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A GENERAL INVERSE SOLUTION FOR 2D EDGE WAVES. (U)  
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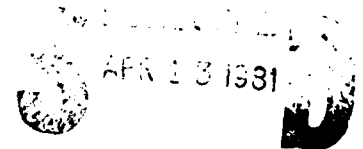
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## A GENERAL INVERSE SOLUTION FOR 2-D EDGE WAVES

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### Abstract

An inverse method is used to determine topographies which will support long period edge waves along a straight coastline. Two examples are given; one corresponds to a known solution and the other appears to be new.

### I: Formulation and Solution

Consider the conditions under which a long wavelength free surface gravity wave may be trapped against a straight coastline by a topography which varies in a direction normal to the coastline. Taking  $x$  in this normal direction and  $y$  along the coastline, the linearized shallow water-long wavelength equations for a homogeneous fluid on a rotating earth are [e.g. LeBlond and Mysak (1978)]:

$$\begin{aligned}\partial u / \partial t + g \partial \eta / \partial x &= f v \\ \partial v / \partial t + g \partial \eta / \partial y &= -f u \\ \partial \eta / \partial t + H(\partial u / \partial x + \partial v / \partial y) + u \partial H / \partial x &= 0\end{aligned}\tag{1}$$

where  $(u, v)$  are velocities,  $\eta$  is the free surface elevation,  $H$  is the water depth and  $f$  is the Coriolis parameter.

Assume solutions which propagate along the coastline, e.g.

$$\begin{aligned}u(x, y, t) &= U(x) \cos (ky - \sigma t) \\ v(x, y, t) &= V(x) \sin (ky - \sigma t) \\ \eta(x, y, t) &= E(x) \sin (ky - \sigma t)\end{aligned}$$

Clearly  $U$  and  $V$  may be eliminated, leaving a single equation on  $E$ ,

e.g. LeBlond and Mysak (1978 - p. 220)

$$(HE')' + \left\{ \frac{\sigma^2 - f^2}{g} - k^2 H - \frac{fk}{\sigma} H' \right\} E = 0 \quad (2)$$

This equation does not force a particular  $x$  dependence on either  $E$  or  $H$ , as is the case for the three dimensional problem, e.g. Shaw (1979). If, for example,  $E$  is required to be exponentially decaying such as  $E_0 \exp(-\ell x)$ , equation (2) requires  $H(x)$  to be

$$H(x) = \frac{\sigma^2 - f^2}{g(k^2 - \ell^2)} + H_0 \exp\left\{-\left[\frac{(k^2 - \ell^2)}{\ell + fk/\sigma}\right]x\right\}$$

where  $H_0$  is an arbitrary constant defined in general either by choosing  $H(0) = 0$  or  $U(0) = 0$  to have no net mass flux across the coastline,  $x = 0$ . If  $H(0)$  is chosen to be zero, the topography is given by

$$H(x) = \frac{\sigma^2 - f^2}{g(k^2 - \ell^2)} \left\{ 1 - \exp\left[-\left(\frac{k^2 - \ell^2}{\ell + fk/\sigma}\right)x\right] \right\} \quad (3)$$

corresponding to the lowest ( $n=0$ ) mode given by Ball (1967).

Certain ranges of parameters give negative topographies which must be excluded. Requiring that  $\ell > 0$ , but recognizing that  $k$  may be either positive or negative, yields the following results:

For  $\sigma > f$ ,  $k > \ell$  gives a positive concave upwards topography, exponentially approaching a uniform depth,  $H_\infty$  equal to  $(\sigma^2 - f^2)/g(k^2 - \ell^2)$ ;  $-\ell < k < \ell$  gives a positive, convex upwards, exponentially increasing depth,  $-\sigma\ell/f < k < -\ell$  gives a positive, concave upwards topography again exponentially asymptotic to  $H_\infty$  and finally  $k < -\sigma\ell/f$  yields a negative topography.

For  $\sigma < f$ ,  $k > \ell$  and  $-\sigma\ell/f < k < \ell$  both yield negative topographies, but  $-\ell < k < -\sigma\ell/f$  yields a positive, concave upwards topography again decaying exponentially to the asymptotic depth  $H_\infty$  while  $k < -\ell$  yields a positive, convex upwards exponentially increasing depth.

The choice of  $U(0)$  equal to zero requires either  $\sigma = -fk/\ell$ , leading to a constant depth,  $H = f^2/g\ell^2$ , or the trivial solution,  $E(0)$  equal to zero and thus  $E(x)$  equal to zero everywhere. This choice is then rejected for this particular form for  $E(x)$ .

Identifying the asymptotic depth as

$$H_{\infty} = (\sigma^2 - f^2)/g(k^2 - \ell^2) \quad (4)$$

and using the slope at the coastline,  $H'_0$  as

$$H'_0 \equiv dH(x)/dx|_{x=0} = H_{\infty}(k^2 - \ell^2)/(\ell + fk/\sigma) \quad (5)$$

provides a topography

$$H(x) = H_{\infty}\{1 - \exp(-H'_0 x/H_{\infty})\} \quad (6)$$

to support an edge wave

$$E(x) = E_0 \exp(-\ell x). \quad (7)$$

The choice of the two parameters  $H_{\infty}$  and  $H'_0$  completely fixes the topography; the remaining four parameters ( $\sigma$ ,  $k$ ,  $\ell$  and  $f$ ) are then related by two equations, i.e. eqs. (4) and (5), leaving two parameters free. If the special case of no rotation,  $f=0$ , is considered, one degree of freedom remains and these equations may be written as a dispersion equation

$$\sigma^4/(gH'_0)^2 + \sigma^2/gH_{\infty} = k^2 \quad (8)$$

and an equation governing the decay parameter

$$\ell^2 = k^2 - \sigma^2/gH_{\infty} \quad (9)$$

Thus, an arbitrary frequency  $\sigma$  determines specific values for  $k$  and  $\ell$  on a given topography. These match equations given by Ball (1967). The important point here is that the same topography exists for all frequencies; only the trapped wave changes as  $\sigma$  changes.

It is of interest to note that the solution due to Ball (n=0 case) represents only part of the solution obtained here. Taking the case of no rotation for simplicity, the Ball (n=0 case) represents the solution for  $k^2 > \ell^2$  which has a topography which decays exponentially to  $H_\infty$ . The solution for  $k^2 < \ell^2$  given here has  $H_\infty < 0$  and  $\exp\{-(k^2 - \ell^2)x/\ell\} \geq 1$  such that the topography is exponentially increasing with  $x$ , Shaw (1981).

In general, equation (2) may be considered to define E for a given H or alternatively H for a given E. While the first view is more realistic physically, i.e. a direct problem, the second is simpler mathematically, i.e.

$$\left\{E' - \frac{fk}{\sigma} E\right\} H' + \left\{E'' - k^2 E\right\} H = \left\{\frac{f^2 - \sigma^2}{g}\right\} E \quad (10)$$

may be solved completely for H(x) for a given E.

$$\begin{aligned} H(x) = C_0 \exp \left[ - \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] \\ + \left\{ \frac{f^2 - \sigma^2}{g} \right\} \exp \left[ - \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] \\ * \int \frac{E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} \cdot \exp \left[ + \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] dx \end{aligned} \quad (11)$$

where  $C_0$  is an arbitrary constant to be chosen to have zero net across the coastline, which is not necessarily chosen to be  $x=0$ , or, as will be seen in the example, to keep finite depths at the critical point of equation (10). While eq. (11) provides a complete solution to the inverse problem, it is more instructive to integrate eq. (10) directly for any given wave form,  $E(x)$ , especially when dealing with the critical point at which  $E' - (fk/\sigma)E$  vanishes.

