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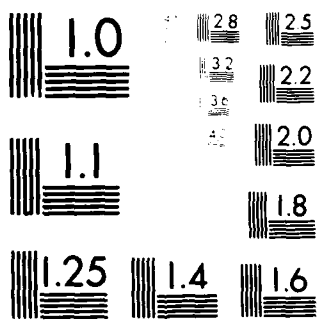

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SYSTEMS ENGINEERING RESEARCH REPORT 81-1

ON SENSITIVITY ANALYSIS IN SYSTEMS FOR PLANNING AND DECISION SUPPORT

Andrew P. Sage

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This paper surveys contemporary research involving error and sensitivity analysis approaches useful for the design of aids for planning and decision support. Discussed are structural sensitivity considerations as well as the effects of errors, for both single and multiattribute cases, in estimation or elicitation of probabilities and utilities. One of the major uses for sensitivity analysis type results is in bounded prioritization of alternatives using ordinal information. This use of sensitivity analysis is discussed and illustrated with examples.

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SENSITIVITY ANALYSIS IN SYSTEMS FOR PLANNING AND DECISION SUPPORT

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ABSTRACT

This paper surveys contemporary research involving error and sensitivity analysis approaches useful for the design of aids for planning and decision support. Discussed are structural sensitivity considerations as well as the effects of errors, for both single and multiattribute cases, in estimation or elicitation of probabilities and utilities. One of the major uses for sensivitity analysis type results is in bounded prioritization of alternatives using ordinal information. This use of sensitivity analysis is discussed and illustrated with examples.

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1. Introduction

A contemporary effort of much interest is the design of evaluation and choice making aids for planning and decision support processes. These adjuvants are sometimes called management information systems; although we feel that the terms decision support system, or planning and decision support system, are more appropriate. A central purpose in use of these systems is not just presentation of information representing facts and values,but the aggregation of this information to aid in evaluation and choicemaking. Research in this area involves many disciplines and perspectives; and thus we have a large scale systems problem. There are a number of sources of error in the design of decision analysis algorithms for planning and decision support systems. We discuss several of these in this paper, namely: errors in the structure of the decision situation, errors in the elicitation of probabilities, and errors in the assessment of single and multiple utility functions. Also, decisionmakers sometimes find it very difficult to provide precise (cardinal) estimates of weights and find it much less stressful to provide ordinal values. As shown in Section 4, sensitivity results can often then be used to infer priorities. We conclude our survey and presentation with a discussion of some contemporary research needs in this area.

2. Sensitivity to Probability Estimation Errors

In decision analysis problems under risk, it is necessary to obtain an objective or subjective estimate of the probability that various outcome states will result from decision alternatives. Once a recommended decision has been established, it is often useful to determine the magnitude of the changes in the state probabilities required for the recommended decision to become less desirable than another decision alternative. This magnitude, coupled with some knowledge of the quality of the state probability estimates, can be used to determine how confident we are in the optimality of the recommended decision.

We consider the three outcome state case first; then we generalize these results to the n outcome state case. For convenience, we assume that the probability associated with each outcome state is independent of the action alternatives. With alternative a^i we associate a utility vector

$$(u^i)^T = [u_1^i \quad u_2^i \quad u_3^i] \tag{2.1}$$

and we write for the outcome state probability

$$p^T = [p_1 \quad p_2 \quad p_3] \tag{2.2}$$

The expected utility of alternative a^i is then

$$EU(a^i) = p^T u^i = p_1 u_1^i + p_2 u_2^i + p_3 u_3^i \tag{2.3}$$

Now suppose that the probabilities p_i, are perturbed. Since we must maintain

$$p_1 + p_2 + p_3 = 1, \; p_i \geq 0 \tag{2.4}$$

we must have

$$\Delta p_1 + \Delta p_2 + \Delta p_3 = 0 \tag{2.5}$$

By substituting Eq. (2.4) into Eq. (2.3), we see that the equation for constant expected utility is that of a straight line

$$EU(a^i) = u_3^i + p_1(u_1^i - u_3^i) + p_2(u_2^i - u_3^i) \tag{2.6}$$

in two dimensions, p_1 and p_2. The difference in expected utility for alternative i and j is some amount Δ^{ij} given by

$$\Delta^{ij} = EU(a^i) - EU(a^j) = p_1(u_1^i - u_1^j) + p_2(u_1^i - u_2^j) + p_3(u_3^i - u_3^j) \tag{2.7}$$

As long as $\Delta^{ij} \geq 0$ we know that alternative a^i is preferred to alternative a^j. The relation $\Delta^{ij} = 0$ graphs as a straight line in the p_1, p_2 plane if we make use of Eq. (2.4) to eliminate p_3. Doing this, however, distorts the planes of interest somewhat. Fortunately, the three dimensional space for p_1, p_2, p_3 becomes a plane when we associate the constraint of Eq. (2.4) with this space.

Figure (2.1) indicates a typical probability triangle. It is straightforward to show that graphs of $\Delta^{ij} = 0$ are straight lines in Figure 2.1. For example, the graphs of $\Delta^{ij} = 0$ for the problem of Table 2.1 are shown in Figure 2.1.

Alternative	Outcome States x_1	x_2	x_3
a^1	1.0	0.7	0.5
a^2	0.9	0.6	0.3
a^3	0.6	0.9	0.2
a^4	0.7	0.8	0.3
a^5	0.0	0.0	1.0

Table 2.1 Illustrative Utilities for a Simple Example

Of interest in this figure is the fact that alternative 1 dominates alternative 2 in that the utility of each outcome state is greater for alternative 1 than for alternative 2. Further, it is not possible for alternative 4 to be the best alternative and the optimum decision regions are as shown in Figure 2.2.

For more than three outcome states, the graphical approach suggested here is infeasible. Isaacs [12] and Fishburn, Murphy and Isaacs [7] describe a general approach that is applicable to the n dimensional case. This approach allows determination of that "second best" alternative which could become the best alternative due to a minimum overall variation in probability.

There are, in general, a variety of possible ways in which probability estimates can be incomplete. Among these are the following [6, 12]:

1. The decision maker provides an estimate of the p_i
2. The decision maker provides a probability density function, $g(p_i)$, for the p_i.
3. There exists no information about the p_i
4. There exists an ordinal measure, or ordering of the p_i. For convenience, and without loss of generality, we may assume that $p_1 \geq p_2 \geq \cdots \geq p_n$.
5. There exists bounded internal measures such that each p_i is bounded, such as $a_i \leq p_i \leq a_i + e_i$ with $a_i \geq 0$, $e_i \geq 0$.
6. There exists a set of inequalities relating the P_i, such as
$$\sum_{j=1}^{k+1} p_{j+1} \geq p_j \geq \sum_{J=1}^{k} p_{j+1}$$
7. Some of the p_i are known whereas others are related by inequalities of the forms given in (4), (5), or (6).

There exists several ways in which the, possibly partial, information concerning probabilities may be processed to assist in evaluation of the alternative courses of action. Among these are:

(a) The estimate of the p_i, if provided, may be used to obtain the subjective expected utility of each alternative a^i from the relation $\sum_{j=1}^{n} p_j u_j^i$. A sensitivity analysis similar to that of our Section 3 may be used to determine the possibility of a decision switch due to probability elicitation errors.

(b) The expected probability of each event outcome may be computed from $\hat{p}_j = \int_{-\infty}^{\infty} p_j g(p_j) dp_j$ and the $\hat{p}_j$ used in place of the p_j. Estimates of subjective expected utility are obtained as in (a) above.

(c) Various dominance relations may be obtained from the ordinal bounds and bounded interval measures provided by the decision maker.

(d) Various minimum changes for a decision switch may be obtained.

(e) Regions in which various alternatives are best may be determined and displayed for the decision maker.

Often,various forms of stochastic dominance [27, 28] can be used to eliminate alternatives from consideration even when there exists little or no information concerning event outcome probabilities. For case 3, in which there exists no probability information, alternative j will dominate alternative i if $u^j \geq u^i$,with the inequality holding for at least one component of the utility vector. In component form this becomes $u_k^j \geq u_k^i$, k=1,2... n. For example,we easily see that alternative 1 dominates alternative 2 for the alternatives and utilities illustrated in Table 2.1. This may be written $a^1 \succ a^2$.

Further,it is often possible to identify an alternative which may not be dominated but which is inferior to or dominated by a mixed strategy consisting of a mass probability $F^T = [F^1, F^2, \ldots, F^m]$ on the set of primary alternatives $a^T = [a^1, a^2, \ldots, a^m]$. If the decision maker adopts mixed strategy F, then alternative a^i is elected with probability F^i. For the example posed by the data Table 2.1 we see that $0.5\, u_j^1 + 0.5\, u_j^3 \geq u_j^4$, j= 1, 2, 3. Thus alternative 4 is dominated by the mixed strategy $F^T = [0.5, 0, 0.5, 0, 0]$. Consequently we may delete alternative 4 from further consideration for the particular case when mixed strategies can be considered and when there is no information available concerning the probabilities of event outcomes. It is dominated by the mixed strategy of choosing alternative 1 with probability 0.5 and alternative 3 with probability 0.5.

