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A GOODNESS-OF-FIT TEST BASED ON SPACINGS.(U)

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BASED ON SPACINGS

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A GOODNESS-OF-FIT TEST BASED ON SPACINGS

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ABSTRACT

→ The difference between consecutive order statistics from a sample is called a spacing. Various tests based on sample spacings have been considered in the literature for testing the hypothesis that the sample is drawn from a specified distribution. Tests based on the spacings are recommended for use when the alternative distribution differs from the hypothetical distribution in the shape of the density function. In this paper, we consider a test based on the spacings designed for the case when the ratio of the two density functions is a piece-wise monotone function. This paper deals mainly with the large sample properties of the test. ←

Key words: Spacings, Goodness-of-fit, Asymptotic Relative Efficiency.

AMS Classification: 62G10, 62G30

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1. Introduction. Let $X_1, \dots, X_n$ be a sample drawn from a certain distribution. In this paper, we consider a test of the hypothesis H_0 that the sample comes from a known distribution F , say, against the alternative hypothesis H_1 that the sample is drawn from a distribution G , say, where G is not known completely. Let A denote the common support of the distributions F and G . We assume that A is a finite or infinite interval and that distributions have continuous density with respect to the Lebesgue measure. Let f and g denote the respective density functions. We further assume that the ratio $Q(x) = g(x)/f(x)$ is a piece-wise strictly monotone function of x inside A . Suppose that the slope of the graph of $Q(x)$ changes sign at k points. Let $\underline{u} = (u_1, \dots, u_k)$ denote the change points. We consider two cases: (i) $\underline{u}$ is known and (ii) $\underline{u}$ is not known.

We shall consider in length the case in which there is a single change of sign ($k=1$) in the slope of the graph of $Q(x)$. This is realized in the following examples, where $\phi(x) = (2\pi)^{-1/2} \exp(-x^2/2)$ denotes the standard normal density function.

$$(a) \quad f(x) = \phi(x); \quad g(x) = (1/e) \phi(x/e)$$

$$(b) \quad f(x) = \phi(x); \quad g(x) = e^{-x} / (1 + e^{-x})^2$$

$$(c) \quad f(x) = \phi(x); \quad g(x) = p\phi(x-\mu_1) + (1-p)\phi(x-\mu_2) \\ \mu_1 < 0 < \mu_2, \quad 0 < p < 1$$

$$(d) \quad f(x) = 1, \quad 0 < x < 1; \quad g(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} \\ 0 < x < 1; \quad \alpha, \beta > 1.$$

Let $Y_i = F(X_i)$, $i = 1, \dots, n$ and let $D_1 = Y_{(1)}$, $D_2 = Y_{(2)} - Y_{(1)}, \dots, D_n = Y_{(n)} - Y_{(n-1)}$ denote the sample spacings of the transformed data where $Y_{(i)}$ denotes the i th smallest value among $Y_1, \dots, Y_n$. Our test is based on the spacings. The test statistic will be denoted by T . Various tests for goodness-of-fit, based on spacings have been considered in the literature. The papers of Pyke (1965, 1972), Proschan and Pyke (1967), Sethuraman and Rao (1970), Kale (1969) and Kirmani and Alam (1974) may be cited for reference. Pyke (1972) points out that tests based on spacings should be used when the alternative distribution differs from the hypothetical distribution in the shape of the density function.

The following statistics have been proposed in the literature for a test of goodness-of-fit:

$$U = \sum_{i=1}^n D_i^r; \quad r = -\frac{1}{2},$$

$$V = \sum_{i=1}^n |nD_i - 1| \quad \text{and}$$

$$W = \sum_{i=1}^n \log D_i.$$

The null hypothesis is rejected when the absolute value of the statistic is large. It is known (see e.g., Cibisov (1961)) that the asymptotic efficiency of any test symmetric in the spacings is equal to zero relative to the Kolmogorov-Smirnov test. Sethuraman and Rao (1970) have compared the relative efficiencies of the tests based on U , V and W .

