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TWO PAPERS ON MAJORITY RULE:

CONTINUITY PROPERTIES OF MAJORITY RULE
WITH INTERMEDIATE PREFERENCES.

by

~~Peter Coughlin and Kuan-Pin Lin~~

and

ELECTORAL OUTCOMES WITH PROBABILISTIC VOTING
AND NASH SOCIAL WELFARE MAXIMA.

by

10 Peter Coughlin and Shmuel Nitzan Kuan-Pin Lin

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ABSTRACT

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This technical report contains two joint (or co-authored) papers on aspects of majority rule.

The first (with Kuan-Pin Lin) studies the continuity properties of majority rule. Specifically, it shows that certain conditions which have previously been shown (by Grandmont) to be sufficient for a society's majority rule relation to be transitive or acyclic are also sufficient for the map from distributions of voter preferences to indices identified with their majority rule relations to be continuous. Applications of this result to societies with certain classical assumptions on preferences reveal that, in such societies, the map from distributions of voter preferences to their majority rule equilibria is also continuous.

The second (with Shmuel Nitzan) analyzes outcomes from electoral competitions with a Luce model of probabilistic voting. These outcomes are shown to be precisely the social alternatives that maximize a Nash-type social welfare function. These outcomes are also shown to be unanimity likelihood maxima when voting is independent. Finally, we show that the model's assumptions also imply the existence and uniqueness of electoral equilibria.

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WITH INTERMEDIATE PREFERENCES

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CONTINUITY PROPERTIES OF MAJORITY RULE
WITH INTERMEDIATE PREFERENCES*

by

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1. Introduction

Grandmont [1978] recently provided a general possibility theorem for majority rule. The theorem assumes two conditions on individual preferences (based on Chichilnisky [1976]) and one on the distribution of these preferences in a society (based on Davis, DeGroot and Hinich [1972]). It states that these three conditions imply that the society's majority rule relation is a particular ("median") alternative which is in the same family of individual relations as the domain of the society's distribution of individual preferences. This, in turn, implies that the society's majority rule relation is transitive or acyclic whenever this domain contains only transitive or acyclic individual preference relations. This paper shows that the assumptions in Grandmont [1978] also imply a continuity property for majority rule (Theorem 1).

The theorem in this paper is closely related to the results of Chichilnisky [1976], [1980 (a)-(b)]. She has shown that, with an unrestricted domain of preferences, it is impossible for a social choice rule which satisfies unanimity and anonymity (e.g. majority rule) to be continuous (viz., as a function from collections of individual preferences to a social preference). However, this paper proves, under the same domain restrictions which Grandmont

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[1978] has shown to be sufficient for a transitive or acyclic majority rule relation, the method of majority decision is a continuous social choice rule (viz., as a function from distributions of indices identified with individuals preference relations to indices identified with their majority rule relations).

In Section 4, we apply our theorem to societies which satisfy certain classical assumptions on preferences which are included as special cases in Grandmont [1978], but which may have measure spaces of voters (as in Grandmont [1978]). In particular, we study societies with "quadratic-based preferences" and ones with the corresponding form of "single-peakedness." Theorem 1 implies that, in such societies, the map from distributions of voter preferences to their majority rule equilibria is also continuous (Corollaries 1 and 2). This provides continuity results which are analogous to the ones derived in Denzau and Parks [1975].

2. Grandmont's Model: Notation and Assumptions

Following Grandmont [1978]: X is a fixed set of alternatives on which each individual has a binary relation. $(R_a)_{a \in A}$ denotes a family of relations indexed by points, a , in an open convex subset A of E^n . This family is assumed to satisfy:

(H.1) (Weak Continuity): For every $x, y \in X$, the set $\{a \in A: xR_a y\}$ is closed in A .

(H.2) (Intermediate Preferences): For every $a', a'' \in A$, R_a is "between" $\frac{1}{R_a}$ and $R_{a''}$ whenever $a = \lambda \cdot a' + (1 - \lambda) \cdot a''$ for $\lambda \in (0,1)$.

A society is specified by a probability measure ν on A . Let A' and A'' be the intersections of A with the two closed half-spaces determined by a hyperplane H . Every ν is assumed to satisfy:

(M.1) There exists $a^* \in A$ such that for every hyperplane H of E^n , $\nu(A') = \nu(A'')$ if and only if $a^* \in H$.

$N(A)$ will denote the collection of probability measures on A which satisfy (M.1). The topology for $N(A)$ will be the relative topology induced by the topology of weak convergence on the collection of all probability measures on A (e.g., see Billingsley [1968]).

The majority rule relation, R_M , for any society $\nu \in N(A)$ is given by $xR_M y$ if and only if $\nu\{a \in A: xR_a y\} \geq \nu\{a \in A: yR_a x\}$.

