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The Velocity Inversion Problem: Present Status, New Directions

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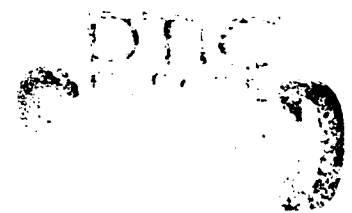
# ABSTRACT

The research program in seismic exploration in progress in the Mathematics Department of the University of Denver is described. This work is identified here by the term <u>velocity</u> <u>inversion</u>. The mathematical formulations employed by this group are outlined and results of computer implementation are depicted. Ongoing research is also presented.

## 1. Introduction

The purpose of this paper is to describe the research efforts of a group of people in the Mathematics Department at the University of Denver and related work, elsewhere. The researchers in the group include the authors, Professor F. G. Hagin of the Mathematics Department, former graduate students, Drs. R. D. Mager, J. A. Armstrong and S. H. Gray, present graduate student, M. Lahlou and programmer, W. S. Grady. The objective of this research is to study the problem of the mapping of the interior of the earth as an _inverse problem_ and to develop methods which yield increasingly more accurate solutions of that inverse problem. This presentation was originally prepared as a lecture which was presented at the Seismic Inversion Workshop at the 50th Annual International Meeting of the SEG, Houston, November 20, 1980.

The methods we use are classical, employing perturbation techniques, transform methods, asymptotic and numerical analysis to arrive at computer algorithms which produce a mapping of the interior of the earth.

We shall describe, here, what we mean by _inversion_ in contrast to _migration_. Our mathematical development and subsequent computer implementation will be presented, followed by a brief discussion of present and ongoing research.

## 2. The Inverse Problem of Seismic Exploration

The mapping of the interior of the earth from observations on the surface of the earth is an _inverse problem_. For this type of inverse problem, the propagation of signals - acoustic, elastic or electromagnetic - into the earth is modelled by the appropriate equation or system of equations in which one or more functions characterizing the interior of the earth (soundspeed, elastic or electromagnetic coefficients) are left free. One or more signals consistent with the model are introduced at or near the surface of the earth in a region of interest. The 'irregularities' of the interior of the earth produce a 'response' to those signals. Observations of those responses are recorded.

The objective of the inverse problem is to determine the free coefficients in the modelling equations from knowledge of the input signal(s) and the response(s) and thereby 'map' the interior of the earth.

This type of inverse problem is known by the fuller title, _inverse scattering problem_. This contrasts with the more familiar _direct scattering problem_ in which the parameters of the equation(s) are known and the objective is to determine the response to the given signal

The mapping of the interior of the earth from observations of the response to a single acoustic source is an inverse problem for the acoustic wave equation. Here, implicit in the model - acoustic wave equation - is a definition of the word 'mapping'. At most, one could hope to

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characterize the interior of the earth from this model in terms of its density and compressibility or its density and acoustic propagation speed.

Under the assumption of constant density, an approximate solution to this inverse problem for the velocity was demonstrated by Claerbout [1971]. Approximate solutions for both velocity and density in a horizontally stratified earth have been presented by Raz [1981c] and Clayton and Stolt [1980].

The mapping of the interior of the earth from 'local' observations on the surface of the responses to an 'ensemble' of acoustic sources is another type of inverse problem. Each experiment in the ensemble is performed separately and the 'locality' of the observations is limited by the extent of the receiver array.

This is the inverse problem which has dominated exploration geophysics in the recent past. Below, this problem will be referred to by the title, the inverse problem of seismic exploration, although we acknowledge the introduction into the repertoire of exploration geophysics of other inverse problems - static and dynamic, acoustic, elastic and electromagnetic.

The seismic exploration problem has recently been treated most successfully by wave equation migration [Claerbout and Doherty, 1972]. This technique treats the inverse problem of seismic exploration as a direct problem in reversed time for the acoustic wave equation with 'halved' propagation speed.

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The following features of wave equation migration are to be noted: It is

i.    a high frequency technique,

ii.   emphasizing phase, but

iii.  neglecting amplitude.


Migration

i.    locates reflectors, but

ii.   does not estimate 'reflection strength'.

Our approach to the inverse problem of seismic exploration is identified by the name, _velocity inversion_ [See, Cohen and Bleistein, 1977, 1979a]. In this approach, the inverse problem is modelled by one or more non-linear integral equations deduced from the direct scattering model. The integral equation(s) are linearized by perturbation methods (or, as the physicists would prefer, by a Born-like approximation). The unknowns in these equations are the free parameters of the model equations. Thus, inversion of the integral equation leads directly to a solution of the inverse problem.

Velocity inversion, as implemented on seismic data, is

i.    a high frequency technique,

ii.   preserving phase and

iii.  preserving amplitude.


Therefore, velocity inversion

i.    locates reflectors and

ii.   _estimates_ reflection strength.


The accuracy of the estimate of reflection strength is limited by the accuracy of the perturbation approximation. Thus, much current research by us and by others, notably Coen [1980], is focused on updating the perturbation, or correcting it, or perturbing about more accurate reference signals.

It should be further noted that high frequency implementation is not a constraint of the basic method, but rather an acknowledgement of the realities of seismic data. For the length scales, velocities and curvatures of interest, 4 Hz is, indeed, a 'high frequency'. For example, for a length scale, L=1000 ft., a velocity of v=5000 ft/sec, and a frequency, f=4 Hz, the relevant parameter,

$$2\pi fL/v=5,$$

is 'large'.

Finally, we remark that estimation of <u>velocity</u> from information about reflection strength requires a further assumption about the relation between density and compressibility. We assume a constant density medium.

## 3. Implementation of Velocity Inversion

For analysis of the inverse problem of seismic exploration by velo-city inversion, the propagation of acoustic waves in the interior of the earth is modelled by the homogeneous acoustic wave equation,

$$\nabla^2 U - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = 0, \quad z > 0, \tag{3.1}$$

subject to the introduction of appropriate sources on the upper surface [See, Cohen and Bleistein, 1977, 1979a]. It is assumed that everywhere on the surface, a 'backscattered response' is observed. That is, the upward travelling signal is observed at the source point. This is the mathematical idealization of an array of CDP stacked responses.

The velocity v is rewritten as

$$\frac{1}{v^2} = \frac{1}{c^2}(1+\alpha). \tag{3.2}$$

An integral equation is then derived for $\alpha$ by a standard perturbation technique. That integral equation is

$$\int_0^\infty dz \int_{-\infty}^\infty dx \, dy \, \alpha(x,y,z)c^{-2}(x,y,z) \int_0^\tau dt \, U_I(t,x,y,z,\xi,\eta,0)U_I(\tau-t,\xi,\eta,0;x,y,z)$$

$$= \int_0^\tau dt \, U_S(t,\xi,\eta,0;\xi,\eta,0)(\tau-t). \tag{3.3}$$

Here $U_I$ is the impulse response due to a source at the point $(\xi,\eta,0)$

at time, t=0, in the absence of the perturbation, $\alpha$. $U_S$ is the observed 'backscattered' wave at $\xi, \eta$ due to the presence of the perturbation, $\alpha$.

When the reference velocity, c, is assumed to be constant and observations of the backscattered field are made 'everywhere' on the upper surface, then this integral equation has the following analytical solution:

$$\alpha(x,y,z) = \frac{2ic^3}{\pi^2} \int_{-\infty}^{\infty} d\xi d\eta \int_{-\infty}^{\infty} dk_1 dk_2 \int_0^{\infty} dk_3 \int_0^{\tau} d\tau \int_0^{\tau} dt \left[ k_3(\tau^2 - \tau t) \right.$$

$$\left. U_S(t,\xi,\eta,0;\xi,\eta,0) \exp\{2i[k_1(x-\xi)+k_2(y-\eta)-k_3 z]+i\omega t\} \right]; \quad (3.4)$$

$$\omega = c[\text{sign } k_3] \left[ k_1^2 + k_2^2 + k_3^2 \right]^{1/2}.$$

Thus, the solution consists of a multi-fold Fourier transform over the observations in $\xi, \eta$ and t, followed by a transformation of transform variables from $k_1, k_2, \omega$, to $k_1, k_2, k_3$, via the indicated dispersion relation. This is a full bandwidth solution for the perturbation in velocity, $\alpha$.

This formula is also quite similar to the result of F-K migration [Stolt, 1978]. Indeed, the methods differ only in the amplitude weighting factor, here deduced by solving exactly an approximate integral equation for the true perturbation in velocity.

Thus, this solution could be implemented by use of fast Fourier transform in three variables, followed by an interpolation to obtain the

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data on the appropriate grid, then followed by the inverse Fourier transform.

The direct calculation of $\alpha$ in this way does not exploit the particular properties of the data or of the nature of the solution we seek. Firstly, as already noted, the data is bandlimited to high frequency. Thus, the Fourier transform of $\alpha$, implicit in (3.4) before calculation of the inverse transform, is <u>bandlimited</u>.

In a series of papers [Bojarski, 1966; Mager and Bleistein, 1978; Armstrong and Bleistein, 1978; Cohen and Bleistein, 1979b], a theory was developed to extract information from a high frequency bandlimited Fourier transform of a piecewise constant function. More precisely, it is shown in this series of references how to locate the discontinuities of such a function and how to estimate the magnitude of the discontinuity from the Fourier inversion of the bandlimited data. To obtain this information, the Fourier data is processed to yield the normal derivative to the surface(s) of discontinuity. This derivative is a bandlimited Dirac delta function, which is readily recognizable. The 'strength' of the delta function at its peak is in known proportion to the magnitude of the discontinuity and to the bandwidth.

For the seismic inverse problem, the output of this process is an array of delta functions which define the 'events' or boundaries between the layers of the subsurface. From the peak values of the output, the reflection strength or velocity increments can be calculated.

For the integral in (3.1), the fact that only high frequency data is

usually collected can be exploited to reduce the number of integrations to be performed. With fewer integrations, it is practical to develop an algorithm which computes the output pointwise. Such a technique has the advantage that the velocity need not be kept constant for each point of processing, but can be replaced by a 'local' estimate of the rms velocity. Implementation of such a variable reference velocity tends to place the depth of events of the output more realistically and also allows diffraction tails 'at depth' to be gathered up more completely. An example of the latter will be shown below.

It should be noted that (3.4) provides a three dimensional output under the assumption that a full two dimensional ensemble of observations is made at the upper surface. More typically in seismic exploration, a line of data is taken. It is then assumed that the earth has only two dimensional variation – along that line and vertically. This is equivalent to assuming y-independence of $\alpha$ and $\eta$-independence of the observations. This results in the elimination of the $\eta$ and $k_2$ integrations in (3.4) and also the elimination of one factor of $\pi$. It should be noted that this is the result of assuming <u>three dimensional</u> propagation from a point source over an earth with two dimensional velocity variation. Thus, the velocity increments are still estimated by the output, but only to the extent that the model is consistent with reality. This is in contrast to a completely two dimensional model in which point sources are equivalent to <u>line sources</u> over the subsurface. In that case, even if the earth <u>did</u> have only two dimensional variation, the output would not properly estimate velocity increments, because the sources

and the nature of propagation in the three dimensional earth was not modelled correctly.

## 4. Implementation of Velocity Inversion on Synthetic Data

A number of synthetic data examples for the velocity inversion for-
malism were presented in [Cohen and Bleistein, 1979a]. Those examples
confirmed the validity of the computer implementation of our method on
data sets generated from simple reflectors and on one example with multi-
ple reflectors, including an interior lens shaped region. Furthermore,
it was seen from those examples that the perturbation method produced
adequate estimates of velocity increments which were 20% of the reference
velocity.

Here, four synthetic examples will be presented. The first two of
these are examples with two dimensional variation, the last two are exam-
ples with three dimensional variation.

Figure 1 is the synthetic timelog for the impulse response from a
two dimensional buried focus. The data was generated from a Kirchhoff
representation of the upward scattered wave. The doublet-like behavior
of the response on the lower curve of the 'bowtie' can be seen. Indeed,
the objective of this example was to test that velocity inversion unrav-
els this doublet.

