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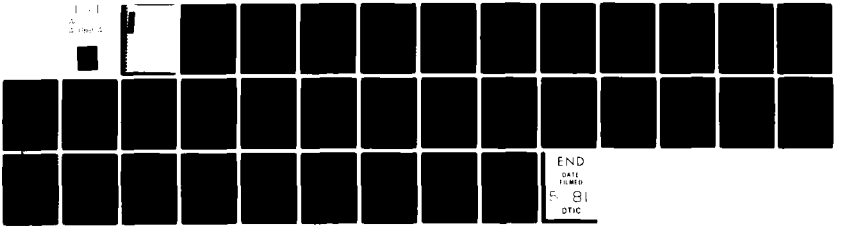

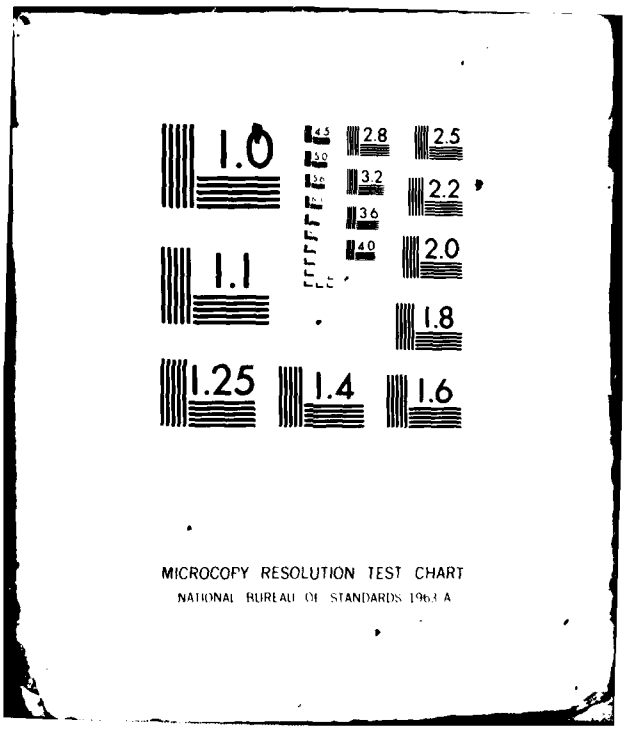
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(6) NOTES ON SOCIAL CHOICE AND VOTING

by

(10) Peter Coughlin

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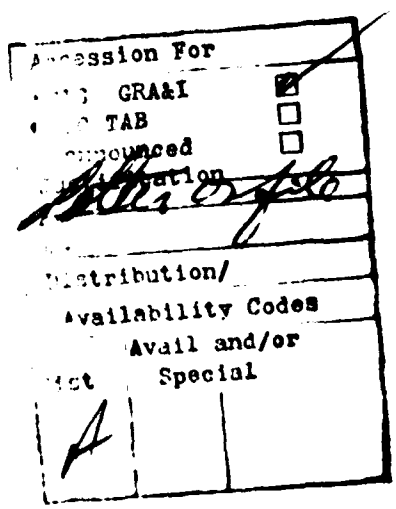
ABSTRACT

This technical report contains four short papers (or notes) on topics in the theory of social choice and voting. The first two develop results on majority rule with probabilistic voting. The last two develop results on relative majority rule and the Borda count, respectively.



NOTES ON SOCIAL CHOICE AND VOTING*

by

Peter Coughlin

I. MULTIDIMENSIONAL MEDIAN RANDOM VOTER RESULTS**

1. Introduction

The first spatial models of economic policy formation through elections (with unidimensional policy spaces) led to the basic median voter result: When a society has single-peaked preferences, there is a convergent equilibrium in pure strategies at the median (of the distribution of voters' "ideal points") for two candidates who maximize votes.

These first models were *generalized for societies* with multidimensional policy spaces by Davis and Hinich [1966], [1967], [1968], and [1971]. They showed that, given certain restrictions on voter preferences, if particular requirements are satisfied by the distribution of these preferences, then there is a convergent equilibrium in pure strategies at the median. This provided the basic "multidimensional median voter results."

Variations on these basic models have been developed and analyzed by both economic and political theorists (see Hinich [1977] for references). The recent work of Comaner [1976], Hinich [1977], [1978] and Kramer [1978]

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has been concerned with the implications of probabilistic voter choices. This generalizes the basic models to include random, non-policy factors in voting decisions.

Comaner [1976] and Hinich [1977] have shown that the one-dimensional median voter result does not hold in general when there is probabilistic voting. Kramer [1978] has subsequently argued that the one-dimensional median voter result is robust when the levels of indeterminancy in voter choices are small.