3. Continuity of Majority Rule

Grandmont [1978] showed that (H.1), (H.2) and (M.1) together imply that $R_M = R_{a^*}$, where $a^* \in A$ satisfies the condition in (M.1). These three conditions therefore define the majority rule correspondence:

$$(1) \quad \phi(\nu) = \{a^* \in A \mid \forall H \subset E^n: \nu(A') = \nu(A'') \text{ if and only if } a^* \in H\}$$

$\frac{1}{R_a}$ is said to be "between" R_a and $R_{a''}$ if for all $x, y \in X$, (i) $xR_a y$ and $xR_{a''} y$ imply $xR_a y$; (ii) $xP_a y$ and $xP_{a''} y$ imply $xP_a y$; (iii) $(xI_a y$ and $xP_{a''} y)$ or $(xP_a y$ and $xI_{a''} y)$ imply $xP_a y$.

from each measure of voters, $\nu \in N(A)$, to a corresponding index or set of indices identified with the majority rule relation.

We prove the following in Section 5:

Theorem 1: Suppose that every society, $\nu \in N(A)$, satisfies (H.1), (H.2) and (M.1). Then the majority rule correspondence defined by (1) is a continuous function.

4. Applications to Majority Rule Equilibria

In this section we apply Theorem 1 to societies in which preferences satisfy classical assumptions from the papers preceding Grandmont [1978]. These applications provide results on the continuity properties of the function from distributions of voters' preferences to their majority rule voting equilibria (as in Denzau and Parks [1975]).

Suppose that R_M is the majority rule relation on the set of alternatives X for a particular society, ν . Then an alternative, $x \in X$, is a majority rule equilibrium (or Condorcet winner) for the society if and only if $xR_M y$ for every $y \in X$. That is, x cannot be defeated by a majority in a pairwise vote against any other alternative.

The assumptions in Grandmont [1978] assure that there is a majority rule equilibrium whenever the individual preferences are restricted to transitive or acyclic relations. However, even when there are majority rule equilibria, his assumptions do not assure that there is any nicely behaved relation between the indices and their maximal elements. However, in the following cases (which provided the standard representation of individual

preferences on public alternatives prior to Grandmont [1978]), it turns out that the map from distributions of voter preferences to their majority rule equilibria is also continuous.

4.a Quadratic-Based Preferences

Tullock [1969] assumed that there is a Euclidean policy space, $X \subset E^n$, and that the preference relation of each individual i satisfies

$$(2) \quad xR_i y \text{ if and only if } \|x - x_i\| \leq \|y - x_i\|$$

for any $x, y \in X$ (where x_i is a unique "ideal point" and $\|\cdot\|$ is the usual Euclidean norm). (2) means that each individual ranks the possible policies according to their Euclidean distance from his ideal point. Such preferences have been labelled "Type I preferences" (e.g., Kramer [1977]). Such preferences can be completely specified by letting the index for each preference relation be its ideal point (i.e. $a = x_i \in E^n$).

The above preferences have been generalized to "ellipsoidal" or "quadratic-based" preferences (e.g., Davis, DeGroot, and Hinich [1972] or Riker and Ordeshook [1973]). Such preferences are given by a binary relation, R_i , which satisfies

$$(3) \quad xR_i y \text{ if and only if } \|x - x_i\|_B < \|y - x_i\|_B$$

for any $x, y \in X$ (where x_i is a unique "ideal point" and $\|x\|_B = x' \cdot B \cdot x$ with B being a symmetric, positive definite, $(n \times n)$ matrix). This means that the indifference contours of an individual are ellipsoids. The ratio of the major axis to the minor axis of an

ellipsoidal indifference curve represents the relative salience of the dimensions. Given B , each preference relation can be completely specified by letting the index be the ideal point (i.e. $a = x_i \in E^n$).

Theorem 1 implies:

Corollary 1: Let $X \subset E^n$ and let B be given. Suppose that each citizen's preferences are indexed by his ideal point and satisfy (3). Then, if each society, v , satisfies (M.1), the correspondence from each society to its majority rule equilibria is a continuous function.

4.b Single-Peaked Preferences

Black [1948], [1958] and Arrow [1963] have shown that "single-peakedness" implies the existence of a majority rule equilibrium. This condition requires a strong ordering of the alternatives such that, for each voter i , if $xR_i y$ and y is between x and z (in the strong ordering) then $yP_i z$ (e.g., see Arrow [1963], p. 77). This is most commonly formulated by assuming $X \subset E^1$ and

$$(4) \quad xR_i y \text{ if and only if } \|x - x_i\| \leq \|y - x_i\|$$

(as in (2)). In this case, voters' preferences are indexed by their ideal points.

We have

Corollary 2: Let $X \subset R^1$. Suppose that each citizen's preferences are indexed by an ideal point and satisfy (4). Then, if each society v has a unique median, the correspondence from each society to its majority rule equilibria is a continuous function.