For applications to reliability and life-testing, Proschan and Pyke (1967) have considered a test of the hypothesis that the given sample comes from an exponential distribution which has a constant failure rate properly against the alternative hypothesis that the distribution has monotone failure rate. The test is based on the statistic

$$(1.1) \quad S = \sum_{i=1}^n \sum_{j=i}^n h(\bar{D}_i, \bar{D}_j)$$

where h is a bounded nonnegative function and $\bar{D}_i = (n-i+1)D_i$ denotes the i th normalized spacing. The authors have shown that the distribution of S is asymptotically normal under the alternative hypothesis. Further, Bickel and Doksum (1969) have shown that the asymptotic normality holds also for a sequence of alternatives G_n approaching the exponential distribution.

The test statistic T is derived from S as follows:

First suppose that n is known. Let $F_i = F(n_i)$ and let π_i denote the set of values of Y_j for which $F_{i-1} \leq Y_j < F_i$, and n_i denote the number of elements in π_i , $i = 1, \dots, k+1$, where $F_0 = 0$ and $F_{k+1} = 1$. Let Y_i^* denote the smallest value in π_i , and let $S_m = Y_i^* - F_{i-1}$ for $m = n_1 + \dots + n_{i-1} + 1$ ($i = 1, \dots, k+1$) and $S_m = D_m$ otherwise, $m = 1, \dots, n$, where $n_0 = 0$. Let

$$(1.2) \quad h(x, y) = \begin{cases} 1 & \text{for } x \leq y \\ 0 & \text{for } x > y. \end{cases}$$

Define

$$T_0 = \sum_{j=1}^n \sum_{i=1}^j h(S_i, S_j), \quad \bar{T}_0 = \sum_{j=1}^n \sum_{i=j}^n h(S_i, S_j)$$

$$T_i = \sum_{j=n_1+\dots+n_{i-1}+1}^{n_1+\dots+n_i} \sum_{i=n_1+\dots+n_{i-1}+1}^j h(S_i, S_j)$$

$$\bar{T}_i = \sum_{j=n_1+\dots+n_{i-1}+1}^{n_1+\dots+n_i} \sum_{i=j}^{n_1+\dots+n_i} h(S_i, S_j).$$

Let $\epsilon_j = 0(1)$ for odd (even) values of $j = 1, \dots, k+1$. We let

$$T = \sum_{j=1}^{k+1} ((1-\epsilon_j)T_j + \epsilon_j \bar{T}_j) \quad \text{or} \quad \sum_{j=1}^{k+1} (\epsilon_j T_j + (1-\epsilon_j) \bar{T}_j)$$

according as the graph of $Q(x)$ has initially a positive or negative slope. If the slope does not change sign, we let

$$T = T_0 \text{ or } \bar{T}_0$$

according as the slope is positive or negative. The hypothesis H_0 is rejected for small values of T .

In case (ii), we estimate $\underline{n}$ from the data and substitute the estimate for $\underline{n}$ in the definition of T , given above. The estimation of $\underline{n}$ is considered in Section 5 below.

In Section 2, we show that the given test is unbiased. To compute the critical value of T and the power of the test, we need to find the distribution of T under H_0 and H_1 . The distribution is shown in Sections 3 and 4. Some results on the relative efficiency of the test are given in Section 6.

2. Unbiasedness of the test. Let $H = GF^{-1}$, $Y'_i = H(Y_i)$ and $D'_i = Y'_{(i)} - Y'_{(i-1)}$, $i = 1, \dots, n$, where $Y'_0 = 0$. Suppose that the slope of the graph of $Q(x)$ is positive inside (η_{m-1}, η_m) . Then H is convex on (ξ_{m-1}, ξ_m) . Therefore, by the mean value theorem we have that for $n_1 + \dots + n_{m-1} + 1 \leq i \leq j \leq n_1 + \dots + n_m$

$$\frac{D'_j}{D'_i} = \frac{Y'_{(j)} - Y'_{(j-1)}}{Y'_{(i)} - Y'_{(i-1)}} = \frac{Y_{(j)} - Y_{(j-1)}}{Y_{(i)} - Y_{(i-1)}} \cdot \frac{H'(C_j)}{H'(C_i)}$$

$$(2.1) \quad \frac{Y_{(j)} - Y_{(j-1)}}{Y_{(i)} - Y_{(i-1)}} = \frac{D_j}{D_i}$$

where $Y_{(j-1)} < C_j < Y_{(j)}$ and $Y_{(i-1)} < C_i < Y_{(i)}$. The inequality in (2.1) follows from the fact that $H'(C_j)/H'(C_i) \geq 1$ due to the convexity of H on (ξ_{m-1}, ξ_m) . From (2.1) we find that

