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A literature review was made in order to recommend a mathematical model to predict jet entrainment in pumped-storage reservoirs. It was found that because of the many different types of inflow situations possible, no one mathematical model would be adequate in all cases. It is recommended that each flow situation be classified and entrainment be predicted for each situation by means of techniques presented in the report.

(Continued)

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20. ABSTRACT (Continued).

The flows are classified according to the source geometry, source buoyancy, ambient stratification, and degree of submergence. A primary and secondary classification scheme resulted in 16 possible flow types. Each situation is reviewed separately and a method for predicting entrainment presented. To use the results, judgment must first be used to classify the flow under consideration and the appropriate technique for that class of flows used. If some of the flows discussed are prevalent or of particular interest, they could be investigated in more depth and mathematical models of them developed.

PREFACE

The study reported herein was sponsored by the Office, Chief of Engineers (OCE), U. S. Army, as part of the Civil Works General Investigations, Environmental and Water Quality Operational Studies (EWQOS) Program. The work unit (CWIS No. 31593) entitled "Develop and Verify Techniques for Describing Pumpback Mixing in Reservoirs" supported the subject study.

The study was conducted during the period November 1978 to October 1979 by Dr. Philip J. W. Roberts of the School of Civil Engineering, Georgia Institute of Technology (Ga. Tech). Mr. David Chin, a graduate student at Ga. Tech, assisted in the literature search. Mr. M. S. Dortch of the Reservoir Water Quality Branch (RWQB), Hydraulic Structures Division, Hydraulics Laboratory, U. S. Army Engineer Waterways Experiment Station (WES) and Dr. D. R. Smith, Chief, RWQB, monitored the effort. This report was written by Dr. Roberts. Program Manager of EWQOS was Dr. J. L. Mahloch.

Directors of WES during this study were COL John L. Cannon, CE, and COL Nelson P. Conover, CE. Technical Director was Mr. Fred R. Brown.

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CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted
to metric (SI) units as follows:

Multiply	By	To Obtain
acres	4046.873	square metres
acre-feet	1233.482	cubic metres
cubic feet per second	0.02831685	cubic metres per second
degrees (angle)	0.01745329	radians
feet	0.3048	metres
miles (U. S. statute)	1.609344	kilometres

JET ENTRAINMENT IN PUMPED-STORAGE RESERVOIRS

1. INTRODUCTION

The quality of water extracted from a reservoir depends strongly
on the vertical temperature structure existing in that reservoir.
Consider, for example, the interrelated typical water quality parameters,
temperature and dissolved oxygen. As a result of surface heating by
the sun and vertical mixing, reservoirs are usually thermally stratified,
with the temperature decreasing downwards from the surface. This causes
a dynamically stable density profile which reduces vertical mixing by
the suppression of vertical motions. The more rapidly that ambient density
changes with depth, the more that these motions are suppressed, and hence
the more that the vertical transfer rates are suppressed. This can cause
the rate of vertical transfer of oxygen to be so low that oxygen is
depleted faster than it is received in the deeper waters, leading to
depressed dissolved oxygen levels there.

This pattern can be significantly different in pumped-storage
reservoirs. For example, if the inflow of pumpback or generation is
discharged as a submerged jet, and is warmer than the water at its level
of injection, buoyancy forces acting on the jet drive it towards the
surface. The combination of the momentum and buoyancy of the jet
causes it to entrain ambient reservoir water. Eventually, this rising column
will reach either the water surface or a level of equal density, at
which point it collapses vertically and spreads laterally. Thus, the
process has redistributed vertically the water in the reservoir, bringing
denser, deeper water towards the surface and altering, both directly
and indirectly, the dissolved oxygen content. The direct change is

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the redistribution of dissolved oxygen in the reservoir by this vertical advection. The indirect changes are due to the altered temperature structure of the reservoir, causing changes in the vertical mixing coefficients and hence changes in the dissolved oxygen content. Depending on whether water is withdrawn from near the surface or bottom of the reservoir, these changes could be either good or bad. Thus, the temperature and dissolved oxygen as well as other water quality parameters of water withdrawn from a pumped-storage reservoir will generally be different from those withdrawn when the reservoir is subject only to natural temperature variations.

Prediction of the change in vertical temperature distribution resulting from pumped-storage operations is therefore a matter of considerable importance. For example, it may be necessary to predict the performance of proposed projects to assess their environmental impact, or to aid in the design of the release structures, or to predict the behavior of an existing project under different operating conditions.

A mathematical model of general applicability, WESTEX, has been developed by the U. S. Army Corps of Engineers Waterways Experiment Station (WES). The model simulates changes in temperature stratification caused by pumped-storage operations. Vertical mixing is simulated by the three processes of entrainment, mixing of inflow and entrained flow, and placement of the mixture at the appropriate reservoir level. The model requires as input the amount of entrainment and vertical reservoir location from which the entrainment occurred. Entrainment is currently expressed as a simple multiple of the incoming flow, where the multiple is based on hydraulic model studies.

The entrained volume depends upon the momentum, buoyancy, and volume fluxes of the source, the discharge structure geometry, and the ambient density stratification. Many studies have been made of entrainment in different fluid flow situations, and various models exist for the prediction of entrainment. The present study was commissioned by WES to review the literature pertaining to jet entrainment and to identify mathematical descriptions of entrainment which offer promise in improving mathematical modeling of pumped-storage reservoirs. WES will investigate candidate descriptions in a subsequent study. The results of the review are presented in this report.

The report is divided into five chapters. In the following chapter, a discussion of typical pumped-storage projects is presented to illustrate the types of flow under consideration. In Chapter 3, the important aspects of the hydrodynamics of pumped-storage reservoirs are discussed and the present problem defined. Chapter 4 presents the methods recommended for entrainment predictions based upon the literature survey. The flows are first classified by a primary and secondary system which is explained in Section 4.1. The primary classification consists of four groups, each of which is discussed in Sections 4.2 through 4.5. The secondary classification also has four parts, each of which is discussed in these sections. Each of these 16 types of flow is reviewed separately and recommendations presented for the prediction of entrainment in each case. Finally, in Chapter 5 a summary of the study and conclusions derived are presented.

2. TYPICAL PUMPED-STORAGE PROJECTS

Pumped-storage projects exist in a wide variety of designs, sizes, and modes of operation, causing a multitude of possible inflow characteristics. The type of system considered here is shown schematically in Figure 2.1; it consists of an upper and a lower reservoir, each having a free surface. The reservoirs are connected by tunnels or pipes within which are generating and pumping units. During periods of peak electric power demand, water is released from the upper reservoir to generate power from turbines. When power demand is low, for example at night, the flow is reversed and water is pumped from the lower to upper reservoir.

Many varieties exist within this basic scheme. Either the upper or the lower reservoir may be much larger than the other, and one reservoir may even be a river. The intake/discharge structures can have various geometries, and may have separate channels for inflow and outflow. The discharge may be near either the bottom or the surface of the receiving water. The flow into the reservoir, which may be either to the lower reservoir during generation or to the upper reservoir during pumpback, is of interest here. Several typical pumped-storage projects are outlined below to illustrate the diversity of flows that must be considered.

2.1 Richard B. Russell Lake

This project is described in Fontane and Bohan (1974). The lake impounds the Savannah River between Clarks Hill Reservoir and the Hartwell Dam and is located on the Georgia-South Carolina border. The Richard B. Russell Lake forms the upper reservoir and Clarks Hill Reservoir the lower when the system is used in the pumped-storage mode. The maximum generation

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UPPER RESERVOIR

INTAKE/DISCHARGE STRUCTURE

GENERATING/PUMPING UNITS

INTAKE/DISCHARGE STRUCTURE

LOWER RESERVOIR
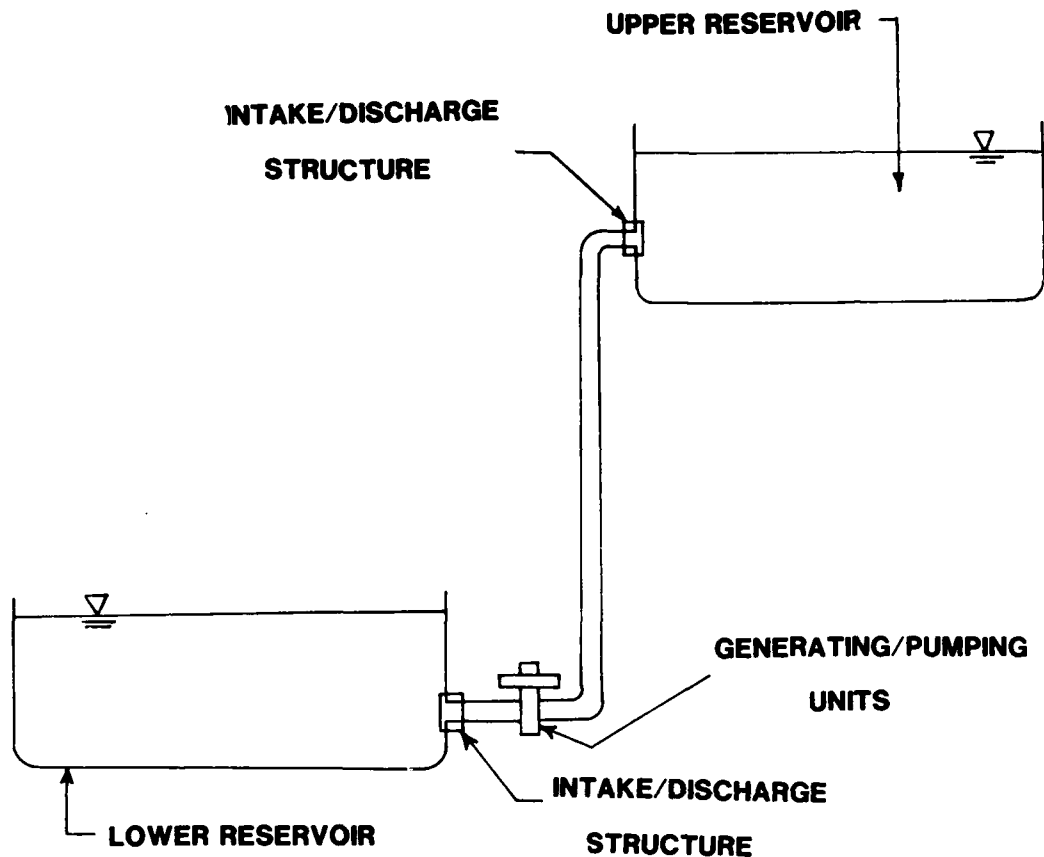
Figure 2.1. Schematic diagram of pumped-storage systems

flow from Richard B. Russell Lake is 60,000 cfs* and the maximum pumpback

flow to the lake is 20,000 cfs.

2.2 Marysville Lake, California

This proposed project, described in Fontane, et al. (1977), is to be

situated in northern California on the Yuba River. The upper reservoir,

Marysville Lake, would be a multipurpose lake. It would be much larger

than the lower reservoir with a storage volume varying from 273,000 to

916,000 acre-feet and a surface area varying from 3,180 to 6,640 acres.

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* A table of factors for converting U. S. customary units of measure-
   ment to metric (SI) units is presented on page 4.

The lower reservoir would have a storage volume of 40,400 acre-feet and an area of 947 acres. Typically, generation and pumpback would occur Monday through Saturday, with a generation flow of 37,700 cfs for 3.71 hours per day, and a pumpback flow of 8,200 cfs for 9 hours per night. Initially six turbines were proposed for a peak generating capacity of 900 MW.

The Waterways Experiment Station physical and mathematical model studies were concerned with definition of the effects of pumpback into Marysville Lake, whose intake/discharge structure configuration is shown in Figure 2.2. Out-flow during generation could be through any of the five selective withdrawal intakes. Each intake consisted of six ports 50 feet wide by 10 feet high to accomodate each of the six turbines. Pumpback flow was through either four of the selective withdrawal ports or a 28-foot square floodgate near the bottom.

Recently, the generating capacity of this system was increased to 2,250 MW (Dortch 1978a), although most of the project features remained unchanged. The previous maximum generation flow of 50,000 cfs was increased to 105,000 cfs and the maximum pumpback flow increased from 9,000 to 50,000 cfs. The storage volumes were increased somewhat, and five turbines are now contemplated, each with vertically staggered selective withdrawal ports 25 feet high by 100 feet wide.