The output of our algorithm is shown in Figure 2. The location of
the surface, as defined at each point by the peak of the bandlimited
delta function, is virtually exact. For this example, the velocity in
the upper medium was taken to be 5000 ft/sec and the increment at the

interface was taken to be 250 ft/sec. Typical estimates of the velocity increment taken from the output were 248 ft/sec to 251 ft/sec.

The timelog for the second example is shown in Figure 3. Here, the reflector has a sharp discontinuity and the objective was to test how well the method 'gathered up' the diffraction tail.

The output of our method is shown in Figure 4. Again, the location of the reflector is exact. The velocity and the velocity increment of the synthetic were as in the previous example. Away from the edge of the reflector, the estimates of velocity increment were as in the previous example. At the edge of the reflector, the estimate decreased to half its true value, as predicted by theory. One trace further to the left, it reduced to one tenth of the value on the reflector, then to one one-hundredth and then was lost in the noise. Thus, the edge was adequately reproduced, along with an estimate of velocity increment.

The final two examples serve as a test of the computer program developed to perform three dimensional velocity inversion. The input was analytically generated data for the reflection from a planar reflector and a spherical reflector. The output is shown in Figure 5 (for the planar reflector) and in Figure 6 (for the spherical reflector).

For reasons of economy, the observation grid was coarsened and an attendant coarsening of the output was observed. Nonetheless, the results do provide a test of the basic theory as implemented by our computer program for three dimensional velocity inversion.

## 5. Implementation of Velocity Inversion on Real Data

Here, we show the results of applying velocity inversion to real data sets. Figure 7 shows the timelog of a data set provided to us by Paul Stoffa of Lamont Observatory. A discussion of this data set as analyzed by wave equation migration can be found in [Herron, etal., 1978]. In Figure 8, the results of applying velocity inversion to this data set are depicted. The figure is normalized by the peak amplitude of the data set with no scaling of the output. Consequently, the picture is completely dominated by the strong reflection at the ocean bottom. In Figure 9, only the part of the depth section below 3000 meters is plotted. Here, a second reflector, sloping downward to the right, is visible. This is the reflector R4 in the above cited reference.

The data provided, here, was not true amplitude data. Furthermore, it is relative amplitude data only to the extent that relative amplitude is preserved by stacking and other preprocessing. Thus, the amplitude of the output is relative amplitude data, subject to the same caveat. Nonetheless, treating it as accurate relative amplitude data, the output can be normalized with respect to the assumed known response at the ocean bottom. It was assumed that the velocity incremented at the ocean bottom from 1500 meters to 3000 meters. On this basis, estimates were made of the velocity change at the reflector below -1625 meters. The estimates varied from -150 m/sec to -250 m/sec.

As a basis of partial comparison of the results of velocity

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inversion and migration, an electrostatic plot of the type used by Lamont, was generated for us by Stoffa. That result is shown along with Stoffa's migrated section, in Figure 10. The similarity of the output is quite apparent. That is, inversion provides as qualitatively 'clean' a depiction of the depth section, while also providing estimates of velocity changes.

Figures 11 and 12 are the left and right halves, respectively, of a timelog provided to us by Marathon Oil Company. Figures 13 and 14 depict the results of velocity inversion. In Figure 13, a long diffraction tail from the edge of the reflector in Figure 14 can be seen. The reason that this diffraction tail 'survived' inversion became apparent in our discussions with the geophysical research group at Marathon. The constant reference velocity used in this first 'pass' at the data was 5000 ft/sec. While this was a good estimate near the surface, 8000 ft/sec was a better estimate at the depth of the discontinuous reflector. Figures 15 and 16 depict a reprocessing of the data below 4900 ft. at this corrected speed. Now, it can be seen that the major diffraction tail has been 'gathered up'. There is, unfortunately, now an 'overmigration' phenomenon apparent with respect to other reflectors at lower depth. A full analysis of this data would require a number of different reference velocities in different regions. We did not have the resources to carry this out. However, the output presented does demonstrate the practicality of using different reference velocities at different depths.

It should be noted that in this latter processing step at the new velocity, it was not necessary to reprocess the portion of the section

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above 4900 ft, but only to process the data in the region of interest. This would be less practical in a multi-fold Fourier inversion, since each separate reference velocity would require a different interpolation grid when transforming from $k_1, k_2, \omega$, to $k_1, k_2, k_3$.

While this extension will help clean up diffraction tails, it will not correct for refractions, since the closed form solution assumes an incident wave $U_I$ which propagates on straight rays.

The number of data points and processing times for this output are listed in the table below:

Processing Time in seconds on a CDC Cyber 76

| Source | # of input data points | # of output data points | Preprocess time(sec) | Process time(secs) | Total time |
|---|---|---|---|---|---|
| Lamont | 480,000 | 60,000 | 31 | 201 | 232 |
| Marathon | 700,000 | 230,000 | 117 | 583 | 700 |

## 6. Recent Developments and Future Research

The objectives of recent research efforts have been to develop methods to invert the integral equation (6.2) when the reference speed, c, is not constant and to more adequately account for the non-linearity of the underlying inverse problem.

### 6.1 Higher Order Accuracy in the One-Dimensional Problem

An important line of research leading to higher order accuracy in the one-dimensional velocity inversion problem was initiated independently and nearly simultaneously by Gray [1980] and Raz [1981c]. While their approach differs, the basic result obtained is nearly identical.

We shall describe Gray's approach. He introduces the travel time as an independent variable. The resulting equation admits an incident wave with constant reference speed and a perturbation proportional to the logarithmic derivative of the true propagation speed. The integral equation for this perturbation reduces to an equation relating its Fourier transform to the observed backscattered data. Fourier inversion and exponentiation then produces the velocity as a function of travel time. Since the true depth is an integral of this velocity with respect to travel time, the velocity is given implicitly in terms of depth.

Gray shows that this result has higher order accuracy in the perturbation, $\alpha$. Gray and Hagin [1980] have shown that an iteration scheme based on this formulation converges even when the velocity has jump

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discontinuities (albeit, not too large), as in a layered earth. In contrast, Prosser [1980] has shown that an iteration scheme based on the previous perturbation technique will converge for sufficiently smooth velocities. However, in private communications, he has indicated pessimism about demonstrating convergence of that scheme for the layered medium case, which arises in the seismic exploration problem.

Figure 17 is taken from Gray, Bleistein and Cohen [1980]. It demonstrates the increased accuracy of this method, both in locating the discontinuities and estimating their magnitude. Figure 18, taken from Hagin [1980] demonstrates the increased accuracy of second iterates in this method.

Raz [1979, 1981a] has provided some extensions of this work to the three-dimensional stratified case. He has shown how observations at offset can be used to invert a tilted stratified earth or could be used to separate density and compressibility. An alternative, but quite similar approach, to the latter problem was also presented at the SEG meeting in Houston by Stolt and Jacobs [1980].

Our research group is presently investigating the extension of this method to the three dimensional seismic inverse problem.

6.2 A Wave Equation for the Propagation of the Ensemble of Backscattered Signals

Research on an alternative approach to the three dimensional velocity inversion problem has recently been initiated. The objective of

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this approach is to derive an equation for the propagation of the ensemble of backscattered signals. That equation should be sufficiently accurate to preserve both phase and amplitude of the ensemble. Clearly, this is the objective of wave equation migration. However, that method is based on travel time arguments alone and thus accurately produces only phase information, except at the first reflector. This will be discussed in further detail, below.

Our method is based on analysis of the Kirchhoff integral representation of the backscattered impulse response. For a single layer, we consider the geometrical configuration depicted in Figure 19. By using geometrical optics approximations in the Kirchhoff integral representation of the solution, one can derive the following integral representation for the backscattered wave (See, for example, Cohen and Bleistein 1979b):

$$u_S(\underline{x},\omega) \sim \frac{i\omega}{8\pi^2 c} \int_S R \, \frac{\tilde{n} \cdot \tilde{r}}{r^2} \exp\{2i\omega/c\} \, dS. \tag{6.1}$$

Here, $\tilde{n}$ is the unit upward normal vector, $\tilde{r}$ is a unit vector from the surface, $S$, to the observation point, $u_S$ denotes the Fourier time transform of $U_S$ and $R$ is the geometrical optics reflection coefficient,

$$R(\underline{x},\underline{\xi}) = \frac{\tilde{n} \cdot \tilde{r} - \{c^2/c_1^2 - 1 + (\tilde{n} \cdot \tilde{r})^2\}^{1/2}}{\tilde{n} \cdot \tilde{r} + \{c^2/c_1^2 - 1 + (\tilde{n} \cdot \tilde{r})^2\}^{1/2}}. \tag{6.2}$$

By applying the wave operator to (6.1) and retaining only terms to

two orders in $\omega$ (i.e., terms multiplied by $\omega^2$ and $\omega$ ), the following partial differential equation can be derived for $U_S$:

$$\left[\nabla^2 + (2\omega/c)^2\right]\left[\frac{\partial u_S}{\partial \omega}\right] \sim - \frac{\omega}{\pi c^2} R_n \delta(\sigma_n). \qquad (6.3)$$

Here, $\delta$ denotes the Dirac delta function; $\sigma_n$ denotes normal distance[*] from the reflector to the observation point. $R_n$ denotes the 'normal' reflection coefficient:

$$R_n = \frac{c_1 - c}{c_1 + c} . \qquad (6.4)$$

Boundary data for $u_S$ is also prescribed at $z=0$.

Equation (6.3) has the interpretation that the ensemble of backscatters is the response to a source distributed over the reflecting surface. Furthermore, that source is proportional to the reflecting strength. Thus, equation (6.3) provides a 'quantification' of the 'explosive reflector' model of backscattering and gives the intensity of the source to leading order in $\omega$.

The inverse problem for the location of the reflector and the reflection strength may now be stated as follows:

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[*]Since the 'support' of the delta function is on the scattering surface, this distance need only be single valued and well defined near that surface, which it will be for a sufficiently 'well behaved' surface.

Let $u_S$ be a solution of (6.3) with unknown source. Given the boundary data for $u_S$, find the source.

Unfortunately, inverse source problems are known to have non-unique solutions. It is our belief, however, that in the class of distributional solutions with uniform strength over a surface, the solution is unique. However, no proof has been developed, as yet.

This result can also be stated in time domain with an alternative interpretation. To deduce this result, we introduce the function

$$v(\underline{x},\omega) = -i \frac{\partial u_S}{\partial \omega} \tag{6.5}$$

and its inverse Fourier transform,

$$V(\underline{x},t) = tU_S(\underline{x},t). \tag{6.6}$$

This function is a solution of the following <u>initial-boundary</u> <u>value</u> <u>problem</u> deduced from the frequency domain problem:

$$\nabla^2 V - \left[\frac{2}{c}\right]^2 \frac{\partial^2 V}{\partial t^2} = 0 ; \tag{6.7}$$

$$V(\underline{x},0) = \frac{R_n \delta(\sigma_n)}{4\pi} \quad , \quad \frac{\partial V}{\partial t}(\underline{x},0) = 0, \tag{6.8}$$

subject to an appropriate boundary condition on the surface z=0.

The inverse problem for the reflector and the reflection strength may now be stated as follows:

Let V be a solution of the wave equation with propagation speed c/2. Given the observations at the upper surface, z=0, for all time and given the second initial condition in (6.8), determine the initial value of V.

This is exactly the problem which is solved by wave equation migration. We conclude, therefore, that <u>at</u> <u>the</u> '<u>first</u> <u>reflector</u>', wave equation migration not only locates the reflector but also, for true amplitude data, <u>accurately</u> <u>estimates</u> <u>the</u> <u>reflection</u> <u>strength</u>.

It is possible to obtain an integral representation of a solution to this problem, namely,

$$V(\underline{\xi},0) = \int_{z=0} dxdy \int_0^\infty dt \ V(x,y,0,t) \ \frac{\partial G}{\partial z}(x,y,0,\underline{\xi},t) \ . \tag{6.9}$$

Here, G is the half space Green's function for the wave equation with source point, $(x,y,0)$ and observation point, $(\underline{\xi})$ and zero boundary value at $z=0$.