In the general case, with no assessment of probabilities, alternative a^i is dominated by a mixed strategy F, assuming the probability vector p is the same for all alternatives, if

$$\sum_{j \neq 1} F^j u_k^j \geq u_k^i \text{ for } k = 1, 2, \ldots, n \tag{2.8}$$

If there exist an ordinal ranking of probabilities, as in case 4, then we can easily show that option alternative a^i is dominated by a mixed strategy if there exists a mixed strategy F such that

$$\sum_{j \neq 1} F^j \left(\sum_{k=1}^{j} u_k^j \right) \geq \sum_{k=1}^{j} u_k^i \text{ for } j = 1, 2, \ldots, m \tag{2.9}$$

Unfortunately there does not appear to be any method that is general and simple to use to determine appropriate mixed strategies. Often, also, the appropriateness of mixed strategies must be questioned for many applications.

For the case where there exists bounded interval measures in the form of case 5, then the primary strategy alternative a^i dominates alternative a^j if

$$\Xi \cdot \left[\sum_{k=1}^{n} \alpha_k \left(u_k^i - u_k^j \right) \right] \geq 0 \tag{2.11}$$

where Ξ is the minimum value of the objective function for the linear programming problem

$$\min \sum_{k=1}^{n} x_j(u_k^i - u_k^j) \tag{2.13}$$

subject to $0 \le x_k \le e_k$

$$\sum_{k=1}^{n} x_k = 1 - \sum_{k=1}^{n} \alpha_k \tag{2.14}$$

There appear to be no general formulae for cases 6 and 7 in which there exists sets of inequalities governing the p_i. For any given set of inequalities, we may write equations involving expected utilities and then equate coefficients. A paper by Barron [1] provides two detailed examples of computations involving these inequalities. Additional details concerning sensitivity of decisions to probability estimation errors may be found in [6, 7, 23, 28, 29].

3. Sensitivity to Variations in Utility-Single Attribute Utility Functions for Decisions Under Uncertainty

In this section, we examine sensitivity relations to changes in utility functions. This section will concern the single attribute case. We will extend the results in this section to the scalar multiattribute case in the next section. The vector multiattribute is considered in [28]. When considering utility function changes, it is convenient but not at all necessary to consider that the same output value or utility function is common to all alternatives and, therefore, represent different alternatives by different probability density or mass functions.

For the continuous state case, we consider a utility function $u(x)$ and alternative a^i defined by the associated probability density function $f^i(x)$. For the finite state case we consider a utility function $u(x_i)$, $i = 1, 2, \ldots, n$ and alternatives a^j defined by an associated probability mass vector function P^j which has n components, $(P^j)^T = [P^j(x_1), P^j(x_2), \ldots P^j(x_n)] = [P_1^0, P_2^j, \ldots, P_n^j]$.

There are a variety of ways in which utility estimates can be stated, possibly incompletely. Among these are the following:

1. The decision maker may provide a complete estimate of the value function $v(x)$ and utility function $u(x)$
2. The utility function $u(x)$ or value function may be completely unspecified.
3. Only ordinal preferences are specified.
4. The value function may be specified, but not the risk aversion coefficient. In this case, the utility function is unspecified.

If the value and utility function are completely specified, the expected utility of each alternative may be computed and a sensitivity analysis similar to that of Section 4 conducted to determine the potential of a likely decision switch due to utility and value function elicitation errors. If the value function is completely unspecified, then the decision maker is inchoate and there is virtually nothing that can be done to aid the decision maker except through procedures that will enhance the value coherence of the decision maker.

Much information concerning alternative option preferences can be obtained from just an ordinal ranking of preference for outcome states. In the sequel we will assume that the decision maker can always express an ordinal preference for the value of event outcomes of the form $v(x_1) \leq v(x_2) \leq \ldots, \leq v(x_n)$.

When ordinal preferences among event outcomes can be elicited, and when probabilitic estimates of occurence of these states can be obtained, then concepts of stochastic dominance can be used to determine bounds on alternative preferences. And if values but not utilities are specified, then it will often be possible to specify risk aversion coefficients which bound alternative preferences. We will illustrate each of these claims by means of a simple example. Prior to doing this, however, it is desirable to establish some fundamental concepts concerning stochastic dominance [29].

We say that alternative a^i is preferred to alternative a^j whenever the expected utility of alternative a^i is greater than that for alternative a^j. In symbols, we have if $a^i \succ a^j$

$$\int_{-\infty}^{\infty} f_i(x)\, u(x)\, dx > \int_{-\infty}^{\infty} f_j(x)\, u(x)\, dx \tag{3.1}$$

where $f_i(x)$ is the probability density function for the event outcome, x, associated with alternative a^i; and u(x) is the utility of the outcome states. We wish to provide for a value function v(x) and will restrict this, for convenience to the interval [0, 1]. The value function is not necessarily a monotone function of the outcome states, x. The utility function, however, should be isotone in v. Thus it is convenient to rewrite Eq. (3.1) as $a^i \succeq a^j$ if

$$\int_0^1 f_i(v)\, u(v)\, dv \geq \int_0^1 f_j(v)\, u(v)\, dv \tag{3.2}$$

where we realize that the utility function is written as u[v(x)] but where we delete the x symbol for convenience.

Stochastic dominance concepts are based upon the imposition of a series of increasing constraints upon the form of the utility function u(v). The most trivial assumption is that utility is a monotone increasing function of increasing value. Thus we require $du(v)/dv = u'(v) \geq 0$. We now integrate Eq. (3.2) by parts. Since we have

$$\int_0^1 f_i(v)\, u(v)\, dv = P_i(v)\, u(v) \Big|_0^1 - \int_0^1 P_i(v)\, u'(v)\, dv$$

$$= 1 - \int_0^1 P_i(v)\, u'(v)\, dv$$

we obtain for Eq. (3.2)

$$\int_0^1 [P_i(v) - P_j(v)]\, u'(v)\, dv \leq 0 \tag{3.3}$$

Without specification of u(v) but with specification that $u'(v) \geq 0$, we see that the inequality of Eq. (3.3) will be satisfied if and only if

$$P_i(v) \leq P_j(v) \quad \forall v \in [0, 1] \tag{3.4}$$

With the inequality holding for at least one i. This is the requirement for first order stochastic dominance. When the inequality of Eq. (3.4) holds we say that a^i dominates a^j by first order stochastic dominance or $a^i \succ_1 a^j$.

We can rewrite the expression for the probability mass function

$$P_i(v_k) = \int_0^{v_k} f_i(v)\, dv$$

in terms of discrete state probabilities $p_i(v_\ell)$, $\ell = 1, 2, \ldots, k$ as

$$P_i(v_k) = \sum_{\ell=1}^{k} p_i(v_\ell)$$

If, in addition to requiring monotonicity of the utility function, we also require risk aversion; then we impose the further requirement that $d^2u(v)/dv^2 = u''(v) \leq 0$. We integrate Eq. (3.3) by parts to obtain

$$u'(1)\, \Gamma(1) - \int_0^1 \Gamma(v)\, u''(v)\, dv \leq 0$$

where

$$\Gamma(v) = \int_0^v [P_i(\alpha) - P_j(\alpha)]\, d\alpha$$

To satisfy this requirement for $u'(1) \geq 0$ and $u''(v) \leq 0$, we must require $\Gamma(v) \leq 0$ in the interval 0 to 1. Our requirement for what is called second order stochastic dominance becomes, therefore,

$$\int_0^v [P_i(\alpha) - P_j(\alpha)]\, d\alpha \leq 0, \quad \forall v \in [0, 1] \tag{3.5}$$

For the finite state case we replace Eq. (3.5) with

$$\sum_{\ell=1}^{k} P_i(v_\ell) - P_j(v_\ell) \quad [v_{\ell+1} - v_\ell] \leq 0, \forall k \in [2, n] \tag{3.6}$$

with the inequality holding for at least one k.

An excellent discussion of stochastic dominance concepts is presented in the chapter by Fishburn and Vickson in [29].

In the general case we can write the requirement for k^{th} order stochastic dominance of alternative a^i over a^j, $a^i \succ_2 a^j$, as

$$\Gamma_k(v) \leq 0 \qquad \forall\, v \in [0,1] \tag{3.7}$$

where

$$\Gamma_k(v) = \int_0^v \Gamma_{k-1}(\alpha)\, d$$

$$\Gamma_1(v) = P_i(v) - P_j(v)$$

Satisfaction of Eq. (3.7) will insure, for increasing k, various increasing requirements on risk aversion of the form $u'(v) \geq 0$, $u''(v) \leq 0, \ldots, (-1)^k$ $u^k(v) \leq 0$.