2.3 Dickey-Lincoln School Lakes, Maine

This proposed project is to be located on the St. John River in north-ern Maine (Dortch, et al. 1976). Again, the upper reservoir, Dickey Lake, would be larger than the lower one, extending about 45 miles upstream. The maximum storage volume of this lake would be 7,700,000 acre-feet, with a surface area of 86,000 acres. The lower reservoir, Lincoln School Lake, will have a total storage of about 85,000 acre-feet. The intakes for
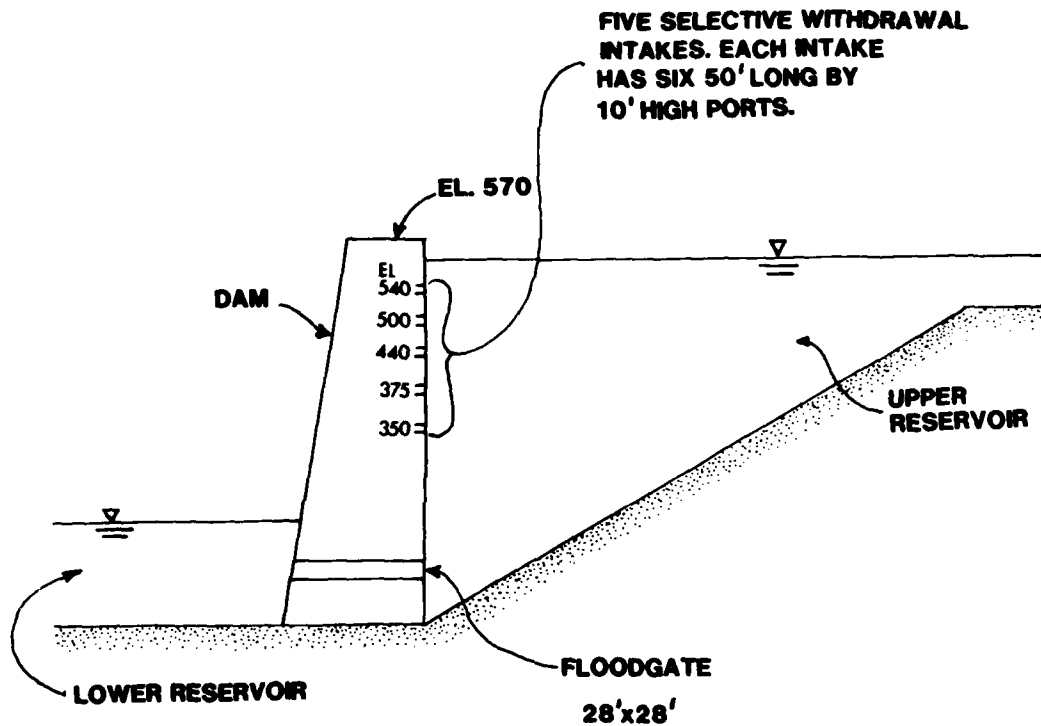
Figure 2.2.   Intake/discharge and reservoir configurations for
Marysville Lake (900-MW capacity)

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generation will consist of a multiple penstock intake structure with selector gates for water quality control. Flows during generation range up to 60,000 cfs. Two modes of operation for pumpback are envisaged; they are shown in Figure 2.3. The modes are: pumping over the top of the selector gates (weirs) and pumping under the raised gates.

2.4 Allegheny Reservoir, Pennsylvania

The Allegheny Reservoir-Kinzua Dam Project is located on the Allegheny River, Pennsylvania (Fontane and Dortch 1978, Dortch 1978b). In this case, the upper reservoir is the smaller of the two; it has a capacity of 7,100 acre-feet and a surface area of 110 acres. The lower reservoir is formed by the impoundment of the Allegheny River by the Kinzua Dam. Its capacity is about 572,000 acre-feet, its surface area 12,080 acres, and its depth 128 ft. The electrical capacity is 380 MW.

2.5 Prattsville, New York

This proposed project would be about 40 miles from Albany, New York (Adams, et al. 1979). The upper reservoir, Schoharie Reservoir, would be smaller with a maximum storage of 30,000 acre-feet. The lower reservoir would have a maximum storage of 70,000 acre-feet. The generating capacity would be 1,000 MW, the maximum generating flow rate would be 16,000 cfs, and the maximum pumpback flow rate would be 10,668 cfs.

2.6 Twin Lakes, Colorado

These lakes are located at an elevation of about 9,000 feet in central Colorado; together they constitute the lower reservoir in this pumped-storage project. They form the larger of the reservoirs, having a combined volume of 170,000 acre-feet, a surface area of about 3,000 acres, and a depth of nearly 100 feet. The upper reservoir has a

12
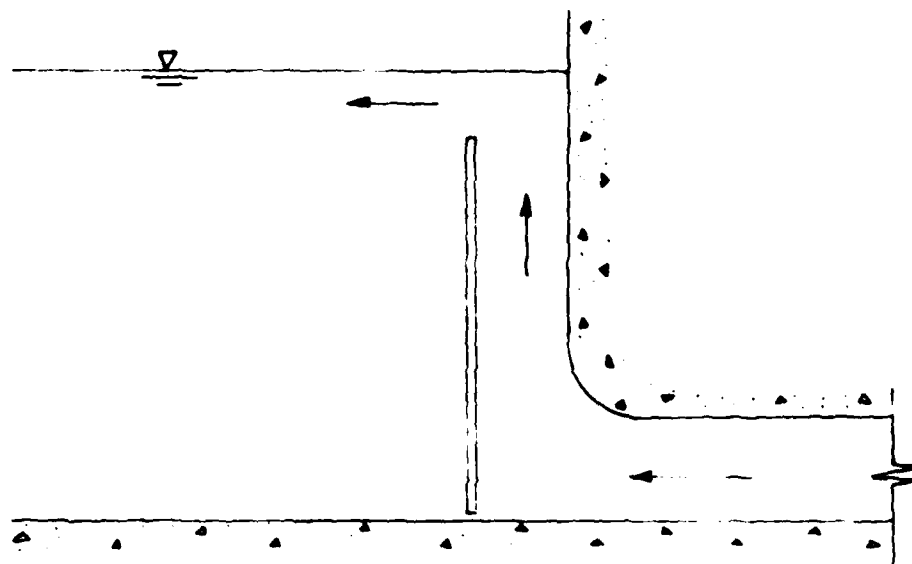
PUMP BACK OVER GATES
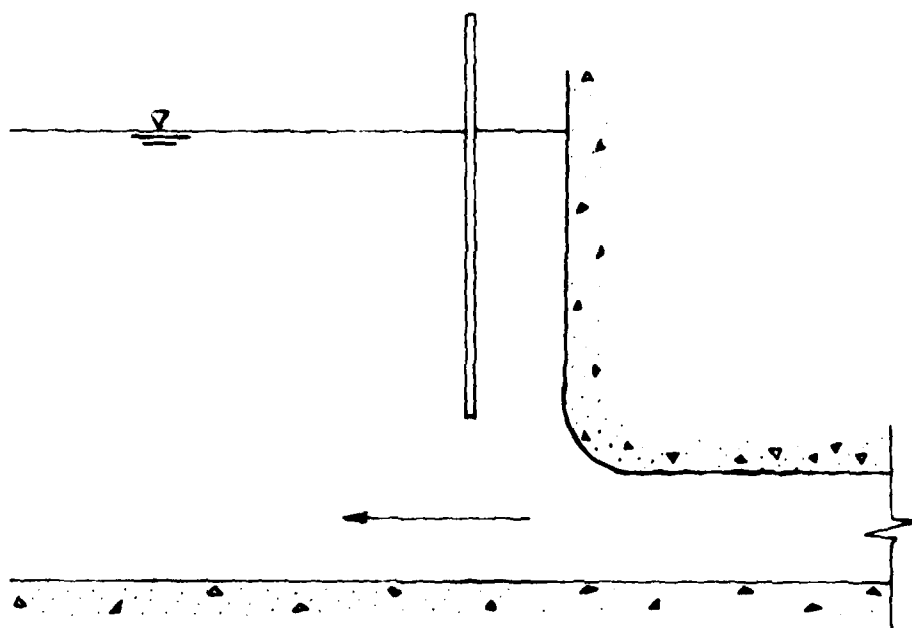
PUMPBACK UNDER GATES

Figure 2.3. Pumpback modes for inflow into Dickey Lake

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volume of 11,000 acre-feet. Ultimately, generation and pumpback are contemplated for Monday through Friday with a generation flow of 7,200 cfs for 8 hours per day and a pumpback flow of 6,180 cfs for 9 hours per night. Two units are proposed for a capacity of 200 MW. For a discussion of this project, see King and Rhone (1977).

3. HYDRODYNAMICS OF PUMPED-STORAGE RESERVOIRS

3.1 Introduction

The flows and mixing induced in pumped-storage reservoirs involve unsteady, three-dimensional, density-stratified, turbulent processes for which no general predictive method exists. These processes and studies of pumped-storage reservoirs are briefly reviewed in this chapter in order to define the problem under consideration and present the assumptions made in solving it.

3.. General Features

The most important hydrodynamic features of reservoirs subject to jetting flows are illustrated in Figure 3.1. The jet can be a result of pumpback or generation, depending on whether an upper or a lower reservoir is involved.

Reservoirs subjected to natural temperature variations usually become density stratified during spring and summer, as shown in Figure 3.1a. Maximum stratifications occur during summer. Minimum stratifications occur during winter when the reservoir may be well-mixed. Usually, there are only small lateral currents, and the shape of the density profile will be similar at different locations within the reservoir.

The flow field shortly after the start of generation or pumpback for the case of a positively buoyant jet is shown in Figure 3.1b. Gravity forces resulting from the buoyancy cause the trajectory of the jet to curve upwards. As the jet ascends, it entrains fluid from the surrounding reservoir, and the volume flow rate within the jet increases with distance from its origin. If the reservoir is density stratified,

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a. Initial conditions,

typical density stratification
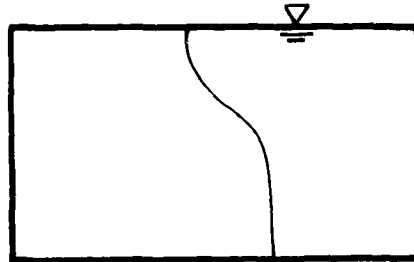
b. Buoyant jetting inflow

Entrainment
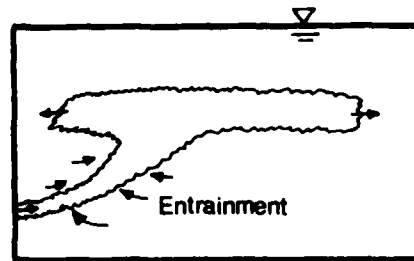
c. Shortly after inflow ceases

Density current
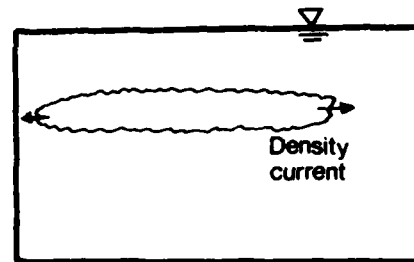
d. Long time after inflow ceases

Final density profile

Initial density profile
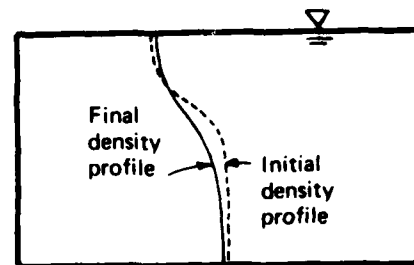
Figure 3.1   Important hydrodynamic features of pumped-storage reservoirs subject to jetting inflows

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the jet can eventually reach a level at which its density is equal to
that of the background water. At this point, the jet collapses vertically
and begins to spread horizontally. If the ambient stratification is not
strong enough to keep the jet submerged, it will reach the surface. In
this situation the deeper waters are "pumped" upwards, causing vertical
mixing within the reservoir.

Shortly after flow ceases (Figure 3.1c) the mixture of inflow and
ambient water continues to spread horizontally as a density current,
during which mixing may continue within the layer. Eventually the layer
will reach the reservoir boundaries, and reflections may occur. Finally,
however, horizontal motions should essentially cease, and the density
profile again become similar at different locations in the reservoir. The
density profile at this time will be different from the original one
(Figure 3.1d), the effect of the process being to make the density profile
more uniform with depth.

The above discussion illustrates only the essential features of
importance here. Other features of secondary importance, for example,
the generation of internal waves in the reservoir, will not be discussed.
Also the horizontal density current may not reach the reservoir boundaries
before withdrawal from the reservoir begins, nor even before the next
cycle begins. Therefore, steady state conditions may never be reached
in the reservoir during pumped-storage operations.

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The present study is concerned only with the entrainment during inflow, the situation shown in Figure 3.1b. A detailed definition sketch of the problem is shown in Figure 3.2 for a positively buoyant discharge.

Conservation of volume for an incompressible fluid yields:

$$Q_c = Q_o + Q_e \qquad (3.1)$$

where Q_c* is the total volume flow rate in the density current, Q_o is the volume flow rate of the inflow, and Q_e is the total volume flow rate of entrainment of ambient water by the jet. An overall entrainment coefficient, E, is now defined by:

$$Q_e = EQ_o \qquad (3.2)$$

The purpose of this study is to provide a method of predicting E and the variation of entrainment with depth.

3.3 Hydraulic Model Studies

Only a few hydraulic model studies of pumped-storage reservoirs of particular geometries or of general applicability have been performed to investigate the effects of pumpback and generation. Some of these studies are summarized below.

Fontane and Bohan (1974) conducted studies on the effects of pumpback flow during pumped-storage operation on the Richard B. Russell Lake. They found an entrainment coefficient, E, of 1.5, where 67 percent of the entrained flow was from the hypolimnion and 33 percent from the epilimnion. The complexity of the entrainment process was illustrated by the fact that following the changeover from generation to pumpback, the level of

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* For convenience, symbols are listed and defined in the Notation (Appendix A).