Consider as an example the form

$$E(x) = E_0 \exp[-\ell(x+a)] \quad (12)$$

which leads to an analytical solution for  $H(x)$  in terms of either an incomplete gamma function or a Kummer function. A nondimensional form using  $\bar{E} = E/E_1 \exp[\lambda(b-a)]$ ,  $\bar{H} = H/H_0$ ,  $\bar{x} = \lambda(x+b)$ ,  $\bar{f} = f/c$ ,  $\bar{k} = k/l$  and  $\bar{\sigma} = (\sigma)/\lambda(gH_0)^{1/2}$ , and then dropping the bars for convenience, gives eq. (5) in the form

$$(1-(1+fk)x)H' + (-2+(1-k^2)x)H = \sigma^2(f^2-1)x \quad (13)$$

The complementary solution to this equation is

$$H_c(x) = G \left\{ [(1+fk)x-1]^{-\alpha} \exp[-(k^2-1)x/(1+fk)] \right\} \quad (14)$$

where  $\alpha$  is  $(1 + 2fk + k^2)/(1 + 2fk + f^2k^2)$ . Variation of parameters, using  $G$  as  $G(x)$ , leads to

$$\frac{dG}{dx} = -\sigma^2 (1-f^2)x \{1-(1+fk)x\}^{-\alpha} \exp[(k^2-1)x/(1+fk)^2] \quad (15)$$

The integration for this is best carried out separately for  $x$  less than and greater than the critical value,  $(1+fk)^{-1}$ . Requiring that  $H$  remain finite at this critical value fixes the constant of integration for  $G(x)$  and the same result is obtained in both regions

$$H(x) = \frac{\sigma^2(1-f^2)}{(k^2-1)} \left\{ 1 - \frac{2\Gamma(\alpha)}{1+fk} \exp(\lambda)\gamma^*(\alpha, \lambda) \right\} \quad (16)$$

where  $\lambda = (k^2-1)(1-(1+fk)x)/(1+fk)^2$  and  $\gamma^*(\alpha, \lambda)$  is that form of the incomplete gamma function which is analytic for all  $\lambda$ ; e.g. Abramowitz and Stegun (1964)

$$\gamma^*(\alpha, \lambda) = \frac{\lambda^{-\alpha}}{\Gamma(\alpha)} \int_0^\lambda \exp(-t)t^{\alpha-1} dt \quad (17)$$

which is especially useful for negative values of  $\lambda$ . In particular,  $\gamma^*(\alpha, \lambda)$  is real for negative values of  $\lambda$ , i.e. for  $x$  greater than the critical value.

For some cases,  $H(x)$  may be shown to be monotonic and asymptotic to  $H_\infty = \sigma^2(1-f^2)/(k^2-1)$  as  $x$  becomes infinitely large. The origin for  $x$  is arbitrary; the coastline is chosen to have  $H\left\{x \begin{matrix} \text{coast} \end{matrix} \right\}$  equal to zero.

For computations, a series valid for all  $|z| < \infty$  is given by Abramowitz and Stegun (1964):

$$\gamma^*(a, z) = \exp(-z) \sum_{n=0}^{\infty} z^n / \Gamma(a + n + 1) \quad (18-a)$$

$$= (\Gamma(a))^{-1} \sum_{n=0}^{\infty} (-z)^n / (a+n)n! \quad (18-b)$$

$\gamma^*(a, z)$  may also be written in terms of a Kummer function as

$$\gamma^*(a, z) = M(1, 1+a, z) / \Gamma(a+1) \exp(z) \quad (18-c)$$

Returning to dimensional variables, using  $\xi = \lambda H_{\infty}$  as the independent spatial variable, leads to a topography

$$H(\xi) = H_{\infty} \{1 - 2\Gamma(\alpha)(1+\nu)^{-1} \exp(\xi/H_{\infty}) \gamma^*(\alpha, \xi/H_{\infty})\} \quad (19-a)$$

$$= H_{\infty} \{1 - 2\alpha^{-1}(1+\nu)^{-1} M(1, 1+\alpha, \xi/H_{\infty})\} \quad (19-b)$$

where  $\nu = (f/\sigma)(k/\ell)$ ,  $\alpha = (1 + 2\nu + (k/\ell)^2)/(1+\nu)^2$  and  $H_{\infty} = (\sigma^2 - f^2)/g(k^2 - \ell^2)$

with all values, e.g.  $f, \sigma, k, \ell$ , dimensional. The trapped wave is

$$E(x) = E_0 \{(x+b) \exp[-\ell(x+a)]\} \quad (20)$$

where

$$x = \ell^{-1}(1+\nu)^{-1} - b - (1+\nu)\ell\xi/H_{\infty}(k^2 - \ell^2) \quad (21)$$

In this case, the topography is fixed by the choice of three parameters,  $H_{\infty}, \nu$  and  $\alpha$ , at least as a function of the variable  $\xi$ . The four parameters  $\sigma, k, \ell$  and  $f$  are then related by three equations

$$\nu = fk/\sigma\ell \quad (22)$$

$$\alpha = (1 + 2\nu + (k/\ell)^2)/(1+\nu)^2 \quad (23)$$

$$H_{\infty} = (\sigma^2 - f^2)/g(k^2 - \ell^2) \quad (24)$$

leaving one degree of freedom, e.g.  $\sigma$  may be specified arbitrarily and

$k$ ,  $l$  and  $f$  then determined from these three equations for a fixed topography given by specific  $H_\infty$ ,  $\nu$  and  $\alpha$ . This is not as convenient as the previous solution which had two degrees of freedom; generally  $f$  is determined by the latitude of the observed wave. If  $\sigma$  is specified here,  $f$  will be determined by the topography rather than the geography. The fact that the topography is given in terms of  $\xi$  simply means that the form of the edge wave must also be expressed in this variable, i.e.

$$E(\xi) = E_1 \left\{ (\ell^{-1}(1+\nu)^{-1} - (1+\nu)\ell\xi H_\infty^{-1}(k^2 - \ell^2)^{-1}) \right. \\ \left. * \exp(-\lambda(a-b+\ell^{-1}(1+\nu)^{-1} - (1+\nu)\ell\xi H_\infty^{-1}(k^2 - \ell^2)^{-1}) \right\} \quad (25)$$

It is of interest to note that the location of the coastline,  $H(\xi_c) = 0$ , gives  $\xi_c/H_\infty$  as a function of  $\alpha$  and  $\nu$  alone;

$$\exp(\xi_c/H_\infty) \gamma^*(\alpha, \xi_c/H_\infty) = (1+\nu)/2\Gamma(\alpha) \quad (26-a)$$

or

$$M(1, 1+\alpha, \xi_c/H_\infty) = \alpha(1+\nu)/2 \quad (26-b)$$

The slope at the coastline,  $H'_c$ , as given by

$$H'_c = dH(\xi)/d\xi \Big|_{\xi_c} = \left( \frac{H_\infty}{\xi_c} \right) \left[ \alpha - \frac{2}{1+\nu} \right] - 1 \quad (27)$$

is also a function of  $\alpha$  and  $\nu$ .

Again, for the special case of no rotation,  $f=0$ , the equations simplify; since the topography depends now on only two parameters,  $H_\infty$  and  $\alpha$ , the three parameters  $\sigma$ ,  $k$  and  $l$  still possess one degree of freedom, i.e. for a given  $H_\infty$  and  $\alpha$ , and thus a fixed topography,  $\sigma$  may still be prescribed arbitrarily and  $k$  and  $l$  determined from

$$k^2 = (\alpha-1)\sigma^2/gH_\infty(\alpha-2) \quad (28)$$

and

$$\ell^2 = k^2/(\alpha-1) = \sigma^2/gH_\infty(\alpha-2) \quad (29)$$

These equations are analogous to eqs. (8) and (9) for the first example, with  $\alpha$  playing the role which  $H_0^1$  played there; the slope at the coastline in this example,  $H_c^1$ , is a function only of  $\alpha$ .

## II: Discussion of Solution

The topography obtained as eq. (19) must be examined to determine under what conditions it would be physically acceptable. One tool to this end is the asymptotic form for large  $|\xi/H_\infty|$ , e.g. Abramowitz and Stegun (1964):

$$\begin{aligned} M(1,1+\alpha,z) &\sim -(\alpha/z)(1-(1-\alpha)/z + \dots); & z < 0 \\ &\sim \Gamma(1+\alpha)\exp(z)(z)^{-\alpha}; & z > 0 \end{aligned} \quad (30)$$

which leads to

$$\begin{aligned} H(\xi)/H_\infty &\sim 1 + 2/[(1+\nu)(\xi/H_\infty)] + \dots; & \xi/H_\infty < 0 \\ &\sim -2\Gamma(\alpha)\exp(\xi/H)(\xi/H_\infty)^{-\alpha} + \dots; & \xi/H_\infty > 0 \end{aligned} \quad (31)$$

Since  $x$  approaching  $+\infty$  defines the appropriate direction for the ocean (a decaying edge wave), the relationship of  $\xi$  to  $x$  must be used to determine the sense of  $\xi$ .