A particularly interesting case occurs for $k = \infty$. The utility curve for infinite risk aversion is given by $u(v) = 0$, $v = 0$ and $u(v) = 1$ $\forall v \in (0,1]$. Generally, it will be relatively easy to determine requirements for infinite order stochastic dominance.* Since the strength of the dominance relation increases with increasing order, we can use this concept to advantage, especially in the multiattribute case. We note that we will be able to determine first order dominance, and infinite order dominance, for many discrete state problems with only ordinal event outcome preference information. Value preference bounds may be determined from ordinal preference information from higher order stochastic domination concepts. This requires the solution of linear programs, much the same as those of Eqs. (2.12) through (2.14) [27-29].

Mean variance dominance, or expected value dominance, is a concept that has often been used, especially in capital budgeting efforts. Alternative a^i dominates alternative a^j in terms of expectation or mean value domination if it has a greater expected value such that

*We can also compute, with relative ease, the expected utility for the modified infinite risk aversion case where $u(v)=0$, $\forall v \in [0, v_1]$ and $u(v)=1$, $\forall v \in (v_1, 1]$.

$$\bar{a}_i = \int_0^1 vp_i(v)\,dv \geq \int_0^1 vp_j(v)\,dv = \bar{a}_j \tag{3.8}$$

Alternative a^i dominates alternative a^j in terms of variance if the variance associated with alternative i is less than that associated with alternative j, or

$$\sigma_i^2 = \int_0^1 v^2\, p_i(v)dv - a_i^2 \leq \int_0^1 v^2 p_j(v)dv - a_j^2 = \sigma_j^2 \tag{3.9}$$

When an alternative dominates another alternative in both expectation and variance, it is said to dominate it in an EV domination sense. Generally, domination in either expectation or variance is not necessarily meaningful. And even EV domination is a less valuable concept than the various stochastic dominance concepts, in that one can easily configure problems for which a non preferred alternative, that is one either stochastically dominated by another alternative or with a lower expected utility than another alternative, may dominate another alternative in an EV domination sense [28]. Thus the EV domination concept must be used with caution.

As an example of sensitivity calculations which use stochastic domination concepts, we consider the problem posed by Table 3.1.

Alternative	state x_1 $v(x_1)=0$	x_2 $v(x_2)=0.25$	x_3 $v(x_3)=0.5$	x_4 $v(x_4)=0.75$	x_5 $v(x_5)=1.0$
a^1	0.25	0.25	0.25	0.25	0
a^2	0	0.25	0.25	0.25	0.25
a^3	0.15	0.15	0.20	0.25	0.25
a^4	0.20	0.80	C	0	0
a^5	0.50	0	0	0	0.50
a^6	0	0	1.0	0	0

Table 3.1 Probability of Occurrence of Various Event Outcome States (shown also are cardinal values, which are used in some calculations)

For first order stochastic dominance we have, from Eq. (3.4), the first order stochastic dominance reachability matrix and digraph [21] shown in Figure (3.1). We note that we need only use the ordinal preferences information among state values to determine this domination relationship.

We must know cardinal values in order to use second and higher order domination relations, however. Bounds on these values can be utilized, and we can consider multivariate multiattribute outcomes, as discussed in [27], [28] and [29]. Unfortunately, this requires resolution of a number of linear programs and this can be computationally rather unattractive. If we use the cardinal values specified in Table 3.1, it is a simple matter to use Eq. (3.5) or Eq. (3.6) to obtain the reachability matrix and minimum edge digraph shown in Figure 3.2. We note that first order stochastic dominance insures second order stochastic dominance. Thus, there is no need to determine dominance for the dominated relations of Figure 3.1. From Figure 3.1 we see that the only possible dominance relations which we need check are

a^1 vs a^4, a^5, a^6

a^2 vs a^5, a^6

a^3 vs a^5, a^6

a^4 vs a^5

a^5 vs a^6

From these candidate relations we determine that

$$a^6 \succ_2 a^1, \quad a^2 \succ_2 a^5, \quad a^3 \succ_2 a^5, \quad a^6 \succ_2 a^5$$

and these are, of course, shown in Figure 3.2.

For infinite order stochastic dominance we easily see, from Table 3.1, that

$$a^2 \sim a^6 \succ_\infty a^3 \succ_\infty a^4 \succ_\infty a^1 \succ_\infty a^5$$

and this dominance pattern is illustrated in Figure 3.3. This dominance digraph indicates that the best two alternatives are a^2 and a^6; just as did the second order stochastic dominance effort. But, infinite order stochastic dominance results, effectively, in a maximization of the probability that the alternative selected will result in other than the minimum return.

*Stochastic dominance of order n guarantees stochastic dominance of order greater than n.

It can be a rather pesimistic criterion, therefore. It may well turn out that alternative a^3 is preferred to alternative a^6 for a more realistic utility function.

We next examine EV domination and easily establish the results shown in Figure 3.4. From this figure we indeed see that domination based on expectation only is a poor indicator to use for choicemaking. Although the EV dominance digraph shown in Figure 3.4C is slightly different from that obtained using the second order stochastic domination results, it does indicate that the best three alternatives are a^2, a^3, a^6.

To determine the final choice alternative we might assume a standard exponential relationship, expressing constant risk aversion r, to relate utility and value. Here we will use

$$u(v) = \frac{1 - e^{-rv}}{1 - e^{-r}}$$

The expected utilities for alternatives a^2, a^3, a^6 are easily determined for various r as shown in Figure 3.5. There is no way, of course, that a^3 can be the preferred alternative since it is dominated by a^2. a^6 can be either the most preferred, the second most preferred, or the third most preferred alternative depending upon the amount of risk aversion. Figure 3.5 indicates transition points where the various optimum decision alternatives change.

4. Sensitivity in Scalar Multiple Attribute Utility Analysis

Much contemporary emphasis has been placed on the evaluation of alternatives using multiple attribute utility theory. The numerical utility that results from use of multiattribute utility theory depends upon the structure of the multiple attribute utility function, the scaling coefficients within this structure, and the individual single attribute utility functions which are aggregated to determine the multiple attribute utility functions.

Fishburn [8,9] has considered approximations of scalar multiattribute utility functions in which u(x) is a continuous real valued scalar cardinal utility function and v(x) is an approximation for u(x). A number of approximations to u(x) of the general form

$$v(x) = v(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{k} f_{1_j}(x_1)\, f_{2_j}(x_2) \ldots f_{n_j}(x_n)$$

are considered. A distance metric in the form of the uniform norm

$$D(u, v) = \sup |\, u(x) - v(x)|$$

is minimized. Among the results of these efforts, the following are especially significant:

1. An additive utility function u(x) can be approximated to arbitrary closeness by the multiplicative form

   $$v(x) = \prod_{i=1}^{n} \nu(x_i)$$

2. If u(x) is multiplicative, then there is a lower bound on the distance between the actual utility and an additive approximation

One of the interesting conclusions of this effort is that simple additive approximations may function as well as the more complicated multiplicative approximations for cases in which the true utility function, $u(x)$, is neither multiplicative nor additive.

Much effort has been devoted to parameter sensitivity in deterministic additive models. Among the most useful results obtained from these studies is the indication that differential weighting of attributes is often not necessary and that equal weighting will perform essentially as well when all attributes have a high positive correlation with each other. Leung [14] provides a survey of much of this work with a number of references to contemporary literature.

Among the more useful of sensitivity type results that can be established for multiattribute decisionmaking under certainty are various types of dominance results for the case where attribute scores, in the range 0 to 1, can be established for each alternative. In the most unspecified case nothing is known about the n weights for an assumed linear multiattribute utility function except that

$$w_i \geq 0, \ i = 1, 2, \ldots, n \tag{4.1}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4.2}$$

$$u^j = u(a^j) = w^T u^j = \sum_{i=1}^{n} w_i u_i(a^j) = \sum_{i=1}^{n} w_i u_i^j \tag{4.3}$$

If it turns out that $u_i^j \geq u_i^k \ \forall \ i$, with the inequality holding for at least one i, then we easily see that alternative j must have, regardless of the weights, a greater utility, $u(a^j)$, than alternative k. We say that a^j dominates a^k. If we have, for example,

$$u^1 = 0.2\ w_1 + 0.2\ w_2$$
$$u^2 = 0.5\ w_1 + 0.2\ w_2$$

$$u^3 = 0.3\, w_1 + 0.2\, w_2$$

$$u^4 = 0.5\, w_1 + 0\, w_2$$

then we see that $a^2 \succeq a^1$, $a^2 \succeq a^3$, and $a^2 \succeq a^4$ for any w_1 and w_2 subject to Eqs. (4.1) and (4.2). Figure 4.1 illustrates these four utilities and the associated dominance relations. We see from this figure that there is no dominance of alternative 3 by alternative 1; nor is there dominance of alternative 3 by alternative 4. Yet we see that there are no values of the weights such that a^3 is preferred to a^1 and a^4. We may establish this fact by noting that the requirements for $a^3 \succeq a^1$, which is $w_1 \geq w_2$, and $a^3 \succeq a^4$, which is $w_2 \geq 2w_1$, are inconsistent.