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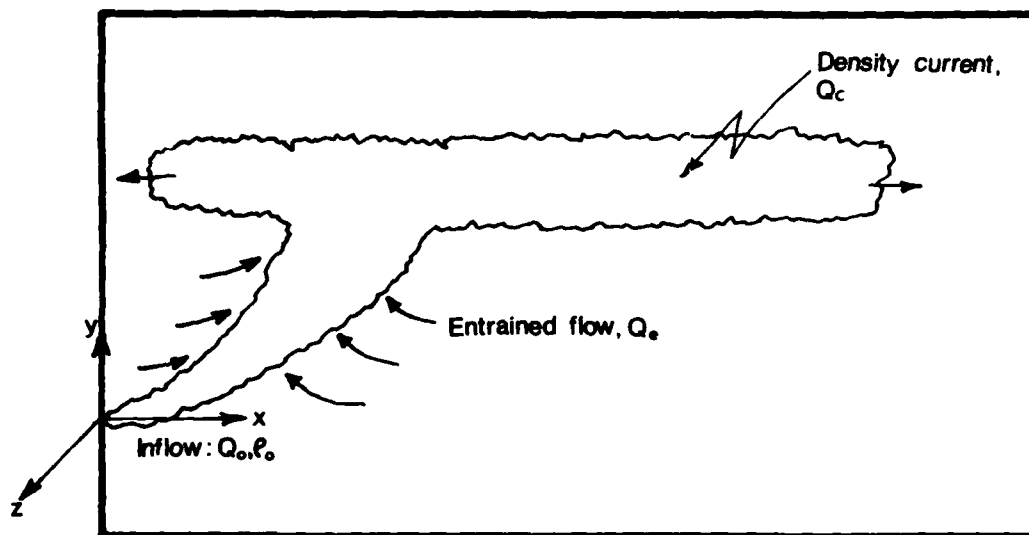
Figure 3.2. Definition sketch of inflow
jet entrainment

entrainment was limited to a layer near the intake level and not to the

bottom reservoir levels.

Dortch, et al. (1976) describe the results of studies on selective

withdrawal and inflow in Dickey-Lincoln School Lakes. The inflow tests

were conducted for flows over and under the gates (see Figure 2.3).

Entrainment coefficients, E, were found to be 0.3 for pumping over the

gates for both negatively and positively buoyant jets and 0.6 for pumping

under the gates with a positively buoyant jet. Stratification in Lincoln

School Lake was completely destroyed within two simulated weeks of opera-

tion. After this period of time the larger reservoir (Dickey Lake) was

still strongly stratified, although some vertical mixing had occurred.

The depth from which the entrained flow originated depended on the flow situation. For a negatively buoyant inflow over the weir, the entrained flow came primarily from above the weir elevation. For positive buoyancy, the entrainment was from all depths below the weir elevation, with the maximum contribution at 20 feet below the weir crest. For positively buoyant flow from below the gates, entrainment was primarily from the deeper waters. In all of these cases, the layers contributing to the entrained flow depend upon the particular density stratification profile present.

Fontane, et al. (1977) report similar tests performed for Marysville Lake, California. The discharge ports in this situation are shown in Figure 2.2. For pumpback from the upper ports, E was found to be about 0.7. The entrained flow came primarily from the levels at and immediately above the port level. For pumpback from the floodgate, E increased to between 2.5 and 3.0, with the entrained flow mainly from the deeper waters. This increased entrainment is due to the positive buoyancy of the inflow.

Tests were also conducted for this lake when the electrical capacity of the project was increased from 900 to 2250 MW, with a corresponding increase in flow rates and port dimensions. The results, presented in Dortch (1978a), suggested an increase in E to 1.0 for discharge through an upper port. The entrained flow was again observed to come mainly from the levels of the port.

Tests on the proposed pumped storage plant at Prattsville were reported by Brocard and Nyquist (1978). Values of entrainment coefficients, E, were evaluated indirectly from these results by Adams, et al. (1979), and a value of E = 1 was chosen.

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Few studies have been performed from which results of general applicability can be derived. Zuluaga-Angel, et al. (1972) and Darden, et al. (1975) report on the circulation created by a turbulent slot jet discharging into a linearly stratified rectangular reservoir. They find complex circulation patterns to be induced, with no true steady state flow existing. Entrainment increased considerably with the buoyancy of the inflow.

3.4 Mathematical Models

Several mathematical models have been developed to predict temperature stratifications due to pumped-storage reservoir operations. All of these models are one-dimensional in their treatment of the reservoir, that is, the reservoir consists of a series of horizontal, well mixed layers; the treatment of the inflow jet may be three-dimensional. These models are summarized below.

The M.I.T. model (Adams, et al., 1979) predicts the vertical distribution of reservoir temperature. The model accounts for heat fluxes to each horizontal element due to solar radiation, surface heat transfer, inflows and outflows, turbulent diffusion, surface wind mixing, and convective overturning. The entrainment due to pumped-storage operations is assumed in this case to come from the layers above the intake and to be uniformly distributed over these layers. By comparing the results of mathematical simulations with the results of physical model tests, a value of $E = 1$ was assumed. The principal finding of the results of simulations of the proposed reservoir was a mixing of the upper layers with a deepening of the epilimnion. The simulation results were found to be insensitive to the value of E in the range $0.5 < E < 2$. The model was also used to simulate temperatures in the operating Blenheim-

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Gilboa Station; reasonable agreements between predicted and measured results were obtained, with most discrepancies occurring in spring, when surface temperatures were overestimated.

A similar model has been developed by the U.S. Army Corps of Engineers (Dortch, et al. 1976). The entrainment coefficient, E, is based on the results of a physical model study for each project, as discussed in Section 3.3. The entrained flow is assumed to come entirely from one layer of the reservoir, the layer being chosen from the results of the physical model study. The average density of the mixture is then computed and the flow placed at its level of neutral buoyancy. Subsequent modifications to the model are described in Dortch (1978a), where computation of entrained flow from several layers is allowed, improving the degree of realism of the model. Applications of the model to the Allegheny Reservoir (Dortch 1978b) produced good simulations of the observed temperature profiles. Further refinement of the entrainment description (Fontane and Dortch 1978) produced even closer simulations to the observed temperatures.

A model was developed by Ditmars (1970) to study mixing in stratified reservoirs. In the situation considered, mixing was intentionally induced by extracting warm water from near the surface and reinjecting it as a positively buoyant jet near the bottom. Thus, the model was not intended specifically for pumped-storage applications, although it can be easily adapted to this case. A three-dimensional integral analysis of the jet is used, and entrainment into the rising jet is allowed continuously from each layer. Ditmars presents general results from which estimates can be made of the time required for complete mixing of a linearly stratified reservoir. The discharge of the warm upper layers of water

as a buoyant jet into the cold lower layers maximizes entrainment and hence maximizes the vertical mixing rate. Therefore, the results can be used to provide a lower bound for the time required for complete mixing of a pumped-storage reservoir. This time is proportional to the ratio of the reservoir volume between the intake and discharge to the pumping rate.

A similar model is presented by Henderson-Sellers (1978).

3.5 Conclusions

The amount of mixing induced in pumped-storage reservoirs can vary widely, ranging from complete mixing induced in time scales on the order of a week to small changes in stratification over months of operation. In general, the smaller the reservoir relative to the daily amount of water pumped, the shorter will be the mixing time. The mixing time also depends on the amount of entrainment; discharge structure designs which increase the amount of entrainment will decrease the mixing time. Thus, the probable reason for the apparent insensitivity to E shown by some of the mathematical simulations discussed above is the large size of the reservoirs considered. For smaller reservoirs the amount of entrainment can have a critical effect on the density stratification.

The process of entrainment is a complex one; the location of the layers from which the entrainment comes is a function of the shape and strength of the ambient density stratification, as well as the buoyancy and momentum fluxes of the discharge and its location. As pumpback or generation occurs for only a few hours, entrainment is assumed constant during this time, and the effects of reservoir geometry are assumed negligible. For continuous discharge these assumptions would not be valid. Methods for predicting entrainment amounts and sources using these assumptions are discussed in the following chapter.

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4. PREDICTION OF JET ENTRAINMENT

4.1 Introduction

The many parameters affecting entrainment in pumped-storage reservoirs result in a complex problem of which many studies have been made. The relevant studies are reviewed in this chapter and potential techniques for entrainment prediction are presented.

Entrainment occurs as a result of turbulent shear flows. As discussed by Roshko (1976), entrainment is the incorporation of nonturbulent, usually irrotational fluid into the turbulent region or, conversely, the diffusion of the turbulent region into the ambient flow. Entrainment appears to be due to the engulfing action of large turbulent eddies near the jet boundaries. Free-stream fluid is drawn in between vortices and ingested into the jet where it is made turbulent and digested by the action of smaller eddies. The rate of the entrainment, therefore, depends on the nature of these large eddies. These depend on the source geometry, the momentum and buoyancy of the discharge, ambient stratification and proximity to local boundaries.

As entrainment is local, the overall reservoir size and geometry should not affect the entrainment rate and their effects will be assumed negligible. The size and geometry will, however, affect overall circulation patterns in the reservoir, and possibly the source of the entrained flow. For the short times over which flows occur, entrainment into the jet will be assumed to be a steady process; this assumption would be invalid for longer times. As the entrainment depends more on the source buoyancy and momentum fluxes than on the volume flux, the effect of source volume flux

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can usually be neglected within the accuracy of engineering estimations. Some of the predictive techniques presented in this chapter neglect the effect of source volume flux on entrainment, but some do not.

Even with these simplifying assumptions, no general technique exists for prediction in all possible solutions. Consider the types of flows considered in Chapter 2, for example. The jet can be near the surface or the bottom (Figure 2.3). It can be from a structure approximating a slot or square orifice (Figure 2.2). Depending on where in the other reservoir the jet came,the jet could be heavier or lighter than or of equal density to the receiving water. And finally, depending on the reservoir size and duration of pumpback or generation, the receiving waters can be density stratified or well mixed. Therefore, to account for all the types of flow which could be conceived, it is necessary to devise a scheme for their classification.

The flows are classified according to a primary and secondary system. The primary classification scheme depends on the source buoyancy and boundary proximity; it is shown in Figure 4.1, in which four classes exist. The secondary classification scheme depends on the ambient stratification and source geometry. The ambient water is either stratified or uniform, and the discharge approximates a slot or round orifice. Thus, each of the four flow schemes shown in Figure 4.1 has four subgroups for a total of 16 flow types. Each of these 16 flow-types is discussed in this chapter.

The chapter is divided according to the flow classification scheme. Each primary group is discussed in Sections 4.2 through 4.5. Each of these sections is further divided into four parts where each flow type is discussed. The parts consist of a brief review of pertinent work, followed by a recommended predictive technique. The techniques exist in either analytical form as

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a. Buoyant immersed jet

b. Nonbuoyant immersed jet

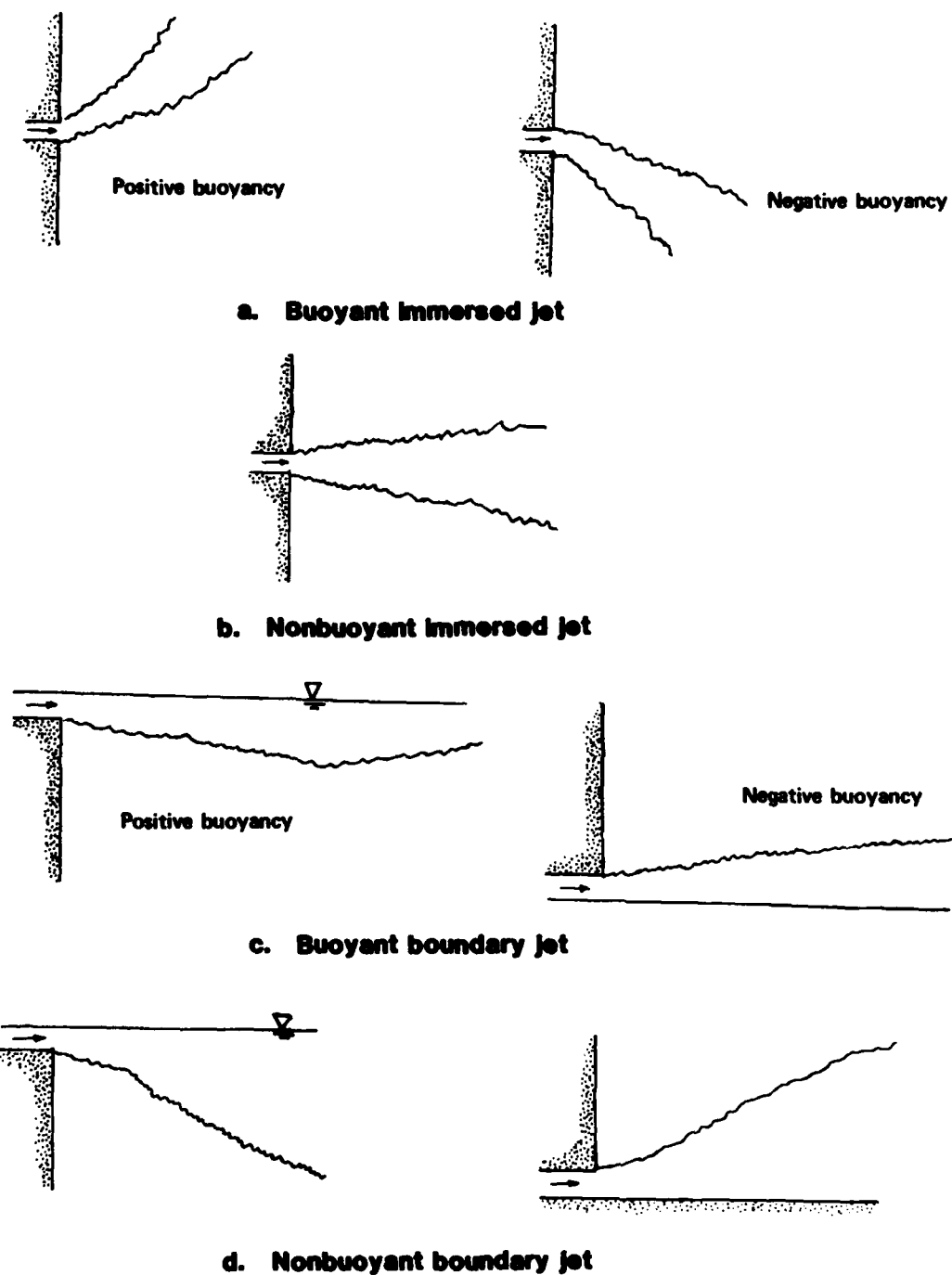c. Buoyant boundary jet

d. Nonbuoyant boundary jet

Figure 4.1. Primary flow classification scheme

equations, as graphs, or as mathematical models.