While there are a number of different cases to be discussed, it may be easiest to first consider the special case of no rotation,  $f=0$ . The parameters  $a$  and  $b$  will be taken to be zero throughout for convenience; their effect could be included without difficulty. In this case, eq. (21) reduces to

$$\ell x = 1 - (\xi/H_\infty)/(k^2/\ell^2 - 1) = 1 - (\xi/H_\infty)/(\alpha-2). \quad (32)$$

For  $\alpha > 2$ ,  $k^2$  must be greater than  $\ell^2$  and  $H_\infty$  is positive; the sense of  $\xi$  will then be opposite to that for  $x$ . For  $1 < \alpha < 2$ ,  $k^2$  must be less than  $\ell^2$  and  $H_\infty$

is negative (although  $H_\infty$  is considered as an asymptote in some situations, in reality it is some convenient combination of symbols which measures a horizontal length scale); in this case, the sense of  $\xi$  will be opposite to that for  $x$  but the sense of  $(\xi/H_\infty)$  will be the same as that for  $x$ . Thus for  $\alpha > 2$ , eq. (31) yields a positive depth,  $H(\xi)$ , which approaches  $H_\infty$  from "below" (shallower depths) as  $\xi$  approaches  $-\infty$ . For  $1 < \alpha < 2$ , eq. (31) yields a positive depth,  $H(\xi)$ , which increases exponentially as  $\xi$  approaches  $-\infty$ .

There are no solutions for  $\alpha < 1$  which have real  $\lambda$  and  $k$ . For  $\alpha = 2$ ,  $k^2$  equals  $\lambda^2$  and the solution for  $H$  must be redone, leading to  $H(x) = (\sigma^2/g)(\lambda x^3/3 - x^2)$ .

Figure 1 sketches the nondimensional depth,  $\bar{H}$ , and edge wave height,  $\bar{E}$ , as a function of  $\bar{\xi}$  for  $\alpha = 4.0$ . This figure is the same for all frequencies;

however, when a dimensional horizontal length scale,  $\xi$ , is used, the dimensional depth,  $H$ , is unchanged but the edge wave height,  $E$ , depends on frequency. Figure 2 indicates this for  $H_\infty = 200$  meters,  $\alpha = 4.0$  with  $E$  plotted on a relative scale for two periods, 48 sec. and 162 sec.

Finally, Figure 3 shows the situation when  $H_\infty = -200$  meters and  $\alpha = 1.5$  for a wave period of 48 sec. This topography is concave downward, i.e. is "dual" to the previous topography. Other wave periods yield different curves for  $E(\xi)$  on the same base topography,  $H(\xi)$ .

The inclusion of rotation introduces some new effects as well as modifications of the non-rotating solutions. Again, the two families ( $H_\infty > 0$  and  $H_\infty < 0$ ) are considered separately. Before proceeding, it is useful to provide some relationships between the parameters, e.g.:

$$k^2/\lambda^2 = v^2 + (\alpha-1)(1+v)^2 = \alpha(1+v)^2 - 1 - 2v$$

$$\sigma^2/\xi^2 = gH_\infty[\alpha(1+v)^2 - 1 - 2v][\alpha(1+v)^2 - 2 - 2v]/(\alpha-1)(1+v)^2$$

$$(k^2/\lambda^2)(1-f^2/\sigma^2) = (\alpha-1)(1+v)^2$$

$$\lambda x = (1+v)^{-1} - (\xi/H_\infty)/[\alpha(1+v)-2]$$

Consider first the concave upward family corresponding to positive  $H_\infty$ . For high frequency waves,  $\sigma > f$ , this implies  $k^2 > \lambda^2$  which can take on two forms. The case  $0 < \lambda < k$  requires  $\nu > 0$  and  $\alpha > 2/(1+\nu)$  and yields  $H(\xi)$  approaching  $H_\infty$  from below as  $\xi$  approaches  $-\infty$  (i.e.  $x$  approaches  $+\infty$ ). The case  $k < -\lambda < 0$  requires  $-1 < \nu < 0$  and  $\alpha > 2/(1+\nu)$ ;  $\alpha > 1$ . Again  $H(\xi)$  approaches  $H_\infty$  from below as  $\xi(x)$  approaches  $-(+)\infty$ .

Next consider low frequency waves,  $\sigma < f$ . This implies  $k^2 < \lambda^2$ . For  $0 < k < \lambda$ ,  $\nu > 0$  and  $\alpha < 1$ ,  $(1+2\nu)/(1+\nu)^2 < \alpha < 2/(1+\nu)$  to have real  $k$  and  $\lambda$ . Here  $x, \xi$  both approach  $+\infty$  and the topography is negative and thus unacceptable. Finally, take  $-\lambda < k < 0$  which requires  $\nu < 0$  and  $\alpha < 1$ ,  $(1+2\nu)/(1+\nu)^2 < \alpha < 2/(1+\nu)$ . For  $-1 < \nu < 0$ ,  $\xi$  approaches  $+\infty$  as  $x$  approaches  $+\infty$  and a negative topography is again obtained. However for  $\nu < -1$ ,  $\alpha < 0$  and  $\xi$  approaches  $-\infty$  as  $x$  approaches  $+\infty$ . This provides a positive topography, but one which approaches  $H_\infty$  from above. At  $\xi = 0$ , however,  $M(1, 1+\alpha, 0) = 1$  and  $H(\xi=0) = H_\infty(1 - 2/\alpha(1+\nu))$ ; while as  $\xi$  approaches  $+\infty$ ,  $H(\xi)$  is negative. Then the topography for this case develops a trough below  $H_\infty$  before approaching  $H_\infty$  from above (deeper values).

These results are described in Figures 4, 5 and 6. For  $H_\infty = 200$  meters,  $\alpha = 2.5$ ,  $\nu = 0.1$ , Figure 4 shows a concave upward topography similar to that of Figure 2 as does Figure 5 for  $H_\infty = 200$  meters,  $\alpha = 2.5$  and  $\nu = -0.1$ . Figure 6 on the other hand, for  $H_\infty = 200$  meters,  $\alpha = -0.77$  and  $\nu = -4.0$  indicates a new kind of topography, i.e. one with a trough. These figures are shown in dimensional variables for a wave period of 48 sec, but could be readily scaled to other periods by "stretching" the E curves. Figures 4 and 5 contain no apparent differences from the results shown in Figure 1; in all of these, the edge wave decays over a reasonable portion of the topography and

has a node near the coastline. Figure 6, however, is quite different in that the topography is not monotonic, but contains a trough. The edge wave is concentrated along the coastal side of the trough, but has non-negligible values on the other side as well. It would be of interest to make a parametric study of this case in particular since analytical solutions for non-monotonic topographies do not appear to currently exist in the literature. This is a low frequency wave traveling "down" the coast ( $k < 0$ ) and may be related to the trench wave of Mysak, LeBlond and Emery (1979); they can not be the same since this is a surface gravity wave and the results of Mysak, et al. refer to non-divergent (rigid lid) trapped waves.

Next, the family of convex upward topographies corresponding to negative  $H_\infty$  must be examined. High frequency waves,  $\sigma > f$ , require  $k^2 < \lambda^2$ . The case  $0 < k < \lambda$  requires  $0 < \nu < 1$  and to have real  $\sigma$  and  $\lambda$  with  $\alpha > 1$  requires  $(1+2\nu)/(1+\nu)^2 < \alpha < 2/(1+\nu)$ . As  $x$  approaches  $+\infty$ ,  $(\xi/H_\infty)$  approaches  $+\infty$  and  $H(\xi)/H_\infty$  is negative implying  $H(\xi)$  is positive and increases exponentially with  $(\xi/H_\infty)$  as  $\xi$  approaches  $-\infty$ . The case  $-\lambda < k < 0$  requires  $-1 < \nu < 0$ ,  $(1+2\nu)/(1+\nu)^2 < \alpha < 2/(1+\nu)$  and  $\alpha > 1$ . As  $x$  approaches  $+\infty$ ,  $(\xi/H_\infty)$  approaches  $+\infty$  and again  $H(\xi)$  is positive and increases exponentially with  $(\xi/H_\infty)$  as  $\xi$  approaches  $-\infty$ . In both of these cases,  $H(\xi=0)$  is negative.

