

LEVEL II

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Split Learning Curves

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HQ AFSC/ACCE

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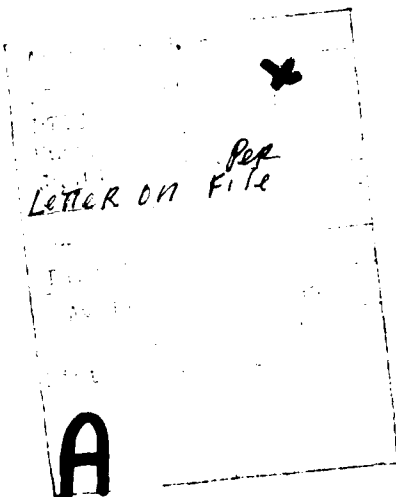
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3780 M(V+1) = ((S2+S3)/(A5*Q1(V+1)))** (1/B6)
3785 T7(V+1) = A5*M(V+1)**B6
3790 IF Q(N-U-1) < N9 GO TO 3910
3800 IF L(N-U-1) > N9 GO TO 3910
3810 S4 = (A5*(N9** (B6+1) - T(N-U-1)** (B6+1)))/(B5+1)
3820 S5 = (A6*(Q(N-U-1)** (B9+1) - N9** (B9+1)))/(B9+1)
3830 M(N-U-1) = ((S4+S5)/(A5*Q1(N-U-1)))** (1/B6)
3835 T7(N-U-1) = A5*M(N-U-1)**B6
3840 IF M(N-U-1) < N9 GO TO 3860
3850 M(N-U-1) = ((S4+S5)/(A5*Q1(N-U-1)))** (1/B9)
3855 T7(N-U-1) = A6*M(N-U-1)**B9
3860 GO TO 3601
3900 M(V+1) = ((Q(V+1)** (B5+1) - L(V+1)** (B5+1))/(Q1(V+1)*(B5+1)))** (1/B5)
3903 T7(V+1) = A4*M(V+1)**B5
3905 N3 = Q(V+1)
3907 GO TO 3790
3910 M(N-U-1) = ((Q(N-U-1)** (B6+1) - L(N-U-1)** (B6+1))/(Q1(N-U-1)*(B6+1)))** (1/B6)
3913 T7(N-U-1) = A5*M(N-U-1)**B6
3915 N9 = Q(N-U-1)
3920 GO TO 3601
7000 FOR I=9 TO 14
7010 R(K,I) = 9999
7020 NEXT I
7030 GO TO 1790
9000 END

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Split Learning Curves

The term "learning curve" describes the improvement in efficiency of producing an item in quantities larger than one. Although the phenomenon has been known by other names and many different mathematical formulations on the learning curve have been developed, the basic concept is accepted throughout government and industry.

Learning curve theory was developed from the observation that as more and more of the same item are produced, less time is spent on subsequent items. In a strict interpretation for a recurring production process, this theory would apply only to hands-on-labor. The theory has been developed and extended through the years to include cost improvements for recurring material costs. In some cases rough analyses are made using only the total cost of an item when the detailed breakouts are not available.

The current methods of calculating recurring production costs assume that actual performance can adequately be described by a single learning curve slope and that this slope can be used as a basis for extrapolation of future performance. This assumption may be valid when using learning curve theory to project labor hours. However, if we extend the theory to include other recurring costs in addition to labor, i.e., material, inconsistencies can occur. Some of these are:

- 1) Initial lots may contain some nonrecurring costs because separation

of recurring and nonrecurring costs is sometimes difficult.

2) Production efficiency during build-up to optimal rate may be different than efficiency thereafter.

3) Design changes in early lots could affect rate of improvement.

4) Data could be aggregated to a higher level of the Work Breakdown Structure (WBS) and, therefore, the learning curve would be a composite of many pieces, each having its own parameters.

Actual "recurring" aircraft production data seems to support the hypothesis that more than one slope may better explain performance than the single slope. The "eyeball technique" for splitting curves was the only method available to account for these inconsistencies when it appeared that a single slope was inappropriate.

The split learning curve models remove much of the subjectivity in determining whether a single curve adequately explains performance. The statistically sound technique was developed to supply the necessary analytical tools to calculate the breakpoint(s) and to fit the "best" two or three log-log linear curves to the data. These models use a relatively new formulation of the log-log linear curve developed by Mr. Karl Berend, formerly of Air Force Systems Command. This formulation assumes continuity in the learning process with the following relationship between the curves.

1) Unit Curve: $y_u = ax^b$

2) Cumulative Total Curve: $y_T = \int_0^x (at^b) dt = (a/(b+1))x^{(b+1)}$

3) Cumulative Average Curve: $y_A = ((a/(b+1))x^b$

The advantage in this formulation, apart from being theoretically sound, lies in the ease of transition between the three curves: unit, cumulative average, and cumulative total. When describing a single curve, all three remain linear on log-log paper, hence, the equation forms remain the same. One point of caution in the use of this formulation is that the unit cost of the xth unit is derived by calculating the total cost of x units and subtracting the total cost of (x-1) units. Because learning is assumed to be continuous, the unit costs are plotted at their respective midpoints. The xth unit cost then should be plotted at the midpoint between the (x-1)st and xth unit. For more detail on this formulation, see "The Unified Linear Progress Formulation" by Mr. Berend.

The two models use essentially the same technique for fitting the best two or three curves to the data. The technique was initially developed by Mr. John Dorsett, currently with Naval Material Command, and has been adapted to calculate the unit curve using the midpoints derived under the Unified Formulation. The methodology will be discussed for the single split (two curves); the double split being an extension of the single split.

The model first regresses the data points to find the conventional

learning curve slope and first unit cost. It uses the midpoints generated from the single curve, partitioning the data points into two sets to begin the splitting process. By forcing at least two data points to be on each curve, the model regresses all the possible sets of data points. For example, with five data points the model will regress forcing the first three data points to be on the first curve and the last two points on the second curve, as a first attempt at splitting the curve. It then regresses the first two points on the first curve and the last three points on the second curve (see Figure 1). For each data point added to the data base an additional regression is needed to exhaust all possible data point partitions. It will then recalculate the midpoints based on the new slopes and repeat the process, eventually zeroing in on a best fit.

Two kinds of phenomena could occur at the breakpoint. As illustrated in Figure 2a, a curve could simply change slopes or, as in Figure 2b, a break in continuity could occur. The model does not force either phenomenon to occur, but chooses only that combination of curves yielding the smallest residual sum of squares.

From a theoretical standpoint each of these phenomena could be explained. The continuity break could be explained by a break in the production run or a significant change in management technique at that point of production. The change in slope could be caused by reaching peak production rate, or a slowing of the production process. There is also

Model Methodology

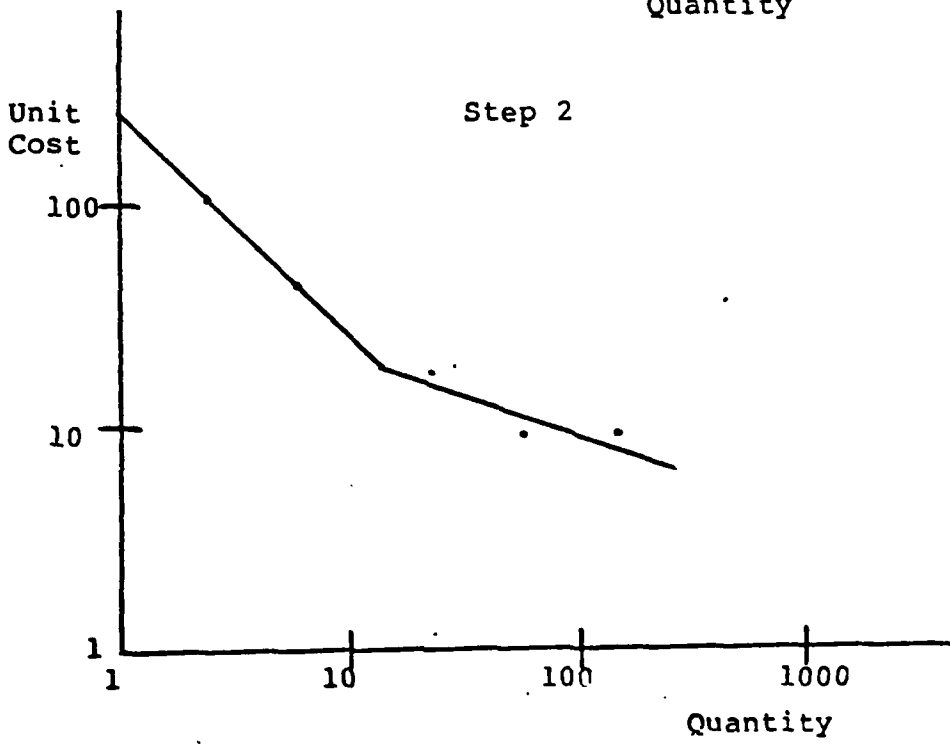
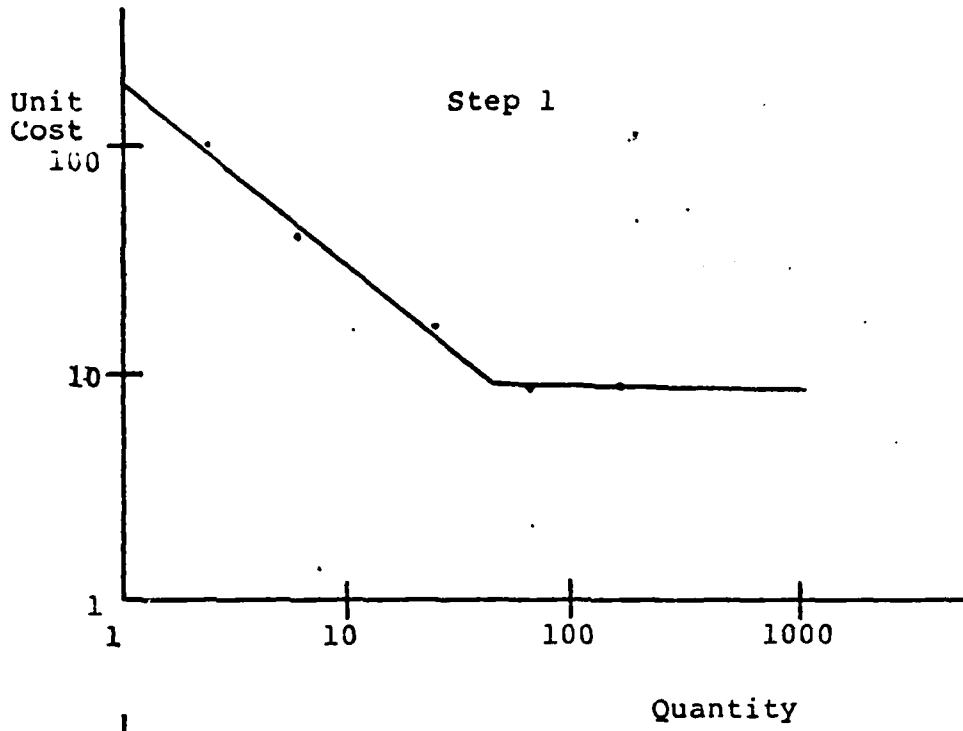
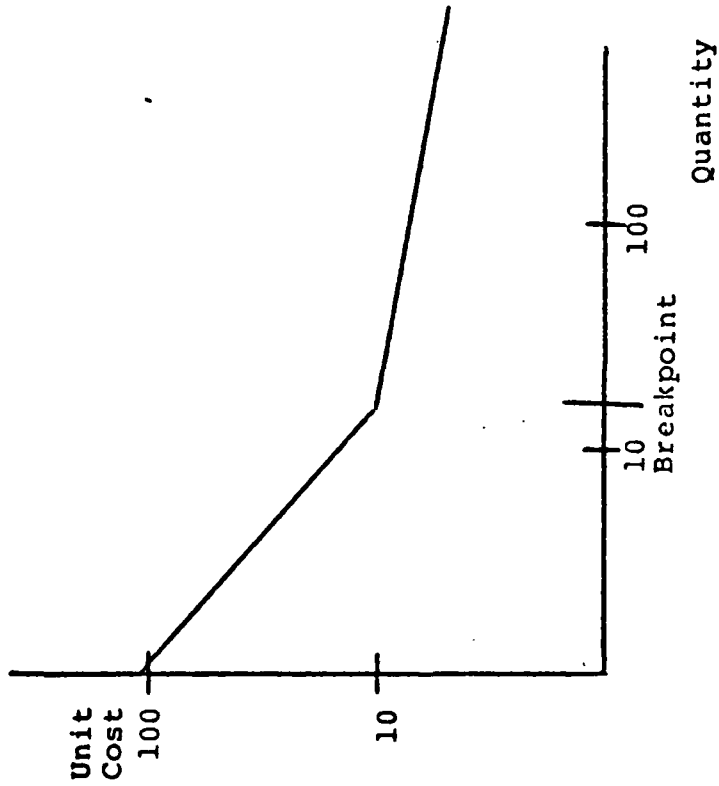
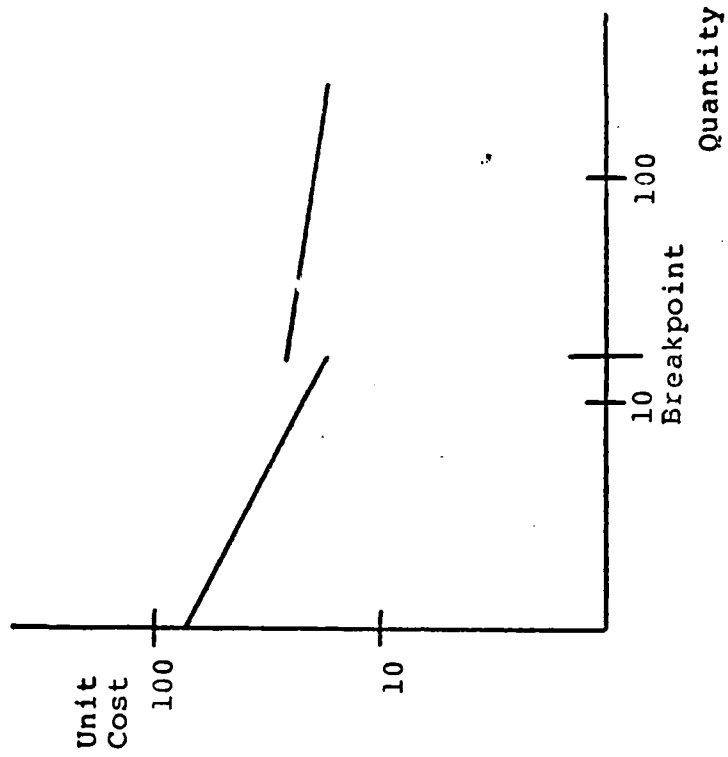


