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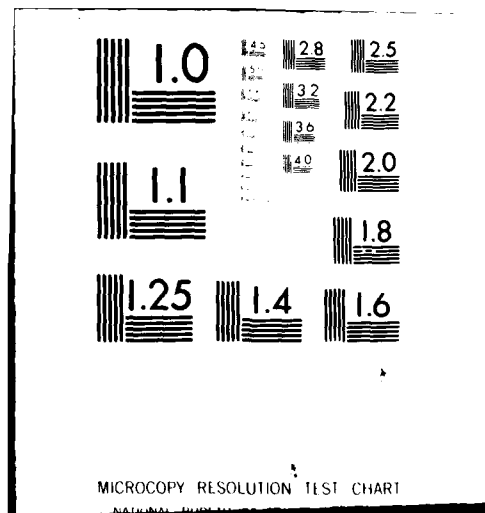
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UNDESIRABLE PERFORMANCE CHARACTERISTICS OF EXISTING MODEL-REFER--ETC(U)
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LIDS-P-1076 AFOSR-TR-81-0389 NL

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ITEM #20, CONTINUED:

content. Novel analytical studies based upon stability theory can be used to predict this high frequency oscillatory behavior, which depends both upon the amplitude and frequency of the reference input. (2) During the transient adaptation phase the excessive control loop bandwidth is detrimental to system performance, because it can excite unmodeled high-frequency dynamics and lead to instability. (3) Similar effects can occur in the presence of stochastic inputs (plant noise and measurement noise).

The analytical techniques that have been developed are constructive, so that modifications to the existing algorithms are suggested to overcome the practical shortcomings of the existing algorithms.

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UNDESIRABLE PERFORMANCE CHARACTERISTICS OF EXISTING
MODEL-REFERENCE ADAPTIVE CONTROL ALGORITHMS*

by

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SUMMARY

The full paper will describe recent theoretical results, backed up by digital computer simulation studies, that relate to the performance of several adaptive control algorithms that are based on model reference techniques. Special emphasis was placed upon the transient performance of these algorithms and the implications upon the bandwidth of the closed-loop adaptive system. The conclusions are as follows:

- (1) During the transient phase of the adaptation procedure, the control signal is characterized by excessive high frequency content. Novel analytical studies based upon stability theory can be used to predict this high frequency oscillatory behavior, which depends both upon the amplitude and frequency of the reference input.
- (2) During the transient adaptation phase the excessive control loop bandwidth is detrimental to system performance, because it can excite unmodeled high-frequency dynamics and lead to instability.
- (3) Similar effects can occur in the presence of stochastic inputs (plant noise and measurement noise).

The analytical techniques that have been developed are constructive, so that modifications to the existing algorithms are suggested to overcome the practical shortcomings of the existing algorithms. Full details will be given in the conference paper.

Submitted as short paper to 20th IEEE Conference on Decision and Control, San Diego, CA, December 1981.

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considerable magnitude, during the early stages of adaptation. Clearly, such high-frequency inputs are undesirable, as they will inevitably excite unmodeled dynamics that can in turn cause instability.

Secondly, it was also shown [2] that, in the presence of an unmodeled high-frequency pole, the adaptive system exhibited wildly oscillatory, and even unstable, closed loop behavior.

Lastly, the presence of even a small amount of observation noise prevented the closed loop system from converging to the desired model by causing it to slowly drift away to an increasingly higher bandwidth system. As a consequence, high frequency unmodeled dynamics will eventually be excited even at an advanced stage of the adaptation process. This increase in the bandwidth of the closed-loop system was also observed by Narendra and Peterson [4] in terms of a scalar example.

In order to obtain practical insights as well as analytical results, a simple scalar system was first studied in a systematic way. The equations that describe it are given below:

$$\text{Plant Dynamics: } \dot{y}(t) = -\alpha y(t) + \beta u(t) \quad (1.a)$$

$$\text{Model Dynamics: } \dot{y}_m(t) = -a y_m(t) + b r(t) \quad (1.b)$$

where the plant control is given by $u(t) = \theta_1(t)y(t) + \theta_2(t)r(t)$, with $r(t)$ being the reference input and $\theta_1(t)$, $\theta_2(t)$ adjustable (control) parameters used in the control signal synthesis. The first algorithm for the control parameter adaptation studied was that proposed by Narendra and Valavani [5]; its scalar version consists of the following system of

nonlinear differential equations for the output error, $e(t)$, and parameter errors $\phi_1(t)$, $\phi_2(t)$, as well as the plant state, $y(t)$.

$$\dot{y}(t) = -ay(t) + \beta u(t) \quad (2.a)$$

$$\dot{e}(t) = -ae(t) + b\phi_1(t)y(t) + b\phi_2(t)r(t) \quad (2.b)$$

$$\dot{\phi}_1(t) = -\gamma_{11}y(t)e(t) - \gamma_{12}e(t)r(t) \quad (2.c)$$

$$\dot{\phi}_2(t) = -\gamma_{21}y(t)e(t) - \gamma_{22}e(t)r(t) \quad (2.d)$$

where $e(t) = y_m(t) - y(t)$ and $\phi_1(t)$, $\phi_2(t)$ are the parameter errors; γ_{ij} , $i, j=1, 2$, are constants and such that $\gamma_{11} > 0$, $\gamma_{11}\gamma_{22} > \gamma_{12}\gamma_{21}$.