In order to generalize this result to a layered earth, it is necessary to analyze the backscattered field from a layer below the first. We start from the same Kirchhoff integral representation of $u_S$. However, the function,

$$[4\pi r]^{-1} \exp\{i\omega r/c\},$$

whose square appears in the integral in (6.1), must now be replaced by the Green's function for an arbitrary medium. This can be simplified somewhat if we are willing to accept only the primary downward propagating part of the Green's function and then further content ourselves with a characterization of that function which has accurate phase and leading order amplitude, only. Thus, each reflector will be treated as if it were the only reflector; i.e., multiple reflections are neglected and, as above, the representation reproduces the phase and amplitude to leading order in $\omega$.

This asymptotic Green's function has the form,

$$G \sim A \exp\{i\omega\varphi\}, \tag{6.10}$$

where $\varphi$ and A are solutions of the eikonal equation and transport equation, respectively,

$$(\nabla\phi)^2 = c^{-2}; \quad 2\nabla\phi \cdot \nabla A + A\nabla^2\phi = 0. \tag{6.11}$$

Furthermore, in order that this be asymptotically the Green's function, it is required that A behave as $[4\pi r]^{-1}$ when r, the distance between source and observation point, approaches zero and that $\varphi$ be zero in that limit. In this case, in analogy with (6.1), the backscattered field has the integral representation,

$$u_S(\underline{x},\omega) \sim 2i\omega\int_S R \; \widetilde{n} \cdot \nabla\phi \; A^2 \; \exp\{2i\omega\varphi\} \; dS. \tag{6.12}$$

Here, the reflection coefficient is as in (6.2) except that $\Upsilon$ must be replaced by $c\nabla\phi$.

Applying the wave operator to (6.12) yields the following equation:

$$\left[\nabla^2 + (2\omega/c)^2\right]\left[\frac{\partial u_S}{\partial\omega}\right] \sim -\frac{\omega}{\pi c^2} R_n\delta(\sigma_n)$$

$$+ \frac{4i\omega}{c}\int_S R(\widetilde{n} \cdot \nabla_\xi\phi) \; A \; \nabla_\xi\phi \cdot (A\nabla_\xi\phi) \; \exp\{2i\omega\phi\} \; dS. \tag{6.13}$$

Were the second line absent from this equation, then the downward propa-

gation of the ensemble of backscatters from any layer would be exactly the same as those from the first layer. In this case, wave equation migration would produce true amplitude at each reflector and would then, indeed, be inversion. The second line in this equation therefore represents the extent to which downward propagation according to the wave equation with speed ($c/2$) of the ensemble of backscattered signals deviates from producing a 'true' ensemble of backscattered signals 'at depth'. It should be noted that this second line has a multiplier of $\omega$ (rather than $\omega^2$) and thus asymptotically affects the leading order amplitude and not the phase of the downward continued ensemble. Thus, neglecting this term will cause errors in amplitude alone and not in phase. This is consistent with the empirical results demonstrating that wave equation migration (i.e., solving the equation represented by the first line only, in (6.13)) locates reflectors accurately.

Research on this full propagation equation is now being carried out by our research group.

## 7. Conclusions

A summary of a research program on a class of methods collectively referred to as <u>velocity</u> <u>inversion</u> has been presented. The mathematical formulation and computer implementation have been described and new and incomplete research in progress has been discussed.

An inspection of the program for the 47th Annual International Meeting of the SEG in Houston in 1976 will reveal a paucity of papers on the subject of <u>inversion</u>. In contrast, the 50th Annual Meeting in 1980 had three sessions and one workshop on inversion. Having presented one of the papers on inversion in that 47th Meeting, we view this as a favorable and gratifying trend.

## ACKNOWLEDGMENTS

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# FIGURE CAPTIONS

Fig. 1     Synthetic timelog from a two dimensional buried focus. Hor-
           izontal spacing, 100 ft.; bandwidth, 12-36 hz. Doublet
           behavior on lower half of bowtie is evident.

Fig. 2     Output of the velocity inversion algorithm for the line of data
           of Fig. 1. Horizontal spacing between traces, 100 ft. Doublet
           in timelog has been replaced by impulse on the output section.

Fig. 3     Synthetic timelog for a half-plane reflector. Horizontal spac-
           ing, 100 ft.; bandwidth, 12-36 hz. Diffraction tail to left of
           reflector is evident.

Fig. 4     The output of the velocity inversion algorithm for the dataset
           of Fig. 3. The diffraction tail has been 'gathered' by the
           algorithm.

Fig. 5     Output of the three dimensional velocity inversion algorithm
           for a planar array of synthetic data over a tilted planar
           reflector. Horizontal spacing is 100 ft. in each transverse
           direction.

Fig. 6     Output of the three dimensional velocity inversion algorithm
           for a planar array of synthetic data over a spherical reflec-
           tor.

Fig. 7     Timelog of a dataset gathered by Lamont-Doherty Observatory
           over the east pacific rise. Horizontal spacing, 50 m;
           bandwidth, 8-31hz. Electrostatic plot provided by Paul Stoffa.

Fig. 8     Output of the two dimensional velocity inversion algorithm
           applied to the dataset of Fig. 7.

Fig. 9     Replotting of the section of Fig. 8 below 3000m.

Fig. 10    Electrostatic plot of output of Fig. 8 and electrostatic plot
           of Stoffa's wave equation migration algorithm applied to the
           same dataset.

Fig. 11    Left half of timelog of a dataset provided by Marathon Oil
           Company. Horizontal spacing, 82 ft.; bandwidth, 5-37hz.

Fig. 12    Right half of dataset connected to left half in Fig. 10.

Fig. 13    Left half output of velocity inversion algorithm. Note long
           diffraction tail below 4900 ft.

Fig. 14    Right half output of velocity inversion algorithm adjacent to left half of Fig. 12.

Fig. 15    Left half output of velocity inversion below 4900 ft. with new reference velocity, 9000ft/sec.

Fig. 16    Right half output of velocity inversion below 4900 ft. with new reference velocity, 9000 ft/sec.

Fig. 17    Comparison of first and second order one-dimensional velocity inversion algorithm. Both amplitude and phase (location of second discontinuity) are more accurate with second order method.

Fig. 18    Comparison of first and second iterates of Gray's second order method.

Fig. 19    The geometry for analysis of the propagation of an ensemble of backscatters from a single layer.