Figure 1

Single Split
Two Variations



(a)

(b)

Figure 2

concern that for long production runs, learning does not continue at the same rate indefinitely.

Of the two models the double split is the most powerful. It not only reacts to discontinuities in a set of data, but also closely approximates an "S" curve (which has been supported by some contractors' data). As with any statistical method, there are limitations that require the application of common sense. First of all, there are the data limitations. The split curve models require at least five data points to split a learning curve once, and at least seven data points to split the curve twice. More data points are desirable just as more than three data points are preferable when deriving a single curve. Secondly, the analyst must keep in mind that three curves will always fit the data better than two curves, and two curves will always fit better than one. Therefore, a split curve should not be chosen over a single curve on the basis of a better coefficient of determination (R^2). A T-test has been incorporated into the model to aid in this determination. Thirdly, the resultant curves should make sense. The splits should have some logical reason for occurring. A break in continuity should be explainable or should serve to indicate that something out of the ordinary is occurring. When estimating the cost of a conceptual weapon system on the basis of an analogy, the exact breakpoints and slopes should make sense in terms of assumptions for the conceptual weapon system.

Equation forms and a list of statistics available from the output of

the model are listed in Appendix A. Some data and graphs for an actual aircraft system have been included in Appendix B to illustrate output and results of the computer programs. A listing of the model, currently in Basic on the Honeywell 6000, is provided in Appendix C. The computer program is currently being converted to Fortran to enhance its capabilities.

Appendix A

The resulting set of curves can be expressed as one equation with the use of dummy variables:

One Split

$$y = (a_1 + a_2d_1)x^{(b_1 + b_2d_2)} \quad \text{where:}$$

y = unit cost at x
 a_1 = Y intercept of unit curve 1 (not first unit cost)
 b_1 = b-value of curve 1
 $a_1 + a_2$ = y intercept of unit curve 2 (not first unit cost)
 $b_1 + b_2$ = b-value of curve 2
 $d_1, d_2 = 0$ if $x <$ breakpoint
 $= 1$ if $x >$ breakpoint

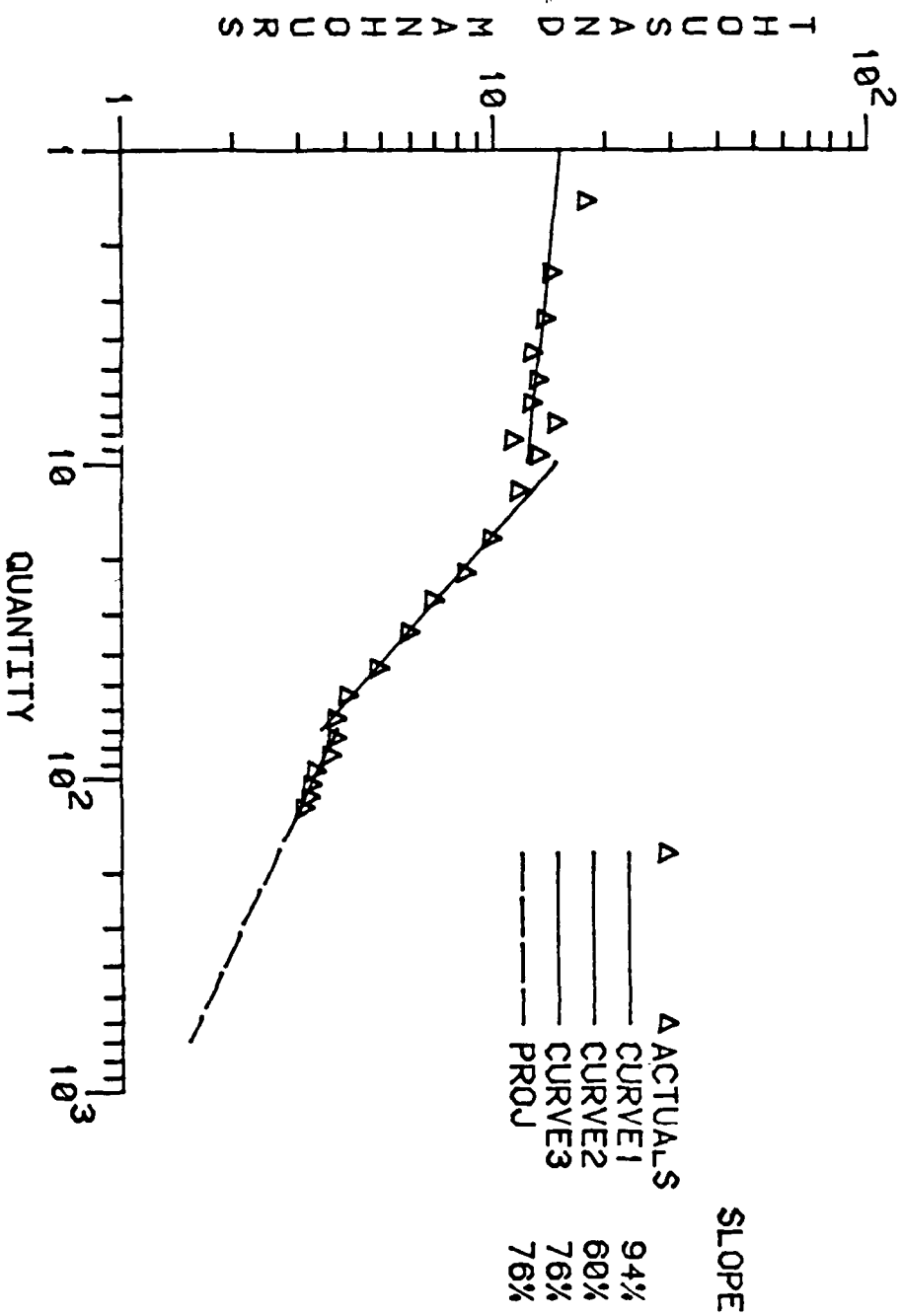
Two Splits

$$y = (a_1 + a_2d_1 + a_3d_2)x^{(b_1 + b_2d_3 + b_3d_4)} \quad \text{where:}$$

y = unit cost at x
 a_1 = y intercept of unit curve 1
 b_1 = b-value of unit curve 1
 $a_1 + a_2$ = y intercept of unit curve 2
 $b_1 + b_2$ = b-value of unit curve 2
 $a_1 + a_2 + a_3$ = y intercept of unit curve 3
 $b_1 + b_2 + b_3$ = b-value of unit curve 3
 $d_1, d_3 = 0$ if $x <$ Breakpoint 1
 $= 1$ if $x >$ Breakpoint 1
 $d_2, d_4 = 0$ if $x <$ Breakpoint 2
 $= 1$ if $x >$ Breakpoint 2

Attachment B
Program Output and Graphs

ACTUAL AIRCRAFT MANHOURS CENTER FUSELAGE



POINTS ON SECOND, NEXT 6 ON THIRD
 ALL VALUES LESS THAN 10 PREDICTED FROM FIRST CURVE
 ALL VALUES LESS THAN 70 BUT GREATER OR EQUAL TO 10
 ARE PREDICTED FROM THE SECOND CURVE, WHILE
 ALL VALUES GREATER THAN 70 PREDICTED FROM THIRD CURVE
 THE PROGRAM HAS COMPLETED 2 ITERATIONS