In many of the studies discussed below, entrainment is described indirectly by dilution. Consider a mixture consisting of a volume V_o of source flow mixed with a volume V_r of receiving water. The dilution, S, is defined by

$$S = \frac{V_o + V_r}{V_o} \qquad (4.1)$$

Similarly, dilutions can be related to volume fluxes Q_o and Q_c; this will be done in the following sections if solutions are given in terms of dilution.

4.2 **Buoyant Immersed Jet**

This situation is case a, Figure 4.1. It occurs frequently in practice, as heated discharges from power plants into lakes and coastal waters, and domestic sewage discharges into coastal waters. Consequently, many studies of the phenomena have been made and predictive techniques for the various flows exist. These flows consist of horizontal discharges from slot or round orifices into a stratified or unstratified fluid. They are reviewed in this section.

A definition sketch showing the details of the problem considered is shown in Figure 4.2. The discharge is horizontal and the ambient fluid may or may not be density stratified. The density of the discharged water, ρ_1, is greater than or less than the density of the ambient fluid, ρ_o, at the level of the discharge, causing buoyancy forces to act on the jet. The particular situation shown in Figure 4.2 is for the case of a positively buoyant $(\rho_1 < \rho_o)$ discharge into a stratified fluid. The ambient fluid density, ρ_a, is a function of depth, y. For a round orifice the diameter is D, and for a slot the width is B. The governing source

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AMBIENT DENSITY
PROFILE

JET GEOMETRY
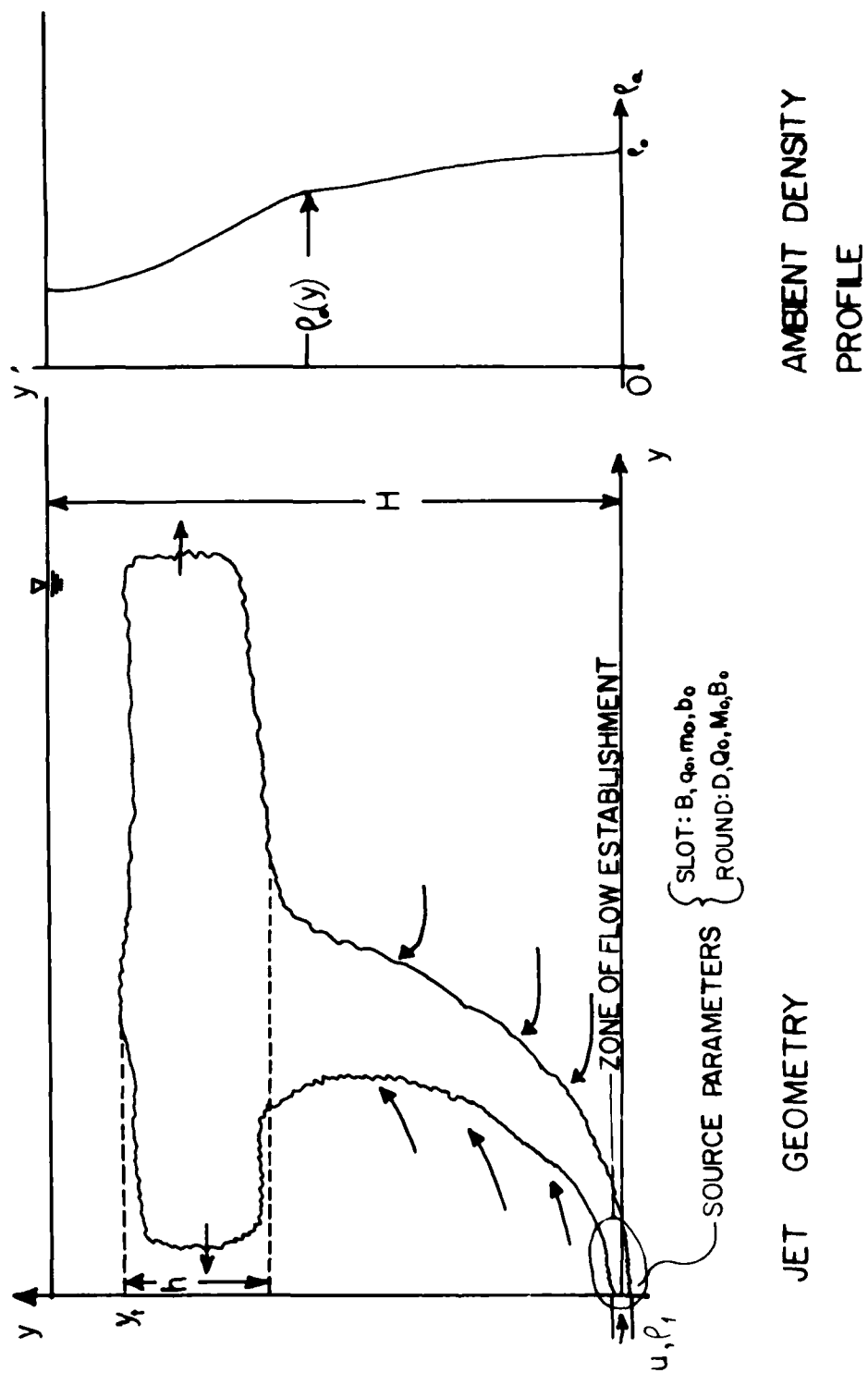
Figure 4.2. Definition sketch for buoyant immersed jet

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parameters can be defined as the source volume, momentum and buoyancy fluxes: Q_o, M_o, and B_o for a round discharge; and q_o, m_o, and b_o per unit width for a slot discharge, where:

$$Q_o = \frac{\pi}{4} u D^2 \tag{4.2}$$

$$M_o = u Q_o \tag{4.3}$$

$$B_o = g' Q_o \tag{4.4}$$

$$q_o = u B \tag{4.5}$$

$$m_o = u q_o \tag{4.6}$$

$$b_o = g' q_o \tag{4.7}$$

$$g' = \frac{\rho_o - \rho_1}{\rho_o} g \tag{4.8}$$

u is the mean initial jet velocity, and g the acceleration due to gravity.

The general features of the flow are also shown in Figure 4.2. These features are discussed in many other papers, for example Koh and Brooks (1975). Near the orifice is a zone of flow establishment in which the velocity profile across the jet width becomes established. This zone extends for about 6.2D for round jets and 5.2B for slot jets (Albertson, et al. 1950). During the zone of flow establishment and beyond, the jet entrains ambient fluid. Near the source the entrainment is primarily due to the source momentum flux; far from the source the buoyancy forces and hence the source buoyancy flux dominate the entrainment. Eventually the rising fluid column either reaches the

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water surface or a level at which its density is equal to the ambient density. (If the ambient fluid is unstratified, the jet will always reach the surface.) Having reached its level of neutral buoyancy, the flow field collapses vertically and begins to spread horizontally. The thickness of the spreading layer, h, can be estimated from the results for a two-dimensional line source of buoyancy flux in an unstratified ambient fluid as (Roberts 1979):

$$h = 0.3y_t \qquad (4.9)$$

where y_t is the terminal rise height of the mixture. Above the height of the bottom of the surface layer, no further dilution or entrainment of ambient water occurs. This phenomenon is known as blocking and must be accounted for. To do this, calculations of entrainment and dilution must cease at a height $y_t - h$, even though results may be predicted up to y_t. The true dilution and entrainment of the density current is that calculated at $y = y_t - h$.

4.2.1 Round discharge, unstratified ambient

Consider a round buoyant jet which discharges horizontally into an otherwise stagnant, unstratified fluid. If a discharge Froude number, F, is defined as

$$F = \frac{u}{\sqrt{g'D}} \qquad (4.10)$$

then it can be shown by means of dimensional analysis that for F greater than about 8, the centerline dilution, S_m, can be expressed as:

$$\frac{S_m}{F} = f\left(\frac{y/D}{F}\right) \qquad (4.11)$$

The functional form of Eq. 4.11 is given in Figure 4.3. For values of discharge Froude number less than 8, a mathematical model discussed in Section 4.2.2 should be used.

In order to use Figure 4.3 to estimate E, E and S_m must be related. It can be shown (Brooks 1972, Fan and Brooks 1969) that the centerline dilution is given by:

$$S_m = 1.15 \frac{Q_c}{Q_o} \qquad (4.12)$$

Eqs. 3.1, 3.2, and 4.12 are combined to give

$$E = \frac{S_m}{1.15} - 1. \qquad (4.13)$$

Hence, Figure 4.3 in conjunction with Eqs. 4.12 and 4.9 can be used to estimate E. The total height of rise is H (see Figure 4.2).

This technique can also be used where the receiving water is of uniform density up to the epilimnion, where the density changes suddenly with a magnitude large enough to trap the rising plume beneath it. In this case the rise height, y_t, is the water depth below the epilimnion.

The layers from which entrainment occurs can be estimated by computing E for various heights y and applying Eq. 3.2.

4.2.2 Round discharge, stratified ambient

This situation could occur in the Marysville project (Chapter 2) if a buoyant inflow was discharged through the 28-foot square floodgate near the reservoir bottom (see Figure 2.2). The square floodgate can be approximated by a round port of equal area, and the techniques of this section applied. There are no general models for discharges other than those from round or slot orifices.
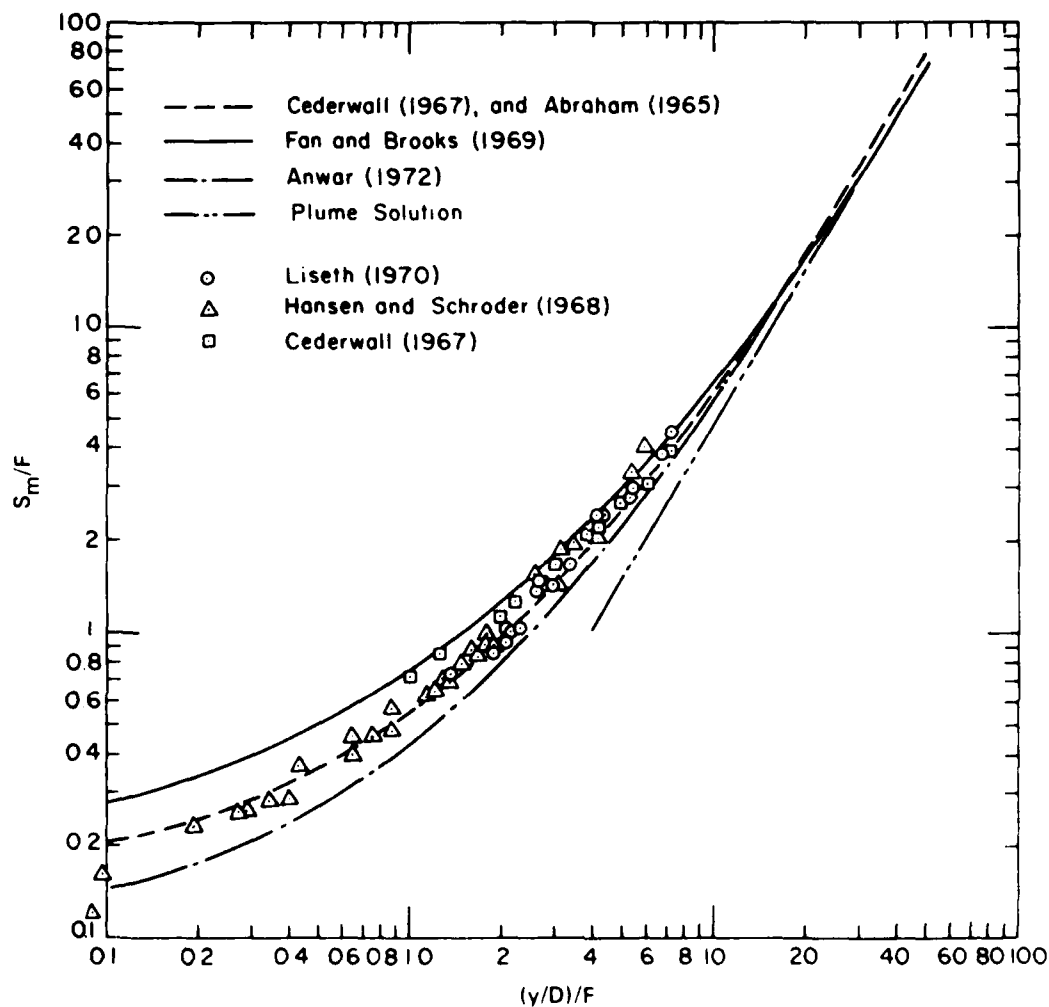
31

Figure 4.3. Centerline dilution of a horizontal, round buoyant jet in a stagnant, uniform fluid (from Roberts 1977)

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Dilution and entrainment can be estimated from the integral solutions of Fan and Brooks (1969). Their report contains the details of the solution method which will not be repeated here. Fan and Brooks show that for linear stratifications, the dilution at the terminal rise height, S_t, can be expressed as:

$$S_t = \frac{\mu_t}{\mu_o} \qquad (4.14)$$

where $\qquad \mu_t = f(m_o, \mu_o) \qquad (4.15)$

m_o and μ_o are dimensionless momentum and volume flux parameters given by Brooks (1972) as:

$$m_o = 0.324 \; F^2 T^{-1} \qquad (4.16)$$

and $\qquad \mu_o = 2.38 \; F^{1/4} T^{-5/8} \qquad (4.17)$

T is a stratification parameter defined as

$$T = \frac{0.87 \; (\rho_o - \rho_1)}{D \; (-d\rho_a/dy)} \qquad (4.18)$$

The functional form of Eq. 4.15 is shown in Figure 4.4. This figure can be used in conjunction with Eqs. 4.13 and 4.14 (with $S_m = S_t$) to estimate E. Again, corrections should be made for the blocking effect (Eq. 4.9).

