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Force-Frequency Compensation Applied to Four-Point Mounting of *AT*-Cut Resonators

ARTHUR BALLATO, SENIOR MEMBER, IEEE

Abstract—For precision applications, where ruggedness is required, it is necessary to mount *AT*-cut resonators at four points. This note considers mounts consisting of pairs of diametric edge attachments. A locus is found for the angles of attachment to circular plates such that radially directed edge forces are compensated. One locus point is found to be particularly frequency insensitive to mounting errors and defines the optimal setting for the four-point mount.

INTRODUCTION

The constraints imposed by stringent performance requirements for digital communication, position location, and sensor systems provided the impetus that produced a new ceramic resonator package [1], [2]. This enclosure is a high-alumina flat-pack that uses special sealing techniques. It is adaptable for various roles, such as 1) high *g*-force and fast warmup and 2) maximum frequency precision with minimum long-term aging.

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The author is with the U.S. Army Electronics Technology and Devices Laboratory, Fort Monmouth, NJ 07703.

With the ceramic flat-pack design and the necessity of using four mounting supports for high-shock applications, a significant contribution to the frequency error arises from forces transmitted from the alumina package to the quartz. Frequency perturbation of quartz vibrators by external forces has received both experimental [3]-[9] and theoretical [10]-[15] attention. It is known that the frequency changes are due to both the static deformation of the crystal lattice and a nonlinear elastic constant effect [11]-[14]. In this note certain results involving the force-frequency effect are applied to the question of mounting an *AT*-cut circular resonator on four points in a manner adaptable to use in the ceramic flat-pack. It is found that an optimum orientation of the mounting supports exists with respect to the crystal axes and that misalignments about the optimum produce minimal frequency shifts.

The solution given here is constructed from the following features: 1) an analytic form for the force-frequency function that is new, simple, and accurate; 2) use of diametric force-pairs, for which superposition holds; and 3) incorporation of the thermoelastic effect.

FORCE-FREQUENCY EFFECT

Fig. 1 defines the geometry under consideration. The crystallographic X axis is the datum from which angle ψ is measured; force F_1 acts along the crystal plate diameter with azimuth ψ, while the diametric force F_2 is applied at azimuth $\psi + \gamma$. The points of application of F_1 and F_2 represent the positions of the mounting supports. The problem is to find the proper values of angles ψ and γ to ensure a minimum sensitivity of frequency to applied force.

Although it is not necessary for the mounts to be diametrically paired as shown in Fig. 1, virtually all of the experimental data, as well as most theoretical results, exist in this form. In addition, the force-frequency effect due to opposed forces acting across the crystal diameter is found to be superposable [6]. The separate contributions to the frequency shift under loads F_1 and F_2 thus add linearly to produce the overall frequency excursion of F_1 and F_2 acting together. This fact allows a simple solution to what is otherwise a very difficult problem.

The force-frequency effect produced in circular crystal plates acted upon by diametric forces F at angle ψ is characterized by Ratajski [7] by means of a force-frequency coefficient $K_f(\psi)$. For a plate of diameter D, thickness $2h$, nominal frequency f_0, and frequency constant N, $K_f(\psi)$ is defined as

$$K_f(\psi) = \frac{\Delta f}{f_0} \cdot \frac{2h \cdot D}{F} \cdot \frac{1}{N}. \tag{1}$$

In (1), Δf is the frequency change brought about by application of compressional force F. For the *AT*-cut of quartz,

$$N = 2h \cdot f_0 = 1.660 \text{ MHz-mm}. \tag{2}$$

$K_f(\psi)$ in (1) can be interpreted as being a proportionality factor relating the fractional frequency change $\Delta f/f_0$ to the average stress acting across the crystal diameter: $F/(2h \cdot D)$. The azimuthal dependence arises from the anisotropic nature of the crystal lattice.

