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DIRECTIONAL AND LOCAL ELECTORAL COMPETITIONS WITH PROBABILISTIC--ETC(U)
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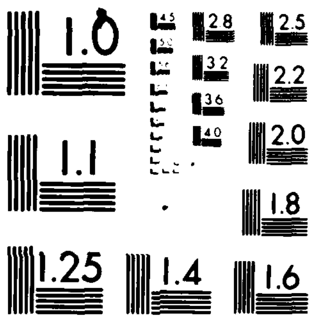


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(6) DIRECTIONAL AND LOCAL ELECTORAL COMPETITIONS
WITH PROBABILISTIC VOTING.

by

(10) Peter Coughlin [redacted] Shmuel Nitzan

(14) TR-303

(9) Technical Report No. 303 ✓
(11) Jan[redacted]80



A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY
✓ Contract (15) ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMICS SERIES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
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ABSTRACT


This paper develops the foundations of spatial models of electoral competitions with probabilistic voting from the existing literature on this topic. We then derive necessary and sufficient conditions for (i) directional electoral equilibria, (ii) stationary electoral equilibria and (iii) local electoral equilibria. These conditions imply general existence results for such equilibria.

These results are derived without using any special concavity conditions or symmetry assumptions on the distribution of voters' preferences. Additionally, they hold for any multi-dimensional policy space. This is in marked contrast to the existing results on electoral equilibria in spatial models with deterministic or probabilistic voting.

The conditions derived in this paper also reveal the equivalence between elections and certain social choice mechanisms involving the social log-likelihood function.

DIRECTIONAL AND LOCAL ELECTORAL COMPETITIONS WITH PROBABILISTIC VOTING*

by

Peter Coughlin and Shmuel Nitzan**

## 1. Introduction

Electoral competitions and simple voting games in which each of the candidates and/or the society is restricted to choosing among "local" options (viz. directions of motion away from the status quo or policies within a small neighborhood of the status quo) have been studied by Plott [1967], Kramer and Klevorick [1974], Schofield [1978], Matthews [1979] and Cohen and Matthews [1980]. Such social choice situations are of interest for a number of reasons. For example, the expense of acquiring information may restrict candidates to learning about voter behavior only near the status quo. Alternatively, institutional restrictions may rule out large changes. These and other reasons have been carefully discussed by the above authors.

The existing results on this problem have been derived for societies with full participation electorates and deterministic voting. These results have established that there are usually directional, stationary and local electoral equilibria when the society's policy space is one-dimensional. However, they have also established that such equilibria

*This work was supported in part by the National Science Foundation and by Office of Naval Research Grant No. ONR-N00014-79-C-0685. We would like to express our thanks for individual comments and suggestions from Ken Arrow, Bob Aumann, Mel Hinich, Bezalel Peleg, Micha Perles, Ariel Rubenstein and an anonymous referee.

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rarely exist when there are two or more dimensions. Local cycles have therefore been studied for societies with such multi-dimensional policy spaces.

Recent work on the theory of economic policy formation through elections has been concerned with the consequences of random voting behavior. Full participation electorates have been studied in Comaner [1976], Hinich [1977], [1978] and Kramer [1978]. Related studies of electorates with abstentions are due to Hinich, Ledyard and Ordeshook, [1972], [1973], McKelvey [1975] and Denzau and Kats [1977].

This paper extends the foundations of spatial models of electoral competition with probabilistic voting from the work of Comaner, Hinich and Kramer (Section 2).

From these foundations, we first derive (in Section 3) a necessary and sufficient condition for a direction from a given status quo to be an equilibrium strategy for a candidate in an electoral competition (Theorem 1). This condition implies that there is such an equilibrium direction at every status quo (Corollary 1).

We then derive a necessary and sufficient condition for a status quo to have an equilibrium direction which is "no change," (i.e., to have a stationary electoral equilibrium) (Theorem 2). This condition implies the existence of a status quo with a stationary electoral equilibrium (Corollary 2).

In Section 4, we are concerned with local electoral equilibria We analyze this problem under conditions which are standard in micro-economic analyses. For these societies we derive necessary and sufficient conditions for a status quo to have a local electoral equilibrium (Theorem 3). This condition implies the existence of a status quo with a local electoral equilibrium (Corollary 3).