FIGURE 1

KILOFEET

3    4    5    6

KILOFEET

2    1    0    -1    -2

FIGURE 2

KILOFEET

3    7    5    9

FIGURE 3

KILOFEET
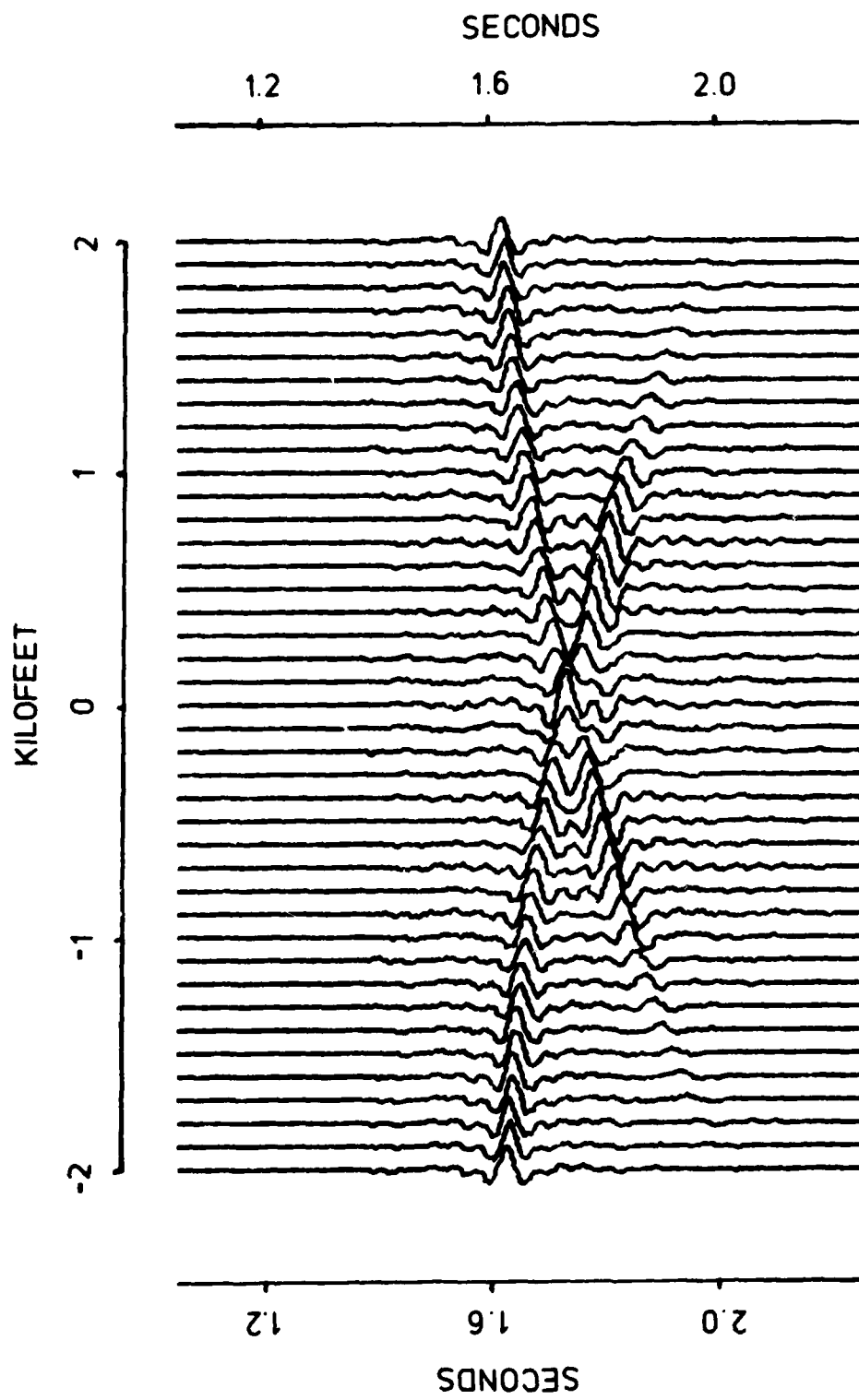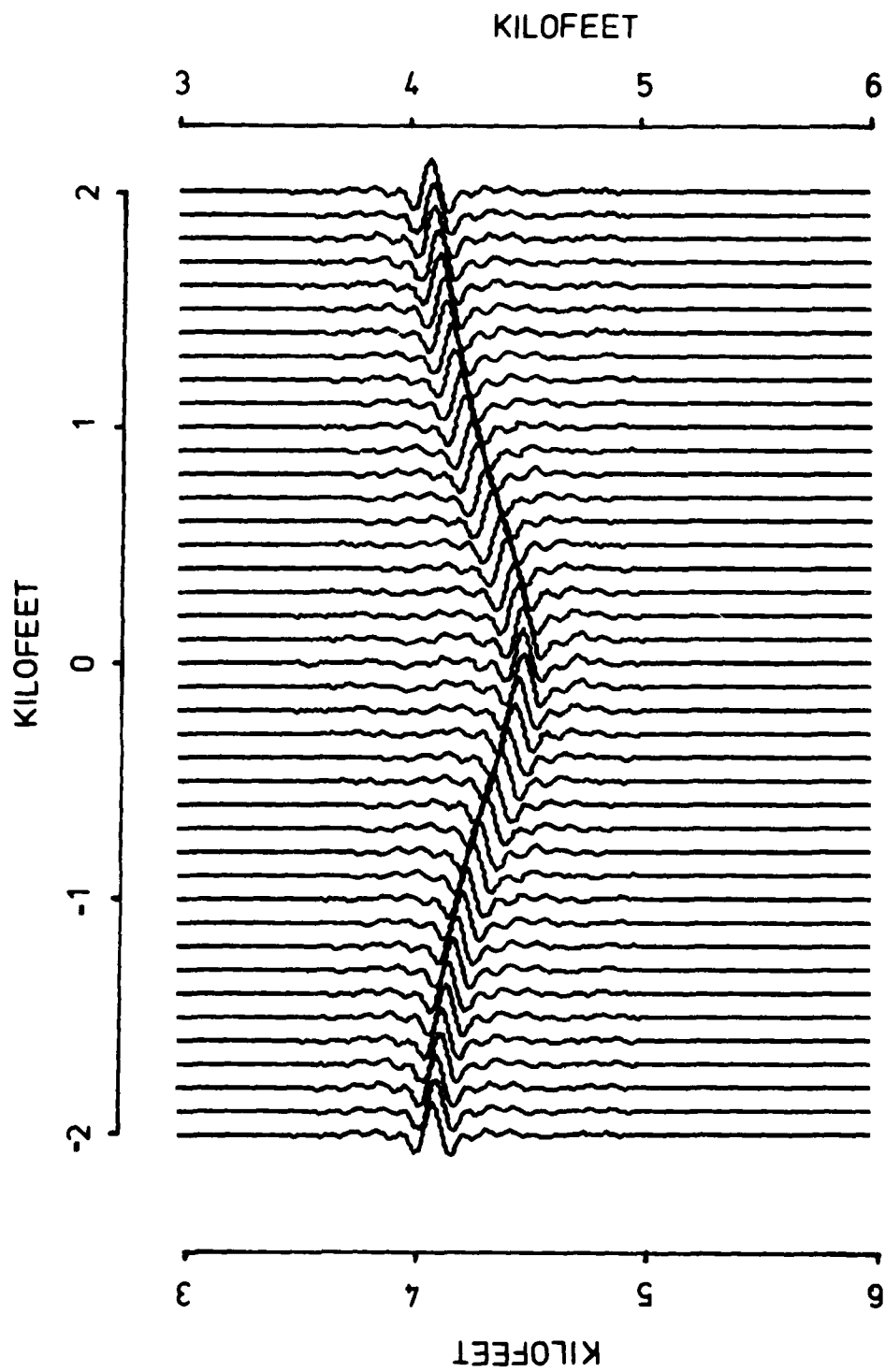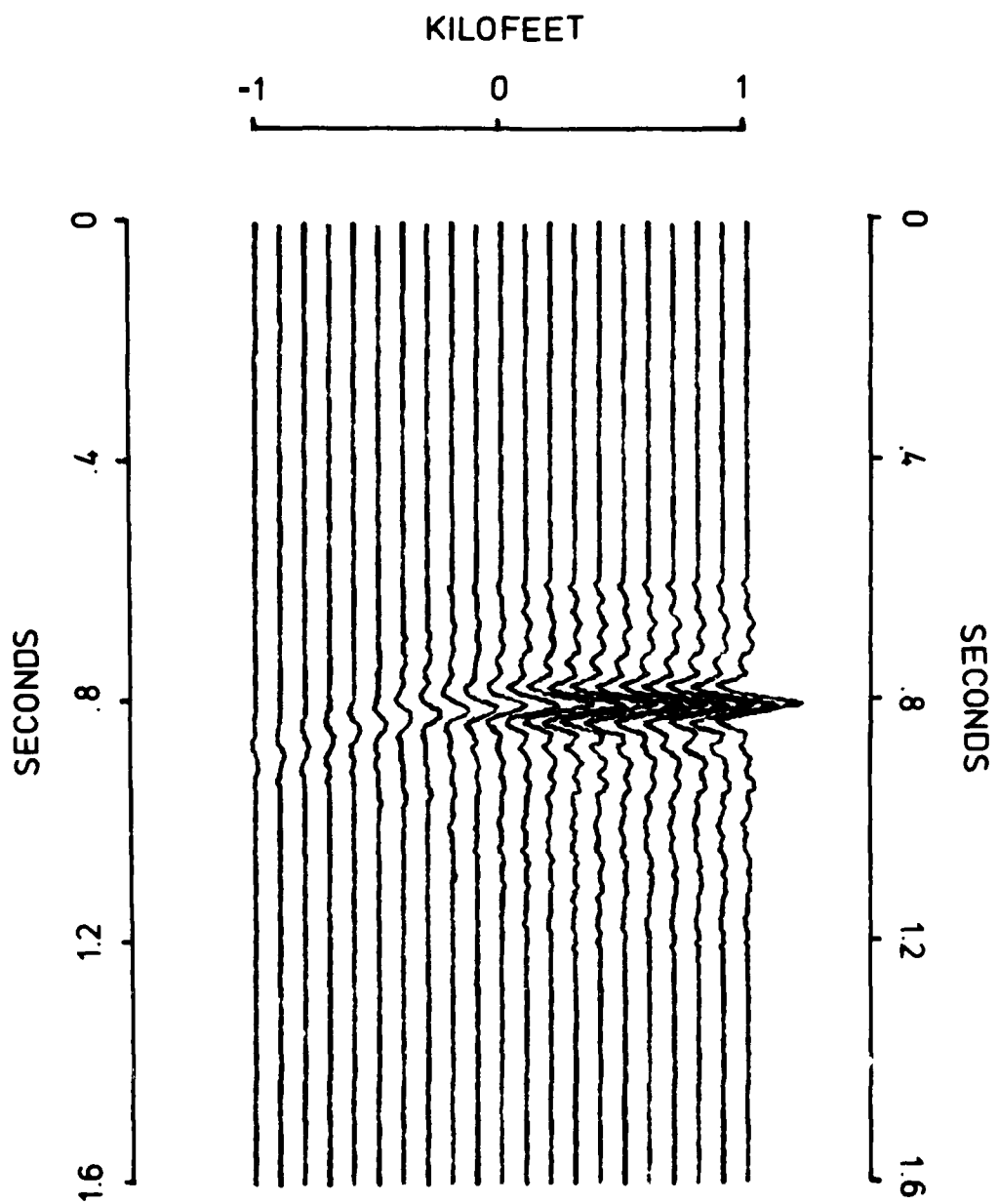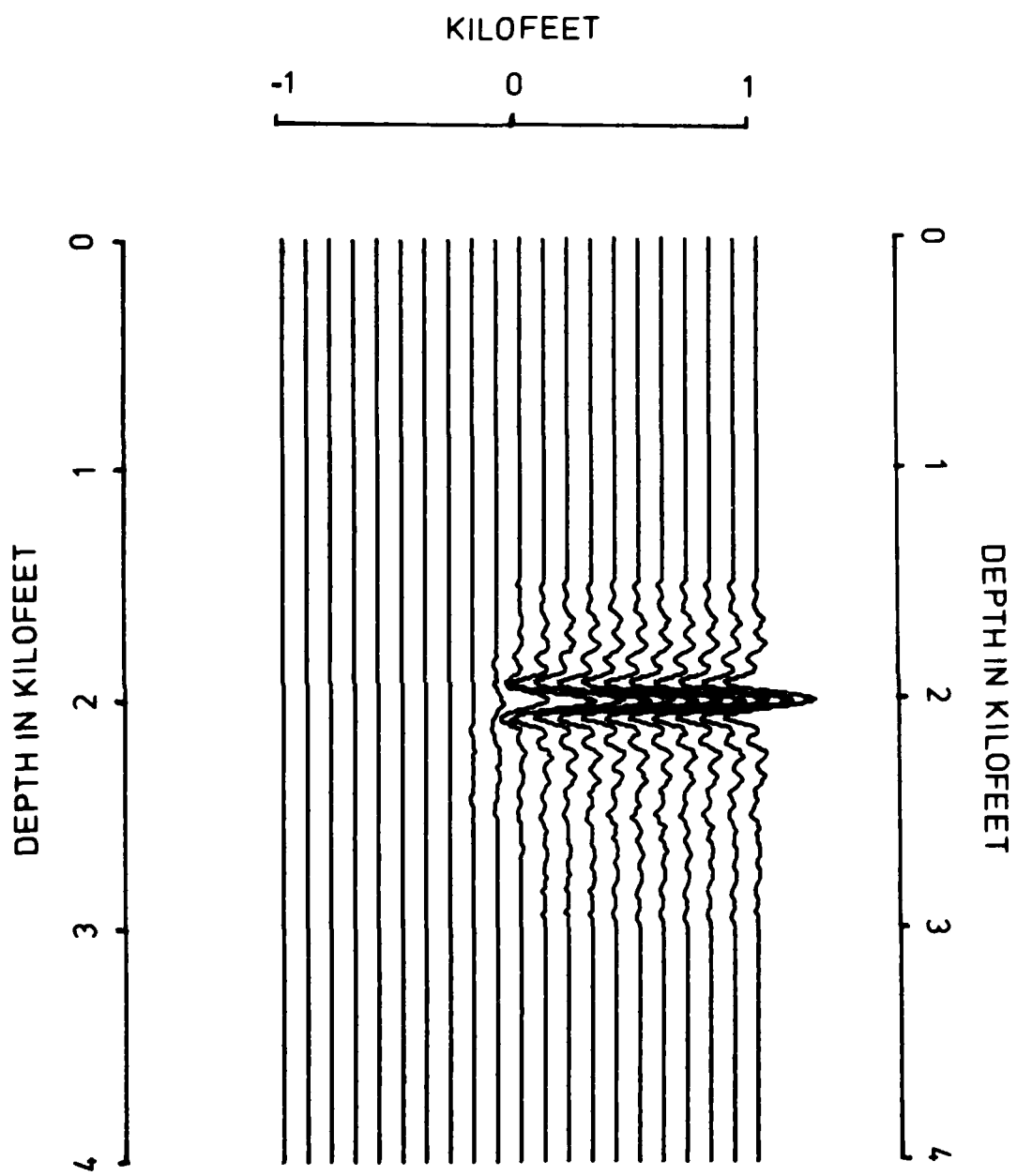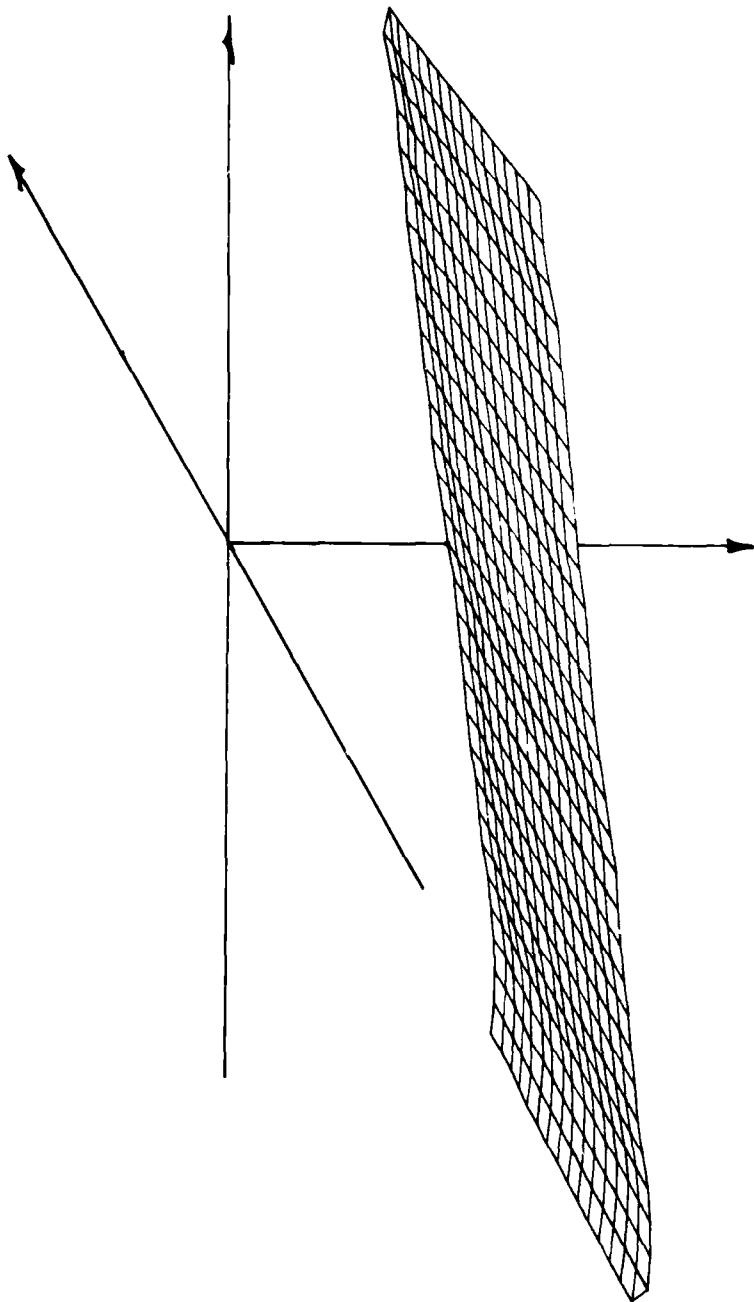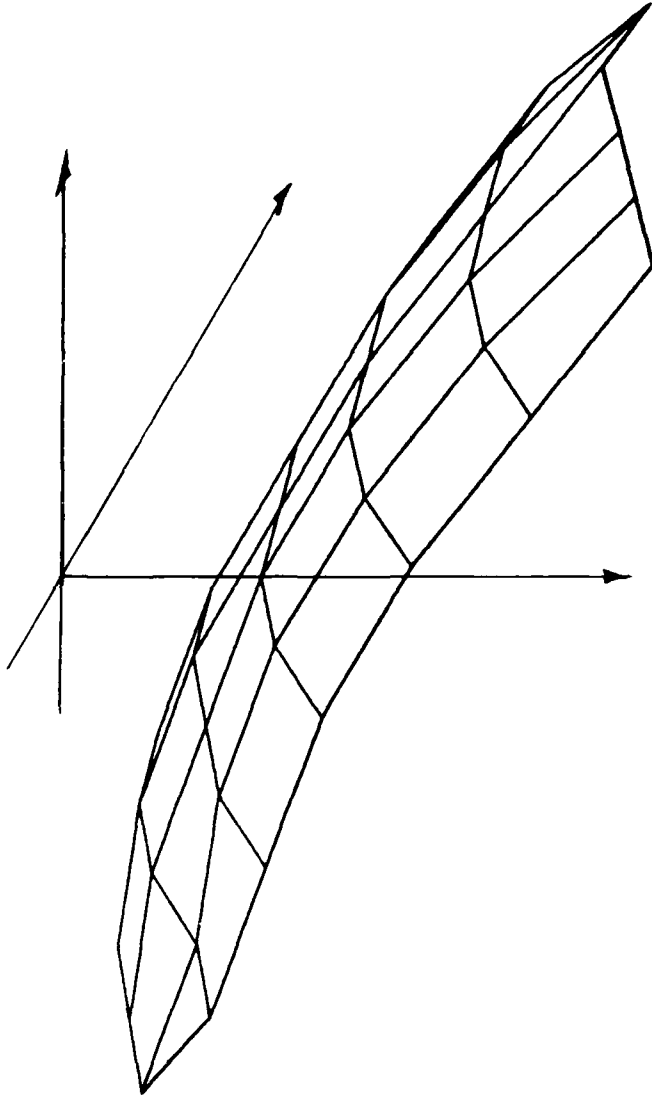
FIGURE 4