The terminal rise height, y_t, is also given by Fan and Brooks. They show that y_t is given by:

$$\frac{y_t}{D} = 1.37 \; \xi_t F^{1/4} T^{3/8} \qquad (4.19)$$

where $\qquad \xi_t$ is a dimensional height of rise and

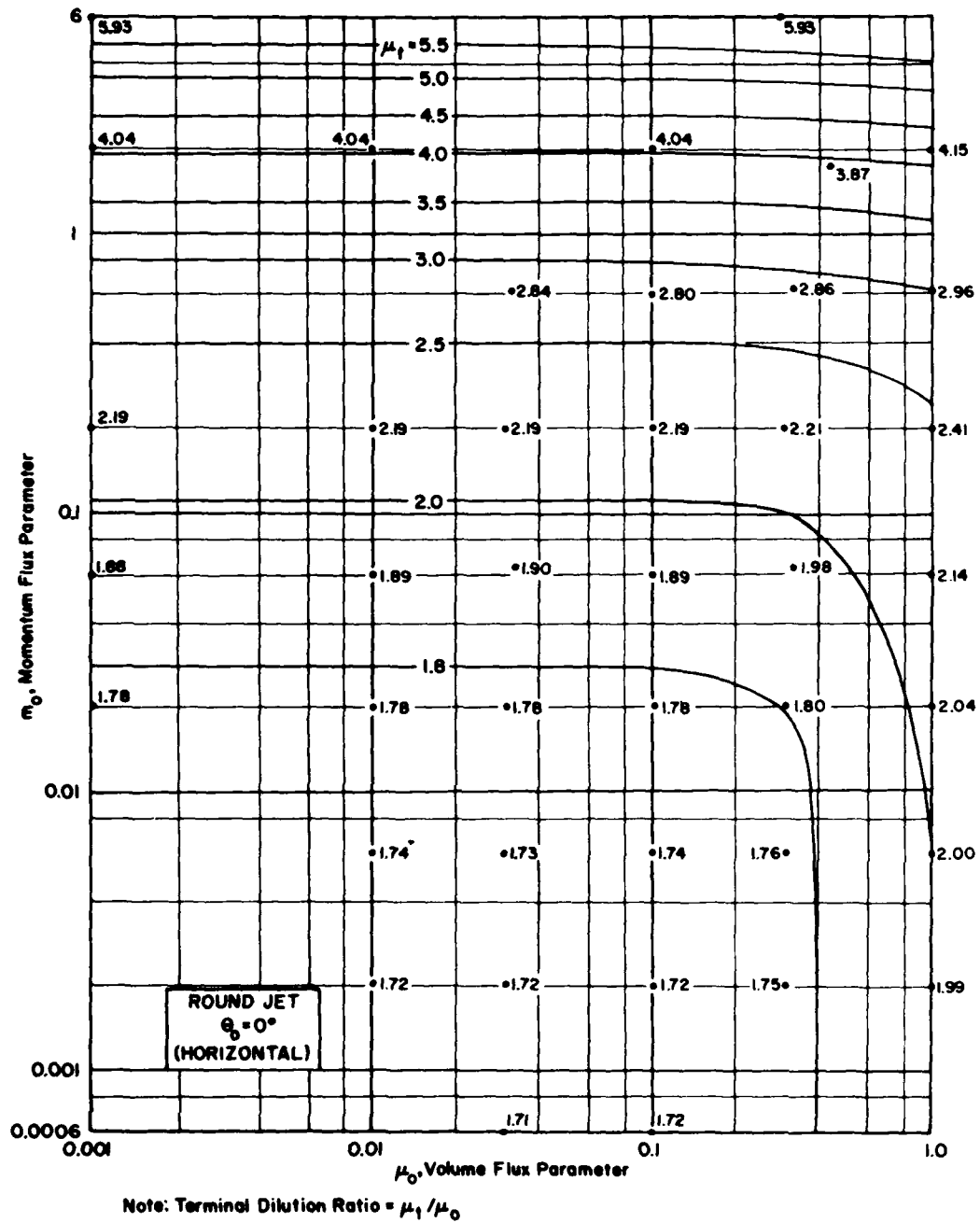$$\xi_t = f(m_o, \mu_o) \qquad (4.20)$$

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Figure 4.4. Terminal volume flux parameter μ_t for horizontal round buoyant jets (from Fan and Brooks 1969)

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The functional form of Eq. 4.20 is shown in Figure 4.5.

To estimate the layers from which entrainment comes, and to predict overall entrainment when the stratification is nonlinear, it is best to use the original computer programs from which these results were generated. The model is described in Ditmars (1969). The levels from which entrainment comes can be obtained by inspection of the output, which gives dilution as a function of depth, or by slight modification of the code to give the result directly.

4.2.3 Slot discharge, unstratified ambient

This problem has been analyzed by similar integral techniques to those discussed in Section 4.2.2. Brooks (1972) shows that the centerline dilution, S_o, can be expressed as

$$S_o = f\left(\frac{y}{B} , F\right), \tag{4.21}$$

where F is a Froude number defined by

$$F = \frac{u}{\sqrt{g'B}} \tag{4.22}$$

and u is the velocity of efflux at the source. The functional form of Eq. 4.21 is shown in Figure 4.6. For a slot jet, the centerline dilution is equal to the volume flux ratio, or

$$S_o = \frac{Q_c}{Q_o} \tag{4.23}$$

Combining Eqs. 3.1, 3.2, and 4.23 gives

$$E = S_o - 1 \tag{4.24}$$

Hence, Figure 4.6 can be used in conjunction with Eq. 4.24 to estimate E after correction is made for blocking (Eq. 4.9). In this unstratified case

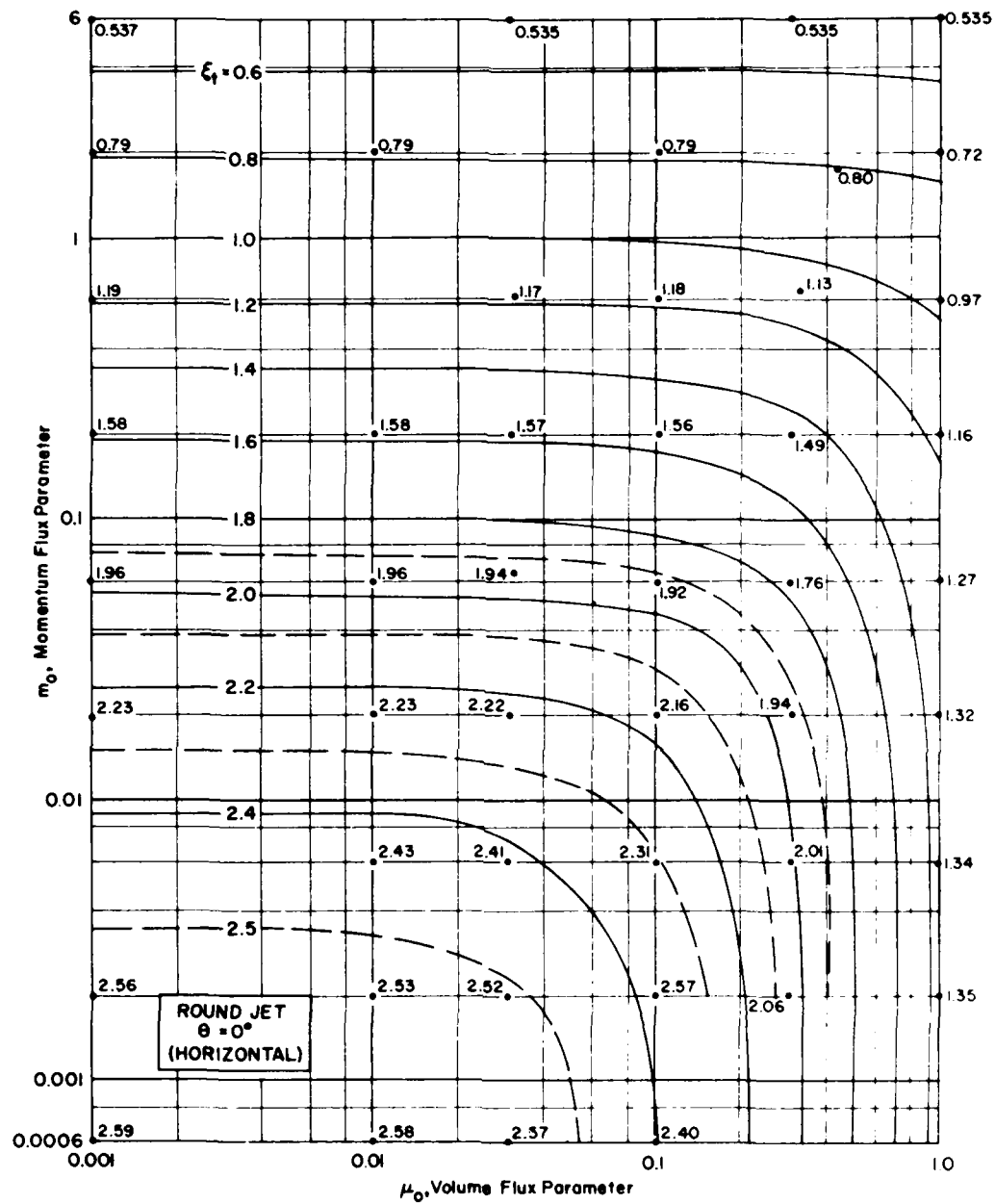35

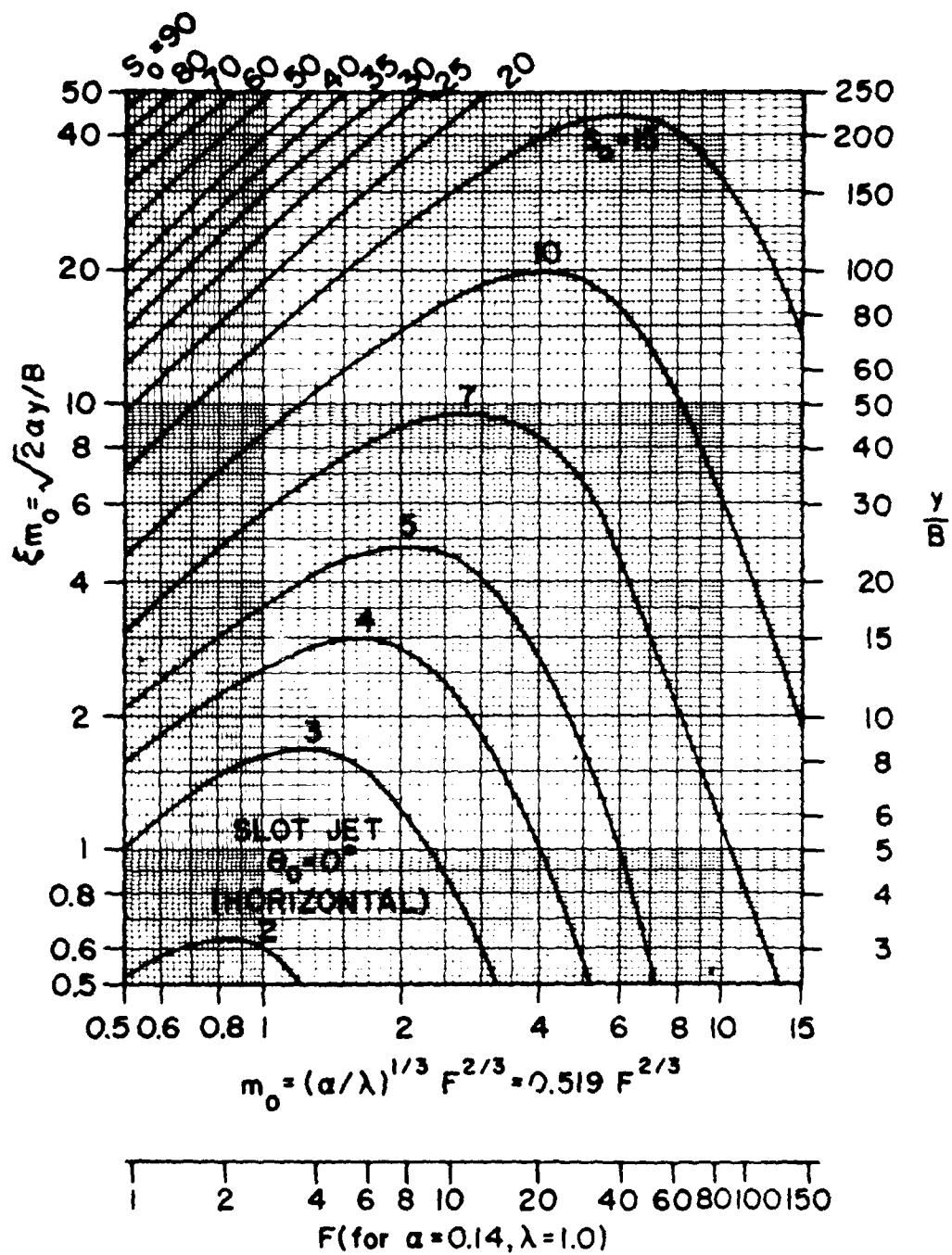Figure 4.5. Terminal rise height ξ_t for horizontal round buoyant jets (from Fan and Brooks 1969)