The results derived in this paper hold for any finite-dimensional policy space and do not use any special symmetry requirements on the distribution of voters' preferences. The general existence results established in Corollaries 1-3 are, therefore, in sharp contrast to the conclusion of Plott [1967], et. al. (for deterministic voting) that directional and stationary electoral equilibria rarely exist when the society's policy space has at least two dimensions. They also do not use any of the special concavity conditions in Hinich, Ledyard and Ordeshook [1972], [1973] and Denzau and Kats [1977].

Theorems 2 and 3 also establish the equivalence between status quos which have stationary equilibriua (respectively, status quos which have local equilibria) and the stationary outcomes (respectively, local maxima) for the society's mean (or social) log-likelihood function. This shows that these alternative social choice mechanisms are equivalent and provides a method for finding the status quos which have electoral equilibria.

Section 5 concludes. The proofs are given in the Appendix.

## 2. Probabilistic Voting and Electoral Competitions

Empirical studies of voting behavior as a function of proposed policies and existing economic conditions leave a significant amount of unexplained variation (e.g., Kramer [1971], Sitgler [1973], and Fair [1978]). This has led to the conclusion that uncertainty and non-policy considerations result in random (or indeterminate) voting behavior when this behavior is viewed as a function of existing and proposed policies. Intriligator [1973], Fishburn [1975] and others have formulated the indeterminateness in voter behavior with individual choice probabilities. For a non-empty Euclidean space of social alternatives, $X \subset R^n$, these individual choice probabilities are summarized by a density function on $X$ (e.g., Nitzan [1975]). This probabilistic voting density function for an individual expresses the probabilities of his choosing an alternative in different possible subsets of $X$, given that he can determine the social choice unilaterally.

Learning the behavior of every individual is impossible. Therefore, political entrepreneurs (or candidates) have to estimate voter behavior. Since the candidates usually have access to the same information (polls, past election data, etc.), we will assume that candidates obtain a common <u>probabilistic voting estimator</u> which estimates the proportions of the population that are described by particular probabilistic voting density functions. To be precise, let $\Theta \subseteq R^\ell$ denote an index set of parameters for a class of density functions. Then we are assuming that the candidates obtain a <u>probabilistic voting estimator</u>, $\hat{g}(\theta)$, which is a (bounded) probability density function on $\Theta$.

For instance, candidates may be willing to use normal probabilistic voting estimators. In this case, the candidates can estimate the proportions of the population that fall into certain combinations of possible means and variances.

$f(x;\theta)$ will denote a real-valued density function on $X$ which has the parameter $\theta \in \Theta$. Since these density functions are only estimates of individual behavior, we will make three regularity assumptions. Specifically,

(i) Each $f(x;\theta)$ is a positive and continuously differentiable function of $x$.

(ii) $f(x;\theta)$ is a bounded, measurable function of $\theta$ whose lower bound is strictly positive.

(iii) $\partial f(x;\theta)/\partial x_h$ $(h = 1,\ldots,n)$, is a bounded, measurable function of $\theta$.

In an election, each voter must choose from a pair of policy-proposing candidates. Therefore, we must learn the consequences of probabilistic voting behavior for pairwise choices. We will let $\psi_1$ and $\psi_2$ denote policies which have been proposed by candidates who are labelled 1 and 2, respectively. Additionally, we will let $P_\theta^j(\psi_1,\psi_2)$ denote the probability that an individual, whose behavior is described by $f(x;\theta)$, votes for candidate $j$, $j = 1,2$.

The following two assumptions provide the basis for calculating individual binary choice probabilities from probabilistic voting behavior on X.

First, we assume that the candidates estimate the behavior of concerned citizens who vote. This is especially plausible in a local equilibrium analysis where only alternatives near the currently proposed positions are of interest. This means that we are concerned with a full participation electorate where

(1) $$P_\theta^1(\psi_1,\psi_2) + P_\theta^2(\psi_1,\psi_2) = 1$$

for every $\theta \in \Theta$ and $(\psi_1,\psi_2) \in X \times X$. This assumption is repeatedly made in the literature on local equilibria.

