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BOUNDED ERROR ADAPTIVE CONTROL

Part I

Benjamin B. Peterson and Kumpati S. Narendra

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Bounded Error Adaptive Control

Part I

Benjamin B. Peterson and Kumpati S. Narendra

Yale University

Abstract

Adaptive identification and control problems when the output error between plant and model cannot be made to tend to zero are discussed. These include cases when the plant output is corrupted with noise and when plant parameters vary with time. The principal aim is to determine adaptive laws so that all the signals in the overall system are bounded.

It is first shown that an error model can be derived which is described by a non-homogeneous differential equation. If the external disturbance, plant parameter variations, and input are bounded and the input is sufficiently rich the parameter error vector is bounded and explicit bounds are presented. If the input is not sufficiently rich, conditions for boundedness and unboundedness of the parameter error vector are derived and examples of unstable systems are given.

Finally, for the control problem where the input cannot be assumed to be bounded, a nonlinear adaptation algorithm is suggested. The need for such an algorithm is shown by considering simple systems in which the parameter error, almost always, becomes unbounded if the disturbance does not tend to zero. By adjusting the parameters of the algorithm it is shown that the boundedness of all the signals in the system can be assured. The method can be extended to more general adaptive control problems which will be considered in greater detail in a subsequent report.

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I. Introduction:

The global stability of single-input single-output systems which had remained an open problem for two decades was recently resolved for both continuous [1] [2] and discrete systems [3]-[6]. It is also now realized that the different adaptive schemes which were developed independently for Model Reference Adaptive Control (MRAC) and Self Tuning Regulators (STR) are all equivalent. The assumptions that have to be made regarding the unknown plant are also the same for the various schemes and are found to be too restrictive. In particular, the assumption regarding an exact knowledge of the relative degree of the plant transfer function is rarely satisfied in practical situations. Hence it was suggested in [7] that the adaptive control problem should be reformulated so that less is demanded of the controller allowing fewer assumptions to be made regarding the plant. This led to the study of bounded error adaptive control in which a bounded error between plant and reference model is all that is required instead of perfect output matching.

All the globally stable schemes described in [1-6] assume that the plant output can be measured exactly. As discussed in [8], such systems lead to error models which can be described by homogeneous differential and difference equations. In this paper, we study in detail error models described by non-homogeneous differential equations. The importance of these error models lies in the fact that they allow adaptive systems with measurement noise, time-varying plant parameters and reduced order models to be analyzed in a unified manner.

In section II the error model is posed. If $u(t)$ is an input vector of time-functions, $v(t)$ a scalar output disturbance and $x(t)$ a parameter error vector, the error equation is given by:

$$\dot{x}(t) = -\gamma u(t)u^T(t)x(t) - \gamma u(t)v(t) \tag{1}$$

For problems arising in identification it can be assumed that both $u(t)$ and $v(t)$ are uniformly bounded. If $u(t)$ is sufficiently rich the homogeneous equation is uniformly asymptotically stable and hence a bounded v(t) produces a bounded x(t). The latter can be derived in terms of the bounds on v(t) as well as the richness of u(t).

It can be demonstrated that x(t) may become unbounded when u(t) is not sufficiently rich. Three examples of second order systems are given which illustrate the type of behavior possible in such cases. It is interesting to note that sufficient conditions for the boundedness of x(t) call for either a sufficiently rich input u(•) or a $u \in L^1$ which does not even assure the asymptotic stability of the homogeneous equation.

More interesting theoretical questions arise when u(t) and v(t) are correlated and cannot be assumed to be uniformly bounded, as for example in the adaptive control problem. While investigation of this problem is still in the initial stages, an attempt is made in section VII to formulate and analyze a specific error model. Using this error model, which contains a nonlinearity (dead-zone) in the adaptive law for updating control parameters, sufficient conditions are derived to assure the boundedness of parameter and output errors.

II. Error Models:

a) The First Error Model

Figure 1 represents the first error model which has been extensively analyzed in the adaptive literature [9]. u(•) is an n-dimensional vector, x(•) is a parameter error vector and e(•) is an output error which is given by $e = x^T u$. The vector x(t) is updated according to the adaptive law $\dot{x} = -\Gamma e(t)u(t)$ where

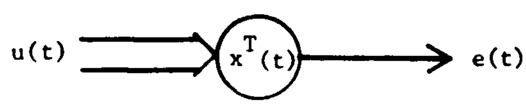

Figure 1

Γ is a constant positive definite matrix. The error equation can be expressed as:

$$\dot{x}(t) = -\Gamma u(t)u(t)^T x(t) \tag{2a}$$

which is a time-varying, homogeneous, differential equation. For the particular case where $\Gamma = \gamma I$ the error equation has the form

$$\dot{x}(t) = -\gamma u(t)u(t)^T x(t) \tag{2b}$$

and the constant γ is the adaptive gain. For ease of exposition, the equation (2b) is used throughout this report. Using a Lyapunov function candidate $V(x) = \frac{x^T x}{2}$ it follows directly from $\dot{V}(x) = -\gamma(u^T x)^2$ that the equilibrium state of (2) is uniformly stable. For $u(\cdot)$ "sufficiently rich" defined by

$$\int_t^{t+T} |u^T(\tau)w|d\tau \geq k_1 T + k_1' \tag{3}$$

(for arbitrary t and $T > 0$ and for all unit vectors w there exist $k_1 > 0$ and k_1' such that (3) is satisfied), it has been shown [10] that (2) is uniformly asymptotically stable (u.a.s.). Bounds on the rate of convergence of $\|x(t)\|$ in such a case have been derived in [11] and [12].

b) <u>Error Model with Output Noise</u>

When the output of the plant is corrupted with noise, or when the plant parameters vary with time, the error model can be described by a non-homogeneous differential equation.

In Figure 2 $v(t)$ represents a disturbance at the output. If the same adaptive law as in the previous case is used to update $x(t)$ the error model is

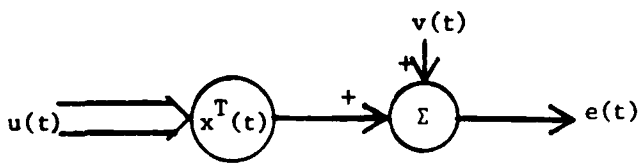

Figure 2

described by

$$\dot{x}(t) = -\gamma e(t)u(t) = -\gamma u(t)u^T(t)x(t) - \gamma u(t)v(t) \qquad (4)$$

Various assumptions can be made regarding v(t) in equation (4). For example if v(t) is stochastic, uncorrelated with u(t) and u(t) is sufficiently rich and $\gamma(t)$ is a time-varying gain which tends to zero in a specific manner it has been shown [13] that $\|x\|$ tends to zero in a mean square sense.

However our interest lies in less restrictive cases where only a bound on v(t) can be assumed and further v(t) and u(t) may be correlated. Also, virtually all applications of adaptive systems theory involve time-varying plant parameters and the adaptive gain $\gamma(t)$ cannot tend to zero. If u(t) is sufficiently rich and the homogeneous equation (2) is u.a.s., the bounded forcing term -u(t)v(t) in (4) will produce a bounded parameter error vector x(t). If u(t) is not sufficiently rich, $\|x\|$ may be unbounded and in such cases our interest lies in classifying the behavior of equation (4) for different types of inputs.