The low frequency solutions,  $\sigma < f$ , require  $k^2 > \lambda^2$ . For  $0 < \lambda < k$ ,  $\nu > 1$  and  $2/(1+\nu) < \alpha < 1$ . This leads to  $(\xi/H_\infty)$  approaching  $-\infty$  as  $x$  approaches  $+\infty$  and thus to an unacceptable negative topography,  $H(\xi)$ . Finally, the case  $k < -\lambda < 0$  requires  $\nu < -1$  and  $2/(1+\nu) < \alpha < 1$ .  $(\xi/H_\infty)$  then approaches  $+\infty$  as  $x$  approaches  $+\infty$  and  $H(\xi)/H_\infty$  approaches  $\text{sgn}[\Gamma(\alpha)]^\infty$ . Then  $H(\xi)$  will be positive (and exponentially increasing with  $-\xi$ ) if  $\Gamma(\alpha) < 0$  which occurs for  $-1 < \alpha < 0$ ,  $-3 < \alpha < -2$ , etc. To keep  $1 + \alpha > 0$  (and thus  $M(1,1+\alpha,z)$  monotonic as  $z$

approaches  $+\infty$ ), only the case  $-1 < \alpha < 0$  will be chosen. Figures 7 and 8 indicate results, in nondimensional form, for  $H_{\infty} = -200$  meters,  $\alpha = 1.1$  and  $\nu = +0.1$  and  $-0.1$  respectively. These results indicate an edge wave which persists over a significant portion of the topography and contains a node near the coastline. Finally, Figure 9 shows results for  $H_{\infty} = -200$  meters,  $\alpha = -0.1$  and  $\nu = -1.1$ , i.e. the last case. Here, the node is far from the coastline and the edge wave is, where it has non-negligible values, negative (i.e. out of phase compared to the other figures) over most of the topography.

### III: Conclusion

An inverse approach to long edge wave trapping has resulted in a new analytical solution as well as an alternative approach to the well-known Ball ( $n=0$ ) solution. These new solutions appear to form only the lowest (or, in view of the single node, possibly the first) modes of what are in reality a spectrum of edge wave modes; this may be seen by analogy to the Ball solution, and a reasonable next step in this research would be to extend these solutions to higher modes.

An equally reasonable next step would be to relate these mathematical solutions to physical observations to determine any value which they might have beyond their novelty as an "inverse" view of the world.

Both of these tasks are left for future study so that the above results may at this time at least be presented for discussion.