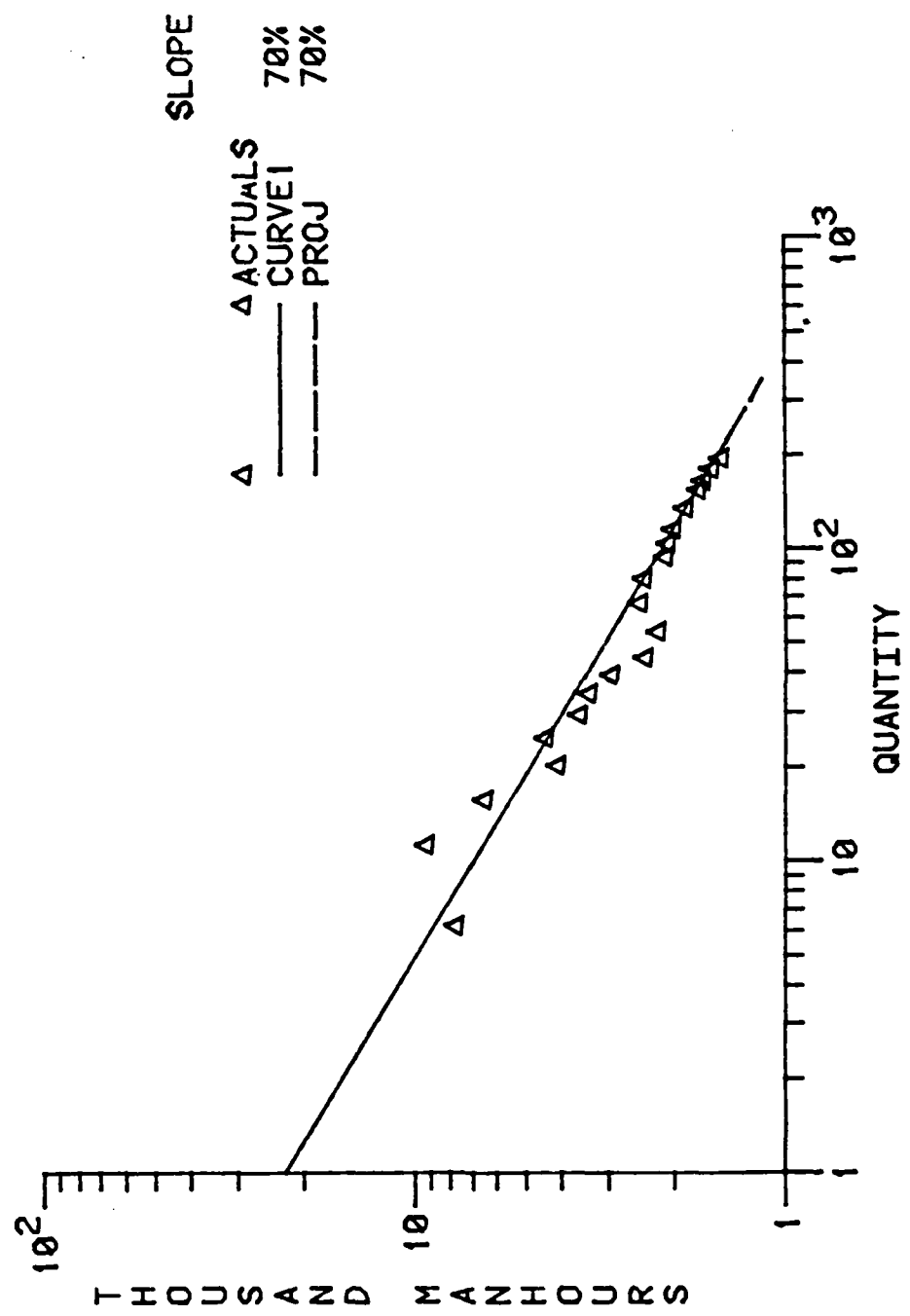
	T1	Y-INTERCEPT	SLOPE	B-VALUE	B+1
CURVE1	16.6365	15.0816	.937	-.093462	.906538
CURVE2	317.4520	81.1276	.597	-.744441	.255559
CURVE3	33.5706	20.3276	.761	-.394482	.605518

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	%DIFF
1.0	1	.3500	17.5000	17.5000	16.6365	-4.93
1.0	2	1.4688	14.0000	14.0000	14.5493	3.92
1.0	3	2.4816	13.5000	13.5000	13.8533	2.62
1.0	4	3.4869	12.5000	12.5000	13.4199	7.36
1.0	5	4.4899	13.0000	13.0000	13.1065	.82
1.0	6	5.4917	12.5000	12.5000	12.8621	2.30
1.0	7	6.4930	13.0000	13.0000	12.6623	-2.60
1.0	8	7.4939	14.5000	14.5000	12.4938	-13.84
1.0	9	8.4946	11.0000	11.0000	12.3483	12.26
1.0	10	9.4952	13.0000	13.0000	12.2204	-6.00
5.0	15	12.3531	57.5000	11.5000	12.4855	8.57
5.0	20	17.3956	48.5000	9.7000	9.6768	-.24
5.0	25	22.4190	41.3000	8.2600	8.0115	-3.01
5.0	30	27.4338	33.8000	6.7600	6.8936	1.98
10.0	40	34.7912	58.0000	5.8000	5.7761	-.41
10.0	50	44.8380	48.0000	4.8000	4.7821	-.37
10.0	60	54.8676	40.0000	4.0000	4.1148	2.87
10.0	70	64.8880	37.0000	3.7000	3.6318	-1.84
10.0	80	74.9224	37.0000	3.7000	3.7033	.09
10.0	90	84.9316	36.0000	3.6000	3.5245	-2.10
10.0	100	94.9388	33.0000	3.3000	3.3730	2.21
10.0	110	104.9447	32.0000	3.2000	3.2423	1.32
10.0	120	114.9495	31.5000	3.1500	3.1279	-.70
10.0	130	124.9534	30.5000	3.0500	3.0266	-.77

PREDICTIONS

LOT	CUM	UNIT	TOT LOT	LOT AVG	CUM TOT	CUM AVG
10.0	10	12.2204	134.1524	13.4152	134.15	13.4152
10.0	20	8.8895	110.8117	11.0812	244.96	12.2482
10.0	30	6.5313	74.5258	7.4526	319.49	10.6497
10.0	40	5.2555	57.7610	5.7761	377.25	9.4313
10.0	50	4.4427	47.8206	4.7821	425.07	8.5014
10.0	60	3.8739	41.1481	4.1148	466.22	7.7703
10.0	70	3.4508	36.3177	3.6318	502.54	7.1791
10.0	80	3.6177	37.0327	3.7033	539.57	6.7446
10.0	90	3.4525	35.2454	3.5245	574.82	6.3868
10.0	100	3.3112	33.7303	3.3730	608.55	6.0855
200.0	300	2.1438	515.7002	2.5787	1124.25	3.7475

ACTUAL AIRCRAFT MANHOURS FORWARD FUSELAGE



FORWARD FUELAGE

UNIFIED LOG-LINEAR PROGRESS CURVE FORMULATION-ALL CURVES ARE LINEAR
 LINEAR LEAST SQUARES ANALYSIS OF UNIT CURVE-NTH UNITS AT MIDPOINTS
 SLOPES: PERCENT = .704; B=-.50638;B+1= .49362;-1/B+1=-2.02586
 FIRST UNIT COST 45.11578; Y-INTERCEPT OF LIN UNIT CURVE= 22.26997
 R= .972842; R-SQUARED= .946421; RBAR-SQUARED= .943601

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	% DIFF
5.0	9	6.2489	38.5000	7.7000	8.8052	14.35
5.0	14	11.3619	46.0000	9.2000	6.5052	-29.29
4.0	18	15.9370	25.8000	6.4500	5.4808	-15.03
5.0	23	20.4232	20.5000	4.1000	4.8339	17.90
4.0	27	24.9598	17.6000	4.4000	4.3670	-.75
5.0	32	29.4467	20.5000	4.1000	4.0163	-2.04
5.0	37	34.4545	17.7500	3.5500	3.7093	4.49
5.0	42	39.4602	16.7500	3.3500	3.4630	3.37
6.0	48	44.9497	17.7000	2.9500	3.2420	9.90
13.0	61	54.3047	31.2000	2.4000	2.9460	22.75
12.0	73	66.8648	26.4000	2.2000	2.6514	20.52
16.0	89	80.8012	39.2000	2.4500	2.4090	-1.67
10.0	99	93.9331	24.0000	2.4000	2.2321	-6.99
11.0	110	104.4273	23.1000	2.1000	2.1156	.74
10.0	120	114.9454	20.5000	2.0500	2.0152	-1.70
30.0	150	134.5803	60.0000	2.0000	1.8606	-6.97
10.0	160	154.9596	18.5000	1.8500	1.7323	-6.36
10.0	170	164.9619	17.0000	1.7000	1.6783	-1.27
20.0	190	179.8603	33.0000	1.6500	1.6064	-2.64
10.0	200	194.9680	15.7500	1.5750	1.5421	-2.09
50.0	250	224.3005	73.7500	1.4750	1.4365	-2.61

SPLIT LEARNING CURVES

EQUATION FORM $Y=e^{-(A1+A2*DUML)*X} (B1+B2*DUML)$

VARIABLE	COEFFICIENT	T-STATISTIC
A1	2.358636	7.080
A2	.468805	1.311
B1	-.135041	-.965
B2	-.312569	-2.184

THE ABSOLUTE VALUE OF THE T-STATISTIC FOR A2 AND/OR B2 DOES NOT EXCEED 2.11 THESE VARIABLES ARE NOT SIGNIFICANT AT THE 95% CONFIDENCE LIMIT AND THEREFORE, A SPLIT LEARNING CURVE DOES NOT EXIST.

R-SQUARED .9736378
 RBAR-SQUARED .9689857
 RESIDUAL SUM OF SQUARES .1468594

FORWARD FUSELAGE (CONT)

FIRST 3 POINTS ON FIRST CURVE, REMAINDER ON SECOND
 ALL VALUES LESS THAN 18 PREDICTED FROM FIRST CURVE
 ALL VALUES GREATER THAN 18 PREDICTED FROM SECOND CURVE

	T1	Y-INTERCEPT	SLOPE	B-VALUE	B+1
CURVE1	12.2278	10.5765	.911	-.135041	.864959
CURVE2	30.5983	16.9021	.733	-.447611	.552389