This paper shows (in Theorem 1) that, under conditions analogous to those originally studied by Davis and Hinich, multidimensional median voter results hold. This is true for any level of indeterminancy in voter choices.

2. Probabilistic Voting and Median Voter Results

The notation and assumptions here are based on Hinich [8].

$X \subset R^n$ will denote the society's set of alternatives (or policy space). An individual voter will be designated by i. Each voter's preferences will be summarized by a utility function, $u_i(\theta) = u_i(\theta; x_i)$, on $\theta \in X$, where x_i is an "ideal point" for individual i (i.e. $u_i(x_i) \geq u_i(\theta)$, $\forall\ \theta \in X$). The two candidates will be designated by $j \in \{1,2\}$. $\theta_j \in X$ will denote a policy proposed by candidate j. When considering a pair of proposals we will always order them as (θ_1, θ_2).

The probability that individual i votes for candidate j when θ_1 and θ_2 are proposed is specified to be

$$(1) \qquad \Pr\{i \text{ votes for } \theta_j\} = P_i(\log u_i(\theta_j; x_i) - \log u_i(\theta_k; x_i))$$

for $j, k \in \{1,2\}$ and $j \neq k$. P_i is assumed to be monotonically non-decreasing and we assume that everyone votes. This strictly generalizes the deterministic voting assumption (viz., i votes for j if $u_i(\theta_j) > u_i(\theta_k)$, $j, k \in \{1,2\}$, $j \neq k$) of the basic spatial models.

We also assume that candidates act to maximize their expected votes.

Davis and Hinich assumed that each individual i's utility function is

$$u_i(\theta; x_i) = \lambda - \|\theta - x_i\|^2_{A(i)} \tag{2}$$

where A(i) is a symmetric, positive definite, $(n \times n)$ matrix, and

$$\|\theta - x_i\|^2_{A(i)} = (x_i - \theta)' \cdot A(i) \cdot (x_1 - \theta) \tag{3}$$

$f(x)$ will be used to denote the density function which summarizes the distribution of voters' ideal points.

Davis and Hinich's first assumption was:

Assumption DH.1: There is some A such that $A(i) = A$ for every voter i and $f(x)$ is multivariate normal.

Their second assumption was:

Assumption DH.2: There is some $\mu \in R^n$ such that, for each A, $f(x_i | A(i) = A)$ is multivariate normal with the common μ. (I.e., μ is a common mean (and median) for all the sub-distributions of voter ideal points).

(2) can be weakened to

$$(4) \qquad u_i(\theta) = \phi_i(\|\theta - x_i\|^2_{A(i)})$$

where ϕ_i is concave. ϕ_c will denote a particular concave function on R_+. Additionally, we can weaken the assumption of a multivariate normal density function to the assumption of "radial symmetry" (viz., there is some "median" μ such that $f(x) = f(2\mu - x)$ for every $x \in X$).

Davis and Hinich have implicitly assumed that all voters have the same probabilistic voting function P_i (viz., the deterministic voting rule). We will let P denote a particular probabilistic voting function which is measurable (almost everywhere) in x for any (θ_1,θ_2).

A voter can be completely characterized by his ideal point x_i together with the triple $h(i) = (A(i),\phi_i,P_i)$. Hence, the most natural generalization of DH.2 to the above conditions is:

Assumption DH.3: There is some $\mu \in R^n$ such that, for each h, $f(x_i|h_i = h)$ is a discrete probability mass function or a continuous density function which is radially symmetric around the median μ. (I.e., μ is a common median for all of the sub-distributions of voter ideal points.)

Theorem 1: Suppose that DH.3 is satisfied by a probabilistic spatial voting model. Then (μ,μ) is an equilibrium in pure strategies for the candidates.

Remark: This also clearly yields the earlier multidimensional median voter results (with DH.1, DH.2 or the weakenings discussed above) as corollaries when there is deterministic voting.

Proof: Let $EV_1(\theta_1,\theta_2|h)$ denote the expected votes for candidate 1 from the citizens with $h(i)$ equal to a particular $h = (A,\phi_c,P)$. We will prove Theorem 1 by showing that, for any specified $h = (A,\phi_c,P)$,

$$(5) \qquad EV_1(\theta_1,\mu|h) \leq EV_1(\mu,\mu|h) \leq EV_1(\mu,\theta_2|h), \ \forall\ \theta_1,\theta_2 \in X$$