$D_i \leq D_j \Rightarrow D'_i \leq D'_j$. Therefore $h(D_i, D_j) \leq h(D'_i, D'_j)$. Let $T'_m, \bar{T}'_m$ and T' be obtained from $T_m, \bar{T}_m$ and T respectively by substituting

Y'_i for Y_i , $i = 1, \dots, n$. Then by the above inequality we have that $T_m \leq T'_m$. Similarly, if the slope of the graph of $Q(x)$ is negative inside (η_{m-1}, η_m) then H is concave on (ξ_{m-1}, ξ_m) and therefore $\bar{T}_m \leq \bar{T}'_m$. It follows that $T \leq T'$. Hence

$$P(T \leq t|G) \geq P(T' \leq t|G) = P(T \leq t|F), \quad \text{for all } t \geq 0.$$

Since H_0 is rejected for small values of T , the test is unbiased.

3. Distribution of T under H_0 . When $G = F$, the sample spacings $D_1, \dots, D_n$ are jointly symmetrically distributed according to the Dirichlet distribution given by the density function

$$(3.1) \quad p(d_1, \dots, d_n) = n! ; \quad d_i \geq 0, \quad 0 \leq d_1 + \dots + d_n \leq 1.$$

Let $Z_1, \dots, Z_n$ be n random variables jointly symmetrically distributed and let

$$R_j = \sum_{i=1}^j h(Z_i, Z_j), \quad j = 1, \dots, n$$

denote the left sequential ranks where the function h is defined as in (1.2). Similarly, let

$$\bar{R}_j = \sum_{i=j}^n h(Z_i, Z_j), \quad j = 1, \dots, n$$

denote the right sequential ranks. It is known (see, e.g., Renyi (1962)) that $R_1, \dots, R_n$ are statistically independent and that the distribution of R_j is uniform, given by

$$(3.2) \quad P(R_j = m) = \frac{1}{j}, \quad m = 1, \dots, j.$$

The same result holds for $\bar{R}_1, \dots, \bar{R}_n$.

The above result is easily generalized as follows: Let $(Z_1, \dots, Z_{\ell_1}), (Z_{\ell_1+1}, \dots, Z_{\ell_1+\ell_2}), \dots, (Z_{\ell_1+\dots+\ell_k+1}, \dots, Z_{\ell_1+\dots+\ell_{k+1}})$ be a partition of $\underline{Z} = (Z_1, \dots, Z_n)$ into $k+1$ sub-vectors, where $\ell_1 + \dots + \ell_{k+1} = n$ and let

$$R_j^m = \sum_{i=1}^j h(Z_{\ell_1+\dots+\ell_{m-1}+i}, Z_{\ell_1+\dots+\ell_{m-1}+j})$$

be the j th left sequential rank of the variables in the m th partition $(Z_{\ell_1+\dots+\ell_{m-1}+1}, \dots, Z_{\ell_1+\dots+\ell_m})$; $j = 1, \dots, \ell_m$;

$m = 1, \dots, k+1$, where $r_0 = 0$. Let the right sequential ranks $\bar{R}_j^m$ be defined in analogous manner. Then the random variables R_j^m , $j = 1, \dots, v_m$, $m = 1, \dots, k+1$ are statistically independent and the distribution of each R_j^m is uniform. Similarly, the random variables $\bar{R}_j^m$, $j = 1, \dots, v_m$; $m = 1, \dots, k+1$ are statistically independent and the distribution of each $\bar{R}_j^m$ is uniform. Moreover, R_i^m and $\bar{R}_j^{m'}$ are independent for $m \neq m'$. These results follow from the property of symmetry of the joint distribution of $Z_1, \dots, Z_n$.

