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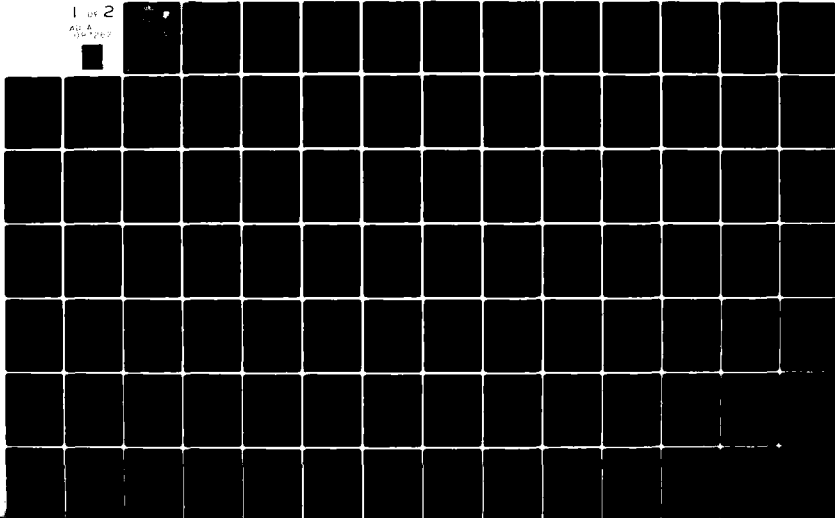
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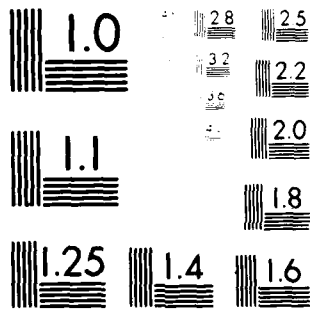
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February 1981

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ENVIRONMENT - ADAPTIVE RADAR TECHNIQUES

Mark Resources, Inc.

J. E. Howard
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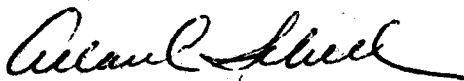
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cont → It is found that, by subdividing the search area into two range zones, it is possible to develop solutions at S-band and higher frequencies. An environment-adaptive radar that incorporates the computer-automated design algorithms is designed to the block diagram level, and the algorithms are detailed in FORTRAN code. Plans for the next phase of the contract include simulations, sensitivity studies, and the design of the environmental sensor and its integration into the radar system. ↗

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TABLE OF CONTENTS

	<u>Page</u>
1. PROBLEM DEFINITION	1-1
1.1 SCOPE	1-1
1.1.1 Background	1-1
1.1.2 Outline of Report	1-2
1.2 PERFORMANCE SPECIFICATIONS	1-3
2. BASELINE DESIGNS FOR AN ADVERSE ENVIRONMENT	2-1
2.1 BASELINE DESIGN APPROACH	2-1
2.1.1 The Blind-Zone Problem and Its Solution	2-1
2.1.2 Waveform Optimization for Blind-Speed Resolution	2-7
2.2 AUTOMATED DESIGN PROCEDURES	2-11
2.2.1 Approach	2-11
2.2.2 Automated Design for Fixed Wavelength	2-15
2.2.3 Automated Design for Fixed Dwell Time	2-18
2.2.4 Impact of Lower than Optimum p	2-23
2.3 SPECIFIC BASELINE DESIGNS	2-26
2.3.1 Design for Rotating 1-D Phased Array	2-26
2.3.2 Design for Stationary 2-D Phased Array	2-34
3. OTHER DESIGN APPROACHES	3-1
3.1 MULTIPLE SIMULTANEOUS FREQUENCIES	3-1
3.1.1 Basic Principles	3-1
3.1.2 Design for Rotating 1-D Phased Array	3-3
3.1.3 Design for Stationary 2-D Phased Array	3-4
3.1.4 Implementation	3-7
3.2 SIMULTANEOUS BLIND-RANGE/BLIND-SPEED RESOLUTION	3-8
3.2.1 Background	3-8
3.2.2 The Basic Principle	3-9
3.2.3 Achievable Performance Improvement	3-13
3.2.4 Application to Radar Design Concepts	3-16
4. DESIGNS FOR ENVIRONMENT-ADAPTIVE RADAR	4-1
4.1 REAL-TIME ADAPTATION OF WAVEFORM DESIGN	4-1
4.1.1 System Approach	4-1
4.1.2 Optimum Design Parameters	4-4
4.1.3 The Use of Subroutines for Determining K,p in Real Time	4-7
4.1.4 New Subroutines for Determining PRF Schedules in Real Time	4-7
4.2 PERFORMANCE IMPROVEMENT FROM ADAPTIVE TECHNIQUES	4-13
5. CONCLUSIONS	5-1
APPENDIX A. SUBROUTINE LISTINGS	A-1

LIST OF FIGURES

	<u>Page</u>
2-1. Example of Blind-Speed Zones for a Uniform Pulse Train	2-2
2-2. Effect of Range on Power of Returned Ground Clutter	2-5
2-3. Typical Doppler Spread of Clutter in a Heavy Storm	2-6
2-4. Assumed Model for Doppler Spread of Clutter	2-6
2-5. Maximum Blind Speed Improvement Factor, K	2-9
2-6. Optimum Number of Pulse Bursts, p, for Blind-Speed Improvement	2-10
2-7. Optimum Parameters for $\rho = 0.02$	2-12
2-8. Optimum Parameters for $\rho = 0.005$	2-13
2-9. Subroutine OPB	2-19
2-10. Subroutine OPK	2-21
2-11. Subroutine SOPK	2-22
2-12. Function FK	2-24
2-13. Function FB	2-25
2-14. Example of Optimization for Long-Range Search	2-29
3-1. Enhancement Factor for Range-Ambiguous Volumetric Clutter	3-11
3-2. Combined Range-Doppler Blind Areas for a 2-PRF Burst	3-12
4-1. Architecture for Adaptive Radar	4-3
4-2. Example of Blind Region	4-12

1. PROBLEM DEFINITION

1.1 SCOPE

1.1.1 Background

This report documents the second phase in a study of minimizing environmental effects on radar sensors. This phase involves the development of environment-adaptive radar techniques that will materially enhance the performance of a ground-based tactical radar.

The baseline radar is intended to handle a realistic airborne threat that is much more demanding than the threat that is handled by current radars like the new G.E. AN/TPS-59 radars. The major features of this threat are:

1. Large search volume--360° azimuth by 20 km altitude by 200 km range.
2. Multiple target tracking--> 50 at 2 Hz update rate.
3. Small, high-speed targets--< 1 sq m RCS, 1000 mps speed.
4. Heavy clutter-- -15 dB terrain, 4 mm/hr rain, 100,000 lb chaff cloud.

A conventional design will not meet the needs of this realistic environment. However, as long as the design is flexible enough to allow adaptive adjustments on the basis of eventual measurements of the environment, degraded performance in the most extreme environments may be acceptable. We have previously developed baseline designs for meeting this environment without the aid of real-time adaptivity. In this report, however, we develop the tools for a fully-adaptive radar system in which the signal processor and waveform generator adapt in real-time to sensed changes in the environment.

The design procedures that are used here go far beyond the procedures commonly found in the literature. The paramount problem in our demanding environment is that unacceptable blind-speed intervals exist when the usual design approaches are followed. Fundamentally, the product of the range interval and range-rate interval to be handled is so large that no single pulse burst is adequate. In general such problems exist whenever the cited product exceeds $c\lambda/4$ (which is $10^7 \text{ m}^2/\text{s}$ at 2.25 GHz). Since the product is $2 \times 10^8 \text{ m}^2/\text{s}$ for our severe environment ($R_{\text{max}} = 200 \text{ km}$, $v_{\text{max}} = 1000 \text{ mps}$) the blind-speed problem overpowers nearly every other design consideration. Consequently the common design procedures that are found in the literature are totally inadequate because they do not provide a solution to the blind-speed problem.

In our baseline designs, however, we found that the problems could be solved without reverting to UHF. By considering how waveform agility can benefit the radar system we have developed a design approach that greatly relieves the remaining problems. This approach involves a recognition that the environment at short ranges includes ground clutter and a need for rapid track updates while the environment at long ranges does not. The consequence of the approach is that a schedule of waveforms that handles all ranges can be developed that is much more efficient than the usual approach which involves a single waveform.

1.1.2 Outline of Report

In this report we first review the two baseline systems which bound the reasonable alternatives for the defined environment. The first system is based on a set of mechanically rotating arrays with electronic scan in elevation only (1-D phased array). The second system is based on a set of stationary arrays with electronic scan in both azimuth and elevation (2-D phased array). The first system is very limited in its tracking capabilities because of a lack of flexibility, but it is retained for consideration because it is so much cheaper than the second system. In the process of reviewing our past design we also develop the tools for an automated design procedure. The resulting computer programs are detailed (and listed in the Appendix). These programs are useful not only for easily generating new designs for new conditions but also for inclusion in a real-time adaptive system.

Before proceeding to the adaptive radar, however, we consider two new design concepts that appear to have potential for further performance improvements. One involves the simultaneous use of multiple carrier frequencies while the other involves waveforms where blind-range zones as well as blind-speed zones must be resolved. The problems with each technique are revealed in the report of results.

Next, the design for a real-time adaptive radar is developed. Automatic algorithms for determining the appropriate PRF schedule are developed, using the results of the previously-developed design tools for determining the unique design parameters. Performance improvements to be gained from the use of such adaptivity are then illustrated with a simple example.

Finally, the plan for the remaining two tasks of the program are reviewed. It is concluded that the next task, which is a simulation task, will result in

a final radar design and a specification of the capabilities of an associated environmental sensor. The final task will then involve the design of this sensor and its integration with the radar system.

1.2 PERFORMANCE SPECIFICATIONS

The baseline systems are designed to meet the requirements for a tactical, ground-based, mobile, air surveillance radar in a mountainous European environment. These requirements are reviewed here, for use in examples that appear throughout the report.

The parameters used for the baseline designs are listed in Table 1-1. The physical constraints are set by the need for mobility. Up to four antenna faces of the specified size are allowed. The two-way antenna efficiency is consistent with -35 dB sidelobes. The surveillance parameters are determined by the envisioned tactical environment which could include attacks from all sides. The cited search and track update rates are specified for the short and medium ranges. Lower rates are permissible at far ranges. The target parameters are based on small tactical aircraft, but a missile threat should be evaluated as well. The clutter environment is based on severe but not unlikely conditions. The cited terrain clutter is typical of dense forests. The rain parameters are typical of heavy storms but not tornados. The chaff is an amount that could be carried by a large transport aircraft.

When a baseline design approach involving two search zones is considered, there is a need for separately specifying the parameters in each zone. In the near-range zone we have clutter consisting of the combination of ground clutter, rain, and chaff. The total clutter therefore has a combined Doppler spread of 20 mps. The search frame time is, as in Table 1-1, 2 sec. In the far-range zone we have clutter consisting *only* of rain and chaff clutter, since the ground clutter is beyond the horizon at far ranges. The total Doppler spread of the clutter is therefore only 5 mps. The far-range zone extends from the radar horizon (typically 80 km) to the 200 km maximum range in Table 1-1. The search frame time at these long ranges can be as long as 8 to 10 seconds, rather than 2 seconds. These sundry parameters form the basis for the designs that follow. It is this natural separation of the environment into two range zones that allow us to develop designs at higher carrier frequencies than has been the common practice in the past.

Table 1-1. Baseline Specifications

Equipment Constraints:

antenna size	2 m high by 3 m wide
transmitted power	100 kw prime, 35 kw avg. radiated
antenna aperture efficiency	0.55
system dissipative losses	10 dB
noise figure	3 dB

Surveillance Parameters:

azimuth coverage	360°
range coverage	5 km to 200 km in search, 1 km to 200 km in track
altitude coverage	50 m to 20 km
max elevation angle	30° in search, 75° in track
search frame time	2 sec
track update rate	2 Hz
tracking accuracy	10 m in range, 0.2° in az/el angle
max no. of simultaneously tracked targets	500
single-look detection probability	0.90
no. of false alarms per frame	5

Target Parameters

target RCS	1 sq m
fluctuation model	Rayleigh
max target velocity	1000 mps
max target acceleration	5 g

Clutter Parameters

backscatter coef of terrain	-15 dB
max rainfall rate	4 mm/hr
max horizontal rain speed	20 mps
chaff cloud weight	100,000 lbs
chaff cloud size	10 km × 10 km × 1 km
chaff cloud speed	20 mps max

2. BASELINE DESIGNS FOR AN ADVERSE ENVIRONMENT

The baseline designs developed in our previous report* were unconventional in the sense that they made efficient use of the prior knowledge that the range extent of the ground clutter is limited. The approach led to much better performance than could be achieved by a design that failed to recognize the environmental phenomena and used a single waveform for all ranges. The details of the baseline design approach are reviewed again here, then recently developed procedures for automating the waveform design are described, and exemplary baseline designs are repeated to illustrate the new design procedures. This work provides the groundwork for our subsequent development of real-time waveform adaptivity for an environment-adaptive radar.

2.1 BASELINE DESIGN APPROACH

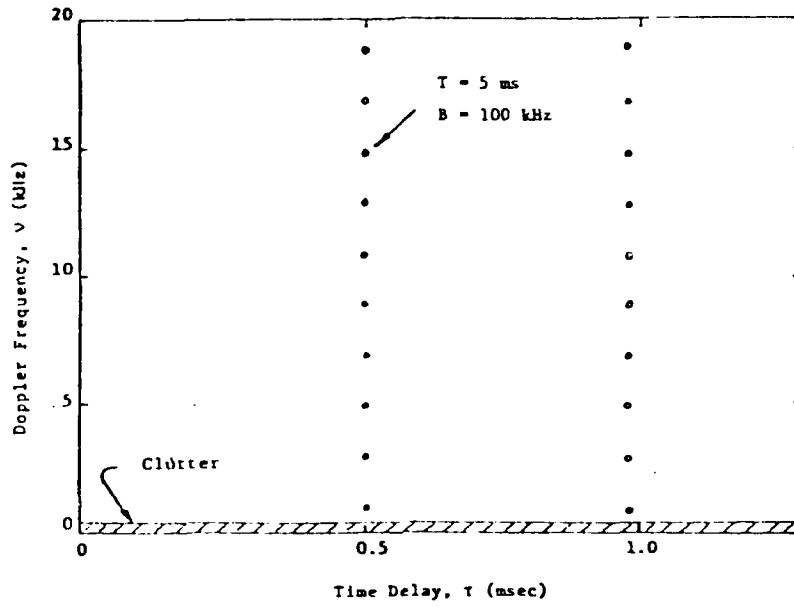
2.1.1 The Blind-Zone Problem and Its Solution

As identified in the earlier stages of this study, the paramount problem for a radar in the defined environment is to avoid interference from ground, rain, and chaff clutter while searching over a broad range of ranges and range rates. It was concluded that the uniform pulse burst is the best choice for a waveform because of its superior clutter suppression capabilities. However, numerous blind zones exist when just a single uniform pulse burst is used. The number of such zones is

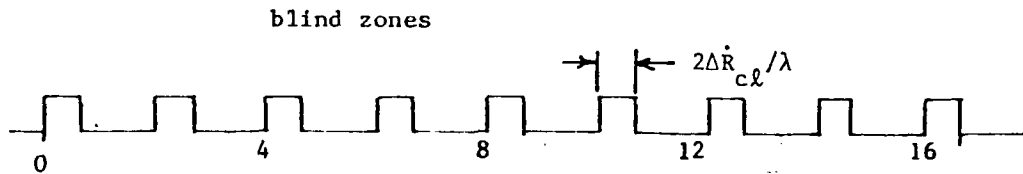
$$N_{\text{blind}} = \frac{4 R_{\text{max}} v_{\text{max}}}{c\lambda}$$

where the radar must handle ranges from 0 to R_{max} , range-rates from 0 to v_{max} , and operates at a wavelength λ . Thus the number of blind zones exceeds 20 for our assumed environment ($R_{\text{max}} = 200$ km, $v_{\text{max}} = 1000$ mps) when the carrier frequency is above 2.25 GHz. A lowering of the PRF shifts the character of the zones into blind speeds, while a raising of the PRF shifts the character of the zones into blind ranges. The blind zones are illustrated in Figure 2-1 for the case where the 20 zones are factored into 10 blind speed zones by 2 blind range zones. The waveform design problem is then to find a sequence

* J. E. Howard, et al, *Baseline Designs for Mobile Tactical Radar*, MARK Resources Report 179-4, January 1980.



(a) Ambiguity Contour



(b) Ambiguous Clutter Spectrum

Figure 2-1. Example of Blind-Speed Zones for a Uniform Pulse Train (2 KHz PRF)

of different PRFs that have non-overlapping blind zones, but with sufficient efficiency that the allowed frame times and available power are not exceeded because of the extra sets of PRFs.

A straightforward approach to solving the blind-zone problem would be to first choose the highest PRF that avoids blind range intervals, and to next find the set of lower PRFs that, when taken together, make all regions of interest free of blind speed intervals as well. The approach follows the basic procedures outlined by Rihaczek.^{*} However, the entire procedure can be simplified into an optimization based on just two parameters summarizing the environment and radar.^{**} The optimization takes into account the limited dwell times for a given beamwidth and update rate as well as the integer properties of the pulse bursts. The optimization objective is to minimize the allowable wavelength for a given search time, or to minimize the search time for a given wavelength.

We previously found that we could systematically develop designs for the defined problem by applying this two-parameter optimization directly. However, we also found that the optimization resulted in designs with relatively low carrier frequencies (below L-band), even though the optimization is designed to minimize wavelength. Since the potential benefits of a higher frequency are so strong, we critically studied the source of the problem and found that the culprit is the use of a single set of bursts for the entire range/range-rate space to be searched. When we later separated the search space into two distinct range zones we found that we could develop acceptable designs at S-band frequencies. In essence, the separate design of waveforms that are customized to each zone results in a combined system that requires less dwell time per search direction, for a fixed set of conditions. Higher carrier frequencies then become admissible even though the corresponding beamwidths are narrower so that the available dwell time per search direction is shorter.

^{*} A. W. Rihaczek, "A Systematic Approach to Blind-Speed Elimination," *IEEE Trans.-AES*, Nov. 1973, pp 940-947.

^{**} G. W. Lank, *Radar Design for Blind-Speed Elimination for Search and Track*, MARK Resources Report 179-3, January 1980.

In review, the baseline approach is centered on a division of the search mode into two range zones in which the environment is distinctly different. The *near-range zone* is defined as the set of ranges for which the ground is not yet beyond the horizon. As illustrated in Figure 2-2, the size of this zone depends on the height of the radar above the ground. A typical compromise range extent for the ground clutter for a mobile tactical radar is 80 km, which is used in several examples in this report. This zero-Doppler ground clutter when combined with rain or chaff clutter produces a relatively wide total clutter spread as illustrated in Figure 2-3 and 2-4, depending on the wind conditions. Since the ratio of PRF to Doppler spread must exceed two in order to solve the blind-speed problem even under ideal conditions, a wide spread implies the need for high PRFs. A high PRF, in turn, implies a short ambiguous range. However, as long as the ambiguous range exceeds the maximum range extent of the ground clutter such a high PRF is acceptable for detecting targets in the near-range zone. The *far-range zone* is the set of ranges from the end of the near-range zone to the maximum detection range of interest. This zone is free of ground clutter so that the *total* clutter spread is much narrower than that for the near-range zone, which is fortunate because only low PRFs are allowed at the long ranges involved. Moreover, the search update time (frame time) for long ranges need not be nearly as short as that for near ranges. As a result the blind-speed resolution can be achieved in this zone with relative ease.

At first it may appear that the two-zone approach consumes time rather than saving time because the waveforms for each zone are transmitted in series. In actual fact, however, the increase in overall efficiency caused by customized PRFs is so high that time is actually saved. For example, the updating of the long-range zone can be slowed down so much that only a small fraction of the radar's time resources is required for the long-range waveforms. And because the clutter spread is so narrow, relatively few PRFs are required to avoid blind speeds. By contrast, the near-range zone requires rapid updates but the ratio of clutter spread to PRF can be kept low because the PRFs can be much higher than is allowed in the long-range zone. Therefore the number of bursts required for blind-speed resolution is again low. The final result is that, overall, the time available for tracking is maximized while fully solving the search problem, and the allowable wavelength is minimized.