Second, we want to relate random voting behavior on a Euclidean policy space to behavior on binary choices, i.e., choices between two proposed policies. We therefore assume that each individual's choice probabilities satisfy (in the terminology of Luce) independence from irrelevant alternatives, i.e.,

(2) $$\frac{P_\theta^1(\psi_1,\psi_2)}{P_\theta^2(\psi_1,\psi_2)} = \frac{f(\psi_1;\ \theta)}{f(\psi_2;\ \theta)}$$

for each $\theta \in \Theta$, $(\psi_1,\psi_2) \in X \times X$.

This is a "separability" condition which merely states that the relative likelihoods of choosing $\psi_1$ and $\psi_2$ from X are preserved when choosing from the set $\{\psi_1, \psi_2\}$. This is the continuous version of the independence from irrelevant alternatives which follows from the basic choice axioms in Luce [1959] (Axiom 1 and Lemma 3) (also in Luce and Suppes [1965] and Ray [1973]).

Finally, since vote totals are random, a candidate could concern himself with his expected plurality or with his probability of winning. However, Hinich ([1977], pp. 212-213) has proven that, for a large electorate with a reasonably large amount of indeterminateness, these two objectives are equivalent. Therefore, we prefer the more tractable objective function, and assume that each candidate is concerned with his expected plurality. We will denote the expected plurality for candidate 1 at the pair of proposed policies $(\psi_1, \psi_2) \in X \times X$ by $P\ell(\psi_1, \psi_2)$.

Usually, electoral competitions are modeled as two-person zero-sum games where the common strategy sets for the two is sone non-empty, compact set, $S \subset X$, of feasible policy proposals, and $P\ell(\psi_1, \psi_2)$ and $-P\ell(\psi_1, \psi_2)$ are the payoff functions to candidates 1 and 2 respectively. This paper focuses, however, on local electoral competitions, i.e., electoral competitions with local options at a given status quo. In these games the payoff functions are either expected pluralities or marginal variations in expected pluralities.

## 3. Directional and Stationary Equilibria

A society always has some current state (or status quo). In this section, we analyze situations in which (i) candidates can, at most, marginally vary their positions from the status quo, and (ii) the status quo is associated with a candidate if he does not choose a direction which varies his position away from it.

The possible directions in which a candidate can move his position from the status quo, together with the choice of "no change," define the candidates' strategy space. The marginal gains in expected plurality from simultaneous variations in positions define the candidates' payoffs. The formal definitions are as follows.

A feasible direction $u$ at a status quo, $\psi \in S$ is a vector in $R^n$ of unit length for which there exists a $\lambda_1 > 0$ such that $\psi + \lambda_1 \cdot u \in S$ for every $\lambda \in (0,\lambda_1)$. The common strategy set, $T(\psi)$, at a status quo $\psi \in S$, consists of all the feasible directions from $\psi$ into $S$ together with the zero vector in $R^n$ (i.e., together with "no change"). $u \in T(\psi)$ and $v \in T(\psi)$ will denote directions selected by candidates 1 and 2, respectively.

The payoff functions at $\psi$ are defined by the net effects of the simultaneous variations $u$ and $v$. In symbols, the payoff function for candidate 1, $P(u,v)$ on $T(\psi) \times T(\psi)$, is

$$P(u,v) = D_{(u,v)}P\ell(\psi_1,\psi_2) = \sum_{h=1}^{n} \frac{\partial P\ell}{\partial \psi_{1h}} u_h + \sum_{h=1}^{n} \frac{\partial P\ell}{\partial \psi_{2h}} v_h \tag{3}$$

at $\psi_1 = \psi_2 = \psi$. The payoff function for candidate 2 is given by $-P(u,v)$. $(u^*,v^*) \in T(\psi) \times T(\psi)$ is a directional electoral equilibrium (in pure strategies) at the status quo $\psi \in X$ if and only if

(4) $$P(u,v^*) \leq P(u^*,v^*) \leq P(u^*,v) \;\forall\; u,v \in S(\psi) \quad .$$

We can now characterize the directional electoral equilibria which occur at a given status quo.