FIGURE 5

FIGURE 6
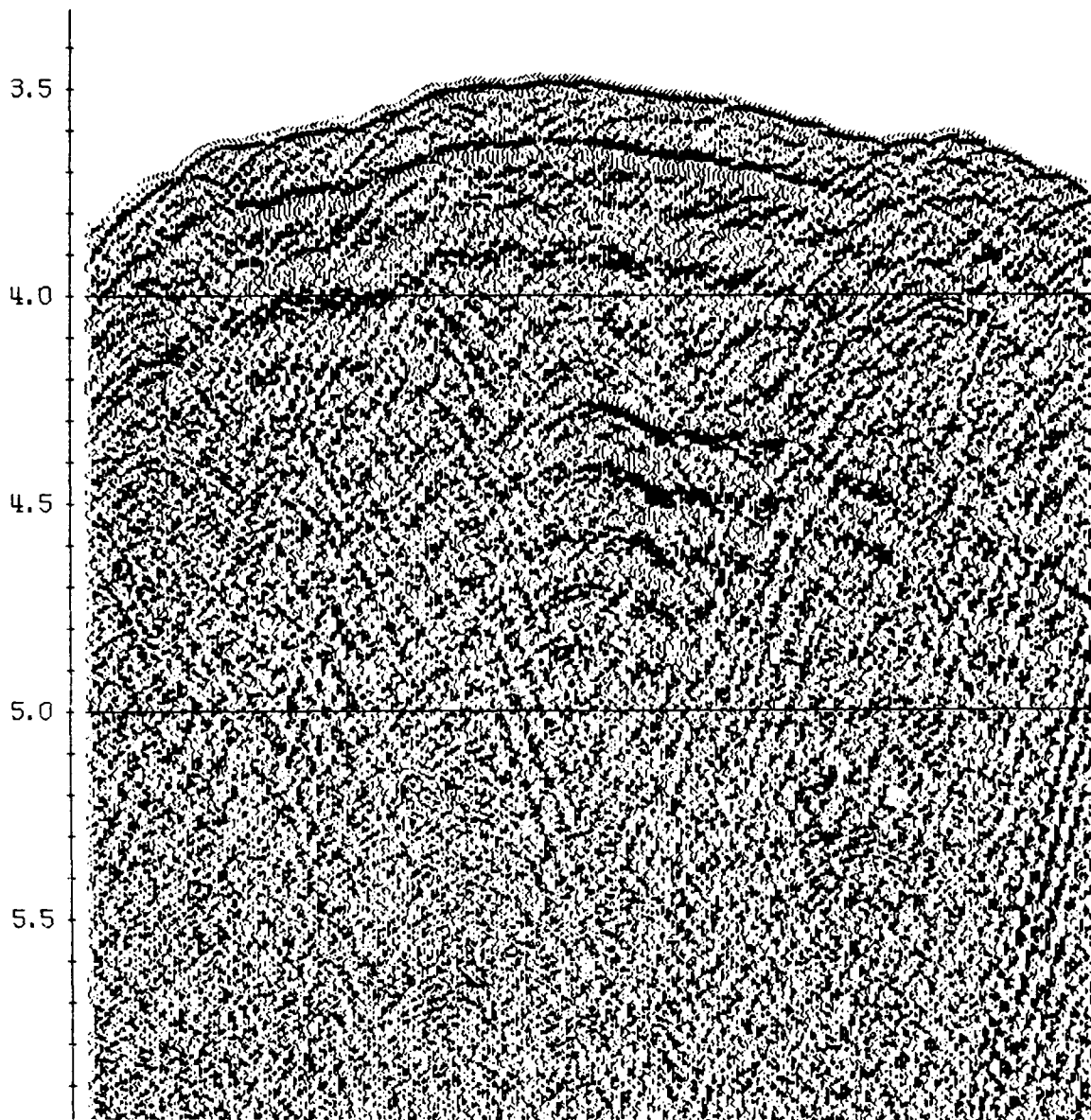
FIGURE 7
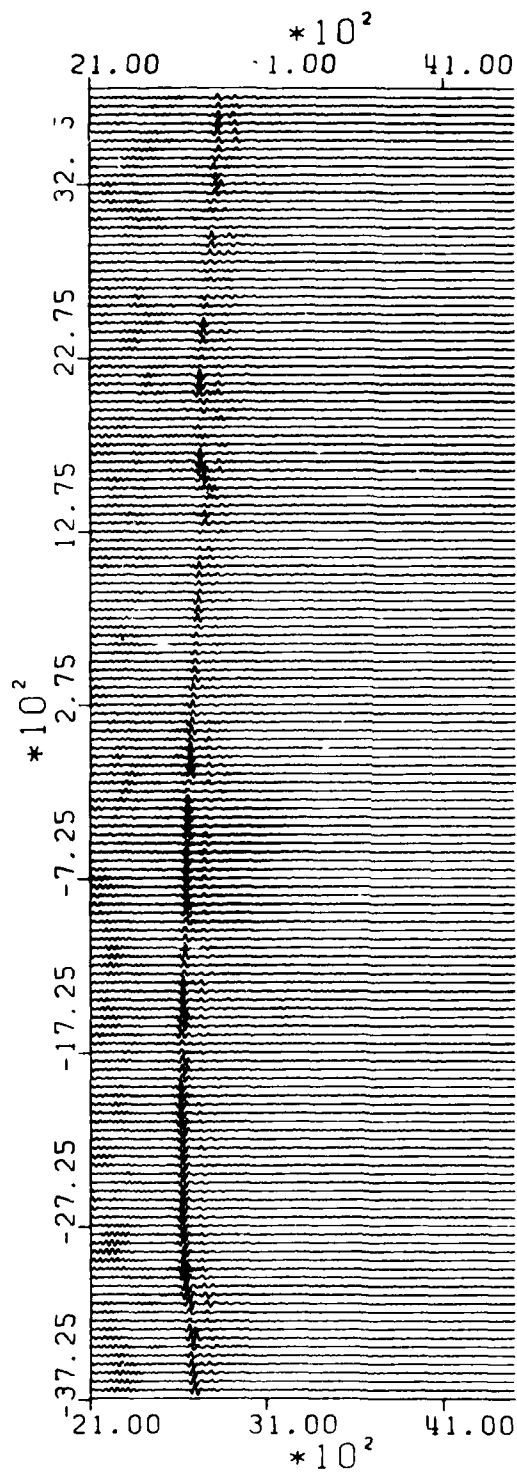
EAST PACIFIC RISE-LINE 17

