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Theory to Incorporate Nonlinear Lift and Drag Coefficients,**

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An Extension of Blade Element Momentum Theory to Incorporate Nonlinear Lift and Drag Coefficients



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An expression for the steady induced inflow velocity is obtained from blade element and momentum theory considerations. The lift coefficient is allowed to be a quadratic function of angle of attack and the drag coefficient, incorporated through a perturbation procedure, can be any arbitrary function of angle of attack. A principal range of interest is identified in which the function inflow versus blade pitch angle is single valued. The results reduce to the classic formula found in the textbooks when the lift coefficient is linear and drag is ignored.

Introduction

THE expressions for induced inflow velocity in hover and vertical flight based on a combination of blade element and momentum theories found in textbooks such as Ref. 1, lack the generality of dealing with nonlinear airfoil lift coefficient c_l versus angle of attack α . When generalized to incorporate nonlinear c_l , the analysis should also include the influence of drag, which may change the inflow by as much as 20% in the stall regime. An additional refinement that is desirable when the formula is to be used in aeroelastic stability analyses, is to ensure that a useful estimate for inflow is available for both positive and negative thrust and in either ascending or slow descending vertical flight. Such a formula has not been published and it is the purpose of this note to present one. This kind of formula will avoid having to solve for the inflow iteratively when the lift coefficient is nonlinear. It should prove to be helpful in aeroelastic stability analyses, such as Ref. 2, when the influence of the stall parameters on the linearized aeroelastic stability is analyzed.

The procedure herein is to combine expressions for the thrust from blade element and momentum theories with the assumption that the lift coefficient is quadratic, over some range, in angle of attack. The influence of drag will be incorporated quite naturally as a perturbation of the vertical component of induced inflow velocity. The effects of wake rotation induced by lift and drag are neglected.

Formulation

From either blade element or momentum theory it is necessary to know the induced velocity to specify the operating condition of the rotor. Consider first the thrust from momentum theory where we define the induced velocity v_i and vertical velocity V positive when the flow is down through the rotor and the thrust per unit blade length dT/dr is positive when air loads produce an upward force on the rotor.

The momentum balance yields

$$\frac{dT}{dr} = 4\pi r v_i |v_i + V| \quad (1)$$

where r is the distance from the center of rotation, being a maximum of R at the blade tip. It is helpful to non-dimensionalize the variables; thus,

$$\tau = \bar{v}_i |\bar{v}_i + \bar{V}| \quad (2)$$

where

$$\begin{aligned} \bar{v}_i &= \frac{v_i}{\Omega R} \\ \bar{V} &= \frac{V}{\Omega R} \end{aligned} \quad (3)$$

$$\tau = \frac{dT/dr}{4\pi r \Omega^2 R^2}$$

and the absolute value signs ensure that thrust τ (positive up) and induced inflow velocity $\bar{v}_i$ (positive down) have the same sign, as they must. A plot of τ versus $\bar{v}_i$ for $\bar{V} > 0$ (climb), $\bar{V} < 0$ (descent), and $\bar{V} = 0$ (hover) is found in Fig. 1. Notice that for a given thrust, the induced inflow may be single valued or assume three different values. The various regimes are identified in the legend of the figure. In the case of descent, for example, when $\bar{v}_i$ is less than $-\bar{V}/2$ (i.e., $\bar{v}_i + \bar{V}/2 < 0$) but $\bar{v}_i > 0$, the flight condition is autorotation and the rotor is operating in the windmill brake state. For $-\bar{V}/2 < \bar{v}_i < -\bar{V}$, momentum concepts do not apply, and, except for small $\bar{V}$, when $\bar{v}_i > -\bar{V}$ they are not generally accurate. Notice for zero rate of climb that the results are antisymmetric about the origin. Gessow and Myers (Ref. 1) do not define the signs in a rigorous manner for these unusual