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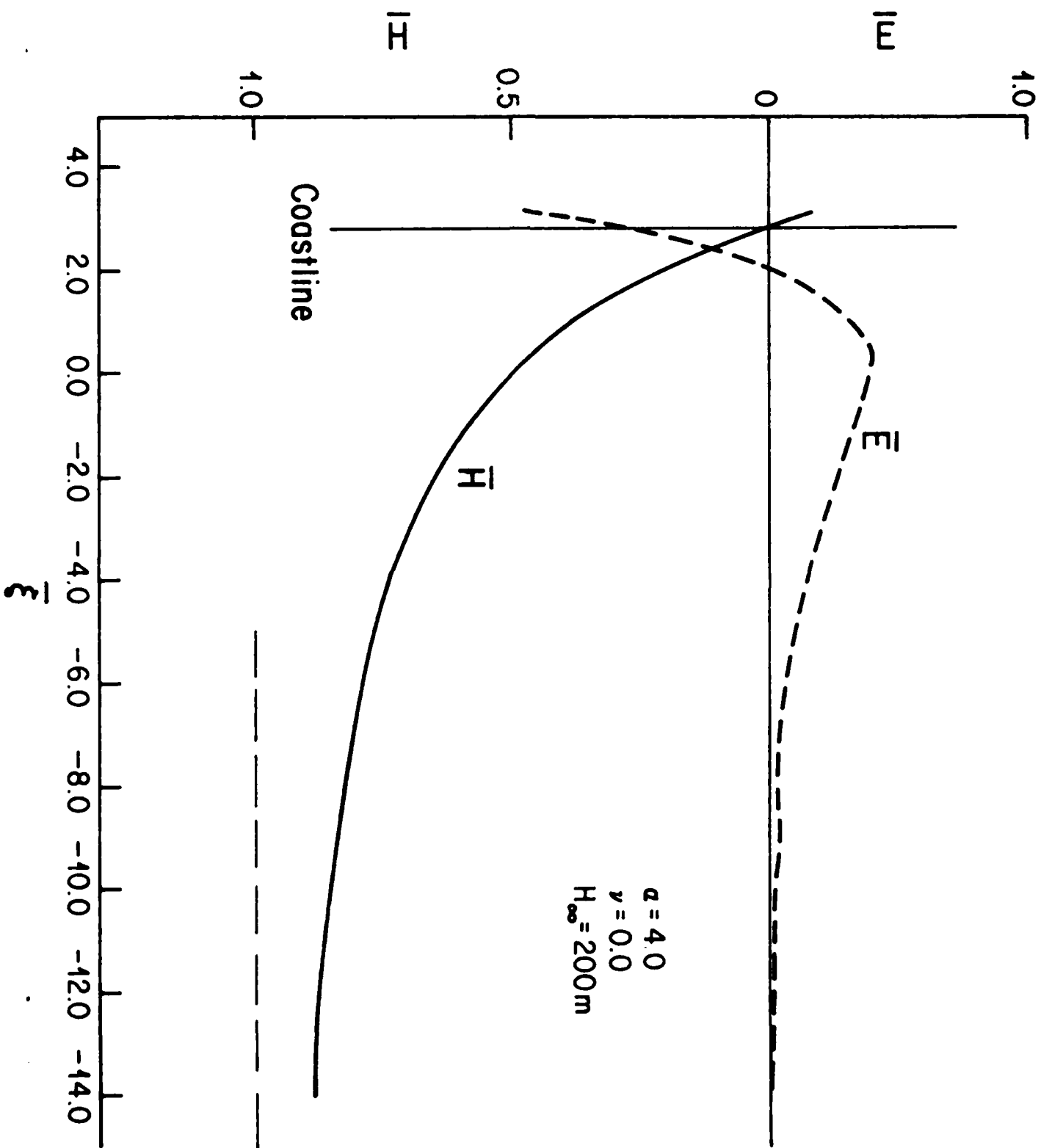
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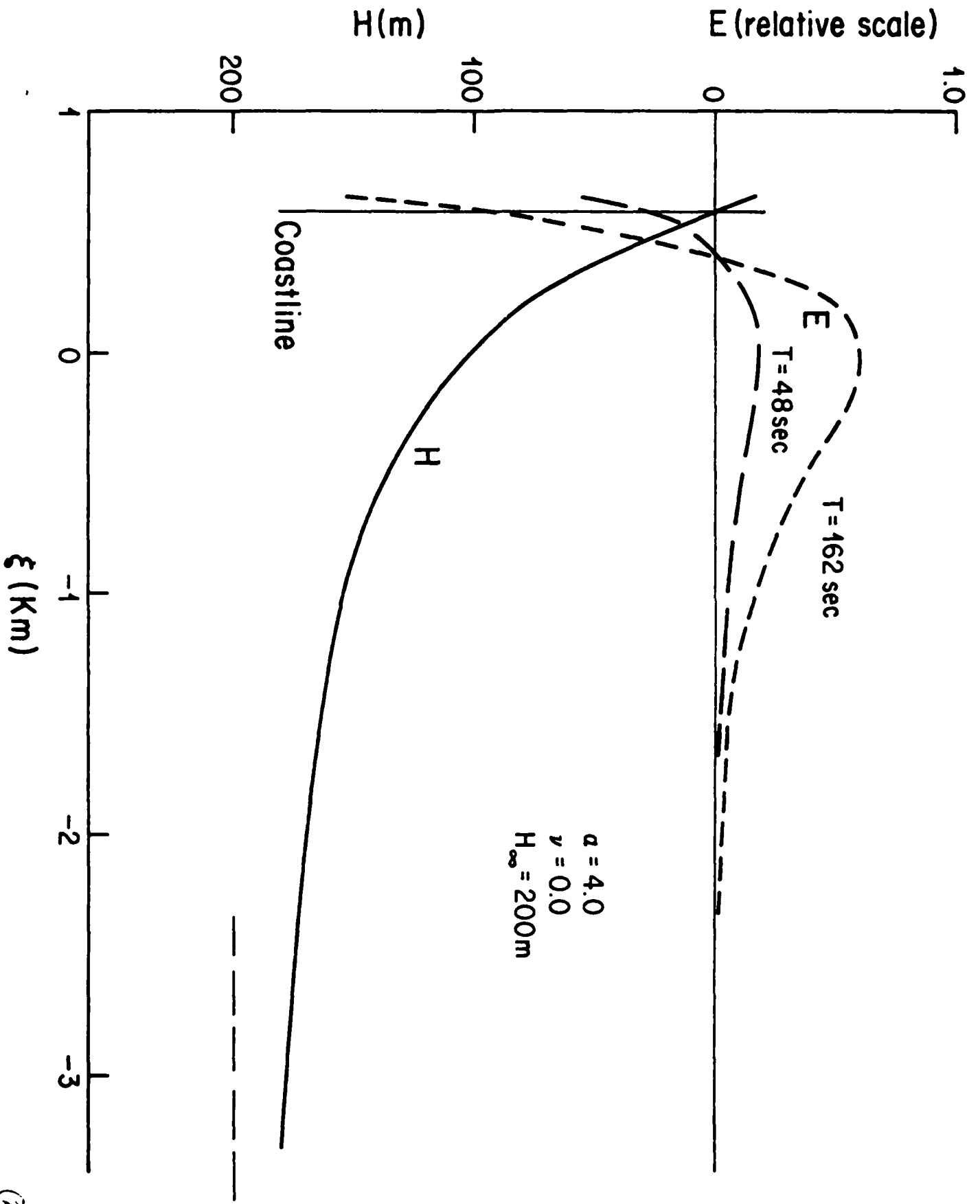
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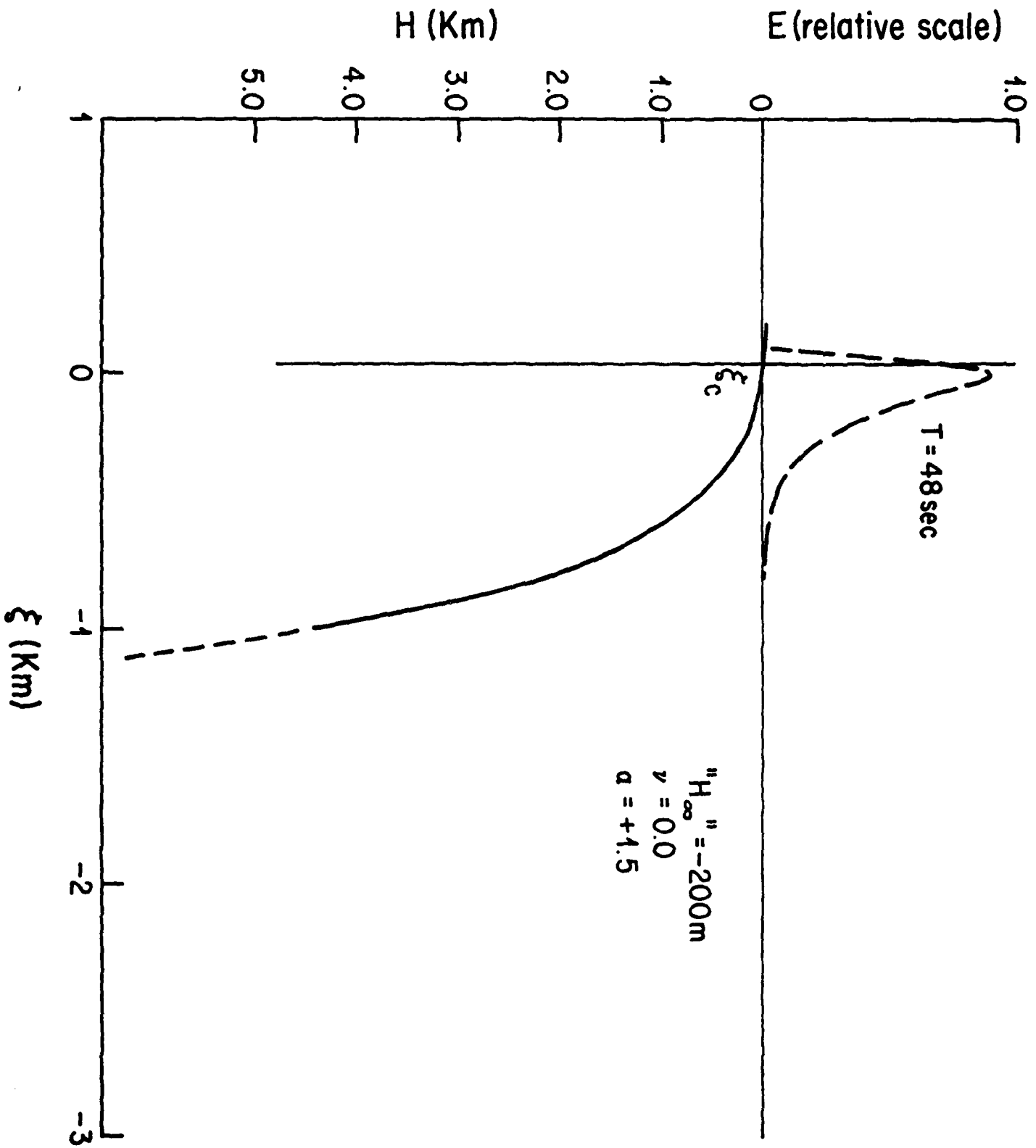
### Figures

- 1: Nondimensional depth  $\bar{H}$  (——) and wave height  $\bar{E}$  (---) as functions of  $\bar{\xi}$  for  $\nu = 0.0$ ,  $\alpha = 4.0$  and  $H_{\infty} = 200$  meters.
- 2: Dimensional depth  $H$  (——) and wave height  $E$  (---) as functions of  $\xi$  for  $\nu = 0.0$ ,  $\alpha = 4.0$ ,  $H_{\infty} = 200$  meters and wave periods of 48 and 162 seconds. Note that  $H$  is independent of period although  $E$  does change (on a relative scale).
- 3: Dimensional depth  $H$  (——) and wave height  $E$  (---) as functions of  $\xi$  for  $\nu = 0.0$ ,  $\alpha = 1.5$  and  $H_{\infty} = -200$  meters with a wave period of 48 seconds. This topography is exponentially increasing with distance from shore, but could be terminated at a flat bottom beyond  $\xi = -1.5$  km with no real change. Other periods would yield the same  $H$  but differing  $E$ .
- 4: Dimensional depth  $H$  (——) and wave height  $E$  (---) for the rotational case with  $\nu = 0.1$ ,  $\alpha = 2.5$ ,  $H_{\infty} = 200$  meters and a period of 48 seconds.  $k = 0.44 \times 10^{-4} \text{ cm}^{-1}$ ,  $\lambda = 0.32 \times 10^{-4} \text{ cm}^{-1}$ .
- 5: Dimensional depth  $H$  (——) and wave height  $E$  (---) for the rotational case with  $\nu = -0.1$ ,  $\alpha = 2.5$ ,  $H_{\infty} = 200$  meters and a period of 48 seconds. Here  $k = -0.68 \times 10^{-4} \text{ cm}^{-1}$  and  $\lambda = 0.62 \times 10^{-4} \text{ cm}^{-1}$ .

