

AD-A097 166

INDIANA UNIV AT BLOOMINGTON DEPT OF MATHEMATICS

F/G 12/1

RANK-ORDER TESTS FOR THE PARALLELISM OF SEVERAL REGRESSION SURF--ETC(U)

FEB 81 C CHIANG, M L PURI

AFOSR-76-2927

UNCLASSIFIED

AFOSR-TR-81-0205

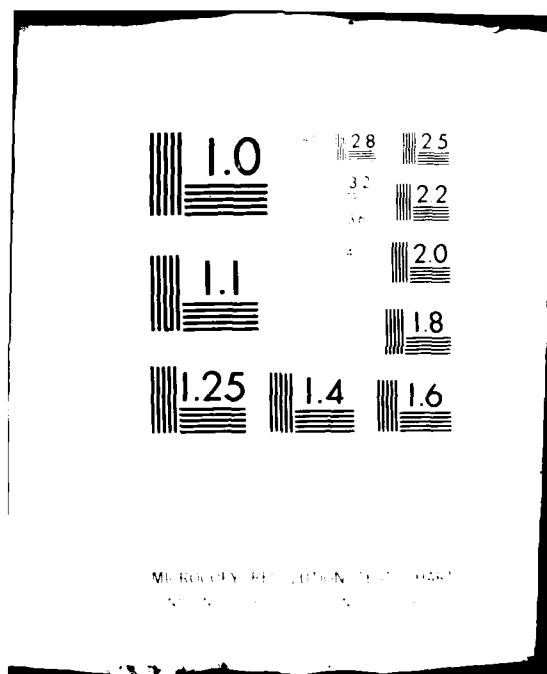
NL

1 OF 1

AD-A097 166



END
DATE
FILMED
5-81
DTIC



18 AFOSR-TR-81-0205

LEVEL II

(4)

AD A097166

RANK-ORDER TESTS FOR THE PARALLELISM
OF SEVERAL REGRESSION SURFACES*

1-AFOSR-76-2927

By Ching-Yuan/Chiang and Madan L. Puri

University of Louisville and Indiana University

FEBRUARY 1981

For testing the hypothesis that several ($s \geq 2$) linear regression surfaces $X_{ki} = \alpha_k + \beta_k C_{ki} + \gamma_k Z_{ki}$ ($k = 1, \dots, s$) are parallel to one another, i.e., $\beta_1 = \dots = \beta_s$, a class of rank-order tests are considered. The tests are shown to be asymptotically distribution-free, and their asymptotic efficiency relative to the general likelihood ratio test is derived. Asymptotic optimality in the sense of Wald is also discussed.

DTIC
ELECTE
S D
APR 1 1981
D

*Work partially supported by the Air Force Office of Scientific Research, AFSC, USAF, under Grant No. AFOSR-76-2927. Reproduction in whole or in part permitted for any purpose of the United States Government.

AMS 1970 subject classification. Primary 62G10; Secondary 62J05.

Key words and phrases. Asymptotic distribution, asymptotic efficiency, asymptotic optimality, linear regression, parallelism, rank-order tests, regression surfaces.

61 3 27

18.1
402522

DTIC FILE COPY

0. Introduction. Consider s (≥ 2) linear regression models

$$(0.1) \quad x_{ki} = \alpha_k + \beta_k c_{ki} + z_{ki}, \quad i = 1, \dots, n_k; \quad k = 1, \dots, s$$

where, for each $k = 1, \dots, s$, α_k is the (unknown) intercept,

$$(0.2) \quad \beta_k = (\beta_{k1}, \dots, \beta_{kq})$$

is a q -dimensional vector of unknown regression parameters,

$$(0.3) \quad c_{ki} = (c_{k1i}, \dots, c_{kqi})$$

is a q -dimensional vector of known regression constants for each $i = 1, \dots, n_k$, and the z_{ki} are all independent (error) random variables with the same (but unknown) continuous distribution

$$(0.4) \quad F(x) = P(Z_{ki} \leq x), \quad k = 1, \dots, s; \quad i = 1, \dots, n_k.$$

A problem of interest is that of testing whether the s regression surfaces are parallel to one another, i.e.,

$$(0.5) \quad H_0 : \beta_1 = \dots = \beta_s = \beta \text{ (unknown)}$$

vs.

$$H : \beta_k \neq \beta_j \text{ for some } 1 \leq k \neq j \leq s.$$

Accession For
NTIS GRA&I
ETIC TAB
Unannounced
Justification

By
Date

10

11

A

For the special case $q = 1$, i.e., testing the parallelism of several regression lines, Sen (1969) has proposed a class of rank order tests. In the present paper we study the problem in the general case $q \geq 1$. Preliminary notations and assumptions are given in

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE: THIS DOCUMENT IS UNCLASSIFIED
THIS DOCUMENT IS UNCLASSIFIED AND IS
APPROPRIATE FOR RELEASE UNDER E.O. 13526 (7b).
Distribution is unlimited.
A. D. BROWN
Technical Information Officer

section 1. In section 2 a class of asymptotically distribution-free aligned rank-order tests are proposed. These are univariate counterparts of a multivariate problem briefly mentioned in Sen and Puri (1977) but not solved. The asymptotic distribution of the test statistics is derived in section 3. In section 4 we derive the asymptotic relative efficiency of the proposed tests with respect to the general likelihood ratio test of the same problem. Finally, in section 5, asymptotic optimality in the sense of Wald (1942) is discussed.

1. Preliminary Notations and Assumptions. For each $k = 1, \dots, s$ let

$$(1.1) \quad \bar{c}_{kn_k} = n_k^{-1} \sum_{i=1}^{n_k} c_{ki} = (\bar{c}_{k1n_k}, \dots, \bar{c}_{kqn_k})'$$

where

$$(1.2) \quad \bar{c}_{kmn_k} = n_k^{-1} \sum_{i=1}^{n_k} c_{kmi}, \quad m = 1, \dots, q.$$