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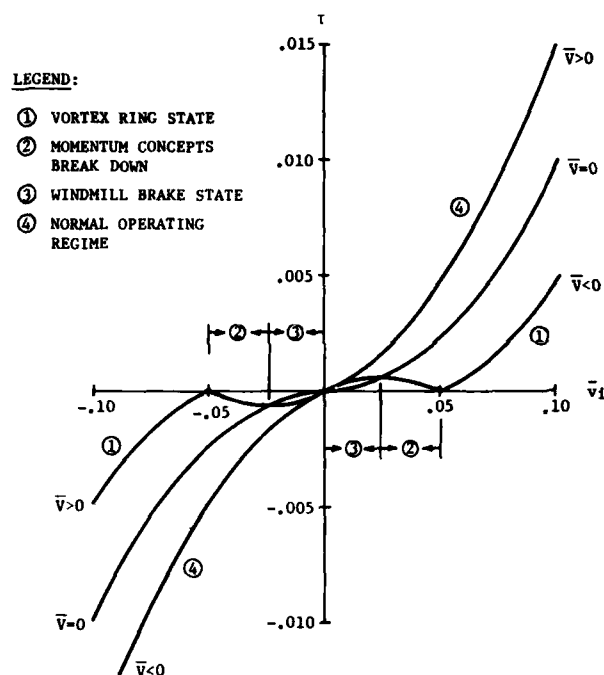


Fig. 1 Thrust τ versus induced inflow velocity $\bar{v}_i$ for various rates of climb $\bar{V}$ based on Eq. (2) with $\bar{V} = -0.05, 0, 0.05$. (All quantities dimensionless).

conditions. These conditions, though certainly unusual, may be encountered in experimental testing or in the course of iterating to solve for the equilibrium operating condition when blade elastic deflections are present (Ref. 3).

Now let us write the thrust from blade element theory, including drag, assuming that the total resultant wind velocity with respect to the blade element U is given by

$$U = \Omega r / \cos \phi + \text{negligible terms}$$

where wake rotation induced by lift and drag forces is neglected. The thrust per unit blade length is thus

$$\begin{aligned} \frac{dT}{dr} &= \frac{b\rho c \Omega^2 r^2}{2\cos^2 \phi} (c_l \cos \phi - c_d \sin \phi) \\ &= \frac{b\rho c \Omega^2 r^2}{2\cos \phi} (c_l - c_d \tan \phi) \end{aligned} \quad (4)$$

where b is the number of blades, c is the local blade airfoil chord length at radial location r , and the inflow angle ϕ is given by

$$\tan \phi = \frac{V + v_i}{\Omega r} = \frac{\bar{V} + \bar{v}_i}{\bar{r}} \quad (5)$$

where $\bar{r} = r/R$.

It is now a simple matter to equate the thrust parameters from the two theories in Eqs. (1) and (4) to obtain an equation for $\bar{v}_i$. In dimensionless variables one obtains

$$\begin{aligned} \tau &= \bar{v}_i |\bar{v}_i + \bar{V}| \\ &= \frac{b\bar{c}}{8\pi} [\bar{r}c_l - (\bar{v}_i + \bar{V})c_d] \left[1 + \left(\frac{\bar{V} + \bar{v}_i}{\bar{r}} \right)^2 \right]^{1/2} \end{aligned} \quad (6)$$

where $\bar{c} = c/R$. We now assume that $\bar{v}_i$ and $\bar{V}$ are of $O(\epsilon)$ where ϵ is in some sense small compared to unity and scale the

dimensionless variables with ϵ so that

$$\begin{aligned} \bar{V} &= \epsilon \nu \\ \bar{v}_i &= \epsilon \lambda \end{aligned} \quad (7)$$

This yields

$$\lambda |\lambda + \nu| = \bar{r}c_l - \epsilon(\lambda + \nu)c_d + O(\epsilon^2) \quad (8)$$

where we have let $\epsilon^2 = b\bar{c}/8\pi$. Terms of order ϵ^2 and higher are assumed to be negligible. Note, however, that this assumption breaks down if $\bar{r}$ is small.