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	% DIFF

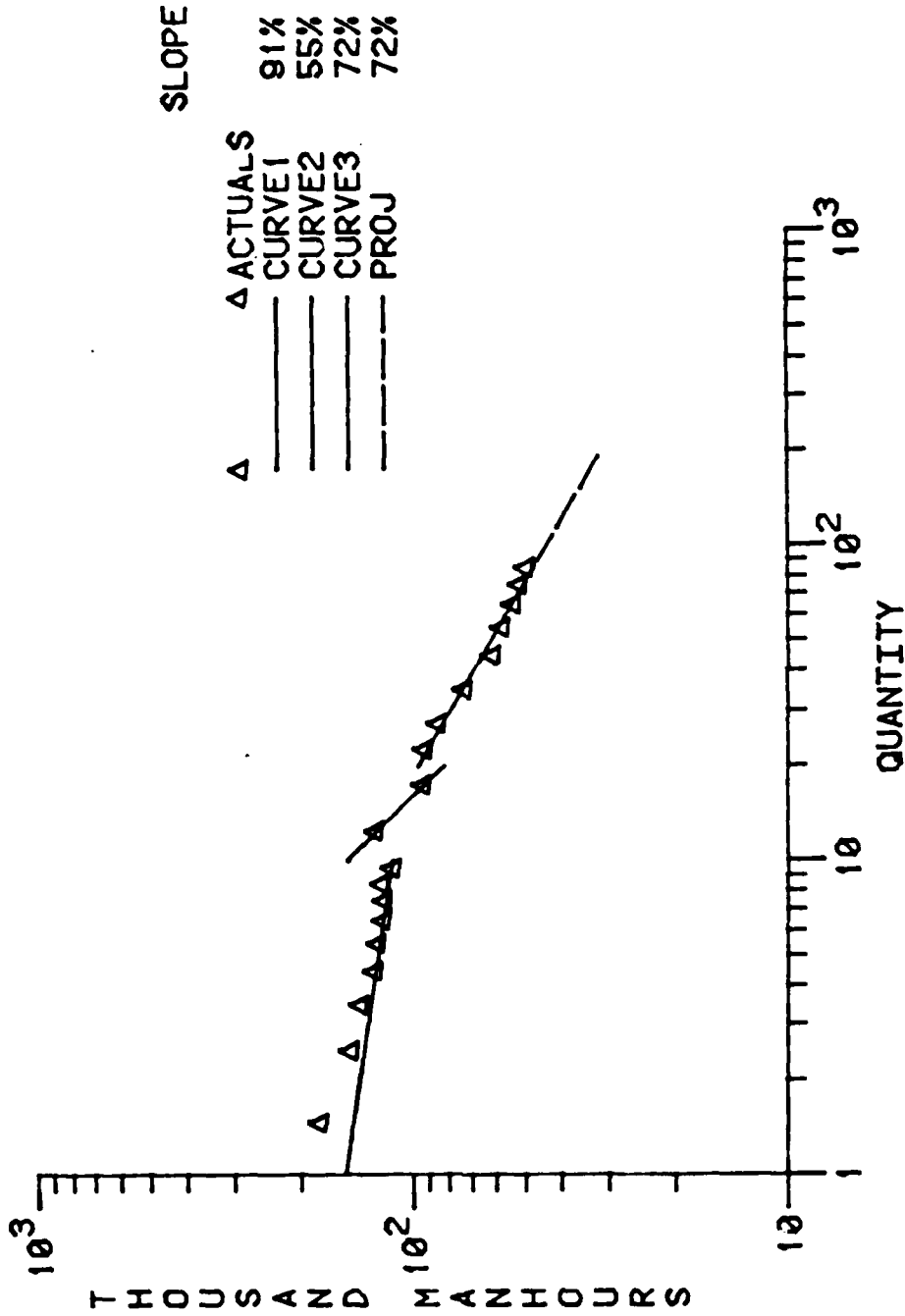
5.0	9	6.3115	38.5000	7.7000	8.2469	7.10
5.0	14	11.3961	46.0000	9.2000	7.6144	-17.23
4.0	18	15.9525	25.8000	6.4500	7.2763	12.81
5.0	23	20.4262	20.5000	4.1000	4.3801	6.83
4.0	27	24.9613	17.6000	4.4000	4.0041	-9.00
5.0	32	29.4488	20.5000	4.1000	3.7185	-9.30
5.0	37	34.4562	17.7500	3.5500	3.4661	-2.36
5.0	42	39.4618	16.7500	3.3500	3.2619	-2.63
6.0	48	44.9517	17.7000	2.9500	3.0772	4.31
13.0	61	54.3123	31.2000	2.4000	2.8274	17.81
12.0	73	66.8701	26.4000	2.2000	2.5760	17.09
16.0	89	80.8089	39.2000	2.4500	2.3667	-3.40
10.0	99	93.9358	24.0000	2.4000	2.2125	-7.81
11.0	110	104.4302	23.1000	2.1000	2.1101	.48
10.0	120	114.9476	20.5000	2.0500	2.0213	-1.40
30.0	150	134.5966	60.0000	2.0000	1.8835	-5.83
10.0	160	154.9611	18.5000	1.8500	1.7684	-4.41
10.0	170	164.9635	17.0000	1.7000	1.7196	1.15
20.0	190	179.8659	33.0000	1.6500	1.6543	.26
10.0	200	194.9693	15.7500	1.5750	1.5956	1.31
50.0	250	224.3277	73.7500	1.4750	1.4985	1.59

PREDICTIONS

LOT	CUM	UNIT	TOT LOT	LOT AVG	CUM TOT	CUM AVG

10.0	10	7.8044	89.5996	8.9600	89.60	8.9600
10.0	20	4.4724	68.4244	6.8424	158.02	7.9012
10.0	30	3.7158	40.1896	4.0190	198.21	6.6071
10.0	40	3.2606	34.4957	3.4496	232.71	5.8177
10.0	50	2.9473	30.7984	3.0798	263.51	5.2702
10.0	60	2.7143	28.1401	2.8140	291.65	4.8608
10.0	70	2.5319	26.1060	2.6106	317.75	4.5393
10.0	80	2.3841	24.4823	2.4482	342.24	4.2780
10.0	90	2.2609	23.1460	2.3146	365.38	4.0598
10.0	100	2.1562	22.0202	2.2020	387.40	3.8740
200.0	300	1.3167	325.0776	1.6254	712.48	2.3749

ACTUAL AIRCRAFT MANHOURS TOTAL AIRFRAME



TOTAL AIRFRAME

UNIFIED LOG-LINEAR PROGRESS CURVE FORMULATION-ALL CURVES ARE LINEAR
 LINEAR LEAST SQUARES ANALYSES OF UNIT CURVE-NTH UNITS AT MIDPOINTS
 SLOPES; PERCENT= .842; B=-.24726; B+1= .7527;-1/B+1=-1.3285
 FIRST UNIT COST 232.62415; Y-INT. OF LIN UNIT CURVE 175.10654

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	%DIFF
1.0	1	.3170	177.0000	177.0000	232.6242	31.43
1.0	2	1.4644	146.0000	146.0000	159.3464	9.14
1.0	3	2.4790	137.5000	137.5000	139.8990	1.74
1.0	4	3.4851	126.0000	126.0000	128.5989	2.06
1.0	5	4.4884	123.0000	123.0000	120.8005	-1.79
1.0	6	5.4905	120.0000	120.0000	114.9287	-4.23
1.0	7	6.4920	119.0000	119.0000	110.2649	-7.34
1.0	8	7.4931	121.0000	121.0000	106.4236	-12.05
1.0	9	8.4939	119.0000	119.0000	103.1753	-13.30
1.0	10	9.4945	113.0000	113.0000	100.3730	-11.17
5.0	15	12.3951	621.0000	124.2000	93.9703	-24.34
5.0	20	17.4254	464.0000	92.8000	86.3800	-6.92
5.0	25	22.4421	460.0000	92.0000	81.1418	-11.80
5.0	30	27.4527	425.0000	85.0000	77.1977	-9.18
10.0	40	34.8508	727.0000	72.7000	72.7749	.10
10.0	50	44.8842	613.0000	61.3000	68.3617	11.52
10.0	60	54.9053	575.0000	57.5000	65.0389	13.11
10.0	70	64.9199	537.0000	53.7000	62.3996	16.20
10.0	80	74.9307	515.0000	51.5000	60.2258	16.94
10.0	90	84.9389	497.0000	49.7000	58.3875	17.48

SPLIT LEARNING CURVES

EQUATION FORM $Y = e^{(A1 + A2 * DUM1 + A3 * DUM2)} * X^{(B1 + B2 * DUM1 + B3 * DUM2)}$

VARIABLE	COEFFICIENT	T-STATISTIC
A1	5.029462	361.49
A2	1.929649	6.62
A3	-.954209	-3.16
B1	-.130789	-15.17
B2	-.719631	-6.62
B3	.368841	3.35

IF ABSOLUTE VALUE OF T-STATISTIC EXCEEDS 2.145
 FOR THE DUMMY VARIABLES A2, A3, B2, AND B3, THEN THESE VARIABLES
 ARE SIGNIFICANT AT 95% CONFIDENCE LIMIT AND THERE EXISTS
 A TWICE SPLIT LEARNING CURVE

R-SQUARED .9960643
 RESIDUAL SUM OF SQUARES .0110168

FIRST 10 POINTS ON FIRST CURVE, NEXT 2
 POINTS ON SECOND, NEXT 8 ON THIRD
 ALL VALUES LESS THAN 10 PREDICTED FROM FIRST CURVE
 ALL VALUES LESS THAN 20 BUT GREATER OR EQUAL TO 10
 ARE PREDICTED FROM THE SECOND CURVE, WHILE
 ALL VALUES GREATER THAN 20 PREDICTED FROM THIRD CURVE
 THE PROGRAM HAS COMPLETED 2 ITERATIONS

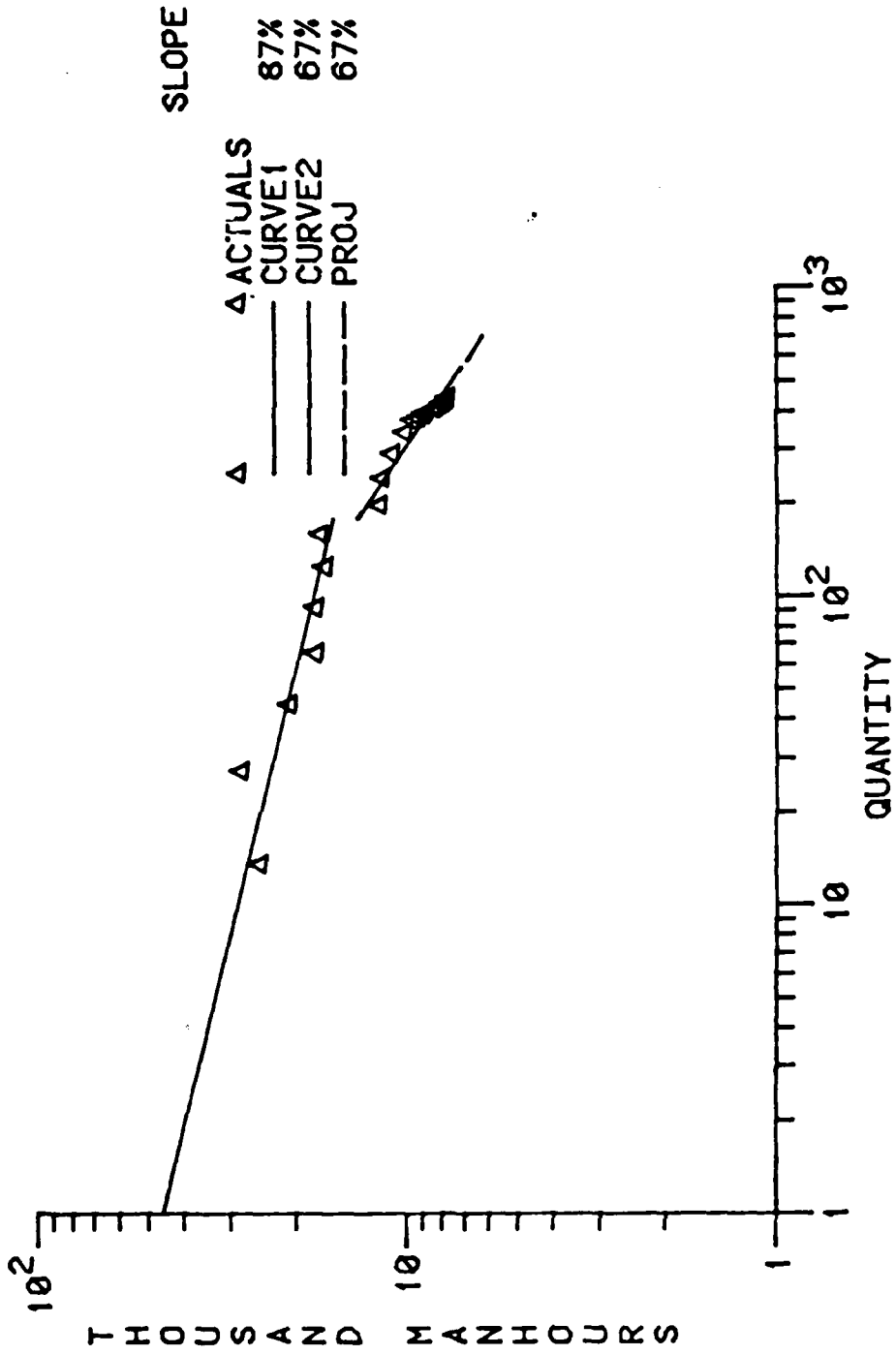
	T1	Y-INTERCEPT	SLOPE	B-VALUE	B+1
CURVE1	175.8500	152.8508	.913	-.130789	.869211
CURVE2	*****	*****	.555	-.850420	.149580
CURVE3	782.0105	405.4110	.716	-.481579	.518421