We assume that the $q \times q$ symmetric matrices

$$(1.3) \quad M_{kn_k} = \sum_{i=1}^{n_k} (c_{ki} - \bar{c}_{kn_k})(c_{ki} - \bar{c}_{kn_k})', \quad k = 1, \dots, s$$

are positive definite and that the limiting matrices

$$(1.4) \quad M_k = \lim_{n_k \rightarrow \infty} n_k^{-1} M_{kn_k}, \quad k = 1, \dots, s$$

exist and are positive definite. Simplifying some of Jurečková's

(1971) conditions on the regression constants, we also assume that each c_{ki} can be expressed as a difference

$$(1.5) \quad \begin{aligned} c_{ki} &= c_{ki(1)} - c_{ki(2)} \\ c_{ki(j)} &= (c_{kli(j)}, \dots, c_{kqi(j)})' \end{aligned}$$

where, for each $k = 1, \dots, s$, $m = 1, \dots, q$ and $j = 1, 2$, $c_{kmi(j)}$ is nondecreasing in i , and the $c_{kmi(j)}$'s satisfy

$$(1.6) \quad \begin{aligned} \lim_{n_k \rightarrow \infty} n_k^{-1} \max_{1 \leq i \leq n_k} [c_{kmi(j)} - \bar{c}_{kmn_k(j)}]^2 &= 0, \\ \bar{c}_{kmn_k(j)} &= n_k^{-1} \sum_{i=1}^{n_k} c_{kmi(j)} \end{aligned}$$

and

$$(1.7) \quad \lim_{n_k \rightarrow \infty} n_k^{-1} \sum_{i=1}^{n_k} [c_{kmi(j)} - \bar{c}_{kmn_k(j)}]^2 \in (0, \infty),$$

which together imply the Noether condition

$$(1.8) \quad \lim_{n_k \rightarrow \infty} \left\{ \max_{1 \leq i \leq n_k} [c_{kmi(j)} - \bar{c}_{kmn_k(j)}]^2 / \sum_{i=1}^{n_k} [c_{kmi(j)} - \bar{c}_{kmn_k(j)}]^2 \right\} = 0.$$

We denote the total (combined) sample size by

$$(1.9) \quad N = \sum_{k=1}^s n_k$$

and assume that the limits

$$(1.10) \quad r_k = \lim_{N \rightarrow \infty} (n_k/N), \quad k = 1, \dots, s$$

exist and satisfy

$$(1.11) \quad r_0 \leq r_k \leq 1 - r_0, \quad k = 1, \dots, s$$

for some $0 < r_0 < 1/s$. Thus we have

$$(1.12) \quad \lim_{N \rightarrow \infty} n_k = \infty, \quad k = 1, \dots, s$$

and

$$(1.13) \quad \sum_{k=1}^s r_k = 1,$$

and the matrices

$$(1.14) \quad M_k^* = \lim_{N \rightarrow \infty} N^{-1} M_{kn_k} = \lim_{N \rightarrow \infty} (n_k/N) M_k = r_k M_k, \quad k = 1, \dots, s$$

are symmetric and positive definite.

For each positive integer n , let the scores $a_n(1), \dots, a_n(n)$ be generated by a non-constant and square integrable function ψ on $(0,1)$ according to one of the following two ways:

$$(1.15) \quad a_n(i) = \psi[i/(n+1)], \quad i = 1, \dots, n$$

or

$$(1.16) \quad a_n(i) = E[\psi(U_{ni})], \quad i = 1, \dots, n$$

where $U_{n1} \leq \dots \leq U_{nn}$ are the order statistics of a random sample of size n from the uniform distribution over $(0,1)$. We assume that ψ can be expressed as the difference $\psi = \psi_1 - \psi_2$ of two

non-decreasing and absolutely continuous functions ψ_1 and ψ_2 on $(0,1)$. Let

$$(1.17) \quad \lambda(\psi) = \left\{ \int_0^1 [\psi(u) - \bar{\psi}]^2 du \right\}^{1/2}, \quad \bar{\psi} = \int_0^1 \psi(u) du.$$

Thus we have $0 < \lambda(\psi) < \infty$.

We assume that the underlying distribution function F has an absolutely continuous density $f = F'$ with finite positive Fisher information

$$(1.18) \quad 0 < I(f) = \int_{-\infty}^{\infty} [f'(x)/f(x)]^2 dF(x) < \infty.$$

We note that

$$(1.19) \quad I(f) = [\lambda(\phi_f)]^2 = \int_0^1 [\phi_f(u)]^2 du$$

where

$$(1.20) \quad \phi_f(u) = -f'[F^{-1}(u)]/f[F^{-1}(u)], \quad u \in (0,1)$$

with

$$(1.21) \quad \bar{\phi}_f = \int_0^1 \phi_f(u) du = 0.$$

2. The Proposed Rank-order Tests. For $b = (b_1, \dots, b_d) \in \mathbb{R}^d$ and $k = 1, \dots, s$, let

$$(2.1) \quad R_{kin_k}(b) = \text{the rank of } X_{ki} - bc_{ki} \text{ among}$$

$X_{k1} - bc_{k1}, \dots, X_{kn_k} - bc_{kn_k}$ in the ascending order of magnitude,

$$(2.2) \quad S_{kmn_k}(b) = \sum_{i=1}^{n_k} (c_{kmi} - \bar{c}_{kmn_k}) a_{n_k} [R_{kin_k}(b)], \quad m = 1, \dots, q$$

where $a_{n_k}(1), \dots, a_{n_k}(n_k)$ are generated according to (1.15) or (1.16) (with n replaced by n_k),

$$(2.3) \quad S_{kn_k}(b) = (S_{k1n_k}(b), \dots, S_{kqn_k}(b)),$$

and define

$$(2.4) \quad S_N(b) = \sum_{k=1}^S S_{kn_k}(b) = (S_{N1}(b), \dots, S_{Nq}(b)).$$

Let

$$(2.5) \quad B_{(N)} = \{b \in \mathbb{R}^q : \sum_{m=1}^q |S_{Nm}(b)| = \text{minimum}\}$$

and choose one element

$$(2.6) \quad \hat{\beta}_N \in B_{(N)}$$

as an estimate of β . Define the s vectors of aligned rank statistics

$$(2.7) \quad \hat{S}_{Nk} = S_{kn_k}(\hat{\beta}_N), \quad k = 1, \dots, s$$

and let

$$(2.8) \quad \lambda_N = \{N^{-1} \sum_{k=1}^s \sum_{i=1}^{n_k} [a_{n_k}(i) - \bar{a}_{n_k}]^2\}^{1/2}$$

where

$$(2.9) \quad \bar{a}_{n_k} = n_k^{-1} \sum_{i=1}^{n_k} a_{n_k}(i), \quad k = 1, \dots, s.$$

Then a class of aligned rank-order tests of (0.5), each determined by a score-generating function ψ , can be based on the statistics

$$(2.10) \quad Q_N = \lambda_N^{-2} \sum_{k=1}^s \hat{S}_{Nk}^{-1} M_{kn_k} \hat{S}_{Nk}^{\prime},$$

whose asymptotic distribution under H_0 is given by Theorem 2.1, which in turn follows from Theorem 3.1 (see section 3).