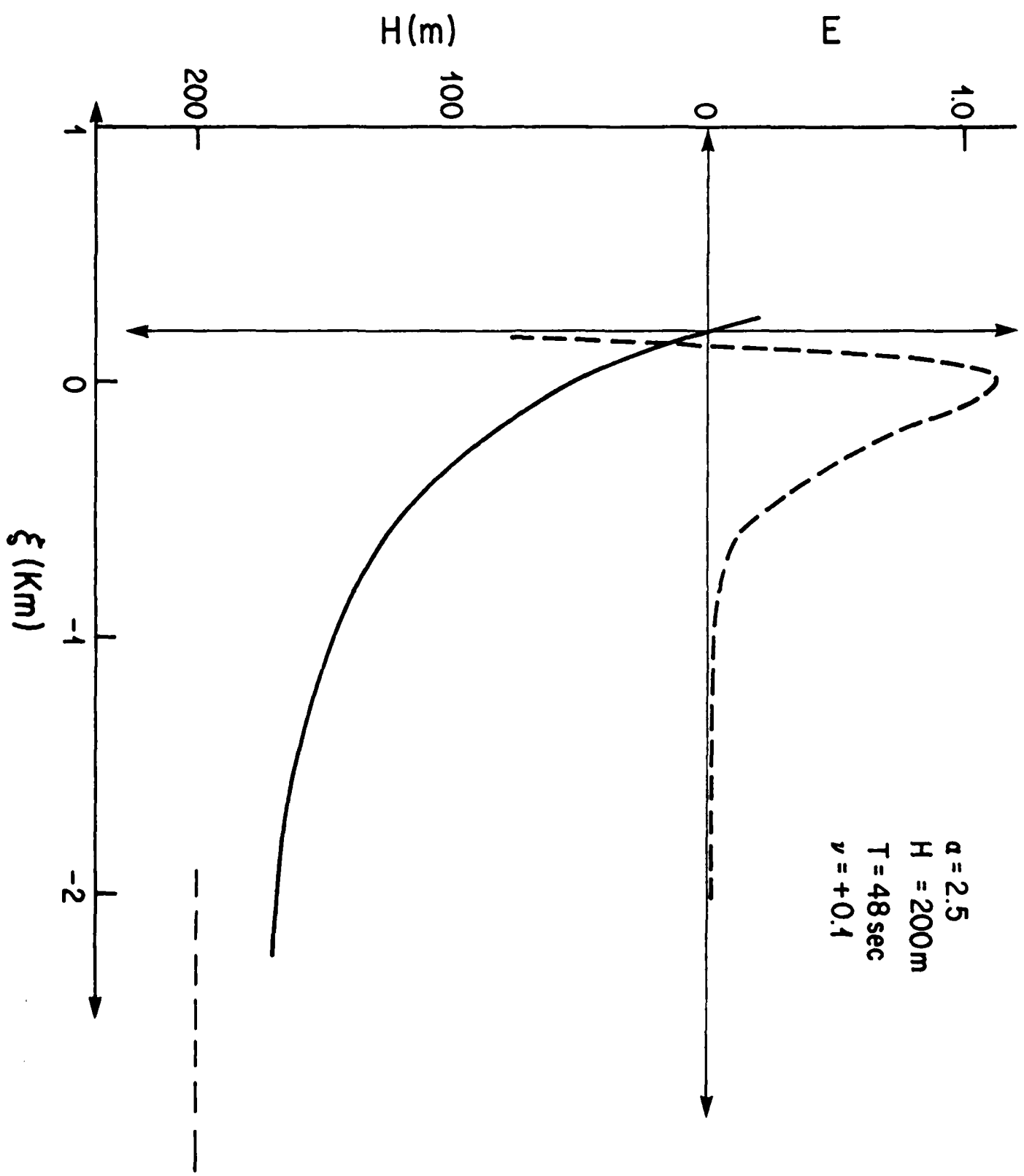
- 6: Dimensional depth  $H$  (————) and wave height  $E$  (---) for  $\nu = -4.0$ ,  $\alpha = -0.77$ ,  $H_\infty = 200$  meters and a period of 48 seconds. There is the "trough" topography: note that  $E$  is still significant past the turning point of the trough, indicating that this topography is "felt" past this point. Note that  $k$  here is negative, i.e. the trapped wave progresses in the negative ( $y$ ) coastline direction with no positive  $k$  counterpart as was the case for Figures 4 and 5.
- 7: Nondimensional depth  $\bar{H}$  (————) and wave height  $\bar{E}$  (---) for  $\nu = 0.1$ ,  $\alpha = 1.1$  and  $H_\infty = -200$  meters. Again, the same curves hold for all wave periods; only when  $\bar{\xi}$  is dimensionalized will  $E$  change with wave period.  $k$  here is positive.
- 8: Nondimensional depth  $\bar{H}$  (————) and wave height  $\bar{E}$  (---) for  $\nu = -0.1$ ,  $\alpha = 1.1$  and  $H_\infty = -200$  meters. This figure repeats Figure 7 with the sign of  $\nu$  reversed. The results are quite similar although here  $k$  will be negative, i.e. a counterpart to Figure 7.
- 9: Nondimensional  $\bar{H}$  (————) and  $\bar{E}$  (---) for  $\nu = -1.1$ ,  $\alpha = -0.1$  and  $H_\infty = -200$  meters. Again,  $k$  is negative with no counterpart for positive  $k$ . The edge wave node is far from the coastline and occurs at extremely small (relative) values of wave height giving the appearance of a monotonic wave height.