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Figure 4.6. Centerline solution of horizontal slot buoyant jets in stagnant uniform environments (from Brooks 1972)

the terminal rise height, y_t, is equal to the water depth, d.

This technique can also be used where the receiving water is of uniform density up to the epilimnion, where the density changes suddenly and with a magnitude large enough to trap the rising plume beneath it. In this case the rise height, y_t, is equal to the water depth beneath the epilimnion.

The layers from which entrainment comes can be estimated by computing E for various heights y and applying Eq. 3.2.

4.2.4 Slot discharge, stratified ambient

A possible occurrence of this situation would be the Marysville project if a buoyant inflow were discharged via one of the lower selective withdrawal intakes (Figure 2.2). Each of these intakes consists of six ports, each 50 feet long by 10 feet high (for the 900-MW capacity). Hence, if the inflow was warmer than the local ambient water, the discharge would approximate the two-dimensional buoyant slot jet into a stratified fluid, the problem discussed in this section.

The situation has been analyzed by the integral techniques previously discussed. Brooks (1972) shows that for a linear stratification, the centerline dilution at the terminal rise height, S_o, can be expressed as:

$$S_o = \sqrt{\mu_t/\mu_o} \tag{4.25}$$

$$\mu_t = f(m_o, \mu_o) \tag{4.26}$$

where m_o and μ_o are dimensionless momentum and volume flux parameters given by

$$m_o = 0.5 \ F^2 \ T^{-1} \tag{4.27}$$

and $\qquad \mu_o = 1.85 \ F^{2/3} \ T^{-1}$ (4.28)

F is defined by Eq. 4.22, and

$$T = \frac{\rho_o - \rho_1}{B(-d\rho_a/dy)}$$ (4.29)

The functional form of Eq. 4.26 is shown in Figure 4.7, which can be used with Eq. 4.24 to estimate E. Again, correction for blocking should be made (Eq. 4.9).

The terminal rise height, y_t, can also be estimated. Brooks (1972) shows that

$$\frac{y_t}{B} = 0.96 \xi_t F^{1/3} \ T^{1/2}$$ (4.30)

where $\qquad \xi_t = f(m_o, \mu_o)$ (4.31)

The functional form of Eq. 4.31 is shown in Figure 4.8.

To estimate the layers from which the entrained water comes and to predict gross entrainment when the stratification is nonlinear, it is best to use the original computer programs from which these results were generated. The programs are given in Sotil (1971). The levels from which entrainment comes can be obtained by inspection of the output, which gives dilution as a function of rise height, or by slight modification of the code to give the result directly.

Other relevant experiments were reported in Zuluaga-Angel, et al. (1972) and Darden, et al. (1975). They measured dilution and observed the flow field caused by discharge of buoyant and nonbuoyant slot jets into linearly stratified model prismatic reservoirs. The results for nonbuoyant discharges are discussed in Section 4.3.4. For buoyant
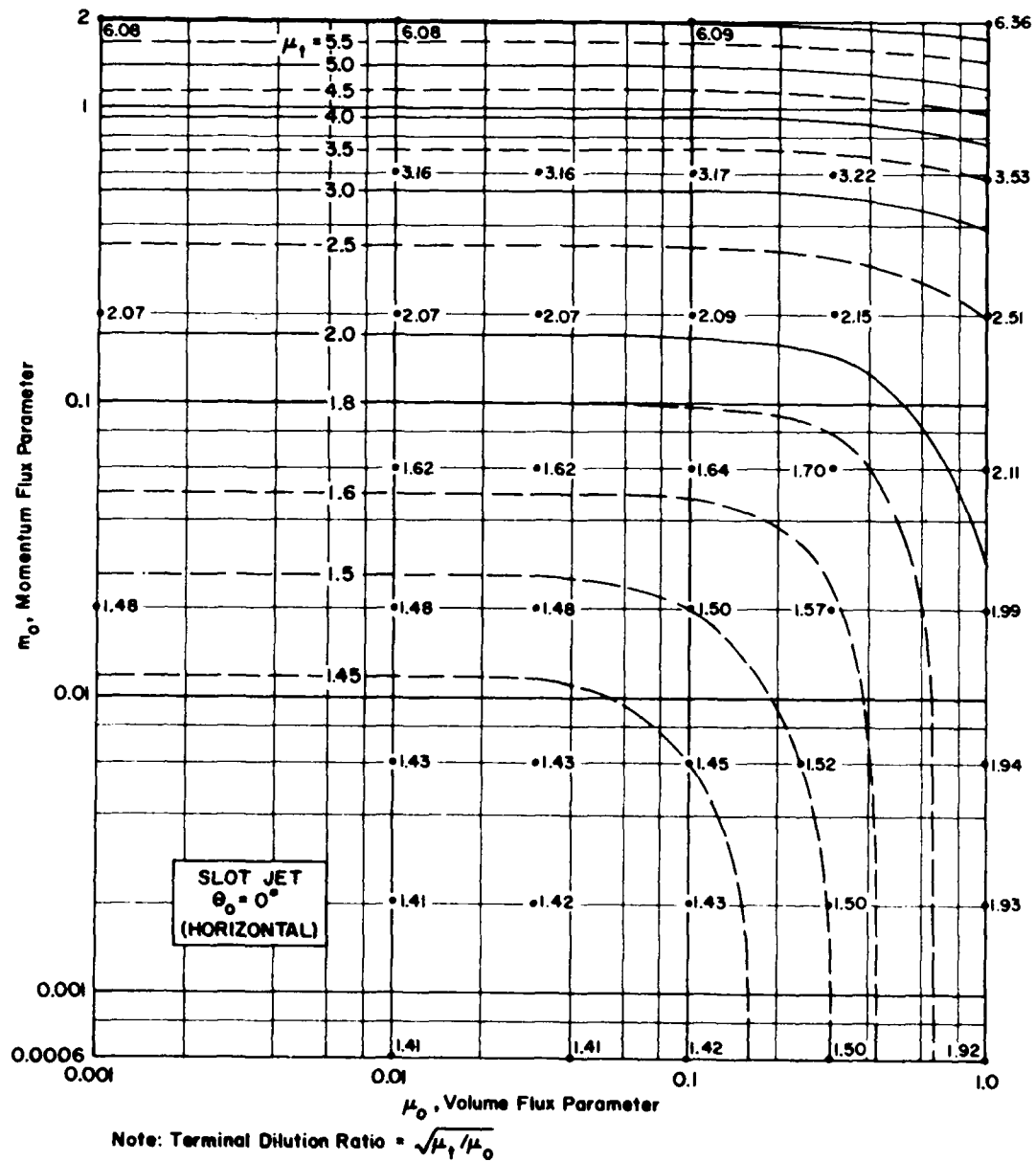
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Figure 4.7. Terminal volume flux parameter μ_t for horizontal slot buoyant jets in linearly density-stratified environments (from Fan and Brooks 1969)

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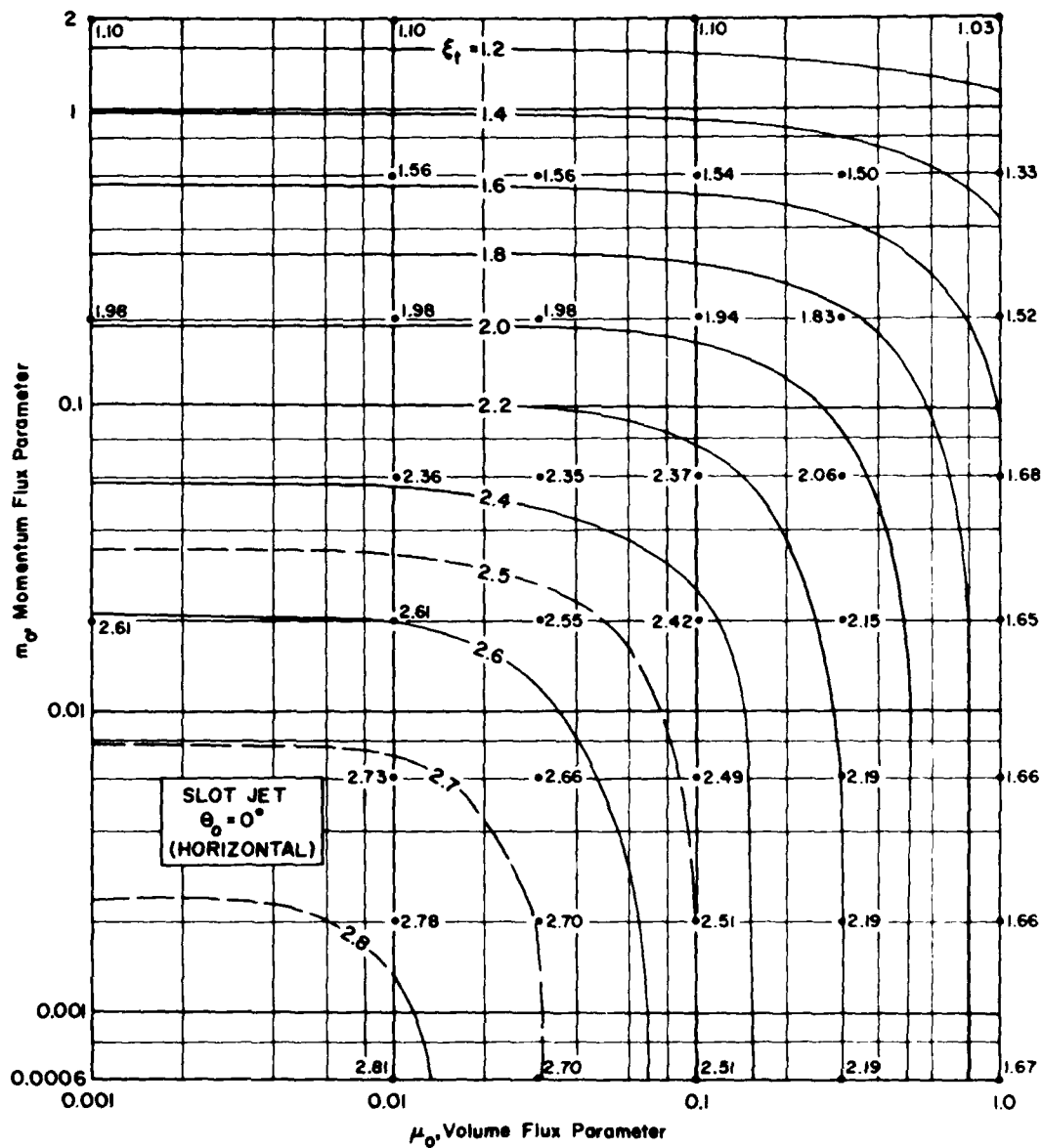
Figure 4.8. Terminal height of rise ξ_t for horizontal slot buoyant jets in linearly density stratified environments (from Fan and Brooks 1969)

discharges, they found the entrainment to be much greater than for nonbuoyant discharges. Although no method is given for predicting entrainment of a buoyant jet, the results provide useful information on the resulting complex reservoir flow patterns, and hence entrainment layers.

4.3 Nonbuoyant Immersed Jet

This situation is case b, Figure 4.1. The density of the jet is equal to the ambient density at its level of injection, so no buoyancy forces exist. For the purpose of engineering calculations, the entrainment can be assumed to be caused entirely by the source momentum flux. This assumption leads to simple analytical solutions in the unstratified case, although the stratified case is more complex.