In the general case where utilities are defined by Eq. (4.3) and the constraints of Eqs. (4.1) and (4.2) hold, we can show that alternative k will be dominated by a mixed strategy of the (m-1) remaining alternatives if there is a non zero (positive) solution J to the linear programming problem

$$J = \min \sum_{\substack{i=1 \\ i \neq k}}^{m} d_i^k \tag{4.4}$$

$$w_i \geq 0 \,,\; i = 1,2,\ldots,n$$

$$\sum_{i=1}^{n} w_i = 1$$

$$d_j^k \geq 0,\; j = 1,2,\ldots,\; k-1,\; k+1,\ldots,\; m \tag{4.5}$$

$$\sum_{i=1}^{n} w_i(u_i^k - u_i^j) + d_j^k \geq 0,\; j=1,2,\ldots,\; k-1,\; k+1,\ldots,\; m \tag{4.6}$$

When we compare alternative 3 with a mixed strategy of alternatives 1 and 4 we see that the foregoing relations become

$$J = \min(d_1^3 + d_4^3)$$

$$w_1 \geq 0$$

$$w_2 \geq 0$$

$$w_1 + w_2 = 1$$

$$d_1^3 \geq 0$$

$$d_4^3 \geq 0$$

$$.1\ w_1 - .1\ w_2 + d_1^3 \geq 0$$

$$-0.2\ w_1 + 0.1\ w_2 + d_4^3 \geq 0$$

The solution to this linear programming problem is

$$d_1^3 = 0.1 - 0.2\ w_1 \quad \forall \quad w_1 \leq 0.5$$

$$d_4^3 = 0.3\ w_1 - 0.1 \ \forall \quad w_1 \geq 0.33$$

We see that there is a non zero J over the entire range of w_1 and thus note again that alternative 3 is dominated by a mixed strategy of alternatives 1 and 4.

In many cases it will be possible for the decisionmaker to express ordinal weights, or bounds, on the weights w_i in the form

$$w_1 \geq w_2 \geq \cdots \quad w_n \geq 0 \tag{4.7}$$

Suppose also that it turns out that

$$u_1^j = u_1^k + \varepsilon_1$$

$$u_2^j = u_2^k + \varepsilon_2 - \varepsilon_1$$

$$u_3^j = u_3^k + \varepsilon_3 - \varepsilon_2$$

$$\vdots$$

$$u_i^j = u_i^k + \varepsilon_i - \varepsilon_{i-1} \tag{4.8}$$

$$\vdots$$

$$u_n^j = u_n^k + \varepsilon_n - \varepsilon_{n-1}$$

where $\varepsilon_i \geq 0 \ \forall \ i$. Then we have for the utility of alternative j

$$u^j = \sum_{i=1}^{n} w_i \ u_i^j = \sum_{i=1}^{n} w_i(u_i^k + \varepsilon_i - \varepsilon_{i-1}) \tag{4.9}$$

where $\varepsilon_0 = 0$. We can write, by changing the summation index,

$$\sum_{i=1}^{n} w_i \ \ \varepsilon_{i-1} = \sum_{i=0}^{n-1} w_{i+1} \ \ \varepsilon_i$$

such that Eq. (4.8) becomes, using Eq. (4.3),

$$u^j = u^k + \sum_{i=1}^{n-1} \varepsilon_i \ (w_i - w_{i+1}) + \varepsilon_n w_n$$

Since we know that $\varepsilon_i \geq 0$ and $w_i \geq w_{i+1} \geq 0$, we have established the fact that if the equalities of Eq. (4.8) hold, we must have $a^j \succ a^i$ regardless of the value of the weights. Equation (4.8) is not in the form most suited for actual use and can easily be rewritten in terms of the cumulative difference inequality

$$\sum_{i=1}^{m} u_i^j \geq \sum_{i=1}^{m} u_i^k \ , \ m = 1,2,\ldots, n \tag{4.10}$$

which is in a form quite suitable for actual use.

For the four alternatives and utilities considered earlier in this section we still have $a^2 \succeq a^1$, $a^2 \succeq a^3$ and $a^2 \succeq a^4$. We also have $a^4 \succeq a^3$, $a^3 \succeq a^1$, and the implied transitive preference, $a^4 \succeq a^1$ as well. In this particular case, specifying an ordinal scale for the attribute weights has completely ordered the (dominance) preference relationship. As indicated in Figure 4.2, the imposition of the constraint $w_1 \geq w_2$ simply eliminates the right half of the space in Figure 4.1 where $w_2 \geq 0.5$.

Even though alternative k may not be dominated by alternative j, it may or may not always have a lower utility than the mixed strategy of alternative i or alternative j. For example in

$$u^5 = 0.6\ w_1 + 0.4\ w_2$$

$$u^6 = 0.3\ w_1 + 0.9\ w_2$$

$$u^7 = 0.7\ w_1 + 0.2\ w_2$$

we see that alternative 5 is not dominated by alternative 6 or alternative 7 if $w_1 \geq w_2$.* However in order to have $a^5 \preceq a^6$ and $a^5 \preceq a^7$ we must have

$$w_1 \geq w_2$$

$$0.6\ w_1 + 0.4\ w_2 \geq 0.3\ w_1 + 0.9\ w_2$$

$$0.6\ w_1 + 0.4\ w_2 \geq 0.7\ w_1 + 0.2\ w_2$$

or

$$1.67 w_2 \leq w_1 \leq 2\ w_2$$

*If, $w_2 > w_1$ then alternative 6 dominates alternative 5 as can be shown.

or, since $w_2 = 1 - w_1$,

$$5/8 \leq w_1 \leq 2/3$$

Thus, dominance of a^5 over a^6 and a^7 is not guaranteed except over a small range of w_1. On the other hand if we have

$$u^8 = 0.8\, w_1 + 0.1\, w_2$$

then we will have $a^5 \succ a^6$ and $a^5 \succ a^8$ only if

$$1.67\ w_2 \leq w_1 \leq 1.5\ w_2$$

and this is in consistent. Thus alternative 5 can not be the alternative with the highest utility in that the utility of alternative 6 or alternative 8 is always greater than this. Figure 4.3 illustrates preference relationships among alternative 5, 6, 7, and 8 as obtained here.

For the general case where the attribute weights are ordered as in Eq. (4.7), it turns out that alternative k will be dominated by a mixed strategy of the (m-1) remaining alternatives if there is a non zero solution J to the linear programming problem

$$J = \min \sum_{\substack{i=1 \\ i \neq k}}^{m} d_i^k \tag{4.11}$$

$$w_1 \geq w_2 \geq \cdots \geq w_n \geq 0$$

$$\sum_{i=1}^{n} w_i = 1$$

$$d_j^k > 0,\ j = 1,2,\ldots,\ k-1,\ k+1,\ldots,\ m$$

$$\sum_{i=1}^{n} w_i(u_i^k - u_i^j) + d_j^k \geq 0,\ j = 1,2,\ldots,\ k-1,\ k+1,\ldots,\ m$$

There have been a number of sensitivity studies of multiattribute utility in the psychological literature [3-5, 24, 30] and the effect of errors, including cognitive bias induced errors, upon risk and uncertainty estimation [22]. A major goal of this psychological research is, a theory of errors. This will allow determination of the effects of poor structuring of decision situation models and poor elicitations of values and uncertainties and the aggregation of these into decision rules. A to be hoped for achievement of all of this research is a theory that explains and clarifies descriptive and prescriptive approaches to substantive and procedural judgment and decision processes such as to enable the design of more efficacious systems for planning and decision support. In this section we have indicated how sensitivity results can be used to this end, and how sensitivity analysis can be used to reduce, often considerably, the needed number of precise weights. This should generally reduce the potential cognitive stress involved in decision analysis efforts. Some rather general results concerning partially identified parameters and associated sensitivity analysis for planning and decison support are given in [28].

5. Sensitivity and the Structure of Decision Situation Models

Sensitivity analysis results can be used to guide the structuring of decision situations. This has been the thrust of recent efforts by Leal, Pearl, and Saleh [13, 16] ; Merkhoffer et. al. [15]; and Rajala and Sage [17-20]. Of interest also is the related work of Chen and Jarboe [2]; Haruna and Komoda [10]; and Howard, Matheson, and North [11].

Use of sensitivity measures to guide the structuring of decision trees, and related structures such as fault trees, requires the availability of preference or utility measurements for event outcomes and uncertainty measures over events. There are four concepts of value in structuring decision trees using sensitivity concepts:

(1) sensitivity differential of a node

(2) relative sensitivity differential of a node

(3) expected value of resolving residual uncertainty

(4) decision sensitivity to outcome variable uncertainty resolution

The sensitivity differential of a node, Δ, is the change that must occor in the value of that node in order to cause a change in the currently best initial decision. In Figure 5.1 for example, a decrease in the value* or utility, $v(x, d)$ at node B of more than 0.95 units will cause the best initial decision to switch from a^1 to a^3. It would require a decrease of -0.125 units in the value at node E, for example, for this to happen. A recursive relation

$$\Delta(i) = \begin{cases} \dfrac{\Delta(i-1)}{p_i} & \text{, node } i \text{ is an event node} \\ \Delta(i-1) + v_{i-1}(x) - v_i(x) & \text{, node } i \text{ is a decision node} \end{cases}$$

may be determined [13]. Here $\Delta(i)$ is the sensitivity differential associated

*We use the value symbol for convenience. All of the discussion in this section applies to utilities as well as to values.

with node i and $\Delta(i-1)$ is the sensitivity differential of the preceeding node. $v_i(x)$ and $v_{i-1}(x)$ refer to the expected values at nodes i and i-1.