Theorem 1: $(w^*,z^*) \in T(\psi) \times T(\psi)$ is a directional electoral equilibrium at the status quo $\psi$ if and only if

$$D_{(z^*,0)}P\ell(\psi_1,\psi_2) = D_{(w^*,0)}P\ell(\psi_1,\psi_2) = \arg\max D_{(u,0)}P\ell(\psi_1,\psi_2),\; \forall\; u \in T(\psi)$$

at $\psi_1 = \psi_2 = \psi$.

This also implies,

Corollary 1: There exists a directional electoral equilibrium at every status quo in $S$.

If, in a directional electoral equilibrium, both candidates choose to remain at the status quo, then the society is in a stationary equilibrium. In our terminology, there is a stationary electoral equilibrium at a status quo $\psi$ if and only if the zero vector in $R^{2n}$ is a directional electoral equilibrium at $\psi$.

We should remark that there are many alternatives to the existing institution of political elections for making a social choice. Various suggestions for aggregating individual choice probabilities and then determining the social choices have been proposed (e.g., see Intriligator [1973], Fishburn [1975], and Nitzan [1975]). We present below one such possible aggregation rule which, it turns out, is closely related to electoral competitions.

Recall that individual choice probabilities are summarized in the density functions $f(x;\theta)$, or, equivalently, in the log-likelihood functions $\ln f(x;\theta)$. The society's <u>mean</u> (or <u>social</u>) <u>log-likelihood function</u> is given by

(5) $$L(x) = \int_{\Theta} \ln\,(f(x;\theta))\hat{g}(\theta)d\theta$$

If a society is concerned with the marginal changes in $L(x)$ which result from variations in the status quo, then they will not vary a status quo $\psi \in X$ (i.e., will choose "no change") if and only if

(6) $$D_u L(\psi) \leq 0 \quad , \quad \forall\, u \in T(\psi) \quad .$$

We will therefore refer to any such point in the society's policy space as a <u>stationary outcome</u> for the social (or mean) log-likelihood function.

Using this alternative social choice mechanism and Theorem 1 we can now characterize the status quos at which a stationary electoral equilibrium occurs.

Theorem 2: There is a stationary electoral equilibrium at a status quo $\psi \in X$ if and only if $\psi$ is a stationary outcome for the society's mean log-likelihood function.

This, in turn, directly implies

Corollary 2: There exists some status quo, $\psi \in X$, at which there is a stationary electoral equilibrium.

## 4. Local Equilibria

Some stationary equilibria have the local instability property that, if the initial position of the candidates is perturbed slightly, we may find the candidates choosing an equilibrium direction which points away from the neighborhood stationary equilibrium point. This is the case, for instance, when candidates are locally minimizing their expected pluralities (taking their rival's position as no change from the status quo). We therefore now consider the stronger concept of local equilibria (e.g., see Kramer and Klevorick [1974]).

We can consider the possible candidate strategies to be "small" neighborhoods of the status quo, and the payoffs to be the candidates' respective expected pluralities rather than marginal changes in their values. Given the expected plurality function, $P\ell(\psi_1,\psi_2)$, the status quo $\psi \in X$ is a local electoral equilibrium if and only if

(7) $$P\ell(\psi_1,\psi) \leq P\ell(\psi,\psi) = 0 \leq P\ell(\psi,\psi_2)$$

for every $\psi_1 \in N_{\varepsilon_2}(\psi)$ and $\psi_2 \in N_{\varepsilon_2}(\psi)$ for some $\varepsilon_1, \varepsilon_2 > 0$.

We will study local electoral equilibria under assumptions on the "social opportunity set" S which are standard for such sets in microeconomic analyses. Specifically, we will assume that S is a compact subset of X which is defined by twice continuously differentiable equations of the form

(8) $g_k(x) = 0 \quad (k = 1,\ldots,m < n)$ .

This will mean that candidates (when they take their rival's position as given) and society will have Lagrangean maximization problems.