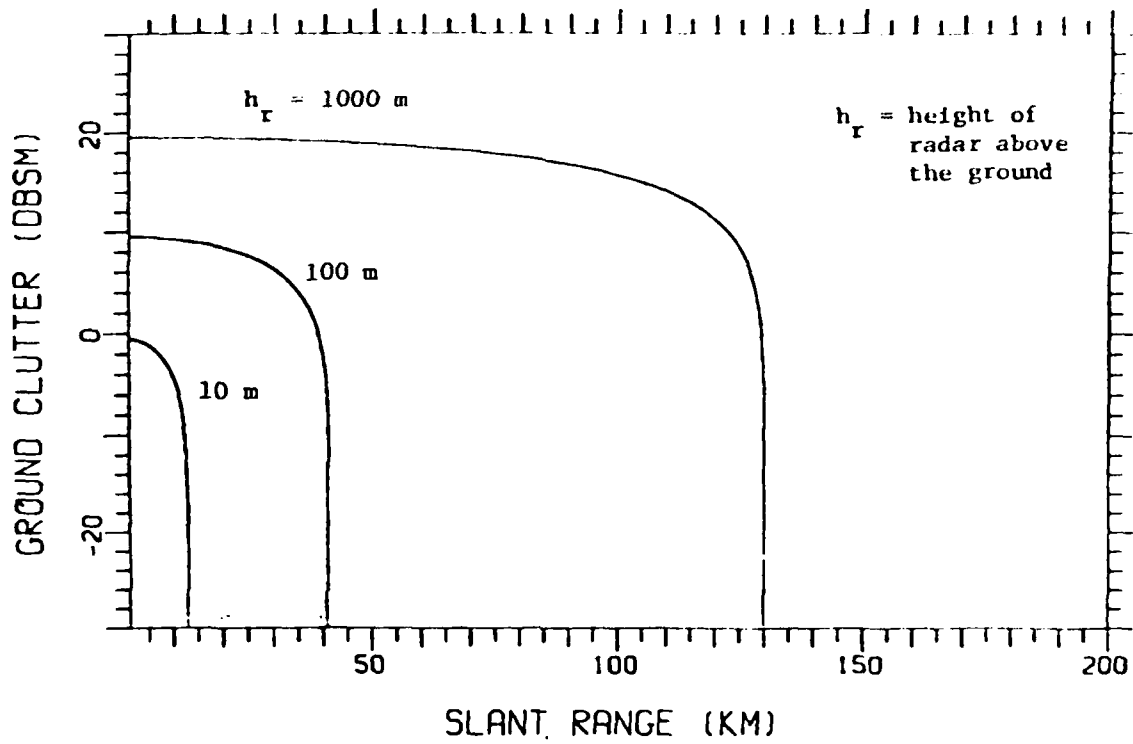


Figure 2-2. Effect of Range on Power of Returned Ground Clutter

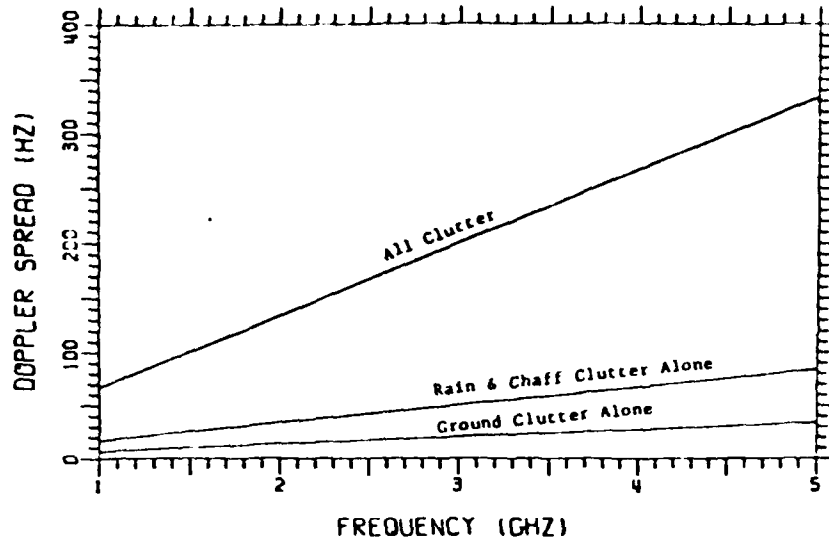


Figure 2-3. Typical Doppler Spread of Clutter in a Heavy Storm

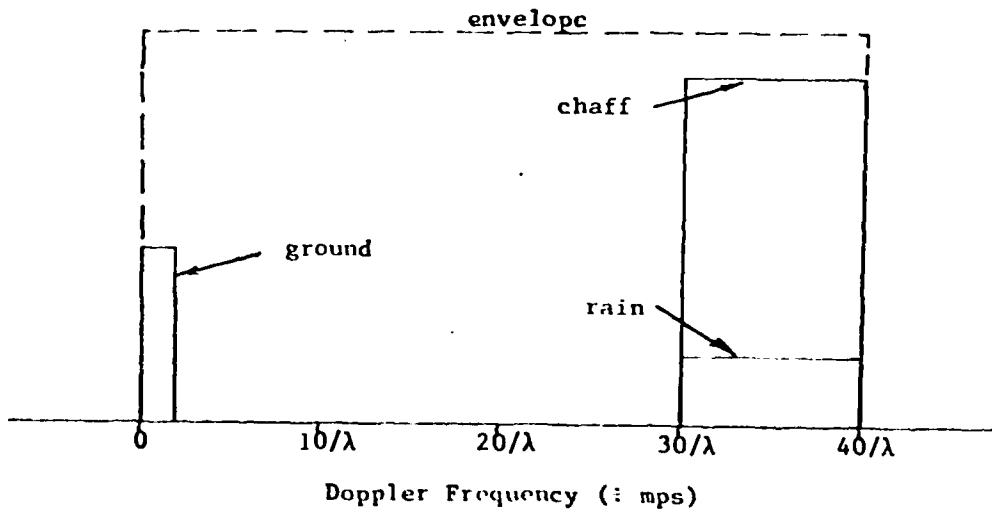


Figure 2-4. Assumed Model for Doppler Spread of Clutter

The baseline design approach therefore involves the reoptimization of the radar waveforms for each new design, with a separate set of optimized waveforms for each of the two range zones. Furthermore, the approach could be extended by similarly dividing the azimuth search region into separate azimuth zones, and again providing different waveforms for each different zone so as to minimize the total dwell required in the search mode. While the latter extension was not a part of our baseline design for a *non-adaptive* radar, it is a part of our approaches to be described later for an *adaptive* radar. *The critical steps in all of our radar designs for the mobile tactical radar are therefore the repeated applications of waveform optimization procedures for blind-speed resolution.*

2.1.2 Waveform Optimization for Blind-Speed Resolution

Since the waveform optimization must be used repeatedly in considering various design alternatives, especially when considering an environment adaptive radar, it is important that the optimization be reduced to a systematic design tool. Our previously-reported initial attempts at a simple procedure resulted in a pair of design graphs that could be used to roughly estimate the required number of bursts and the allowable carrier frequency for the optimum solution to the blind-speed problem in a specified zone. Since that time we have refined the graphical procedures to make them more useful in practice and we have developed new computer programs that allow one to bypass the graphical procedures and take advantage of computer-aided design procedures. The graphical procedures are described here in order to introduce the objectives and accomplishments of the waveform optimization.

The waveform optimization can be performed graphically as follows. First, two fundamental input parameters must be defined from the environment:

$$\beta = \frac{k_a v_{\max} T_s}{k_p w_a \theta_s}$$

$$\rho = \Delta \dot{R}_{cl} / v_{\max}$$

where

- k_a = antenna beamwidth factor (which is 0.85 for the baseline antenna)
- k_p = filter broadening factor (which is 1 to 1.7 depending on Doppler sidelobe levels required)
- v_{max} = maximum target speed of interest (which is 1000 mps for the baseline)
- w_a = antenna width (3m baseline)
- T_s = total available dwell time in one search frame (less than 2 seconds baseline for the near-range zone)
- θ_s = angular extent to be searched in time T_s (2π baseline)
- ΔR_{cl} = actual range rate extent of clutter to be suppressed (20 mps baseline)

The optimization that minimizes wavelength (or, equivalently, minimizes dwell time) results in two parameters which uniquely define the solution:

- K , the ratio of maximum target Doppler to the maximum PRF in the set of PRFs
- p , the number of separate bursts required.

This optimum can be found directly from Figures 2-5 and 2-6. The solution can then be translated into radar parameters as follows. The basic (maximum) PRF is based on the maximum range of the radar operations R_{max} , such that

$$f_r = c/2R_{max}$$

The wavelength is then

$$\lambda = 2v_{max}/f_r K$$

and the dwell time required for all p bursts in one beam position is

$$T_d = k_p \beta / f_r K$$

so that the number of pulses in the basic burst is

$$N = k_p \beta / K p$$

A practical design procedure is actually quite different from the above procedure because the penalty for a large number of bursts p is not factored into the optimization. First, a large value of p implies a small number of pulses N per burst. Since the optimization does not recognize that N is only an integer, a practical application of the results causes one to round up to

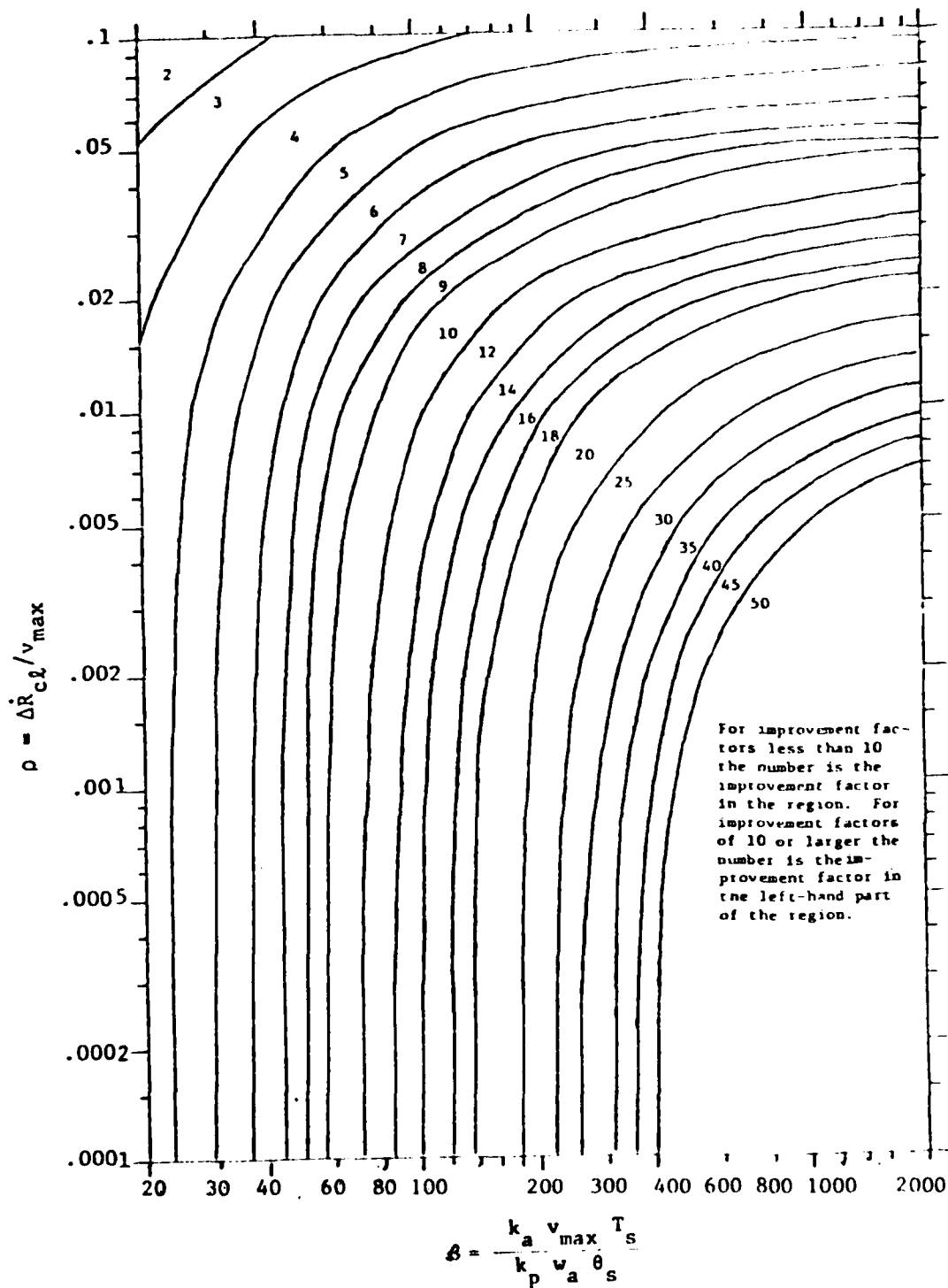


Figure 2-5. Maximum Blind Speed Improvement Factor, K

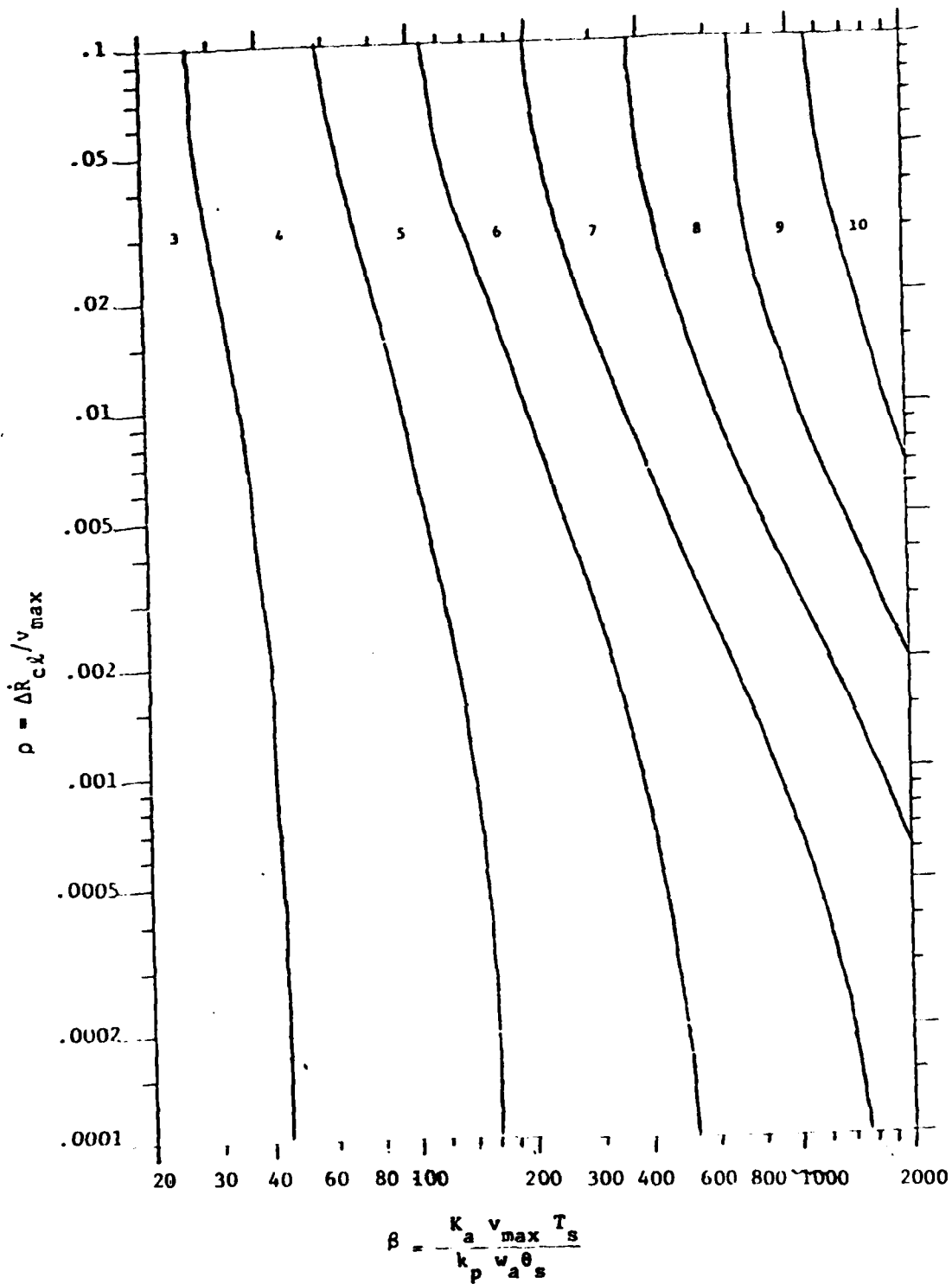


Figure 2-6. Optimum Number of Pulse Bursts, p , for Blind-Speed Improvement

the next higher integer value of N . If N is small then this rounding up can represent a large increase in dwell time beyond what was used in the optimization. Second, a large value of p implies the radiation of a great deal of radar power just to solve the blind-speed problem. That is, p times more power is required than is required when only one burst is used. It is therefore important to recognize that a natural bias away from the optimum and toward lower p is preferable. An easy means of recognizing the impact of lower than optimum p is therefore required.

Just such a format for the graphical solution has been developed, as illustrated in Figures 2-7 and 2-8. In this format each new value of ρ represents a separate graph, and a family of curves representing a variety of values of β is shown on each graph. It is then an easy matter to inspect the curve for the proper β and to see not only the optimum (maximum K) but also how much K must be sacrificed in order to arrive at a lower p . Since K is proportional to carrier frequency this tradeoff is easily interpreted in its impact on the radar system.

While the above graphical solutions and formulae are sufficient for sizing the radar system, they do not define the actual schedule of PRFs precisely. It has been found, however, that Johnson's tables^{*} can be used along with Table 2-1 to produce the actual PRFs to a reasonable accuracy.

When many different parameters and designs must be considered, however, a more automated approach is preferable. We therefore describe next the computer program that has been developed for automatically solving the blind-speed problem.

2.2 AUTOMATED DESIGN PROCEDURES

2.2.1 Approach

The need for a computer implementation of the waveform optimization is clear when one considers that a wide variety of environments must be investigated. A main objective of this project is to determine whether a radar sensor that

^{*}S. M. Johnson, *Multiple Repetition Frequency Radar Coverage (Maximum No-Gap Coverage)*, Rand Corp. Report R-969-PR. August 1972.

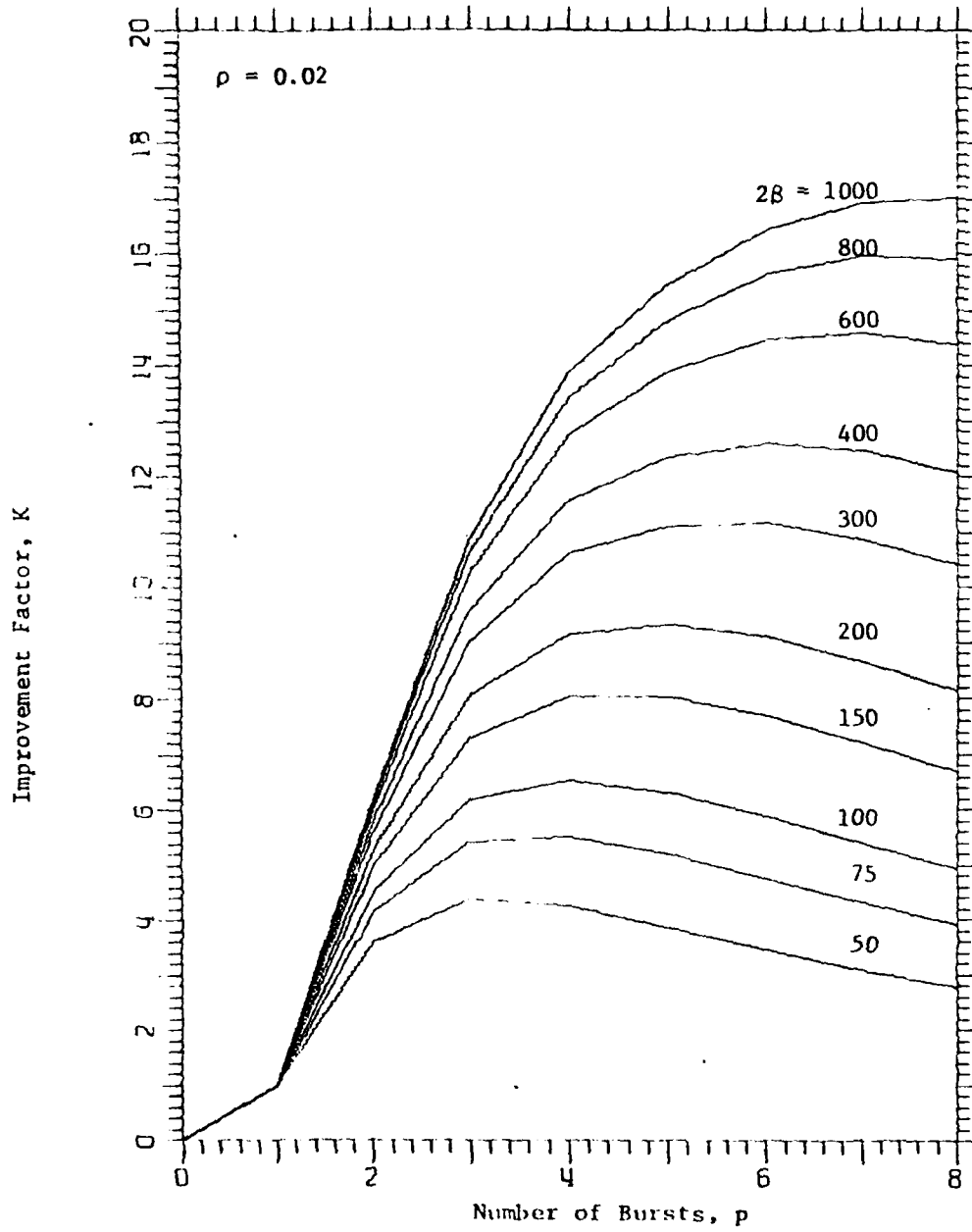


Figure 2-7. Optimum Parameters for $\rho = 0.02$

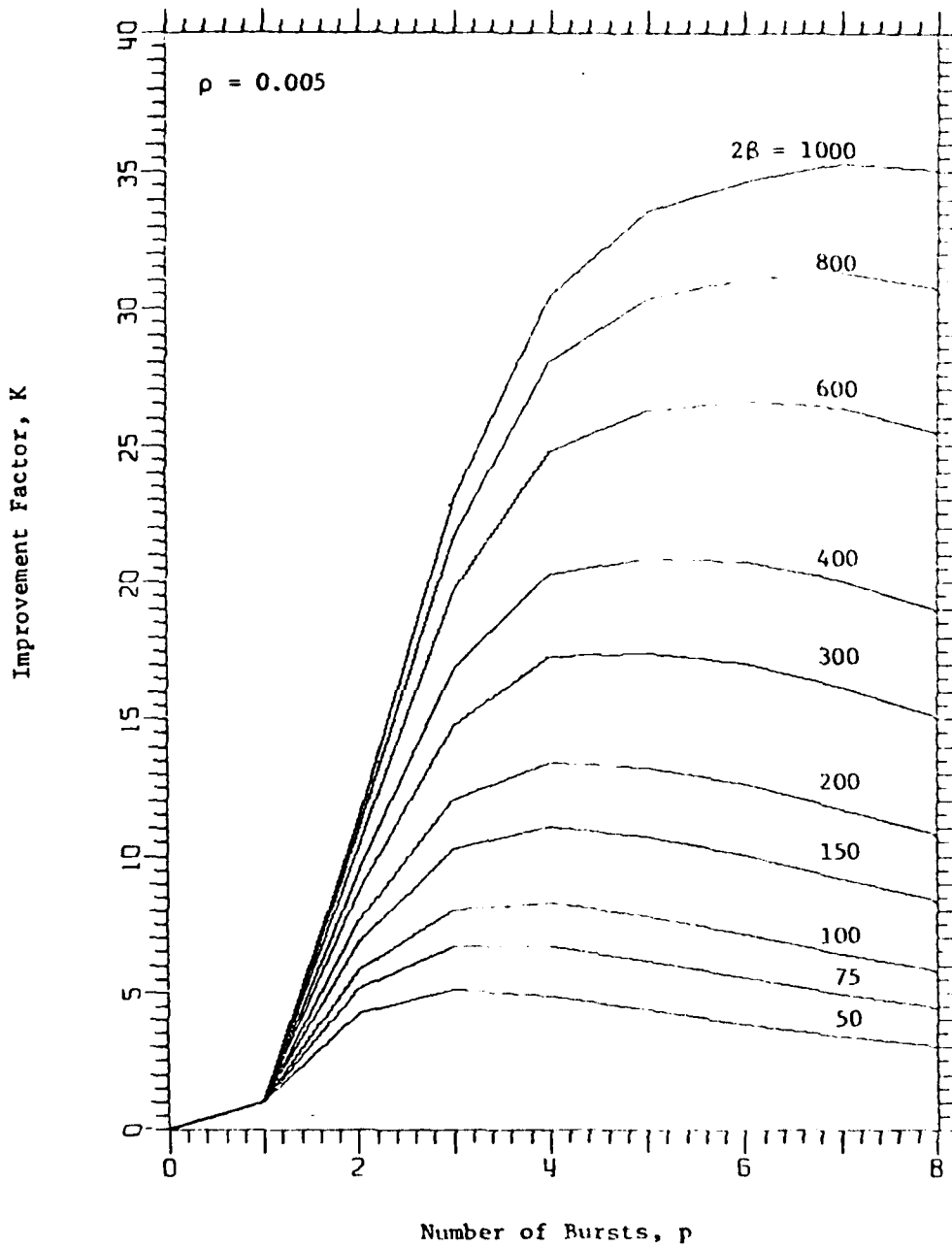


Figure 2-8. Optimum Parameters for $\rho = 0.005$

adapts to its environment has a major advantage over a nonadaptive radar. This adaptive radar must therefore be able to reoptimize its waveforms to environmental changes. Accordingly either the radar must perform a reoptimization in real time or a large number of waveform sets must be precomputed and stored in the computer. In either case the generation of a large number of waveform designs is implied.