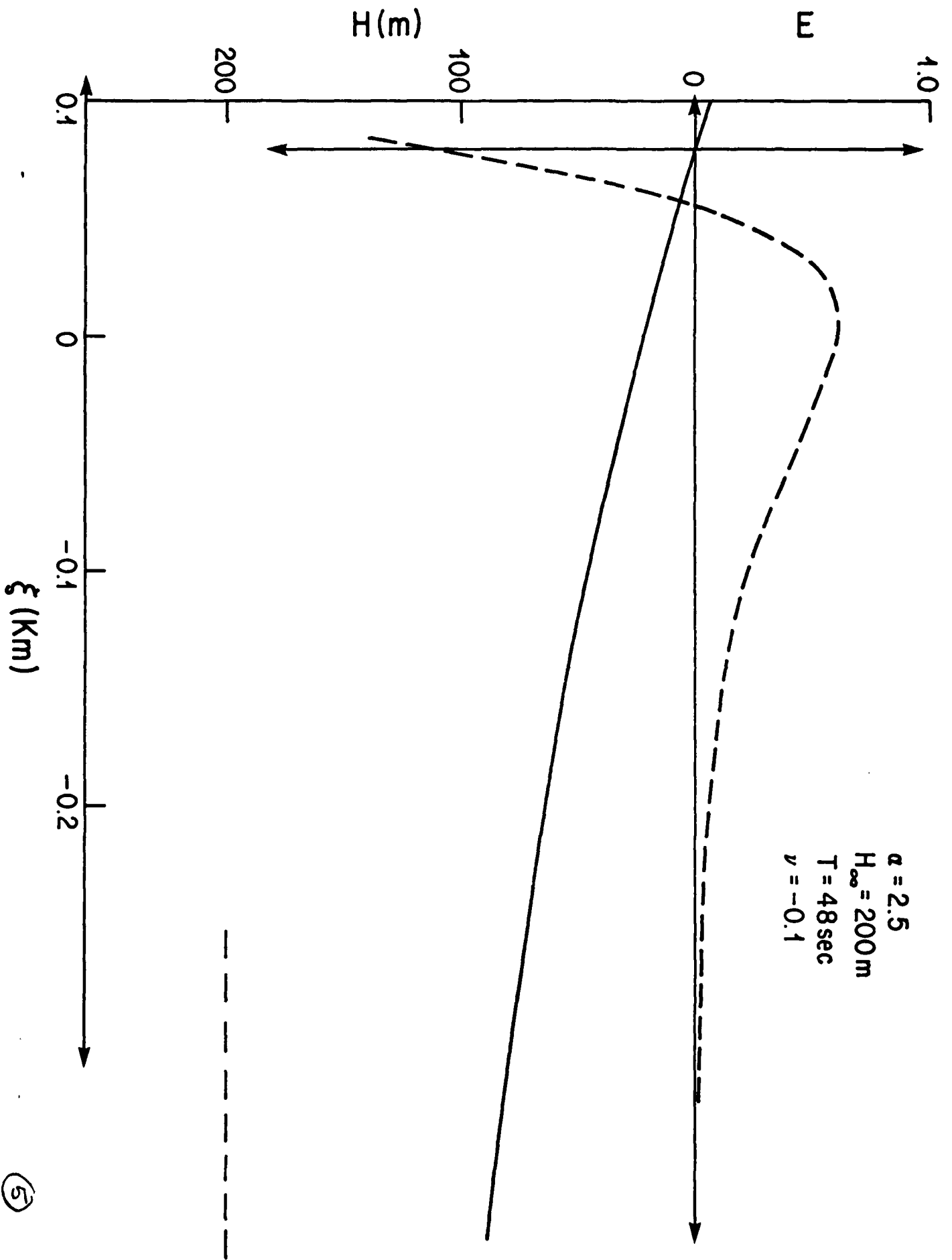


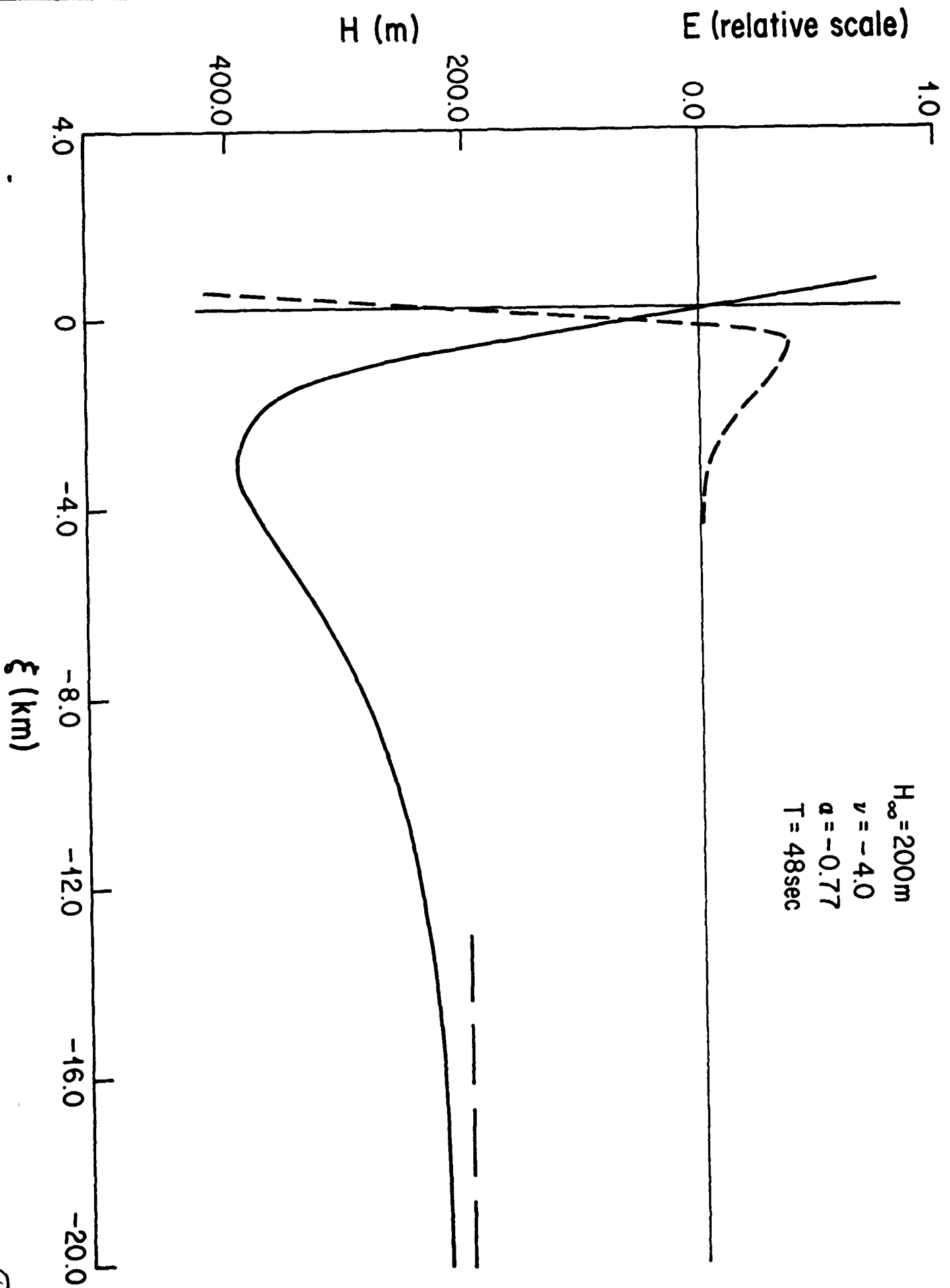


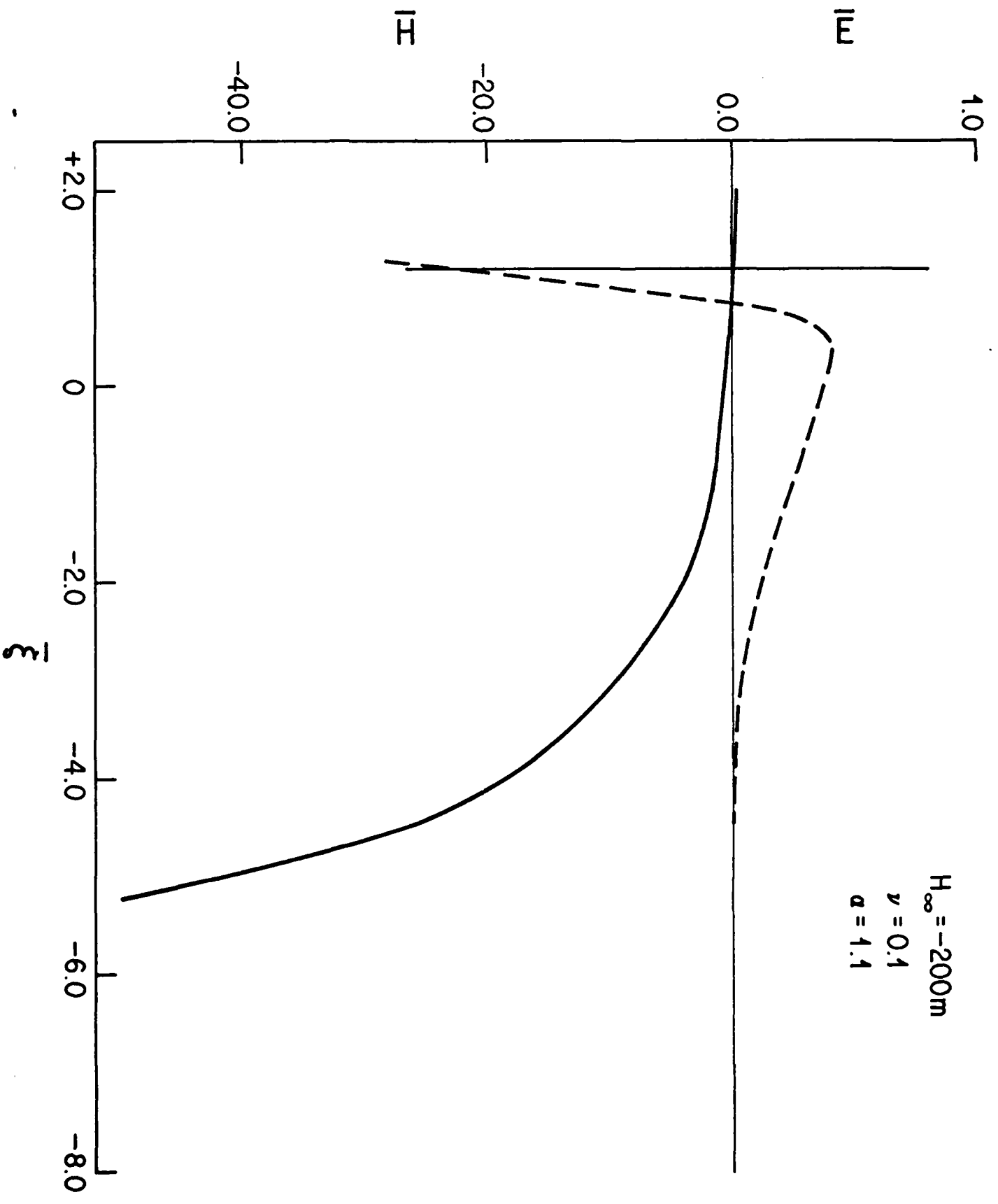


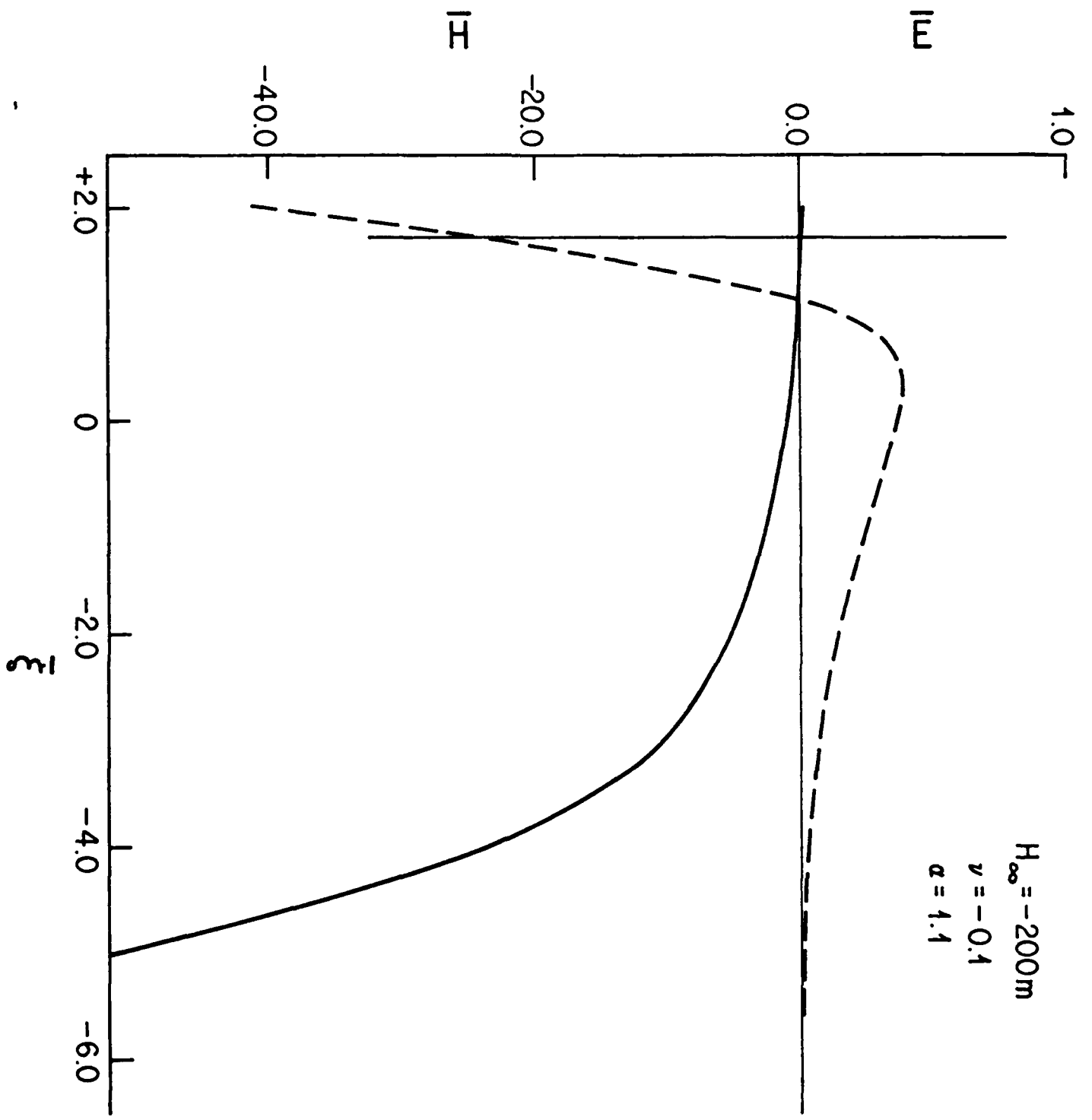