5. Proofs

In this section, we give proofs of the main theorem and its corollaries. We first develop properties of the majority rule correspondence ϕ defined by (1), culminating in the result that Grandmont's conditions (H.1), (H.2) and (M.1) imply that ϕ is a continuous function.

For notational convenience, denote a hyperplane which contains $a \in E^n$ by $H(a)$. The disjoint open half-spaces determined by this hyperplane will be denoted by $H^+(a)$ and $H^-(a)$. Their closures will have the usual notation of $\bar{H}^+(a)$ and $\bar{H}^-(a)$.

Lemma 1: The majority rule map $\phi: N(A) \rightarrow A$ defined by (1) is a (single-valued) function.

Proof: From Grandmont's main theorem ([1978], p. 324), $\phi(v) = \{a^* \in A \mid \forall H(a) \subset E^n: v(A') = v(A'') \text{ if and only if } a^* \in H(a)\} \neq \emptyset$ for each $v \in N(A)$. Suppose that there exist $a, b \in A$, $a \neq b$, where a and b are both a^* 's for the same $v \in N(A)$. Since $a \neq b$, there is a family F of parallel hyperplanes such that $a \in H(a) \in F$ and $b \in H(b) \in F$ while $H(a) \neq H(b)$. Since a is an a^* , $v(A') = v(A'')$ for H only if $a \in H$. Since $H(a) \neq H(b)$, we have $a \notin H(b)$. Therefore, $v(A') \neq v(A'')$ for $H(b)$. But this contradicts b being an a^* . Hence $\phi(v)$ is single-valued for each $v \in N(A)$. Q.E.D.

Lemma 2: Let $H(b)$ and $H(c)$ be from the same family F of parallel hyperplanes in E^n with $H^+(b) \cap H^-(c) \neq \emptyset$. Then $a = \phi(v) \in H^+(b) \cap H^-(c)$ if and only if $v(H^+(b) \cap A) > 1/2$ and $v(H^-(c) \cap A) > 1/2$.

Proof: Suppose that $v(H^+(b) \cap A) > 1/2$ and $v(H^-(c) \cap A) > 1/2$, but $a = \phi(v) \notin H^+(b) \cap H^-(c)$. Then $\bar{H}^-(a) \subset E^n \setminus H^+(b)$ or $\bar{H}^+(a) \subset E^n \setminus H^-(c)$. Therefore, $v(\bar{H}^-(a) \cap A) \leq 1 - v(H^+(b) \cap A) < 1/2$ or $v(\bar{H}^+(a) \cap A) \leq 1 - v(H^-(c) \cap A) < 1/2$. But (M.1) implies that $v(\bar{H}^+(a) \cap A) = v(\bar{H}^-(a) \cap A) \geq 1/2$. A contradiction.

To show the converse, let $a \in H^+(b)$. Then $\bar{H}^+(a) \subset H^+(b)$. So $v(\bar{H}^+(a) \cap A) \leq v(H^+(b) \cap A)$. But (M.1) implies $v(\bar{H}^+(a) \cap A) \geq 1/2$. Therefore, $v(H^+(b) \cap A) \geq 1/2$. Suppose $v(H^+(b) \cap A) = 1/2$. Write $H^+(b) = [H^+(b) \cap H^-(a)] \cup \bar{H}^+(a)$. Then $1/2 = v(H^+(b) \cap A) = v(H^+(b) \cap H^-(a) \cap A) + v(\bar{H}^+(a) \cap A)$. But $v(\bar{H}^+(a) \cap A) \geq 1/2$ by (M.1). Therefore, $v(H^+(b) \cap H^-(a) \cap A) = 0$. Since A is open and convex, then $v(H^+(b) \cap H^-(a) \cap A) = 0$ says that there is some $d \in H^+(b) \cap H^-(a) \cap A$ with $v(A') = v(A'')$ for $H(d) \in F$. But then $v(A') = v(A'')$ does not occur only if $a \in H(a)$, which contradicts (M.1). Hence $v(H^+(b) \cap A) > 1/2$. A similar argument establishes $v(H^-(c) \cap A) > 1/2$ (see Figure 1 for the graphical interpretation of the proof for the case $a \in H^+(b)$ in E^2).

Q.E.D.