In automating the design procedures it is not enough to merely automate the previously described graphical procedures. Instead one must consider that two range zones must be covered by the same radar. Thus, when a minimum wavelength that is achievable in one zone is lower than the minimum wavelength that is achievable in the other zone, the former minimum cannot be used. Instead the longer wavelength must be considered as a constraint in determining the waveforms for the former zone.

The automated design procedures must therefore be separated into two approaches. In practice one may first determine the minimum allowable wavelength for the near-range zone and then the minimum for the far-range zone. The actual wavelength will have to be the greater of these two minima. Then the waveform for the zone that results in the smaller wavelength will have to be redesigned with the greater wavelength as a constraint. The overall result will be a two-zone design that provides both short search times and small wavelengths.

The first design approach is therefore to find the solution that provides the minimum wavelength for given environmental parameters β and ρ . We shall hereafter call this approach "Case 1." The second design approach is to find the solution that provides the minimum required search dwell time for given wavelength (and hence improvement factor K) and parameter ρ . We shall call this approach "Case 2." The computer solution for each approach is detailed next.

2.2.2 Automated Design for Fixed Wavelength

Formulation of "Case 2"

We shall detail "Case 2" first because it will be easier to appreciate the details of "Case 1" once the solutions for "Case 2" are at hand. "Case 2" is the design approach in which the system wavelength has already been selected and now we want to find the radar waveform that minimizes the required search dwell time for a given range rate extent ΔR_{cl} for the clutter and maximum speed v_{max} for the target.

Expressed mathematically, we want to minimize T_s for a fixed λ and ρ . Recall, however, that wavelength is related to the blind-speed improvement factor K as

$$\lambda = 2 v_{\max} / f_r K$$

where f_r is that maximum allowable pulse repetition frequency (PRF)

$$f_r = c/2R_{\max}$$

where R_{\max} is the maximum range for the zone under design. Thus, since λ , v_{\max} , and f_r are fixed, K is also fixed. Recall that the parameter β is:

$$\beta = \left(\frac{k_a v_{\max}}{k_p w_a \theta_s} \right) T_s$$

which is proportional to the total dwell time T_s . Usually the parameters within the parenthesis will be prespecified so that they may be considered fixed for the purposes of the automated design. The entire design objective for "Case 2" can therefore be stated parametrically as: "minimize β for a given K and ρ ."

Optimization Approach

The optimum solution is subject to two design equations (as described in Howard, *et al*, *op cit*):

$$K = \frac{x^p - 2x^{p-1} + 1}{x - 1} \quad (3-1)$$

$$K \leq \frac{2\beta}{x(2\rho\beta + p)} \quad (3-2)$$

where x is a normalized PRF (maximum allowed PRF divided by Doppler spread of the clutter) given by

$$x = \lambda f_r / 2\Delta R_{cl}$$

(Recall also that K and p must be integers.) The constraint given by (2) can be written also as

$$\beta \geq \frac{pKx(K,p)}{2[1 - K\rho x(K,p)]} \quad (3-3)$$

where we have explicitly shown the dependence of x on K and p . β is therefore minimum when equality holds in (3) and the right-hand side of (3) is minimized. Since K and p are prespecified, however, the only parameter that can be adjusted to minimize β is the number of bursts p . We will therefore proceed to describe the optimization for the range of parameters of interest. (From experience this range is p from 1 to 12 and K from 1 to 60.)

If K is unity then we should of course use p as unity. If K is an integer larger than one then, due to the nature of $x(K,p)$, the right-hand side of (3) will have a single positive minimum with respect to p only at some $p \geq 2$. Thus we may implement a computer search for a minimum by starting at $p=2$ and evaluating (3), then increasing p one at a time and reevaluating (3) until the value of (3) starts increasing. Then the minimum of (3) will be recognized. This "brute force" approach is reasonable because there are at most only eleven values of p to consider. (Recall that the transmission efficiency is only $1/p$ when p bursts are required for blind-speed resolution. The smaller values of p are therefore preferred, and $p > 12$ is unacceptable.)

In evaluating (3), however, we need not *compute* the function $x(K,p)$ anew eleven times! Since only 59 values of K and 11 values of p are ever of interest for the subject radar, we may precompute the corresponding 649 (59 times 11) possible values of $x(K,p)$ and store them in a table. Then the evaluation of (3) is reduced to a small amount of arithmetic and a serial table reading. Such a table has been generated off-line by using a recursive algorithm:

$$x_{i+1} = x_i + \Delta x_i \quad i=1,2, \dots$$

where

$$\Delta x_i = \frac{x_i^p - 2x_i^{p-1} - Kx_i + K + 1}{px_i^{p-1} - 2(p-1)x_i^{p-2} - K}$$

and

$$x_1 = K^{1/(p-1)} + 1/K^{1/(p-1)}$$

We iterate until $\Delta x_i < 10^{-12}$ and use the resultant value at x_{i+1} as our value for $x(K,p)$.

Subtleties

A few details in the solutions must be handled as follows. For smaller values of p the right-hand side of (3) can be negative depending upon the values of K and ρ . In those cases there is no value of β which satisfies the conditions. However, for a fixed K and ρ as soon as a non-negative value of β exists it will be non-negative for any larger value of p . Thus we can determine the minimum p to be used to start the search for the minimum β .

One can show that if $\rho \geq 1$ or $K \geq 1/2\rho$ then no solution exists. On the computer output we shall denote this condition by assigning a value of $p=0$ and $\beta=0$. If the optimum value of p is larger than 12 we shall denote this condition by assigning $p=13$, $\beta=10^5$.

If K is 1 then p is 1. Under these circumstances x is undetermined in (1) and can have a value between zero and one. We use the value $x=1$ in the right-hand side in this case and thus have the computer output $p=1$, $\beta=1/[2(1-\rho)]$.

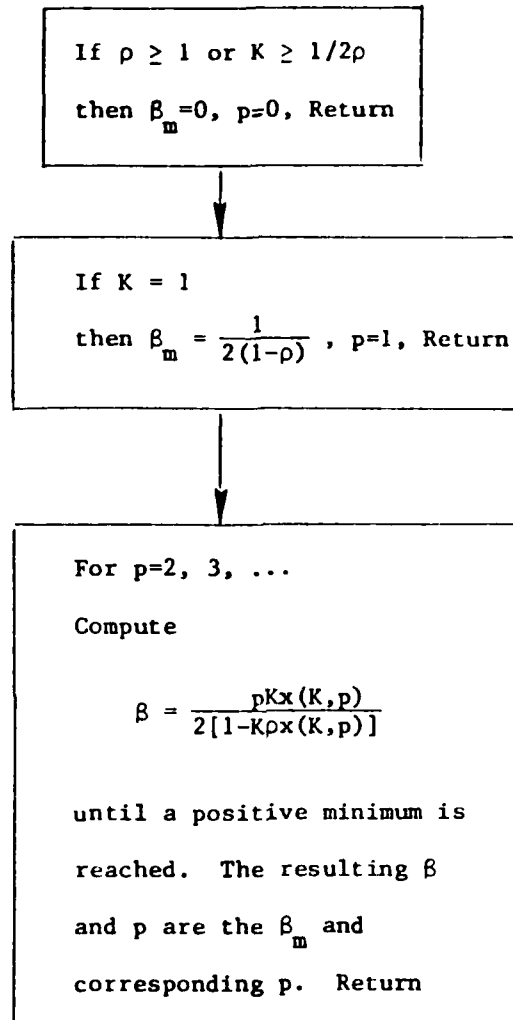
Subroutine OPB

Using the above procedure a FORTRAN subroutine called OPB (K, ρ, β_m, p) has been generated. For a given K and ρ it generates the minimum β ($\equiv \beta_m$) and the corresponding p . The flow diagram for this subroutine is shown in Figure 2-9.

2.2.3 Automated Design for Fixed Dwell TimeOptimization Approach

We now consider "Case 1" where dwell time is fixed and wavelength must be minimized. In this case β and ρ are given and we want the maximum integer K along with its associated p that satisfy the design equations (1) and (2). Since K is an integer it is possible that more than one p will provide the same integer value for K . In that case we choose the smallest of the alternative values of p because small p corresponds to high transmission efficiency.

Another way of stating the problem is that we wish to find the integer K which, for the given ρ , yields a β (and associated p) as close as possible to the given β . (The reason the problem can be stated in this form is the following. A given K and ρ will have a minimum β and corresponding p . Then these values of β and ρ will give a non-integer value for the maximum K . This is why we use the largest minimum β which does not exceed the actual β and yet gives an integer K .)



K and ρ are inputs to the subroutine. The minimum β (denoted β_m) and its associated p are outputs.

Figure 2-9. Subroutine OPB (K, ρ, β_m, p)

Subroutine OPK

A subroutine to compute the maximum K has been developed, and is called OPK $(\beta, \rho, K, p, \beta_m)$. (See Figure 2-10.) For inputs β and ρ it generates the maximum integer K and corresponding integer p . It also generates β_m , which is the value of β needed to obtain the integer value of K . That is, $\beta_m \leq \beta$, β_m yields as high an integer value for K as β , and at the same time the p corresponding to β_m is the smallest value which will yield this integer value for K .

The subroutine OPK $(\beta, \rho, K, p, \beta_m)$ uses the subroutine OPK (K, ρ, β_m, p) in the following way. It can be shown from (3) and the nature of $x(K, p)$ that β_m increases as K increases for a fixed ρ . We can therefore determine, by increasing K one by one, which K yields β_m closest to β but no larger than β . The subroutine OPB also determines the corresponding p .

This then gives us the maximum value of K and the corresponding p . Since β_m is a minimum and in general less than β , the form of (3) then implies that lowering p will increase β above β_m for K fixed at its maximum integer value. Thus for the fixed K and p we test lower values of p to find the smallest p for which the β is no larger than the β given in OPK.

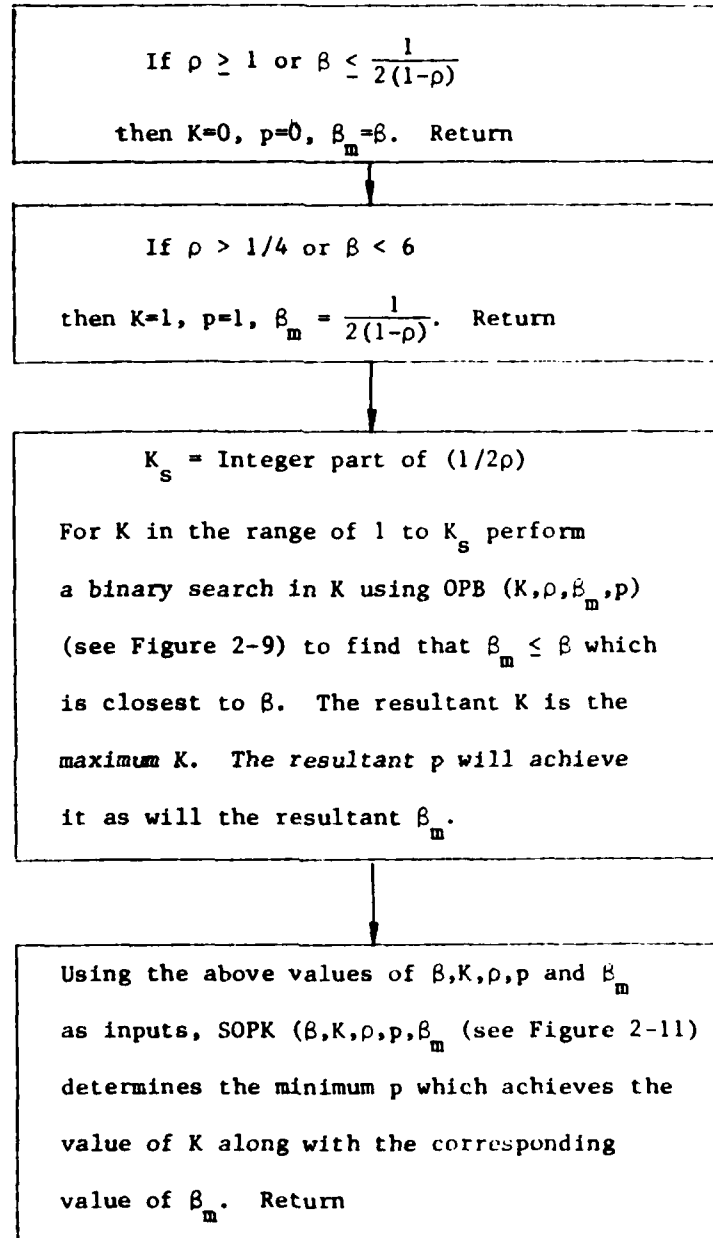
Subroutine SOPK

This is done using the equality sign in (3) in a subroutine called SOPK $(\beta, K, \rho, p, \beta_m)$. (See Figure 2-11.) The inputs to SOPK are the given β and ρ along with the maximum integer K and corresponding p and β_m found from OPK. The outputs from SOPK are β, K, ρ and the smallest p and associated β (still called β_m) which yield the maximum integer K and yet has $\beta_m \leq \beta$.

Since the value of K that we are seeking lies in the range of 1 to 60, it is more efficient to perform a binary search for K than to search sequentially starting at $K=1$ and proceeding upwards using OPB (K, ρ, β_m, p) . From (1) and (2) we can show that

$$K < 1/2\rho$$

so that the largest value to use for K in the binary search is bounded. The lowest value of K we use in the binary search is 2. If the search indicates the resultant K is less than 2 we take $K=1$. If the search indicates the resultant K is larger than 60 we use $K=60$. Again p is assumed no larger than 12 so that a $p=12$ will result if in reality $p > 12$. In this case the value of K obtained from OPK may be smaller than the maximum K .



β and ρ are inputs to subroutines. The maximum integer K and corresponding p are outputs. Also generated is the actual $\beta_m \leq \beta$ needed to achieve this K .

Figure 2-10. Subroutine OPK $(\beta, \rho, K, p, \beta_m)$

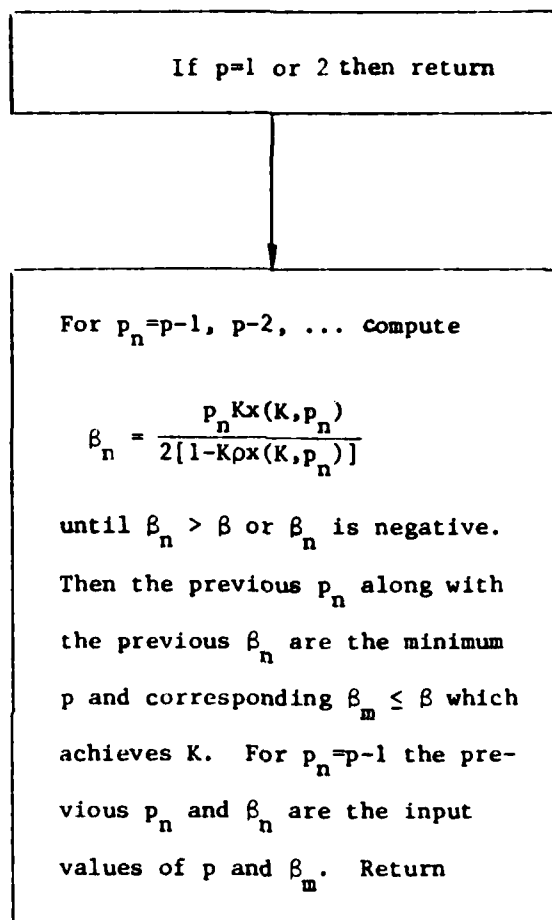


Figure 2-11. Subroutine SOPK ($\beta, K, \rho, p, \beta_m$)

As with OPB we have no solution if $\rho \geq 1$. We denote no solution by producing $K=0$, $p=0$ and $\beta_m = \beta$ in OPK. Also, from the form of (3) and the fact that $x < 1$ (even if $K=1$), we can show that no solution exists when $\beta \leq 1/[2(1-\rho)]$.

If the maximum allowable K is 1 then the corresponding p is 1. We then set $\beta_m = 1/[2(1-\rho)]$ because any β larger than this will be large enough to produce $K=1$, $p=1$.

2.2.4 Impact of Lower than Optimum p

OPK ($\beta, \rho, K, p, \beta_m$) yields the maximum integer K and corresponding p for a given β and ρ . Also OPB (K, ρ, β_m, p) yields the minimum $\beta = \beta_m$ and corresponding p for a given K and ρ . In these two optimization problems it is of interest to know how much is lost in the optimizations if smaller values of p than the optimum are used. Due to average energy constraints and/or processor complexity there are designs where it is advantageous to lower the value of p despite a sacrifice from optimality.

Thus given β, ρ and p (less than the optimum p) we wish to solve for K after we have used OPK. Given K, ρ and p (again less than the optimum p) we wish to solve for β .

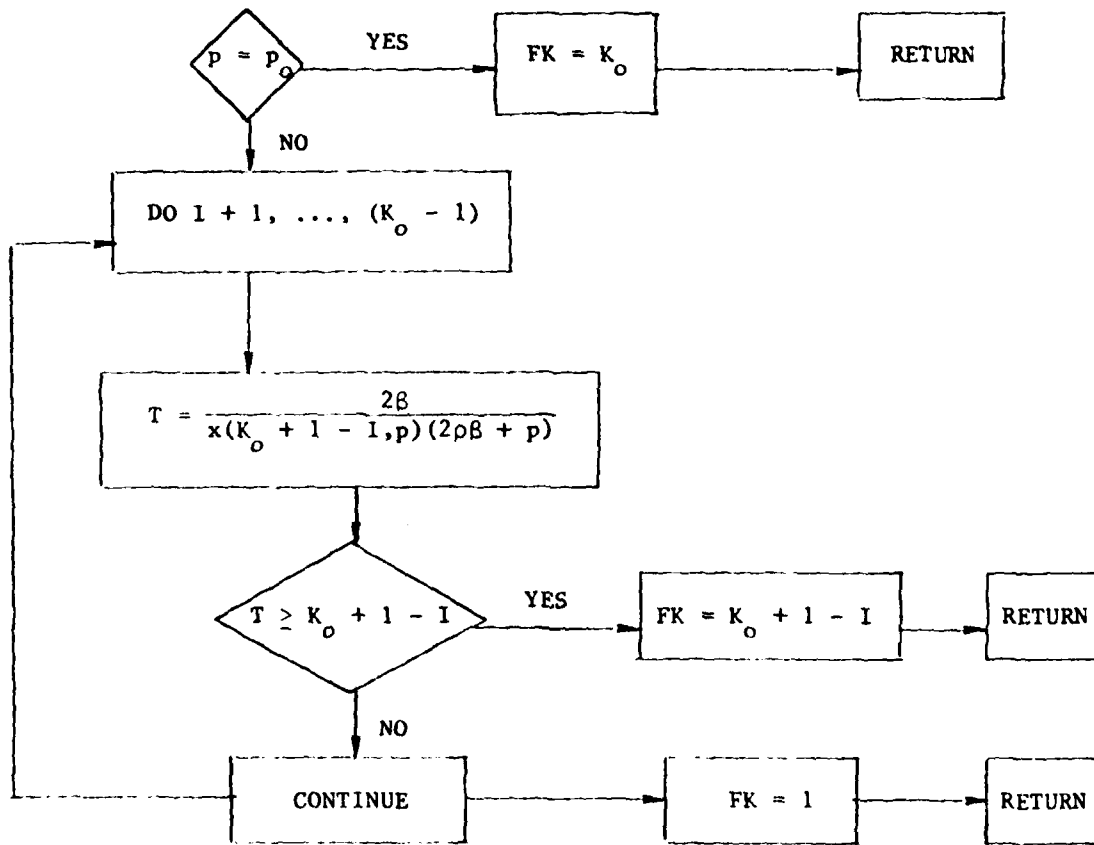
The former problem can be most easily solved by using (2). Substituting β, ρ and the p of concern into the right-hand side of (2) we put decreasing integer values of K into the right-hand side (beginning with the optimum K) until we reach a K where the inequality holds. The resulting K is the achievable K .

This suboptimum K is found by using the defined function called $FK(\beta, \rho, p_o, K_o, p)$. The flow chart for this function is shown in Figure 2-12. The inputs are the values of β and ρ . Also required are the resultant optimum values of p and K (called p_o and K_o) and the value of p for which we wish to calculate K (called FK).

For the latter problem we can simply calculate β using the equality sign in (3) (i.e., K, ρ and p are given). We do this using the FORTRAN function $FB(K, \rho, p)$ shown in Figure 2-13.

If p is made too small we may have a situation where β does not exist. In this case the right-hand side of (3) is negative. If this situation occurs we output $FB=0$ in Figure 2-13.

In Appendix A we have listed the code for the subroutines OPB (K, ρ, β_m, p), OPK ($\beta, \rho, K, p, \beta_m$) and SOPK ($\beta, K, \rho, p, \beta_m$) as well as the functions $FK(\beta, \rho, p_o, K_o, p)$ and $FB(K, \rho, p)$.