Partition X into $\{X_0,X_1,X_2\}$ so that $X_0 = \{\mu\}$, $x \in X_1 \Leftrightarrow 2\mu - x \in X_2$ and both P and $f(x|h)$ are measurable (almost everywhere) in x on X_1 and X_2. We will also let $U(\theta; x)$ denote $\log \phi_c(\|\theta - x\|_A^2)$. Then,

$$(6) \qquad EV_1(\theta_1,\mu|h) = \sum_{k=0}^{2} \int_{X_k} P(U(\theta_1; x) - U(\mu; x)) \cdot f(x|h) \cdot dx$$

If $f(x|h)$ is continuous, then the first term in (6) equals zero. If $f(x|h)$ is discrete, then, since P is monotonically non-decreasing, the first term satisfies

$$(7) \qquad P(U(\theta_1; \mu) - U(\mu; \mu)) \cdot f(\mu|h) \leq P(0) \cdot f(\mu|h) = \frac{1}{2} \cdot f(\mu|h)$$

for every $\theta_1 \in X$.

Consider any $x \in X_1$ and the corresponding $z = 2 \cdot \mu - x \in X_2$. We have $\|\theta - z\|_A = \|z - \theta\|_A = \|x + z - \theta - x\|_A$. Therefore, $U(\theta; z) = \log \phi_c(\|\theta - z\|_A) = \log \phi_c(\|2\mu - \theta - x\|_A) = U(2\mu - \theta; x)$. Consequently, the last two terms of (6) sum to

$$(8) \qquad T_2(\theta_1,\mu) = \int_{X_1} [P(U(\theta_1; x) - U(\mu; x)) + P(U(2\mu - \theta_1; x) - U(\mu; x))] \cdot f(x|h) \cdot dx$$

Now, since everyone votes, (1) implies

(9) $P(U(\theta; x) - U(\mu; x)) + P(U(\mu; x) - U(\theta; x)) = 1$

Additionally, since $\|\theta - x_i\|^2_{A(i)}$, $\phi_i(\cdot)$ and $\log(\cdot)$ are concave functions, $U(\theta)$ is concave. Therefore, $U(\mu; x) \geq 1/2\, U(\theta; x) + 1/2\, U(2\mu - \theta; x)$. So $2 \cdot U(\mu; x) \geq U(\theta; x) + U(2\mu - \theta; x)$. So,

(10) $U(\mu; x) - U(2\mu - \theta; x) \geq U(\theta; x) - U(\theta; x)$.

Hence, since P is monotonically non-decreasing,

$$P(U(\mu; x) - U(2\mu - \theta; x)) \geq P(U(\theta; x) - U(\mu; x)) ,$$

by (10). Therefore, letting $\theta = 2\mu - \theta_1$ in (9), we obtain

$$P(U(\theta_1; x) - U(\mu; x)) + P(U(2\mu - \theta_1; x) - U(\mu; x)) \leq 1 .$$

Consequently, (8) implies

(11) $T_2(\theta_1, \mu) \leq \int_{X_1} f(x|h) \cdot dx = \frac{1}{2}\left[\int_{X_1} f(x|h) \cdot dx + \int_{X_2} f(x|h) \cdot dx\right]$.

Finally, (9) implies $EV(\mu, \mu|h) = 1/2$. Therefore, by (7) and (11),

$$EV_1(\theta_1, \mu|h) \leq EV_1(\mu, \mu|h) = \frac{1}{2} , \quad \forall\, \theta_1 \in X .$$

A similar argument establishes

$$EV_1(\mu, \theta_2|h) \geq EV_1(\mu, \mu|h) = \frac{1}{2} , \quad \forall\, \theta_2 \in X . \qquad \text{Q.E.D.}$$

3. Conclusion

It should be observed that (as in all of the papers on multidimensional median voter results) the conditions which have been shown here to be sufficient for the existence of a global electoral equilibrium at the median are very restrictive and can hardly be expected to be satisfied in most empirical situations. Additionally, it is easy to show, by example, that similar conditions which are analogous to sufficient conditions for multidimensional median voter results which have been developed elsewhere do not guarantee such results when voting is probabilistic. Finally, since the purpose of this paper is to show that the multidimensional median voter results of Davis and Hinich hold under analogous conditions in probabilistic spatial voting models, it still leaves open the possibility that there are electoral equilibria elsewhere in the policy space under other conditions--for instance, even when the distribution of preferences is not symmetric.

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II. PARETO OPTIMALITY OF ELECTORAL TRAJECTORIES IN PROBABILISTIC VOTING MODELS*

1. Introduction

Public choice theorists have long been interested in whether outcomes from majority rule and electoral decision processes are Pareto optimal or not (e.g., Cohen [1979], Kramer [1977], McKelvey [1979], Ordeshook [1971], and Wittman [1977]). Most of the existing literature on this question has been concerned with societies in which each individual always votes for the candidate whose proposed policy has the greatest utility for that individual. However, public choice theorists have also been interested in electorates where there is a positive probability that a citizen, drawn from a collection of individuals with a common utility function or ideal point, will vote against the candidate whose proposed policy has the greatest utility for him or will abstain. This formulation incorporates indifference, alienation and non-policy factors into voter decisions.