Given $\underline{\xi}$ and $\underline{n} = (n_1, \dots, n_{k+1})$, π_i represents a sample of n_i observations from a uniform distribution on (ξ_{i-1}, ξ_i) . Moreover, the $k+1$ subsamples are conditionally independent. The following properties of the conditional distribution of $T_1, \dots, T_{k+1}$; $\bar{T}_1, \dots, \bar{T}_{k+1}$, given $\underline{n}$ follow from the results given above: (a) $T_1, \dots, T_{k+1}$ are independent, (b) $\bar{T}_1, \dots, \bar{T}_{k+1}$ are independent, (c) T_i and $\bar{T}_j$ are independent for $i \neq j$ and (d) T_i and $\bar{T}_i$ have the same distribution for each i . Therefore conditionally,

$$(3.3) \quad T \stackrel{d}{=} \sum_{i=1}^{k+1} T_i$$

where $\stackrel{d}{=}$ means "distributed as".

Let $P_{i,n_i}(t) = P(T_i = t | \underline{n})$, $i = 1, \dots, k+1$. Clearly, $P_{i,n_i}(t) = 0$ for $t < n_i$. For $t \geq n_i$, the probability can be computed recursively from the relation

$$(3.4) \quad n_i P_{i,n_i}(t) = P_{i,n_{i-1}}(t-1) + P_{i,n_{i-1}}(t-2) + \dots + P_{i,n_{i-1}}(t-n_i).$$

Since $T_1, \dots, T_{k+1}$ are conditionally independent given $\underline{n}$, from (3.3) we get

$$(3.5) \quad P(T = \underline{t} | \underline{n}) = \sum_{t_1 + \dots + t_{k+1} = n} \prod_{i=1}^{k+1} p_{i, n_i}(t_i).$$

As $\underline{n}$ is distributed according to the multinomial distribution with the associated probability vector $p = (p_1, \dots, p_{k+1})$, $p_i = \xi_i - \xi_{i-1}$, $i = 1, \dots, k+1$, we obtain

$$P(T = \underline{t}) = \sum_{n_1 + \dots + n_{k+1} = n} P(T = \underline{t} | \underline{n}) n! \prod_{i=1}^{k+1} \frac{p_i^{n_i}}{n_i!}.$$

Kendall (1938) and Mann (1945) have tabulated the distribution of $p_{i, n_i}(t)$, for $n_i \leq 10$.

Since T_i is the sum of independent random variables $R_1^i, \dots, R_{n_i}^i$, it follows from Liapounov's theorem that for large values of n_i , T_i is asymptotically normally distributed with mean $n_i(n_i+3)/4$ and variance $n_i(n_i-1)(2n_i+5)/72$. Therefore, T is asymptotically normally distributed for large n , with mean μ_n and variance σ_n^2 , given by

$$(3.6) \quad \mu_n = \sum_{i=1}^{k+1} n_i(n_i+3)/4 \sim (n^2/4) \sum_{i=1}^{k+1} p_i^2$$

$$(3.7) \quad \sigma_n^2 = \sum_{i=1}^{k+1} n_i(n_i-1)(2n_i+5) \sim (n^3/36) \sum_{i=1}^{k+1} p_i^3.$$

where $\sim$ means "asymptotically equivalent to".

4. Distribution of T under H_1 . The small sample theory of the distribution of T under H_1 is mathematically intractable.

Therefore, we consider the asymptotic theory. Let $Z_1, \dots, Z_n$ be a sample from the exponential distribution whose cdf is given by $A(z) = 1 - \exp(-z)$, and let $Z_{(i)}$ denote the i th order statistic in the sample. Similarly, let $U_{(i)}$ denote the i th order statistic in a sample of n observations from the uniform distribution in $(0,1)$. It is known that

$$Z_{(i)} \stackrel{d}{=} \sum_{j=1}^i Z_j / (n-j+1).$$

Let

$$\lambda(u) = (1-u) / H'(H^{-1}(u)),$$

where $H = G^{-1}$. We have

$$\begin{aligned} (4.1) \quad nD_i &\stackrel{d}{=} n(H^{-1}A(Z_{(i)}) - H^{-1}A(Z_{(i-1)})) \\ &= n(Z_{(i)} - Z_{(i-1)}) e^{-Z_{(i-1)}} / g(G^{-1}(A(Z_{(i-1)}))), \\ &\quad \text{by mean value theorem} \\ &= n(Z_{(i)} - Z_{(i-1)}) \lambda(u) \\ &\stackrel{d}{=} (n/(n-i+1)) Z_i^{(i)}(U_i) \end{aligned}$$

where $Z_{(i-1)} < z < Z_{(i)}$, $u = A(z)$ and $U_{(i-1)} < u_i < U_{(i)}$.