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	%DIFF
1.0	1	.3424	177.0000	177.0000	175.8500	-.65
1.0	2	1.4678	146.0000	146.0000	145.3688	-.43
1.0	3	2.4810	137.5000	137.5000	135.7236	-1.29
1.0	4	3.4865	126.0000	126.0000	129.8164	3.03
1.0	5	4.4895	123.0000	123.0000	125.5935	2.11
1.0	6	5.4914	120.0000	120.0000	122.3278	1.94
1.0	7	6.4927	119.0000	119.0000	119.6771	.57
1.0	8	7.4937	121.0000	121.0000	117.4538	-2.93
1.0	9	8.4945	119.0000	119.0000	115.5439	-2.90
1.0	10	9.4950	113.0000	113.0000	113.8733	.77
5.0	15	12.3441	621.0000	124.2000	124.1949	-.00
5.0	20	17.3893	464.0000	92.8000	92.7992	-.00
5.0	25	22.4312	460.0000	92.0000	90.6471	-1.47
5.0	30	27.4438	425.0000	85.0000	82.2568	-3.23
10.0	40	34.8227	727.0000	72.7000	73.3445	.89
10.0	50	44.8624	613.0000	61.3000	64.9209	5.91
10.0	60	54.8875	575.0000	57.5000	58.9118	2.46
10.0	70	64.9049	537.0000	53.7000	54.3428	1.20
10.0	80	74.9176	515.0000	51.5000	50.7150	-1.52
10.0	90	84.9273	497.0000	49.7000	47.7428	-3.94

PREDICTIONS

LOT	CUM	UNIT	TOT LOT	LOT AVG	CUM TOT	CUM AVG
10.0	10	113.8733	1301.2280	130.1228	1301.23	130.1228
10.0	20	84.1992	1084.9706	108.4971	2386.20	119.3099
10.0	30	79.4467	864.5199	86.4520	3250.72	108.3573
10.0	40	69.0267	733.4452	73.3445	3984.16	99.6041
10.0	50	61.9178	649.2089	64.9209	4633.37	92.6674
10.0	60	56.6669	589.1182	58.9118	5222.49	87.0415
10.0	70	52.5821	543.4276	54.3428	5765.92	82.3703
10.0	80	49.2858	507.1499	50.7150	6273.07	78.4134
10.0	90	46.5522	477.4279	47.7428	6750.50	75.0055
10.0	100	44.2372	452.4907	45.2491	7202.99	72.0299
200.0	300	26.0203	6532.9906	32.6650	13735.98	45.7866

ACTUAL AIRCRAFT MANHOURS
TOTAL FABRICATION



TOTAL FABRICATION

UNIFIED LOG-LINEAR PROGRESS CURVE FORMULATION-ALL CURVES ARE LINEAR
 LINEAR LEAST SQUARES ANALYSIS OF UNIT CURVE-NTH UNITS AT MIDPOINTS
 SLOPES: PERCENT = .766; B=-.38376;B+1= .61624;-1/B+1=-1.62276
 FIRST UNIT COST 145.77862; Y-INTERCEPT OF LIN UNIT CURVE= 89.83397
 R= .956506; R-SQUARED= .914904; RBAR-SQUARED= .910425

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	% DIFF
12.6	21	13.6263	312.0350	24.7647	32.9691	33.13
14.6	35	27.4519	408.1040	27.9523	25.1981	-9.85
19.8	55	44.5928	411.9450	20.8053	20.9176	.54
22.8	78	65.9453	402.9020	17.6711	18.0012	1.87
30.5	108	92.4698	540.7600	17.7298	15.8110	-10.82
34.5	143	125.0009	573.3130	16.6178	14.0838	-15.25
35.8	179	160.2388	616.9750	17.2339	12.8035	-25.71
40.9	220	198.5642	485.8820	11.8798	11.7920	-.74
45.9	265	241.9479	532.2040	11.5949	10.9308	-5.73
49.9	315	289.8547	546.0040	10.9420	10.1987	-6.79
49.7	365	339.7309	503.0280	10.1213	9.5958	-5.19
9.1	374	369.5377	87.1420	9.5760	9.2910	-2.98
9.4	384	378.7855	86.9690	9.2520	9.2033	-.53
9.3	393	388.1382	83.3030	8.9573	9.1176	1.79
9.7	403	397.6349	84.8750	8.7500	9.0334	3.24
7.1	410	406.0431	60.1170	8.4672	8.9611	5.83
6.4	416	412.7954	53.0480	8.2887	8.9046	7.43
7.2	423	419.5934	58.2470	8.0899	8.8490	9.38
7.2	430	426.7933	57.1410	7.9362	8.7914	10.77
6.1	437	433.4460	47.6180	7.8062	8.7393	11.95
8.6	445	440.7913	66.6660	7.7519	8.6832	12.01

SPLIT LEARNING CURVES

EQUATION FORM $Y=e^{(A1+A2*DUM1)} * X^{(B1+B2*DUM1)}$

VARIABLE	COEFFICIENT	T-STATISTIC
A1	3.826407	28.278
A2	1.773740	3.571
B1	-.204730	-6.249
B2	-.370901	-4.236

THE ABSOLUTE VALUE OF THE T-STATISTIC FO A2 AND B2
 EXCEEDS 2.11 THESE VARIABLES, THEN, ARE
 SIGNIFICANT AT THE 95% CONFIDENCE LIMIT AND THERE EXISTS
 A SPLIT LEARNING CURVE

R-SQUARED .975408
 RBAR-SQUARED .9710682
 RESIDUAL SUM OF SQUARFS .0832367

TOTAL FABRICATION (CONT)

FIRST 7 POINTS ON FIRST CURVE, REMAINDER ON SECOND
 ALL VALUES LESS THAN 178.6 PREDICTED FROM FIRST CURVE
 ALL VALUES GREATER THAN 178.6 PREDICTED FROM SECOND CURVE

	T1	Y-INTERCEPT	SLOPE	B-VALUE	B+1
CURVE1	57.7132	45.8976	.868	-.204730	.795269
CURVE2	637.3261	270.4616	.671	-.575631	.424369

LOT	CUM	X (M)	ACTUAL LOT TOT	ACTUAL LOT AVG	CALC LOT AVG	% DIFF
12.6	21	13.7150	312.0350	24.7647	26.8517	8.43
14.6	35	27.5102	408.1040	27.9523	23.2853	-16.70
19.8	55	44.6587	411.9450	20.8053	21.0865	1.35
22.8	78	66.0043	402.9020	17.6711	19.4656	10.15
30.5	108	92.5450	540.7600	17.7298	18.1642	2.45
34.5	143	125.0721	573.3130	16.6178	17.0780	2.77
35.8	179	160.2987	616.9750	17.2339	16.2320	-5.81
40.9	220	198.4968	485.8820	11.8798	12.8661	8.30
45.9	265	241.8782	532.2040	11.5949	11.4824	-.97
49.9	315	289.7858	546.0040	10.9420	10.3480	-5.43
49.7	365	339.6728	503.0280	10.1213	9.4438	-6.69
9.1	374	369.5354	87.1420	9.5760	8.9967	-6.05
9.4	384	378.7838	86.9690	9.2520	8.8696	-4.13
9.3	393	388.1358	83.3030	8.9573	8.7459	-2.36
9.7	403	397.6348	84.8750	8.7500	8.6251	-1.43
7.1	410	406.0412	60.1170	8.4672	8.5218	.65
6.4	416	412.7948	53.0480	8.2887	8.4413	1.84
7.2	423	419.5921	58.2470	8.0899	8.3623	3.37
7.2	430	426.7907	57.1410	7.9362	8.2808	4.34
6.1	437	433.4451	47.6180	7.8062	8.2074	5.14
8.6	445	440.7902	66.6660	7.7519	8.1284	4.86

PREDICTIONS

LOT	CUM	UNIT	TOT LOT	LOT AVG	CUM TOT	CUM AVG
10.0	10	28.9514	360.2009	36.0201	360.20	36.0201
10.0	20	24.9857	264.8923	26.4892	625.09	31.2547
10.0	30	22.9550	237.8551	23.7855	862.95	28.7649
10.0	40	21.6232	221.8394	22.1839	1084.79	27.1197
10.0	50	20.6469	210.6434	21.0643	1295.43	25.9086
10.0	60	19.8835	202.1307	20.2131	1497.56	24.9594
10.0	70	19.2610	195.3155	19.5316	1692.88	24.1840
10.0	80	18.7381	189.6649	18.9665	1882.54	23.5318
10.0	90	18.2891	184.8590	18.4859	2067.40	22.9711
10.0	100	17.8968	180.6919	18.0692	2248.09	22.4809
200.0	300	10.1534	2734.1233	13.6706	4982.22	16.6074

Appendix C
Program Listings

One Split

```
10 DIM T(30)
12 FOR I=1 TO 30
14 READ T(I)
16 NEXT I
17 DATA 12.706,4.303,3.182,2.776,2.571,2.447,2.365,2.306,2.262,2.228
18 DATA 2.201,2.179,2.16,2.145,2.131,2.12,2.11,2.101,2.093,2.086
19 DATA 2.08,2.074,2.069,2.064,2.06,2.056,2.052,2.048,2.045,2.042
40 REM   SPLIT LEARNING CURVE PROGRAM
41 REM   AT LEAST FIVE AND PREFERABLY MORE DATA POINTS NEEDED
42 REM   THIS PROGRAM FIRST REGRESSES DATA TO FIT A LOG-LOG LINEAR
43 REM   UNIT CURVE ACCORDING TO THE UNIFIED LINEAR PROGRESS CURVE
44 REM   FORMULATION. IT PLOTS ALL UNIT COSTS AT THEIR RESPECTIVE
45 REM   MIDPOINTS, INCLUDING LOTS OF ONE. IT THEN FITS THE BEST TWO
46 REM   LOG-LOG LINEAR CURVES TO THE SET OF DATA, CREATING A SPLIT
47 REM   CURVE.
50 REM   USE DATA STATEMENTS BETWEEN LINES 100 AND 1000 TO INPUT DATA
52 REM   ENTER THE NUMBER OF DATA POINTS
55 REM   FIRST PUT IN ALL LOT COST OR HOURLY DATA. NEXT ENTER IN SETS,
56 REM   THEIR RESPECTIVE CUMULATIVE BEGINNING POINTS AND END POINTS
57 REM   OF THE LOTS. THIRDLY, INPUT THE CUMULATIVE QUANTITIES FROM
65 REM   WHICH YOU WISH TO PREDICT
70 REM   DO NOT RESAVE !!!!!!!!!!!!!!!
72 REM   EXAMPLE: 100 DATA 9
74 REM           110 DATA 6.53,6.31,6.14,5.99,5.87,5.59,5.35,5.15,4.96
76 REM           120 DATA 0,50,50,100,100,150,150,200,200,250,250,300
77 REM           121 DATA 300,350,350,400,400,450
78 REM           130 DATA 40,60,80,100,500,1000
80 DIM A7(50),A8(50),Q(50),M(50),Z(50,2),Z1(2,50),Z2(2,2)
82 DIM Z3(2,2),Z4(2,50),B7(2)
84 DIM P(7),R(40,10)
86 DIM E6(50,50),E7(50,50),F1(50),F3(1,50)
88 DIM P1(2),C5(50),C6(50)
90 DIM L(50),Q1(50)
91 DIM R1(1,50)
92 DIM Q2(50)
93 DIM Q3(50)
94 DIM X1(50,50),X4(50,50),Y1(50,50),Y(50),X(50,4),D(50)
100 DATA 15
110 DATA 496.585,397.268,366.787,348.296,335.162,325.054,316.882,310.051
111 DATA 304.199,294.926,282.912,272.419,263.146,254.867,247.415
120 DATA 0,100,100,200,200,300,300,400,400,500,500,600,600,700
121 DATA 700,800,800,900,900,1000,1000,1100,1100,1200,1200,1300,1300
122 DATA 1400,1400,1500
```

```

130 DATA 1,500,1000,1500,1800
500 READ N
505 FILES M2;M3
510 FOR I=1 TO N
520 READ A7(I)
530 NEXT I
535 SCRATCH #2
540 FOR I=1 TO N
550 READ L(I),Q(I)
552 Q1(I)=Q(I)-L(I)
560 A8(I)=A7(I)/Q1(I)
565 A8(I)=LOG(A8(I))
567 WRITE #2,A8(I)
570 NEXT I
572 FOR I=1 TO 1000
573 READ Q2(I)
574 IF Q2(I)=9010000001 THEN 580
576 NEXT I
580 S1=0
590 FOR I=1 TO N
596 RESTORE #2
597 MAT READ #2,Y(N)
610 S1=S1+Y(I)
612 NEXT I
615 S1=S1**2
620 S1=S1/N
630 B5=-.9
640 FOR I=1 TO N
650 M(I)=((Q(I)**(B5+1)-L(I)**(B5+1))/((Q(I)-L(I))*(B5+1)))**(1/B5)
660 Z(I,2)=M(I)
670 Z(I,2)=LOG(Z(I,2))
680 Z(I,1)=1
690 NEXT I
691 SCRATCH #2
692 FOR I=1 TO N
693 FOR J=1 TO 2
694 WRITE #2,Z(I,J)
695 NEXT J
696 NEXT I
697 RESTORE #2
698 MAT READ #2,Z(N,2)
700 MAT Z1=TRN(Z)
710 MAT Z2=Z1*Z
720 MAT Z3=INV(Z2)
730 MAT Z4=Z3*Z1
740 MAT B7=Z4*Y