Theorem 2.1. Under H_0 , Q_N has asymptotically the (central) chi-square distribution $\chi^2_{(s-1)q}$ with $(s-1)q$ degrees of freedom.

For $0 < \epsilon < 1$, let $\chi^2_{(s-1)q, \epsilon}$ be the upper 100% point of the $\chi^2_{(s-1)q}$ distribution. Then for large N we have the following asymptotically distribution-free test of approximately size ϵ :

(2.11) Reject H_0 (in favor of H) if and only if

$$Q_N \geq \chi^2_{(s-1)q, \epsilon}.$$

3. Asymptotic Distribution of the Test Statistics. Consider the sequence of hypotheses

$$(3.1) \quad H_N : \beta_k = \beta_{kN} = \beta + N^{-1/2} b_k^*, \quad k = 1, \dots, s$$

where the s vectors $b_k^* \in \mathbb{R}^q$, $k = 1, \dots, s$ are such that

$$(3.2) \quad \sum_{k=1}^s b_k^* M_k^* = 0.$$

Then the asymptotic distribution of Q_N under H_N is given by the following theorem.

Theorem 3.1. Under H_N , Q_N has asymptotically the non-central chi-square distribution $\chi^2_{(s-1)q}(\Delta_Q)$ with $(s-1)q$ degrees of freedom and noncentrality parameter

$$(3.3) \quad \Delta_Q = [\gamma(\psi, f) / \lambda(\psi)]^2 \sum_{k=1}^s b_k^* M_{k-k}^* b_k^*$$

where

$$(3.4) \quad \gamma(\psi, f) = \int_0^1 \psi(u) \phi_f(u) du.$$

Remark. Clearly for $b_1^* = \dots = b_s^* = 0$, which satisfy (3.2), H_N reduces to H_0 , and Δ_Q reduces to 0. Thus Theorem 2.1 is a special case of Theorem 3.1.

For later purpose we also estimate the β_k 's separately. For each $k = 1, \dots, s$, let

$$(3.5) \quad B_{kn_k} = \{b \in \mathbb{R}^q : \sum_{m=1}^q |S_{kmn_k}(b)| = \text{minimum}\}$$

and choose one element

$$(3.6) \quad \hat{\beta}_{kn_k} \in B_{kn_k}$$

as an estimate of β_k based on the k -th sample

$$(3.7) \quad X_{kn_k} = (X_{k1}, \dots, X_{kn_k})$$

We note that since the s samples $X_{1n_1}, \dots, X_{sn_s}$ are independent so are the estimates $\hat{\beta}_{1n_1}, \dots, \hat{\beta}_{sn_s}$. By Jurečková's (1971)

results (see Theorems 3.1 and 4.1, and Lemmas 4.1 and 4.5), the distribution (denoted by $\mathcal{D}$) of $n_k^{-1/2}(\hat{\beta}_{kn_k} - \beta_k)$ is asymptotically normal (denoted by N_q), i.e.,

$$(3.8) \quad \mathcal{D}[n_k^{-1/2}(\hat{\beta}_{kn_k} - \beta_k)] \rightarrow N_q(0, [\lambda(\psi)/\gamma(\psi, f)]^2 M_k^{-1}) \quad (k = 1, \dots, s),$$

and

$$(3.9) \quad n_k^{-1/2} S_{kn_k}(\hat{\beta}_{kn_k}) = o_p(1) \quad (k = 1, \dots, s).$$

Similarly we have

$$(3.10) \quad N^{-1/2} S_N(\hat{\beta}_N) = o_p(1).$$

We need the following lemmas to prove Theorem 3.1.

Lemma 3.2. For each $k = 1, \dots, s$ we have

$$(3.11) \quad N^{-1/2} S_N(\hat{\beta}_N) = \gamma(\psi, f) N^{-1/2}(\hat{\beta}_{kn_k} - \beta_N) M_k^* + o_p(1).$$

Proof. By Theorem 3.1 of Jurecková (1971), for each $k = 1, \dots, s$ we have

$$(3.12) \quad \begin{aligned} n_k^{-1/2} S_{kn_k}(\hat{\beta}_{kn_k}) &= n_k^{-1/2} S_{kn_k}(\beta_k) \\ &\quad - \gamma(\psi, f) n_k^{-1/2}(\hat{\beta}_{kn_k} - \beta_k) M_k + o_p(1) \end{aligned}$$

and

$$(3.13) \quad \begin{aligned} n_k^{-1/2} S_{kn_k}(\hat{\beta}_N) &= n_k^{-1/2} S_{kn_k}(\beta_k) \\ &\quad - \gamma(\psi, f) n_k^{-1/2}(\hat{\beta}_N - \beta_k) M_k + o_p(1). \end{aligned}$$

Subtracting (3.12) from (3.13) and using (2.7) and (3.9), we have

$$(3.14) \quad n_k^{-1/2} \hat{S}_{Nk} = \gamma(\psi, f) n_k^{1/2} (\hat{\beta}_{kn_k} - \hat{\beta}_N) M_k + o_p(1) .$$

Multiplying both sides of (3.14) by $(n_k/N)^{1/2}$ and using (1.14), we obtain (3.11).

For later use we also define the $q \times q$ matrix

$$(3.15) \quad D = \sum_{k=1}^S M_k^* = (d_{\ell m})_{\ell, m=1, \dots, q}$$

which, being a sum of symmetric and positive definite matrices, is itself symmetric and positive definite and hence has a symmetric inverse

$$(3.16) \quad A = D^{-1} = (a_{\ell m})_{\ell, m=1, \dots, q} .$$

Thus we have

$$(3.17) \quad DA = AD = \sum_{k=1}^S M_k^* A = A \sum_{k=1}^S M_k^* = I_q ,$$

where I_q is the $q \times q$ identity matrix.