The relative sensitivity differential of node i is given by

$$s_r(i) = \frac{\sigma_v(i)}{\Delta(i)}$$

where $\sigma_v(i)$ is the anticipated change in the value at node i which may result from further refinement. Unfortunately there is no general way to determine $\sigma_v(i)$ except by elicitations from the decision maker. It is reasonable that $\sigma_v(i)$ is linearly proportional to $v_i(x)$ since greater inaccuracies typically are associated with larger values.

The expected value of resolving residual uncertainty (EVRRU) may be easily computed [15]. We assume that, in Figure 6.2, the current best decision is $[a^1, a^4]$. The following results are obtained:

1. The EVRRU is zero if we consider a node, such as node 1, in which the node is along a path leading from the current best <u>initial</u> decision, a^1, but not the best current decision $[a^1\ a^4]$.
2. The EVRRU is zero if we consider a node such as node 2, in which the node is along a path leading from the current best initial decision, a^1, but where

   $$P_i\ [v(x, a^1, a^4) - v(x, a^1, a^5)] \leq v(x, a^1) - v(x, a^2)$$

3. For case 2, the EVRRU is given by

   $$\text{EVRRU} = P_i P_j\ f\ [v^2(x) + \Delta - m_*^2]$$

   where $f = \text{prob}\ [v^{*2}(x) \leq v^2(x) + \Delta]$

   $$m_*^2 = E\ [v^{*2}(x) \mid v^{*2}(x) \leq v(x) + \Delta]$$

   if the inequality in case 2 is reversed.

4. If we consider node 3 which is not along the path leading from the best initial decision, then we have

$$EVRRU = - P_L\ P_m\ g\ [v^3(x) + \Delta - m^*]$$

$$\text{where } g = \text{prob}\ [v^{*3}(x) \geq v^3(x) + \Delta]$$

$$m^* = E\ [v^{*3}(x) |\ v^{*3}(x) > v(x) + \Delta]$$

Rajala and Sage [18] have developed a 9 step tree expansion structuring procedure based upon these sensitivity relations. Steps in this procedure are:

1. Identify the initial decision alternatives and represent them by branches emanating from the first decision node.
2. Identify state variables of importance in determining the value of each alternative.
3. Elicit a value or utility function.
4. Encode provisional probability distributions on each state variable for each alternative course of action
5. Using the value model compute probability distributions, f and g, on the rollback value at the terminal nodes and each interior node using the state variable distributions.
6. Determine the next appropriate node for expansion. Either the EVRRU and/or the relative sensitivity, $s_r(i)$, is appropriate as an aid in this determination. If either, or both of these are below some threshold for all remaining nodes we stop expansion of the tree. Otherwise we go to step 7.
7. The sensitivity of the current best initial decision to uncertainty resolution in each state variable is determined. Multiattribute

utility functions are especially appropriate to aid in this task.

8. We verify that the best course of action may be affected by incorporating factors determined in steps 6 and 7 into the decision model. Often this can be accomplished by determining whether the event probability required for a decision switch converges to an amount less than or equal to the amount elicited from the decisionmaker. If it does we go to step 9. If it does not we return to step 7 and repeat this step until we are convinced that no switch in the decision is feasible.

9. Incorporate relevant features in the decision tree to obtain a new and improved representation of the decision situation. We return to step 4 and continue the process until the EVRRU and/or $s_r(i)$ are sufficiently low at all remaining nodes such that we conclude that no further improvement in the decision situation structural model is feasible.

We shall limit our discussion concerning the value of information measures to a primary decision situation that involves only a single decision d whose uncertain outcomes are represented by the discrete state variables $(x_1, \ldots, x_n)$. The uncertainty on this state vector x is encoded in the distribution f(x) and the value of the outcome is measured by the value model v(x, d).

The purpose behind considering the possibility of acquiring additional information, given this complete model of the primary decision situation, is to determine the worth of eliminating remaining uncertainty on the state variables. An important quantity which establishes an upper bound to the amount that the decisionmaker should pay to eliminate all uncertainty on a state vector is the value of perfect information.

If a clairvoyant, an individual capable of indicating the exact outcome of an uncertain quantity, were to report to the decisionmaker that a particular state vector x^i would occur, then the decisionmaker could select the appropriate course of action d_* that gives the maximimum value, denoted by $v(x^i, d_*)$. However, since the decisionmaker does not know what x^i the clairvoyant would report, the value that would be realized from a particular outcome must be weighted by the probability that the outcome will be reported. The probability that is assigned is just the prior probability, $f(x^i)$. To the decisionmaker, the expected value of the decision situation with perfect information is

$$E[v(x, d_*)] = \sum_i f(x^i)v(x^i, d_*)$$

The well known expected value of perfect information, EVPI, which is a measure of the upper bound the decisionmaker should be willing to pay to resolve all uncertainty on a state vector x, is the difference between the quantity determined in the equation above and the expected value of the optimal course of action based only on information encoded from the decisionmaker's prior knowledge and experience, that is

$$\begin{aligned} EVPI &= E[v(x, d_*)] - \max_d E[v(x, d)] \\ &= \sum_i f(x^i)v(x^k, d_*) - \max_d \sum_i f(x^i)v(x^k, d) \end{aligned}$$

The magnitude of EVPI can assist the analyst in determining the level of effort to be directed toward identifying and organizing information gathering decisions into the decision model. A lower magnitude of EVPI generally warrants a lower level of structuring activity. These methods provide a basis which allows the analyst to prompt the decisionmaker into identifying information gathering alternatives, with EVPI providing additional incentive to the decisionmaker. The expected value of information from these methods is, of course, bounded above by EVPI, assuming no structural modelling errors.

The information z obtained about the state vector x from an information gathering alternative causes the decisionmaker's experience and knowledge to change, and its effect may be completely accounted for by a revision in the probability distribution on x. The revised distribution on x is determined from Bayes' rule

$$f(x|z) = \frac{f(z|x)\ f(x)}{f(z)}$$

where the probability functions are appropriately defined. The expected value of information, EVI, is the difference between the optimal course of action with additional information and the optimal course of action without additional information. It is computed as

$$EVI = E_x[\max_{d(z,c_z)} E\{v[x, d(z, c_z]\}\ ] - \max_d E[v(x, d)]$$

$$= \sum_z [\ \max_{d(z,c_z)} \sum_z f(x|z)\ v[x, d(z, c_z)]]\ f(z) - \max_d \sum_x f(x)v(x,d)$$

where $d(z, c_z)$ indicates the primary decision d is based on information z obtained at cost c_z. If EVI is positive, then the expected value of the decision situation will increase when the secondary decision to gather additional information is made. If EVI is negative, the expected value will decrease, reflecting that information costs more than it is worth. If $c_z=0$, then EVI cannot be negative.

Rajala and Sage [17-20] present several examples of these procedures. These approaches are especially capable for use as structuring tools to determine parsimonious decision trees which are reflective of the decisionmaker's perception of the decision situation. Also, the approach provides a general method to use in "pruning" already structured trees. Of particular interest, in this connection, would be the ability to deal with multi parametric sensitivity issues [25].

6. Conclusions

This paper has presented a discussion of contemporary efforts involving error and sensitivity analysis of decision analysis algorithms for evaluation and choicemaking associated with planning and decision support. We have examined sensitivity to probability estimation errors, sensitivity to utility elicitation errors, sensitivity to variations in the structure and parameters of multiattribute utility functions, and sensitivity to variations in the structure as well as the probability and utility parameters. This structural sensitivity to expansion of the decision tree is especially useful in that it provides an aid in the determination of parsimonious models of decision situations.

Our recent research has also concerned a mixed scanning based planning and decision support system which involves a vector multiple attribute utility function [26, 27]. In this approach, which is believed behaviorally relevant, we intentionally avoid elicitation of attribute weights and only elicit those weights which can be shown to be most beneficial in increasing the domination pattern among alternatives. Sensitivity results of the type obtained in this paper have been found useful in guiding the partial aggregation of values. This results in a planning and decision support system in which the attribute weight elicitation procedure is guided by an interaction process involving the judgment and desires of the decision maker and suggestions concerning the efficiency of value aggregation that are determined by sensitivity analysis approaches.