4.3.1 Round discharge, unstratified ambient

The volume flux in the jet, Q_c, is assumed to depend only on the initial momentum flux, M_o, and the distance, x, from the source, thus

$$Q_c = f(M_o, x) \tag{4.32}$$

A dimensional analysis of Eq. 4.32 yields

$$\frac{Q_c}{M_o^{1/2} x} = C_1 \tag{4.33}$$

where C_1 is a constant which must be measured experimentally. For a round port:

$$u = \frac{4Q_o}{\pi D^2} \tag{4.34}$$

and a combination of Eqs. 4.3, 4.33, and 4.34 yields

$$\frac{Q_c}{Q_o} = C_1 \sqrt{\frac{4}{\pi}} \frac{x}{D} \qquad (4.35)$$

Thus, the volume flux in the jet increases linearly with distance from the source. The experimental work of Albertson, et al. (1950) yields $C_1 = 0.284$, so Eq. 4.35 becomes

$$\frac{Q_c}{Q_o} = 0.32 \frac{x}{D} \qquad (4.36)$$

Substitution of Eqs. 3.1 and 3.2 into Eq. 4.36 yields

$$E = 0.32 \frac{x}{D} - 1 \qquad (4.37)$$

It is important to note that in this case, E is not equal to a finite value but increases with x, and therefore time. To estimate the total volume of entrainment for the duration of the pumped-storage inflow cycle, Eq. 4.37 must be integrated with respect to time. To do this, the length of the jet as a function of time is required; this can be obtained from the results of Albertson, et al. (1950).

As the density of the ambient water is uniform, the density of the mixture is indifferent to the location of the entrained flow.

4.3.2 Round discharge, stratified ambient

The effect of stratification is to suppress vertical turbulent fluctuations, leading to eventual collapse of the jet and suppression of entrainment. The problem is analogous to that of wake collapse behind a submarine in a stratified fluid, reviewed by Lin and Pao (1979).

E is finite and should depend on the strength of the stratification. No experimental data were found from which E would be predicted. From

the experimental data on pumped-storage reservoirs discussed in Section 3.3, E would not be expected to differ significantly from unity for conditions typical of pumped-storage projects.

Because of the suppression of vertical motions by the stable stratification, most of the entrained flow should come from the same level as the discharge port.

4.3.3 Slot discharge, unstratified ambient

The volume flux is again assumed to depend only on the source momentum flux, m_o:

$$q_c = f(m_o, x) \tag{4.38}$$

where q_c is the volume flux per unit width. A dimensional analysis of Eq. 4.38 yields:

$$\frac{q_c}{m_o^{1/2} x^{1/2}} = C_2 \tag{4.39}$$

or

$$q_c = C_2 m_o^{1/2} x^{1/2} \tag{4.40}$$

where C_2 is a constant whose value must be experimentally determined. The experimental work of Albertson, et al. (1950) yields $C_1 = 0.62$, and combination of Eqs. 3.1, 3.2, 4.5, 4.6, and 4.40 yields

$$E = 0.62\left(\frac{x}{B}\right)^{1/2} - 1 \tag{4.41}$$

Again, E does not tend to a finite value but increases continuously with x, and therefore time. To estimate the total volume entrained during the pumped-storage inflow cycle, Eq. 4.41 must be integrated with respect to time. To do this, the length of the inflow jet as a function

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of time is needed; this can be obtained from the experiments of Albertson, et al. (1950).

As the density of the inflow and ambient water is uniform, the density of the jet is indifferent to the location of the entrained flow.

4.3.4 Slot discharge, stratified ambient

This situation could occur in the Marysville project (Figure 2.2) if inflow and outflow were the same set of selective withdrawal intakes. The effect of the stratification is to suppress vertical motions so that the jet must eventually collapse and entrainment cease.

Studies of the problem have been reported by Zuluaga-Angel, et al. (1972) and Darden, et al. (1975). A sketch of a typical flow distribution found by them is shown in Figure 4.9. An internal hydraulic jump forms shortly after the jet enters the tank; most of the entrainment takes place in the hydraulic jump. The entering jet pushes ahead of it a layer of reservoir water distinct from the jet itself. As a result, forward and reverse currents and rotors were set up as shown. Downstream of the hydraulic jump, the turbulence collapses and the flow became laminar and stable.

The results of these experiments enable entrainment to be estimated. Zuluaga-Angel, et al. (1972) expressed their results in terms of the total amount of fluid, V_e, entrained in a time t after the discharge was started. Their results were expressed as:

$$E^* = K_1 (t^*)^{3/4} \qquad\qquad (4.42)$$

where E^* is a normalized entrainment parameter, t^* is a scaled time, and K_1 is a constant.
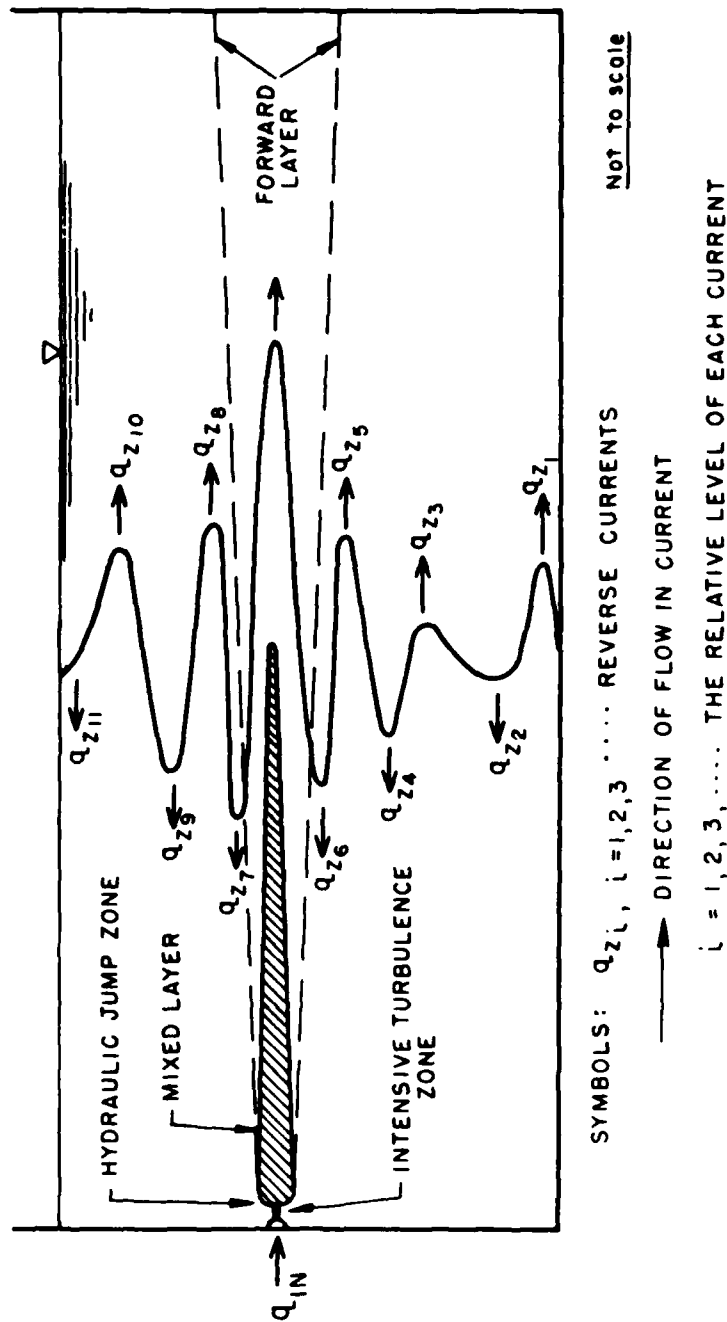
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Figure 4.9. Typical flow distribution induced by horizontal nonbuoyant slot jet into a linearly density-stratified reservoir (from Zuluaga-Angel, et al. 1972)

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$$E^* = \frac{V_e \, (\varepsilon g)^{2/3} \nu^{11/18}}{q_o^{3/2} g^{2/9}} \qquad (4.43)$$

and
$$t^* = t(\varepsilon g)^{1/2} \qquad (4.44)$$

where
$$\varepsilon = \frac{1}{\rho_o} \frac{d\rho_a}{dy} \qquad (4.45)$$

and ν is the kinematic viscosity. Substitution of Eqs. 4.44 and 4.45 into 4.42 yields

$$V_e = K_1 \, t^{3/4} (\varepsilon g)^{-7/24} q^{3/2} g^{2/9} \nu^{-11/18} \qquad (4.46)$$

To obtain the rate of entrained flow, q_e, Eq. 4.46 is differentiated with respect to time, yielding

$$q_e = \frac{dV_e}{dt} = \frac{3}{4} K_1 \, t^{-1/4} \, (\varepsilon g)^{-7/24} \, q_o^{3/2} \, g^{2/9} \nu^{-11/18} \qquad (4.47)$$

Finally, a combination of Eqs. 4.47 and 3.2 yields

$$E = \frac{3}{4} K_1 \, t^{-1/4} \, (\varepsilon g)^{-7/24} \, q_o^{1/2} \, g^{2/9} \, \nu^{-11/18} \qquad (4.48)$$

Zuluaga-Angel, et al. (1972) found two values for K_1, depending on the value of a source Frounde number, F', where

$$F' = \frac{q_o}{\sqrt{\varepsilon g B^2}} \qquad (4.49)$$

For $210 \le F' \le 985$, $K_1 = 560$

and for $61 \le F' \le 105$, $K_1 = 250$
$$\left. \right\} \qquad (4.50)$$

The different values of K_1 depend on the strength of the internal hydraulic jump. The higher value of K_1 corresponds to a strong jump,

the lower value to a weak jump.

The difficulty of estimating the source of the entrained fluid is illustrated by the complexity of the circulation patterns induced (Figure 4.9). More flow patterns are shown in Darden, et al. (1975), from which crude estimates of the sources of the entrained flow can be made.

Caution should be exercised in applying these results to a prototype situation. A dimensional analysis of the problem yields more parameters for the entrainment than given in Zuluaga-Angel, et al. (1972). Also, the small scale and laminar flow in their experiments, contrasted with the turbulent flows in the prototype, render their results questionable.

No studies were found dealing with inflows into a reservoir of arbitrary stratification.

4.4 Buoyant Boundary Jet

This situation is case C, Figure 4.1. The buoyancy forces are directed towards the boundary; the flow is therefore either a warm surface jet or a cold bottom jet. As the buoyancy forces oppose the vertical turbulent fluctuations, the jet collapses at some point downstream, so that E is always finite. As this flow situation occurs frequently in the discharge of heated water from power plants, many studies were found. These studies, however, concerned only the discharge into unstratified ambients; no studies of discharges into stratified ambients were found.

4.4.1 Round discharge, unstratified ambient

The studies in this category were made of discharges from rectangular channels. If the discharges of interest are from

similar rectangular channels, the results can often be used directly,
without recourse to an equivalent semicircular discharge structure.
Baddour and Chu (1978) present a listing of the more important
studies of this phenomenon.

The general features of this flow are shown in Figure 4.10. The
discharge velocity is u, and the channel depth and width h_o and $2b_o$,
respectively. Near the source, the jet entrains ambient fluid with
the width and depth of the jet growing similar to a nonbuoyant jet.
The effect of the jet buoyancy, however, is to suppress the vertical
turbulent fluctuations, so that the vertical growth rate becomes less
than that of a nonbuoyant jet. Eventually, the turbulent flow ceases
entirely, and the jet collapses into a thinner layer which spreads
due to its buoyancy. In the buoyant spreading layer, the amount of
entrainment is negligible. Thus, the total flow entrained by the jet
is finite and occurs within a finite distance from the source.