Notation. Let $\{U_n\}$ and $\{V_n\}$ be two sequences of random vectors of the same dimension. Then

$$(3.18) \quad U_n \sim V_n \text{ if and only if } U_n - V_n = o_p(1) .$$

Lemma 3.3.

$$(3.19) \quad N^{1/2} \hat{\beta}_N \sim N^{1/2} \sum_{k=1}^S \hat{\beta}_{kn_k} M_k^* A .$$

Proof. By (2.4), (2.7), (3.11) and (3.15) we have

$$\begin{aligned}
 N^{-1/2} S_N(\hat{\beta}_N) &= \sum_{k=1}^S N^{-1/2} S_{Nk} \\
 &= \sum_{k=1}^S \gamma(\psi, f) N^{1/2} (\hat{\beta}_{kn_k} - \hat{\beta}_N) M_k^* + o_p(1) \\
 &= \gamma(\psi, f) [N^{1/2} \sum_{k=1}^S \hat{\beta}_{kn_k} M_k^* - N^{1/2} \hat{\beta}_N D] + o_p(1) .
 \end{aligned}$$

Since $\gamma(\psi, f)$ is a non-zero constant, by (3.10) we have

$$(3.20) \quad N^{1/2} \hat{\beta}_N D \sim N^{1/2} \sum_{k=1}^S \hat{\beta}_{kn_k} M_k^* ,$$

which, together with (3.17), implies (3.19).

Lemma 3.4. Under H_N , for each $k = 1, \dots, s$ we have

$$(3.21) \quad D[N^{1/2} (\hat{\beta}_{kn_k} - \beta) | H_N] \rightarrow N_q(b_k^*, [\lambda(\psi)/\gamma(\psi, f)]^2 M_k^{*-1}) .$$

Proof. By (3.1), under H_N for each $k = 1, \dots, s$ we have

$$n_k^{1/2} (\hat{\beta}_{kn_k} - \beta) = n_k^{1/2} (\hat{\beta}_{kn_k} - \beta_k) + (n_k/N)^{1/2} \hat{\beta}_k$$

and so by (1.10) and (3.8) we have

$$D[n_k^{1/2} (\hat{\beta}_{kn_k} - \beta) | H_N] \rightarrow N_q(r_k b_k^*, [\lambda(\psi)/\gamma(\psi, f)]^2 M_k^{*-1}) .$$

Hence, by (1.14), under H_N the random vector

$$N^{1/2}(\hat{\beta}_{kn_k} - \beta) = (n_k/N)^{-1/2} n_k^{1/2}(\hat{\beta}_{kn_k} - \beta)$$

is asymptotically q -variate normal with mean b_k^* and covariance matrix

$$[\lambda(\psi)/\gamma(\psi, f)]^2 r_k^{-1} M_k^{-1} = [\lambda(\psi)/\gamma(\psi, f)]^2 M_k^{*-1}.$$

Lemma 3.5. Under H_N , the sq -dimensional random vector

$$(3.22) \quad T_N = N^{1/2}(\hat{\beta}_{1n_1} - \hat{\beta}_N, \dots, \hat{\beta}_{sn_s} - \hat{\beta}_N)$$

is asymptotically normal $N_{sq}(b^*, [\lambda(\psi)/\gamma(\psi, f)]^2 J)$, where

$$(3.23) \quad b^* = (b_1^*, \dots, b_s^*)$$

and J can be partitioned as

$$(3.24) \quad J = (J_{kj})_{k,j=1,\dots,s}$$

with

$$(3.25) \quad J_{kj} = \delta_{kj} M_j^{*-1} - A$$

(δ_{kj} being the Kronecker delta).

Proof. We prove Lemma 3.5 by showing that any linear combination of the components of T_N is asymptotically normal under H_N , with the appropriate mean and variance. Let

$$(3.26) \quad t = (t_1, \dots, t_s) \in \mathbb{R}^{sq}$$

where

$$(3.27) \quad t_k = (t_{k1}, \dots, t_{kq}) \in \mathbb{R}^q, \quad k = 1, \dots, s,$$

and let

$$(3.28) \quad u = \sum_{k=1}^s t_k.$$

Then by (3.19) we have

$$\begin{aligned} (3.29) \quad T_N t' &= \sum_{k=1}^s N^{1/2} (\hat{\beta}_{kn_k} - \hat{\beta}_N) t'_k \\ &= \sum_{k=1}^s N^{1/2} \hat{\beta}_{kn_k} t'_k - N^{1/2} \hat{\beta}_N u' \\ &\sim \sum_{k=1}^s N^{1/2} \hat{\beta}_{kn_k} t'_k - N^{1/2} \sum_{k=1}^s \hat{\beta}_{kn_k} M_k^* Au'. \end{aligned}$$

By making the substitution $\hat{\beta}_{kn_k} = (\hat{\beta}_{kn_k} - \beta) + \beta$ on the right-hand side of $\sim$ in (3.29) and then making cancellation (using (3.17)), we have

$$(3.30) \quad T_N t' \sim \sum_{k=1}^s N^{1/2} (\hat{\beta}_{kn_k} - \beta) (t'_k - M_k^* Au').$$

Now by (3.21) and the symmetry of M_k^* and A , under H_N the random variable

$$N^{1/2} (\hat{\beta}_{kn_k} - \beta) (t'_k - M_k^* Au')$$

is asymptotically normal with mean $b_k^* (t'_k - M_k^* Au')$ and variance

$$(\lambda(\psi)/\gamma(\psi, f))^{-2} (t'_k - u A M_k^*) M_k^{*-1} (t'_k - M_k^* Au').$$

So, by independence of the $\hat{\beta}_{kn_k}$'s, under H_N the right-hand

side of (3.30) is asymptotically normal with variance $[\lambda(\psi)/\gamma(\psi, f)]^2 c^2$ where

$$(3.31) \quad c^2 = \sum_{k=1}^S (\tilde{t}_k - u A M_k^*) M_k^{*-1} (\tilde{t}_k - M_k^* A u') ,$$

and mean

$$(3.32) \quad \sum_{k=1}^S b_k^* (\tilde{t}_k - M_k^* A u') = \sum_{k=1}^S b_k^* \tilde{t}_k - \left(\sum_{k=1}^S b_k^* M_k^* \right) A u' \\ = b^* \tilde{t}' ,$$

the last equality in (3.32) being a consequence of (3.2).

