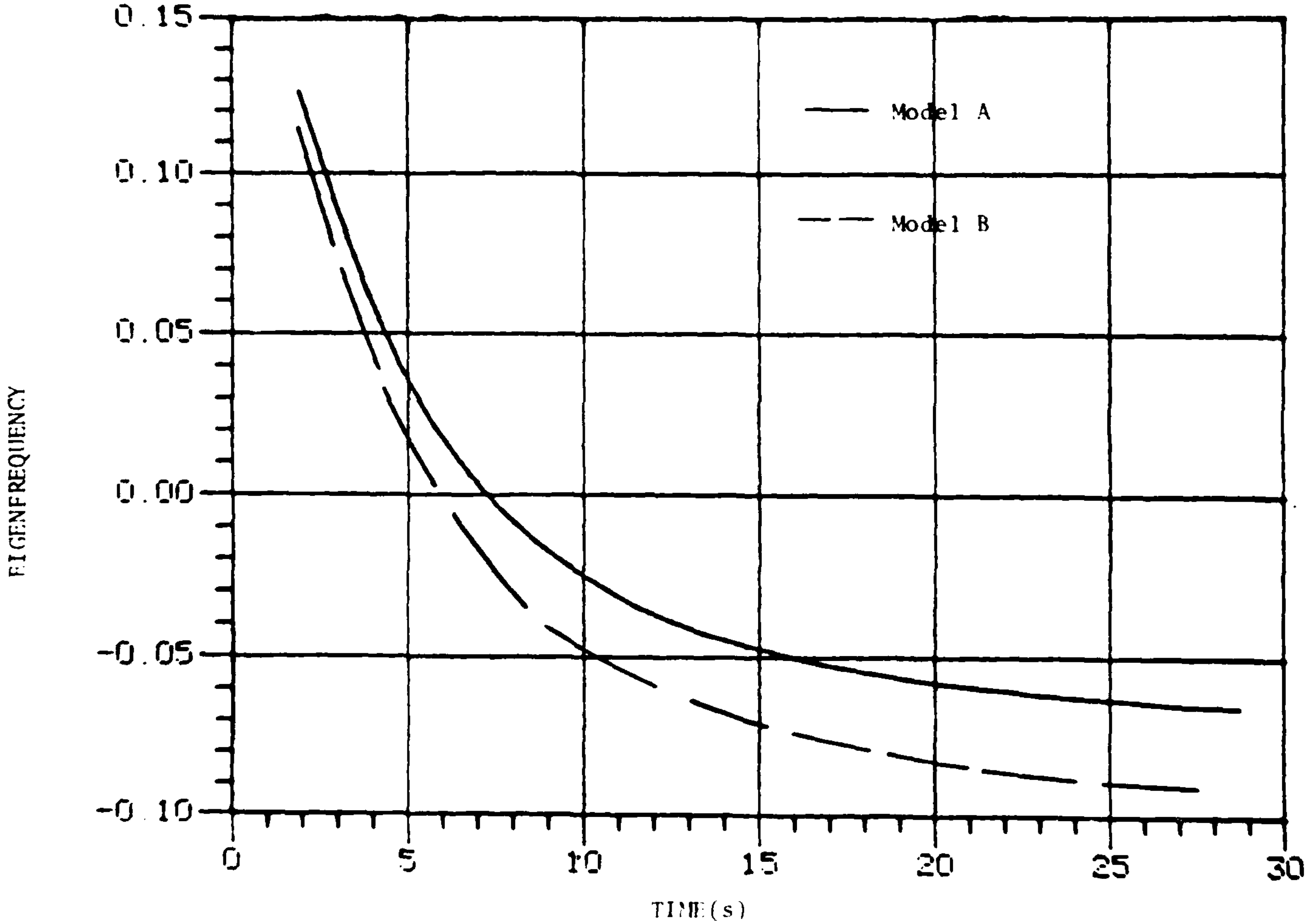

Figure 7. Eigenfrequency histories during spin-up.

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13. R.D. Whiting, "An Experimental Study of Forced Asymmetric Oscillations in a Rotating Liquid-Filled Cylinder," Ballistic Research Laboratories Report in publication.

LIST OF SYMBOLS

a	Radius of cylindrical cavity.
$\bar{a}$	average radius, $\bar{a} = (V_T/2c\pi)^{\frac{1}{2}}$
b	radial coordinate of free surface in partially-filled cylinder
c	half-height of cylindrical cavity (cm)
d	interior radius of an annulus (Fig. 1)
E	Ekman number (ν/c^2p)
j	index of axial perturbation mode
n	index of radial perturbation mode
p	spin of the gyroscope or projectile
t_c	Ekman spin-up time, Eq. (2)
V_T	total available volume of a liquid-filled container
ν	kinematic viscosity of liquid
ρ	density of liquid
$\dot{\phi}_1$	fast precessional frequency of motion of the gyroscope or projectile
τ_{nj}	inviscid liquid eigenfrequency (non-dimensional)
τ_I	yaw growth rate of the gyroscope or TAU I (non-dimensional)
τ_{NU}	fast precessional (nutational) frequency of empty gyroscope (non-dimensional)
τ_R	coning frequency of liquid-filled gyroscope or TAU R (non-dimensional)

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