DEPTH SECTION 2100-4500 METERS

FIGURE 8

EAST PACIFIC RISE-LINE 17

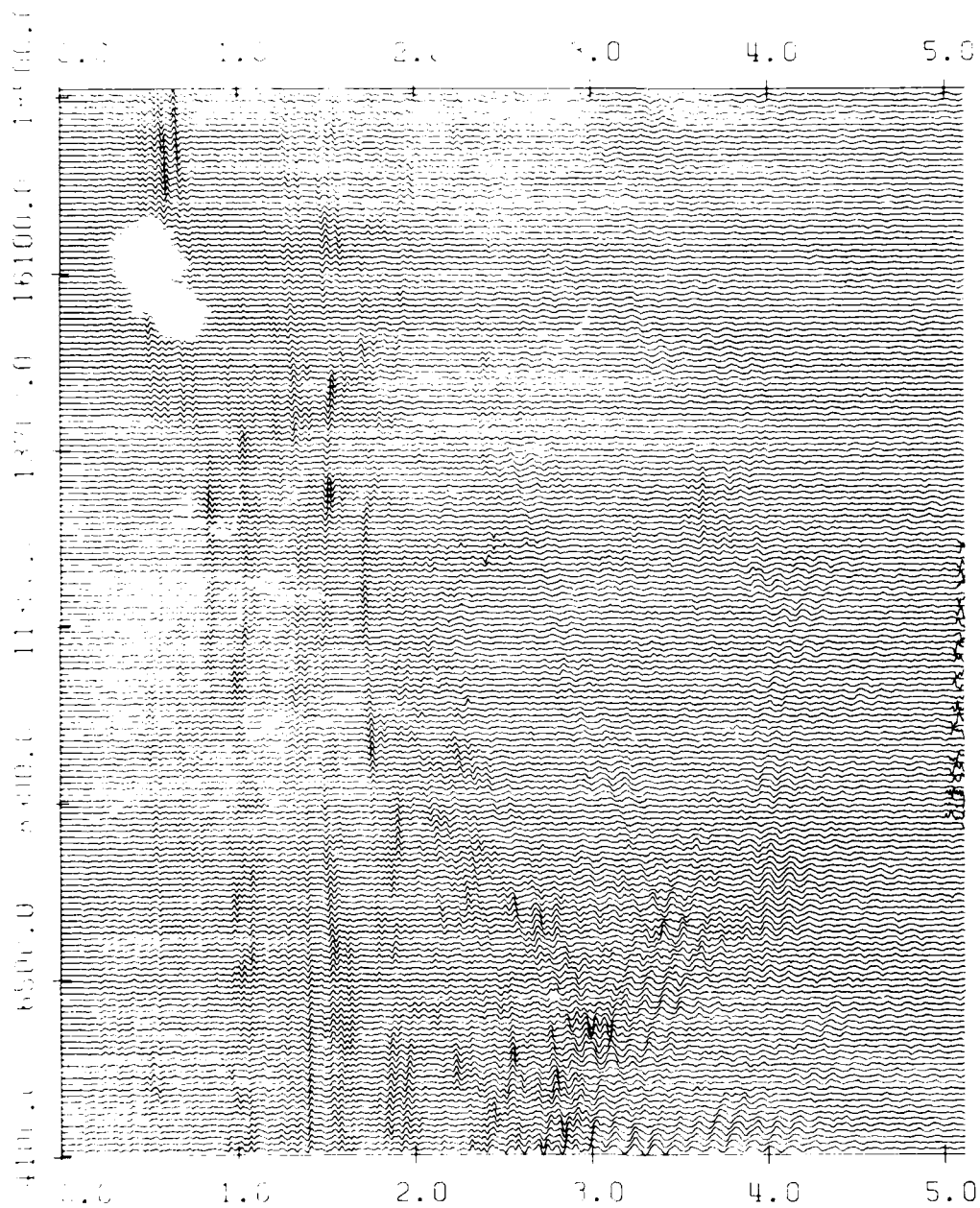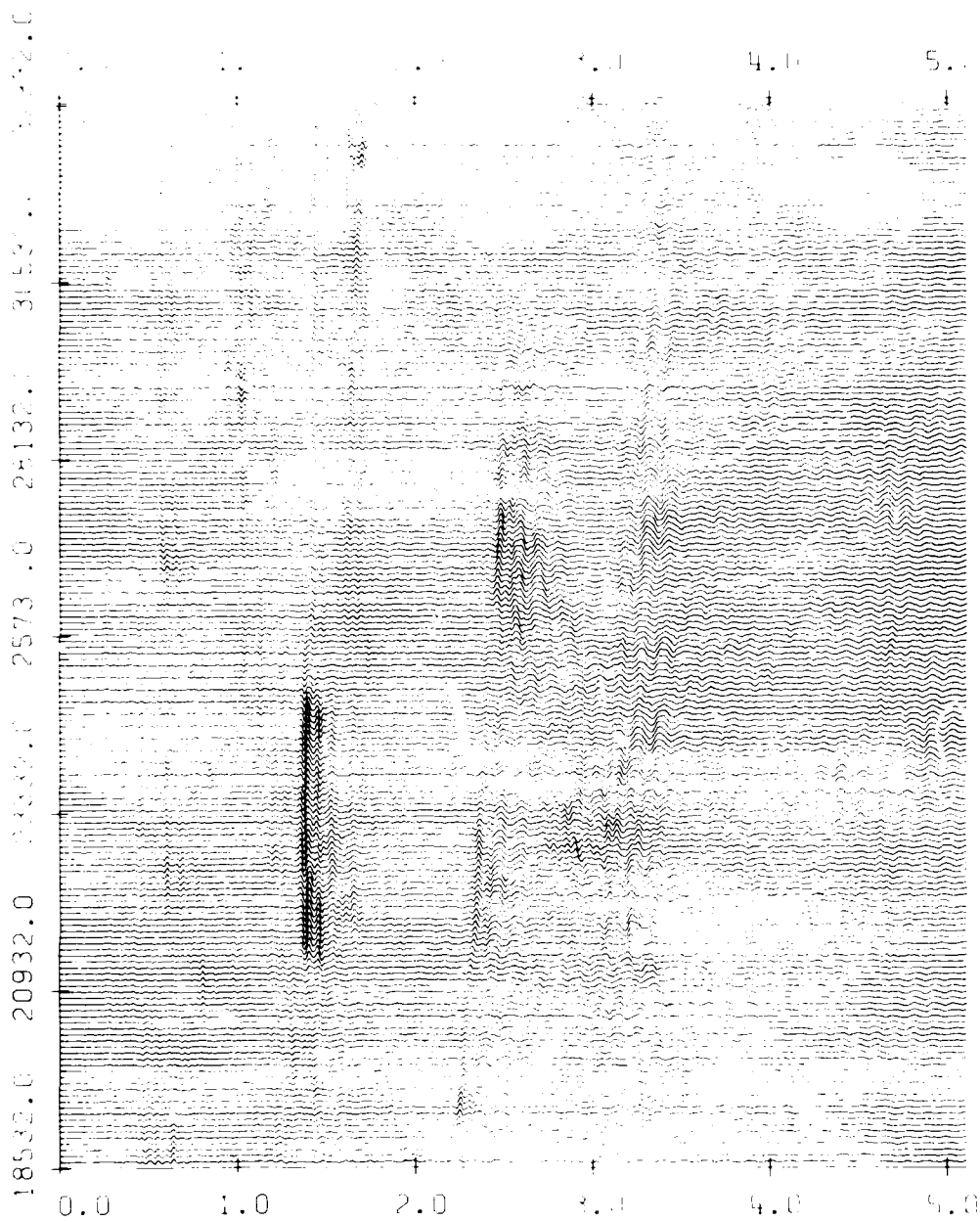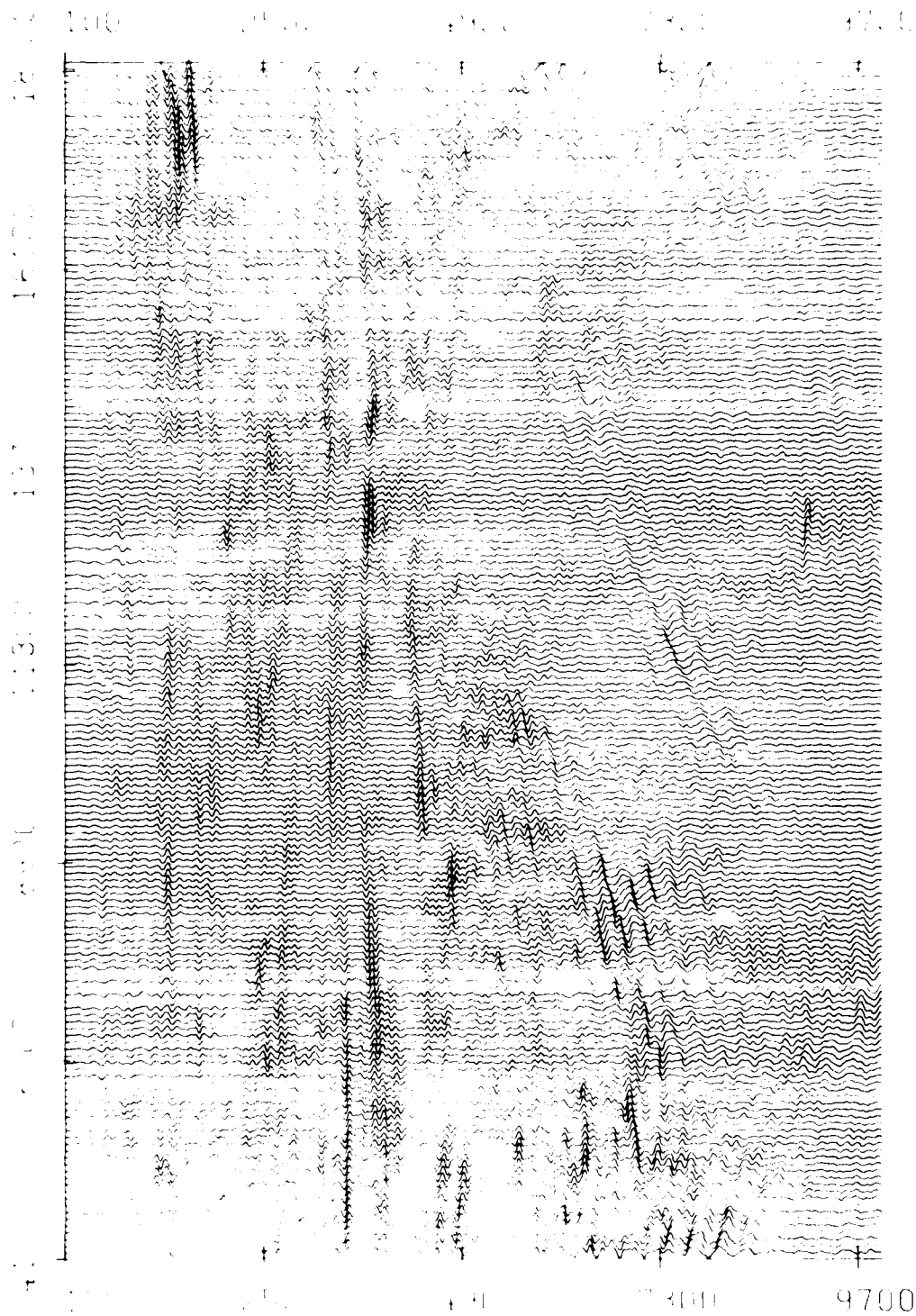DEPTH SECTION  3000-4500 METERS
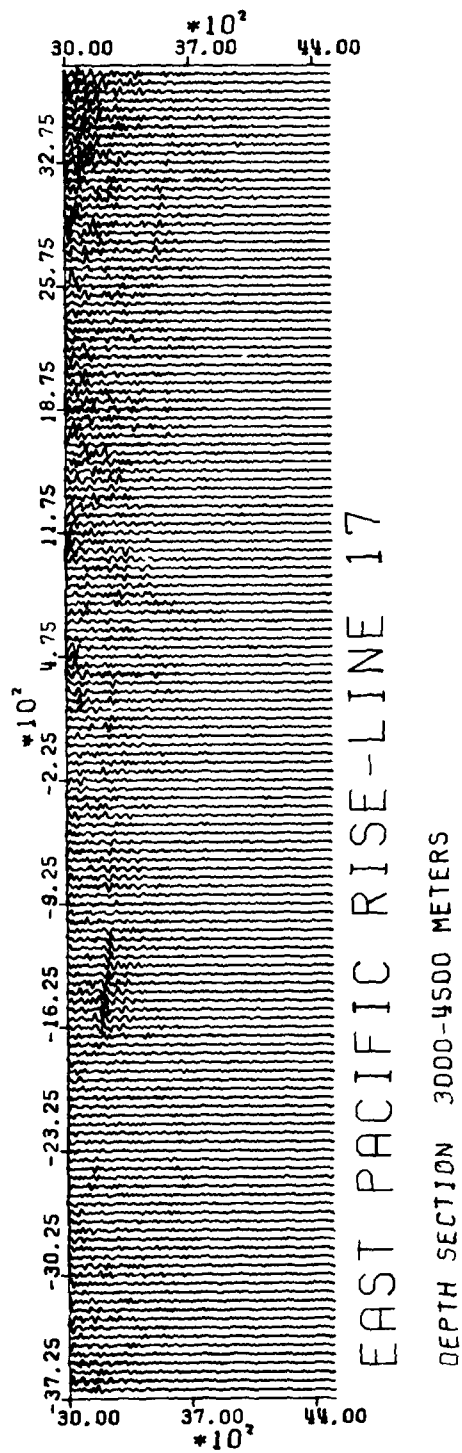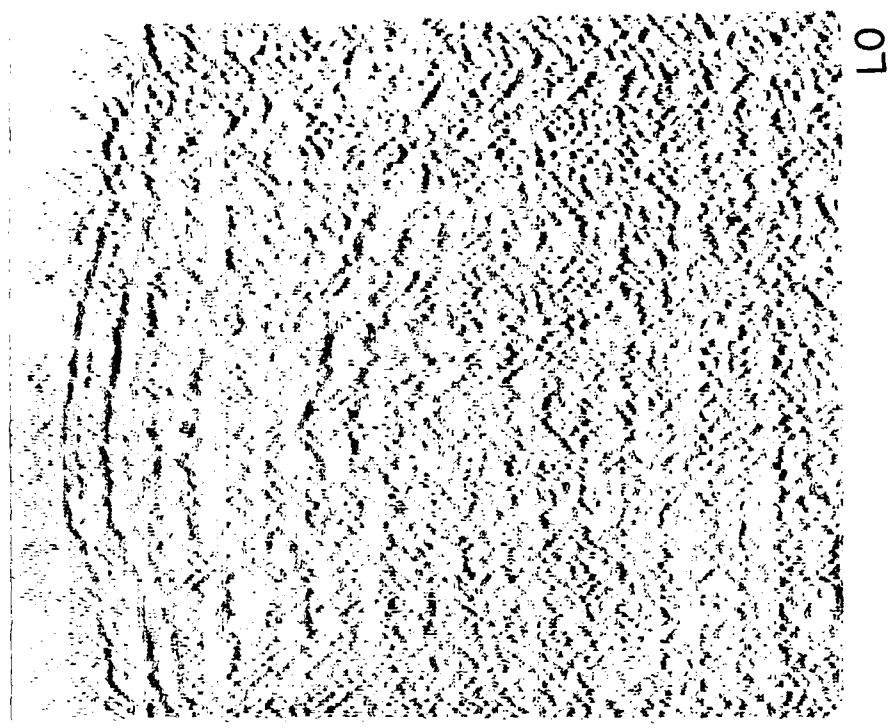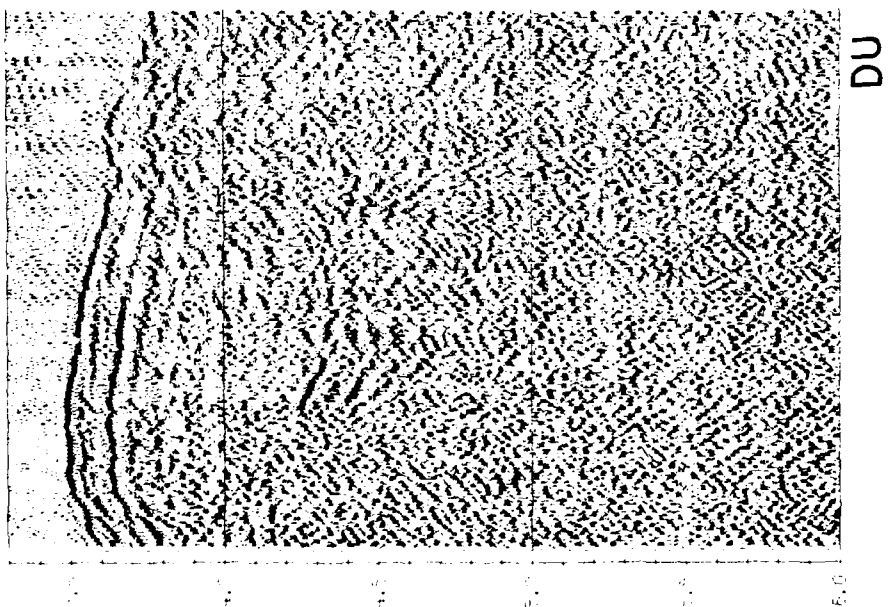
FIGURE 9