Expanding the right-hand side of (3.31) and using (3.17), we have

$$c^2 = \sum_{k=1}^S \tilde{t}_k M_k^{*-1} \tilde{t}_k - \sum_{k=1}^S \tilde{t}_k A u' \\ = \sum_{k=1}^S \tilde{t}_k \sum_{j=1}^S (\delta_{kj} M_j^{*-1} - A) \tilde{t}_j \\ = \tilde{t} J \tilde{t}' .$$

Thus, for any $\tilde{t} \in \mathbb{R}^{sq}$, $T_N \tilde{t}'$ under H_N has asymptotically a (possibly degenerate) normal distribution with mean $b^* \tilde{t}'$ and variance $[\lambda(\psi)/\gamma(\psi, f)]^2 \tilde{t} J \tilde{t}'$. It follows that

$$(3.33) \quad \mathcal{D}(T_N | H_N) \rightarrow N_{sq}(b^*, [\lambda(\psi)/\gamma(\psi, f)]^2 J) .$$

Lemma 3.6. Under H_N , the random variable

$$(3.34) \quad Q_N^* = N [\gamma(\psi, f) / \lambda(\psi)]^2 \sum_{k=1}^s (\hat{\beta}_{kn_k} - \hat{\beta}_N) M_k^* (\hat{\beta}_{kn_k} - \hat{\beta}_N)^*$$

is asymptotically $\chi^2_{(s-1)q}(\Delta_Q)$.

Proof. Let

$$(3.35) \quad Y_N = [\gamma(\psi, f) / \lambda(\psi)] T_N = (Y_{N1}, \dots, Y_{Ns})$$

where

$$(3.36) \quad Y_{Nk} = [\gamma(\psi, f) / \lambda(\psi)] N^{1/2} (\hat{\beta}_{kn_k} - \hat{\beta}_N), \quad k = 1, \dots, s.$$

Then by (3.33) we have

$$(3.37) \quad D(Y_N | H_N) \rightarrow N_{sq}([\gamma(\psi, f) / \lambda(\psi)] b^*, J).$$

Define the $(sq) \times (sq)$ symmetric matrix

$$(3.38) \quad K = (K_{kj})_{k,j=1,\dots,s} = (\delta_{kj} M_k^*)_{k,j=1,\dots,s}.$$

Then (3.34) and (3.3) can be rewritten respectively as

$$(3.39) \quad Q_N^* = \sum_{k=1}^s Y_{Nk} M_k^* Y_{Nk}^* = Y_N K Y_N^*$$

and

$$(3.40) \quad \Delta_Q = \{[\gamma(\psi, f) / \lambda(\psi)] b^*\} K \{[\gamma(\psi, f) / \lambda(\psi)] b^*\}^*.$$

So, to prove Lemma 3.6, it suffices to show that KJ , or equivalently its transpose

$$(3.41) \quad W = JK,$$

is idempotent with trace equal to $(s-1)q$ and that

$$(3.42) \quad b^* K J K b^{**} = b^* K b^{**}$$

(see Pearle (1971), Corollary 2s.1). By direct computation we have

$$(3.43) \quad W = (w_{kj})_{k,j=1,\dots,s}$$

where

$$(3.44) \quad w_{kj} = \delta_{kj} I_q - A M_j^* \quad , \quad k, j = 1, \dots, s \quad .$$

By further computation and (3.17) we have $W^2 = W$. On the other hand

$$K J K = K W = (\delta_{kj} M_k^* - M_k^* A M_j^*)_{k,j=1,\dots,s}$$

and so

$$(3.45) \quad \begin{aligned} b^* K J K b^{**} &= \sum_{k=1}^s b_k^* M_k^* b_k^{**} - \left(\sum_{k=1}^s b_k^* M_k^* A \sum_{j=1}^s M_j^* b_j^{**} \right) \\ &= b^* K b^{**} \quad , \end{aligned}$$

where the last equality in (3.45) follows from (3.2).

It remains to compute the trace of W . Let

$$(3.46) \quad M_k^* = (c_{km\ell})_{m,\ell=1,\dots,q} \quad , \quad k = 1, \dots, s \quad .$$

Then by (3.15) we have

$$(3.47) \quad d_{m\ell} = \sum_{k=1}^s c_{km\ell} \quad , \quad m, \ell = 1, \dots, q$$

and by (3.16) and (3.17) we have

$$(3.43) \quad \sum_{m=1}^q a_{km} c_{km} = 1, \quad k = 1, \dots, q.$$

Now for $k = 1, \dots, s$, by (3.44) we have

$$W_{kk} = I_{qk} - AM_k^*$$

$$(a_{km} c_{km})_{m=1}^q, \quad k = 1, \dots, q.$$

with $\text{tr}(W)$

$$\text{tr}(W_{kk}) = q - \sum_{m=1}^q a_{km} c_{km}.$$

It follows that

$$(3.45) \quad \text{tr}(W) = \sum_{k=1}^s \text{tr}(W_{kk}) = sq - q = (s-1)q.$$

Thus Lemma 3.6 is established.

Proof of Theorem 3.1. By Lemma 3.6 it suffices to show that

$$(3.50) \quad C_N = J_N^*.$$

By (1.12), (1.15) - (1.17) and (2.9) we have

$$\lim_{N \rightarrow \infty} \frac{1}{n_k} \sum_{i=1}^{n_k} a_{ik}(\psi) = \bar{a}_{nk}^2 = \lambda^2(\psi), \quad k = 1, \dots, s$$

and so by (2.8), (1.10) and (1.13) we have

$$(3.51) \quad \lim_{N \rightarrow \infty} \lambda_N = \lambda(\psi).$$

It follows from (3.11) and (3.36) that

$$(3.52) \quad (N\lambda_N^2)^{-1/2} S_{Nk} = Y_{Nk} M_k^*, \quad k = 1, \dots, s.$$

Now by (1.14) we have

$$(3.53) \quad \lim_{N \rightarrow \infty} N M_{kn_k}^{-1} = M_k^{*-1}, \quad k = 1, \dots, s.$$

It follows from the symmetry of M_k^* that

$$(3.54) \quad \lambda_N^{-2} \hat{S}_N M_{kn_k}^{-1} \hat{S}_N \sim Y_{Nk} M_k^* Y_{Nk}'.$$

Summing up both sides of (3.54) over $k = 1, \dots, s$ and using (2.10) and (3.39), we obtain (3.50). Thus Theorem 3.1 is proved.

4. Asymptotic Efficiency. Using (3.7), we rewrite (0.1) as

$$(4.1) \quad X_{kn_k} = x_k l_{n_k} + e_k C_k + Z_{kn_k}, \quad k = 1, \dots, s$$

where

$$(4.2) \quad l_{n_k} = (1, \dots, 1) \in \mathbb{R}^{n_k},$$

$$(4.3) \quad Z_{kn_k} = (Z_{k1}, \dots, Z_{kn_k})$$

and

$$(4.4) \quad C_k = (c_{k1}, \dots, c_{kn_k})$$

is a $q \times n_k$ matrix.