Electoral equilibria have been shown to exist in societies with the first type of electorate only when special symmetry assumptions are satisfied. Similarly, electoral equilibria have been shown to exist in societies with the second type of electorate only when special symmmetry or concavity assumptions have been made (e.g., Denzau and Kats [1977], Hinich, Ledyard and Ordeshook [1972], [1973] and Hinich and

* I would like to acknowledge helpful comments and suggestions from Ken Arrow, Gene Mumy, Shmuel Nitzan, Kotaro Suzumura and an anonymous referee.

Ordeshook [1971]). However, these special assumptions have been criticized for being unduly restrictive. As a result, the outcomes from sequences of social decision processes have subsequently been studied (rather than static equilibria).

This paper considers the Pareto optimality properties of electoral outcomes from sequences of elections in societies with the second type of electorate (without any symmetry or concavity assumptions). It shows that, if challengers maximize their expected pluralities or their expected votes, then the sequence of electoral choices will be in the Pareto set within one step, and will remain in that set ever after. This is the opposite of the established Pareto non-optimality of basic majority rule trajectories for electorates of the first type (e.g., Cohen [1979], McKelvey [1979]). It is also stronger than the Pareto optimality properties of electoral trajectories for these electorates (e.g., Kramer [1977] and Wittman [1977]).

2. Dynamical Probabilistic Voting Models

An electorate is

(1) a set, S, of feasible social alternatives,

(2) an index set A such that for each $\alpha \in A$ there is an associated utility function $U_\alpha : S \to R_+$;

(3) a probability measure space $(A, \mathcal{A}, \mu)$.

This formulation includes both finite and continuous populations as special cases, of course.

A <u>two-candidate contest</u> with <u>probabilistic voting</u> is

(1) an electorate

(2) two candidates indexed by $i \in C = \{1,2\}$.

(3) a function for each $i \in C$, $P_\alpha^i: S \times S \to [0,1]$.

We will use θ_i to denote a policy proposed by candidate $i \in C$, and the ordered pair (θ_1,θ_2) to represent a pair of policies proposed by the two candidates. $P_\alpha^i(\theta_1,\theta_2)$ denotes the probability that an individual randomly drawn from the collection of individuals indexed by α will vote for candidate i when θ_1 and θ_2 are proposed.

Hinich, Ledyard and Ordeshook [1973] discussed the behavioral heuristics which should be expressed in assumptions about voters' choices. Aggregate assumptions which correspond to their formalizations of these heuristics are:

<u>Assumption 1</u>: For $i \in C$, P_α^i is a strictly monotone increasing function of $U_\alpha(\theta_i)$.

<u>Assumption 2</u>: For $i \in C$, P_α^i is a monotone decreasing function of $U_\alpha(\theta_j)$ for $j \in C$, $j \neq i$.

Assumption 1 does exclude certain models with deterministic voting where the P_α^i move in steps from 0 to 1/2 to 1 (e.g., the basic assumptions in Kramer [1977]). However, it could be satisfied if everyone votes deterministically, but for each α there is a continuous distribution of utility differences at which the individuals switch their votes. It will

also be satisfied if most citizens vote deterministically, but for each α there is at least one individual who votes probabilistically (and satisfies the corresponding assumption in Hinich, Ledyard and Ordeshook [1973]). Assumptions 1 and 2 merely express the idea that voters' behavior should be minimally responsive to changes in proposals which would lead to changes in voters' utilities. This response need be nothing more than a change in the likelihoods of abstentions.

Kramer [1977] has developed a dynamical model for studying sequences of elections. The analogous dynamical process of policy formation here is as follows. Let the society be at any feasible status quo, $s_0 \in S$, with some incumbent in office. In each election, the challenger proposes a feasible alternative and the incumbent must defend the status quo. The winner's proposal then becomes the status quo. Any sequence (s_j), where s_j is selected by a challenger against s_{j-1}, is an <u>electoral trajectory</u> of policies which can occur. It should be observed that the sequence is not indexed by time (unlike Kramer [1977]) since the number of times an incumbent wins is randomly determined.

Candidates could be concerned with maximizing their expected votes, their expected pluralities or their probabilities of winning. However, Hinich [1977] has shown that the last two are equivalent when there is a large population. Therefore, only the first two will be considered here.