Figure 2-12. Function $FK(\beta, \rho, p_0, K_0, p)$

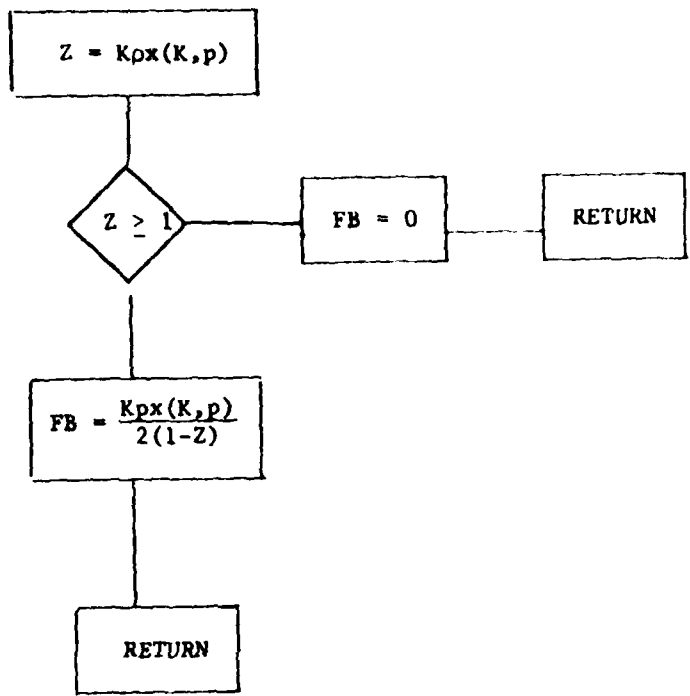


Figure 2-13. Function FB (K, rho, p)

2.3 SPECIFIC BASELINE DESIGNS

In order to illustrate the use of the new automated design procedures we use here the baseline parameters from our first report (Howard, *et al.*, *op cit*). Two distinct designs based on two different antenna concepts were developed. The first concept used a set of four rotating antennas with electronic scan in elevation. The second concept, which is clearly more costly but also more capable in tracking, used a set of four stationary antennas with electronic scan in both elevation and azimuth.

2.3.1 Design for Rotating 1-D Phased Array

Background

For the mechanically rotating antennas we are greatly constrained in the use of the antennas for various tasks. *Each* of the four antennas must be used partly for tracking in order to achieve the required 2 Hz tracking rate. (The antenna rotation rate is 30 rpm.) Roughly half of the time load for each antenna face is devoted to tracking. For one of the antenna faces the other half of the load is reserved for long-range search (search in the far-range zone) in which the frame time is allowed to be as many as 5 or so antenna revolutions. For the other three faces the other half of the load is reserved for short-range search (in the near-range zone) in which the frame time is only 2 seconds so that *all* bursts required for the blind-speed resolution must be implemented by the three faces in just one revolution of the antenna.

Far-range Zone

Let us consider the far-range zone first. For search in this zone we use just *one* antenna and devote only *half* of its time to search. Thus, in every antenna revolution we have available only 1 second for search. However, we allow the frame time to be 2 p seconds where p is the number of bursts required for blind-speed resolution, as long as p does not exceed about 5 or so. Thus the total search time available for this mode is p seconds for a search over 2π in azimuth. We also have no ground clutter in the long-range search, so that the clutter spread is just the range-rate spread of wind-driven rain, or 5 mps. The parameters for the long-range search are therefore

$$\begin{aligned} k_a &= 0.85 \\ k_p &= 1.0 \\ v_{\max} &= 1000 \text{ mps} \end{aligned}$$

$$\begin{aligned}
 w_a &= 3 \text{ m} \\
 T_s &= p \text{ sec} \\
 \theta_s &= 2\pi \\
 \Delta \dot{R}_{cl} &= 5 \text{ mps}
 \end{aligned}$$

so that the inputs to the computer design programs are

$$\begin{aligned}
 \beta &= 45.1 \text{ p} \\
 \rho &= 0.005
 \end{aligned}$$

The problem could now be stated that we wish to find the optimum K, p for a given ρ, β but where β is dependent on the solution for p . Clearly, if β were fixed, we could use the programs to find the optimum K, p . However, since β is proportional to p a different procedure is required. Recall that only a few values of the integer p are of interest. We could therefore approach the problem by finding K for a given ρ, β , and p , first for $p=1$ then repeating the procedure for all integer p 's of interest. We could then choose the K, p pair that is optimum or at least acceptable. This procedure is performed by repeatedly using subroutine OPK and function FK. For a preselected value of p we compute β (as 45.1 p in the example) and the use β, ρ as inputs to subroutine OPK. The output of the subroutine is the optimum pair K_o, p_o for the given *fixed* values for β, ρ . This optimum pair is now used, along with the pre-selected values for p, β , and ρ , as the input to the function FK. The value of the function will then be the value of K for the given values of p, β and ρ .

When the above procedure is followed for the parameters of the example, Table 2-2 and Figure 2-14 result. Evidently the achievable improvement factor K increases without limit as the number of bursts p is increased! In other words, by *not* constraining the search time and instead allowing it to grow with the number of bursts used for blind-speed resolution we find the very high improvement factors (and hence short wavelengths) are achievable without limit.

Evidently this conclusion is unrealistic. We had previously stated that we allow the search frame time to increase with p . A high p therefore implies not a need for more transmitted power but a need for a long frame time. The

Table 2-2. Solutions for Far-Range Zone

α	ρ	actual p	optimum p	optimum k	x	actual k
180.4	.005	2	4	12	2.896	7
270.6	.005	3	4	16	3.096	14
360.8	.005	4	5	20	2.649	19
451.0	.005	5	5	22	2.691	22
541.2	.005	6	5	25	2.748	25

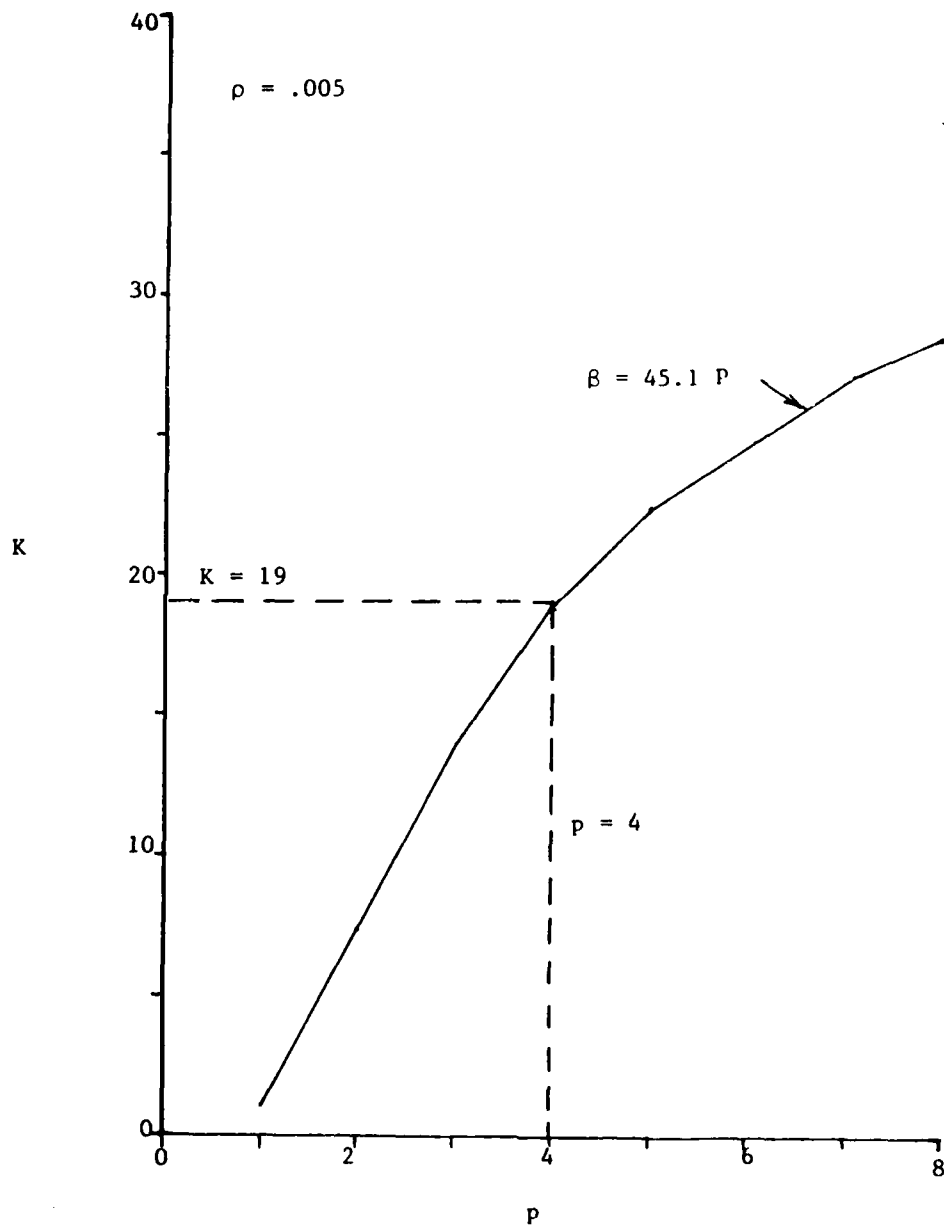


Figure 2-14. Example of Optimization for Long-Range Search

frame time is $2 p$ seconds for the example. An increased frame time is therefore acceptable for the far-range zone as long as p is not greater than 5. By contrast, frame times longer than 10 seconds would probably not be acceptable because of the possible extensive maneuvers of the targets within such a long frame. Thus $p < 5$ is a realistic constraint for the example.

If we choose $p=4$ we find from Figure 2-14 that the achievable improvement factor is 19. This solution results in an 8-second frame time, as desired. Since the maximum range for the far-range zone is 200 km, the maximum allowable PRF for the bursts is

$$f_r = c/2 R_{\max} = 750 \text{ Hz}$$

Accordingly the minimum achievable wavelength is

$$\lambda = 2 v_{\max} / f_r = 14.0 \text{ cm}$$

Near-range Zone

In the near-range zone we are even more constrained because we must perform the search within a single revolution of the antennas. Recall that one of the four antennas has been reserved for the search of the far-range zone. The remaining three antennas are therefore potentially available for the search of the near-range zone. However, half of the available time resources is reserved for tracking. Therefore each antenna has available for near-range search a 1-second dwell time out of the specified 2-second near-range frame time. Since three antennas are operating simultaneously for performing the search, it is possible to transmit separate bursts on separate antennas at the same time. On the other hand, since coherency must be maintained within each burst so that Doppler filtering is provided, the dwell time per burst is limited to 2 second for 1, 2, or 3 bursts ($p=1,2, \text{ or } 3$), 0.5 seconds for 4,5, or 6 bursts, 0.33 seconds for 7,8, or 9 bursts, etc. Evidently the available time is not efficiently used when p is not a multiple of 3. That is, the *total* dwell available is really 3 seconds because there are 3 antennas. But only 2 seconds are used when $p=2$, 2.5 seconds are used when $p=5$, etc., because of the constraint on dwell time per burst. Thus, for the mechanically rotating antenna we are biased toward the solutions for $p=3, 6, \text{ and } 9$.

Keeping in mind this constraint on p we may now attempt to achieve a satisfactory design as follows. As far as the mathematics and computer programs are concerned we can now develop the design by considering one antenna searching a $2\pi/3$ azimuth sector rather than three parallel antennas searching a 2π sector. The relevant design parameters are therefore:

$$\begin{aligned} k_a &= 0.85 \\ k_p &= 1.0 \\ v_{\max} &= 1000 \text{ mps} \\ w_a &= 3 \text{ m} \\ T_s &= 1.0 \\ \theta_s &= 2\pi/3 \\ \Delta \dot{R}_{cl} &= 20 \text{ mps (including ground clutter)} \end{aligned}$$

so that the inputs to the computer subroutine are

$$\begin{aligned} \beta &= 135.3 \text{ (equivalent to } \alpha = 270.6) \\ \rho &= 0.02 \end{aligned}$$

The optimum solution that results from an application of subroutine OPK is $K=10$, $p=5$. However, a nearly optimum solution exists at $K=9$, $p=4$. Furthermore, in light of the 3-antenna constraint a better solution is actually $K=8$, $p=3$. The maximum range in the near-range zone is 80 km so that the PRF is nominally

$$f_r = c/2R_{\max} = 1875$$

Accordingly, the minimum allowable wavelength is

$$\lambda = 2 v_{\max} / f_r K = 13.3 \text{ cm}$$

(Since we have not selected the mathematical optimum the actual dwell time needed for search may be less than the originally allowed time. In order to find the actual time now needed we may recompute the parameter β for the selected values of K and p . When we do this we find that β is 98 rather than 135.3. The time needed for the near-range search is therefore only 73% of the originally allocated time of 1 second per antenna per search frame.)

The 13.3 cm wavelength of this solution is slightly shorter than the minimum allowable wavelength for the far-range zone. It is therefore evident that the far-range zone determines the operating frequency in this example. Thus, we must now reoptimize the search in the near-range zone by relaxing the wavelength to $\lambda = 14.0$ cm and then solving for the actual allowable search time. We expect to find that a somewhat lower β is optimum for $p=3$, thereby allowing a reduction in the time that must be devoted to the short-range search. This reduction would make available more time for tracking, thereby improving the design beyond what would result if we merely retained the bursts determined for 13.3 cm wavelength.

When we attempt the reoptimization, however, we find that a relaxation of wavelength from 13.3 cm to 14 cm is not enough to materially alter the solution. In fact, we recall that K is related to λ through the parameters f_r and v_{\max} so that, for $\lambda=14$ cm we have

$$K = 2v_{\max} / \lambda f_r = 7.6$$

Thus, since λ has not been increased sufficiently to change K by a full integer, we must retain the preceding solution of $K=8$, $p=3$. However, we may improve the design by adjusting the maximum PRF downwards such that K is exactly equal to 8. That is, we select $f_r = 1786$ Hz (recalling that v_{\max} is 1000 mps, λ is 14 cm). The range ambiguities are now at a farther range than previously so that the maximum range of the near-range zone can be increased to

$$R_{\max} = c/2f_r = 84 \text{ km}$$

This increase in maximum range is the only practical benefit from the slightly lowered wavelength.

Summary of the Design

The baseline design for the mechanically rotating set of antennas can now be summarized from the preceding results along with Johnson's tables. The far-range zone is searched with one antenna in four successive revolutions of the antenna. A different PRF is used in each revolution in order to resolve all

blind speeds within the four revolutions (or 8 seconds). The PRFs for the solution ($K=19$, $p=4$, $x=3.23$) are, using an automatic algorithm for computing PRFs (to be described later)*

$$\begin{aligned} f_{r1} &= 750 \text{ Hz} \\ f_{r2} &= 722.75 \text{ Hz} \\ f_{r3} &= 668.26 \text{ Hz} \\ f_{r4} &= 522.94 \text{ Hz} \end{aligned}$$

(This set of PRFs leads to just one blind region at a Doppler frequency of 636 Hz and 5.25 Hz wide.) The number of pulses at each PRF are

$$\begin{aligned} n_1 &= 5 \\ n_2 &= 5 \\ n_3 &= 5 \\ n_4 &= 4 \end{aligned}$$

so that the total dwell time for all 4 bursts in one beam position is 28.7 milliseconds (rather than the theoretical $T_d = 25.3$ milliseconds). Thus 56.5% of the time load for one antenna is devoted to the long-range search.

The near-range zone is searched with three antennas in one revolution of the antenna. A different PRF is used for each successive look at the search area. The PRFs for the solution ($K=8$, $p=3$, $x=3.54$) are, using the automatic algorithm once again

$$\begin{aligned} f_{r1} &= 1786 \text{ Hz} \\ f_{r2} &= 1625.79 \text{ Hz} \\ f_{r3} &= 1345.42 \text{ Hz} \end{aligned}$$

(This solution produces a single 63.75 Hz wide blind zone centered at 1566 Hz.) The number of pulses at each PRF are

$$\begin{aligned} n_1 &= 12 \\ n_2 &= 11 \\ n_3 &= 9 \end{aligned}$$

so that the total dwell time for all 3 bursts in one beam position is 20.2 msec (rather than the theoretical $T_d = 18.9$ msec). Thus 53.4% of the time load for each antenna is devoted to the near-range search zone.

* At this point we could have used Johnson's tables, as described in our earlier report. However, when we developed techniques for determining the set of PRFs in real time, we found a better design approach, as detailed in Section 4.

2.3.2 Design for Stationary 2-D Phased Array

Background

We are much less constrained with a 2-D phased array in that the same antenna provides *all* bursts so that we have complete flexibility in the number of bursts allowed. Thus we need not define allowable search times consistent with the periodic reappearance of rotating antennas. If we assume a total frame time in the far-range zone of 8 sec, then we may perform the design for a total dwell of 1.5 seconds for a search over a $\pi/2$ sector. (The other three stationary antennas are designed identically.) Similarly, in the short-range zone we no longer find that $p=3$ and $p=6$ are the only easily implemented solutions. Thus such solutions as $p=4$ are allowed.

Far-range Zone

For the long-range search we now have the parameters:

$$\begin{aligned} k_a &= 0.85 \\ k_p &= 1.0 \\ v_{\max} &= 1000 \text{ mps} \\ w_a &= 3\text{m} \\ T_s &= 1.5 \text{ sec (for 8 sec frame time)} \\ \theta_s &= \pi/2 \\ \Delta R_{cl} &= 5 \text{ mps} \end{aligned}$$

so that the inputs to the computer programs are

$$\begin{aligned} \beta &= 270.6 \\ \rho &= 0.005 \end{aligned}$$

The computer solutions (or graphical solutions from Figure 2-5) are $K=25$, $p=5$, $x=2.75$. However, we also find little sacrifice in K by choosing $p=4$, for which $K=23$. Moreover, a 25% increase in power and time efficiency results, and the longer attendant wavelength makes the elevation beam sufficiently wide that a major gain loss from cosecant-squared beam shaping does not result. The PRF and wavelength corresponding to the solution are

$$\begin{aligned} f_r &= c/2R_{\max} = 750 \text{ Hz} \\ \lambda &= 2v_{\max}/f_r K = 11.6 \text{ cm} \end{aligned}$$

Evidently we have gained a great deal in terms of reducing λ merely by allowing more overall time for the long-range search because the constraints of the mechanical array have been removed. (Because we do not have to rely on revolving

antennas to achieve the required 2 Hz tracking rate, we no longer have to restrict the search time to half the load of one antenna, or 1.0 seconds per revolution per antenna. Thus the increase in allowance to 1.5 seconds per antenna is a consequence of the removal of the mechanical constraint.) Overall, 18.75% of the time load for the antenna is devoted to long-range search.

Near Range Zone

In the near-range zone we may work backwards from the design wavelength. That is, for the 80 km maximum range in the near-range zone we have the maximum PRF of

$$f_r = 1875 \text{ Hz}$$

Thus we have for the selected 11.6 cm wavelength a value for improvement factor K of

$$K = 2v_{\max} / f_r \lambda = 9.2$$

Since λ is fixed at 11.6 cm, we must use an integer value of K larger than 9.2. Otherwise v_{\max} would have to decrease and/or f_r would have to increase. Neither situation is desirable. On the other hand we do not wish to use a K any larger than necessary as the larger K, the larger α so that the scanning time would increase unnecessarily. Thus we take

$$K = 10$$

Since 10 is larger than 9.2 we can lower the PRF and thus extend the maximum range in the short-range mode. The result is

$$f_r = \frac{2v_{\max}}{\lambda K} = 1724 \text{ Hz}$$

The corresponding maximum range is

$$R_{\max} = \frac{c}{2f_r} = 87 \text{ km}$$

The value for K of 10, coupled with the value for ρ of 0.02 in the near-range zone, leads to an optimum solution for β and p . The result is $\beta=115$, $p=5$. The amount of time devoted to the short-range search is therefore (for one antenna)

$$T_s = \frac{\beta k_w \theta}{k_a v_{\max}} = \frac{115 \times 1 \times 3 \times \pi/2}{.85 \times 1000} = 0.638 \text{ sec}$$

out of the available 2-second frame time. Thus, only 31.9% of the total time for each antenna need be devoted to the short-range search.

Summary of the Design

In summary we use 18.75% of the time for long-range search, 31.9% of the time for short-range search, leaving 49.35% of the time for tracking. Moreover, this has been done with a wavelength of just 11.6 cm. This contrasts with the mechanical antenna set where *less than* 50% of the time remains for tracking and the wavelength is 14.8 cm!

We may now determine the actual schedule of bursts to be used for blind-speed resolution. The set of PRFs for the long-range search is found from our new algorithm as

$$\begin{aligned} f_{r1} &= 750 \text{ Hz} \\ f_{r2} &= 727.02 \text{ Hz} \\ f_{r3} &= 676.46 \text{ Hz} \\ f_{r4} &= 562.70 \text{ Hz} \end{aligned}$$

where a single blind zone of 34 Hz width remains. The number of pulses at each PRF are

$$\begin{aligned} n_1 &= 6 \\ n_2 &= 6 \\ n_3 &= 6 \\ n_4 &= 5 \end{aligned}$$

From these results we find a total dwell time for all 4 bursts of 34.7 msec (rather than the theoretical $T_d = 31.4$ msec). In this case 20.7% of the time load for one antenna is devoted to the long-range search.

The set of PRFs for the short-range search is

$$\begin{aligned} f_{r1} &= 1724 \text{ Hz} \\ f_{r2} &= 1623.61 \text{ Hz} \\ f_{r3} &= 1510.68 \text{ Hz} \\ f_{r4} &= 1313.05 \text{ Hz} \\ f_{r5} &= 1016.60 \text{ Hz} \end{aligned}$$

In this case *no* blind zones remain. The number of pulses at each PRF are

$$n_1 = 5$$

$$n_2 = 5$$

$$n_3 = 4$$

$$n_4 = 4$$

$$n_5 = 3$$

These results lead to a total dwell time for all 5 bursts of 14.6 msec (rather than the theoretical $T_d = 13.3$ msec). Then 35% of the time load for each antenna is devoted to the short-range search.

3. OTHER DESIGN APPROACHES

Two other design approaches were also studied in a further search for extended performance. The first approach is a variation of the baseline blind-speed resolution scheme in which multiple bursts are transmitted simultaneously on different frequencies. The main question in its application is whether the added performance warrants the added equipment required. The second approach is an extension of the baseline scheme to the resolution of blind ranges as well as blind speeds. The main question here is whether the added performance warrants the added complexity and sidelobe requirements.