Let

$$(4.5) \quad X_N = (X_{1n_1}, \dots, X_{sn_s})$$

$$(4.6) \quad Z_N = (Z_{1n_1}, \dots, Z_{sn_s})$$

$$(4.7) \quad \underline{\alpha} = (\alpha_1, \dots, \alpha_s) ,$$

$$(4.8) \quad \underline{\theta} = (\alpha, \beta_1, \dots, \beta_s) ,$$

and let

$$(4.9) \quad E_k = (0', \dots, \underset{\sim}{1}_{n_k}', \dots, 0')' \quad , \quad k = 1, \dots, s$$

be the $s \times n_k$ matrix with $\underset{\sim}{1}_{n_k}$ as the k -th row and all the other rows being 0. Then the s linear models in (4.1) can be combined into one linear model

$$(4.10) \quad \underline{X}_N = \underline{\theta} C_N^* + \underline{Z}_N$$

where

$$(4.11) \quad C_N^* = \begin{bmatrix} E_1 & E_2 & \dots & E_s \\ C_1 & 0 & \dots & 0 \\ 0 & C_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & C_s \end{bmatrix}$$

is an $[s(q+1)] \times N$ matrix. The parameter space for $\underline{\theta}$ is the $s(q+1)$ -dimensional Euclidean space

$$(4.12) \quad \Omega = \mathbb{R}^{s(q+1)}$$

and H_0 can be expressed as

$$(4.13) \quad H_0 : \underline{\theta} \in \Omega_0 = \{(\underline{a}, \underline{b}_1, \dots, \underline{b}_s) \in \Omega : \underline{b}_1 = \dots = \underline{b}_s\} .$$

The likelihood ratio test of (0.5) rejects H_0 (in favor of H)

if the likelihood ratio

$$\Lambda_N = \sup_{\{a_k, b_k\}} \prod_{k=1}^s \prod_{i=1}^{n_k} f(x_{ki} - a_k - b_k c_{ki}) : a_k \in \mathbb{R}, k = 1, \dots, s; b_k \in \mathbb{R}^q \} /$$

$$\sup_{\{a_k, b_k\}} \prod_{k=1}^s \prod_{i=1}^{n_k} f(x_{ki} - a_k - b_k c_{ki}) : a_k \in \mathbb{R}, b_k \in \mathbb{R}^q, k = 1, \dots, s \}$$

is small, or equivalently if

$$(4.14) \quad L_N = -2 \log \Lambda_N$$

is large. Here f (or equivalently F) is assumed to be known. Under Assumptions I - V and VII of Wald (1943), but no assumption concerning the shape of F , the asymptotic distribution of L_N under H_N is given by Theorem 4.1, which will be proved later in this section.

Theorem 4.1. Under H_N , L_N is asymptotically $\chi^2_{(s-1)q}(\Delta_L)$ with

$$(4.15) \quad \Delta_L = I(f) \sum_{k=1}^s b_k^* M_k^* b_k^*.$$

To compare the proposed rank-order tests with the likelihood ratio test, we make the additional assumption that

$$(4.16) \quad b_k^* \neq 0 \quad \text{for some } 1 \leq k \leq s,$$

which makes the right-hand side of (4.15) strictly positive. Combining Theorems 3.1 and 4.1, we have the asymptotic relative efficiency.

Corollary 4.2. The asymptotic efficiency of the aligned rank-order test of (0.5) (based on Q_N) relative to the likelihood ratio test (based on L_N) is

$$(4.17) \quad e_{Q,L}(F) = \gamma^2(\psi, f) / [I(f) \lambda^2(\psi)]$$

$$= \left[\int_0^1 \psi(u) \phi_f(u) du \right]^2 / \left[\int_0^1 [\phi_f(u)]^2 du \int_0^1 [\psi(u) - \bar{\psi}]^2 du \right] .$$

Clearly if the score-generating function ψ is the same as ϕ_f , then the right-hand side of (4.17) reduces to unity.

Corollary 4.3. With the score-generating function $\psi = \phi_f$, the aligned rank-order test of (0.5) has asymptotic relative efficiency one with respect to the likelihood ratio test.

Examples. If F is the standard logistic distribution function, then $\psi(u) = \phi_f(u) = 2u - 1$ generates Wilcoxon-type scores; and if $F = \Phi$ is the standard normal distribution function, then $\psi = \phi_\Phi = \phi^{-1}$ generates normal scores.

Proof of Theorem 4.1. Consider the map

$$(4.18) \quad \xi = (\xi_0, \xi_1, \dots, \xi_s) : \Omega \rightarrow \Omega$$

defined by

$$(4.19) \quad \begin{aligned} \xi_0(\theta) &= \alpha, \quad \xi_1(\theta) = \beta_1, \\ \xi_k(\theta) &= \beta_k - \beta_1 \quad \text{for } k = 2, \dots, s, \quad (\theta = (\alpha, \beta_1, \dots, \beta_s) \in \Omega) . \end{aligned}$$

Let

$$\begin{aligned}
 \xi(1) &= (\xi_0, \xi_1) = (\xi_1, \dots, \xi_{s+q}) \\
 \xi(2) &= (\xi_2, \dots, \xi_s) = (\xi_{s+q+1}, \dots, \xi_{s(q+1)})
 \end{aligned}
 \quad (4.20)$$

Then H_0 can be expressed as

$$\xi(2)(\theta) \equiv 0 \quad (4.21)$$

Clearly ξ is a homeomorphism and, with the identification

$$\theta = (\alpha, \beta_1, \dots, \beta_s) = (\theta_1, \dots, \theta_{s(q+1)}) \quad (4.22)$$

has a positive Jacobian $\det(\partial \xi / \partial \theta)$ not depending on θ ; moreover, the first two partial derivatives of $\xi_1(\theta), \dots, \xi_{s(q+1)}(\theta)$ are uniformly continuous and bounded functions of θ . Indeed, the inverse $\theta = \xi^{-1}$ of ξ is given by

$$\theta(\xi) = (\xi_0, \xi_1, \xi_1 + \xi_2, \dots, \xi_1 + \xi_s) \quad (4.23)$$

with Jacobian matrix

$$M = (\partial \theta / \partial \xi) = \begin{bmatrix} I_s & 0 & 0 & \dots & 0 \\ 0 & I_q & 0 & \dots & 0 \\ 0 & I_q & I_q & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & I_q & 0 & \dots & I_q \end{bmatrix} \quad (4.24)$$

(whose determinant is equal to unity).