3. One-Step Pareto Optimality

The Pareto relation R is defined by $(x,y) \in R \Leftrightarrow U_\alpha(x) \geq U_\alpha(y)$ a.e. in A. An alternative $x \in S$ is in the Pareto set means $\forall\, y \in S$: $(y,x) \notin R$ or $(x,y) \in R$. The strong Pareto relation R_S is defined by $(x,y) \in R_S \Leftrightarrow$ $(x,y) \in R$ and $[\exists\, A_1 \in A$: $\mu(A_1) > 0$ and $U_\alpha(x) > U_\alpha(y)$ a.e. in $A_1]$. An alternative $x \in S$ is in the strong Pareto set means $\forall\, y \in S$: $(y,x) \notin R_S$. Of course, if x is in the strong Pareto set it is also in the Pareto set. These definitions are from Hildenbrand [1969] and [1974].

Theorem: Let s_0 be any initial policy. If there is a sequence of two-candidate contests with probabilistic voting which satisfies Assumptions 1 and 2 and challengers maximize their expected votes or their expected pluralities, then any s_t, $t = 1,2,\ldots$, in any electoral trajectory is in the strong Pareto set.

Proof: Suppose, without any loss of generality, that candidate 2 is the incumbent and the status quo is s_{t-1}. Then the expected votes which the candidates get at the possible proposals of candidate 1 are given by

$$EV_i(\theta_1, s_{t-1}) = \int_A P_\alpha^i(\theta_1, s_{t-1}) \cdot d\mu(\alpha) \quad \text{for} \quad i \in C \ .$$

Suppose that s_t is not in the strong Pareto set. Then $\exists\, y \in S$: $(y,s_t) \in R_S$. Therefore, $U_\alpha(y) \geq U_\alpha(s_t)$ a.e. in A and $\exists\, A_1 \in A$: $\mu(A_1) > 0$ and $U_\alpha(y) > U_\alpha(s_t)$ a.e. in A_1. Therefore, by Assumption 1, $P_\alpha^1(y,s_{t-1}) \geq$

$P^1_\alpha(s_t, s_{t-1})$ a.e. in A and $P^1_\alpha(y, s_{t-1}) > P^1_\alpha(s_t, s_{t-1})$ a.e. in

Therefore,

$$EV_1(y, s_{t-1}) = \int_{A_1} P^1_\alpha(y, s_{t-1}) \cdot d\mu(\alpha) + \int_{A \setminus A_1} P^1_\alpha(s_t, s_{t-1}) \cdot d\mu(\alpha)$$

$$> \int_{A_1} P^1_\alpha(s_t, s_{t-1}) \cdot d\mu(\alpha) + \int_{A \setminus A_1} P^1_\alpha(s_t, s_{t-1}) \cdot d\mu(\alpha) \tag{1}$$

$$= EV_1(s_t, s_{t-1})$$

i.e., candidate 1 did not maximize his expected votes.

The expected pluralities which candidate 1 could get are given by $EPl_1(\theta_1, s_{t-1}) = EV_1(\theta_1, s_{t-1}) - EV_2(\theta_1, s_{t-1})$. Now, Assumption 2 implies $P^2_\alpha(y, s_{t-1}) \leq P^2_\alpha(s_t, s_{t-1})$ a.e. in A. Hence $EV_2(y, s_{t-1}) \geq EV_2(s_t, s_{t-1})$. Therefore, by (1),

$$EPl_1(y, s_{t-1}) = EV_1(y, s_{t-1}) - EV_2(y, s_{t-1})$$

$$> EV_1(s_t, s_{t-1}) - EV_2(s_t, s_{t-1})$$

$$= EPl_1(s_t, s_{t-1}) .$$

i.e., candidate 1 did not maximize his expected plurality. Q.E.D.

Remark 1: In the case of a finite electorate, Assumptions 1 and 2 imply that EV_1 and EPl_1 are functions of $U_1(\theta_1), \ldots, U_n(\theta_1)$ and are strictly increasing in each of these arguments. I.e., for any s_{t-1}, EV_1 and EPl_1 are Bergson-Samuelson Social Welfare functions whose optima are well known to be Pareto optimal.

Remark 2: If we relax Assumption 1 to non-strict monotonicity (as in Denzau and Kats [1977]), then the standard deterministic voting models are included. It is therefore easy to show, by example, that there is not an analogous theorem with the same strong Pareto optimality property for this case.

The Pareto optimality properties established in this theorem are stronger than those in Kramer [1977] since in his framework (i) from any status quo which is not Pareto optimal, every subsequent social choice is closer to the Pareto optimal set, but every choice could be outside of this set or the society may take a large finite number of steps to reach this set, and (ii) a Pareto optimal alternative may be replaced by one which is not Pareto optimal.