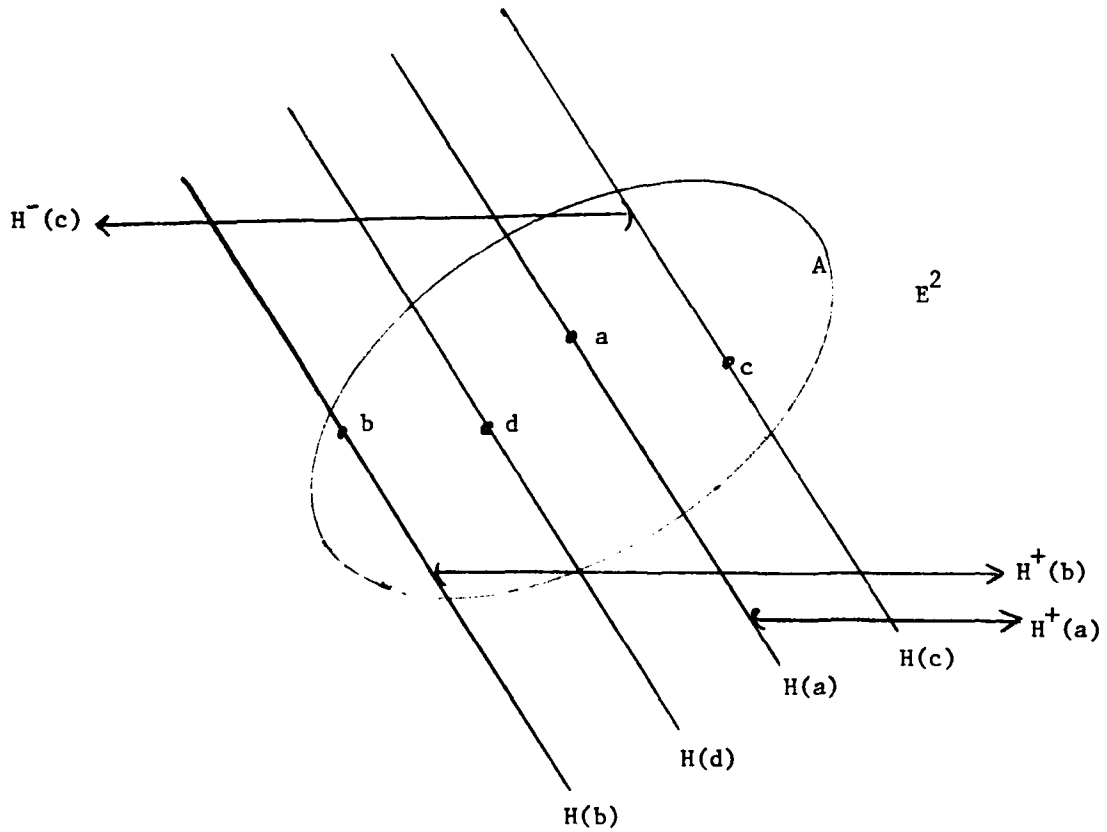


Figure 1

We are now in a position to prove our main theorem about the continuity of the majority rule correspondence defined by (1).

Proof of Theorem 1: First, by Lemma 1, ϕ is a single-valued function. Therefore, what we must prove is that for any a in the range of ϕ and any neighborhood of a , $U_\delta(a) = \{a' \in A: \|a - a'\| < \delta\}$ (with $\|\cdot\|$ being the Euclidean norm and δ a positive real number), the set $\phi^{-1}(U_\delta(a))$ is open in $N(A)$. For any such pair, choose any $v \in \phi^{-1}(U_\delta(a))$. A basic open neighborhood of v is given by any set $B_\epsilon(v) = \{v' \in N(A): v'(G_i) > v(G_i) - \epsilon, i = 1, \dots, k\}$ where the G_i are open and $\epsilon > 0$. We will give $2n$ open sets G_i and show how to choose $\epsilon > 0$ so that the resulting $B_\epsilon(v)$ is contained in $\phi^{-1}(U_\delta(a))$.

First, the construction of the G_i 's is as follows (see Figure 2 for the case $n = 2$): Since $U_\delta(a)$ is open, there exists an n -dimensional open set $I(b,c) = \{a' \in A: b_j < a'_j < c_j, j = 1, \dots, n\}$ such that $v(I(b,c)) \neq 0$ and $a \in I(b,c) \subset U_\delta(a)$. Let β_j be the vector whose j th component is b_j while every other component is zero. Similarly, let γ_j be the vector whose j th component is c_j while every other component is zero. We now define the G_i ($i = 1, \dots, k$) as $G_{2j-1} = H^+(\beta_j) \cap A$ and $G_{2j} = H^-(\gamma_j) \cap A, j = 1, \dots, n$, where $H(\beta_j)$ and $H(\gamma_j)$ are parallel to the hyperplane $\{y \in E^n: y_j = 0\}$ and $H^+(\beta_j) \cap H^-(\gamma_j) \neq \emptyset$. We notice that $k = 2n$.