FIGURE 10

TIME SECTION  0 - 5 SEC

LEFT HALF

**FIGURE 11**

TIME SECTION  0 - 5 SEC

RIGHT HALF

**FIGURE 12**

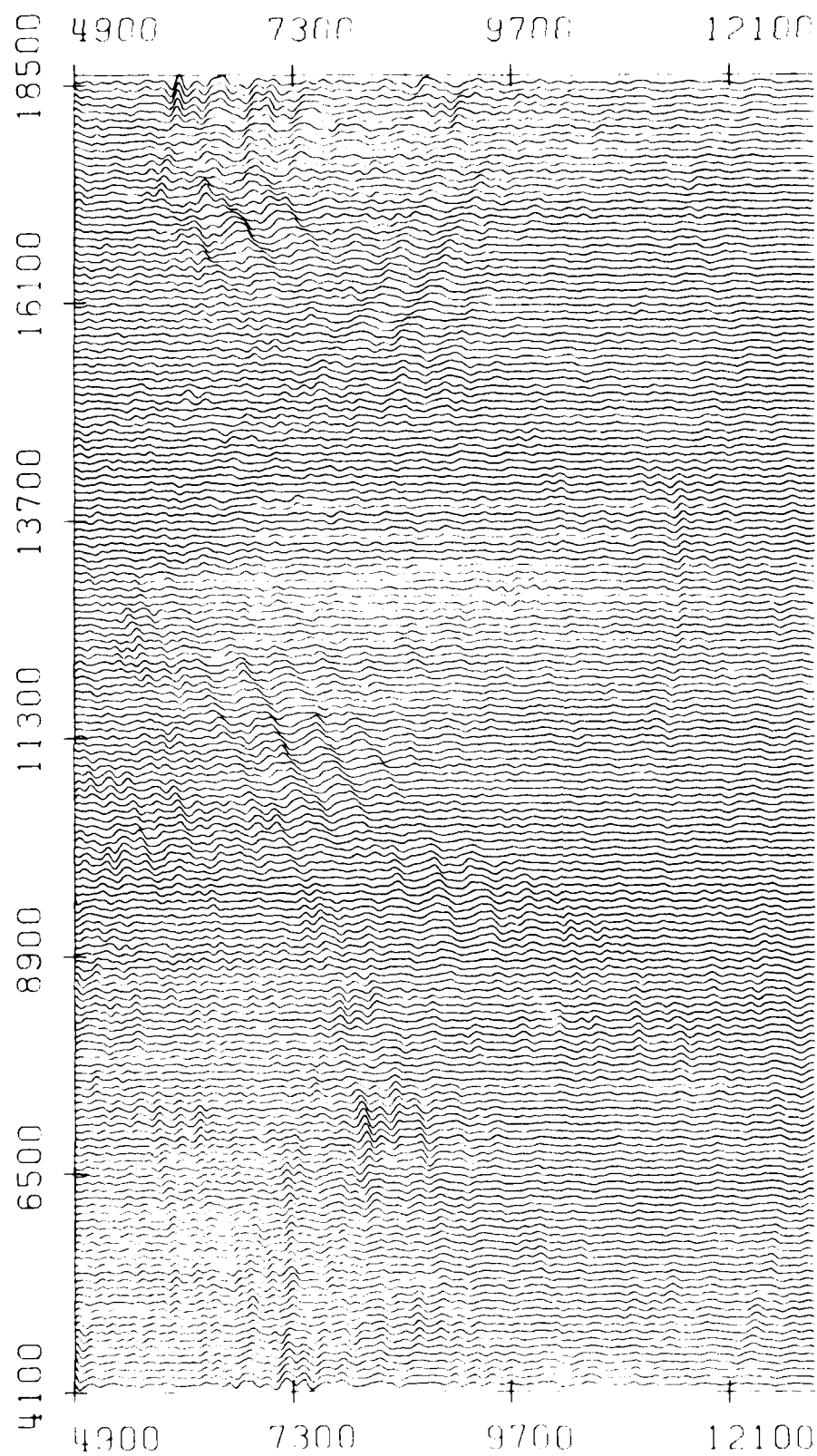DEPTH SECTION  100- 10000 FEET

LEFT HALF

**FIGURE 13**
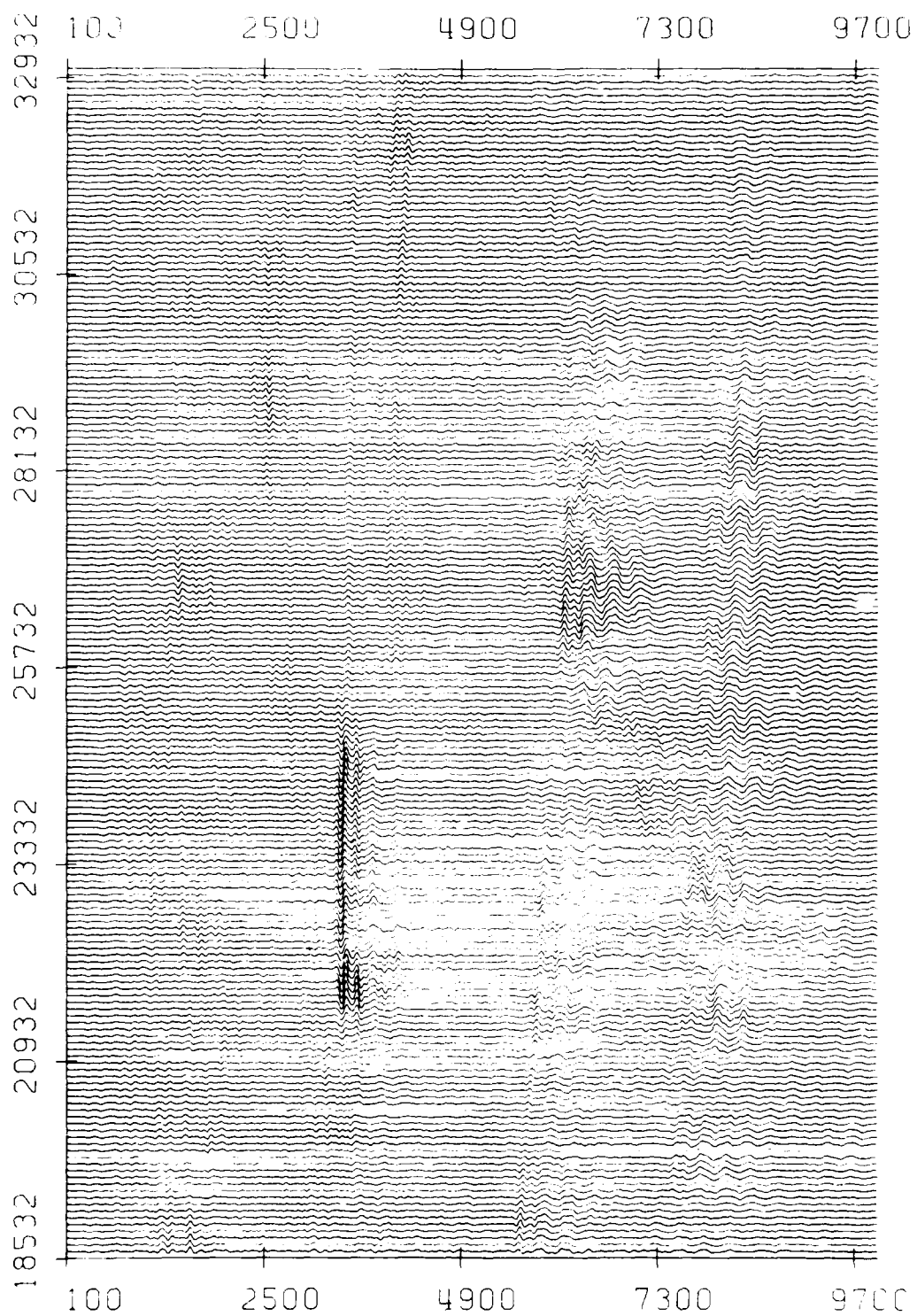
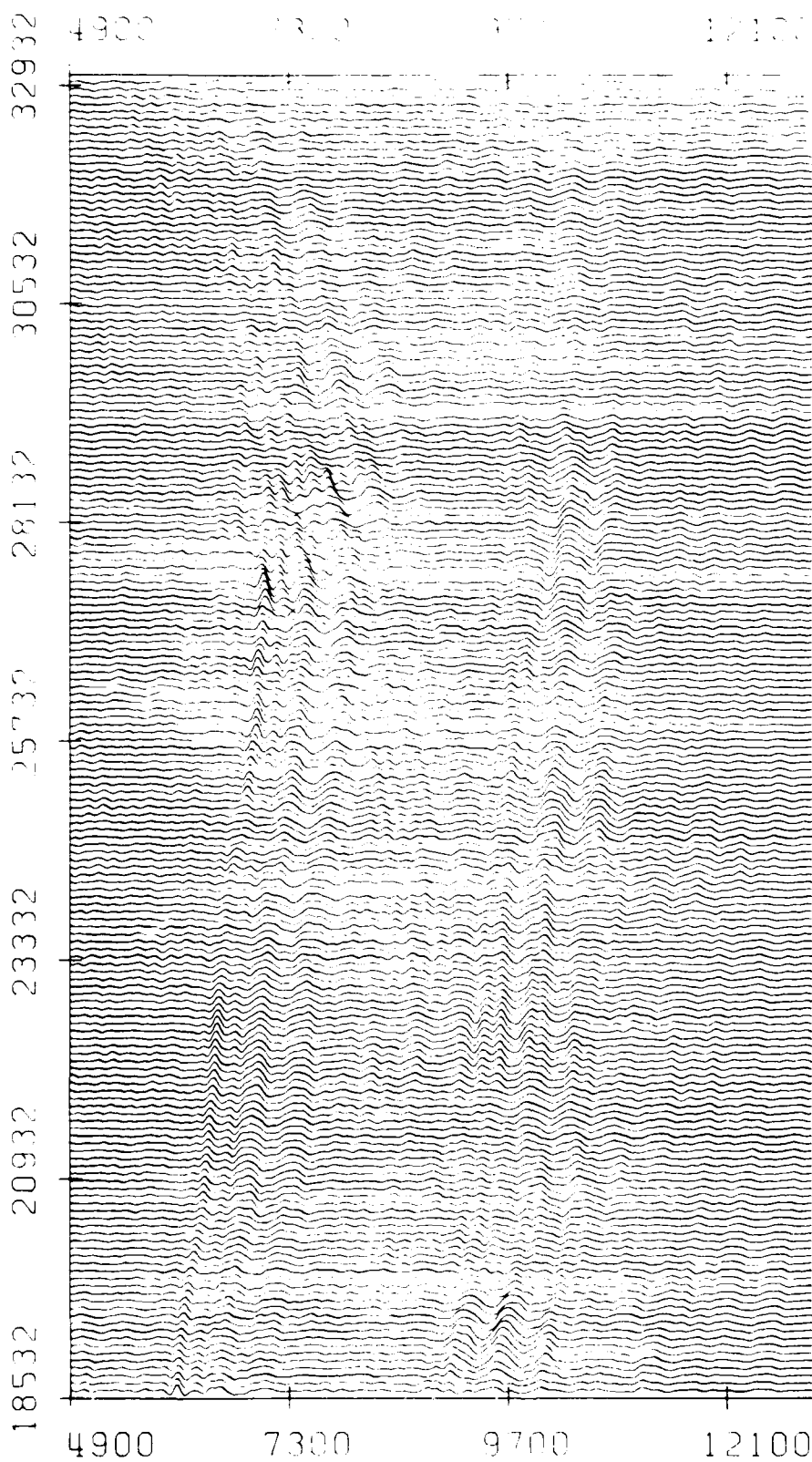DEPTH SECTION 100 - 10000 FEET