We will also strengthen our regularity conditions (see p. 5) on the individuals' probabilisitc voting estimators and additionally assume:

(1) Each $f(x; \theta)$ is twice continuously differentiable in x, and

(2) $\partial^2 f(x; \theta)/\partial x_h \partial x_\ell (h,\ell \in \{1,\ldots,n\})$ is a bounded, measurable function of $\theta$.

Finally, in order to study this problem for $C^2$ payoff functions, we will analyze those situations in which $P\ell(x,\psi)$ and $L(x)$ are non-degenerate function with respect to the constraints given by (8) (see Hestenes [1975], p. 153). It should be observed that this imposes essentially no further restriction on the class of electoral competitions we are analyzing since, generically, every $C^2$ function is non-degenerate (e.g., see Hirsch ([1976], Theorem 6.1.2)).

For this standard microeconomic decision-making context we can obtain a complete characterization of the status quos at which there are local electoral equilibria:

Theorem 3: There is a local electoral equilibrium at a status quo $\psi \in S$ if and only if $\psi$ is a local maximum for the society's mean log-likelihood function.

This, again, gives us a general equilibrium result:

Corollary 3: There exists some status quo, $\psi \in S$ at which there is a local electoral equilibrium.

## 5. Conclusion

This paper has extended the foundations of spatial models of electoral competitions with probabilistic voting. In particular, we have included infinite voter populations, estimators of voting behavior for the candidates and multidimensional policy spaces. We then derived necessary and sufficient conditions for directional, stationary and local electoral equilibria. These conditions establish general existence results for such electoral equilibria. They also reveal the equivalence between holding an election and using certain social choice mechanisms which select stationary outcomes or local maxima of the society's mean log-likelihood function.

## Appendix

<u>Proof of Theorem 1</u>: First, we obtain the expected plurality function. By (1) and (2),

$$P_\theta^1(\psi_1,\psi_2) = \frac{f(\psi_1;\ \theta)}{f(\psi_1;\ \theta) + f(\psi_2;\ \theta)} \tag{9}$$

for every $\theta \in \Theta$ and $(\psi_1,\psi_2) \in X \times X$.

The expected plurality for candidate 1 from an individual with the probabilistic voting density function indexed by $\theta$ is $P_\theta(\psi_1,\psi_2) = P_\theta^1(\psi_1,\psi_2) - P_\theta^2(\psi_1,\psi_2) = 2P_\theta^1(\psi_1,\psi_2) - 1.$

Therefore, the expected plurality for candidate 1 from the entire population is

$$\begin{aligned} Pl(\psi_1,\psi_2) &= \int_\Theta P_\theta(\psi_1,\psi_2)g(\theta)d\theta \\ &= \int_\Theta \{2 \cdot \frac{f(\psi_1;\ \theta)}{f(\psi_1;\ \theta) + f(\psi_2;\ \theta)} - 1\}g(\theta)d\theta \end{aligned} \tag{10}$$

Since $f(x;\theta)$ is a measurable function of $\theta$ for any given $x \in X$, $f(x;\theta) + f(y;\theta)$ is measurable for any given $x,y \in X$. $f(x;\theta) > 0$ for every $x \in X$ and $\theta \in \Theta$, and, therefore,

$$\{\theta \in \Theta \mid \frac{1}{f(x;\,\theta) + f(y;\,\theta)} < k\} = \{\theta \in \Theta \mid f(x;\,\theta) + f(y;\,\theta) > \frac{1}{k}\}$$

for any $k > 0$, and

$$\{\theta \in \Theta \mid \frac{1}{f(x;\,\theta) + f(y;\,\theta)} < k\} = \emptyset$$

for any $k \leq 0$.

Hence, $1/[f(x;\,\theta) + f(y;\,\theta)]$ and, in turn $P_\theta(\psi_1,\psi_2)$ are measurable functions of $\theta$. Additionally, by (9), $P_\theta(\psi_1,\psi_2)$ is bounded by 0 and 1. Finally, since $\hat{g}(\theta)$ is a (bounded) probability density function on $\Theta$, the product $P_\theta(\psi_1,\psi_2) \cdot \hat{g}(\theta)$ is bounded and measurable and hence, Lesbegue integrable. Therefore, the integral $P\ell(\psi_1,\psi_2) = \int_\Theta P_\theta(\psi_1,\psi_2)\hat{g}(\theta)d\theta$ exists.