3.1 MULTIPLE SIMULTANEOUS FREQUENCIES

3.1.1 Basic Principles

We have seen that the successful resolution of blind speeds consumes a considerable amount of extra dwell time. Our past approach has always been to transmit *sequentially* the p bursts that are required. However, if we can provide isolated parallel transmitting channels at different frequencies then we can conceivably transmit the bursts in parallel and thereby save a considerable amount of dwell time. Variations of this approach are detailed here.

The parallel use of multiple carrier frequencies in place of the sequential use of a single carrier frequency is acceptable in principle as long as one recognizes that it is range rate, not Doppler frequency, that must be made blind-zone free. Since range rate is related to Doppler frequency as

$$\dot{R} = \lambda f_d / 2 \quad ,$$

the ambiguous range rate is related to PRF as

$$\dot{R}_a = \lambda f_r / 2 \quad .$$

Thus a change in wavelength is equivalent to a change in PRF as far as blind-speed resolution is concerned. Moreover, any new wavelength that is sufficiently far for isolation from the basic wavelength could be used as long as the PRF were adjusted accordingly. The use of multiple frequencies therefore provides an easy means of cutting the dwell time to reasonable levels.

The conceptually most simple implementation would be to use p carrier frequencies with wavelengths proportional to the multiple PRFs that were

formerly required for blind-speed resolution. With this concept the same pulsing and PRF would be used for all carrier frequencies simultaneously. The approach therefore appears simple and attractive. However, a number of shortcomings become clear upon further examination. First, the range of carrier frequencies required is usually too large for most radars. For example, the PRFs required for the baseline design for the rotating antenna differ by factors up to 1.5. This means that the multi-frequency approach would require a very wide range of operating frequencies, such as 2 GHz to 3 GHz. Second, the tuning of the multiple frequencies is rather critical for successful elimination of blind zones. This means that when the environment changes such that a new set of PRFs would conventionally be required, the multiple frequencies must be precisely retuned. A truly adaptive radar would therefore be considerably more difficult to implement with multiple frequencies than with multiple PRFs.

The more reasonable approach from the standpoint of equipment is the use of separate carrier frequencies *for isolation only*, and the use of PRF adjustments for the actual blind-speed resolution. With this approach there are actually several levels of sophistication. For example, one could use a two-channel system in which two PRFs are sequentially and independently transmitted in each channel such that the blind-speed resolving capabilities of a four-PRF system are achieved. Or one could use a four-channel system with just one PRF for each channel (albeit each PRF is different). We therefore can achieve different levels of dwell time reduction depending on the different number of parallel channels available.

Designs for a multi-frequency radar can be developed with the aid of the automatic design procedures as follows. First it must be assumed that the multiple frequencies do not differ in wavelength by more than about 10%. With this assumption the change in Doppler resolution with change in wavelength can be ignored. Second, the basic (highest) PRF will be used at the lowest carrier frequency (longest wavelength). Denoting this PRF and wavelength as f_{r0} and λ_0 , we first design the system for this wavelength alone. However, we replace the total dwell time available for search, T_s , by the quantity qT_s where q is the number of parallel channels available. (Note that $q \leq p$ is required, and furthermore it is assumed that p/q is an integer so that all channels are used to their full capacity.) The set of PRFs

that results from a blind-speed optimization is then modified by the wavelength of the available channels. In general the lowest PRF is used with the shortest wavelength so that the *adjusted* PRF is raised toward the basic PRF. In other words, the multiple wavelengths available will have a normalizing effect on the PRFs so that the range of PRFs required is less extreme. We will illustrate these design procedures by redesigning the baseline systems under the assumption that four isolated parallel channels are available at frequencies that are spaced about 3% apart. (The relationship between the new adjusted PRF and the original PRF is, for the *i*-th burst,

$$f_{ri}^{new} = \lambda_o f_{ri}^{orig} / \lambda_i$$

where $\lambda_o > \lambda_i$.)

3.1.2 Design for Rotating 1-D Phased Array

We may redesign the waveforms for the long-range zone by merely multiplying the parameter β by q , the number of channels. (As long as the resulting number of bursts, p , is an integer multiple of q then the analysis is valid.) Thus we have

$$\beta = \begin{cases} 721.6 & \text{for } p=4, q=4 \\ 1443.2 & \text{for } p=8, q=4 \end{cases}$$

$$\rho = .005$$

The impact on the design is phenomenal. We find for $\beta=721.6$ that $K=43$, $p=7$ is optimum and for $\beta=1443.2$ that $K=57$, $p=8$ is optimum. Since we are constrained to $p=8$ for efficiency, β must lie in the range 800 to 1950, and K must be 45 to 57. If we assume K is 50 and that the basic PRF is 750 Hz at the longest wavelength then we find that this wavelength must be

$$\lambda = 2v_{max} / f_{ro} K = 5.3 \text{ cm}$$

which is much less than half the admissible wavelength for a single-channel system. The frame time is, however, 16 seconds for the long-range search!

It is more reasonable to restrict the number of bursts to 4 so that the frame time is 8 seconds or less. In this case β is 721.6 and the optimum p is 7, but we are constrained to the suboptimum choice of $p=4$. The achievable improvement factor K is therefore reduced from 50 to about 34. The corresponding wavelength is

$$\lambda = 2v_{\max} / f_{ro} K = 7.8 \text{ cm}$$

which is at least 40% shorter than the wavelength that results for the one-channel radar.

For the near-range mode, we have the parameters

$$\beta = 541.2$$

$$\rho = 0.02$$

for the four-channel system. Again restricting the number of bursts to $p=4$ we find that the achievable improvement factor is 14. In this mode, however, we may choose $p=8$ without an increase in search dwell time, and the more nearly optimum solution of $K=17$ results. For the 1875 Hz basic PRF of the near-range mode, the achievable wavelength is 6.3 cm, which is shorter than the wavelength allowable in the far-range zone. Thus we must constrain ourselves to the latter wavelength ($\lambda = 7.8$ cm) and work backwards. We find that the resulting K factor is 13.6. As with the near-range designs for the conventional approaches in Section 2, we must raise K to the next integer, which is $K=14$. The basic PRF then becomes 1832 Hz and the near-range search is extended to 82 km. Since $K=14$ can be achieved with $p=4$ this solution is acceptable. It leads to $\beta = 525$, which is 97% of the value used when $T_s = 1$ sec (i.e., $T_s = 0.97$ sec).

3.1.3 Design for Stationary 2-D Phased Array

We may redesign the system with the fully-phased array in an analogous manner. If we retain the allowance of 1.5 sec (out of 8 sec) for the long-range search, then we merely need to multiply the parameter β by the number of channels, $q=4$. The design parameters are therefore

$$\beta = 1082.2$$

$$\rho = 0.005$$

resulting in the optimum solution $K=51$, $p=7$. Since p must be an integer multiple of q we will use $p=8$ rather than $p=7$. The corresponding K is still 51, however, so $p=8$, $K=51$ is the solution. The resulting wavelength is $\lambda = 5.2$ cm, which is 35% lower than the mechanically rotating case.

If we now use this wavelength as a constraint for the near-range search, then the required improvement factor is $K=20.5$ for a 1875 Hz basic PRF. We may therefore relax the basic PRF to 1832 Hz so that K is 21 and the maximum range is now 82 km. The corresponding β for $p=8$ and $K=21$ is 1608. This solution implies that 2.2 seconds are needed for short-range search, which is more than the allowed 2-second frame time. This solution is therefore clearly inadmissible!

The difficulty is that the near-range search now controls the choice of wavelength, as we shall see. We choose $T_s = .638$ sec for the short-range search at each antenna. This is the time used for the short-range search and 2-D phased array when only one channel can be used. The corresponding β was 115. Since we have four channels this is multiplied by four so that in the near-range design we have

$$\beta = 460$$

$$\rho = 0.02$$

These parameters lead to $K=16$ at $p=6$. For the allowable solutions we have $K=16$, $p=8$ and $K=13$, $p=4$. Using $p=8$ we obtain $\lambda = 6.7$ cm. This is longer than the value $\lambda = 5.2$ cm found for the long-range search. Thus using the same search times as we did for the one-channel case we find that the short-range wavelength must be used.

If we use $p=4$ (which was used in the mechanical, four-channel design) we find that the short-range case now yields $\lambda = 8.2$ cm while the long-range case yields $\lambda = 7.2$ cm. Again the short-range wavelength of 8.2 cm must be used. But this is *larger* than the wavelength that was allowed with the mechanically scanned antenna using 4 channels and $p=4$. That is, we end up with a *worse* solution with the *better* antenna!

This difficulty can be overcome by using less time in the long-range search and thereby providing more time in the short-range search. Thus we shall use 1 sec (out of 8 sec) for the long-range search. For the long-range search we now have

3-6

$$\beta = 721.8$$

$$\rho = 0.005$$

resulting in the optimum solution $K=43$, $p=7$. For the allowable values of $p=4$ and $p=8$, we have $K=34$ and $K=43$, respectively.

For $p=8$ the resultant wavelength is 6.2 cm which is about 50% of the value which can be achieved with one channel. For the wavelength of 6.2 cm we find in the short-range search that we must use a $K=18$. For $p=8$ the resultant β is 625, which corresponds to a scan time of 0.85 sec (out of 2 sec) for the short-range case. We therefore devote 42.5% of the available time to short-range search and 12.5% of the time to long-range search, for a total of 55% of the time being devoted to search modes altogether.

In sum by using a four-channel system we have reduced the wavelength from 11.6 cm to 6.2 cm by devoting an equal amount of dwell time to both the long-range and short-range searches. The tradeoff is not all advantageous, however, because we now use about twice as many bursts for blind-speed resolution. This means that the average power required for detection is doubled, which could exceed practical constraints.

Suppose we instead constrain the number of bursts to $p=4$ in light of the possible power problem. Now we can design the four-channel system using one burst per channel. Under this constraint the long-range mode requires $K=34$. The allowable wavelength for the basic PRF of 750 Hz is 7.8 cm. Thus we still have a very worthwhile improvement over the 11.6 cm of the one-channel case.

Using 7.8 cm as the wavelength constraint on the near-range search, we obtain $K=14$, and a PRF of $f_r = 1832$ Hz. The short-range search is now extended to $R_{\max} = 82$ km. The corresponding β for $\rho=0.02$ and $p=4$ is $\beta=525$. Using the fact that we are using four channels this corresponds to a search time of 0.72 seconds for the short-range case. Now we are using 36% of the time for short-range search and 12.5% of the time for long-range search. This means we are using 48.5% of the time for search which leaves 51.5% of the time for tracking. This is an improvement over the 49.35% of the time left for tracking in the one-channel, while the wavelength of 7.8 cm is 33% smaller than the one-channel case.

Schedule of Bursts

The long-range search uses the following PRFs:

$$\begin{aligned} f_{r1} &= 750.00 \text{ Hz} \\ f_{r2} &= 733.82 \text{ Hz} \\ f_{r3} &= 692.75 \text{ Hz} \\ f_{r4} &= 569.53 \text{ Hz} \end{aligned}$$

A single blind zone of 19.4 Hz width remains with this set of PRFs. The number of pulses at each PRF are

$$\begin{aligned} n_1 &= 11 \\ n_2 &= 11 \\ n_3 &= 10 \\ n_4 &= 8 \end{aligned}$$

The short-range search uses the following PRFs

$$\begin{aligned} f_{r1} &= 1832 \text{ Hz} \\ f_{r2} &= 1744.76 \text{ Hz} \\ f_{r3} &= 1582.75 \text{ Hz} \\ f_{r4} &= 1258.72 \text{ Hz} \end{aligned}$$

A single blind zone of 37.4 Hz width remains with this set.

3.1.4 Implementation

We have seen that significant performance improvement can be achieved by using a multi-channel radar system in which the bursts for blind-speed resolution are implemented in parallel on different RF carriers. We have not addressed, however, the difficulty of implementing such a system.

Two difficulties are evident. First, a multi-channel system evidently requires a duplication of RF equipment such that all channels can be used at the same time. This requirement may be difficult to meet with a mobile system. Moreover, the equipment complexity may be excessive. Second, there is a severe problem of isolation among the separate channels. Since different PRFs are used in the different channels, a lack of adequate isolation could easily lead to eclipsing problems. That is, reception on one channel may be required at the same time as transmission on another channel. The only source of isolation is the separation in carrier frequency among the channels. Our experience indicates that adequate isolation is not achieved by frequency alone. Instead, it is usually necessary to use *separate* antennas for transmission and reception.

Since weight and size are already problems for this mobile radar, such a solution is probably not admissable.

In conclusion, unless the technology of the radar equipment can be advanced far beyond its present state, particularly in the area of transmit/receive isolation, the multi-channel approach outlined here will be impractical.

3.2 SIMULTANEOUS BLIND-RANGE/BLIND-SPEED RESOLUTION

3.2.1 Background

In the past there has existed a consensus that ground-based radars which must operate in a heavy clutter environment must be designed without range ambiguities within the maximum detection range. In the presence of range ambiguities, a target to be detected near the maximum detection range would have to compete against clutter folded over from areas very close to the radar. The R^3 law for ground clutter, and in the case of rain the R^2 law for rain clutter, would enhance the clutter relative to the signal to such a degree that clutter suppression would be impossible. Thus it has become common practice to design ground-based radars without range ambiguities.

On the other hand, the consequence of a range-unambiguous design is the appearance of a large number of blind-speed regions within which aircraft and missiles cannot be detected. Since targets may go at constant range rate for relatively long times, in particular if they are headed toward the radar, blind speeds are unacceptable for a high-performance radar. Thus one must introduce schemes for avoiding blind speeds, which means multiple PRFs. If this is done inefficiently, one is wasting such important radar resources as average transmitter power and dwell time on the target. In particular, with respect to dwell time, it is found that avoiding blind speeds may increase the required dwell time to such an extent that one is forced to choose a low carrier frequency, such as L-band or even GHF. These radars then have poor angular tracking accuracy, they are subject to severe multipath effects, they are more readily jammed, and they have an insignificant capability for target identification.

It appears that the optimum solution (that is, the solution that allows the highest carrier frequency) is to allow *both* blind ranges and blind speeds and then provide sets of PRFs that allow the resolution of both types of ambiguous clutter. The problem of the R^3 -enhanced strength of range-ambiguous

clutter presents a new requirement for low sidelobes which may not always be achievable in practice. Moreover, the elimination of *all* blind-speed and blind-range zones may not be possible in general. But, as we shall see, the performance potentials are so high from a combined system that it warrants our serious consideration at the least.

3.2.2 The Basic Principle

The conventional approach to PRF selection in a heavy clutter environment for a ground-based radar is to choose the fundamental PRF so that the first range ambiguity will be at the maximum detection range. If multiple PRFs are required to avoid blind speeds, then all other PRFs will be lower than the fundamental one so that there will be no range ambiguities closer than the maximum detection range. The wisdom behind this choice is that range ambiguous operation causes an enhancement of the clutter, which is to be avoided.

However, in spite of the enhancement of clutter there is much to be gained by range-ambiguous operation. The additional clutter could conceivably be suppressed by Doppler filtering, and the higher PRFs make the blind-speed avoidance problem much easier.

In order to illustrate how clutter is enhanced by range-ambiguous operation, let us assume that the fundamental PRF is 5 kHz, so that the first range ambiguity will be at 30 km. In volumetric clutter the received power from clutter will be proportional to $(1/R)^2$ for the first 30 km. At $R=35$ km the received power will be proportional to $(1/35)^2$ plus the ambiguous component at $R=5$ km, so that the total clutter power will be proportional to $(1/35)^2 + (1/5)^2$. In general, we can define

$$\begin{aligned} R &= \text{target range} \\ R_A &= \text{range of first ambiguity} \\ N &= \text{largest integer less than } (R/R_A) \\ R' &= R - NR_A \end{aligned}$$

The received clutter power will then be proportional to

$$P_c \propto \sum_{n=0}^N \frac{1}{(R' + nR_A)^2}$$

If we compare the received clutter power with the component at range R , we can define an enhancement factor

$$E = \sum_{n=0}^N \left(\frac{R}{R + nR_A} \right)^2$$

which is illustrated in Figure 3-1.

The Doppler filtering processor must be designed to suppress the enhanced clutter. For example, if the processor is capable of handling an additional 20 dB of clutter, we can read from Figure 3-1 and determine that the particular pulse burst will be blind at:

$$\begin{aligned} R_A &\leq R \leq 1.10 R_A \\ 2R_A &\leq R \leq 2.25 R_A \\ 3R_A &\leq R \leq 3.50 R_A \\ 4R_A &\leq R \leq 4.70 R_A \end{aligned}$$

If the processor could handle an additional 30 dB of clutter (10 more than above) we can limit the blind-range interval to about 10% of R_A up to the fourth range ambiguity. In Figure 3-2 we show the combined range-Doppler blind zones for a 2-PRF burst of 7 and 5 kHz with a blind-range interval of $0.10 R_A$ and a blind-Doppler interval of 2 kHz. Note that the first range at which both PRFs are blind is 7 times the first blind-range of the fundamental PRF (7 kHz), which is 150 km. The first Doppler at which both PRFs are blind is 14 kHz. There are also local zones at which both PRFs will be blind, but these zones amount to only about 5% of the total range-Doppler area of interest. Additional PRFs can be used to improve coverage, or to reduce the clutter enhancement.

We have expended a great deal of effort in attempting to develop systematic procedures based on these principles, but the choice of PRFs for operation in range-ambiguous clutter is not yet solved. There are many considerations such as the maximum detection range, first blind speed, Doppler width of clutter, and clutter suppression capabilities via Doppler filtering. We believe that a systematic approach could be evolved, and the result would be superior to the conventional approach of unambiguous operation. We will demonstrate the possible improvements through the following intuitive look at the approach.

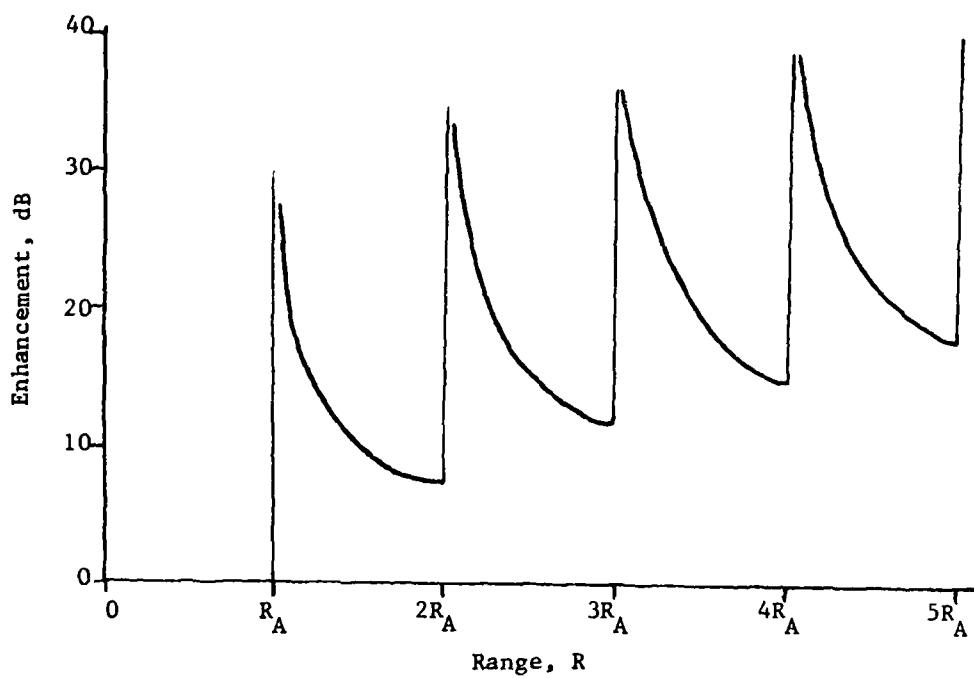


Figure 3-1. Enhancement Factor for Range-Ambiguous Volumetric Clutter

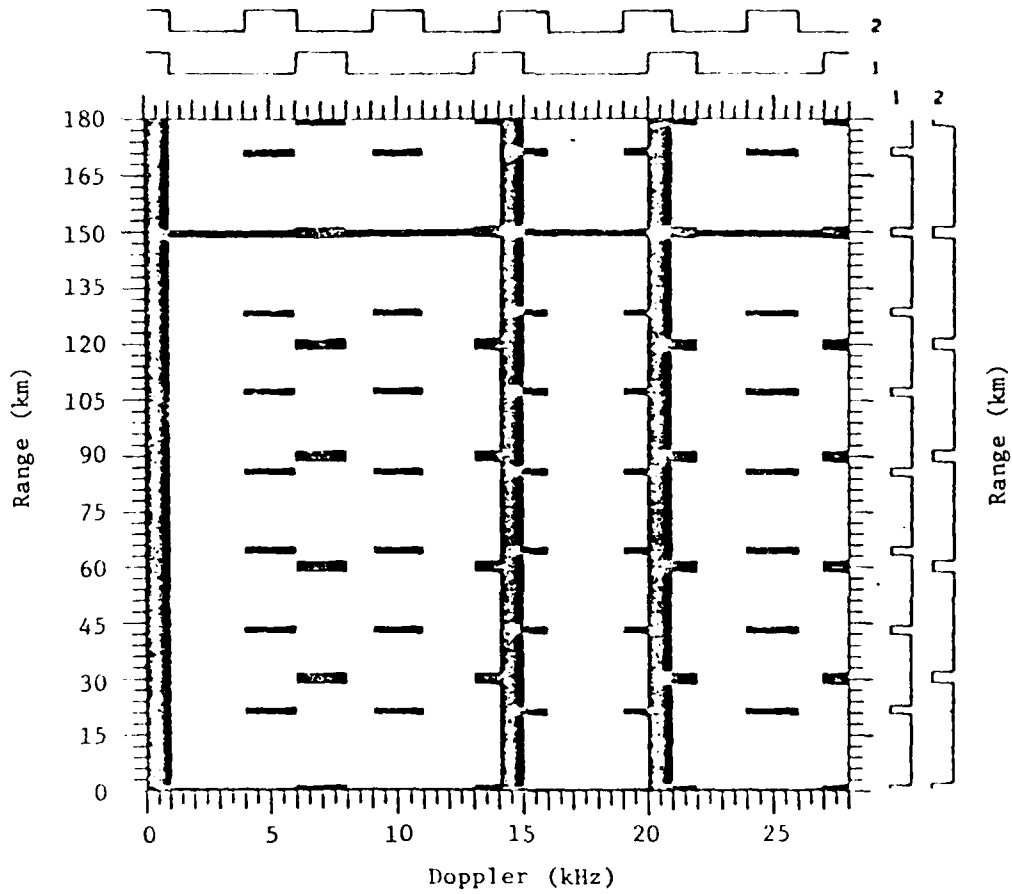


Figure 3-2. Combined Range-Doppler Blind Areas for a 2-PRF Burst

$$PRF_1 = 7.0 \text{ kHz}$$

$$PRF_2 = 5.0 \text{ kHz}$$

10% blind range interval

$$\Delta f_{B1} = 2 \text{ kHz}$$

3.2.3 Achievable Performance Improvement

The given quantities in any such system are the maximum detection range, R_{\max} , the maximum target speed to be accommodated, v_{\max} , and the maximum range rate spread of the clutter, $\dot{\Delta R}_{cl}$. We assume that the ambiguous range is to be some fraction of the maximum detection range,

$$R_a = R_{\max}/A, \quad (3-1)$$

where it would be desirable for A to be no larger than unity. The case to be discussed is where A must of necessity be larger than unity.