RIGHT HALF

**FIGURE 14**

DEPTH SECTION

LEFT HALF

**FIGURE 15**

FIGURE 16

DEPTH SECTION

RIGHT HALF

FIRST ORDER METHOD △
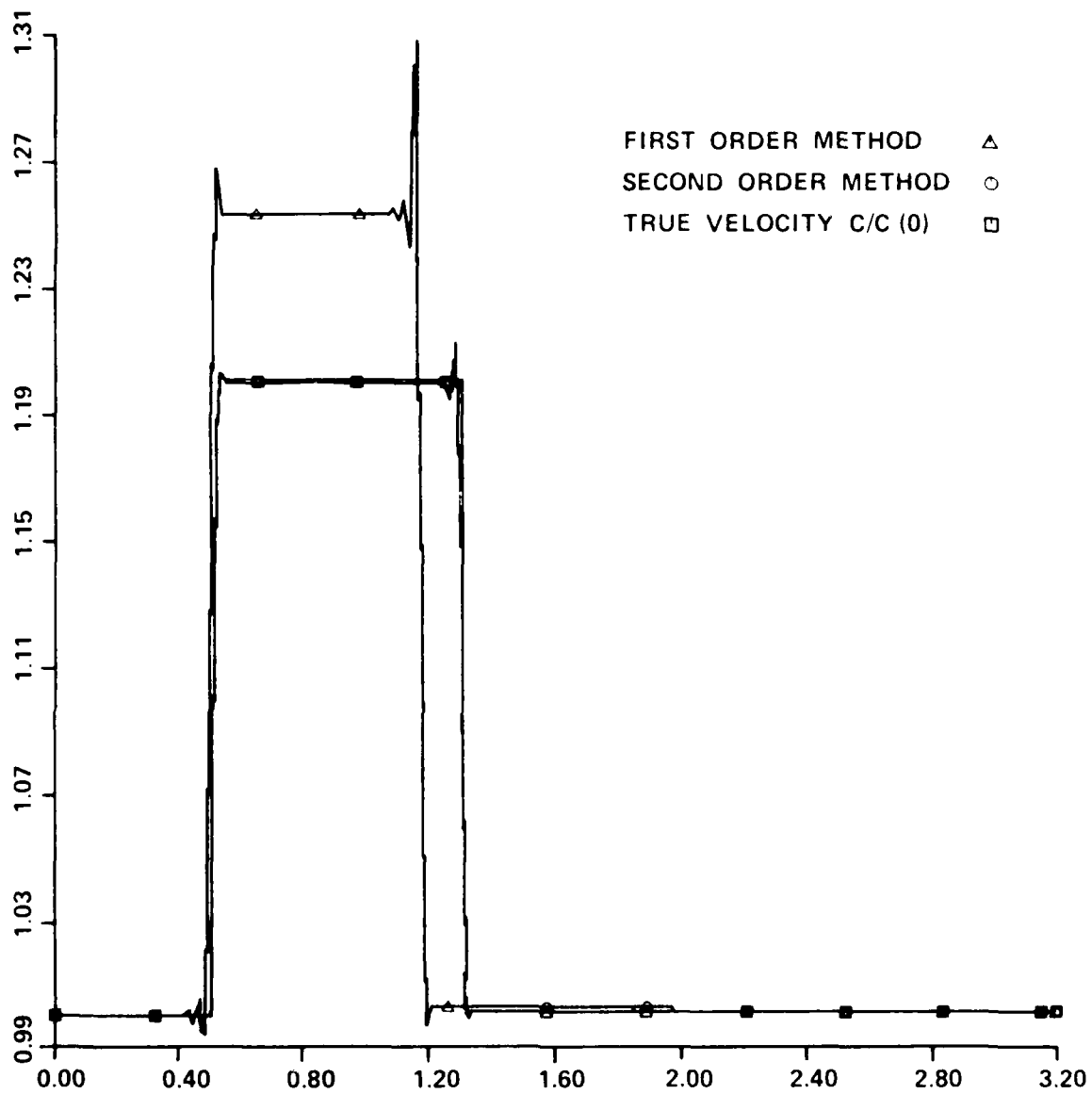SECOND ORDER METHOD ○
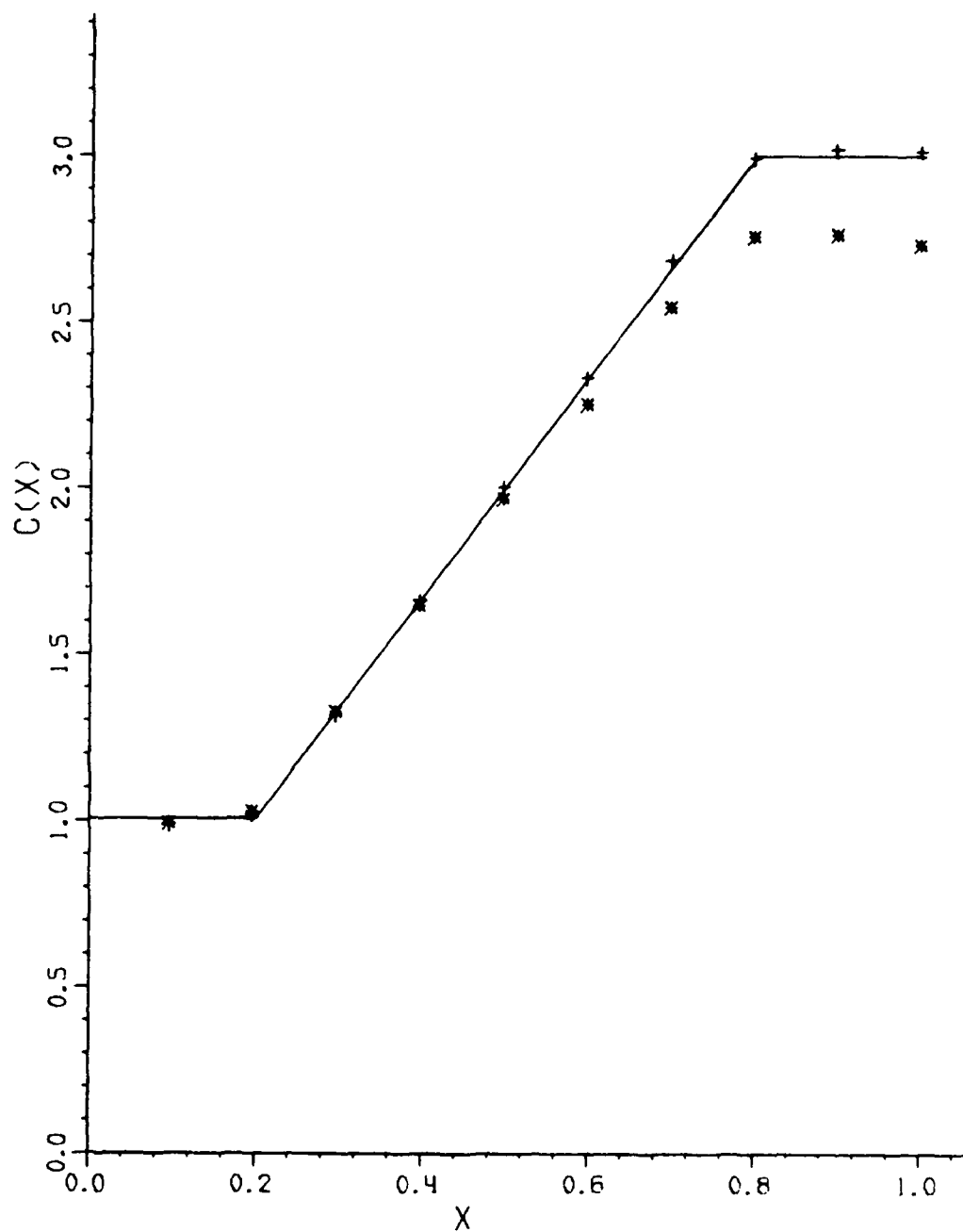TRUE VELOCITY C/C (0) □
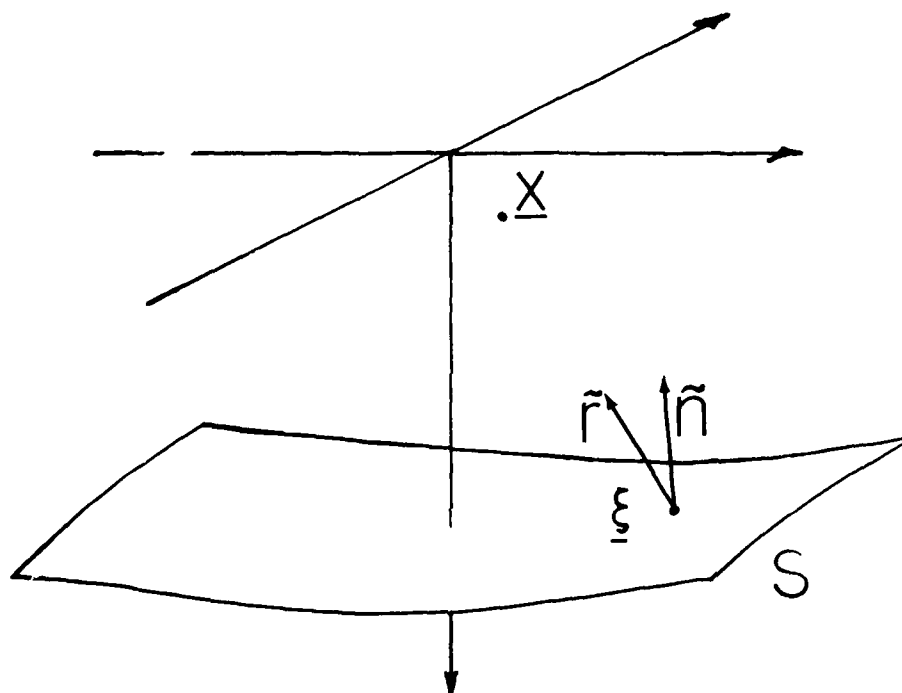
FIGURE 17

FIGURE 18

FIGURE 19

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| velocity inversion | geophysics |
| wave equation migration | velocity profile |
| seismic | |

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research program in seismic exploration in progress in the Mathematics
Department of the University of Denver is described. This work is identified
here by the term velocity inversion. The mathematical formulations employed
by this group are outlined and results of computer implementation are
depicted. Ongoing research is also presented.

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