By Corollary 5.9 in Bartle [1966] and the regularity conditions on the $f(x;\,\theta)$, the gradient of $P\ell(x,\psi)$ is given by the terms

$$(11)\quad \frac{\partial P\ell(x,\psi)}{\partial x_h} = \int_\Theta \frac{\partial}{\partial x_h}\left(\frac{2f(x;\,\theta)}{f(x;\,\theta) + f(\psi;\,\theta)} - 1\right)\hat{g}(\theta)d\theta$$

for $h = 1,\ldots,n$.

It should be remarked that, whenever $x$ is a boundary point of $X$, the partial derivatives of $P\ell(x,\psi)$ are interpreted as the partial derivatives of an (appropriately continuously differentiable) extension of $P\ell$ onto some open set which contains $X$.

By (10), $P\ell(x,y) = -P\ell(y,x)$. Therefore, (3) implies $P(w,z) = -P(z,w)$ and $P(z,z) = 0$ for all $w,z \in T(\psi)$. In other words, the candidates' directional game is symmetric and zero-sum. Consequently, $(w^*,z^*) \in T(\psi) \times T(\psi)$ is a directional electoral equilibrium if and only if both $(w^*,w^*)$ and $(z^*,z^*)$ are also directional electoral equilibria. Additionally, this implies that $(w^*,w^*)$ is a directional electoral equilibrium at $\psi$ if and only if

$$(12) \qquad D_{(u,w^*)}P\ell(\psi,\psi) \leq D_{(w^*,w^*)}P\ell(\psi,\psi) = 0$$

for every $u \in T(\psi)$.

By (3), (12) is equivalent to

$$(13) \qquad \sum_{h=1}^{n} \frac{\partial P\ell(\psi,\psi)}{\partial \psi_{1h}} u_h + \sum_{h=1}^{n} \frac{\partial P\ell(\psi,\psi)}{\partial \psi_{2h}} w_h^* \leq 0$$

for every $u \in T(\psi)$.

Now, $P\ell(x,y) = -P\ell(y,x)$ implies

$$\frac{\partial P\ell(\psi,\psi)}{\partial \psi_{2h}} = \frac{-\partial P\ell(\psi,\psi)}{\partial \psi_{1h}} .$$

Therefore, (13) is equivalent to

$$(14) \qquad \sum_{h=1}^{n} \frac{\partial P\ell(\psi,\psi)}{\partial \psi_{1h}} u_h - \sum_{h=1}^{n} \frac{\partial P\ell(\psi,\psi)}{\partial \psi_{1h}} w_h^* \leq 0$$

for every $u \in T(\psi)$.

Finally, (14) is equivalent to

(15) $$D_{(u,0)}P\ell(\psi,\psi) \leq D_{(w^*,0)}P\ell(\psi,\psi)$$

for every $u \in T(\psi)$.

By a similar argument, $(z^*,z^*)$ is a directional electoral equilibrium at $\psi$ if and only if

(16) $$D_{(u,0)}P\ell(\psi,\psi) \leq D_{(z^*,0)}P\ell(\psi,\psi)$$

for every $u \in T(\psi)$.

Hence the theorem follows. Q.E.D.

Proof of Corollary 1: At any $\psi \in X$,

$$D_u P\ell(x,\psi)\Big|_{x=\psi} = \sum_{h=1}^{n} \frac{\partial P\ell(x,\psi)}{\partial \psi_{1h}}\Big|_{x=\psi} \cdot u_h$$

is a linear, and hence continuous function of $u$. Additionally, since $S$ is compact, the set of feasible directions, at any $\psi \in S$, is compact. Therefore, $D_u P\ell(\psi,\psi)$ achieves a maximum over this set. Either a maximizing direction in this set or the zero vector must, therefore, maximize $D_u P\ell(\psi,\psi)$. Q.E.D.