In section III bounds are derived on $\|x(t)\|$ for the noisy case using the explicit form of the error equation (4). In section IV similar bounds on $\|x(t)\|$ are derived using the results of [11] and [12] when the forcing term w(t) is independent of the input u(t). Such a situation arises when no output disturbance is present but the plant parameters vary with time. Section V deals with situations in which bounded inputs u(t) and v(t) result in an unbounded output x(t). Specific examples of such behavior are also given. Section VI briefly describes identification problems which can be analyzed using the results in sections III-V. The last section makes a first attempt at formulating an error model derived from equation (4) which applies to the control problem.

III. <u>Bounds on $\|x\|$ when u is sufficiently rich:</u>

In the non-homogeneous equation (4)

$$\dot{x}(t) = -\gamma u(t)u^T(t)x(t) - \gamma u(t)v(t)$$

the following assumptions are made:

For all $t \in [0,\infty)$

(i) $|v(t)| \leq v_0$

(ii) constants $k_1 > 0$ and $0 < T < \infty$ exist such that

$$\int_t^{t+T} |u^T(\tau)w|\,d\tau \geq k_1 \text{ for all constant unit vectors } w. \tag{5}$$

(iii) A constant k_2 exists such that $\int_t^{t+T} \|u(\tau)\|\,d\tau \leq k_2 < \infty$.

Although assumption (ii) is not identical in form to the "richness", "mixing", or "persistently exciting" conditions that have appeared in the literature [10-12] it is equivalent to them [10]. In lieu of the constraint on $\|u(\cdot)\|$ generally used the less restrictive integral constraint (iii) is used in the following discussion; if $\|u(t)\| \equiv 1$, $k_2 = T$. The richness of $u(t)$ is a function of k_1, k_2, and T. If any two of these are held constant and the third varied, richer $u(t)$ is implied by larger k_1, smaller k_2 and smaller T.

If $\overline{k_1}$ and $\overline{k_2}$ are defined by;

$$\overline{k_1} \triangleq k_1/T \text{ and } \overline{k_2} \triangleq k_2/T$$

they represent average values of $|u^T w|$ and $\|u\|$ over the interval $[t,t+T)$.

Since

$$\frac{d}{dt}[x^T(t)x(t)] = -2\gamma[(u^T(t)x(t))^2 + u^T(t)x(t)v(t)] \text{ ,}$$

it follows that $\Delta W(t) \triangleq [x^T(t+T)x(t+T) - x^T(t)x(t)]$

$$= \int_t^{t+T} -2\gamma[(u^T(\tau)x(\tau))^2 + u^T(\tau)x(\tau)v(\tau)]d\tau \text{ .} \tag{6}$$

$$\forall\ t \in [0,\infty)$$

Since $\int_t^{t+T} (u^T(\tau)x(\tau))^2 d\tau \geq \frac{1}{T}\left[\int_t^{t+T} |u^T(\tau)x(\tau)|d\tau\right]^2$ and by assumption 5(i)

$$\Delta W(t) \leq \frac{-2\gamma}{T}\int_t^{t+T} |u^T(\tau)x(\tau)|d\tau\left[\int_t^{t+T} |u^T(\tau)x(\tau)|d\tau - v_0 T\right]$$

Hence $\Delta W(t) \leq 0$ or $\|x(t)\|$ is non-increasing in the interval $[t,t+T]$ if

$$\int_t^{t+T} |u^T(\tau)x(\tau)|d\tau \geq v_0 T \tag{7}$$

From equation (6) it also follows that if $|u^T x| > v_0$ or if $\mathrm{sgn}(u^T x) = \mathrm{sgn}(v)$, $\frac{d}{dt}(x^T x) < 0$ or $\|x\|$ decreases. Hence for $\|x\|$ to increase it is necessary that $|u^T x + v| \leq v_0$. In such a case we have, using equation (4) and assumption 5(iii),

$$\|x(\tau) - x(t)\| = \left\| \int_t^{\tau} \gamma u(s)[u^T(s)x(s) + v(s)]ds \right\|$$

$$\leq \gamma k_2 v_0 \quad \text{for } \tau \in [t,t+T) \tag{8}$$

Equation (8) sets a bound on the change in x(t) in an interval of length T when $\|x(t)\|$ is increasing.

From equations (4) and (8) and assumptions 5(ii) and 5(iii) it follows that

$$\int_t^{t+T} |u^T(\tau)x(\tau)|d\tau \geq \int_t^{t+T} |u^T(\tau)x(t)|d\tau - \int_t^{t+T} |u^T(\tau)[x(\tau) - x(t)]|d\tau$$

$$\geq k_1 \|x(t)\| - \gamma v_0 k_2^2 \quad . \tag{9}$$

Equivalently, if

$$\|x(t)\| \geq \frac{v_0[T + \gamma k_2^2]}{k_1} \tag{10}$$

then $\|x(t+T)\| \leq \|x(t)\|$.

Further since $\frac{d}{dt}[x^T x] \leq \frac{\gamma v_0}{2}$, finite escape times are not possible and hence $\|x(t)\|$ is bounded. The bound on $\|x(t)\|$ can be expressed equivalently as

$$\|x(t)\| \leq \frac{v_0[1 + \gamma T\overline{k_2}^2]}{\overline{k_1}} \tag{11}$$

where $\overline{k_1}$ and $\overline{k_2}$ are the average values defined above.

Equations (10) and (11) reveal clearly the relation between the bound on $\|x(t)\|$ and the richness of the input u(t) as well as v_0 and the adaptive gain γ. Tighter bounds are obtained when richer inputs are used i.e. T and k_2 are small and k_1 is large. From (11) it is seen that as $\overline{k_1} \to 0$ the bound on $\|x(t)\|$ becomes arbitrarily large.

In the non-homogeneous equation (4) the forcing term $-\gamma u(t)v(t)$ depends on u(t). In the following section we consider the case where the forcing term, represented by w(t), is an independent bounded time function.

IV. The Error Equation $\dot{x}(t) = -u(t)u^T(t)x(t) + w(t)$:

Error equations of this form arise when the parameters of the plant being identified vary with time. If u(t) is sufficiently rich a bounded w(t) results in $\|x(t)\|$ being bounded. Bounds on $\|x(t)\|$ with different integral constraints on w(t) have been analyzed in [11] by Sondhi and Mitra and are included here for the sake of completeness.

Given an input vector u(t) and an unknown time-varying parameter vector k(t), let the output $y_p(t)$ be described by

$$y_p(t) = k^T(t)u(t).$$

Let $\hat{k}(t)$ denote the estimate of k(t) and let

$$y_m(t) = \hat{k}^T(t)u(t)$$

where $y_m(t)$ denotes the output of a model.

If $y_m(t) - y_p(t) \triangleq e(t)$ and $\hat{k}(t) - k(t) \triangleq x(t)$, we have the first error model

$$x^T(t)u(t) = e(t)$$

However, in this case the adaptive law is implemented as

$$\dot{\hat{k}}(t) = -\gamma e(t)u(t)$$

and yields the parameter error equation:

$$\dot{x}(t) = -\gamma e(t)u(t) - \dot{k}(t)$$

which can also be expressed as

$$\dot{x}(t) = -\gamma u(t)u^T(t)x(t) - \dot{k}(t) \tag{12}$$

or more generally

$$\dot{x}(t) = -\gamma u(t)u^T(t)x(t) + w(t) \tag{13}$$

where $w(\cdot)$ is a vector function of time with specified characteristics. Two cases are of special interest and can be denoted by the constraint inequalities

$$\frac{1}{T}\int_t^{t+T} \|w(\tau)\| \, d\tau \leq w_0 \tag{14a}$$

where T and w_0 are positive constants and $t \in [0,\infty)$,

and
$$\left\| \int_{t_1}^{t_2} w(\tau)d\tau \right\| \leq w_1 \tag{14b}$$

where t_1 and $t_2 \in [0,\infty)$ and $t_2 > t_1$

In the context of equation (12), the constraint inequality (14a) limits the rate of change of the parameters k(t) while (14b) places a bound on the total change over an arbitrary interval.