From (3-1), the repetition frequency is

$$f_r = c/2R_a = cA/2R_{\max}, \quad (3-2)$$

and by multiplication by $\lambda/2$ we obtain the ambiguous range rate

$$\dot{R}_a = cA\lambda/4R_{\max}. \quad (3-3)$$

The normalized basic PRF of the system is simply the ratio of \dot{R}_a and the range rate spread of the clutter, $\dot{\Delta R}_{cl}$, so that from (3-3) we have

$$x = \dot{R}_a/\dot{\Delta R}_{cl} = cA\lambda/4\dot{\Delta R}_{cl}R_{\max}. \quad (3-4)$$

It is seen from (3-4) that the normalized PRF decreases with the wavelength. Since blind-speed resolution is impossible when x is much smaller than two, a decrease of the wavelength must beyond some point be offset by an increase in the value of A , since the other parameters in (3-4) are fixed. A value of A exceeding unity, however, implies the introduction of range ambiguities, so that the system of multiple PRFs must be designed to avoid both blind speeds and blind ranges. The interesting question is whether there are any limits on the maximum value of A , and what these limits are if they do exist. In other words, how do we find sets of PRFs for combined blind-speed and blind-range resolution which allow us to achieve high values of A , and are there any fundamental restrictions?

On the basis of a limited study of the problem, we have come to the following conclusions. Although there is a systematic way of designing a set of PRFs for complete elimination of blind speeds, this is not possible for the two-dimensional case of combined blind-speed and blind-range

elimination. Blind regions in the range/range-rate plane can be eliminated, or made insignificantly small, for close ranges. With increasing range an increasing percentage of the range/range-rate plane will be blind, until beyond some range essentially the entire region is blind. Thus there appears to be a fundamental restriction on how large a range interval can be accommodated with acceptable blind regions.

It turns out that, with a good design of the set of PRFs, the range interval over which blind regions can be reduced to acceptable level is inversely proportional to the range-rate spread of the clutter. The larger the range-rate spread, the smaller the range interval that can be "cleared". This maximum operational range can be written as

$$R_{op} = \eta \frac{c\lambda}{4\Delta\dot{R}_{c\ell}} \quad , \quad (3-5)$$

where the factor η depends on the design of the PRF set.

We have found in the limited research that a factor $\eta=1$ is easy to achieve, but that the blind regions increase rather rapidly in relative size when the value of unity is exceeded. There appear to be no problems of finding PRF sets for $\eta=1.2$, and perhaps $\eta=1.3$. It is more difficult, but possible, to find sets with $\eta=1.5$. The general trend is that larger values of η can be achieved with a larger number of PRFs, evidently at the price of increased system complexity and power inefficiency. We shall postpone a discussion of how the PRF sets are found, and simply assume here that the value of η that can be realized is limited. Although our research is not broad enough to permit a completely reliable estimate for the maximum value of η , we shall here assume as a reasonable estimate

$$\eta_{max} = 1.5 \quad . \quad (3-6)$$

We return to our system design problem, which means finding the value of the normalized PRF x of (3-4). Since x must be larger than two but a small wavelength is desired, we must increase the value of A . Thus we consider next the question of how large A can be made.

We evidently must make the operational range R_{op} of (3-5) equal to the maximum detection range R_{max} . Substituting R_{op} for R_{max} in (3-4) gives

$$x = A/\eta \quad , \quad (3-7)$$

so that

$$A_{\max} = x \eta_{\max} \quad . \quad (3-8)$$

If we substitute A_{\max} into (3-4), we find the minimum wavelength for which combined blind-range and blind-range rate resolution is possible,

$$\lambda_{\min} = 4\dot{\Delta R}_{cl} R_{\max}/c\eta_{\max} \quad . \quad (3-9)$$

With $\eta_{\max} = 1.5$, we have

$$\lambda_{\min} = 2.7 \dot{\Delta R}_{cl} R_{\max}/c \quad . \quad (3-10)$$

As an example, suppose $\dot{\Delta R}_{cl} = 20$ m/sec and $R_{\max} = 200$ km. The minimum wavelength for which the problem of blind ranges and blind speeds can be overcome then is $\lambda_{\min} = 3.6$ cm. Since the minimum value of x is two, we see from (3-8) that such a design would have at least a triple range ambiguity.

In practice, the wavelength may be fixed. If it is not fixed, then we may estimate from (3-10) the minimum usable wavelength from the point of view of avoiding clutter problems, and assess other problems at this wavelength (such as rain clutter and rain attenuation strengths). This will then lead to a decision about the wavelength to be used.

Once the wavelength is chosen, we can from (3-4) find the relation between x and A . A must be large enough to yield a value of x exceeding two, and the larger the value of x , the fewer the number of PRFs that must be used. Thus we want A large. On the other hand, the more closely A approaches its maximum value as given by (3-8), the more serious the residual clutter problems will be. Thus the value of A is bracketed between A_{\min} corresponding to a value of $x=2$, which from (3-4) is

$$A_{\min} = 8\dot{\Delta R}_{cl} R_{\max}/c\lambda \quad , \quad (3-11)$$

and the value A_{\max} of (3-8). However, the latter constraint means only that x must be made large enough to accommodate the chosen value of A . A high value of A means that there are many effective range ambiguities. The simplified derivation on which the above conclusions are based does not fully take into account the increase in clutter problems with increasing order of

the range ambiguity. Thus we want to keep A , which means the number of range ambiguities, as small as possible; that is, close to the value of A_{\min} of (3-11).

In the design procedure, we find from (3-11) the value of A_{\min} , and from (3-7) the corresponding value of x . We decide at this point how critical the clutter problem and the situation are, by deciding whether we want to design for $\eta=1.2$, 1.2, or a higher value. The more PRFs that are used, the higher the admissible value of η , but also the higher the cost. We survey the available solutions for the sets of PRFs, and decide on the best compromise between x and η . What is still needed is the set of "best" solutions for x and the associated other normalized PRF values.

It appears that the design of an *optimum* set of PRFs is not possible. We have found one set of solutions by a combination of reasoning and computer trials. We have found other solutions by starting with the sets of PRFs used for resolution of blind speeds only, and multiplying the normalized PRFs by some factor. This has revealed the general trend of results as discussed earlier, yet some solutions are better than others and locally reverse the trend. The best procedure for a generalized system of solutions appears to be the following: Start with the blind-speed only sets of PRFs for three, four, and five PRFs, and in each case for different values of the extension factor for the blind speed. Also multiply these normalized PRFs by some selected factors. For these diverse sets of PRFs we then find the value of η for which percentage blindness at the largest range exceeds a certain threshold. We then select those solutions for which η is largest for a given number of PRFs and a given extension factor (or equivalent measure). The resulting table of solutions is the one to be used in the system design. The same solutions are used in an adaptive system, where the clutter conditions are changing and range ambiguous operation is necessary under some conditions and in some beam positions.

3.2.4 Application to Radar Design Concepts

The difficulty of finding a systematic solution to the combined blind-speed/blind-range problem has led us to the conclusion that a further study is beyond the scope of the present program. However, we have recently had the opportunity to study the problem of a low-probability-of-intercept (LPI)

radar in which the principles just discussed are applicable. Our study of the LPI problem has led to the conclusion that the combined blind-range/ blind-speed problem *must* be solved if future radars are to be made significantly less vulnerable to anti-radiation missiles. We therefore review the tradeoffs here to demonstrate that a separate program is needed.

Background

Our study of the LPI problem has shown that a common assumption made in these types of investigations is that the ARM will use essentially the same type of receiver against LPI radars as is being used against existing radars. Specifically, the assumption is made that the ARM will not use a high degree of noncoherent integration of the radar signal. We believe that this is an entirely unrealistic assumption. Since the signal of a conventional radar can be detected with extreme ease by an intercept receiver, there is no justification for designing any but the simplest type of intercept receiver. However, with LPI developments under way, it is reasonable to assume that better intercept receivers will be designed. In particular, it is simple and inexpensive to design the intercept receiver so that it uses long-term *non-coherent* integration. However, with current radar designs and conventional design approaches one can show it to be impossible to prevent easy homing on part of the ARM. Although it is true that one can deploy radar decoys in order to protect the radar, these decoys become rather expensive and vulnerable if designed for conventional radars.

At MARK Resources we have studied the problem and have found that LPI performance can be obtained if nonconventional design approaches are being used for the radars. In fact, these approaches appear to be so effective that it may be unjustified to design modern radars in accordance with the old methods, including the ones we have developed at MARK Resources. Hence it appears to us extremely important to extend the present problem on radar adaptivity to the inclusion of LPI. We see a way of modifying our design methods to where effective LPI radars can be designed just as efficiently and systematically as we have done previously for "ordinary" radars.

Brief Description of the New Approach to Radar Design

There are several departures from conventional radar design which, taken together, allow the implementation of a true LPI radar. By this we mean a

radar that prevents successful homing by an ARM. We shall briefly describe these new design features.

As perhaps the most important features, the radar must be designed so that it operates intermittently. For example, it might search for half a second and not transmit at all for the next half second. It is, of course, obvious that such intermittent operation is highly desirable. What is not so obvious is how to perform all the required radar functions within the reduced time, where even with conventional designs it is difficult to provide sufficient time for all functions.

Our study of the problem has shown that dwell time requirements can be drastically reduced by going to range-ambiguous operation. The ambiguous range is chosen much smaller than the maximum detection range. We have found that the unsolvability of the clutter enhancement problem due to the ambiguous foldover is a myth. Although penalties must be paid, these penalties can be made to be acceptable. The overall result is that the dwell time requirements are reduced by a factor roughly in the order of the multiplicity of the range ambiguity. In other words, if we introduce five range ambiguities within the maximum detection range, we would expect a dwell time reduction by a factor of roughly five.

The scheme for resolving blind speeds for range-unambiguous operation must be replaced by one for resolving the combination of blind speeds and blind ranges. We have done enough work to know that the problem is solvable, and we have investigated particular approaches. We also understand the limitations, but these limitations are beyond the typical practical combinations of maximum target speeds and maximum detection ranges. What is not available is a unified theory of optimum algorithms for selecting multiple PRFs for combined blind-speed/blind-range resolution for minimizing dwell time requirements.

A second important feature is the use of many range zones during the search. The farthest range zone, for example, requires the highest transmitter power, but the search frame time for this far zone can be very long, perhaps close to a minute for really long search ranges. A high-power signal that is observed only once every minute evidently is of no use for an ARM. As the search zones get closer to the radar, the search frame time

must be reduced. However, the transmitter power will be reduced in accordance with the R^4 law. Thus, as the frame times become shorter, the signal will be more and more difficult to detect. We may use a minimum-range search zone for which the largest detection range is, say, only 20 km, with a search frame time of only one second. The radar power then will be many orders of magnitude lower than for the maximum range; so low in fact, that successful homing on the part of the ARM can be prevented.

We have used different search zones in our earlier designs, but for different purposes and subject to different constraints. For LPI radar, for example, we want to time the search over all zones so that when the search for different zones coincides, it can be done with a single transmission, just as with an ordinary radar. This implies, however, that the multiple PRF scheme for blind-speed/blind-range resolution is appropriately designed. A consecutive search over different search zones must be avoided in order to achieve a longer dead time for the radar.

Another important design constraint concerns the presence of multiple radars. It is usually argued, with much hand-waving, that a radar with only a minimal LPI capability might prevent homing by an ARM since there will be more than one radar in the battlefield. A closer examination of this point shows that the presence of multiple radars can indeed be utilized for improved LPI performance, but only if this is taken into account in the design procedures. For example, use of a very high carrier frequency will allow the ARM to implement a sharp beam, for better angular resolution of multiple radars. On the other hand, much of this advantage can be offset by the tremendous differences in the power levels used by these radars for the different search zones. A radar in the outskirts of the ARM beam and transmitting at the power level for the far zone can appear much stronger for the ARM than a radar within the beam and transmitting at the power level for the close zone. The resulting phase center wander is another contribution to the defeat of the ARM.

Recommendations

These preliminary results indicate that a program should be initiated for extending our design procedures to radar with LPI capability. The central issue is that of combined blind-speed and blind-range avoidance by multiple

PRFs, and its interrelation with clutter enhancement due to range ambiguities. Optimum methods of dividing the range interval into multiple search zones, with a particular view toward the compromise between reduction of the dwell time and increased clutter enhancement, must be developed. The interrelation with search frame times as a function of distance from the radar and power levels as seen by the ARM must be studied, and optimum design procedures must be developed. In essence, we should arrive at a point where the design of an LPI radar can be carried out in the same systematic fashion as for a conventional system.

4.0 DESIGNS FOR ENVIRONMENT-ADAPTIVE RADAR

The designs reviewed thus far have been cognizant of the environment but have not been adaptive to it. For example, the systems have been designed to handle ground clutter to some maximum range (say, 80 or 100 km), but have not been designed to readjust themselves as this maximum range changes with time (or with time-varying azimuth angle). The final big step in performance improvement is expected from this and other adaptive readjustments to the environment. Just how this is done is detailed in this section along with a preliminary estimate of the expected performance improvement.

4.1 REAL-TIME ADAPTATION OF WAVEFORM DESIGN

4.1.1 System Approach

The Need for Adaptivity

We have previously determined that the solution of the blind-speed problem dominates the radar design. It is therefore natural that a readjustment of the solution to a changing environment would dominate the design of an adaptive radar. Since the blind-speed problem is solved through proper waveform design, the basic problem in developing an adaptive radar therefore lies in developing a means of redetermining and readjusting the waveforms as the environment changes.

There are basically three environmental characteristics that greatly influence the waveform design: range extent of the ground clutter, Doppler spread of the clutter, and intensity of the clutter backscatter. These characteristics are clearly time varying.

On a short-term basis they are time varying because of the rapidly changing azimuth look angle of the radar antenna. The search is performed with a beam that is narrow in azimuth and scans sequentially through the search volume (e.g., through 360 degrees of azimuth). Each azimuth angle will generally have different clutter characteristics so that the waveform must be capable of rapid readjustment *within* each search frame. For example, a northerly wind will cause rain to present a high mean Doppler and Doppler spread to the radar when the radar is pointed to north or south, but a low mean Doppler and Doppler spread when pointed to east or west. As a consequence the total clutter spread, including the ground clutter which has zero Doppler, will be maximum at north and south azimuths and minimum at east and west azimuths.

The clutter characteristics are also time-varying on a long-term basis. That is, over several search frames the weather may change enough to cause the clutter characteristics to change even at a fixed azimuth angle. High winds may increase clutter spread, and high moisture may increase the clutter intensity of vegetation. Moreover, an adversary may dispense chaff, which will alter the required waveforms tremendously.

Implementation of Adaptivity

Evidently the earth-referenced clutter conditions change relatively slowly with time even though the radar-referenced conditions change rapidly because of a rapidly changing azimuth angle. This fact suggests that the waveform changes with azimuth angle should be implemented with a rapid look-up table, where the contents of the table are modified relatively slowly as the earth-referenced environment changes. In other words, real-time waveform redesign is a relatively slow procedure because of a slowly changing environment, whereas real-time waveform resetting is a rapid process that occurs many times within a search frame but is based on waveforms that have been previously designed.

This two-pronged approach to the adaptive radar is illustrated in Figure 4-1. An auxiliary sensor, which may actually be integrated with the radar, measures the environment and thereby determines the design parameters for the waveform designs. For an azimuth angle θ the relevant parameters will be $\rho(\theta)$, $R_{amb}(\theta)$, and $k_p(\theta)$, where ρ is the ratio of clutter spread to maximum target speed of interest, R_{amb} is the range extent of the ground clutter, and k_p is the filter broadening required to provide adequately low sidelobes for suppressing the sensed clutter. A 360° search sector may, for example, be divided into 16 sectors of $22\ 1/2^\circ$ each, with separately measured values for ρ , R_{amb} , and k_p in each sector. Whenever the measured values in a sector change significantly from those that resulted in the current waveform set, a new set is computed by means of a real-time algorithm for waveform design. This new set then replaces the old set in the memory for that sector so that the new set will be used the next time the corresponding azimuth is passed.

The payoff for these real-time readjustments is two-fold. First, the dwell time required per search frame is minimized for each environment so that the remaining time available for tracking is maximized. Second, because

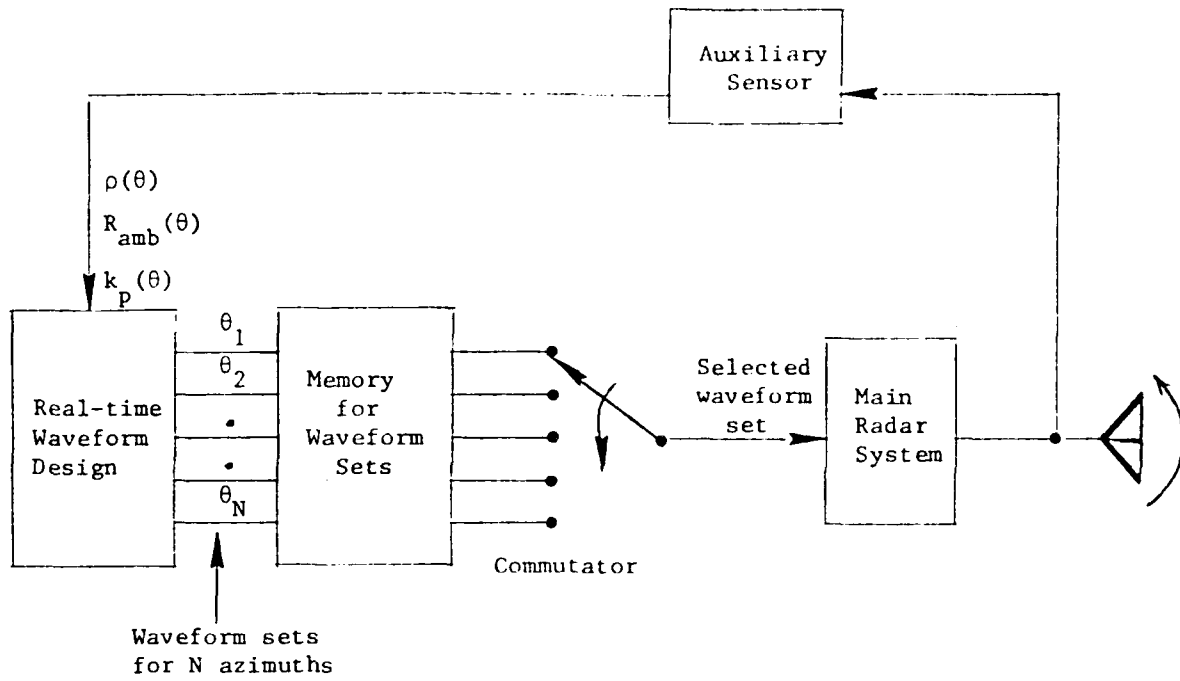


Figure 4-1. Architecture for Adaptive Radar

the time is used so efficiently the allowable wavelength for a given performance level is reduced below the wavelength allowable in a *non*-adaptive system. In theory, the judicious use of adaptivity allows one to design for the average conditions rather than the worst-case conditions with the knowledge that the adaptive networks will adjust to adverse conditions when needed. In practice, the radar will be designed such that the search frame time is not exceeded by the search dwell time even in the worst case. Then as conditions become more benign the waveforms will be redesigned, thereby freeing more and more time for tracking.

In the following we will develop the formalism for the waveform redesign. First we will outline how the optimum design parameters are determined (e.g., search time, number of bursts, basic PRF), then we will demonstrate a real-time algorithm for deriving the full PRF schedule from these parameters.

4.1.2 Optimum Design Parameters

The adaptive radar is controlled by spatio-temporal changes in the environment as summarized by just three environmental parameters:

$$\begin{aligned} R_a &= \text{range extent of ground clutter} \\ \Delta R_{cl} &= \text{range-rate spread of total clutter} \\ S_{cl} &= \text{Doppler sidelobe level required for clutter suppression} \end{aligned}$$

We shall differentiate between long-term time variations and short-term time variations as described previously by redesigning the waveforms as a function of

$$\begin{aligned} R_a(\theta, t) \\ \Delta R_{cl}(\theta, t) \\ S_{cl}(\theta, t) \end{aligned}$$

where θ is the azimuth angle and t is time. Although azimuth angle is a strictly rapidly varying function of time for a sequential scan, we shall treat each azimuth independently and thereby treat azimuth as a constant. The time-varying azimuth angle will then be handled by the commutator illustrated in Figure 4-1. In other words, we will divide the search volume into N azimuth sectors and perform independent waveform design for each sector on the basis of the parameters

$$\begin{aligned} R_a(\theta_n, t) \\ \Delta R_{cl}(\theta_n, t) \\ S_{cl}(\theta_n, t) \end{aligned}$$

for $n=1$ to N .

In developing the real-time procedures for waveform reoptimization we must first translate the three environmental parameters into parameters that are recognizable in our automatic blind-speed algorithms. Recall that we have two search zones: long-range zone and short-range zone. The parameters that are measured by the auxiliary sensor must also be measured separately in each zone. The parameters to be measured for the short-range zone are

$$\begin{aligned} & R_a(\theta_n, t) \\ & \Delta R_{cl}(\theta_n, t)_S \\ & S_{cl}(\theta_n, t)_S \end{aligned}$$

where the subscript S indicates the short-range zone. The parameters for the long-range zone are

$$\begin{aligned} & \Delta R_{cl}(\theta_n, t)_L \\ & S_{cl}(\theta_n, t)_L \end{aligned}$$

(The maximum operating range, R_{max} , is also relevant, but it is fixed, unlike the maximum range for the short-range zone, R_a .)