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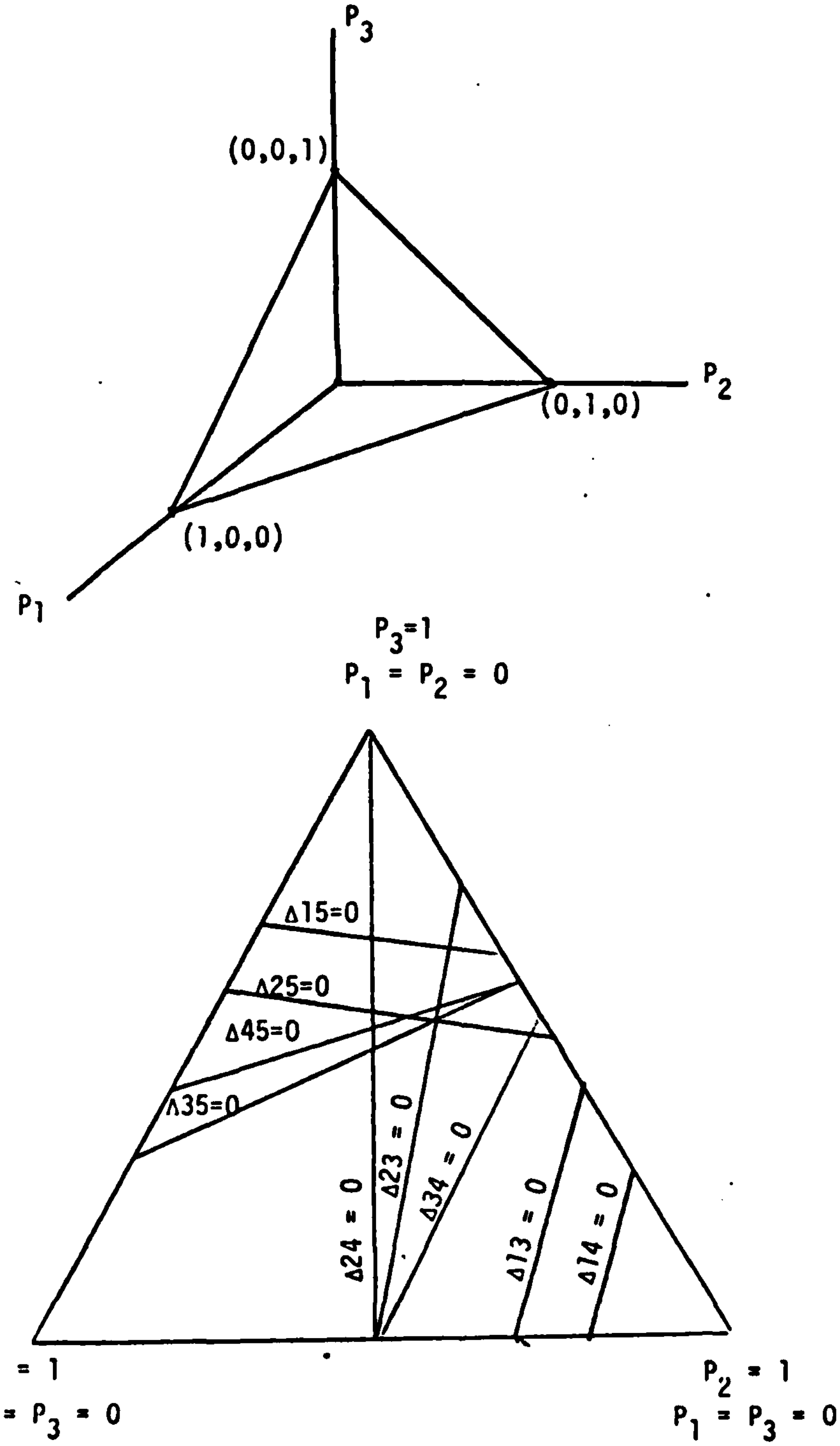


Figure 2.1 Probability Triangle and Planar Projection with Decision Switch Points from Table 2.1.

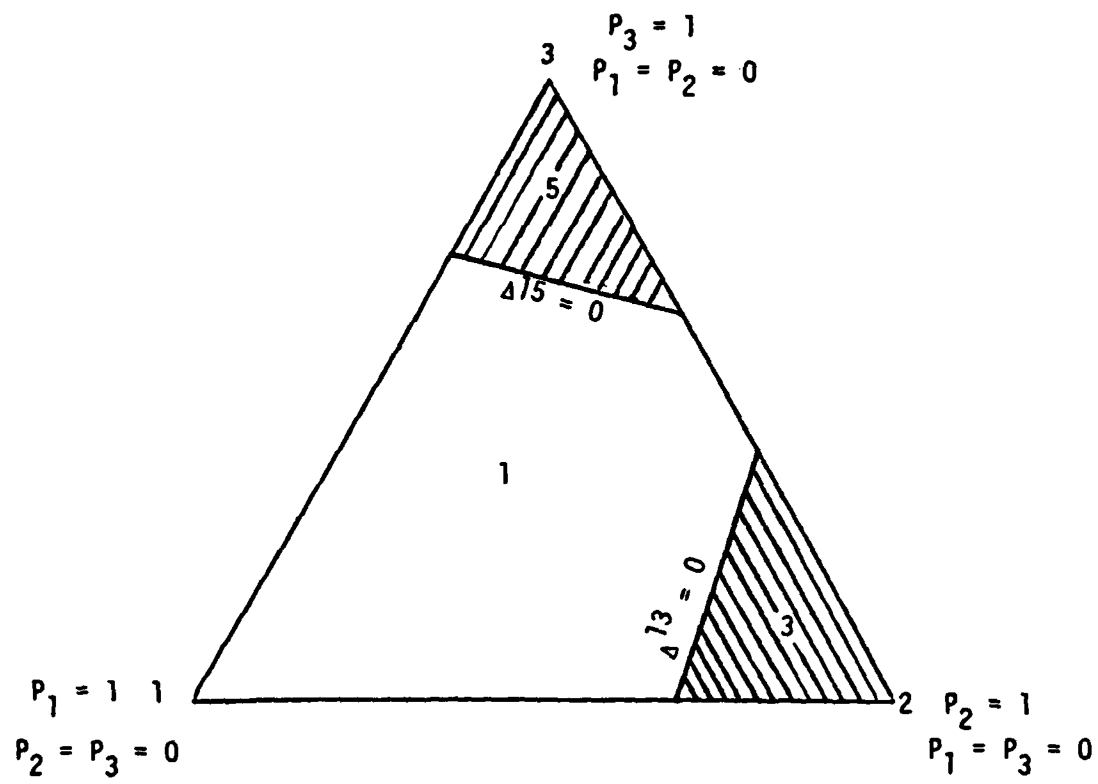


Figure 2.2 Optimum Decision Regions as a Function of Probabilities

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	0	0
a^2	1	1	1	1	0	0
a^3	1	0	1	1	0	0
a^4	0	0	0	1	0	0
a^5	0	0	0	0	1	0
a^6	0	0	0	1	0	1

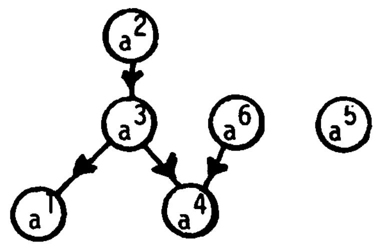


Figure 3.1 Reachability Matrix and Minimum Edge Digraph for First Order Stochastic Domination

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	0	0
a^2	1	1	1	1	1	0
a^3	1	0	1	1	1	0
a^4	0	0	0	1	0	0
a^5	0	0	0	0	1	0
a^6	1	0	0	1	1	1

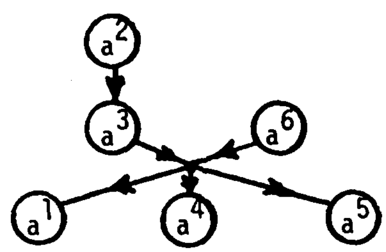


Figure 3.2 Reachability Matrix and Minimum Edge Digraph for Second Order Stochastic Dominance.