If the homogeneous equation (2) is u.a.s. and b is the minimum rate of exponential convergence, it is shown in [11] that

$$\lim_{t\to\infty} \|x(t)\| \triangleq \|x\|_\infty \leq \frac{w_0 T}{1-e^{-bT}} \tag{15}$$

if the constraint on w(t) is given by (14a). Explicit expressions for b are given in [11] and [12]. In general, for small values of adaptive gain γ, b is proportional to γ, while for large values of γ, b is proportional to $1/\gamma$.

If the constraint equation for w(t) is (14b) rather than (14a) and in addition the condition

$$\frac{1}{T}\int_t^{t+T} u(\tau)u(\tau)^T d\tau \leq k_2 I \qquad \forall\ t \in [0,\infty) \text{ and some } T < \infty$$

is satisfied, the following bound can also be derived.

$$\|x\|_\infty \leq \frac{\gamma k_2 w_1 T}{1-e^{-bT}} \leq \frac{\gamma k_2 w_1}{b} \tag{16}$$

From inequalities (15) and (16) it is clear that the asymptotic bounds on $\|x(t)\|$ depend on the nature of the plant parameter variations. If only the rate of change of the plant parameters is bounded, $\|x\|_\infty$ is inversely proportional to γ for small values of γ and proportional to γ for large γ. If, on the other hand, the plant parameter variations are restricted to a compact region in parameter space, the bound on $\|x\|_\infty$ is independent of γ for small values of γ and proportional to γ^2 for large γ.

If both output disturbance $v(t)$ and plant parameter variations ($\dot{k}(t) \neq 0$) are present the analysis in sections III and IV can be used to determine the bounds on $\|x(t)\|$. The non-homogeneous equation has the form

$$\dot{x}(t) = -\gamma u(t)u^T(t)x(t) - \gamma u(t)v(t) - \dot{k}(t)$$

Due to linearity the total effect on $x(t)$ is the sum of the effects of the two inputs.

In practice the choice of the adaptive gain γ must therefore be a compromise between tracking time varying parameters with a small error and filtering the effects of output disturbances.

V. Behavior of Error Model When u(t) is Not Sufficiently Rich:

In sections III and IV it was assumed that $u(t)$ is sufficiently rich so that the homogeneous equation is uniformly asymptotically stable. This, in turn, assures the boundedness of the parameter error vector $x(t)$ when $v(t)$ and $w(t)$ are uniformly bounded. When $u(t)$ is not sufficiently rich but $v(t),w(t) \equiv 0$ it was also shown that the system is uniformly stable. The question naturally arises whether the error models considered so far can result in unbounded $x(t)$ for bounded functions $v(t)$

and w(t). In this section some specific examples are given where such is indeed the case.

a) Scalar Equation

We first consider the scalar equation

$$\dot{x}(t) = -u^2(t)x(t) - u(t)v(t) \tag{17}$$

where u and v are bounded scalar functions. If $u \in L^2$ the homogeneous system is stable but not asymptotically stable; $u \notin L^2$ assures asymptotic stability and $\int_0^t u^2(\tau)d\tau > at + b$ for $t \in [0,\infty)$ and for some constants $a > 0$ and b, assures uniform asymptotic stability. In the last case the input is sufficiently rich and the system is also exponentially stable.

The non-homogeneous case can be classified as follows.

Case (i) If $u \in L^1$, the output x(t) is bounded. Examples include $u \equiv 0$ and $u(t) = 1/t^\alpha$ $(\alpha > 1)$.

Case (ii) If $u \in L^2$ but $u \notin L^1$

$$x(t) = -\int_{t_0}^{t} \Phi(t,\tau)u(\tau)v(\tau)d\tau$$

where $|\Phi(t,\tau)| \geq M > 0$ for some constant M.

Choosing v(t) = sgn u(t), we have

$$|x(t)| \geq M\int_{t_0}^{t} |u(\tau)|d\tau$$

Since $u \notin L^1$, $\lim_{t\to\infty} \|x(t)\| = \infty$.

Examples include $u(t) = 1/t^\alpha$ for $1/2 < \alpha \leq 1$.

Case (iii) If $u \notin L^2$ but is not sufficiently rich two possible situations can arise.

1) No v(t) exists such that the system has unbounded outputs. An example of this is a binary valued function u(t) which assumes the values 0 or 1 for all $t \in [0,\infty)$ and satisfies the above conditions.

2) As in case (ii) a v(t) can be determined which results in x(t) being unbounded.

An example is $u(t) = 1/t^{1/2}$

Case (i) provides sufficient conditions for bounded outputs. Case (ii) provides sufficient conditions for realizing unbounded outputs; the homogeneous system is not asymptotically stable and $u \notin L^1$. At the present time necessary and sufficient conditions for unbounded outputs are not available.

b) <u>Vector Equation</u> $\dot{x} = -u(t)u^T(t)x(t) - u(t)v(t)$ (18)

Conditions similar to those stated for the scalar case can also be derived for the vector case.

(i) Let $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$ where u_1 and u_2 are n_1 and n_2 dimensional vectors respectively. Let u_1 be sufficiently rich in $\mathbb{R}^{n_1}$ and $u_2 \in L^1$. Then the output x(t) is uniformly bounded. Only a brief outline of the proof is given below.

$$\dot{x}_1 = -u_1(t)u_1^T(t)x_1(t) - u_1(t)v(t) - u_1(t)u_2(t)x_2(t) \qquad (19a)$$

$$\dot{x}_2 = -u_2(t)u_2^T(t)x_2(t) - u_2(t)v(t) - u_2(t)u_1(t)^T x_1(t) \qquad (19b)$$

From equation (19) it follows that x_1 and x_2 are either both bounded or both unbounded. In the latter case, from (19a) $x_1(t)$ grows slower than $x_2(t)$ since $u_2 \in L^1$. From (19b) it can be shown that x_1 and x_2 grow at the same rate and hence this results in a contradiction.

An example of this is when u is a constant vector. This can be cast in the form $u^T = [u_1, 0 \ldots 0]$ where u_1 is a scalar and $u_2 \equiv 0$ an $n - 1$ dimensional vector.

(ii) To determine situations in which the output x(t) grows in an unbounded fashion we consider conditions similar to case (ii) in (a).

If $u \notin L^1$ and $u \in L^2$, the bounded function v can be chosen so that x(t) is unbounded. If $\Phi_1(t,\tau), \ldots \Phi_n(t,\tau)$ are the rows of the transition matrix $\Phi(t,\tau)$ of (18)

$$x(t) = -\int_{t_0}^{t} \begin{bmatrix} \Phi_1(t,\tau)u(\tau) \\ \Phi_2(t,\tau)u(\tau) \\ \cdot \\ \cdot \\ \cdot \\ \Phi_n(t,\tau)u(\tau) \end{bmatrix} v(\tau)d\tau$$

The homogeneous system is uniformly stable but not asymptotically stable and $\|\Phi(t,\tau)\| \geq M > 0$ for all $t, \tau \in [0,\infty)$. Further since $u \notin L^1$, there exists at least one function $\Phi_i(t,\tau)u(\tau)$ such that

$$\int_{t_0}^{t} |\Phi_i(t,\tau)u(\tau)|d\tau \text{ is unbounded}$$

as $t \to \infty$. Hence choosing $v(\tau) = \text{sgn } \Phi_i(t,\tau)u(\tau)$ the output may be made unbounded.