The input parameters for the automatic design algorithms are $K(\theta_n, t)$ and $\rho(\theta_n, t)$, which must be defined separately for the long- and short-range zones. The algorithms will then result in the parameters $\alpha(\theta_n, t)$ and $p(\theta_n, t)$. The search dwell time for the sector is then determined from $\alpha(\theta_n, t)$ from a knowledge of $k_p(\theta_n, t)$. The time remaining for tracking is then evident. The required translations are therefore, for a given system wavelength,

$$\begin{aligned} R_a(\theta_n, t) & \longrightarrow K(\theta_n, t)_S \\ \Delta R_{cl}(\theta_n, t)_S & \longrightarrow \rho(\theta_n, t)_S \\ S_{cl}(\theta_n, t)_S & \longrightarrow k_p(\theta_n, t)_S \\ \Delta R_{cl}(\theta_n, t)_L & \longrightarrow \rho(\theta_n, t)_L \\ S_{cl}(\theta_n, t)_L & \longrightarrow k_p(\theta_n, t)_L \end{aligned}$$

and $K(\theta_n, t)_L$ is fixed by R_{max} .

The translation of maximum ranges into the improvement factor K is based on the implicit assumption of avoiding range ambiguities. Thus the basic PRF is

$$f_r = \begin{cases} c/2R_a(\theta_n, t) & \text{for short-range mode} \\ c/2R_{\max} & \text{for long-range mode} \end{cases}$$

The improvement factor is therefore

$$K = \frac{2v_{\max}}{f_r \lambda} = \begin{cases} \frac{4v_{\max}}{c\lambda} R_a(\theta_n, t) & \text{for short-range mode} \\ \frac{4v_{\max}}{c\lambda} R_{\max} & \text{for long-range mode} \end{cases}$$

The translation of clutter spread into the parameter ρ is direct:

$$\rho(\theta_n, t) = \Delta \dot{R}_{c\ell}(\theta_n, t) / v_{\max}$$

with an S subscript for the short-range zone and a L subscript for the long-range zone.

The translation of required sidelobe level $S_{c\ell}$ into filter broadening factor k_p is much less direct. The precise translation depends on the precise filter synthesis used. When deep sidelobe suppression is required over only a narrow fraction of the sidelobe region, the broadening is relatively benign and k_p is near unity. However, when the suppression is required uniformly over the entire sidelobe region, the broadening can be wide. Moreover, a restriction to relatively simple filter types leads to much worse than minimum broadening. For the design purposes we will conservatively assume a simple cosine-on-a-pedestal class of filters. The broadening can then be written as

$$k_p(\theta_n, t) = 0.886 \{ 1 + 0.636 [- .3409 + \sqrt{.04545(-S_{dB}) - .47466}]^2 \}$$

where S_{dB} is the required (negative) sidelobe level in dB, or

$$S_{dB} = 10 \log_{10} S_{c\ell}(\theta_n, t)$$

(again with an S subscript for the short-range zone and a L subscript for the long-range zone).

4.1.3 The Use of Subroutines for Determining K,p in Real Time

For each new R_a , $\Delta R_{c\ell}$ and $S_{c\ell}$ that is sensed, we proceed to compute the new K and p as follows.

$$K = \frac{4v_{\max} R_a}{c\lambda}$$

From the initial design v_{\max} and λ are fixed so that we determine K from the new R_a alone. In general the resulting K will not be an integer. Since K must be an integer we use the next highest integer as already discussed. Then we have

$$K = \text{smallest integer no smaller than } \frac{4v_{\max} R_a}{c\lambda}$$

The value of p is then obtained from subroutine OPB. In this subroutine we have inputs K and

$$\rho = \Delta R_{c\ell} / v_{\max}$$

Thus ρ is determined from our new value of $\Delta R_{c\ell}$. The subroutine outputs the optimum value of p. Thus we have the required values of K and p.

The new value of $S_{c\ell}$ is not needed in order to determine the new values of K and p. It does however, determine search dwell time in the following manner. The subroutine OPB outputs the minimum β along with the corresponding optimum value of p. We can relate k_p to $S_{c\ell}$ from the relations in Section 4.1.2. The dwell time required for search is then determined from

$$T_s = \frac{k_w \theta}{k v_{\max}} \frac{p a_s}{a_{\max}} \beta$$

4.1.4 New Subroutines for Determining PRF Schedules in Real Time

Need for a New Approach

Once the improvement factor K and number of bursts p have been determined we still have the problem of determining the precise set of PRFs. In the past we have relied on Johnson's* tables for defining this set. However, there are a

* Johnson, *op cit.*

number of reasons why this is not the best approach for incorporation in our environment-adaptive radar.

First, a table of all sets of PRFs from Johnson's tables would be very unwieldy. Numerous sets of PRFs would have to be stored by the radar when all useful pairs K, p are considered.

Second, the tables are not complete in that not all possible K, p pairs are included. The reason for this shortcoming is that some pairs have no solution for which all speeds are unblinded. Johnson has tabulated only those sets of PRFs for which there are *absolutely* no blind speeds from 0 to v_{\max} . In practice, however, we would allow a few narrow blind-speed zones because such narrow zones really involve only the skirts of the clutter spectrum. As a result their inclusion would degrade detection performance only slightly overall, especially considering that detection probabilities no greater than 0.9 are expected anyway. Had Johnson included solutions with a few narrow unblinded regions, his tables would have been complete and useful to us. On the other hand, since he maintained a stricter definition of the blind zone, we find that we must develop more practical solutions of our own.

Third, not only does a looser definition allow completion of the tables but it also leads to more practical solutions in the cases where Johnson does have solutions. For our application a few narrow blind-speed regions would be admissible if these regions occurred only at low speeds. With this allowance for some blind zones we find that we can evolve sets of PRFs which use fewer bursts and a narrower range of PRFs, thereby increasing the practicality of the solutions.

Example of the New Approach

Before mathematically detailing our new approach to finding the sets of PRFs, let us outline the philosophy and give an example. The automatic algorithm for computing the PRFs picks PRFs in decreasing values such as to unblind the higher ambiguities first. Thus if we do not use the lowest PRFs, then the lower ambiguities may be partially or wholly blinded. It is of interest in some situations to see what regions are blinded when the lower PRFs are not used. Because these regions correspond to low velocity rates, it is unlikely that threatening targets will lie in these regions. If this is the case then we clearly gain by using fewer PRFs.

As an example we consider the baseline design for the mechanically rotating set of antennas in the far-range zone. By using our new algorithm (to be described next) for determining the PRFs, we find that an acceptable set is

$$f_{r1} = 750 \text{ Hz}$$

$$f_{r2} = 722.75$$

$$f_{r3} = 668.26$$

$$f_{r4} = 522.94$$

This solution differs from the set that would be found from Johnson's tables in that a narrow blind-speed zone remains. This zone is, however, only 5.25 Hz wide, which is certainly small by comparison with the 15 kHz Doppler spectrum being searched. Moreover, the zone is centered at a low speed (about 50 m/s). The existence of this residual blind zone is therefore of little practical consequence.

Our new solution also has the property that the omission of the lowest PRF in the schedule introduces blind zones at the lower speeds first. This property is illustrated in Table 4-1. Evidently the omission of each successive PRF, starting at the lowest PRF, merely introduces more blind zones at the lower speeds. A good tradeoff therefore exists between the number of bursts used and the size of residual blinded regions at low speeds. Thus, when transmitter power or dwell time is at a premium one may opt to omit the PRF of lowest frequency.

Description of the PRF Selection Algorithm

We can now describe our new procedure for selecting the PRFs once the parameters K and p have been selected. We shall choose the frequencies so as to first eliminate the higher blind speeds as these are the most important regions in practice. Thus if after using the p frequencies there are still blind regions, they will be at the lowest blind speeds. With this procedure we can also examine how many extra blind regions are created at the lower end if we use $p-1$ instead of p frequencies, etc.

For a given K and maximum PRF f_r , the highest blind-speed region which we wish to unblind is centered at $(K-1)f_r$. We are given p and K from our automatic design procedure. The maximum PRF is $f_r = c/(2R_a)$ and we always

Table 4-1. Blinded Regions Versus Number of Frequencies Used

Frequencies (Hz)	Ambiguity Blinded (Hz)	Extent Blinded (Hz)
$f_1 = 750$ $f_2 = 722.75$ $f_3 = 668.26$ $f_4 = 522.94$	#1	5.25
$f_1 = 750$ $f_2 = 722.75$ $f_3 = 668.26$	#1 #2	150.56 68.82
$f_1 = 750$ $f_2 = 722.75$	#1 #2 #3 #4 #5 #6 #7 #8	205.06 177.81 150.56 123.32 96.07 68.82 41.58 14.33

(Solution for rotating antenna operating in the far-range mode is shown.)

choose this as our first PRF. We pick our second PRF in the following manner. It is chosen so that it blinds the region just above the blind region centered at $(K-1)f_r$. Thus we unblind the region centered at $(K-1)f_r$. This second PRF (f_2) is chosen to be as large as possible but no larger than f_r .

The blind region centered at $(K-2)f_r$ is checked to see if it is now unblinded when f_2 is present. If it is unblinded, we similarly check $(K-3)f_r$, etc., until we reach a region where there is some blinding, say at jf_r . We compute where the upper end of the blinded region lies. The frequency f_3 is chosen so that it blinds the region just above this upper end. Thus we unblind the region in the vicinity of jf_r . Again f_3 is chosen to be as large as possible but no larger than f_r .

This procedure will generate decreasing frequencies f_i for $i=1, \dots, p$ (where $f_1 = f_r$). When a blind region exists in the vicinity of jf_r the above procedure implies we choose the frequency f_i as

$$f_i = f_r - \frac{(f_r - \Delta + v)}{j + 1} \quad (4-1)$$

Here Δ is the width of the blind zone and is given by

$$\Delta = \frac{c}{2R_a x(K,p)}$$

We thus use R_a and the computer routine which computes $x(K,p)$ to obtain Δ as a function of the real-time values for (K,p) .

The above procedure assumes that if there was blinding in the vicinity of jf_r , the choice of f_i in (4-1) totally unblinded the region near jf_r . This total unblinding is true except for cases where both Δ is large and j is small. When these cases exist we repeat our unblinding procedure in the vicinity of jf_r until enough f_i are used to totally unblind the region near jf_r .

In Figure 4-2 we show the blinded region in the vicinity of jf_r . Thus in the expression for f_i we have that v is the width of the unblinded part at the top of the original blinded region. Also u is similarly the width of the unblinded part at the bottom of the original blinded region. The region we show blinded may not be totally blinded. It may have gaps which are not blinded. However, u and v define the region where there is some blinding.

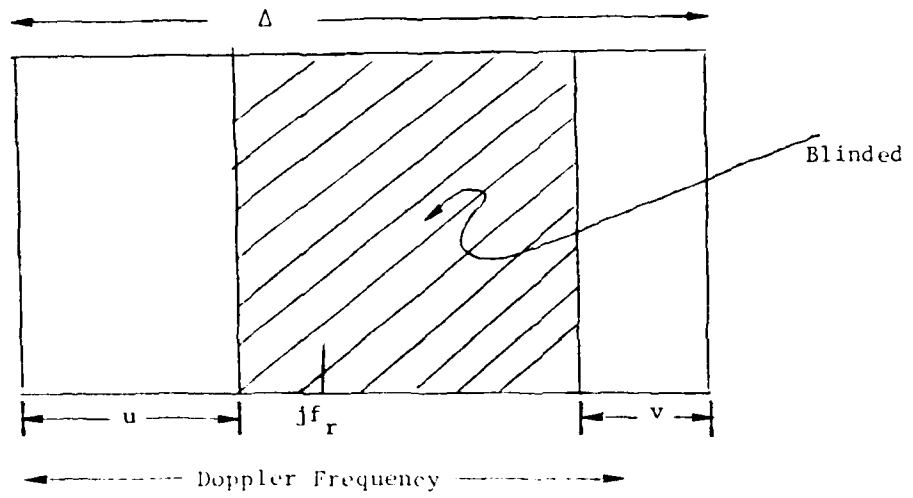


Figure 4-2. Example of Blind Region

We have generated a FORTRAN program which determines u and v at jf_r given that frequencies f_1, \dots, f_M are present. We have used this routine and (4-1) to determine the frequencies needed to unblind the regions of concern. The program can also be used to determine the u and v which exist when we do not use enough frequencies to unblind the total region below Kf_r .

The FORTRAN program is shown in the Appendix. The main subroutine is called BLIND(K,P,FR,DI). For inputs of K, p and FR it determines the frequencies to be used, any regions which remain blinded and the number of frequencies actually used if less than p . If $DI=0$ then the program computes Δ from the optimum relationship, i.e.,

$$\Delta = \frac{FR}{x(K,p)} \quad (4-2)$$

If $DI \neq 0$ then it uses this value as the value for Δ . The philosophy in using (4-1) to determine the f_i is that we wish to unblind a region using as high a frequency as possible and at the same time have it blind regions that we know previous frequencies have unblinded. Our experience with the algorithm in the run using many cases is that at worst it blinds 20% of the lower end of the lowest blind-speed region. None of the higher blind-speed regions ever remained blinded. This of course assumes that we use the optimum relationship for Δ in (4-2).

In most cases that were run, the blinding did not exist at all or was well below the 20% value in the lowest blind-speed region. For many cases, particularly for higher p ($p \geq 6$) we did not need to use all the frequencies available in order to unblind the total region of concern.

4.2 PERFORMANCE IMPROVEMENT FROM ADAPTIVE TECHNIQUES

We wish to illustrate the payoff from an adaptive radar by giving some examples of nonhomogeneous environments. The payoff occurs in the reduction of the dwell time required per search frame. We consider the case of a short-range search which in the worst case is designed for a range R_a of 80 km and a clutter width ΔR_{cl} of 20 m/sec. This clutter width corresponds to ground clutter in the presence of the worst expected rain clutter. The remaining parameters which are fixed are given as

$$\begin{aligned}
 v_{\max} &= 1000 \text{ m/sec} \\
 w_a &= 3 \text{ m} \\
 k_p &= 1 \\
 k_a &= 1 \\
 \theta_s &= \pi/2 \\
 T_s &= < 2 \text{ sec}
 \end{aligned}$$

If we take $T_s = 1.56 \text{ sec}$ (leaving .44 sec for track) then we have

$$\beta = \frac{k_a v_{\max} T_s}{k_p w_a \theta_s} = 331$$

while

$$\rho = \Delta \dot{R}_{cl} / v_{\max} = .02$$

For these values of β and ρ the subroutine OPK yields the maximum value of K-15 along with the corresponding value of $p=6$. The wavelength of transmission is then given by

$$\lambda = \frac{4v_{\max} R_a}{cK} = 7.1 \text{ cm}$$

Thus we operate the transmitter at a wavelength of 7.1 cm and in the worst case need a dwell time for search of 1.56 sec.

We next divide the search sector into two equal parts so that each sector has a width

$$\Delta\theta = \theta_s / 2 = \pi/4$$

and we perform a two-zone search. We do this to examine how a nonhomogeneous environment can appreciably reduce the dwell time required per search frame.

As our first example, suppose R_a is 80 km in the first zone while, due to the change in terrain, it is only 50 km in the second zone. We still assume the worst clutter width of 20 m/sec for both zones. The wavelength of the transmitter must stay fixed at 7.1 cm. Thus we obtain K from

$$K = \frac{4v_{\max} R_a}{c\lambda}$$

In the first zone K is still 15. In the second zone $R_a = 53$ km. Thus we must use $K=10$ so that we extend R_a to 53 km in the second zone.

In the first zone we then have $K=15$, $\rho=.02$ while in the second zone $K=10$, $\rho=.02$. We now use subroutine OPB to find the minimum τ and corresponding p in each zone for the given K and ρ . We then use

$$T_{si} = \frac{\beta k w \Delta \theta}{k v \frac{p a}{a \max}} \quad i=1,2$$

to find the dwell time needed for search in each azimuth zone. The total dwell time is then given by $T_s = T_{s1} + T_{s2}$ for the short-range bursts.

The result of the above procedure for this example as well as other examples are given in Table 4-1. We see from this table that the dwell time can be decreased by anywhere from about one-half second to one second if the nonhomogeneous nature of the environment is taken into account. We further see that for the four cases considered the maximum of R_a and $\Delta R_{c\lambda}$ for each case are 80 km and 20 m/sec. These are the values in the worst case design. Thus we have achieved significant improvement over the worst case even though one or the other zone has worst case conditions associated with it.

We have given an example of dividing a region into two zones. Ideally we should divide the region into many zones so that in the limit

$$T_s = \int_{\text{region}} \frac{dT_s}{d\theta} d\theta \quad ,$$

where

$$\frac{dT_s}{d\theta} = \frac{\beta(\theta) k w}{k v \frac{p a}{a \max}} \quad .$$

Here $\beta(\theta)$ is obtained by substituting (K, ρ) into subroutine OPB. We have that

$$K = \frac{4v \max R_a(\theta)}{c\lambda} \quad ,$$

Table 4-2. Dwell Times for Adaptive Radar ($\lambda = 7.1$ cm)

		ZONE 1	ZONE 2	T_{db} sec
Initial Worst Case Design	Condition	Worst Case	Worst Case	1.56
	R_{am} k	80	80	
	ΔR_{cl} m/sec	20	20	
	K	15	15	
	p	6	6	
	T_{s1} sec	0.78	0.78	
One	Condition	Worst Case	Smaller Ground Clutter Range	1.05
	R_{am} k	80	53	
	ΔR_{cl} m/sec	20	20	
	K	15	10	
	p	6	5	
	T_{s1} sec	0.78	0.27	
Two	Condition	Worst Case	Smaller Rain Clutter Extent	1.06
	R_{am} k	80	80	
	ΔR_{cl} m/sec	20	5	
	K	15	15	
	p	6	5	
	T_{s1} sec	0.78	0.28	
Three	Condition	Worst Case	Smaller Ground Clutter Range & Smaller Rain Clutter Extent	0.93
	R_{am} k	80	53	
	ΔR_{cl} m/sec	20	5	
	K	15	10	
	p	6	4	
	T_{s1} sec	0.78	0.15	
Four	Condition	Smaller Ground Clutter Range	No Rain Clutter	0.49
	R_{am} k	53	80	
	ΔR_{cl} m/sec	20	0	
	K	10	15	
	p	5	4	
	T_{s1} sec	0.27	0.22	

while

$$\rho = \frac{\Delta R_{ct}(0)}{v_{\max}}$$

As a practical matter, we use sums and zones (rather than integrals and differentials) because we require integer mathematics (e.g., integer number of pulses). However, as seen from the examples in Table 4-1, even dividing the region into a small number of zones can yield appreciable improvement in the required dwell time for scanning.

5. CONCLUSIONS

The major conclusion that is evident from the preceding results is that the potential payoff from adaptivity is great. The radar can be designed to meet average conditions rather than worst-case conditions, with the knowledge that the radar will adapt to changing conditions. We have found in our examples that large reductions in search dwell time and/or operating wavelength can result from the adaptivity.

The major remaining question is how much improvement can be realized in practice. Although we have already defined the basic design for the adaptive radar, we cannot yet answer the difficult performance question. The problem is that the degree of performance improvement depends on the range of environments to be encountered. It therefore becomes important to simulate the adaptive radar under a broad variety of conditions in order to define performance.

The simulation of the radar system is precisely the next task that is planned. As illustrated in Table 5-1, the next task (Task 3) is about eight months in duration and will involve a testing of the simulated system against realistic environments. We expect to gain from this simulation not only an estimate of performance but also a feeling for the measurement accuracies required of the environmental sensor that measures the adverse clutter environment. The final task (Task 4) will then involve the actual design of the entire system with such an environmental sensor included.