To determine a compensated mounting configuration, the

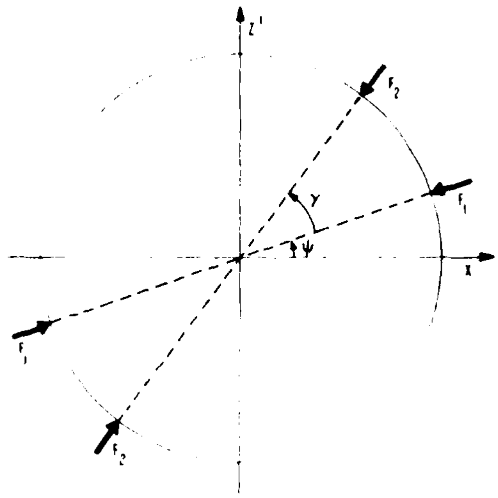

Fig. 1. Definition of mounting angles with respect to crystal axes.

variation of $K_f(\psi)$ with ψ must first be accurately determined. Although the theoretical curves have become increasingly better fits to the experimental data with time, at present the greatest accuracy in characterizing $K_f(\psi)$ versus ψ is to be had by using the experimental results.

To this end, the data for the *AT*-cut given by Ratajski [7], representing a compilation from a number of sources, were subjected to a least-squares fit. From symmetry considerations the function $K_f(\psi)$ must satisfy the relations:

$$K_f(-\psi) = K_f(+\psi) \tag{3}$$

and

$$K_f(\pi/2 - \psi) = K_f(\pi/2 + \psi). \tag{4}$$

With due regard for these symmetries the functional form adopted for $K_f(\psi)$ is

$$K_f(\psi) = \sum_{n=0}^{N} A_n \cos^{2n} \psi. \tag{5}$$

A five-term least-squares fit gives the coefficients listed in Table I. The resulting curve is shown in Fig. 2. A comparison between the latest theoretical results and the fit to the experimental data using (5) is given in Table II.

Four-Point Mounts

The anisotropy of quartz with respect to thermal expansion will produce unequal forces on the mounting supports because the ceramic flat-pack holder is isotropic, so differential strains will be azimuth-dependent. Since the Δf produced by a force is proportional to both the force and to the value of $K_f(\psi)$ at the azimuth of the force, and since it is desired that the algebraic sum of both frequency shifts equals zero, we have

$$K_f(\psi) + \rho(\psi, \gamma) \cdot K_f(\psi + \gamma) = 0. \tag{6}$$

In (6)

$$\rho(\psi, \gamma) = F_2/F_1 \tag{7}$$

is the force ratio. Because the forces depend on the differential expansion coefficients, for a fixed temperature change, the

TABLE I
Coefficients of (5) Determined by Least Squares Fit to Data

A_n	Value (10^{-15} m·s/N)
A_0	-9.21
A_1	31.82
A_2	64.51
A_3	-95.15
A_4	32.27

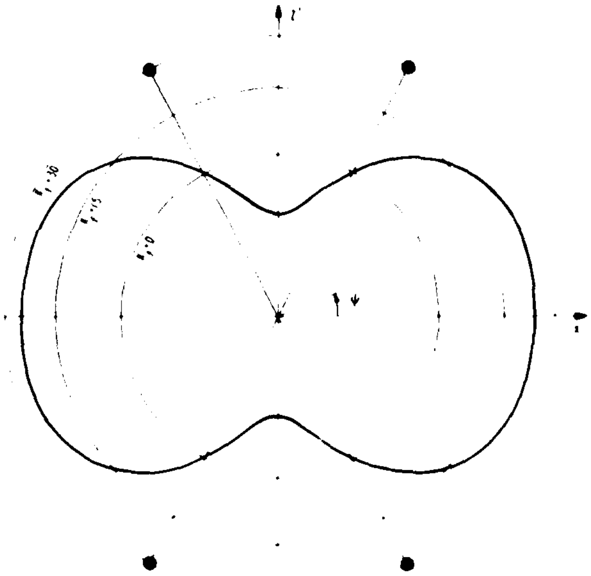

Fig. 2. Polar plot of $K_f(\psi)$ versus angle ψ. Units of $K_f(\psi)$ are 10^{-15} m · s/N.