(iii) As in the scalar case, if u is not sufficiently rich but $u \notin L^2$ the resulting system can exhibit both bounded and unbounded behavior. Two examples of unbounded behavior are given below.

If $u(t) = \begin{bmatrix} \cos\sqrt{2t} \\ \sin\sqrt{2t} \end{bmatrix}$ and $v(t) = -2$ then $x(t) = \begin{bmatrix} \cos\sqrt{2t} + \sqrt{2t}\sin\sqrt{2t} \\ \sin\sqrt{2t} - \sqrt{2t}\cos\sqrt{2t} \end{bmatrix}$

is an unbounded solution with $\|x\|$ growing as $\sqrt{t}$. A computer simulation of such a system is shown in Figure 3. Figure 4 shows the computer simulation for the case when

$$u(t) = \begin{bmatrix} 1 \\ \cos\sqrt{t} \end{bmatrix} \quad v(t) = -2\,\text{sgn}[\sin\sqrt{t}\,]$$

and once again $\|x\|$ grows as $\sqrt{t}$. In both cases u(t) is such that the homogeneous equation is asymptotically stable.

c) <u>Vector Equation</u> $\dot{x} = -u(t)u^T(t)x(t) + w(t)$

In this case the forcing term w(t) is independent of u(t) and is assumed to be uniformly bounded. It follows directly that if u(t) is sufficiently rich x(t) will also be uniformly bounded. If u(t) is not sufficiently rich it is well-known that bounded inputs w(t) can be found which result in unbounded x(t).

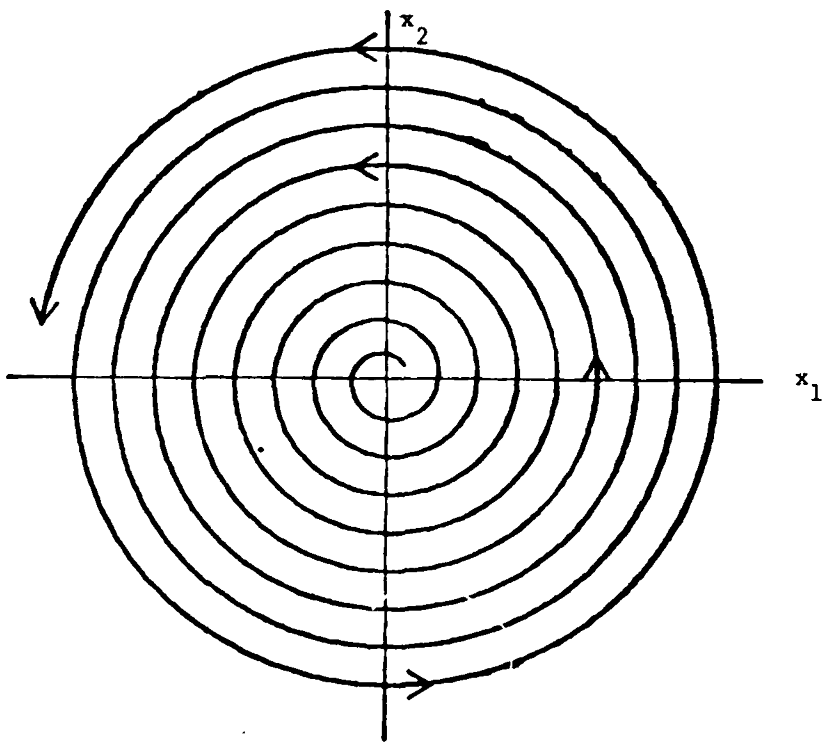

Figure 3

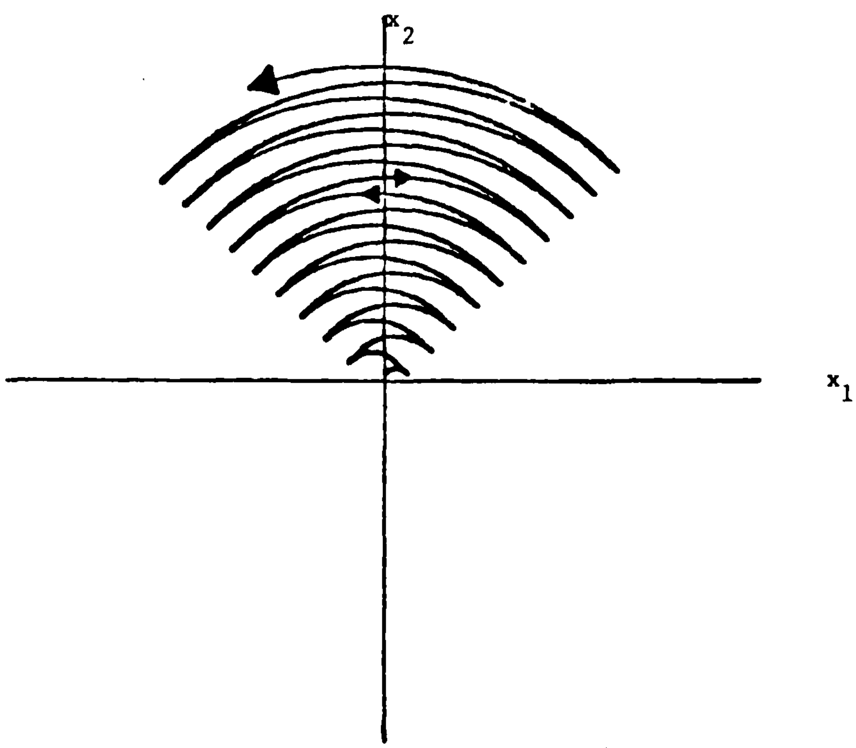

Figure 4

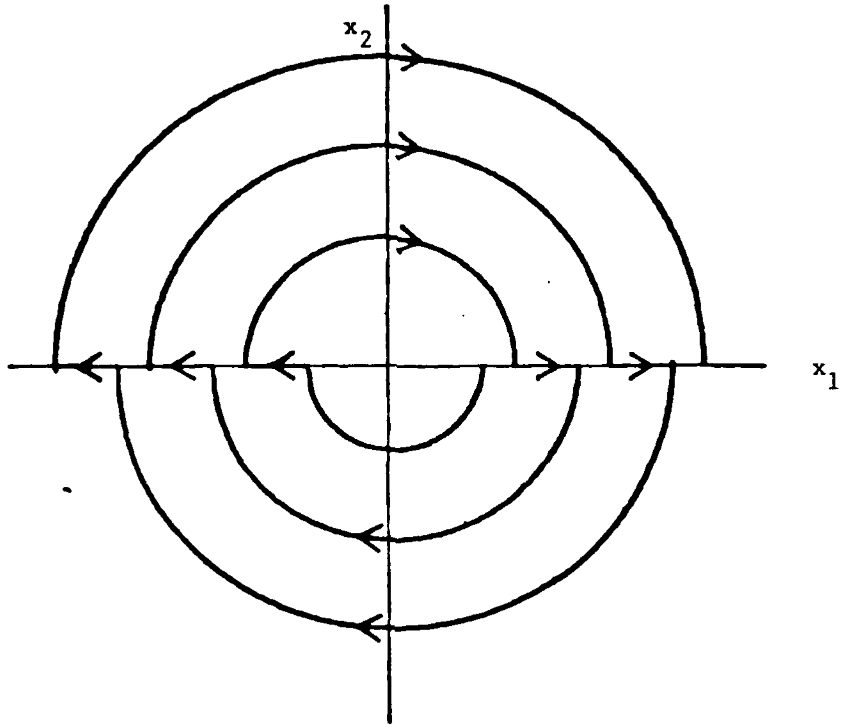

Figure 5

While many simple examples of such behavior can be given, we present below one which is particularly relevant for the case of time varying parameters described earlier. In Figure 5, if u(t) is defined by

$$u(t) = \begin{bmatrix} \sin\sqrt{2(t_i-t)} - \frac{1}{\sqrt{2(t_i-t)}} \cos\sqrt{2(t_i-t)} \\ -\cos\sqrt{2(t_i-t} - \frac{1}{\sqrt{2(t_i-t)}} \sin\sqrt{2(t_i-t)} \end{bmatrix}$$