Next, we will show that we can choose $\epsilon > 0$ such that $B_\epsilon(v) \subset \phi^{-1}(U_\delta(a))$. In particular, choose an $\epsilon > 0$ which satisfies:

$$0 < \epsilon < \min \{v(H^+(\beta_j) \cap A) - \frac{1}{2}, v(H^-(\gamma_j) \cap A) - \frac{1}{2}, j = 1, \dots, n\} .$$

Such an ϵ exists by Lemma 2 since, by construction, $a \in H^+(\beta_j) \cap H^-(\gamma_j)$ for each $j = 1, \dots, n$. Now, by definition, for any $v' \in B_\epsilon(v)$, we have

$v'(H^+(\beta_j) \cap A) > v(H^+(\beta_j) \cap A) - \epsilon$ and $v'(H^-(\gamma_j) \cap A) > v(H^-(\gamma_j) \cap A) - \epsilon$ for $j = 1, \dots, n$. From the chosen ϵ , then, $v'(H^+(\beta_j) \cap A) > 1/2$ and $v'(H^-(\gamma_j) \cap A) > 1/2$. By Lemma 2, the above inequalities imply that

$$d = \phi(v') \in H^+(\beta_j) \cap H^-(\gamma_j) ,$$

i.e., $b_j < d_j < c_j$. Therefore, $d \in I(b, c) \subset U_\delta(a)$. Hence $v' \in \phi^{-1}(U_\delta(a))$. This shows that $B_\epsilon(v) \subset \phi^{-1}(U_\delta(a))$. This means that every $v \in \phi^{-1}(U_\delta(a))$ is an interior point, so $\phi^{-1}(U_\delta(a))$ is open. Q.E.D.

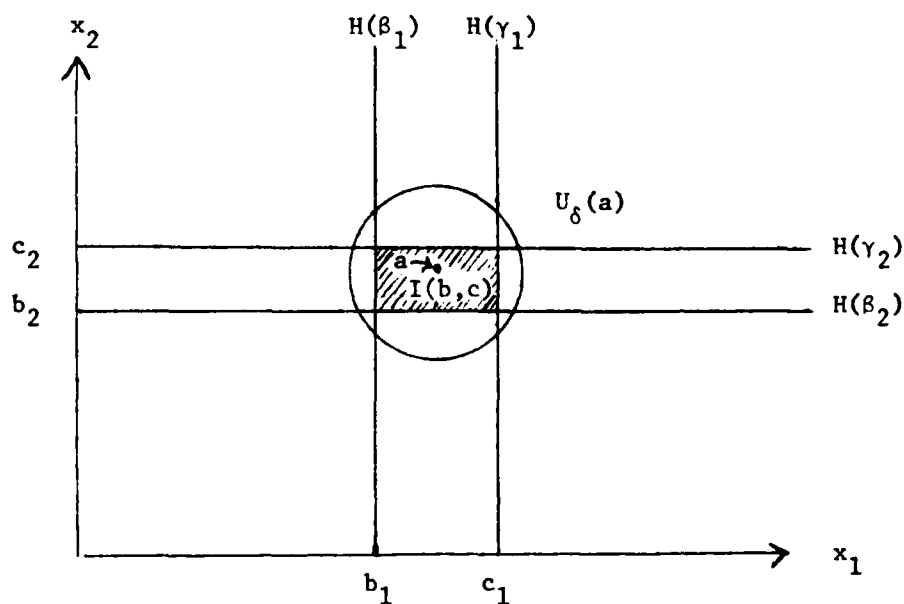


Figure 2

Proof of Corollary 1: Quadratic based preferences on a Euclidean policy space indexed by their ideal points satisfy (H.1) and (H.2). Therefore, $\phi: N(A) \rightarrow A$ is continuous. Since, for $R_{a^*} = R_M$, the index a^* is the unique majority rule equilibrium, the corollary follows. Q.E.D.

Proof of Corollary 2: Follows directly from Corollary 1.

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ELECTORAL OUTCOMES WITH PROBABILISTIC VOTING
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1. Introduction

Electoral competition and the maximization of a Nash social welfare function are two alternative methods of social decision making. The properties of Nash social welfare functions have recently been studied in Yaari [1978] and in Kaneko and Nakamura [1979]. Recent work on electoral competition has been concerned with the implications of probabilistic voting behavior. Some of the implications for full participation electorates have been developed in Comaner [1976], Hinich [1977, 1978] and Kramer [1978]. Related work on probabilistic abstention is due to Hinich, Ledyard and Ordeshook [1972, 1973] and Denzau and Kats [1977]. Additional social choice studies with probabilistic voting include Intriligator [1973, 1979], Nitzan [1975], Fishburn [1975] and Fishburn and Gehrlein [1977].

This paper studies spatial models of electoral competition with probabilistic voting where there is a very close connection between each voter's preferences and his choice probabilities. We show that in any such society

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a policy is an electoral outcome if and only if it maximizes the society's Nash social welfare function (Theorem 1). An election is therefore a game which implements the Nash social welfare function in such societies. This result also implies that, when voting choices are made independently, electoral outcomes are policies which have the maximum likelihood of receiving unanimous support - and vice versa (Corollary 1). Additionally, Theorem 1 implies that there is always an equilibrium in pure strategies for the societies which we have studied (Corollary 2).