While strong Pareto optimality properties have been established here, it should be remarked that in most of these models the Pareto set is often quite large. Therefore, it should also be remarked that the theorem in this paper is not a direct extension of Kramer's main results. Finally, since this paper has been concerned with the Pareto optimality properties of the electoral trajectories defined in Section 2, it still leaves open the questions of whether they converge to a small subset of the strong Pareto set and whether they have further optimality properties.

4. Conclusion

This paper has answered the question of whether electoral outcomes from sequences of elections are Pareto optimal when Assumptions 1 and 2 are satisfied and challengers maximize their expected votes or their expected pluralities. Its most important point is the answer that the social choices move into the strong Pareto set within one step--never to leave again.

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III. NECESSARY AND SUFFICIENT CONDITIONS FOR δ-RELATIVE MAJORITY VOTING EQUILIBRIA*

Greenberg [1979] recently provided necessary and sufficient conditions on absolute majority rules for the existence of a voting equilibrium for every possible profile in a society (Theorems 1 and 2). He then provided a sufficient condition for the existence of a relative majority voting equilibrium for all possible profiles in a society (Theorem 3) and showed that this condition is necessary when there is exactly one more citizen than dimension in the policy space (Theorem 4). This led him to conclude that "no bound...lower than (the given sufficient conditions) will, in general, assure the existence of a δ-relative equilibrium" (p. 632).

The theorem below provides the necessary and sufficient condition which the relative majority rule must satisfy for the existence of a δ-relative majority voting equilibrium for every possible profile in a society. This bound is derived from Greenberg's first two theorems (under the exact assumptions used in his results on δ-relative equilibria).

This theorem shows that the sufficient bound in Greenberg [1979] is, in fact, not necessary in most societies with "large" populations (i.e. where the number of citizens exceeds the number of dimensions in the society's policy space). It also specifies which of these societies do have Greenberg's sufficient conditions as a necessary condition. This theorem also implies that Greenberg's sufficient condition is necessary when the number of

*I would like to acknowledge helpful comments and suggestions from Jerry Green, Ken Arrow and an anonymous referee.

citizens does not exceed the number of dimensions in the policy space by more than one. Finally, Theorems 3 and 4 in Greenberg [1979] follow as corollaries of this theorem.

The reader is referred to Greenberg [1979] for notation and definitions.

The following assumptions were used in Greenberg's results on δ-relative equilibria:

(A) X is a nonempty, compact and convex subset of R^m of dimension m, and

(B) for each $i \in N$, there is a continuous, strictly quasi-concave utility function defined over X.

We will additionally use $[a]$ to denote the greatest integer $\leq a$ and $\{a\}$ to mean the smallest integer $\geq a$.

Theorem: Suppose that a society's alternative space satisfies (A). Then there exists a δ-relative equilibrium for every profile which satisfies (B) if and only if

$$\delta \geq \frac{[(m \cdot n)/(m+1)]}{\{n/(m+1)\}} \quad . \tag{1}$$

Proof: First, by Theorems 1 and 2 in Greenberg [1979], there exists a d-majority equilibrium for every profile if and only if

$$d > \frac{m}{m+1} \cdot n \quad . \tag{2}$$

Second, by definition, a profile has a δ-relative majority equilibrium if and only if $\exists x \in X$ such that $p(x,y) \leq \delta \cdot p(x,y)$, $\forall y$ or equivalently, $p(x,y) \cdot (1 + \delta) \leq \delta \cdot (n - i(x,y))$, $\forall y$. Hence, a particular profile has a δ-relative majority equilibrium if and only if there is some $x \in X$ such that

$$p(x,y) \leq \frac{\delta}{1 + \delta} \cdot (n - i(x,y)) \quad , \quad \forall y \quad . \tag{3}$$

We will now show that there exists a δ-relative majority equilibrium if and only if

$$\frac{\delta}{1 + \delta} \cdot n \geq \left[\frac{m}{m + 1} \cdot n\right] \quad . \tag{4}$$

First, "if." Assume $\delta/(1 + \delta) \cdot n \geq [m/((m + 1) \cdot n)]$. Then, since $p(x,y)$ is an integer, (2) implies that for each profile there is a d-equilibrium x (for some $d = (m/((m + 1) \cdot n)) + \varepsilon$, $\varepsilon > 0$) which satisfies

$$p(x,y) \leq \left[\frac{m}{m + 1} \cdot n\right] \leq \frac{\delta}{1 + \delta} \cdot n \quad , \quad \forall y \quad . \tag{5}$$

We will show that, for each profile, this d-equilibrium is also a δ-relative equilibrium. Suppose not. Then there is a profile for which x has $p(x,y) > (\delta/(1 + \delta)) \cdot (n - i(x,y))$ for some $y \in X$ (by (3)). Define $z = 1/2(x) + 1/2(y)$. Then, since the individuals' utility functions are strictly quasi-concave, $p(x,z) = p(x,y) + i(x,y)$. Therefore, $p(x,z) > ((\delta/(1 + \delta)) \cdot (n - i(x,y))) + i(x,y) = (\delta/(1 + \delta)) \cdot n + (1/(1 + \delta)) \cdot i(x,y)$.