For $t_i - \frac{(i\pi)^2}{2} \leq t < t_i - \frac{(i-1)^2\pi^2}{2}$

and $t_i = t_{i-1} + 2(i-1)\pi^2$

For $i = 2\ldots\infty$, arbitrary t_1 and $w(t)$ is a sequence of impulses which are alternately positive and negative and occur when $x(t)$ has rotated through an angle π it is seen that the norm of $x(t)$ can be made arbitrarily large.

VI. Application to Identifiers and Adaptive Observers:

The error models described so far find direct application in identifiers and adaptive observers. Two simple typical cases are described in this section.

a) Identifier

Let $w_p(s)$ be the stable unknown transfer function of a plant which has to be identified. If $w_p(s)$ is parametrized as $w_p(s) = \sum_{i=1}^{N} w_i(s)k_i$ where $w_i(s)$ are known stable transfer functions, the identification problem is to estimate the constants k_i. This is carried out by constructing a model whose transfer function is $\sum_{i=1}^{N} w_i(s)\hat{k}_i$ and adjusting $\hat{k}_i(t)$ so that the error between model and plant outputs tends to zero. Defining the bounded reference input to both plant and model as r(t), plant output as $y_p(t)$ and model output as $\hat{y}_p(t)$, and $w_i(s)r(t) \overset{*\Delta}{=} u_i(t), \hat{k}_i(t)-k_i \overset{\Delta}{=} \phi_i$ $\hat{y}_p(t) - y_p(t) = e(t)$, the error equation may be expressed as

$$\sum_{i=1}^{N} \phi_i(t)u_i(t) \overset{\Delta}{=} \phi^T(t)u(t) = e(t)$$

which is the first error model.

Case (i): If the output of the plant is corrupted with observation noise v(t) we have

$$\phi^T(t)u(t) + v(t) = e(t)$$

which is the error model considered in section II.

Case (ii): Let a reduced order model of the form

$$\sum_{i=1}^{N_1} w_i(s)\hat{k}_i \qquad\qquad N_1 < N$$

be used to represent the plant. If $\sum_{i=N_1+1}^{N} k_i w_i(s)r(t) = v_1(t)$, then $v_1(t)$ represents the residual error. The output of the plant can be expressed as

$$y_p(t) = \bar{y}_p(t) + v_1(t)$$

---

*'s' is used as both the Laplace transform variable and the operator $\frac{d}{dt}$.

and $\sum_{i=1}^{N_1} \hat{k}_i w_i(s)r(t) = \hat{y}_p(t)$ is the estimate of $\bar{y}_p(t)$. In this case the error equation is given by

$$\phi^T(t)u(t) - v_1(t) = e(t) \qquad (20)$$

(where ϕ and u are now N_1 dimensional vectors). From equation (20) it is seen that the residual error $v_1(t)$ enters the error model as a disturbance.

<u>Case (iii)</u>: If in case (i) $v(t) \equiv 0$ but the plant parameters $k_i(t)$ vary with time the error equation has the form

$$\phi^T(t)u(t) = e(t)$$

but the law for adjusting $\hat{k}(t)$ reduces to

$$\dot{\phi}(t) = -\gamma u(t)u^T(t)\phi(t) - \dot{k}(t).$$

If a plant with time-varying coefficients is identified in the presence of output noise by a reduced order model all the effects discussed in cases (i)-(iii) must be included. In such a case the overall error equation has the form

$$\dot{\phi}(t) = -\gamma u(t)u^T(t)\phi(t) - \gamma[v(t) + v_1(t)]u(t) - \dot{k}(t)$$

and can be analyzed using the results of the earlier sections.

The error models described in sections 3 and 5 apply directly to cases (i) and (iii). Hence, by the proper choice of the input u(·) and the disturbance v(·), or k(·), it is possible to make the parameters of the model unbounded as described in section V. In case (ii) the input vector u(t) and the residual error $v_1(t)$ are correlated. If $r(\cdot) \in L^\infty \cap L^1$ or r(·) is sufficiently rich it follows from the results of section V that the parameters of the model will be bounded. It can also be shown that u(·) and v(·) do not satisfy the sufficient conditions given in section 5 for unbounded parameter errors. However, so far, it has not been demonstrated conclusively that the parameters of the model will remain bounded.

b) <u>Adaptive Observers</u>

The adaptive observer of Lüders and Narendra with observation noise is shown in Figure 6. The unknown plant has a stable rational transfer function N(s)/D(s). The output of the plant is $y_p(t)$, but can be observed only in the presence of additive noise v(t). The input and noisy output are processed through state variable filters (denoted by the operator F) to generate the vectors $u_1(t)$ and $u_2(t)$. If $\theta^T(t) \triangleq [\theta_1^T(t), \theta_2^T(t)]$ and $u^T(t) \triangleq [u_1^T(t), u_2^T(t)]$, then the output of the observer is $\hat{y}_p(t)$ where

$$\hat{y}_p(t) = \theta^T(t)u(t)$$

If the vector θ^* represents the plant parameter vector using this parametrization, the parameter error vector x(t) may be defined as $x(t) \triangleq \theta^T(t) - \theta^*$. It is easily

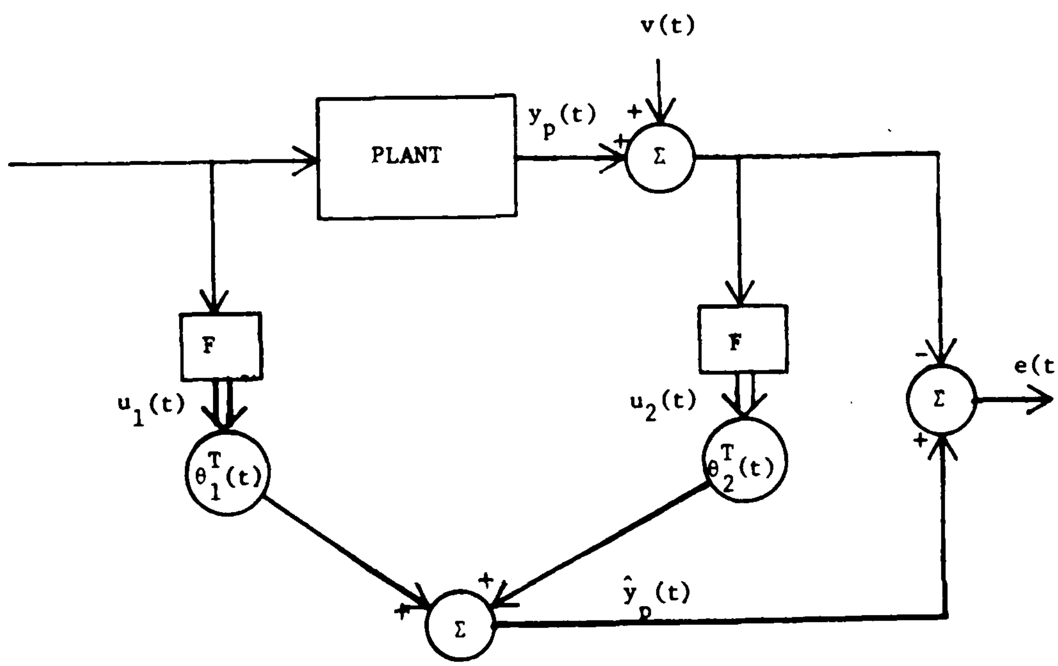

Figure 6

shown that the output error between observer and plant outputs (i.e. $e(t) \stackrel{\Delta}{=} \hat{y}_p(t) - y_p(t) - v(t)$) satisfies the equation