Proof of Theorem 2: First of, $\ln f(x;\theta)$ is a bounded function of $\theta$ which is defined for every $x \in X$, since $f(x;\theta) > 0$ is a bounded function of $\theta$ whose lower bound is strictly positive, for any $x \in X$ (see p. 5). Additionally, for any given $x$

$$\{\theta \in \Theta \mid \ln f(x;\theta) > k = \{\theta \in \Theta \mid f(x;\theta) > e^k\}$$

for any $k \in R$. Therefore, since $f(x;\theta)$ is a measurable function of $\theta$. $\ln f(x;\theta)$ is measurable. Consequently the integral defining $L(x)$ exists.

By Corollary 5.9 in Bartle [1966] and the regularity conditions on the $f(x;\theta)$, for $h = 1,\ldots,n$,

$$\frac{\partial L(x)}{\partial x_h} = \int_\Theta \frac{\frac{\partial f(x;\theta)}{\partial x_h}}{f(x;\theta)} \hat{g}(\theta)d\theta \tag{17}$$

By (11),

$$\frac{\partial P\ell(x,\psi)}{\partial x_h} = \int_\Theta \frac{\partial}{\partial x_h} \left(\frac{2f(x;\theta)}{f(x;\theta) + f(\psi;\theta)} - 1\right)\hat{g}(\theta)d\theta \ . \tag{18}$$

Therefore

$$\left.\frac{\partial P\ell(x,\psi)}{\partial x_h}\right|_{x=\psi} = 2 \cdot \int_\Theta \left.\frac{f(\psi;\theta) \cdot \frac{\partial f(x;\theta)}{\partial x_h}}{(f(x;\theta) + f(\psi;\theta))^2}\right|_{x=\psi} \hat{g}(\theta)d\theta \tag{19}$$

$$= \int_\Theta \frac{\left.\frac{\partial f(x;\theta)}{\partial x_h}\right|_{x=\psi}}{2f(\psi;\theta)} \hat{g}(\theta)d\theta \ .$$

Hence

(20) $$\left.\frac{\partial P\ell(x,\psi)}{\partial x_h}\right|_{x=\psi} = \frac{1}{2}\left.\frac{\partial L(x)}{\partial x_h}\right|_{x=\psi} .$$

Therefore, for any $u \in T(\psi)$,

(21) $$D_{(u,0)}P\ell(\psi,\psi) = \sum_{h=1}^{n} \frac{\partial P\ell(\psi,\psi)}{\partial x_h} u_h = \frac{1}{2} \sum_{h=1}^{n} \frac{\partial L(\psi)}{\partial x_h} u_h = \frac{1}{2} D_u L(\psi)$$

By Theorem 1, $0 \in T(\psi)$ is an equilibrium direction at $\psi$ if and only if $D_{(u,0)}P\ell(\psi,\psi) \leq 0$ for every feasible direction at $\psi$ and, hence, if and only if $D_u L(\psi) \leq 0$ for every feasible direction at $\psi$. Q.E.D.

<u>Proof of Corollary 2</u>: Consider any $\psi \in X$. Since $f(x;\theta)$ is a continuous function of $x$ (by regularity condition (i)), $\ln f(x;\theta)$ is a continuous function of $x$. Therefore, $L(x)$ is continuous (by the Lebesgue Dominated Convergence Theorem). Hence, since $S$ is compact, there is some $\psi \in S$ such that $\psi$ is a critical point or a boundary maximum of $L(x)$. Any such point must satisfy (6). Therefore there is some stationary outcome for the social log-likelihood function. Hence, by Theorem 1, there must be a stationary electoral equilibrium. Q.E.D.

Proof of Theorem 3: First, in some neighborhood $\psi \in X$,

$$(22) \quad \frac{\partial^2 P\ell(x;\psi)}{\partial x_h \partial x_\ell} = \int_\Theta \left\{ \frac{2(f(x;\theta) + f(\psi;\theta))^2 f(\psi;\theta) \frac{\partial^2 f(x;\theta)}{\partial x_h \partial x_\ell}}{(f(x;\theta) + f(\psi;\theta))^4} - \frac{4f(\psi;\theta)(f;\theta) + f(\psi;\theta)) \frac{\partial f(x;\theta)}{\partial x_h} \cdot \frac{\partial f(x;\theta)}{\partial x_\ell}}{(f(x;\theta) + f(\psi;\theta))^4} \right\} \hat{g}(\theta) d\theta$$