A stability-based time-varying analysis of the properties of the nonlinear time-varying system described by eqns. (2) resulted in the development of a new analytical method that explains and even predicts the behavior of the adaptive system observed in the simulations. The analytical results thus obtained are valid in the vector case as well; they also provide further insights into the mechanism of adaptation. Due to the inherent nonlinearity of the closed-loop, it was found that the behavior of the system is sensitive to both the magnitude of the reference input and the input frequencies. A large reference input magnitude can create arbitrarily high frequency content in the adjustable (control) parameters, $\phi_1(t)$ and $\phi_2(t)$, and, therefore, also in the control signal, $u(t)$.

Moreover, this analysis demonstrates that the parameter errors, $\phi_1(t)$ and $\phi_2(t)$, do not necessarily converge to zero but, rather, to a linear

subspace in the parameter space where the output error is identically zero. This subspace evolves in time, and its evolution is dependent on the time-varying characteristics of the reference input and the plant output. If the reference input is such that the subspace remains fixed, no changes in the parameters occur. If, however, the subspace varies with time, the parameter errors will approach it from a direction orthogonal to it. It is precisely here that the so called "richness" of the adaptive signals could play an important role, insofar as parameter error convergence is concerned. It is becoming increasingly apparent, however, that the closed loop, given the present algorithms, cannot satisfy the "richness" conditions required for asymptotic convergence. This has obvious implications for certain adaptive [6,7] schemes whose overall global stability hinges heavily on exact parameter convergence. On the other hand, stability results have been obtained recently for the above mentioned class of algorithms, irrespective of "sufficient excitation" and parameter convergence [8]; but they are valid only locally.

When the algorithm described in [5] was studied with an unmodeled high frequency pole, the same analytical results successfully explained the exhibited oscillatory, and eventually unstable, behavior and, moreover, demonstrated the same dependence of such behavior on the reference input.

In the case of observation noise, $n(t)$, our analysis shows that a term containing $n^2(t)$ will in effect keep driving the error system and, therefore, the parameter error can increase within the zero output subspace discussed above. But if the variance of the measurement noise is known, this term can be subtracted out. Unfortunately, the problem still remains

of white noise being input into a marginally stable system in the same subspace. However, since the noise also drives (affects) the time-evolution of the subspace, it may very well be the case that the decaying effect of the time evolution of the subspace on the parameter errors is enough to keep their variance bounded.

The analytical framework mentioned in the foregoing was employed to study different adaptive algorithms and consistency of the results was shown. Specifically, our study so far has concentrated on the algorithms obtained using a stability point of view for their derivation, the most important representatives of which are Monopoli's [9], Narendra, Valavani and Lin's [5,10], Feuer and Morse's [11], Landau and co-workers' [12], both for discrete as well as continuous-time systems. In the analysis, the above algorithms all shared the same characteristics, although some exhibited marginally better behavior in some numerical cases. For example, the algorithm described in [10] remained stable in the presence of one unmodeled pole, with increasing reference input amplitude, although the control signal was heavily oscillatory. However, the presence of two unmodeled high-frequency poles resulted in its instability. Conversely, the observation noise enters here in a much more complicated way than in the rest of the algorithms examined.

A class of adaptive algorithms which are currently being investigated in the framework established here is that containing Åström and Wittenmark's basic self-tuning regulator scheme [13] and those that followed it along the same lines, i.e. some schemes contained in [14], and the algorithms obtained

by Goodwin, Ramadge and Caines [15]. These are basically dead-beat algorithms designed for discrete-time systems. Although at first glance they seem to have some advantage over the others with respect to the high frequency content of the control input, we anticipate a much worse response in the presence of unmodeled dynamics. Work is currently in progress to verify this preliminary conjecture.

Finally, based on our theoretical results, we are developing modifications to the existing schemes that avoid the undesirable behavior described so far. The full paper will describe the results obtained.

A lot of future research is required along these lines, before adaptive algorithms can have practical impact, apart from the purely theoretical appeal they have enjoyed so far.

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