The angle of attack is given by

$$\begin{aligned} \alpha &= \theta - \phi = \theta - \tan^{-1} \left(\frac{\bar{V} + \bar{v}_i}{\bar{r}} \right) \\ &= \frac{\epsilon}{\bar{r}} (\bar{r}\beta - \nu - \lambda) + O(\epsilon^2) = \delta\epsilon \end{aligned} \quad (9)$$

where the pitch angle θ is scaled as $\epsilon\beta$ and the angle of attack α is scaled as $\delta\epsilon$. Now let $\lambda = \lambda_0 + \lambda_1\epsilon$ so that

$$\delta = \delta_0 + \delta_1\epsilon = \frac{1}{\bar{r}} (\bar{r}\beta - \nu - \lambda_0 - \lambda_1\epsilon) \quad (10)$$

The lift and drag coefficients in terms of δ are now

$$\begin{aligned} c_l(\delta) &= c_l(\delta_0) + \delta_1\epsilon \frac{dc_l}{d\delta}(\delta_0) \\ c_d(\delta) &= c_d(\delta_0) + \delta_1\epsilon \frac{dc_d}{d\delta}(\delta_0) \end{aligned} \quad (11)$$

Substitution of Eqs. (10) and (11) into Eq. (8) yields

$$\begin{aligned} \bar{r}c_l(\delta_0) + \bar{r}\delta_1\epsilon \frac{dc_l}{d\delta}(\delta_0) - \epsilon(\nu + \lambda_0)c_d(\delta_0) &= \\ (\lambda_0 + \epsilon\lambda_1) |\nu + \lambda_0 + \lambda_1\epsilon| + O(\epsilon^2) \end{aligned} \quad (12)$$

The lift coefficient can be of no higher degree than quadratic if we expect a closed form solution to Eq. (12) without the ϵ terms. Thus, let

$$\begin{aligned} c_l &= c_{l0} + c_{l1}\alpha + c_{l2}\alpha^2 \\ &= t_0 + t_1\delta + t_2\delta^2 \end{aligned} \quad (13)$$

Here c_{l0} could contain the effects of steady coning as well as airfoil camber. Now, the equations for ϵ^0 and ϵ^1 in Eq. (12) with c_l from Eq. (13) are

$$\begin{aligned} \epsilon^0: \quad & \left[1 - \frac{t_2}{\bar{r}} \text{sgn}(\lambda + \nu) \right] \lambda_0^2 + \left\{ \nu + \left[t_1 + \frac{2t_2}{\bar{r}} (\bar{r}\beta - \nu) \right] \right. \\ & \times \text{sgn}(\lambda + \nu) \left. \right\} \lambda_0 - \text{sgn}(\lambda + \nu) \left[t_0 + t_1 (\bar{r}\beta - \nu) \right. \\ & \left. + \frac{t_2}{\bar{r}} (\bar{r}\beta - \nu)^2 \right] = 0 \end{aligned} \quad (14)$$

$$\epsilon^1: \quad \lambda_1 = \frac{-(\nu + \lambda_0)c_d(\delta_0)\text{sgn}(\lambda + \nu)}{2\lambda_0 + \nu + (t_1 + 2t_2\delta_0)\text{sgn}(\lambda + \nu)} \quad (15)$$

The solution of Eq. (14) is then

$$\lambda_0 = \left\langle -\frac{\nu}{2} - \text{sgn}(\lambda + \nu) \left[\frac{t_1}{2} + \frac{t_2}{f} (\bar{r}\beta - \nu) \right] \right. \\ \left. \pm \left\{ \left[\frac{\nu}{2} - \frac{t_1}{2} \text{sgn}(\lambda + \nu) \right]^2 - t_0 t_2 + \text{sgn}(\lambda + \nu) [\bar{r} t_0 \right. \right. \right. \\ \left. \left. \left. + \bar{r} t_1 \beta + t_2 \beta (\bar{r}\beta - \nu) \right] \right\}^{1/2} \left[1 - \frac{t_2}{f} \text{sgn}(\lambda + \nu) \right]^{-1} \right\rangle \quad (16)$$