```

```

750 IF ABS(B7(2)-B5) < ABS(.0000005) GO TO 780
760 B5=B7(2)
770 GO TO 640
780 MAT B8=TRN(B7)
782 MAT R1=B8*Z1
784 MAT R3=R1*Y
786 MAT Y1=TRN(Y)
788 MAT Y2=Y1*Y
795 SCRATCH #2
796 MAT WRITE #2,Y2,R3
797 RESTORE #2
798 MAT READ #2,P1
810 R2=(P1(2)-S1)/(P1(1)-S1)
812: UNIFIED LOG-LINEAR PROGRESS CURVE FORMULATION-ALL CURVES ARE LINEAR
814: LINEAR LEAST SQUARES ANALYSIS OF UNIT CURVE-NTH UNITS AT MIDPOINTS
816: SLOPES: PERCENT =###.###; B=#.#####;B+1=#.#####;-1/B+1=##.#####
817: ACTUAL ACTUAL CALC
818: FIRST UNIT COST #####.#####; Y-INTERCEPT OF LIN UNIT CURVE=#####.#####
820: R=#.#####; R-SQUARED=#.#####; RBAR-SQUARED=#.#####
821 PRINT USING 812
822 PRINT USING 814
823 PRINT USING 816,2**B7(2),B7(2),B7(2)+1,-1/(B7(2)+1)
824 PRINT USING 818,EXP(B7(1))/(B7(2)+1),EXP(B7(1))
825 PRINT USING 820,R2**(1/2),R2,1-((N-1)/(N-2))*(1-R2)
826:-----
827: LOT CUM X(M) LOT TOT LOT AVG LOT AVG % DIFF
828:*****
829: ###.# ##### ###.### ###.### ###.### ###.### ###.###
830 PRINT USING 826
831 PRINT USING 817
832 PRINT USING 827
834 PRINT USING 828
838 FOR I=1 TO N
839 C6(I)=EXP(B7(1))*(M(I))**B7(2)
840 C5(I)=A7(I)/Q1(I)
841 PRINT USING 829,Q1(I),Q(I),M(I),A7(I),C5(I),C6(I),((C6(I)-C5(I))/C5(I))*1
00
842 NEXT I
898 M1=1
899 GO TO 1030
900 B5=R(U,2)
902 B6=R(U,2)+R(U,4)
903 A5=R(U,1)
904 A6=R(U,1)+R(U,3)
905 A5=EXP(A5)
906 A6=EXP(A6)
907 B9=R(U,2)

```

```

909 IF M(U+2)<N8 GO TO 3400
910 IF M(U+1)>N8 GO TO 3400
915 FOR I=1 TO N
920 IF Q(I) <N8 GO TO 950
930 B5=R(U,2)
940 M(I)=((Q(I)**(B5+1)-L(I)**(B5+1))/((Q(I)-L(I))*(B5+1)))**(1/B5)
945 GO TO 1000
950 IF L(I)<N8 GO TO 980
960 B5=R(U,2)+R(U,4)
970 GO TO 940
980 S2=((A5)*(N8**(B5+1)-L(I)**(B5+1)))/((B5+1)*Q1(I))
990 S3=((A6)*(Q(I)**(B6+1)-N8**(B6+1)))/((B6+1)*Q1(I))
996 M(I)=((S2+S3)/A5)**(1/B5)
997 IF M(I)<N8 GO TO 1000
998 M(I)=((S2+S3)/A6)**(1/B6)
1000 NEXT I
1030 DIM C(4)
1040 U=1
1080 FOR I=1 TO N
1090 X(I,2)=M(I)
1100 X(I,1)=1
1105 D(I)=1
1110 NEXT I
1115 FOR I=1 TO N
1120 M(I)=X(I,2)
1125 NEXT I
1200 FOR K=1 TO N-3
1220 FOR I=1 TO 1+K
1230 D(I)=0
1240 NEXT I
1400 FOR I=1 TO N
1405 X(I,2)=M(I)
1410 X(I,3)=D(I)
1412 X(I,4)=X(I,2)
1420 NEXT I
1425 SCRATCH #2
1431 FOR I=1 TO N
1432 FOR J=1 TO 4
1434 WRITE #2,X(I,J);
1435 NEXT J
1436 NEXT I
1439 RESTORE #2
1440 MAT READ #2,X(N,4)
1450 FOR I=1 TO N
1452 X(I,2)=LOG(X(I,2))
1453 X(I,4)=D(I)*LOG(X(I,4))

```

```

1470 NEXT I
1500 MAT X1=TRN(X)
1510 MAT X2=X1*X
1520 MAT X3=INV(X2)
1530 MAT X4=X3*X1
1540 MAT E=Y4*Y
1545 MAT Y1=TRN(Y)
1550 MAT E1=Y1*Y
1560 MAT E2=X1*Y
1565 MAT B1=TRN(B)
1570 MAT E3=B1*E2
1575 MAT E=E1-E3
1580 MAT E4=(2)*E3
1585 MAT E5=E1-E4
1590 MAT E6=B1*X1
1595 MAT E7=TRN(E6)
1600 MAT E8=E6*E7
1605 MAT E9=E5+E8
1610 MAT F1=X*B
1615 MAT F2=E6*F1
1620 MAT F3=E6*Y
1625 MAT F3=(2)*F3
1630 MAT F4=F3-E1
1635 MAT F5=E6*F1
1640 MAT F6=F3-F5
1710 SCRATCH #1
1720 MAT WRITE #1,B,E9,F6,E1
1723 RECTORE #1
1725 MAT READ #1,P
1730 FOR I=1 TO 5
1735 R(K,I)=P(I)
1740 NEXT I
1750 R(K,6)=(P(6)-S1)/(P(7)-S1)
1760 R(K,7)=R(K,2)/((R(K,5)/(N-4)).5*(X3(2,2)).5)
1770 R(K,8)=R(K,3)/((R(K,5)/(N-4)).5*(X3(3,3)).5)
1780 R(K,9)=R(K,4)/((R(K,5)/(N-4)).5*(X3(4,4)).5)
1785 R(K,10)=R(K,1)/((R(K,5)/(N-4)).5*(X3(1,1)).5)
1790 IF K=1 THEN 1820
1800 IF R(K,5)>R(U,5) THEN 1820
1810 U=K
1820 NEXT K
1900 N8=EXP((-R(U,3))/R(U,4))
1910 N8=INT(1000*N8+.5)/1000
1920 M1=M1+1
1925 IF M1=30 GO TO 1980
1926 PRINT M1,B9,B6

```

```

1927 PRINT R(K,5),R(K,6)
1930 IF A5=0 GO TO 900
1931 IF M1=30 GO TO 1990
1932 IF ABS(R(U,2)-B9) >.0000005 GO TO 900
1933 IF ABS(R(U,2)+R(U,4)-B6) >.0000005 GO TO 900
1980 IF M(U+2) <N8 GO TO 3500
1981 IF M(U+1) <N8 GO TO 3500
1985 N9=N8
1990 PRINT
1991 PRINT
1992 PRINT
1993 PRINT
2000 PRINT "          SPLIT LEARNING CURVES"
2005 PRINT
2010 PRINT " EQUATION FORM  $Y=e^{(A1+A2*DUM1)} * X^{(B1+B2*DUM1)}$ "
2015 PRINT
2019:      'LLLLLLLL      'LLLLLLLLLLLL      'LLLLLLLLLLLL
2020:      'LLLLLLLL      #####.#####      #####.###
2021 U1$="VARIABLE"
2022 U2$="COEFFICIENT"
2023 U3$="T-STATISTIC"
2025 PRINT USING 2019,U1$,U2$,U3$
2029 U$=" A1"
2030 PRINT USING 2020,U$,R(U,1),R(U,10)
2039 U$=" A2"
2040 PRINT USING 2020,U$,R(U,3),R(U,8)
2049 U$=" B1"
2050 PRINT USING 2020,U$,R(U,2),R(U,7)
2059 U$=" B2"
2060 PRINT USING 2020,U$,R(U,4),R(U,9)
2070 PRINT
2071 IF ABS(R(U,8)) <T(N-4) GO TO 3340
2072 IF ABS(R(U,9)) >T(N-4) GO TO 3340
2073 PRINT "THE ABSOLUTE VALUE OF THE T-STATISTIC FO A2 AND B2"
2074 PRINT "EXCEEDS ";T(N-4) "THESE VARIABLES, THEN, ARE"
2075 PRINT "SIGNIFICANT AT THE 95% CONFIDENCE LIMIT AND THERE EXISTS"
2077 PRINT " A SPLIT LEARNING CURVE"
2079 PRINT
2080 PRINT " R-SQUARED ";R(U,6)
2085 PRINT " RBAR-SQUARED ";1-((N-1)/(N-4))*(1-R(U,6))
2090 PRINT " RESIDUAL SUM OF SQUARES ";R(U,5)
2100 PRINT
2110 PRINT " FIRST ";U+1;" POINTS ON FIRST CURVE,REMAINDER ON SECOND"
2120 PRINT " ALL VALUES LESS THAN ";N9;" PREDICTED FROM FIRST CURVE"
2130 PRINT " ALL VALUES GREATER THAN ";N9;" PREDICTED FROM SECOND CURVE"
2140 PRINT