2. The Model

Societies with Probabilistic Voting

Let $X \subset \mathbb{R}^m$ denote a non-empty compact and convex set of feasible social alternatives. The elements of X could be given different interpretations - such as alternative tax structures, alternative amounts of various public goods, alternative institutional arrangements or combinations of these. The individuals in a society are indexed by the $N = \{1, \dots, n\}$. Uncertainty in voters' behavior is represented by density functions, $f_i(x)$, on X for the individuals, $i \in N$. Each individual's probabilistic voting density function summarizes his choice probabilities. In particular, for any (measurable) subset $A \subset X$, $\int_A f_i(x) \cdot dx$ is the probability that individual i chooses some member of A (given that he can unilaterally determine the social choice). This generalizes the treatment of individual choice probabilities in Intriligator [1973, 1979] to Euclidean policy spaces.

In our discussion we follow the suggestion of Intriligator [1973, p. 553] that an individual's choice probabilities should be "proportional to his strength of preferences." To be precise, we assume that each individual's density function, f_i , is also his differentiable utility function, $U_i(x) > 0$ (up to a positive scalar multiple). This is referred to as a Luce model (e.g. Becker, DeGroot and Marschak [1963], Luce [1959] or Luce and Suppes [1965]). We also assume that these public sector preferences are concave (e.g. as in Hinich, Ledyard and Ordeshook [1972, 1973], Denzau and Kats [1977] and other studies).

Probabilistic Voting in Elections

In our elections there are two types of participants - individuals who vote and two candidates. Candidates try to win the support of voters by proposing alternatives which they will enact if elected. $\theta_1 \in X$ and $\theta_2 \in X$ denote the policies proposed by candidate 1 and candidate 2, respectively. $P_i^1(\theta_1, \theta_2)$ denotes the probability of individual i voting for candidate 1 when θ_1 and θ_2 have been proposed. $P_i^2(\theta_1, \theta_2)$ denotes the probability that i votes for candidate 2 under the same circumstances.

We consider electorates where everyone votes (as in Comaner [1976], Hinich [1977, 1978], Kramer [1978] and elsewhere). This means that

$$(1) \quad P_i^1(\theta_1, \theta_2) + P_i^2(\theta_1, \theta_2) = 1 .$$

We also assume that each individual's choice probabilities on every binary

set $\{\theta_1, \theta_2\}$ correspond to the likelihoods of his choosing these points from X . To be precise, we assume

$$(2) \quad \frac{P_i^1(\theta_1, \theta_2)}{P_i^2(\theta_1, \theta_2)} = \frac{f_i(\theta_1)}{f_i(\theta_2)}$$

This merely says that the relative likelihoods of choosing θ_1 and θ_2 from X are preserved when the choice set consists only of θ_1 and θ_2 . This is the continuous analog of the independence from irrelevant alternatives which follows from the basic choice axiom of Luce [1959] (see also Ray [1973]).

The individual choice probabilities on any pair of proposed policies are therefore given by

$$(3) \quad P_i^1(\theta_1, \theta_2) = \frac{f_i(\theta_1)}{f_i(\theta_1) + f_i(\theta_2)} = \frac{U_i(\theta_1)}{U_i(\theta_1) + U_i(\theta_2)}$$

$$(4) \quad \text{and} \quad P_i^2(\theta_1, \theta_2) = \frac{f_i(\theta_2)}{f_i(\theta_1) + f_i(\theta_2)} = \frac{U_i(\theta_2)}{U_i(\theta_1) + U_i(\theta_2)}$$

This is a binary Luce model.

Electoral Outcomes

We are concerned with candidates (or political entrepreneurs) who are primarily office seekers. Such candidates will attempt to maximize their expected plurality (or margin of victory). We could, alternatively, assume that a candidate is seeking to maximize his probability of winning. However, Hinich [1977, pp. 212-213] has established that these two objectives are

equivalent when the electorate is large.

By (3) and (4), the expected plurality for candidate 1 is

$$(5) \quad P(\theta_1, \theta_2) = \sum_{i=1}^n \frac{f_i(\theta_1) - f_i(\theta_2)}{f_i(\theta_1) + f_i(\theta_2)} .$$

By definition, the expected plurality for candidate 2 is $-P(\theta_1, \theta_2)$. We will let $M(\theta_1, \theta_2) = (P(\theta_1, \theta_2), -P(\theta_1, \theta_2))$ denote the candidates' payoff function.

When all of the above assumptions are satisfied we obtain the two-person, zero sum game,

$$(6) \quad \Gamma = \Gamma(X, X, M)$$

where X is the set of strategies available to each candidate, and M gives the candidates payoff functions. This game is referred to as the electoral competition.