But, since $\delta > 0$ and $i(x,y) \geq 0$, this contradicts (5).

Now, "only if." Assume there is a δ-relative majority equilibrium for every profile. Then, for each profile, there is an $x \in X$ such that $p(x,y) \leq ((\delta/(1 + \delta)) \cdot n) - ((\delta/(1 + \delta)) \cdot i(x,y))$, $\forall$ y.

Therefore, for each profile there is an $x \in X$ such that $p(x,y) \leq \delta/(1 + \delta) \cdot n$, $\forall$ y. but, by (2), this occurs only if $(\delta/(1 + \delta)) \cdot n \leq [m/(m + 1) \cdot n]$.

The theorem now follows directly from (4) by using the equality

$$[\frac{m}{m + 1} \cdot n] + \{\frac{n}{m + 1}\} = n \quad .$$

Q.E.D.

Remark: This theorem implies the following. If $n \geq m + 1$, then $\delta \geq m$ is a necessary and sufficient condition for the existence of a δ-relative equilibrium for all profiles satisfying (B) if and only if $n = k \cdot ((m + 1)/m)$ for some integer k. Otherwise some $\delta < m$ are sufficient. If $m + 1 \geq n$, then $\delta \geq n - 1$ is a necessary and sufficient condition. Finally, Theorems 3 and 4 in Greenberg [1979] follow directly from this result.

Reference

Greenberg, J. [1979], "Consistent Majority Rules Over Compact Sets of Alternatives," Econometrica, 47, 627-636.

IV. A DIRECT CHARACTERIZATION OF BLACK'S FIRST BORDA COUNT*

The Borda rule has been studied as a possible method for aggregating individual preferences (e.g., see the references). Black [1976], in particular, has proven an important theorem which provides a justification and interpretation of the Borda count in terms of a majority rule. This note shows that an equivalent theorem can be proven directly with his Method I (which he has neglected because "it is clumsier and needs much more labour") by using two characteristic functions. We first recall the notation of Black and then prove our assertion. The note closes by obtaining Black's original theorem as a corollary and giving an alternative "closeness to unanimity" interpretation.

Let $N = \{1,\ldots,n\}$ be a finite set of individuals and $A = \{a_1,\ldots,a_m\}$ a finite set of alternatives. Let $R_i \subset A \times A$ denote a preference ordering on A for individual i, with P_i and I_i the asymmetric (strict preference) and symmetric (indifference) parts of R_i. $\#S$ will denote the cardinality of the set S.

The Borda count for a particular $a_h \in A$ is defined as follows. Let $r_i = \#\{a_k \in A | (a_h,a_k) \in P_i\}$ and $s_i = \#\{a_k \in A | (a_h,a_k) \in I_i\}$. Then <u>i's Borda count for</u> a_h is $B'_i(a_h) = r_i + 1/2(s_i - 1)$ (as in Black's Method I), and Black's first <u>Borda count for</u> a_h is $B'(a_h) = \sum_{i \in N} B'_i(a_h)$.

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*I would like to thank Kotaro Suzumura for suggesting this problem to me.

An alternative a_k is a Borda choice (i.e., is in the Borda choice set) iff $B'(a_k) \geq B'(a_h)$ for all $a_h \in A$.

Black also defined a fraction for use in a majority rule. Let $p_{hk} = \#\{i \in N | (a_h, a_k) \in P_i\}$ and $t_{hk} = \#\{i \in N | (a_h, a_k) \in I_i\}$. Then Black's majority rule fraction is $f_{hk} = (p_{hk} + 1/2 \cdot t_{hk})/n$. The mean fraction of votes for a_h is given by

$$F(a_h) = \frac{1}{m-1} \cdot \sum_{a_k \in A_h} f_{hk} \tag{1}$$

where $A_h = A \backslash \{a_h\}$.