$$x^T(t)u(t) - v_1(t) = e(t)$$

where

$$v_1(t) = v(t) - \theta_2^{*T} F v(t)$$

$$= \frac{D(s)}{R(s)} v(t)$$

where R(s) is the characteristic polynomial of the filter. Therefore, the analysis of the adaptive observer of Figure 6 reduces to the analysis of the error model of section IIb.

VII. Adaptive Control in the Presence of Disturbance:

In sections II to VI it was assumed that the input vector u and the disturbance v are uniformly bounded. While this assumption on u is a reasonable one for identification, it is no longer valid in the control case. The principal problem here is to establish the boundedness of all signals and parameters as was done in [1-6] for the disturbance free case. In this section, we propose a nonlinear adaptive law which appears to have potential for the resolution of the general adaptive control problem with a bounded disturbance. The case of a simple system described by a scalar differential equation is considered first to illustrate the principal concepts.

a) The Scalar Control Problem

An unstable plant is described by the linear time-invariant differential equation

$$\dot{y}_p = -a_p y_p + w(t) \qquad a_p < 0$$

It is desired to stabilize it using feedback such that

$$w(t) = -\theta(t) y_p(t)$$

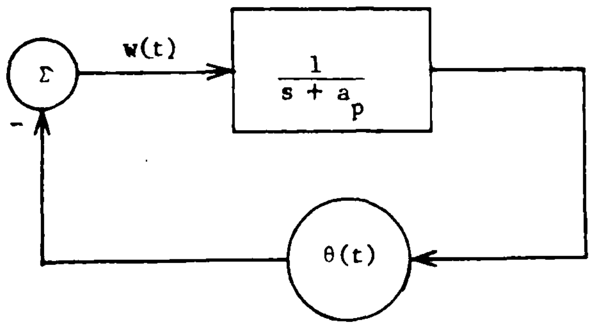

Figure 7

as shown in Figure 7. The plant parameter a_p is unknown and the control parameter $\theta(t)$ has to be adjusted using all available signals in the system.

(i) If no disturbance is present and the adaptive law

$$\dot{\theta}(t) = + \gamma y_p^2(t) \qquad \gamma > 0 \tag{21}$$

suggested in [16] is used, it follows that $\lim_{t\to\infty} y_p(t) = 0$ and $\lim_{t\to\infty} \theta(t) = \theta^*$, where θ^* is a constant such that $- a_p - \theta^* < 0$. In other words, the parameter $\theta(t)$ is a non-decreasing function of time and converges to some constant θ^* (depending on initial conditions) such that the feedback loop is asymptotically stable.

(ii) For Model Reference Adaptive Control (MRAC), a stable model described by

$$\dot{y}_m(t) = -a_m y_m(t) + r(t), \qquad a_m > 0$$

(where the reference input $r(t)$ is uniformly bounded) is specified and it is desired to track $y_m(t)$ with the output of the plant.

If

$$e(t) \triangleq y_p(t) - y_m(t)$$

and the control input to the plant is

$$w(t) = r(t) - \theta(t)y_p(t)$$

where $\theta(t)$ is adjusted using the adaptive law

$$\dot{\theta}(t) = +\gamma e(t)y_p(t) \tag{22}$$

then

$$\lim_{t\to\infty} e(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} a_p + \theta(t) = a_m$$

if the input is sufficiently rich.

(iii) If in case (i) the observed output of the plant $y(t)$ is corrupted with additive noise $v(t)$, then

$$y(t) = y_p(t) + v(t)$$

If the same adaptive law (21) is used with $y_p(t)$ replaced by $y(t)$, the overall system is described by the equations

$$\begin{aligned} \dot{y}(t) &= -(a_p + \theta(t))y(t) + v_1(t) \\ \dot{\theta}(t) &= \gamma y^2(t) \end{aligned} \tag{23}$$

where

$$v_1(t) = \dot{v}(t) + a_p v(t)$$

From equation (23) it is seen that $\theta(t)$ is a non-decreasing function of time and will be unbounded if y does not belong to L^2. It can be shown that a necessary and sufficient condition for $\lim_{t\to\infty} \theta(t) = \infty$ is $v_1 \notin L^2$.

The above result implies that the parameter $\theta(t)$ will grow in an unbounded fashion in most practical situations where the disturbance does not tend to zero with time. It is perhaps also worth noting that the observation noise $v(t)$ can be replaced by an input noise $v_1(t)$ without affecting the analysis carried out so far.

The above example shows the need for some provision for stopping the adaptive process if the control parameter is to remain bounded. The nonlinear adaptive law suggested in case (iv) realizes this objective.

(iv) In case (iii) $\theta(t)$ is a monotonic non-decreasing function of time. If $v_1 \notin L^2$ there exists some time $t = t_1$ at which $-a_p - \theta(t_1) < 0$. The homogeneous system

$$\dot{y}(t) = -[a_p + \theta(t)]y(t)$$

is asymptotically stable for all $t > t_1$. If $v_1(t)$ is uniformly bounded and $|v_1(t)| \leq v_0$ it can be shown that

$$\lim_{t\to\infty} |y(t)| = 0$$

Hence, if the adaptive law (21) is modified as follows

$$\begin{aligned} \dot{\theta}(t) &= \gamma y^2(t) \qquad & |y(t)| > \varepsilon \\ &= 0 & |y(t)| \leq \varepsilon \end{aligned} \tag{24}$$

for any constant $\varepsilon > 0$, all signals $y(t)$, $\theta(t)$ and $w(t)$ remain bounded. The smaller the value of ε chosen, the larger is the limiting value of the parameter $\theta(t)$. Figure 8 shows the nature of the trajectories in the (y,θ) space.

(v) The results of case (iv) can also be extended to the MRAC problem treated in (ii), when observation noise is present. The objective in this case is to track the output of the reference model with output $y(t)$ of the plant, with a bounded error.