Now consider the $[s(q+1)] \times [s(q+1)]$ matrix

$$(4.25) \quad A_N = C_N^* C_N^{*'} = \begin{bmatrix} E & F_1' & F_2' & \dots & F_s' \\ F_1 & D_1 & 0 & \dots & 0 \\ F_2 & 0 & D_2 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ F_s & 0 & 0 & \dots & D_s \end{bmatrix}$$

where

$$(4.26) \quad E = (\delta_{kj} n_k)_{k,j=1,\dots,s}$$

$$(4.27) \quad D_k = C_k C_k' = \sum_{i=1}^{n_k} c_{ki} c_{ki}' , \quad k = 1, \dots, s$$

and

$$(4.28) \quad F_k = (0', \dots, n_k \bar{c}_{kn_k}, \dots, 0') , \quad k = 1, \dots, s$$

is a $q \times s$ matrix with $n_k \bar{c}_{kn_k}$ as the k -th column and 0 elsewhere. By routine computation we have the $[s(q+1)] \times [s(q+1)]$ matrix

$$(4.29) \quad A_N^* = M' A_N M = \begin{bmatrix} A_{N11}^* & A_{N12}^* \\ A_{N21}^* & A_{N22}^* \end{bmatrix}$$

where

$$(4.30) \quad A_{N11}^* = \begin{bmatrix} E & \sum_{k=1}^s F_k' \\ \sum_{k=1}^s F_k & \sum_{k=1}^s D_k \end{bmatrix}$$

is $(s+q) \times (s+q)$,

$$(4.31) \quad A_{N12}^* = \begin{bmatrix} F_2' & \dots & F_s' \\ D_2 & \dots & D_s \end{bmatrix}$$

$$(4.32) \quad A_{N21}^* = A_{N12}^{*'} ,$$

and

$$(4.33) \quad A_{N22}^* = (\delta_{kj} D_k)_{k,j=2,\dots,s} .$$

We note that by assumption A_N is positive definite, hence so is A_N^* . Consider the matrix

$$(4.34) \quad M_N = \sum_{k=1}^s M_{kn_k} ,$$

which is symmetric and positive definite and hence has a symmetric inverse

$$(4.35) \quad G_N = M_N^{-1} ,$$

and define the $q \times s$ matrix

$$(4.36) \quad \bar{C}_N = (\bar{c}_{1n_1}, \dots, \bar{c}_{sn_s}) .$$

Then by routine computation and the obvious identity

$$(4.37) \quad M_{kn_k} = D_k - n_k \bar{c}_{kn_k} \bar{c}_{kn_k}' , \quad k = 1, \dots, s$$

it can be checked that

$$(4.38) \quad A_{N11}^{*-1} = \begin{bmatrix} \bar{C}_N' G_N \bar{C}_N + E^{-1} & -\bar{C}_N' G_N \\ -G_N \bar{C}_N & G_N \end{bmatrix}$$

By further computation and the additional identity

$$(4.39) \quad F_k E^{-1} F_j' = \delta_{kj} n_k \bar{c}_{kn_k} \bar{c}_{jn_j}' , \quad k, j = 1, \dots, s$$

we have

$$(4.40) \quad A_{N21}^* A_{N11}^{*-1} A_{N12}^* = (\delta_{kj} n_k \bar{c}_{kn_k} \bar{c}_{jn_j}' + M_{kn_k} G_N M_{jn_j})_{k,j=2,\dots,s} .$$

So we have the $[(s-1)q] \times [(s-1)q]$ matrix

$$(4.41) \quad \begin{aligned} \bar{A}_N^* &= A_{N22}^* - A_{N21}^* A_{N11}^{*-1} A_{N12}^* \\ &= (\delta_{kj} M_{kn_k} - M_{kn_k} G_N M_{jn_j})_{k,j=2,\dots,s} . \end{aligned}$$

Now H_N can be expressed as

$$(4.42) \quad H_N : \theta = \theta_N = (\alpha, \beta_{1N}, \dots, \beta_{sN}) .$$

We also note that

$$(4.43) \quad \xi_{(2)}(\theta_N) = N^{-1/2} [(b_2^*, \dots, b_s^*) - (b_1^*, \dots, b_1^*)] .$$

Then, by Theorem IX of Wald (1943), L_N under H_N is asymptotically noncentral chi-square with $s(q+1) - (s+q) = (s-1)q$ degrees of freedom and noncentrality parameter

$$(4.44) \quad \Delta_{L_N}^* = I(f) \xi_{(2)}(\theta_N) \bar{A}_N^* \xi_{(2)}(\theta_N)' = I(f) \Delta_N^*$$

where

$$(4.45) \quad \Delta_N^* = N^{-1} [(I) - (II) - (III) + (IV)]$$

with

$$\begin{aligned}
 (4.46) \quad (I) &= (b_2^*, \dots, b_s^*) \bar{A}_N^* (b_2^*, \dots, b_s^*)' \\
 &= \sum_{k=2}^s b_k^* M_{kn_k} b_k^{*'} - \left(\sum_{k=2}^s b_k^* M_{kn_k} \right) G_N \left(\sum_{k=2}^s M_{kn_k} b_k^{*'} \right) ,
 \end{aligned}$$

$$\begin{aligned}
 (4.47) \quad (II) &= (b_2^*, \dots, b_s^*) \bar{A}_N^* (b_1^*, \dots, b_1^*)' \\
 &= \left(\sum_{k=1}^s b_k^* M_{kn_k} \right) G_N M_{1n_1} b_1^{*'} ,
 \end{aligned}$$

$$(4.48) \quad (III) = (II)'$$

and

$$\begin{aligned}
 (4.49) \quad (IV) &= (b_1^*, \dots, b_1^*) \bar{A}_N^* (b_1^*, \dots, b_1^*)' \\
 &= b_1^* M_{1n_1} b_1^{*'} - b_1^* M_{1n_1} G_N M_{1n_1} b_1^{*'} .
 \end{aligned}$$

By (1.14) and (3.2) we have

$$(4.50) \quad \lim_{N \rightarrow \infty} N^{-1} \sum_{k=2}^s b_k^* M_{kn_k} = -b_1^* M_{1n_1}^* .$$

And by (3.15) - (3.16) and (4.34) - (4.35) we have

$$(4.51) \quad \lim_{N \rightarrow \infty} N G_N = A .$$

It follows that

$$\lim_{N \rightarrow \infty} \Lambda_N^* = \sum_{k=1}^s b_k^* M_{kn_k}^* b_k^{*'} \text{ and so } \lim_{N \rightarrow \infty} \Delta_{LN} = \Delta_L .$$

The proof is complete.