We will additionally define two characteristic functions for a specified $a_h \in A$:

$$\chi_P(a_k, i) = \begin{cases} 1 & \text{if } (a_h, a_k) \in P_i \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$\chi_I(a_k, i) = \begin{cases} 1 & \text{if } (a_h, a_k) \in I_i \\ 0 & \text{otherwise} \end{cases} . \tag{3}$$

Using this alternative notation, $r_i = \sum_{a_k \in A} \chi_P(a_k, i)$ and $s_i = \sum_{a_k \in A} \chi_I(a_k, i)$. Therefore,

$$(4) \qquad B'(a_h) = \sum_{i \in N} \left[\sum_{a_k \in A} \chi_P(a_k,i) + \frac{1}{2} \cdot \left(\sum_{a_k \in A} [\chi_I(a_k,i)] - 1 \right) \right] .$$

We also could have introduced the alternative definitions: $p_{hk} = \sum_{i \in N} \chi_P(a_k,i)$ and $t_{hk} = \sum_{i \in N} \chi_I(a_k,i)$. We then have, by (1),

$$(5) \qquad F(a_h) = \frac{1}{m-1} \cdot \sum_{a_k \in A_h} \left[\left[\sum_{i \in N} \chi_P(a_k,i) + \frac{1}{2} \cdot \sum_{i \in N} \chi_I(a_k,i) \right] / n \right] .$$

Using these alternative definitions we can directly prove:

Theorem: For any $a_h \in A$, $B'(a_h) = n \cdot (m-1) \cdot F(a_h)$.

Proof: By (5),

$$F(a_h) = \frac{1}{(m-1) \cdot n} \cdot \sum_{a_k \in A_h} \sum_{i \in N} [\chi_P(a_k,i) + \frac{1}{2} \cdot \chi_I(a_k,i)] .$$

By (4),

$$B'(a_h) = \sum_{i \in N} \sum_{a_k \in A_h} [\chi_P(a_k,i) + \frac{1}{2} \cdot \chi_I(a_k,i)]$$

since by (3), $\chi_I(a_h,i) = 1$ and $\chi_P(a_h,i) = 0$ for every i.

Therefore, $B'(a_h) = (m-1) \cdot n \cdot F(a_h)$. Q.E.D.

Black's second Borda count for a_h was

$$(6) \quad B^2(a_h) = \sum_{i \in N} [\#\{a_k \in A | (a_h, a_k) \in P_i\} - \#\{a_k \in A | (a_k, a_h) \in P_i\}] \ .$$

Corollary [Black's Theorem ([1976], p. 9)]: For any $a_h \in A$, $B^2(a_h) = 2 \cdot n \cdot (m - 1) \cdot (F(a_h) - 1/2)$.

Proof: Black ([1976], p. 6) showed: $B^2(a_h) = 2 \cdot B'(a_h) - n \cdot (m - 1)$. Q.E.D.

Finally, unanimous support for a_h against any other proposal gives $F(a_h) = 1$ and unanimous support against a_h for every other proposal gives $F(a_h) = 0$. Therefore, $F(a_h)$ gives a measure of "closeness to unanimity" which differs computationally from the one proposed by Farkas and Nitzan [1979]. The conclusion is the same:

Corollary: *Let $F(a_h)$ measure "closeness to unanimity." Then* $a_k \in A$ is the proposal which is closest to unanimity.

Proof: By our theorem, "closeness to unanimity" (i.e., $F(a_h)$) is a positive linear function of $B'(a_h)$. Q.E.D.

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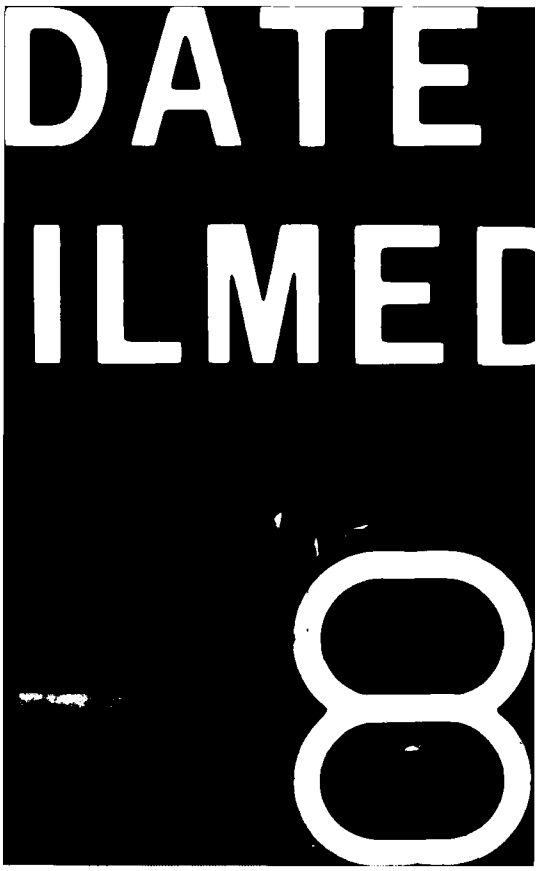
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