The feedback gain $\theta(t)$ in this case cannot be adjusted to make the observed output error arbitrarily small, since the latter is determined both by the noise $v_1(t)$ as well as the mismatch between plant and model. Hence, the dead-zone cannot be made arbitrarily small. The equations describing the observed error $e(t)$ and the parameter error $\phi(t)$ are:

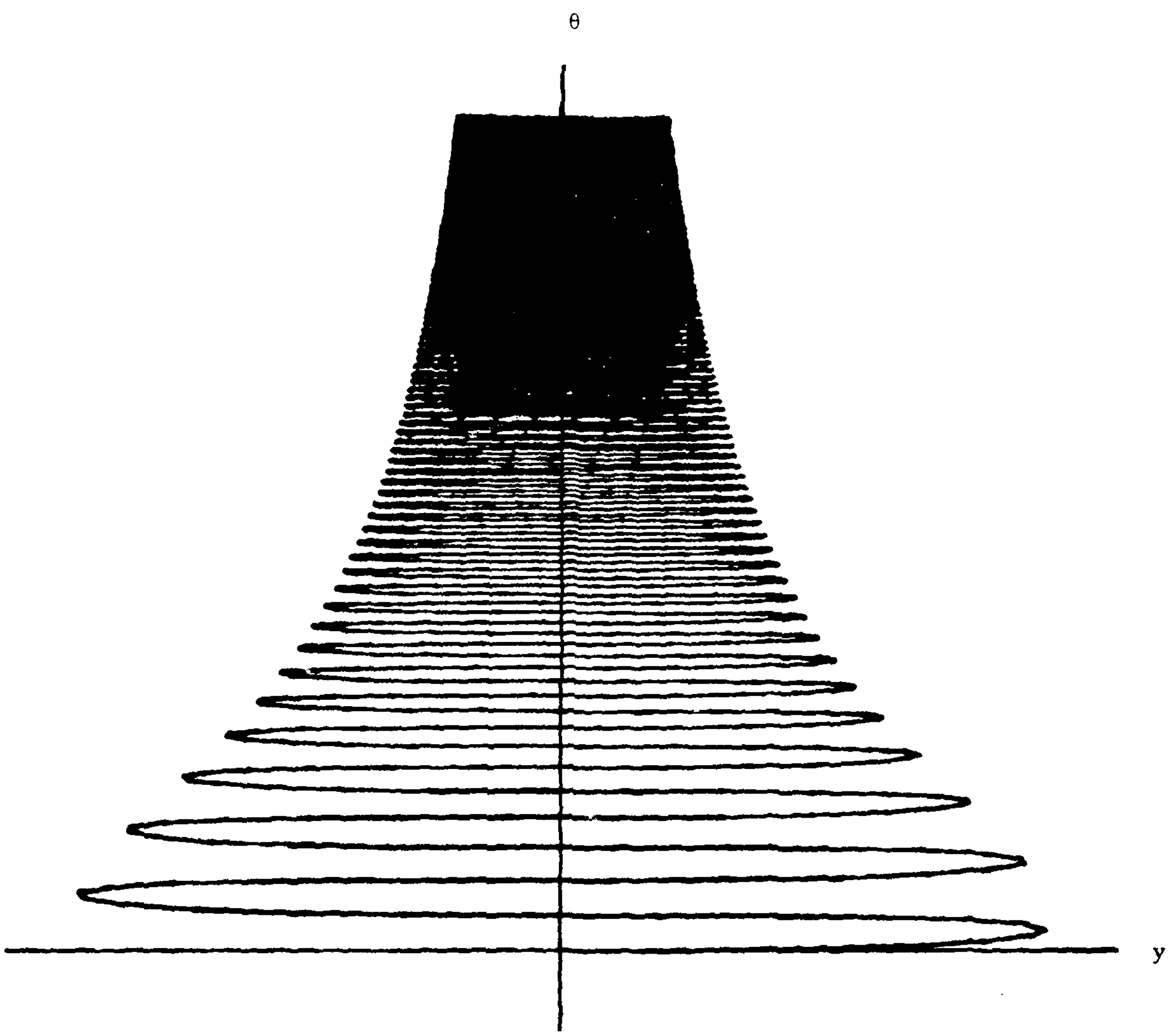

Figure 8

$$e(t) = y_p(t) + v(t) - y_m(t) = y(t) - y_m(t)$$

$$\phi(t) = \theta(t) - \theta^* = \theta(t) - a_m + a_p$$

and it is easily shown that

$$\dot{e}(t) = -a_m e(t) - \phi(t)y(t) + v_1(t) \qquad |v_1(t)| \le v_0 - \varepsilon,\ \varepsilon > 0 \tag{25}$$

where $y(t)$ and $v_1(t)$ are as defined in (iii). Using the nonlinear adaptive law

$$\dot{\phi}(t) = \dot{\theta}(t) = \gamma e(t)y(t) \qquad |e(t)| \geq \frac{v_0}{a_m}$$
$$= 0 \qquad |e(t)| < \frac{v_0}{a_m} \tag{26}$$

it is seen that the function $V(e,\phi) = \frac{1}{2}(e^2 + \frac{1}{\gamma}\phi^2)$ decreases when $|e(t)| \geq \frac{v_0}{a_m}$ and adaptation is effective. When $|e(t)| < \frac{v_0}{a_m}$ no adaptation takes place, θ is a constant and $V(e,\phi)$ is bounded.

The analysis of cases (i)-(v) indicates that the use of a nonlinear adaptive law may be effective in achieving a bounded error in more general cases, when observation noise is present. This is treated in the next section.

b) Adaptive Control in the Presence of Disturbance

The results of section (a) (iv) carry over directly to the adaptive control of a single input-single output time-invariant plant of order n and relative degree $n^* = 1$. The plant is assumed to satisfy the following conditions:

(i) the transfer function of the plant is $w_p(s)$ where

$$w_p(s) \triangleq \frac{N(s)}{D(s)} .$$

$N(s)$ is assumed to be a monic polynomial of degree $(n-1)$ with all its roots in the left half plane. The characteristic polynomial $D(s)$ is of degree n.

(ii) the transfer function of the model is $w_M(s)$ where

$$w_M(s) \triangleq \frac{1}{s+\alpha} \qquad \alpha > 0$$

The assumption that $N(s)$ is monic is for ease of exposition and the results that follow do not depend on this property.

As in the noise free case [1-6], the uniformly bounded reference input $r(t)$ and the model are specified. $y_p(t)$ and $y_m(t)$ are respectively the plant and model