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	1	0
a^2	1	1	1	1	1	0
a^3	1	0	1	1	1	0
a^4	1	0	0	1	1	0
a^5	0	0	0	0	1	0
a^6	1	0	1	1	1	1

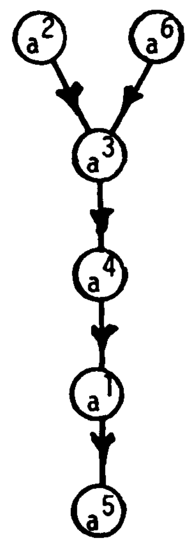
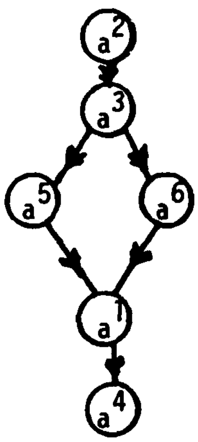


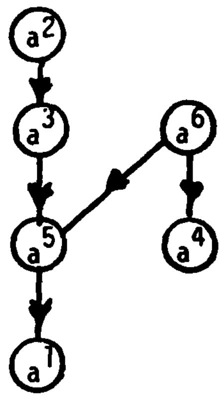
Figure 3.3 Reachability Matrix and Minimum Edge Digraph for Infinite Order Stochastic Dominance

	mean	variance
a^1	0.375	0.7813
a^2	0.625	0.7813
a^3	0.575	0.11938
a^4	0.2	0.01
a^5	0.5	0.25
a^6	0.5	0

a) Expectation Variance Values for Example



b) Domination based on expectation only



c) Domination based on expectation and variance

Figure 3.4 EV Dominance Results

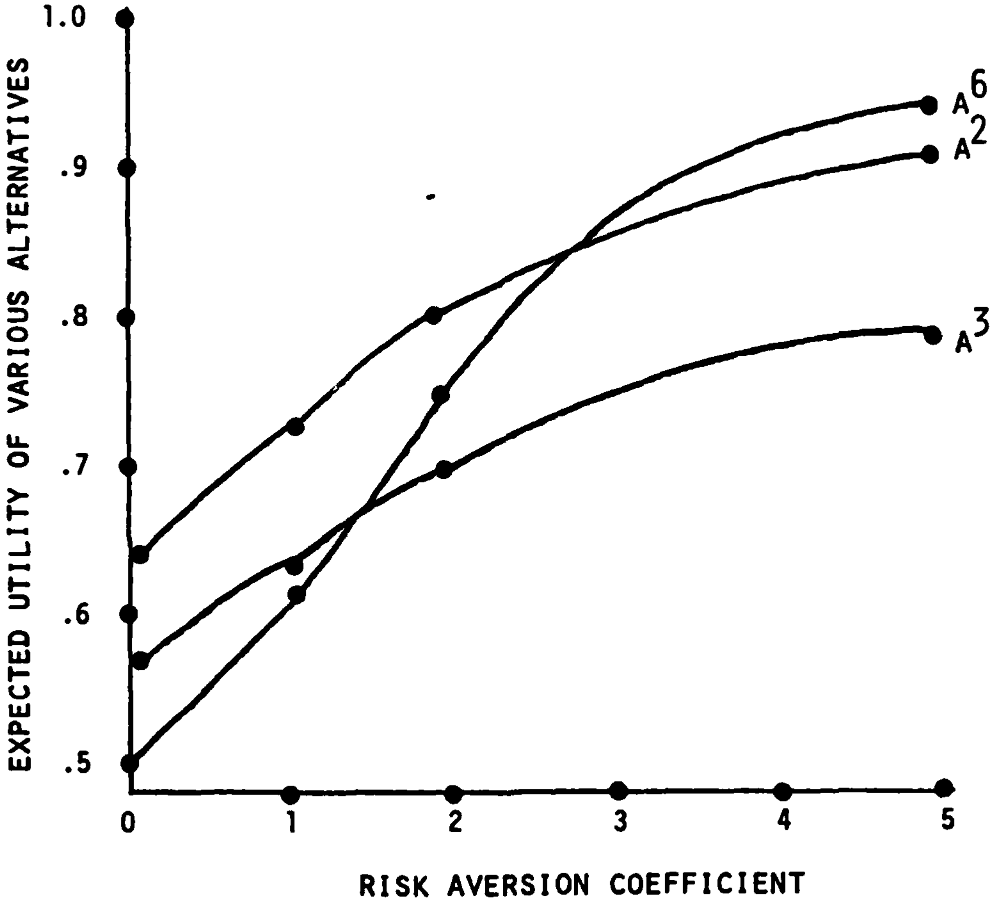
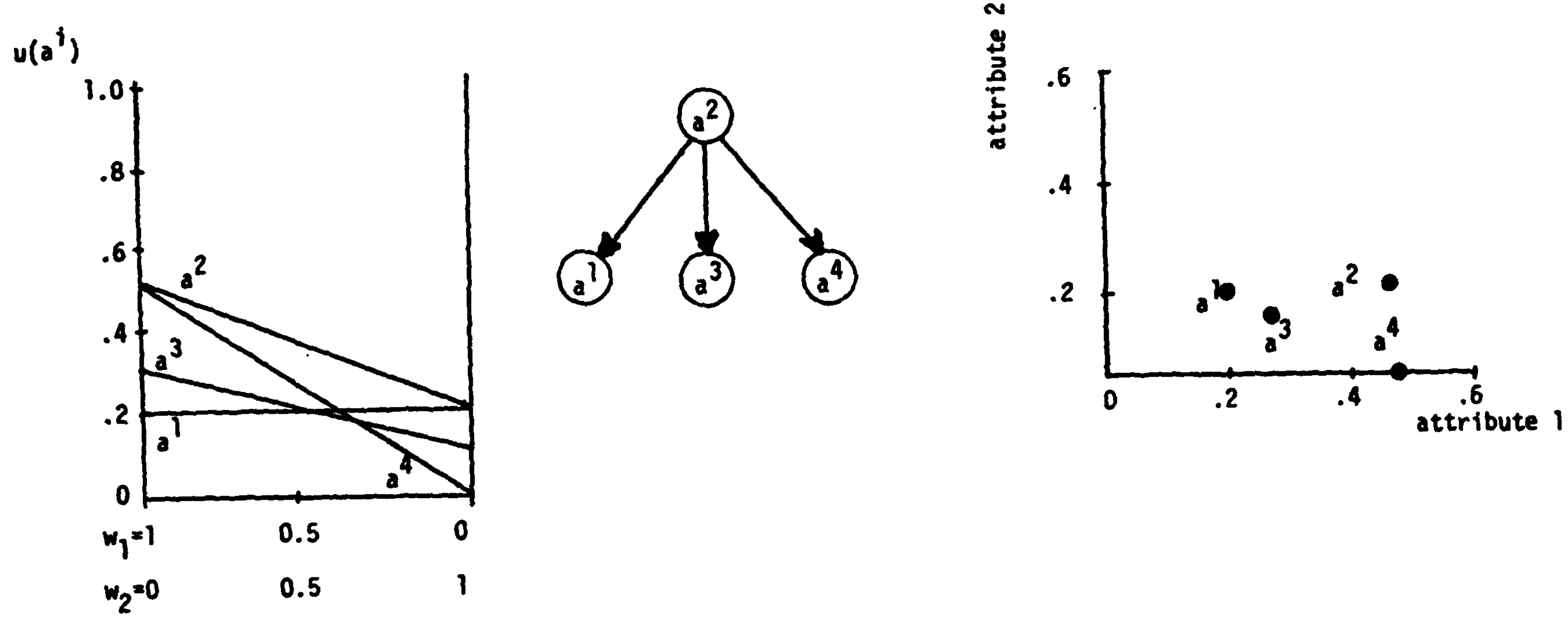


Figure 3.5 Sensitivity Effect of Risk Aversion Coefficient in Determining Best Alternatives

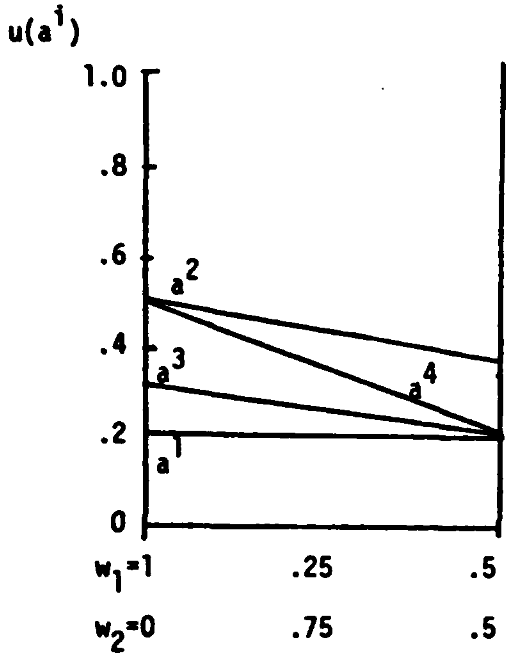


a) Utilities as a function of attribute weights

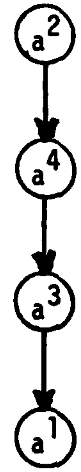
b) Domination Digraph

c) Value Scores

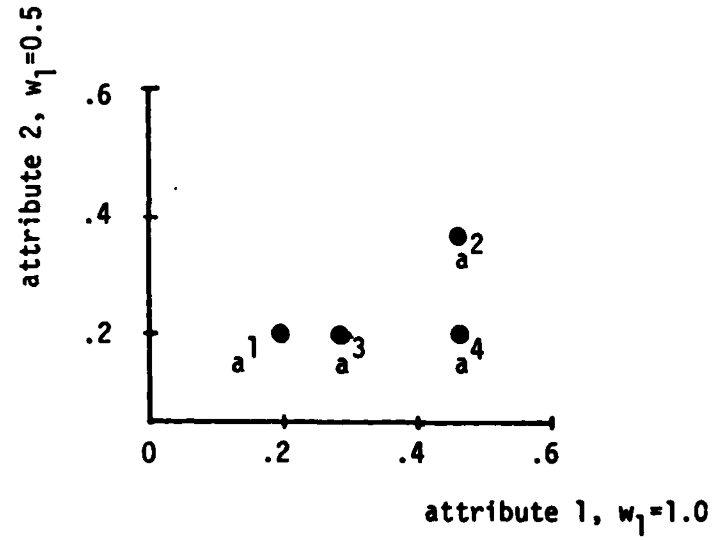
Figure 4.1 Preference Relations for a Simple Example