```

```

2150:          'LL          'LLLLLLLLLLLLL          'LLL          'LLLLLL          'LLL
2160:  'LLLLLL  ###.####  ###.####  #.###  ##.#####  ##.#####
2170  U1$="T1"
2172  U2$="SLOPE"
2174  U3$="B-VALUE"
2175  U6$="Y-INTERCEPT"
2176  U4$="CURVEL"
2178  U5$="CURVE2"
2179  U7$="B+1"
2180  PRINT USING 2150,U1$,U6$,U2$,U3$,U7$
2190  PRINT USING 2160,U4$,A5/(B9+1),A5,EXP(B9*LOG(2.0)),B9,B9+1
2200  PRINT USING 2160,U5$,A6/(B6+1),A6,EXP(B6*LOG(2.0)),B6,B6+1
2500:          'LLLLL          'LLLLLLLLLLLLLLLLL
2510:          #####.#  #####.###
2590  PRINT
2595  PRINT
2600  PRINT USING 817
2602  PRINT USING 827
2603  PRINT USING 828
2605  FOR I=1 TO N
2610  IF Q(I)>N9 GO TO 2640
2630  PRINT USING 829,Q1(I),Q(I),M(I),A7(I),A7(I)/Q1(I),A5*M(I)**B9,((A5*M(I)*
*B9-A7(I)/Q1(I))/(A7(I)/Q1(I)))*100
2635  GO TO 2650
2640  PRINT USING 829,Q1(I),Q(I),M(I),A7(I),A7(I)/Q1(I),A6*(M(I)**B6,((A6*M(I)
)*B6-A7(I)/Q1(I))/(A7(I)/Q1(I)))*100
2650  NEXT I
2900  PRINT
2920:          PREDICTIONS
2925  PRINT USING 2920
2930  PRINT USING 826
2940:  LOT      CUM      UNIT      TOT LOT      LOT AVG      CUM TOT      CUM AVG
2945  PRINT USING 2940
2947  PRINT USING 828
2950:  #####.#  #####  ###.####  #####.####  ###.####  #####.##  ###.####
2955  I=1
2959  N1=B9+1
2960  N2=B6+1
2961  N3=A5/N1
2962  N4=A6/N2
2975  I=1
3000  C=Q2(I)
3005  IF C=9010000001 GO TO 9000
3006  DATA 9010000001
3010  IF I=1 GO TO 3040
3020  Q3(I)=C-Q2(I-1)
3030  GO TO 3050
3040  Q3(1)=Q2(1)

```

```

3050 IF C>N9 GO TO 3160
3060 C1=N3*C**N1-N3*(C-1)**N1
3080 C2=N3*C**N1
3090 IF I=1 GO TO 3120
3100 C3=C2-N3*Q2(I-1)**N1
3110 GO TO 3130
3120 C3=C2
3130 C4=C3/Q3(I)
3140 C5=C2/Q2(I)
3150 GO TO 3300
3160 C1=N4*C**N2-N4*(C-1)**N2
3170 D3=N4*C**N2-N4*N9**N2
3180 D4=N3*N9**N1
3190 C2=D3+D4
3200 C5=C2/Q2(I)
3210 IF I=1 GO TO 3260
3220 IF Q2(I-1) <N9 GO TO 3280
3230 C3=C2-(N4*Q2(I-1)**N2-N4*N9**N2+N3*N9**N1)
3240 C4=C3/Q3(I)
3250 GO TO 3300
3260 C3=C2
3270 C4=C3/Q2(I)
3275 GO TO 3300
3280 C3=C2-N3*Q2(I-1)**N1
3290 C4=C3/Q3(I)
3300 PRINT USING 2950,Q3(I),Q2(I),C1,C3,C4,C2,C5
3310 I=I+1
3320 GO TO 3000
3330 GO TO 900
3340 PRINT "THE ABSOLUTE VALUE OF THE T-STATISTIC FOR A2 AND/OR"
3342 PRINT "B2 DOES NOT EXCEED ";T(N-4) "THESE VARIABLES ARE NOT"
3344 PRINT "SIGNIFICANT AT THE 95% CONFIDENCE LIMIT AND THEREFORE,"
3346 PRINT "A SPLIT LEARNING CURVE DOES NOT EXIST."
3350 GO TO 2079
3400 FOR I=1 TO N
3410 IF I>U+1 GO TO 3450
3420 M(I)=((Q(I)**(B9+1)-L(I)**(B9+1))/(Q1(I)*(B9+1)))**(1/B9)
3430 GO TO 3480
3450 M(I)=((Q(I)**(B6+1)-L(I)**(B6+1))/(Q1(I)*(B6+1)))**(1/B6)
3480 NEXT I
3490 GO TO 1030
3500 N9=Q(U+1)
3510 GO TO 1990
9000 END

```

Two Splits

```
10 DIM T(30)
12 FOR I=1 TO 30
14 READ T(I)
16 NEXT I
17 DATA 12.706,4.303,3.182,2.776,2.571,2.447,2.365,2.306,2.262,2.228
18 DATA 2.201,2.179,2.16,2.145,2.131,2.12,2.11,2.101,2.093,2.086
19 DATA 2.08,2.074,2.069,2.064,2.06,2.056,2.052,2.048,2.045,2.042
40 REM SPLIT LEARNING CURVE PROGRAM
45 REM AT LEAST SEVEN AND PREFERABLY MORE DATA POINTS NEEDED
50 REM USE DATA STATEMENTS BETWEEN LINES 100 AND 1000 TO INPUT DATA
52 REM ENTER THE NUMBER OF DATA POINTS THEN
55 REM FIRST PUT IN ALL COST OR HOUR DATA(LOT TOTALS),NEXT THEIR RESPECTIVE
60 REM CUMULATIVE QUANTITIES (BEGINNING AND END) ,THIRDLY, CUMULATIVE
65 REM QUANTITIES FROM WHICH YOU WISH TO PREDICT
80 DIM A7(50),A8(50),Q(50),M(50),Z(50,2),Z1(2,50),Z2(2,2),Z3(2,2)
81 DIM Z4(2,50),B7(2),P(9),X1(50,50),X4(50,50),Y1(50,50),Y(50)
82 DIM X(50,6),D(50),P1(2),C5(50),C6(50),L(50),Q1(50),R4(1,50)
83 DIM Q2(50),Q3(50),R(40,14),R1(40,14),D2(50)
84 DIM P7(50),T7(50)
85 DIM R2(40,14)
86 DIM E6(50,50),E7(50,50)
87 DIM F1(50)
88 DIM F3(1,50)
89 DIM H1(1,4)
90 DIM C1(50),C3(50),C4(50),C2(50)
100 DATA 24
110 DATA 17.5,14.0,13.5,12.5,13.0,12.5,13.0,14.5,11.0,13.0,57.5,48.5
111 DATA 41.3,33.8,58.0,48.0,40.0,37,37,36,33,32,31.5,30.5
120 DATA 0,1,1,2,2,3,3,4,4,5,5,6,6,7,7,8,8,9,9,10,10,15,15,20,20,25
121 DATA 25,30,30,40,40,50,50,60,60,70,70,80,80,90,90,100,100,110,110,120,120
,130
130 DATA 10,20,30,40,50,60,70,80,90,100,300
500 READ N
505 FILES M2;M3
510 FOR I=1 TO N
520 READ A7(I)
530 NEXT I
535 SCRATCH #2
540 FOR I=1 TO N
550 READ L(I),Q(I)
552 Q1(I)=Q(I)-L(I)
560 A8(I)=A7(I)/Q1(I)
565 A8(I)=LOG(A8(I))
567 WRITE #2,A8(I)
```

```

570 NEXT I
572 FOR I=2 TO 1000
573 READ Q2(I)
574 IF Q2(I)=9010000001 THEN 580
576 NEXT I
580 S1=0
590 FOR I=1 TO N
596 RESTORE #2
597 MAT READ #2,Y(N)
610 S1=S1+Y(I)
612 NEXT I
615 S1=S1**2
620 S1=S1/N
630 B5=-.9
640 FOR I=1 TO N
650 M(I)=((Q(I)**(B5+1)-L(I)**(B5+1))/(Q1(I)*(B5+1)))**(1/B5)
660 Z(I,2)=M(I)
670 Z(I,2)=LOG(Z(I,2))
680 Z(I,1)=1
690 NEXT I
691 SCRATCH #2
692 FOR I=1 TO N
693 FOR J=1 TO 2
694 WRITE #2,Z(I,J)
695 NEXT J
696 NEXT I
697 RESTORE #2
698 MAT READ #2,Z(N,2)
700 MAT Z1=TRN(Z)
710 MAT Z2=Z1*Z
720 MAT Z3=INV(Z2)
730 MAT Z4=Z3*Z1
740 MAT B7=Z4*Y
750 IF ABS(B7(2)-B5) < .0000005 GO TO 780
760 B5=B7(2)
770 GO TO 640
780 MAT B8=TRN(B7)
782 MAT R4=B8*Z1
784 MAT R3=R4*Y
786 MAT Y1=TRN(Y)
788 MAT Y2=Y1*Y
789 MAT H1=B8*Z2
790 MAT H2=H1*B7
791 MAT H3=(2)*R3
792 MAT H4=Y2-H3
793 MAT H5=H4+H2

```

```

794 MAT H6=Y2-R3
795 SCRATCH #2
796 MAT WRITE #2,Y2,R3
797 RESTORE #2
798 MAT READ #2,P1
810 R2=(P1(2)-S1)/(P1(1)-S1)
812: UNIFIED LOG-LINEAR PROGRESS CURVE FORMULATION-ALL CURVES ARE LINEAR
814: LINEAR LEAST SQUARES ANALYSES OF UNIT CURVE-NTH UNITS AT MIDPOINTS
816: SLOPES; PERCENT=###.###; B=#.#####; B+1=#.#####;-1/B+1=##.####
817:
      ACTUAL    ACTUAL    CALC
818: FIRST UNIT COST #####.#####; Y-INT. OF LIN UNIT CURVE #####.#####
820: R=#.#####; R-SQUARED=#.#####; REAR-SQUARED=#.#####
821 PRINT USING 812
822 PRINT USING 814
823 PRINT USING 816,2**B7(2),B7(2),B7(2)+1,-1/(B7(2)+1)
824 PRINT USING 818,EXP(B7(1))/(B7(2)+1),EXP(B7(1))
826:
-----
827:  LOT      CUM      X(M)      LOT TOT      LOT AVG      LOT AVG      %DIFF
828:  *****
829:  #####.#  #####  #####.####  #####.####  ###.####  ###.####  ###.##
830 PRINT USING 826
831 PRINT USING 817
832 PRINT USING 827
834 PRINT USING 828
838 FOR I= 1 TO N
839 C6(I)=EXP(B7(1))*(M(I)**B7(2))
840 C5(I)=A7(I)/Q1(I)
841 PRINT USING 829,Q1(I),Q(I),M(I),A7(I),C5(I),C6(I),((C6(I)-C5(I))/C5(I))*1
00
842 NEXT I
843 M1=1
845 GO TO 1030
900 B5=R1(U,2)
901 B6=B5+R1(U,4)
902 B9=B6+R1(U,6)
903 A4=EXP(R1(U,1))
904 A5=EXP(R1(U,1)+R1(U,3))
905 A6=EXP(R1(U,1)+R1(U,3)+R1(U,5))
915 GO SUB 3600
1030 DIM C(6)
1040 U=1
1080 FOR I=1 TO N
1090 X(I,2)=M(I)
1100 X(I,1)=1
1110 NEXT I
1115 V=1
1170 FOR L=1 TO N-5