Though (Z_i, Z_j) and (u_i, u_j) are statistically dependent,

Proschan and Pyke (1967) have shown that the degree of dependence is negligible in relation to the distribution of T for

for large n . Under the condition of independence, we have

$$\begin{aligned} E h(D_i, D_j) &= P(D_i \leq D_j) \\ &= E \frac{\lambda^{(n)}(j)}{n-j+1} \left[\frac{\lambda^{(n)}(i)}{n-i+1} + \frac{\lambda^{(n)}(j)}{n-j+1} \right]^{-1}. \end{aligned}$$

Then for large n , we have

$$\begin{aligned} ET_i &= E \sum_{n_{i-1}+1 \leq i \leq n_i} h(D_i, D_j) \\ (4.2) \quad &= n^2 \int_{i-1}^i \int_u^i \frac{H'(H^{-1}(u))}{H'(H^{-1}(u)) + H'(H^{-1}(v))} du dv \\ &= n^2 \sigma_i^2, \text{ say.} \end{aligned}$$

From Theorem 4.2 of Proschan and Pyke (1967) it follows that T_i is asymptotically normally distributed. The mean of the asymptotic distribution is given by (4.2). The variance of the asymptotic distribution equal to $n^3 \sigma_i^2$, say, can be obtained from formulas (4.59) and (4.60) of the paper. We do not give the expression for the variance, since it is involved. The distribution of T is asymptotically normal with mean equal to $n^2 \sum_{i=1}^{k+1} \mu_i$ and variance equal to $n^3 \sum_{i=1}^{k+1} \sigma_i^2$.

5. Estimation of μ . We have considered the case when μ or equivalently ε is known, giving the points of inflection in the

graph of $Q(x)$. If ξ is unknown, we estimate ξ as follows. We shall describe the method of estimation when there is a single point of inflection ξ_1 , though the method carries through to the case in which there are several points of inflection. Let

$$L_m = \sum_{j=1}^m \sum_{i=1}^j h(D_i, D_j) + \sum_{j=m+1}^n \sum_{i=j+1}^n h(D_i, D_j)$$

and let m^* denote the value of m , maximizing (minimizing) L_m as m varies from 0 to 1, if the slope of $Q(x)$ changes sign from negative to positive (positive to negative). If the sign changes from negative to positive then $r(u)/(1-u)$ is first increasing then decreasing as u varies from 0 to 1. From the representation (4.1) of the sample spacings, it is seen that the values of D_i tend to increase then decrease as i varies from 1 to n and therefore the value of m^* maximizing L_m is approximately given by

$$(5.1) \quad m^* = [\xi_1, n]$$

where $[x]$ denotes the nearest integer value of x . If the slope of $Q(x)$ changes sign from positive to negative then m^* minimizes L_m . The estimate of ξ_1 is given by the largest value of ξ_1 satisfying the relation (5.1).

It can be shown that $m^*/n = \xi_1 + O_p(m^{-1/2})$. Therefore, the asymptotic theory developed in the previous section remains valid when ξ_1 is replaced by its estimated value in the definition of T .

6. Asymptotic relative efficiency. In this notion, we compare the test based on T with a likelihood ratio test, using the criterion of asymptotic relative efficiency (ARE) for the comparison. We consider below two examples for the comparison. In Example 1 we test an exponential distribution against a Weibull distribution. In Example 2 we test a uniform distribution against a beta distribution. For a specified set of alternatives indexed by θ , say, the formula for the ARE of a sequence of tests (based on a sequence of asymptotically normal test statistics $\{T_n\}$) against a sequence of tests (based on the asymptotically normal test statistics $\{t_n\}$) is given by the formula (see e.g., Gibbons (1971))

$$(6.1) \quad \text{ARE} = \lim_{n \rightarrow \infty} \left[\frac{\mu'_{T_n}(\theta_0)}{\sigma_{T_n}(\theta_0)} / \frac{\mu'_{t_n}(\theta_0)}{\sigma_{t_n}(\theta_0)} \right]^2$$