(by Corollary 5.9 in Bartle [1966] and the regularity conditions on the $f(x;\theta)$). Therefore,

$$(23) \quad \left.\frac{\partial^2 P\ell(x,\psi)}{\partial x_h \partial x_\ell}\right|_{x=\psi} = \int_\Theta \left\{ \frac{\frac{\partial^2 f(x;\theta)}{\partial x_h \partial x_\ell} f(x;\theta) - \frac{\partial f(x;\theta)}{\partial x_h} \cdot \frac{\partial f(x;\theta)}{\partial x_\ell}}{2f(x;\theta)^2} \right\}_{x=\psi} \hat{g}(\theta) d\theta$$

But,

$$(24) \quad \frac{\partial^2 L(x)}{\partial x_h \partial x_\ell} = \int_\Theta \left\{ \frac{f(x;\theta) \frac{\partial^2 f(x;\theta)}{\partial x_h \partial x_\ell} - \frac{\partial f(x;\theta)}{\partial x_h} \cdot \frac{\partial f(x;\theta)}{\partial x_\ell}}{f(x;\theta)^2} \right\} \hat{g}(\theta) d\theta$$

(by Corollary 5.9 in Bartle [1966] and the regularity conditions on the $f(x;\theta)$). Therefore,

$$(25) \quad \frac{\partial^2 P(x,\psi)}{\partial x_h \partial x_\ell} = \frac{1}{2} \frac{\partial^2 L(x)}{\partial x_h \partial x_\ell} \quad \text{at} \quad x = \psi \ .$$

Now, suppose that there is a local electoral equilibrium at $\psi \in S$. Then, since $P\ell(x,\psi)$ is non-degenerate with respect to the constraints given by (8), there exist unique multipliers $\lambda_1,\ldots,\lambda_m$ such that, if we set $L_1(x) = P\ell_1(x,\psi) + \lambda_1 g_1(x) + \ldots + \lambda_m g_m(x)$, then $\nabla L_1(\psi) = 0$. Furthermore,

$$L_1''(\psi) = \sum_{\ell=1}^{n} \sum_{h=1}^{n} \frac{\partial L_1(\psi)}{\partial x_h x} \cdot w_h \cdot w_\ell > 0$$

for every non-zero vector $w \in R^n$ which satisfies the equation $\nabla g_k(\psi) \cdot w = 0$ $(k = 1,\ldots,m)$. (E.g., see Theorem 3.3.2 in Hestenes [1975].)

Now consider $L(x)$. Since $\nabla L(\psi) = 2 \cdot \nabla P\ell(x)$, we must have $L_2(x) = L(x) + 2 \cdot \lambda_1 g_1(x) + \ldots + 2 \cdot \lambda_2 g_m(x)$ implies $\nabla L_2(\psi) = 0$. Additionally, since (25) implies

$$\frac{\partial^2 L(\psi)}{\partial x_h \partial x_\ell} = 2 \cdot \frac{\partial^2 P\ell(x,\psi)}{\partial x_h \partial x_\ell} \quad ,$$

we must have

$$L_2''(\psi) = \sum_{\ell=1}^{n} \sum_{h=1}^{n} \frac{\partial L_2(\psi)}{\partial x_h \partial x_\ell} \cdot w_h \cdot w_\ell > 0$$

for every non-zero vector $w \in R^n$ which satisfies $\nabla g_k(\psi) \cdot w = 0$ $(k = 1,\ldots,m)$. Therefore, since $L(x)$ is non-degenerate with respect to the constraints given by (8), $\psi$ must be a strict local maximum of $L(x)$ (e.g., see Theorem 3.3.2 in Hestenes [1975]).

The converse follows similarly. Q.E.D.

<u>Proof of Corollary 3</u>: By the argument of Corollary 2, $L(x)$ is a continuous function of $x$. Therefore, since $S$ is compact, there is some $\psi \in S$ such that $\psi$ is a local maximum of $L(x)$. By Theorem 3, this $\psi$ must also be a status quo at which there is a local electoral equilibrium. Q.E.D.

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