a) Utilities as a function of attribute weights for $w_1 \geq w_2$



b) Domination Digraph with $w_1 \geq w_2$



c) Value scores for attributes, $w_{1min} = 0.5$

Figure 4.2 Preference Relations for a Simple Example, $w_1 \geq w_2$

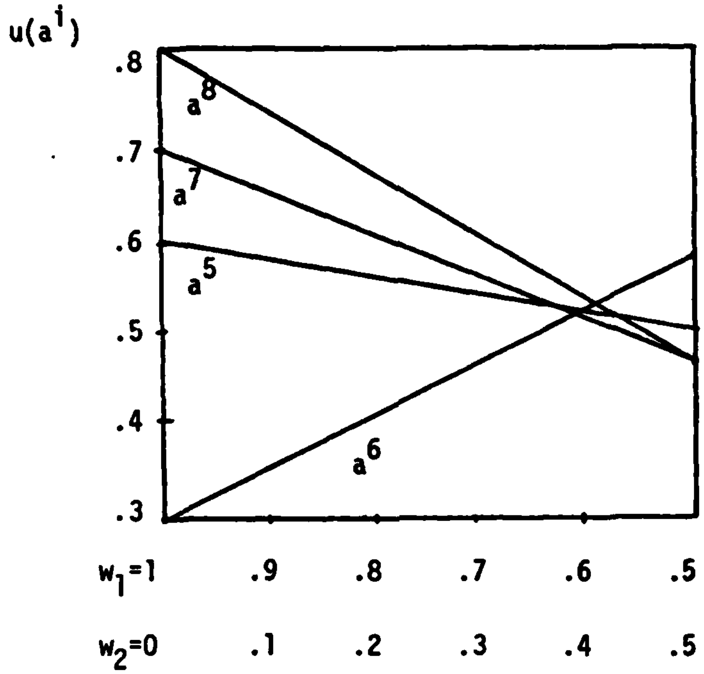


Figure 4.3 Utilities as functions of attribute weights. Note that a mixed strategy of alternatives 8 and 6 will always be better than alternative 5 or 7.

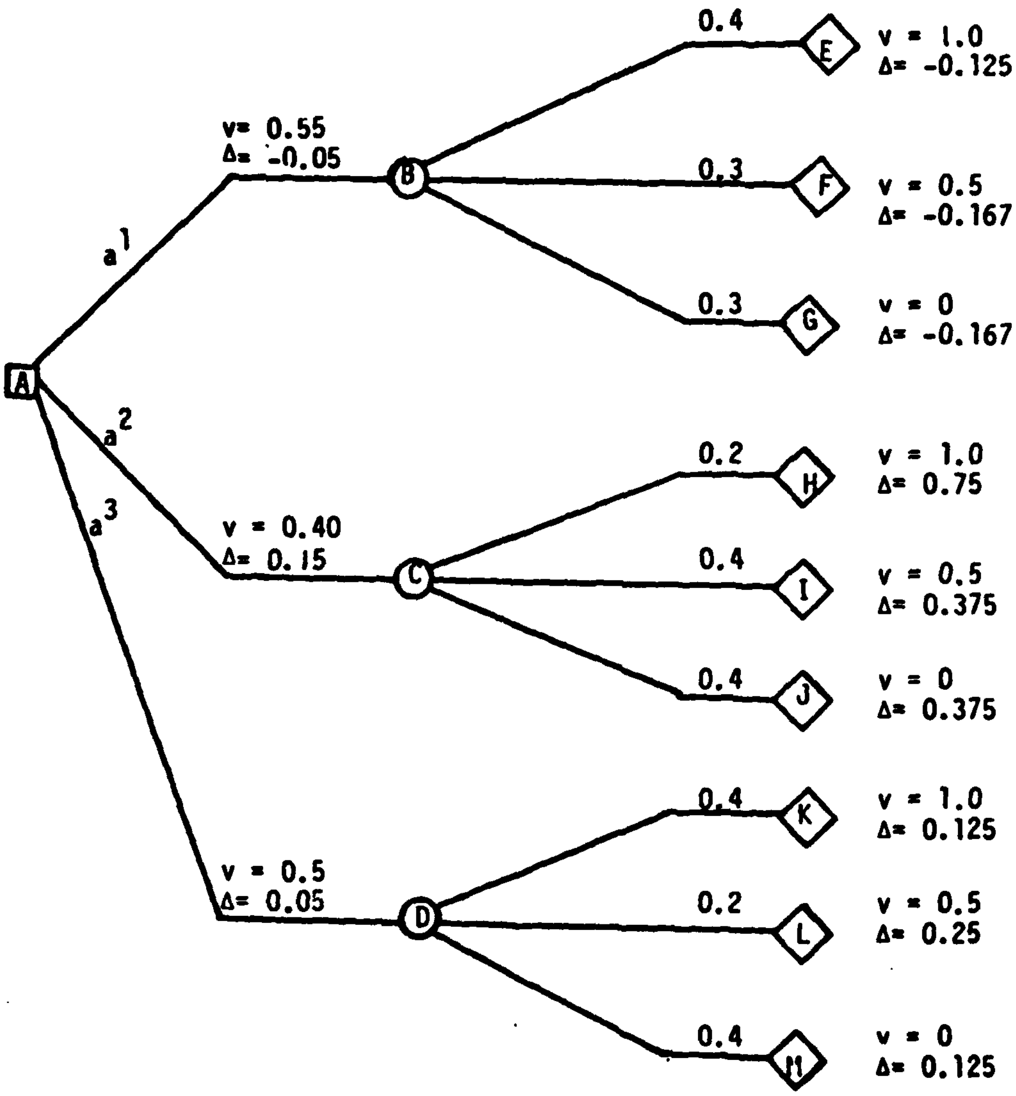


Figure 5.1 Decision Tree with Expected (rollback) values and sensitivity Differentials

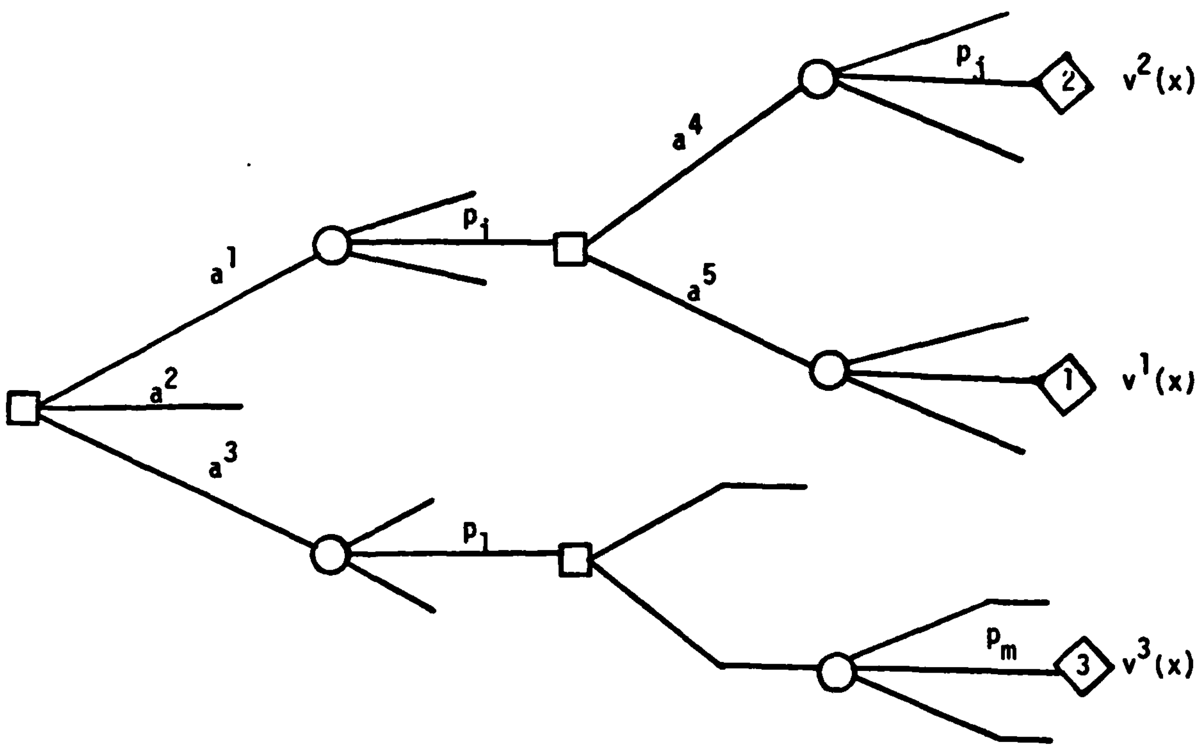


Figure 5.2 Prototypical Decision Tree for Computation of The Expected Value of Resolving Residual Uncertainty

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