APPENDIX A. SUBROUTINE LISTINGS

Following are the codes for the various subroutines and functions that are described in the text.

```

SUBROUTINE CPKIBETA,RHO,K,P,BETAP)
INTEGER P
COMMON /TABLES/ X(60,12)
IF (RHO.GE.1.) THEN
  K=C
  P=C
  BETAP=BETA
  RETURN
  END IF
BTEST=1./(2.*(1.-RHO))
IF (BETA.LE.BTEST) THEN
  K=C
  P=C
  BETAP=BETA
  RETURN
  END IF
IF (RHO.GT..25.CR.BETA.LT.6.) THEN
  K=1
  P=1
  BETAP=1./(2.*(1.-RHO))
  RETURN
  END IF
IF (RHO.EC.0.) THEN
  KS=60
  CCTC 100
  END IF
KS=INT(1./(2.*RHO))
KS=MIN0(KS,60)
T=1./(RHO*X(KS,12))
IF (T.LE.FLOATE(KS)) THEN
  KS=KS-1
  IF (KS.EQ.1) THEN
    K=1
    P=1
    BETAP=BETA
    RETURN
    ELSE
    CCTC 200
    END IF
  END IF
  IF (KS.EC.2) THEN
    CALL CPB(2,RHO,BETAP,P)
    IF (BETAP.LE.BETA) THEN
      K=2
      CALL SCPKIBETA,K,RHO,P,BETAP)
      RETURN
      ELSE
      K=1
      P=1
      BETAP=1./(2.*(1.-RHO))
      RETURN
      END IF
    END IF
  100  KL=2
  KH=KS
  CALL CPB(KS,RHO,BETAP,P)
  IF (BETAP.LE.BETA) THEN
    K=KS
    CALL SCPKIBETA,K,RHO,P,BETAP)
    RETURN
    END IF
  300  KN=(KL+KH)/2
  CALL CPB(KN,RHO,BETAP,P)
  IF (KN.EC.KL.ANC.BETAP.LE.BETA) THEN
    K=KN
    CALL SCPKIBETA,K,RHO,P,BETAP)
    RETURN
    END IF
  IF (KN.EC.KL.ANC.BETAP.GT.BETA) THEN
    K=1
    P=1
    BETAP=1./(2.*(1.-RHO))
    RETURN
    END IF
  IF (BETAP.GT.BETA) THEN
    KH=KN
    CCTC 300
    ELSE
    KL=KN
    CCTC 300
    END IF
  END IF
END

```

```

SUBROUTINE OPB(K, RHO, BETAM, P)
C   CALLED BY OPK.
REAL KREAL
INTEGER P, PP
COMMON /TABLES/ X(60, 12)
KREAL=FLOAT(K)
PREAL=FLOAT(P)
IF (RHO. EQ. 0. ) GOTO 200
TESTK=1. / (2. *RHO)
IF (RHO. GE. 1. . OR. KREAL. GE. TESTK) THEN
    BETAM=0.
    P=0
    RETURN
END IF
200  IF (K. EQ. 1) THEN
    BETAM=1. / (2. * (1. -RHO))
    P=1
    RETURN
END IF
DO 400 J=2, 12
COMP=KREAL*RHO*X(K, J)
IF (COMP. GE. 1. ) GOTO 400
BETA=FLOAT(J)*KREAL*X(K, J)/(2. * (1. -COMP))
IF (J. EQ. 12) THEN
    BETAM=BETA
    P=J
    RETURN
END IF
PP=J
BETAP=BETA
GOTO 300
400  CONTINUE
BETAM=1. E5
P=13
300  NN=J+1
DO 10 I=NN, 12
    BETA=FLOAT(I)*KREAL*X(K, I)/(2. * (1. -KREAL*RHO*X(K, I)))
    IF (BETA. GE. BETAP) GOTO 100
    PP=I
    BETAP=BETA
10   CONTINUE
P=PP+1
BETAM=BETA
RETURN
100  BETAM=BETAP
P=PP
RETURN
END

```

```

SUBROUTINE SOPK(BETA, K, RHO, P, BETAM)
C   CALLED BY OPK.
   INTEGER P, PP
   COMMON/TABLES/X(60, 12)
   IF (P. EQ. 2. OR. P. EQ. 1) RETURN
   PP=P
   BETAP=BETAM
   N=P-2
   DO 10 I=1, N
     J=P-I
     TEST=FLOAT(K)*RHO*X(K, J)
     IF (TEST. GE. 1. ) THEN
       P=PP
       BETAM=BETAP
       RETURN
     END IF
     BETAN=FLOAT(J*K)*X(K, J)/(2. -2. *TEST)
     IF (BETAN. LE. BETA) THEN
       PP=J
       BETAP=BETAN
       GOTO 10
     ELSE
       P=PP
       BETAM=BETAP
       RETURN
     END IF
10  CONTINUE
   P=PP
   BETAM=BETAP
   RETURN
END

```

```
INTEGER FUNCTION FK(BETA, RHO, PNOT, KNOT, P)
INTEGER P, PNOT
COMMON /TABLES/X(60,12)
IF (P.EQ.PNOT) THEN
    FK=KNOT
    RETURN
END IF
N=KNOT-1
DO 100 I=1,N
    T=2.*BETA/(X((KNOT+1-I),P))*(2.*RHO*BETA+FLOAT(P))
    IF (T.GE.FLOAT(KNOT+1-I)) THEN
        FK=KNOT+1-I
        RETURN
    END IF
100 CONTINUE
FK=1
RETURN
END
```

```
FUNCTION FB(K,RHO,P)
  INTEGER P
  COMMON /TABLES/X(60,12)
  CRUD=FLOAT(K)*RHO*X(K,P)
  IF (CRUD. GE. 1. ) THEN
    FB=0.
  ELSE
    FB=FLOAT(K*P)*X(K,P)/(2. *(1. -CRUD))
  END IF
  RETURN
END
```

```
      SUBROUTINE TABLE
C     THIS SUBROUTINE SETS UP A TABLE OF VALUES OF THE
C     FUNCTION XX(K,P) FOR K=2,...,60 AND P=2,...,12.
C     THIS TABLE IS THEN PUT INTO THE COMMON BLOCK
C     'TABLES'. THIS SUBROUTINE MUST BE CALLED BEFORE
C     USING ANY OTHERS.
      INTEGER P
      COMMON /TABLES/ X(60,12)
      DO 10 I=1,60
         X(I,1)=0.
10     CONTINUE
      DO 20 I=1,12
         X(1,I)=0.
20     CONTINUE
      DO 40 P=2,12
         DO 30 K=2,60
            X(K,P)=XX(K,P)
30         CONTINUE
40     CONTINUE
      RETURN
      END
```

```
FUNCTION XX(K,P)
C   THIS FUNCTION IS DEFINED FOR K=2,...,60, AND
C   P=2,...,12. THE VALUES OF THIS FUNCTION ARE
C   USED IN MOST ALL OF THE SUB-PROGRAMS.
  INTEGER P
  REAL KK
  KK=FLOAT(K)
  PP=FLOAT(P)
  X=KK**(1./(PP-1.))+1./(KK**(1./(PP-1.)))
100  TOP=X**P-2.*X**(P-1)-KK*X+KK+1.
  BOTTOM=PP*X**(P-1)-2.*(PP-1.)*X**(P-2)-KK
  DELTAX=-TOP/BOTTOM
  X=X+DELTAX
  IF (ABS(DELTAX).GT.1.E-12) GOTO 100
  XX=X
  RETURN
END
```

```

SUBROUTINE BLIND(K,F,FR,DI,M,URL,DBL,F)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION F(1),S1B(13),S2A(13),S3B(13),S3A(13),URL(1),DBL(1)
INTEGER F
COMMON/TABLES/X(60,12)
CALL XMITD(-60,0.,DBL)
CALL XMITD(-60,0.,URL)
LL=2
IF (K.NE.0) GOTO 510
TYPE 500
500  FORMAT (1X,7HF1 = 0.)
RETURN
510  IF (K.NE.1) GOTO 530
TYPE 520, FR,F
520  FORMAT (1X,4HFR = ,D16.8,2X,4HF = ,I2)
RETURN
530  IF (DI.EQ.0.) DI=FR/X(K,F)
J=K-1
U=0.
V=0.
F(1)=FR
F(2)=FO(U,V,J,F(1),DI)
M=2
LO=2
IF (F(1).LT.(DI/J+2.*DI)) LO=LO-1
IF (LO.EQ.1) GOTO 550
IF (J.NE.1) GOTO 550
CALL CLEARJ(DI,F,M,J,ZU,ZD)
RETURN
550  IF (M.EQ.F) LL=1
DO 580 L=LO,(K-1)
J=K-L
560  CALL STATES(DI,F,M,J,S1B,II,S2A,JJ,S3B,S3A,KK)
NN=NS(II,JJ,KK)
IF (NN.EQ.8) GOTO 580
CALL EXT(S1B,II,S2A,JJ,S3B,S3A,KK,B,A,BCC,ACC)
CALL UV(DI,B,A,BCC,ACC,S3B,S3A,KK,NN,U,V)
IF (U.EQ.DI.OR.(U+V).EQ.DI) GOTO 580
IF (LL.NE.1) GOTO 570
URL(J)=U
DBL(J)=DI-U-V
GOTO 580
570  M=M+1
F(M)=FO(U,V,J,F(1),DI)
IF (M.EQ.F) LL=1
IF (V.LT.(DI/J+2.*DI F(1))) GOTO 560
580  CONTINUE
RETURN
END

```

```

SUBROUTINE CLEAR(DI,F,M,JS,JF,URL,DBL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION F(1),URL(1),DBL(1)
TYPE 500,DI,M,(F(I),I=1,M)
FORMAT (///,' DI =',D16.8,' M =',I3,/, ' F =',13(D16.8,1X))
TYPE 510
510 FORMAT (//,' J',10X,' URL',10X,' DBL',/,1X,29(1H-))
DO 530 J=JS,JF
CALL CLEARJ(DI,F,M,J,ZU,ZD)
URL(J)=ZU
DBL(J)=ZD
IF (ZD.EQ.0.) GOTO 530
TYPE 520,J,URL(J),DBL(J)
520 FORMAT (1X,I3,2(5X,D16.8))
530 CONTINUE
RETURN
END

```

```

SUBROUTINE CLEARJ(DI,F,M,J,ZU,ZD)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION F(1),S1B(13),S2A(13),S3B(13),S3A(13)
CALL STATES(DI,F,M,J,S1B,II,S2A,JJ,S3B,S3A,KK)
NN=NS(II,JJ,KK)
IF (NN.EQ.8) GOTO 500
CALL EXT(S1B,II,S2A,JJ,S3B,S3A,KK,B,A,BCC,ACC)
CALL UV(DI,B,A,BCC,ACC,S3B,S3A,KK,NN,U,V)
IF (U.EQ.DI.OR.U+V.EQ.DI) GOTO 500
ZU=U
ZD=DI-U*V
RETURN
500 ZD=0.
ZU=0.
RETURN
END

```

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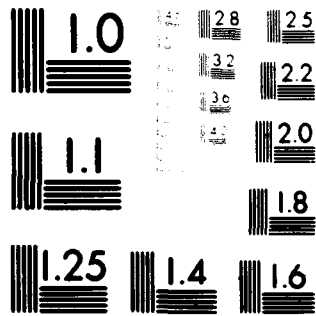
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SUBROUTINE EXT(S1B,I,S2A,J,S3B,S3A,K,H,A,BCC,ACC)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S1B(13),S2A(13),S3A(13),S3B(13)
R=0.
A=0.
BCC=0.
ACC=0.
IF (I.EQ.0) GOTO 510
R=S1B(1)
IF (I.EQ.1) GOTO 510
DO 500 II=2,I
    R=DMIN1(R,S1B(II))
500 CONTINUE
510 IF (J.EQ.0) GOTO 530
A=S2A(1)
IF (J.EQ.1) GOTO 530
DO 520 JJ=2,J
    A=DMIN1(A,S2A(JJ))
520 CONTINUE
530 IF (K.EQ.0) RETURN
BCC=S3B(1)
ACC=S3A(1)
IF (K.EQ.1) RETURN
DO 540 KK=2,K
    BCC=DMIN1(BCC,S3B(KK))
    ACC=DMIN1(ACC,S3A(KK))
540 CONTINUE
RETURN
END

```

```

FUNCTION FB(K,RHO,P)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
INTEGER P
COMMON /TABLES/X(60,12)
CRUD=FLOAT(K)*RHO*X(K,P)
IF (CRUD.LT.1.) GOTO 500
FB=0.
GOTO 510
500 FB=FLOAT(K*P)*X(K,P)/(2.*(1.-CRUD))
510 RETURN
END

```

```

INTEGER FUNCTION FK(BETA,RHO,PNOT,KNOT,P)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
INTEGER P,PNOT
COMMON /TABLES/X(60,12)
IF (P.NE.PNOT) GOTO 500
FK=KNOT
RETURN
500 N=KNOT-1
DO 510 I=1,N
    T=2.*BETA/(X((KNOT+1-I),P)*(2.*RHO*BETA+FLOAT(P)))
    IF (T.LT.FLOAT(KNOT+1-I)) GOTO 510
    FK=KNOT+1-I
    RETURN
510 CONTINUE
FK=1
RETURN
END

```

```

FUNCTION NS(I,J,K)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IF (I+J+K.NE.0) GOTO 500
NS=8
RETURN
500 NS=1
IF (I.GT.0.AND.J.GT.0.AND.K.GT.0) RETURN
NS=2
IF (I.GT.0.AND.K.GT.0.AND.J.EQ.0) RETURN
NS=3
IF (K.GT.0.AND.J.GT.0.AND.I.EQ.0) RETURN
NS=4
IF (I.GT.0.AND.J.GT.0.AND.K.EQ.0) RETURN
NS=5
IF (K.GT.0.AND.J.EQ.0.AND.I.EQ.0) RETURN
NS=6
IF (I.GT.0.AND.J.EQ.0.AND.K.EQ.0) RETURN
NS=7
RETURN
END

```

```

SUBROUTINE OPB(K,RHO,BETAM,P)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C CALLED BY OPK.
REAL KREAL
INTEGER P,PP
COMMON /TABLES/ X(60,12)
KREAL=FLOAT(K)
PREAL=FLOAT(P)
IF (RHO.EQ.0.) GOTO 200
TESTK=1./(2.*RHO)
IF (RHO.LT.1..AND.KREAL.LT.TESTK) GOTO 200
BETAM=0.
P=0
RETURN
200 IF (K.NE.1) GOTO 201
BETAM=1./(2.*(1.-RHO))
P=1
RETURN
201 DO 400 J=2,12
COMP=KREAL*RHO*X(K,J)
IF (COMP.GE.1.) GOTO 400
BETA=FLOAT(J)*KREAL*X(K,J)/(2.*(1.-COMP))
IF (J.NE.12) GOTO 202
BETAM=BETA
P=J
RETURN
202 PP=J
BETAP=BETA
GOTO 300
400 CONTINUE
BETAM=1.E5
P=13
300 NN=J+1
DO 10 I=NN,12
BETA=FLOAT(I)*KREAL*X(K,I)/(2.*(1.-KREAL*RHO*X(K,I)))
IF (BETA.GE.BETAP) GOTO 100
PP=I
BETAP=BETA
10 CONTINUE
P=PP+1
BETAM=BETA
RETURN
100 BETAM=BETAP
P=PP
RETURN
END

```

```

SUBROUTINE DPX(BETA,RHO,K,F,BETAM)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
INTEGER I
COMMON /IAHLS/ X(60,12)
IF (RHO.LT.1.) GOTO 400
K=0
F=0
BETAM=BETA
RETURN
400 BTEST=1./(2.*(1.-RHO))
IF (BETA.GT.BTEST) GOTO 405
K=0
F=0
BETAM=BETA
RETURN
405 IF (RHO.LT..25.AND.BETA.GE.6.) GOTO 410
K=1
F=1
BETAM=1./(2.*(1.-RHO))
RETURN
410 IF (RHO.NE.0.) GOTO 415
KS=60
GOTO 100
415 KS=INT(1./(2.*RHO))
KS=MIN0(KS,60)
200 T=1./(RHO*X(KS,12))
IF (T.GT.FLOAT(KS)) GOTO 420
KS=KS-1
IF (KS.NE.1) GOTO 200
K=1
F=1
BETAM=BETA
RETURN
420 IF (KS.NE.2) GOTO 100
CALL OPK(2,RHO,BETAM,F)
IF (BETAM.GT.BETA) GOTO 450
K=2
CALL SOPK(BETA,K,RHO,F,BETAM)
RETURN
450 K=1
F=1
BETAM=BETA
RETURN
100 KL=2
KM=KS
CALL OPK(KS,RHO,BETAM,F)
IF (BETAM.GT.BETA) GOTO 300
K=KS
CALL SOPK(BETA,K,RHO,F,BETAM)
RETURN
300 KN=(KL+KM)/2
CALL OPK(KN,RHO,BETAM,F)
IF (KN.NE.KL.OR.BETAM.GT.BETA) GOTO 460
K=KN
CALL SOPK(BETA,K,RHO,F,BETAM)
RETURN
460 IF (KN.NE.KL.OR.BETAM.IE.BETA) GOTO 470
K=1
F=1
BETAM=1./(2.*(1.-RHO))
RETURN
470 IF (BETAM.LT.BETA) GOTO 480
KN=KN
GOTO 300
480 KL=KN
GOTO 300
END

```

```

FUNCTION OPK(X,Y,D,A,KK)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION Y(13),Y(13)
DO 10 I=1,KK
  IF (Y(I).GT.D) GOTO 20
10 CONTINUE
  KK=I
  RETURN
20 IF (X(I).LE.B) GOTO 40
  KK=I
  RETURN
40 IF (Y(I).LT.A) GOTO 50
  KK=A
  RETURN
50 KK=Y(I)
  IF (KK.EQ.1.OR.I.EQ.KK) RETURN
  ITEMF=I+1
  DO 30 K=ITEMF,KK
    IF (X(K).GT.KK) RETURN
    IF (Y(K).LE.KK) GOTO 30
    IF (Y(K).LT.A) GOTO 60
    KK=A
    RETURN
60 KK=Y(K)
30 CONTINUE
  RETURN
END

```

```

SUBROUTINE SOPK(BETA,K,RHO,P,BETAM)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C CALLED BY OPK.
INTEGER P,PP
COMMON/TABLES/X(60,12)
IF (P.EQ.2.OR.P.EQ.1) RETURN
PP=P
BETAF=BETAM
N=P-2
DO 10 I=1,N
  J=P-I
  TEST=FLOAT(K)*RHO*X(K,J)
  IF (TEST.LT.1.) GOTO 20
  P=PP
  BETAM=BETAF
  RETURN
20 BETAN=FLOAT(J*K)*X(K,J)/(2.-2.*TEST)
  IF (BETAN.GT.BETA) GOTO 30
  PP=J
  BETAF=BETAN
  GOTO 10
30 P=PP
  BETAM=BETAF
  RETURN
10 CONTINUE
  P=PP
  BETAM=BETAF
  RETURN
END

```

```

SUBROUTINE SORT(S,I,UR,UA,KK,DI)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S(13),K(13),UR(13),UA(13)
DO 10 J=1,KK
  K(J)=DI-K(J)
10 CONTINUE
  UR(J)=S(J)
  UA(J)=K(J)
  IF (KK.EQ.1) RETURN
  DO 20 I=2,KK
    DO 20 K=1,(I-1)
      IF (S(I).LT.UR(K)) GOTO 30
20 CONTINUE
    UR(I)=S(I)
    UA(I)=K(I)
    GOTO 50
30 DO 40 L=K,(I-1)
    UR(L+1)=UR(L)
    UA(L+1)=UA(L)
40 CONTINUE
    UR(K)=S(I)
    UA(K)=K(I)
50 CONTINUE
  RETURN
  END

```

```

SUBROUTINE STATES(DI,F,M,JJ,S1R,I,S2A,J,S3R,S3A,K)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION F(1),S1R(13),S2A(13),S3R(13),S3A(13)
I=0
J=0
K=0
DO 100 II=2,M
  Q=FLOAT(JJ)*F(1)/F(II)
  N=INT(Q)
  RC=FLOAT(N)*F(II)+DI
  AC=RC+F(II)-2*DI
  TEMP=FLOAT(JJ)*F(1)
  IF (RC.GT.TEMP.OR.AC.LT.TEMP) GOTO 10
  I=0
  J=0
  K=0
  RETURN
10 IF (RC.LE.TEMP.OR.AC.GE.TEMP) GOTO 20
  K=K+1
  S3R(K)=RC-TEMP
  S3A(K)=-AC+TEMP
  GOTO 100
20 IF (RC.LE.TEMP) GOTO 30
  I=I+1
  S1R(I)=RC-TEMP
  GOTO 100
30 J=J+1
  S2A(J)=-AC+TEMP
100 CONTINUE
  RETURN
  END

```

```

SUBROUTINE TABLE
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
THIS SUBROUTINE SETS UP A TABLE OF VALUES OF THE
C      FUNCTION XX(K,F) FOR K=2,...,60 AND F=2,...,12.
C      THIS TABLE IS THEN PUT INTO THE COMMON BLOCK
C      'TABLES'. THIS SUBROUTINE MUST BE CALLED BEFORE
C      USING ANY OTHERS.
      INTEGER P
      COMMON /TABLES/ X(60,12)
      DO 10 I=1,60
         X(I,1)=0.
10      CONTINUE
      DO 20 I=1,12
         X(1,I)=0.
20      CONTINUE
      DO 40 F=2,12
         DO 30 K=2,60
            X(K,F)=XX(K,F)
30      CONTINUE
40      CONTINUE
      RETURN
      END

SUBROUTINE UV(DI,B,A,BCC,ACC,S3B,S3A,K,N,U,V)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S3A(13),S3B(13)
IF (N.GT.7.OR.N.LT.1) GOTO 80
GOTO (10,20,30,40,50,60,70) N
10  CALL UV1(DI,B,A,BCC,ACC,S3B,S3A,K,U,V)
    RETURN
20  CALL UV2(DI,B,BCC,ACC,S3B,S3A,K,U,V)
    RETURN
30  CALL UV3(DI,A,BCC,ACC,S3B,S3A,K,U,V)
    RETURN
40  CALL UV4(DI,B,A,U,V)
    RETURN
50  U=0.
    V=0.
    RETURN
60  U=0.
    V=DI-B
    RETURN
70  U=DI-A
    V=0.
    RETURN
80  TYPE 90, N
90  FORMAT (1X,37HN OUT OF BOUNDS IN SUBROUTINE UV. N =,I3)
    STOP
    END

```

```

SUBROUTINE UV1(DI,B,A,HCC,ACC,S3B,S3A,K,U,V)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S3B(13),S3A(13),UB(13),UA(13)
DAC=DI-ACC
DBC=DI-BCC
IF ((A+B).GT.DI) GOTO 10
U=DI
V=0.
RETURN
10  IF (B.LE.DAC.OR.A.LE.DBC) GOTO 40
    U=DI-A
    V=DI-B
    RETURN
20  IF (B.GT.DAC.OR.A.GT.DBC) GOTO 40
    CALL SORT(S3B,S3A,UB,UA,K,DI)
    U=RB(UB,UA,DI-A,B,K)
    IF (U.NE.B) GOTO 30
    U=DI
    V=0.
    RETURN
30  CALL SORT(S3A,S3B,UB,UA,K,DI)
    V=RB(UB,UA,DI-B,DI-U,K)
    RETURN
40  IF (B.LE.DAC.OR.A.GT.DBC) GOTO 50
    CALL SORT(S3B,S3A,UB,UA,K,DI)
    U=RB(UB,UA,DI-A,DAC,K)
    V=DI-B
    RETURN
50  CALL SORT(S3A,S3B,UB,UA,K,DI)
    V=RB(UB,UA,DI-B,DBC,K)
    U=DI-A
    RETURN
END

```

```

SUBROUTINE UV2(DI,B,BCC,ACC,S3B,S3A,K,U,V)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S3B(13),S3A(13),UB(13),UA(13)
U=0.
DAC=DI-ACC
IF ((B.GT.DAC.OR.B.GT.BCC).AND.B.LE.DAC) GOTO 10
V=DI-B
RETURN
10  CALL SORT(S3A,S3B,UB,UA,K,DI)
    V=RB(UB,UA,DI-B,DI-BCC,K)
    RETURN
END

```

```

SUBROUTINE UV3(DI,A,BCC,ACC,S3B,S3A,K,U,V)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION S3B(13),S3A(13),UB(13),UA(13)
V=0.
DBC=DI-BCC
IF ((A.GT.DBC.OR.A.GT.ACC).AND.A.LE.DBC) GOTO 10
U=DI-A
RETURN
10 CALL SORT(S3B,S3A,UB,UA,K,DI)
U=RB(UB,UA,DI-A,DI-ACC,K)
RETURN
END

```

```

SUBROUTINE UV4(DI,B,A,U,V)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IF ((B+A).GT.DI) GOTO 20
U=DI
V=0.
RETURN
20 U=DI-A
V=DI-B
RETURN
END

```

```

FUNCTION XX(K,P)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
REAL KK,PF
C THIS FUNCTION IS DEFINED FOR K=2,...,60, AND
C P=2,...,12. THE VALUES OF THIS FUNCTIFON ARE
C USED IN MOST ALL OF THE SUB-PROGRAMS.
INTEGER P
KK=FLOAT(K)
PF=FLOAT(P)
100 X=KK**(1./(PF-1.))+1./(KK**(1./(PF-1.)))
TOP=X**P-2.*X**(P-1)-KK*X+KK+1.
BOTTOM=PF*X**(P-1)-2.*(PF-1.)*X**(P-2)-KK
DELTAX=-TOP/BOTTOM
X=X+DELTAX
IF (ABS(DELTAX).GT.1.E-12) GOTO 100
XX=X
RETURN
END

```



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