The length of the turbulent entraining region can be estimated from
the experimental results of Baddour and Chu (1978). They find the layer
to collapse in a longitudinal distance x_c, where

$$x_c \approx 5L_s \qquad\qquad (4.51)$$

L_s is a length scale defined by

$$L_s = \frac{M^{3/4}}{B_o^{1/2}} \qquad\qquad (4.52)$$

where M is the kinematic flow force (momentum flux plus excess hydro-
static pressure force):

$$M = uQ_o + \bar{Z}_o g' A_o \qquad\qquad (4.53)$$

SIDE
VIEW

h_0 u →

ENTRAINMENT

$X = X_c$

JET-LIKE
FLOW

BUOYANT
SPREADING

ENTRAINMENT
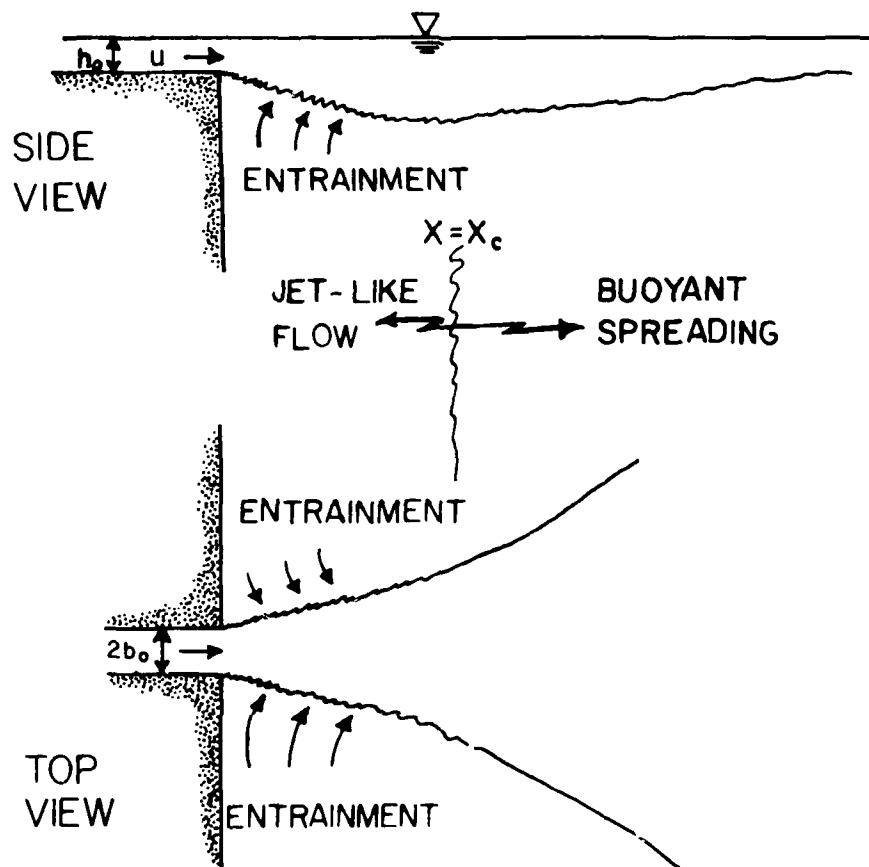
$2b_0$

TOP
VIEW

ENTRAINMENT

Figure 4.10.   General features of buoyant surface jet flow

B_o the source buoyancy flux, and Q_o is the source volume flux:

$$Q_o = u A_o \qquad (4.54)$$

and
$$B_o = g' Q_o \qquad (4.55)$$

where A_o is the channel area, $\overline{Z}_o$ is the centroid of the discharge, and g' is defined by Eq. 4.8.

The amount of entrainment can be estimated by the theoretical analysis of Stolzenbach and Harleman (1973), who considered discharge from a rectangular channel. They express entrainment in terms of a maximum dilution D_m:

$$D_m = \frac{Q_c}{Q_o} \qquad (4.56)$$

where
$$D_m = f(F_r, \; h_o/b_o) \qquad (4.57)$$

F_r is a Froude number defined by

$$F_r = \frac{u}{\sqrt{g' h_o}} \qquad (4.58)$$

Stolzenbach and Harleman find solutions for Eq. 4.57 by integration of the continuity, momentum, and energy equations and assumption of similarity profiles. Their results are shown in Figure 4.11. Combination of Eqs. 3.1, 3.2, and 4.56 yields

$$E = D_m - 1 \qquad (4.59)$$

Thus E can be estimated from Fig. 4.11 and Eq. 4.59. It is cautioned, however, that the predictions of Figure 4.11 are unconfirmed by experiment.
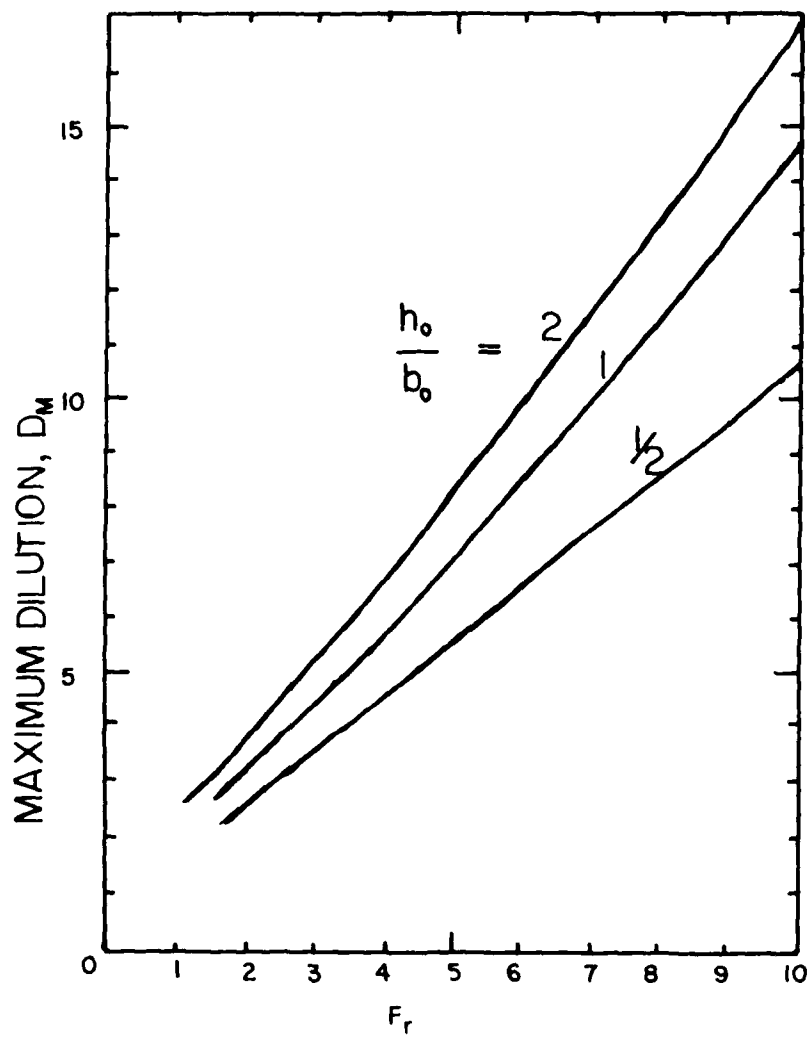
Figure 4.11. Calculated maximum dilution, D_m, for buoyant surface jet of finite width (from Stolzenbach and Harleman 1973)

Furthermore, the velocity distributions were found by Wiuff (1978) not to be similar, as assumed by Stolzenbach and Harleman. The reliability of estimates based on Figure 4.11 are therefore questionable. Stefan and Shanmugham (1975) also give some values of entrainment for $x/h_o = 10$. These results cannot be used here, however, as entrainment is not complete according to Baddour and Chu's criterion.

The density of the entrained flow is indifferent to the location of the entrainment in this uniform density case.

4.4.2 Round discharge, stratified ambient

No experimental or theoretical studies of this situation were found. It would be expected, however, that the amount of entrainment would not be significantly different from the unstratified case. As the effect of stratification is to suppress vertical motions, most of the entrainment would come from the levels of the discharge. Entrainment can thus be estimated as for the unstratified ambient (Section 4.4.1).

4.4.3 Slot discharge, unstratified ambient

Many studies of this problem have been made; some of them are listed in Chu and Vanvari (1976).

The basic properties of the flow were analyzed by Koh (1971) and are are shown in Figure 4.12. Koh found that several different flow regimes were possible, depending on the relative importance of source buoyancy and momentum fluxes, interfacial shear, and surface heat exchange. Following the initial jet-mixing region, an internal hydraulic jump can occur, downstream of which entrainment is negligible. Even if the jump does not occur, the effect of the buoyancy forces is to eventually suppress vertical turbulence and entrainment. Thus, the amount of entrainment is always finite.
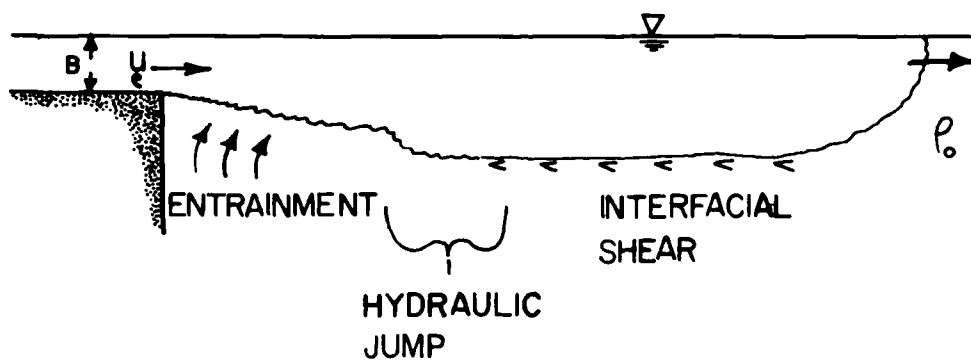
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Figure 4.12. General features of two-dimensional buoyant
surface jet

An upper limit to the amount of entrainment occurring in pumped-storage flows can be found by assuming that no internal jump occurs. The flow is then of the maximum entraining type, as discussed by Wilkinson (1971). Wilkinson expresses the amount of entrainment in this case as:

$$\frac{Q_c}{Q_o} = \frac{2F^2 + 1}{3F^{4/3}} \tag{4.60}$$

where F is defined by Eq. 4.22. Comparison of Eqs. 3.1, 3.2, and 4.60 yields

$$E = \frac{2F^2 + 1}{3F^{4/3}} - 1 \tag{4.61}$$

The density of the entrained flow is indifferent to its source in this uniform density case.

4.4.4 Slot discharge, stratified ambient

No experimental or theoretical studies of this situation were found. The effect of stratification would be expected to constrain the entrainment to come from the layers immediately below the discharge. The amount of entrainment would probably not be reduced considerably from the unstratified case.

4.5 Nonbuoyant Boundary Jet

This situation is case d, Figure 4.1. As there are no buoyancy forces exerted on the jet, the free surface or the bottom can be regarded as planes of symmetry, and the solution techniques are the same as those presented in Section 4.3, with the amount of entrainment reduced by one half.

4.5.1 Round discharge, unstratified ambient

See Section 4.3.1.

55

4.5.2 Round discharge, stratified ambient

See Section 4.3.2.

4.5.3 Slot discharge, unstratified ambient

See Section 4.3.3.

4.5.4 Slot discharge, stratified ambient

See Section 4.4.4.

5. SUMMARY AND CONCLUSIONS

The objective of the present study was to identify mathematical
descriptions of jet entrainment which offer promise in improving math-
ematical modeling of pumped-storage reservoir inflows. It was found that
a wide variety of inflow types could exist, whose entrainment is influenced
to varying degrees by source geometry, boundary proximity, source buoyancy
and momentum fluxes, and ambient stratification. Because of the inter-
action of these many variables, no one model exists which will predict
entrainment in all possible situations.

The flows are classified according to a primary and secondary system
discussed in Section 4.1. The primary classification groups the flows
according to source buoyancy and boundary proximity. The four flow types
are shown in Figure 4.1. The secondary classification divides each of
these flows into those discharging into uniform or stratified ambient
water and those discharging from a slot or round orifice. Thus, a total
of 16 possible flow types exist; each of these flows is discussed individually
in Sections 4.2 through 4.5.

Some of the flow types have been thoroughly studied, and some have
not been studied at all. For each flow type, the important features are
discussed and a candidate method for predicting entrainment presented. In
general, not all of the studies are cited, nor are detailed discussions
presented. Potential methods of prediction are presented either as analytical
solutions, graphical solutions, or mathematical models. The analytical
and graphical solutions are given in the appropriate section; where can-
didate mathematical models are available, references to their sources are
cited.

The reader is cautioned that often only fairly crude estimates of entrainment can be made. This is partly due to the errors involved in approximating complex source geometries by slot or round orifices. Also, not all of the idealized flows used for entrainment prediction have been thoroughly studied. As discussed in the text, the reliability of the predictions for some of these flows is questionable. Some of the flows discussed may have wide applicability to the prediction of pumped-storage effects; these flows should be investigated in greater detail, and better mathematical models of them developed.

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APPENDIX A: NOTATION

A_o Channel area

b_o Buoyancy flux per unit length; also half channel width

B Width of slot orifice

B_o Source buoyancy flux of round discharge

C_1, C_2 Experimental constants

D Diameter of round orifice

D_m Maximum dilution of three-dimensional surface buoyant jet

E Entrainment coefficient

E^* Normalized entrainment parameter

F, Fr, F' Froude number

g Acceleration due to gravity

g' Modified gravitational acceleration

h Thickness of established density current (Figure 4.2)

h_o Depth of channel

K_1 Experimental constant

L_s Length scale

m_o Momentum flux per unit length; also dimensionless momentum flux

M Kinematic flow force

M_o Source momentum flux of round discharge

q_o Volume flux per unit length

Q_c, Q_e, Q_o Volume flow rates (see Figure 3.2)

S Dilution

S_m, S_o Centerline dilution

S_t Dilution at terminal rise height

t Time

A1

$t*$	Scaled time	
T	Stratification parameter	
u	Velocity of pumpback or generation flow at source	
V_o	Volume of pumpback or generation flow	
V_e, $'_r$	Volume of entrained water	
x, y, z	Coordinates (Figure 3.2)	
x_c	Length of entraining flow region	
y_t	Terminal rise height	
$\bar{Z}_o$	Centroid of discharge	
ε	Stratification parameter	
μ_o	Dimensionless volume flux parameter	
μ_t	Terminal volume flux parameter	
ν	Kinematic viscosity	
ξ_t	Dimensionless terminal rise height	
ρ_a	Ambient water density	
ρ_o	Ambient water density of level of pumpback or generation flow and initial density of efflux in Figure 3.2	
ρ_1	Density of pumpback or generation flow	

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