where θ_0 denotes the null hypothesis, $\mu_{T_n}(\theta)$ and $\sigma_{T_n}^2(\theta)$ denote the limiting mean and variance, respectively, of $\{T_n\}$, $\mu'_{T_n}(\theta)$ denotes the derivative of $\mu_{T_n}(\theta)$ with respect to θ . The parametric functions $\mu_{t_n}(\theta)$, $\sigma_{t_n}^2(\theta)$ and $\mu'_{t_n}(\theta)$ are defined similarly, as for $\{t_n\}$.

Example 1. Let $f(x) = e^{-x}$, $x > 0$ and $g(x) = \theta x^{\theta-1} e^{-x^\theta}$, $x > 0$, $\theta > 1$. It is seen that $Q(x)$ is increasing (decreasing) in x for $x \leq (>)$ for all $\theta > 1$. We have $\xi_1 = F(1) = 1 - e^{-1}$. The spacing-test statistic is

$$T = T_1 + \bar{T}_2.$$

From (4.2) we have, after simplification,

$$(6.2) \quad \mu_T'(1) \sim n^2(I_1 + I_2)$$

where I_1 and I_2 are given by

$$(6.3) \quad I_1 = \int_{0 < y < z < 1} \left[\frac{y^2 z^2 e^{-y-z} [(1+y) \log y - (1+z) \log z]}{(y^2 + z^2)^2} \right] dy dz$$

= , approximately.

$$(6.4) \quad I_2 = \int_{0 < y < z < 1} \left[\frac{y^2 z^2 e^{-(y+z)/yz} [((1+z)/z) \log z - ((1+y)/y) \log y]}{(y^2 + z^2)^2} \right] dy dz$$

= , approximately.

The approximate values of I_1 and I_2 given above are obtained by numerical integration. From (3.7) we have

$$(6.5) \quad \sigma_T^2(1) \sim \frac{n^3}{36} (1 - 2\varepsilon_1 (1 - \varepsilon_1)).$$

Proschan and Pyke (1967) have shown that the likelihood ratio test rejects the null hypothesis when T_W is large where

$$(6.6) \quad T_W = \sum_{i=1}^n (1 - X_i) \log X_i$$

and

$$(6.7) \quad \mu_{T_W}'(1) = n[(\gamma - 1)^2 + \pi^2/6], \quad \gamma = .5772, \text{ approximately.}$$

$$(6.8) \quad \sigma_{T_W}^2(1) = n[(\gamma-1)^2 + \pi^2/6].$$

From (6.1) the ARE is given by

$$\text{ARE} = 36(I_1 + I_2)^2 / (1 - 2\xi_1(1 - \xi_1))((\gamma-1)^2 + \pi^2/6)$$

= approximately.

Example 2. Let $f(x) = 1$, $0 < x < 1$ and $g(x) = x^{b-1}$, $b > 1$.

It is seen that $Q(x)$ is increasing in x inside the interval $(0,1)$.

Therefore $T = T_0$. From (4.2) and (3.7) we get after some simplification

$$\mu_{T_0}'(1) = -\frac{n^2}{2} \left[\frac{1}{3^2} - \frac{2}{5^2} + \frac{3}{7^2} - \dots \right] = -(0.031)n^2$$

$$\sigma_{T_0}^2(1) = n^3/36.$$

The likelihood ratio test rejects the null hypothesis when

$T_B = \sum_{i=1}^n \log x_i$ is large. By direct computation

$$\mu_{T_B}'(1) = -n \quad \text{and} \quad \sigma_{T_B}^2(1) = n.$$

From (6.1) the value of the ARE turns out to be equal to .0345 approximately.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The difference between consecutive order statistics from a sample is called a spacing. Various tests based on sample spacings have been considered in the literature for testing the hypothesis that the sample is drawn from a specified distribution. Tests based on the spacings are recommended for use when the alternative distribution differs from the hypothetical distribution in the shape of the density function. In this paper, we consider a test based on the spacings designed for the case when the ratio of the two density functions is a piece-wise monotone function. This paper deals mainly with the large sample properties of the test.		

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