An electoral equilibrium (or an equilibrium in pure strategies) for this game is a pair of proposed policies, $(\theta_1^*, \theta_2^*) \in X \times X$ which satisfies

$$(7) \quad P(\theta_1, \theta_2^*) \leq P(\theta_1^*, \theta_2^*) \leq P(\theta_1^*, \theta_2)$$

for every $\theta_1 \in X$ and $\theta_2 \in X$.

The actual outcome is determined by chance. However, since the game is symmetric and zero sum, we have the following two properties of Γ_1 : first, $P(\theta_1^*, \theta_2^*) = 0$ at any electoral equilibrium (i.e., a tie is expected).

Second, if $\theta_1^* \neq \theta_2^*$ in an electoral equilibrium, then both (θ_1^*, θ_1^*) and (θ_2^*, θ_2^*) are pure strategy equilibria. We therefore refer to any policy in an equilibrium pair as an electoral outcome (or outcome).

Nash Social Welfare Maxima

Following Kaneko and Nakamura [1979] let each society have some distinguished alternative, x_0 , which represents one of the worst alternatives for all individuals (the origin). Suppose x_0 is excluded from the feasible set, X , being considered. The set $X^* = X \cup \{x_0\}$ is called the society's basic space of alternatives. The Nash social welfare function over the mixed alternatives (or lotteries) on X^* is

$$(8) \quad W_0(U_1(x), \dots, U_n(x)) = \sum_{i=1}^n \log (U_i(x) - U_i(x_0))$$

for individual utility functions, U_i , defined on these alternatives.

We are concerned with choices from the alternatives in X (rather than with lotteries on X^*). Additionally, we can set $U_i(x_0) = 0$ and have $U_i(x) > 0$ for every $x \in X$. The Nash social welfare function on X is then given by

$$(9) \quad W(x) = \sum_{i=1}^n \log (U_i(x))$$

(as in Example 4.1 in Kaneko and Nakamura [1979]). A Nash social welfare maximum is any $x \in X$ which maximizes (9).

3. Results

Electoral Outcomes and Nash Social Welfare Maxima

We are now in a position to state our main theorem:

Theorem 1: An alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X, X, M)$ if, and only if, it is a Nash social welfare maximum over X .

The proof of this theorem is in the appendix.

The Probabilistic Unanimity Rule

Some possible alternatives to electoral competition and to the maximization of the Nash social welfare function have been based on unanimity rules. When voting is random, a natural extension of such rules can be given by the selection of an alternative which has the maximum likelihood of receiving unanimous support.

If individuals make independent voting decisions, i.e., their density functions, the f_i , are independent, then the likelihood that a policy x receives unanimous support (when voters can choose any policy in X) is

$$(10) \quad L(x) = \prod_{i=1}^n f_i(x) = \prod_{i=1}^n U_i(x)$$

for any $x \in X$.

We can now observe that the maxima of $L(x)$ are also maxima of

$$(11) \quad \log L(x) = \sum_{i=1}^n \log U_i(x)$$

which is the Nash social welfare function. Therefore we have:

Corollary 1: Suppose that individuals make independent voting decisions. Then an alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X,X,M)$ if, and only if, it is a unanimity likelihood maximum.

Existence of Electoral Equilibria

Theorem 1 has given a necessary and sufficient condition for an alternative to be an electoral outcome. Since the f_i (and hence the U_i) are continuous and the set of feasible alternative, X , is compact, this condition is always satisfied for some $x \in X$. Namely, there is always a Nash social welfare maximum. Hence,

Corollary 2: Every electoral competition $\Gamma(X,X,M)$ has an electoral equilibrium,

and clearly,

Corollary 3: Suppose that at least one individual has a strictly concave utility function. Then $\Gamma(X,X,M)$ has a unique electoral equilibrium.

These general existence results for global electoral equilibria do not use any special symmetry assumptions for the distribution of voter preferences (unlike Plott [1967], Sloss [1973], Matthews [1979] and others). They also follow without introducing abstentions and special assumptions about non-voting behavior (unlike Hinich, Ledyard and Ordeshook [1972, 1973] and Denzau and Kats [1977]).

4. Conclusion

This paper has studied societies with probabilistic voting. In particular, it has focused on societies where individual choice probabilities and preferences have a close relation so that a Luce model describes voting behavior. In these societies, policies are selected in an election if, and only if, they maximize the society's Nash social welfare function. If voting decisions are independent, such policies are also ones which are most likely to receive unanimous support when the society chooses from its entire set of feasible alternatives. Finally, these societies always have a global equilibrium, in candidate strategies (without including any of the special symmetry or non-voting assumptions of preceding papers).