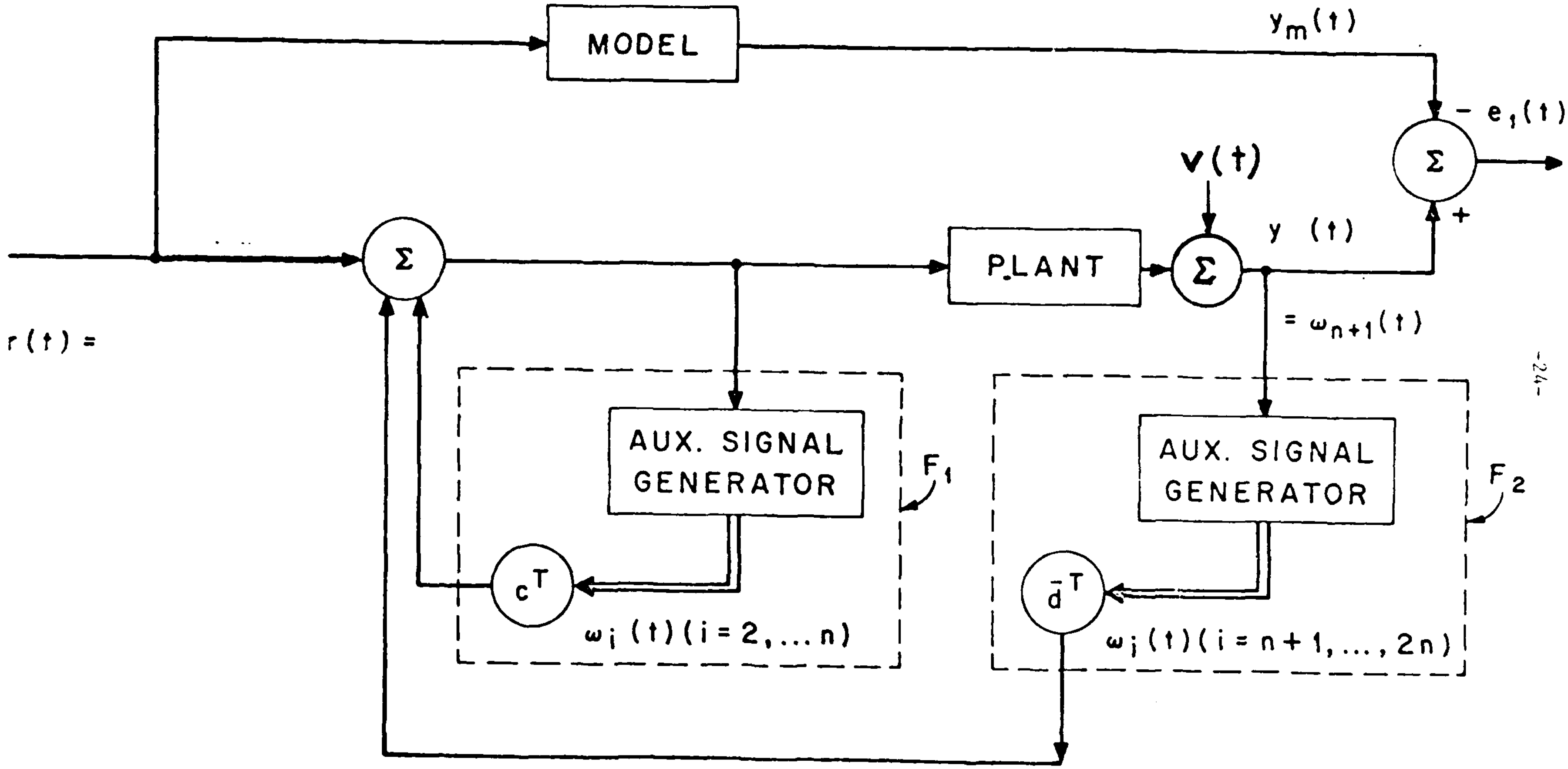

Figure 9. Basic Structure of the Adaptive System.

outputs and the observed output, $y(t) \overset{\Delta}{=} y_p(t) + v(t)$, contains additive noise. It is desired to determine the input to the plant u(t) such that

$$e_1(t) \overset{\Delta}{=} y(t) - y_m(t)$$

is uniformly bounded.

The structure of the controller used in this case is identical to that suggested in [16] for the noise free case. The input u(t) and the observed output y(t) of the plant generate the auxiliary vector valued output $\omega(t)$ $(\omega(t)^T \overset{\Delta}{=} [\omega_2(t), \omega_3(t), \ldots \omega_{2n}(t))$ and a linear combination of these is the desired feedback signal (i.e. $\theta^T(t)\omega(t)$). The entire structure of plant and controller is shown in Figure 9.

The signals $\omega_i(t)$ $(i = 2,3,\ldots,2n)$ are correlated with the observation noise v(t). From [16] it is known that constants θ_i^* exist such that when $\theta_i(t) \equiv \theta_i^*$ and $v(t) \equiv 0$ the plant output $y_p(t)$ asymptotically approaches the model output $y_m(t)$. Our objective in the noisy case is to modify the adaptive law so that the error $e_1(t)$ remains bounded even when v(t) is present.

If $\theta(t) - \theta^* \overset{\Delta}{=} \phi(t)$, the output error $e_1(t)$ is related to $\phi(t),\omega(t),v(t)$ and the plant transfer function as shown in Figure 10.

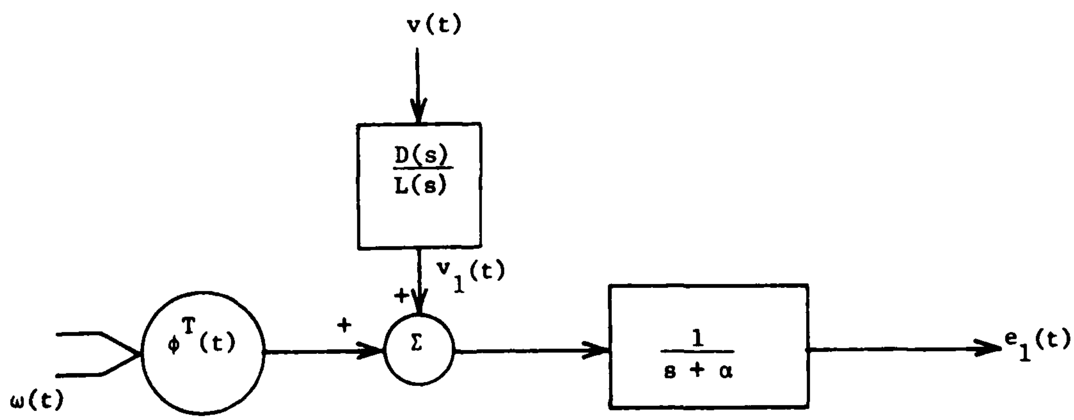

Figure 10

The differential equation describing $e_1(t)$ is

$$\dot{e}_1(t) = -\alpha e_1(t) + \phi^T(t)\omega(t) + v_1(t) \tag{27}$$

[Note that $\frac{D(s)}{L(s)}$ is not proper and that D(s) is unknown. However it is assumed that $v_1(t)$ is bounded and that prior information is available to calculate its bound.] Equation (27) is similar to (25) in section VII(a) and can be analyzed along the same lines. If $|v_1(t)| \leq v_0 - \epsilon$ for some constant $\epsilon > 0$, the adaptive laws used in the noise free case are modified as follows:

$$\dot{\theta}_i(t) = -\gamma e_1(t)\omega_i(t) \qquad |e_1(t)| \geq \frac{v_0}{\alpha}$$

$$= 0 \qquad |e_1(t)| < \frac{v_0}{\alpha}$$

If $V(e_1,\phi) \triangleq \frac{1}{2}[e_1^2(t) + \frac{1}{\gamma}\phi^T(t)\phi(t)]$ we obtain

$$\dot{V}(e_1,\phi) = -\alpha e_1^2(t) + e_1(t)v_1(t)$$

Let the region $|e_1(t)| \leq \frac{v_0}{\alpha}$ in the (e_1,ϕ) space be denoted by Ω. If $(e_1,\phi) \in \Omega$ $\phi(t)$ is a constant. Outside this region $\dot{v}(e,\phi) < 0$. If t_1 and t_2 denote two instants of time at which $(e_1(t_i),\phi(t_i)) \in \delta\Omega$ $(i=1,2)$, then $\|\phi(t_2)\| \leq \|\phi(t_1)\|$ for $t_2 > t_1$.

$\|\phi(t)\|$ decreases monotonically when $(e_1(t),\phi(t)) \notin \Omega$ and adaptation is effective. Both output and parameter errors are bounded and in the limit $\phi(t)$ tends to a constant ϕ^* such that $|e_1(t)| \leq \frac{v_0}{\alpha}$.

We have introduced in this section a nonlinear adaptive law for controlling the parameters of a general n^{th} order plant with single input and single output and observation noise. A preliminary study of such a system when the plant transfer function has a relative degree $n_1^* = 1$ has been presented here. A more detailed study of this system, as well as the extension of the adaptive law to the general case where $n_1^* \geq 2$ will be discussed in a following report.

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