TABLE II
Comparison of $K_f(\psi)$ Values

Condition on $K_f(\psi)$	Ratajski [7] ψ	Ratajski [7] $K_f(\psi)$	Lerhisse, et al. [5] ψ	Lerhisse, et al. [5] $K_f(\psi)$
maximum	0	24.7	0	24.5
zero	62.0	0	64.7	0
minimum	90	-9.2	90	-11.5

(ψ in degrees, $K_f(\psi)$ in 10^{-15} m·s/N)

force ratio is given by

$$\rho(\psi, \gamma) = (\alpha''_{11}(\psi + \gamma) - \alpha_0)/(\alpha''_{11}(\psi) - \alpha_0) \tag{8}$$

where

$$\alpha''_{11}(\psi) = \alpha_{11}(\cos^2\psi + \sin^2\psi \sin^2\theta) + \alpha_{33}(\sin^2\psi \cos^2\theta) \tag{9}$$

is the thermal expansion coefficient of the plate in the radial direction at azimuth ψ, expressed in terms of the unrotated constants α_{ij} and the rotation angle $\theta = +35°25'$ for *AT*-cut quartz. The quantity α_0 in (8) is the thermal expansion coefficient of alumina.

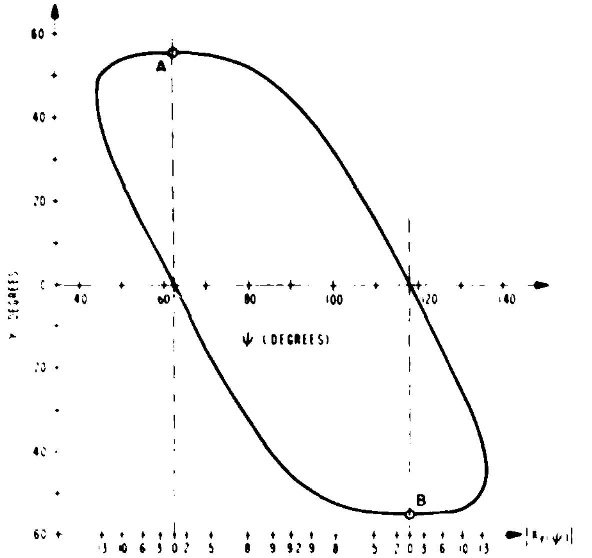

Fig. 3. Angle ψ versus angle γ on locus of points having zero frequency shift.

Insertion of (9) into (8) and the result into (6) yields the desired relation between ψ and γ for which the total frequency change is zero. The result of solving (6) is shown in Fig. 3. The locations where the curve crosses the $\gamma = 0°$ axis represent the case where the four-point mount degenerates into a two-point mount. About those locations the mounting points are close together and the sensitivity to mounting misalignment is large. At the points marked "A" and "B," which are equivalent because of the two-fold symmetry, the individual contributions to Δf are each zero, and the mounting locations are symmetrically disposed about the X and Z' axes. Furthermore, the sensitivity to errors in mounting is minimized, and the location is independent of α_0. This defines the optimum mounting configuration for this problem. Angles ψ and γ are related by

$$2\psi + \gamma = \pi \tag{10}$$

for the optimum configuration. The variation of $K_f(\psi)$ in the vicinity of point "A" with the angle constraint of (10) is given in Table III. From Fig. 3 it is seen that when

- $\gamma = 56°$ (11)

substantial errors in ψ may be tolerated and the frequency change will still remain quite small.

Ceramic flat-pack enclosures are not subject to large manufacturing errors, (11) is, accordingly, not difficult to maintain. By reasonable attention to keep

- $\psi \sim 62°$ (12)

the force-frequency effect contribution to the resonator frequency error may be readily minimized. The resulting angular spread between the four mounting pads should be sufficient to permit this design to be used in high-shock applications.

In Fig. 2, diameters are drawn through the locations $K_f(\psi) = 0$, defining the positions of the optimal mounting configuration. Black circles simulate the positions of fixation of the quartz plate.

TABLE III
ψ AND γ IN THE VICINITY OF $K_f = 0$

ψ	γ	$K_f(\psi)$
61.98	56.04	-1.15
61.99	56.02	-0.46
62.00	56.00	0.24
62.01	55.98	0.93
62.02	55.96	1.62

(ψ and γ in degrees, $K_f(\psi)$ in 10^{-17} m·s/N)

Considerations similar to those given in this note apply to the design of mounting supports for SC-cut [9], [15] crystals. In the case of doubly rotated cuts in general, the $K_f(\psi)$ versus ψ curve does not exhibit symmetry, and the resulting sensitivities to errors in ψ and γ are increased.

CONCLUSIONS

The four-point mounting problem for circular AT-cut quartz resonator plates subjected to thermally induced mounting forces has been solved. A locus of acceptable configurations has been determined, and from this locus the position of minimum sensitivity to mounting errors has been found.

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