APPENDIX

Proof of Theorem 1:

The theorem follows from lemmata 1-3.

Lemma 1: An alternative, $\theta \in X$, is an outcome of the electoral competition $\Gamma(X, X, M)$ if, and only if, it is a local maximum of $P(x, \theta)$ given the strategy $\theta_2 = \theta$.

Proof: Γ_1 is a symmetric and zero-sum game. Therefore, if (θ_1, θ_2) with $\theta_1 \neq \theta_2$ is an electoral equilibrium, then (θ_1, θ_1) and (θ_2, θ_2) are electoral equilibria. Hence, $\theta \in X$ is an electoral outcome if, and only if, (θ, θ) is an electoral equilibrium. In other words, if, and only if,

$$P(x, \theta) \leq P(\theta, \theta) \leq P(\theta, y) \quad \forall x, y \in X .$$

Put differently, $\theta \in X$ is an electoral outcome if, and only if, it is a global maximum of $P(x, \theta)$ for candidate 1 who chooses a strategy against $\theta_2 = \theta$.

Consider the payoff function,

$$P(x, \theta) = \sum_{i=1}^n \frac{f_i(x) - f_i(\theta)}{f_i(x) + f_i(\theta)} .$$

Each term, $\frac{f_i(x) - f_i(\theta)}{f_i(x) + f_i(\theta)}$, is a strictly monotone increasing concave function of $f_i(x)$. Therefore, since $f_i(x) = U_i(x)$ is concave, each

term $\frac{f_1(x) - f_1(\theta)}{f_1(x) + f_1(\theta)}$ is a concave function of x . Consequently, since $P(x, \theta)$ is the sum of such terms, $P(x, \theta)$ is a concave function of x . Now, since every local maximum of a concave function on X is also a global maximum, the lemma follows. Q.E.D.

Lemma 2: An alternative, $\theta \in X$, is a global Nash social welfare maximum if, and only if, it is a local Nash social welfare maximum.

Proof: The Nash social welfare function is $W(x) = \sum_{i=1}^n \log U_i(x)$. Each term, $\log U_i(x)$, is a strictly monotone increasing concave function of $U_i(x)$. Therefore, since $U_i(x)$ is concave in x each term is concave in x . Consequently, since $W(x)$ is a sum of these terms, it is itself a concave function of x and the lemma is obtained. Q.E.D.

Lemma 3: An alternative, $\theta \in X$, is a local maximum of $P(x, \theta)$ if, and only if, it is a local maximum of $W(x)$.

Proof: Choose any $\theta \in X$. Then $b \in R^n$ is a permissible direction from θ if, and only if, there is some real number, $\lambda_1 > 0$, such that $(\theta + \lambda \cdot x) \in X$ for every $\lambda \in (0, \lambda_1)$. The alternative $\theta \in X$ is a local maximum of $P(x, \theta)$ if, and only if, the directional derivative, $D_b P(x, \theta)$ is non-positive for every permissible direction b . Additionally, $\theta \in X$ is a local maximum of $W(x)$ if, and only if, $D_b W(x) \leq 0$ for every permissible direction b .

Since each $f_1(x)$ is differentiable at every $x \in X$, we know that the directional derivatives of $P(x, \theta)$ and $W(x)$ are given by

$$D_b P(x, \theta) \Big|_{x=\theta} = \nabla P(x, \theta) \cdot b \Big|_{x=\theta}$$

and

$$D_b W(x) \Big|_{x=\theta} = \nabla W(x) \cdot b \Big|_{x=\theta}$$

for every permissible direction b . We therefore evaluate the partial derivatives of $P(x, \theta)$ and $W(x)$ to obtain

$$\frac{\partial P(x, \theta)}{\partial x_h} \Big|_{x=\theta} = \left[\sum_{i=1}^n \frac{2 \cdot f_i(\theta) \cdot \partial f_i(x) / \partial x_h}{[f_i(x) + f_i(\theta)]^2} \right]_{x=\theta} = \left[\sum_{i=1}^n \frac{\partial f_i(x) / \partial x_h}{2 \cdot f_i(\theta)} \right]_{x=\theta}$$

and

$$\frac{\partial W(x)}{\partial x_h} \Big|_{x=\theta} = \left[\sum_{i=1}^n \frac{\partial U_i(x) / \partial x_h}{U_i(x)} \right]_{x=\theta} = \left[\sum_{i=1}^n \frac{\partial f_i(x) / \partial x_h}{f_i(\theta)} \right]_{x=\theta}$$

for $h = 1, \dots, m$.

This now implies

$$D_b P(x, \theta) \Big|_{x=\theta} = \frac{1}{2} D_b W(x) \Big|_{x=\theta}$$

so that $D_b P(x, \theta) \leq 0$ if, and only if, $D_b W(x) \leq 0$ for every permissible direction b . Q.E.D.

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