```

```
1171 FOR I=1 TO N
1172 D2(I)=1
1173 NEXT I
1175 FOR I=1 TO N-(L+1)
1180 D2(I)=0
1185 NEXT I
1200 FOR K=1 TO N-(4+L)
1205 FOR I=1 TO N
1206 D(I)=1
1207 NEXT I
1220 FOR I=1 TO 1+K
1230 D(I)=0
1240 NEXT I
1400 FOR I=1 TO N
1405 X(I,2)=M(I)
1410 X(I,3)=D(I)
1412 X(I,4)=X(I,2)
1414 X(I,5)=D2(I)
1416 X(I,6)=X(I,2)
1420 NEXT I
1425 SCRATCH #2
1431 FOR I=1 TO N
1432 FOR J=1 TO 6
1434 WRITE #2,X(I,J);
1435 NEXT J
1436 NEXT I
1439 RESTORE #2
1440 MAT READ #2,X(N,6)
1450 FOR I=1 TO N
1452 X(I,2)=LOG(X(I,2))
1453 X(I,4)=D(I)*LOG(X(I,4))
1460 X(I,6)=D2(I)*LOG(X(I,6))
1470 NEXT I
1500 MAT X1=TRN(X)
1510 MAT X2=X1*X
1520 MAT X3=INV(X2)
1530 MAT X4=X3*X1
1540 MAT B=X4*Y
1545 MAT Y1=TRN(Y)
1550 MAT E1=Y1*Y
1560 MAT E2=X1*Y
1565 MAT B1=TRN(B)
1570 MAT E3=B1*E2
1580 MAT E4=(2)*E3
1585 MAT E5=E1-E4
1590 MAT E6=B1*X1
```

```

1595 MAT E7=TRN(E6)
1600 MAT E8=E6*E7
1605 MAT E9=E5+E8
1640 MAT F6=E4-E8
1710 SCRATCH #1
1720 MAT WRITE #1,B,E9,F6,E1;
1723 RESTORE #1
1725 MAT READ #1,P
1730 FOR I=1 TO 7
1735 R(K,I)=P(I)
1740 NEXT I
1750 R(K,8)=(P(8)-S1)/(P(9)-S1)
1755 IF R(K,7)=0 GO TO 7000
1760 R(K,9)=R(K,2)/((R(K,7)/(N-4))^.5*(X3(2,2))^.5)
1770 R(K,10)=R(K,3)/((R(K,7)/(N-4))^.5*(X3(3,3))^.5)
1780 R(K,11)=R(K,4)/((R(K,7)/(N-4))^.5*(X3(4,4))^.5)
1782 R(K,12)=R(K,5)/((R(K,7)/(N-4))^.5*(X3(5,5))^.5)
1783 R(K,13)=R(K,6)/((R(K,7)/(N-4))^.5*(X3(6,6))^.5)
1785 R(K,14)=R(K,1)/((R(K,7)/(N-4))^.5*(X3(1,1))^.5)
1790 IF K=1 THEN 1820
1791 IF R2(V,7)=0 THEN 1800
1792 IF R(K,7)>R2(V,7) THEN 1820
1793 GO TO 1810
1800 IF R(K,7)>R(V,7) THEN 1820
1810 V=K
1815 R2(V,7)=R(V,7)
1820 NEXT K
1825 FOR Q=1 TO 14
1830 R1(L,Q)=R(V,Q)
1835 NEXT Q
1850 IF L=1 THEN 1890
1859 IF R1(L,7)=R1(U,7) THEN 1890
1860 IF R1(L,7)>R1(U,7) THEN 1890
1865 U=L
1890 NEXT L
1900 N8=EXP((-R1(U,3))/R1(U,4))
1920 N9=EXP(-R1(U,5)/R1(U,6))
1935 IF M1=2 GO TO 1980
1940 M1=M1+1
1950 IF A5=0 GO TO 900
1955 IF ABS(R1(U,2)-B5)>.0000005 GO TO 900
1960 IF ABS(R1(U,2)+R1(U,4)-B6)>.0000005 GO TO 900
1965 IF ABS(R1(U,2)+R1(U,4)+R1(U,6)-B9)>.0000005 GO TO 900
1980 B5=R1(U,2)
1981 B6=B5+R1(U,4)
1982 B9=B6+R1(U,6)

```

```

1983 A4=EXP(R1(U,1))
1984 A5=EXP(R1(U,1)+R1(U,3))
1985 A6=EXP(R1(U,1)+R1(U,3)+R1(U,5))
1990 GO SUB 3600
1991 PRINT
1992 PRINT
1993 PRINT
1994 PRINT
2000 PRINT "          SPLIT LEARNING CURVES"
2005 PRINT
2010 PRINT " EQUATION FORM  $Y=e^{(A1+A2*DUM1+A3*DUM2)*X}$  (B1+B2*DUM1+B3*DUM2) "
2015 PRINT
2019:      'LLLLLLLL      'LLLLLLLLLLLL      'LLLLLLLLLLLL
2020:      'LLLLLLLL      #####.#####      #####.##
2021 U1$="VARIABLE"
2022 U2$="COEFFICIENT"
2023 U3$="T-STATISTIC"
2025 PRINT USING 2019,U1$,U2$,U3$
2029 U$=" A1"
2030 PRINT USING 2020,U$,R1(U,1),R1(U,14)
2039 U$=" A2"
2040 PRINT USING 2020,U$,R1(U,3),R1(U,10)
2042 U$=" A3"
2044 PRINT USING 2020,U$,R1(U,5),R1(U,12)
2049 U$=" B1"
2050 PRINT USING 2020,U$,R1(U,2),R1(U,9)
2059 U$=" B2"
2060 PRINT USING 2020,U$,R1(U,4),R1(U,11)
2062 U$=" B3"
2064 PRINT USING 2020,U$,R1(U,6),R1(U,13)
2070 PRINT
2072 PRINT " IF ABSOLUTE VALUE OF T-STATISTIC EXCEEDS ";T(N-6)
2074 PRINT " FOR THE DUMMY VARIABLES A2,A3,B2,AND B3, THEN THESE VARIABLES"
2075 PRINT " ARE SIGNIFICANT AT 95% CONFIDENCE LIMIT AND THERE EXISTS"
2077 PRINT " A TWICE SPLIT LEARNING CURVE"
2079 PRINT
2080 PRINT " R-SQUARED ";R1(U,8)
2090 PRINT " RESIDUAL SUM OF SQUARES ";R1(U,7)
2100 PRINT
2110 PRINT "FIRST ";V+1;"POINTS ON FIRST CURVE, NEXT ";N-V-U-2
2115 PRINT "POINTS ON SECOND, NEXT ";U+1;" ON THIRD"
2120 PRINT " ALL VALUES LESS THAN ";N8;" PREDICTED FROM FIRST CURVE"
2125 PRINT " ALL VALUES LESS THAN ";N9;" BUT GREATER OR EQUAL TO ";N8
2127 PRINT " ARE PREDICTED FROM THE SECOND CURVE, WHILE"
2130 PRINT " ALL VALUES GREATER THAN ";N9;" PREDICTED FROM THIRD CURVE"
2135 PRINT "THE PROGRAM HAS COMPLETED ";M1;" ITERATIONS"

```

```

2140 PRINT
2150:      'LL      'LLLLLLLLLLLL 'LLLLL 'LLLLLLL      'LLL
2160: 'LLLLLL ###.####      ###.#### #.### ##.##### ##.#####
2170 U1$="T1"
2172 U2$="SLOPE"
2174 U3$="B-VALUE"
2175 U6$="Y-INTERCEPT"
2176 U4$="CURVEL"
2177 U7$="B+1"
2178 U5$="CURVE2"
2179 Q9$="CURVE3"
2180 PRINT USING 2150,U1$,U6$,U2$,U3$,U7$
2190 PRINT USING 2160,U4$,A4/(B5+1),A4,EXP(B5*LOG(2.0)),B5,B5+1
2200 PRINT USING 2160,U5$,A5/(B6+1),A5,EXP(B6*LOG(2.0)),B6,B6+1
2210 PRINT USING 2160,Q9$,A6/(B9+1),A6,EXP(B9*LOG(2.0)),B9,B9+1
2500:      'LLLL      'LLLLLLLLLLLLLLLLL
2510:      ###.#      #####.###
2590 PRINT
2591 PRINT
2592 PRINT
2600 PRINT USING 817
2602 PRINT USING 827
2603 PRINT USING 828
2605 FOR I=1 TO N
2610 P7(I)=((T7(I)-EXP(A8(I)))/EXP(A8(I)))*100
2620 PRINT USING 829,Q1(I),Q(I),M(I),A7(I),EXP(A8(I)),T7(I),P7(I)
2630 NEXT I
2900 PRINT
2903 PRINT
2905 PRINT
2910:      PREDICTIONS
2915 N8=INT(N8+.5)
2916 N9=INT(N9+.5)
2920 PRINT USING 2910
2930 PRINT USING 826
2940:  LOT      CUM      UNIT      TOT LOT      LOT AVG      CUM TOT      CUM A
VG
2945 PRINT USING 2940
2947 PRINT USING 828
2950:  ###.#  #####  ###.####  #####.####  ##.####  #####.##  ###.####
2959 M5=B5+1
2960 M6=B6+1
2961 M9=B9+1
2962 N4=A4/M5
2963 N5=A5/M6
2964 N6=A6/M9
2975 I=2

```

```

2976 Q2(I)=0
2977 C2(I)=0
2980 F1=N4*N8**M5
2990 F2=N5*N8**M6
3000 F3=N5*N9**M6
3010 F4=N6*N9**M9
3020 IF Q2(I)=9010000001 GO TO 9000
3030 Q3(I)=Q2(I)-Q2(I-1)
3040 DATA 9010000001
3050 IF Q2(I)>N9 GO TO 3130
3060 IF Q2(I)>N8 GO TO 3100
3070 C2(I)=N4*Q2(I)**M5
3080 C1(I)=C2(I)-N4*(Q2(I)-1)**M5
3090 GO TO 3150
3100 C2(I)=N5*Q2(I)**M6-F2+F1
3110 C1(I)=C2(I)-N5*(Q2(I)-1)**M6+F2-F1
3120 GO TO 3150
3130 C2(I)=N6*Q2(I)**M9-F4+F3-F2+F1
3140 C1(I)=C2(I)-N6*(Q2(I)-1)**M9+F4-F3+F2-F1
3150 C5(I)=C2(I)/Q2(I)
3160 C3(I)=C2(I)-C2(I-1)
3170 C4(I)=C3(I)/Q3(I)
3500 PRINT USING 2950,Q3(I),Q2(I),C1(I),C3(I),C4(I),C2(I),C5(I)
3510 I=I+1
3520 GO TO 3000
3600 GO TO 3720
3601 FOR I=1 TO V
3625 M(I)=((Q(I)**(B5+1)-L(I)**(B5+1))/(Q1(I)*(B5+1)))** (1/B5)
3627 T7(I)=A4*M(I)**B5
3629 NEXT I
3630 FOR I=V+2 TO N-U-2
3650 M(I)=((Q(I)**(B6+1)-L(I)**(B6+1))/(Q1(I)*(B6+1)))** (1/B6)
3652 T7(I)=A5*M(I)**B6
3655 NEXT I
3660 FOR I=N-U TO N
3700 M(I)=((Q(I)**(B9+1)-L(I)**(B9+1))/(Q1(I)*(B9+1)))** (1/B9)
3705 T7(I)=A6*M(I)**B9
3710 NEXT I
3715 RETURN
3720 IF Q(V+1) < N8 GO TO 3900
3730 IF L(V+1) > N8 GO TO 3900
3740 S2=(A4*(N8** (B5+1)-L(V+1)** (B5+1)))/(B5+1)
3750 S3=(A5*(Q(V+1)** (B6+1)-N8** (B6+1)))/(B6+1)
3760 M(V+1)=((S2+S3)/(A4*Q1(V+1)))** (1/B5)
3765 T7(V+1)=A4*M(V+1)**B5
3770 IF M(V+1) < N8 GO TO 3790

```

END

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