5. Asymptotic Optimality. Let Γ_N and B_N be non-singular square matrices of orders $s + q$ and $(s - 1)q$, respectively, satisfying

$$(5.1) \quad \Gamma'_{N1} \Gamma_{N1} = A_{N11}^*$$

and

$$(5.2) \quad B'_N B_N = \bar{A}_N^*,$$

and define the $(s + q) \times (s - 1)q$ matrix

$$(5.3) \quad \Gamma_{N2} = (\Gamma'_{N1})^{-1} A_{N12}^*.$$

Then the square matrix

$$(5.4) \quad K_N = \begin{bmatrix} \Gamma_{N1} & \Gamma_{N2} \\ 0 & B_N \end{bmatrix}$$

of order $s(q + 1)$ is nonsingular and satisfies

$$(5.5) \quad K_N A_N^{*-1} K'_N = I_{s(q + 1)}.$$

For $\omega = (a, b, \dots, b) \in \Omega_0$ and $c > 0$ define the surface

$$(5.6) \quad S(\omega, c) = \{ \theta \in \Omega : I(f) \xi_{\theta}(2) (\theta) \bar{A}_N^* \xi_{\theta}(2) (\theta)' = c, \xi_{\theta}(\theta) \Gamma_N' = (a, b) \Gamma'_{N1} \}$$

where

$$(5.7) \quad \Gamma_N = (\Gamma_{N1}, \Gamma_{N2})$$

is $(s + q) \times (s(q + 1))$. Consider the transformation of Ω

$$(5.8) \quad \theta = (\alpha, \beta_1, \dots, \beta_s) \rightarrow (\alpha^*, \beta_1^*, \dots, \beta_s^*) = [I(f)]^{\frac{1}{2}} \xi(\theta) K_N'$$

where

$$(5.9) \quad (\alpha^*, \beta_1^*) = [I(f)]^{\frac{1}{2}} \xi(\theta) \Gamma_N'$$

and

$$(5.10) \quad (\beta_2^*, \dots, \beta_s^*) = [I(f)]^{\frac{1}{2}} \xi(2)(\theta) B_N'$$

which maps $S(\omega, c)$ into

$$(5.11)$$

$$S^*(\omega, c)$$

$$= \{(\alpha^*, \beta_1^*, \dots, \beta_s^*) \in \Omega : (\alpha^*, \beta_1^*) = [I(f)]^{\frac{1}{2}}(a, b) \Gamma_{N1}', \sum_{k=2}^s \beta_k^* \beta_k^* = c\}.$$

For $\theta_0 \in \Omega$ and $\rho > 0$ let

$$(5.12)$$

$$\Omega(\theta_0, \rho)$$

$$= \{\theta \in \Omega : \theta, \theta_0 \in S(\omega, c) \text{ for some } \omega \in \Omega_0 \text{ and } c > 0, \text{ and } \|\theta - \theta_0\| \leq \rho\}$$

($\|\cdot\|$ being the Euclidean norm on Ω), and let $\Omega^*(\theta_0, \rho)$ be its image under the transformation (5.8). For $\theta \in \Omega$ let

$$(5.13) \quad \eta(\theta) = \lim_{\rho \rightarrow 0} \{A[\Omega^*(\theta, \rho)] / A[\Omega(\theta, \rho)]\}$$

where A denotes area. Then by Theorem VIII of Wald (1943) the likelihood ratio test of (0.5) is asymptotically optimal in the sense that it

- (a) has asymptotically best average power with respect to the weight function $\eta(\theta)$ and the family of surfaces

$$(5.14) \quad S = \{S(\underline{\omega}, c) : \underline{\omega} \in \Omega_0, c > 0\};$$

- (b) has asymptotically best constant power on the surfaces in S ;

and

- (c) is an asymptotically most stringent test.

By Corollary 4.3., with the score-generating function $\psi = \phi_F$ the proposed rank-order test is asymptotically power-equivalent to the Wald-optimal likelihood ratio test. Thus if the underlying distribution F is logistic, then the Q_N -test using Wilcoxon-type scores is asymptotically optimal; and if F is normal, then the normal-scores rank-order test is asymptotically optimal.

REFERENCES

- [1] Jurečková, J. (1971). Nonparametric estimate of regression coefficients. Ann. Math. Statist. 42, 1328-1338.
- [2] Searle, S.R. (1971). Linear Models. Wiley, New York.
- [3] Sen, P.K. (1969). On a class of rank order tests for the parallelism of several regression lines. Ann. Math. Statist. 40, 1668-1683.
- [4] Sen, P.K. and Puri, M.L. (1977). Asymptotically distribution-free aligned rank order tests for composite hypotheses for general multivariate linear models. Z. Wahrscheinlichkeitstheorie und verw. Gebiete 39, 175-186.
- [5] Wald, A. (1943). Tests of statistical hypotheses concerning several parameters when the number of observations is large. Trans. Amer. Math. Soc. 54, 426-482.

Department of Mathematics
Naval Science Building
University of Louisville
Louisville, KY 40292

Department of Mathematics
Swain Hall East
Indiana University
Bloomington, IN 47405

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 81 - 0205	2. GOVT ACCESSION NO <i>AD-A097166</i>	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Rank-Order Tests for the Parallelism of Several Regression Surfaces		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ching-Yuan Chiang and Madan L. Puri		8. CONTRACT OR GRANT NUMBER(s) AFOSR-76-2927 v
9. PERFORMING ORGANIZATION NAME AND ADDRESS Indiana University Department of Mathematics Bloomington, Indiana 47405		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/WNM Bolling Air Force Base, D.C. 20332		12. REPORT DATE February, 1981
		13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Asymptotic distribution, asymptotic efficiency, asymptotic optimality, linear regression, parallelism, rank-order tests, regression surfaces.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) For testing the hypothesis that several ($s \geq 2$) linear regression surfaces $X_{ki} = \alpha_k + \beta_k C_{ki} + Z_{ki}$ ($k = 1, \dots, s$) are parallel to one another, i.e. $\beta_1 = \dots = \beta_s$, a class of rank-order tests are considered. The tests are shown to be asymptotically distribution-free, and their asymptotic efficiency relative to the general likelihood ratio test is derived. Asymptotic optimal- ity in the sense of Wald is also discussed.		

**DTIC
ELECTE**
S APR 1 1981 **D**
D

DATE
FILMED
-8