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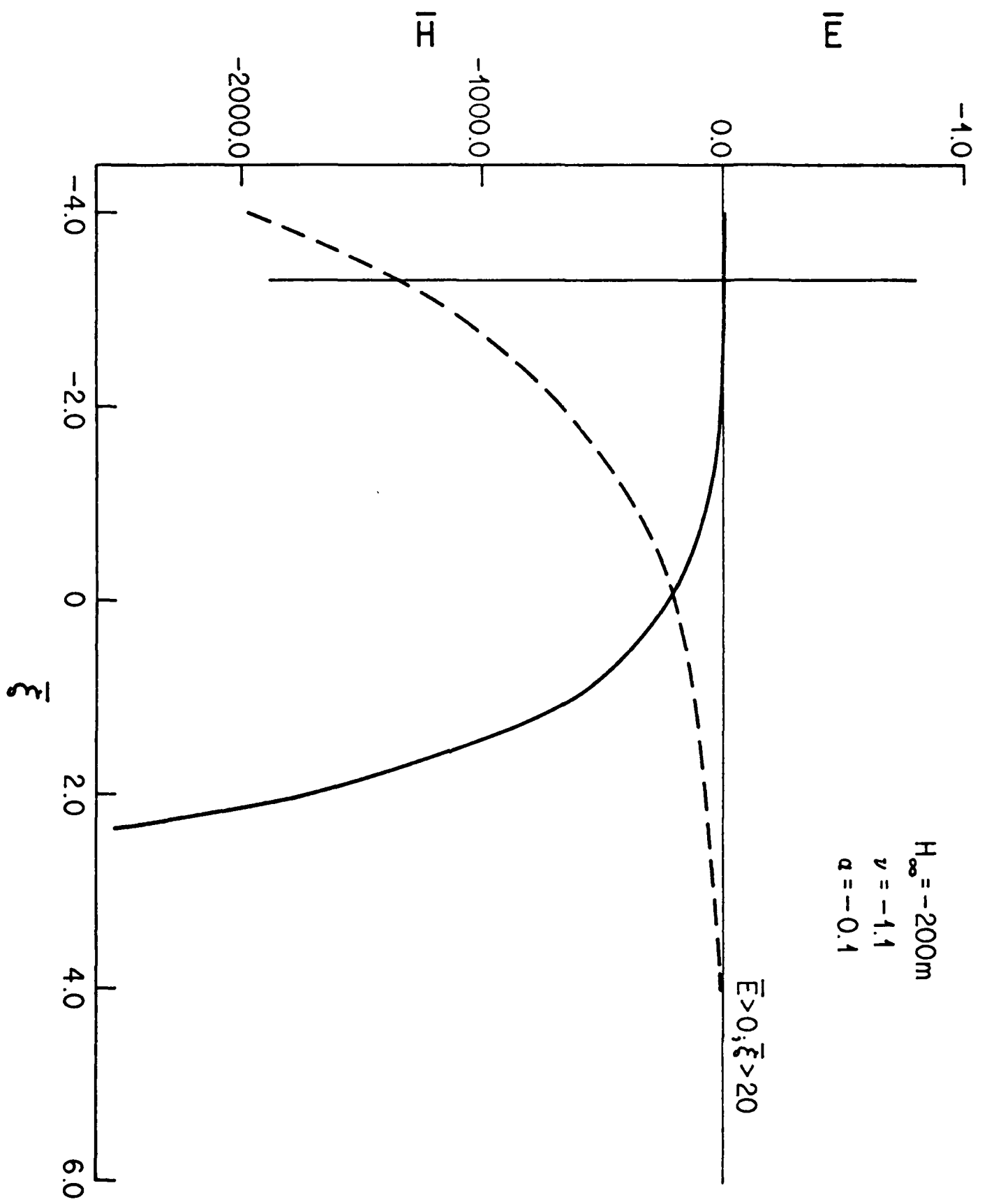












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