Now, $\bar{v}_i$ can be expressed from Eqs. (15) and (16)

$$\bar{v}_i = \epsilon \lambda = \epsilon (\lambda_0 + \epsilon \lambda_1) = \bar{v}_{i0} + \bar{v}_{i1} \quad (17)$$

with

$$\bar{v}_{i0} + \bar{V} = \frac{\frac{1}{2} \bar{V} - \sigma (\frac{1}{2} c_{l1} + c_{l2} \theta) \pm Z}{1 - (\sigma c_{l2} / \bar{r})} \quad \bar{r} > \sigma c_{l2} \quad (18)$$

$$\bar{v}_{i1} = \frac{-\sigma (\bar{v}_{i0} + \bar{V}) c_d (\alpha_0)}{\pm 2Z} \quad (19)$$

where

$$Z^2 = \left(\frac{\bar{V}}{2} - \frac{\sigma c_{l1}}{2} \right)^2 + \sigma \left[\bar{r} c_{l0} \left(1 - \frac{\sigma c_{l2}}{\bar{r}} \right) \right. \\ \left. + \bar{r} \theta c_{l1} + \theta c_{l2} (\bar{r} \theta - \bar{V}) \right] \quad (20)$$

and

$$\sigma = \frac{b\bar{c}}{8\pi} \text{sgn}(\bar{v}_i + \bar{V}) \quad (21)$$

Straightforward numerical evaluation of $\bar{v}_i$ for representative values of $b\bar{c}/8\pi$, θ , $\bar{V}$, c_{l0} , c_{l1} , c_{l2} , c_d , and $\bar{r}$ yields the simple result that the sign of the quantity Z must be taken the same as the sign of $\bar{v}_i + \bar{V}$. The choice of this sign is commensurate with the thrust and inflow being of the same sign. Note that $\bar{r}$ must not be small and, if $\sigma c_{l2} > 0$, we have a lower limit on its value in Eq. (18).

With $\bar{Z} = Z \text{sgn}(\bar{v}_i + \bar{V})$ we have

$$\bar{v}_i + \bar{V} = (\bar{v}_{i0} + \bar{V}) \left[1 - \frac{\sigma c_d (\alpha_0)}{2\bar{Z}} \right] \quad (22)$$

and the drag correction term (underlined> is normally smaller than unity so that $\text{sgn}(\bar{v}_i + \bar{V}) = \text{sgn}(\bar{v}_{i0} + \bar{V})$. It should be noted that the drag coefficient can assume any functional form without precluding a closed form solution. Changes in sign for $\bar{v}_{i0} + \bar{V}$ occur at $\theta = \theta_{\sigma 1}$, where

$$\theta_{\sigma 1} = \frac{c_{l1}}{2c_{l2}} \left(-1 + \sqrt{1 - \frac{4c_{l2}c_{l0}}{c_{l1}^2}} \right) \quad c_{l1}, c_{l2} \neq 0 \quad (23a)$$

$$\theta_{\sigma 1} = -\frac{c_{l0}}{c_{l1}} \quad c_{l1} \neq 0, c_{l2} = 0 \quad (23b)$$

If $c_{l1} = c_{l2} = 0$, $\bar{v}_{i0} + \bar{V}$ has the sign of c_{l0} . When $c_{l1}, c_{l2} \neq 0$ the principal range of interest for θ is when $\theta < \theta_{\sigma 2}$ ($\alpha > 0$) or $\theta > \theta_{\sigma 2}$ ($\alpha < 0$). Here $\bar{v}_i$ may be regarded as a single-valued function and $\theta_{\sigma 2}$ is given by

$$\theta_{\sigma 2} = \frac{c_{l1}}{2c_{l2}} \left(-1 - \sqrt{1 - \frac{4c_{l2}c_{l0}}{c_{l1}^2}} \right) \quad (24)$$

Of course, Z^2 must be positive for all cases as well.

In summary, $\text{sgn}(\bar{v}_i + \bar{V}) = \text{sgn}(\theta - \theta_{\sigma 1})$ with $\theta_{\sigma 1}$ given in Eq. (23) and $\text{sgn}(\theta_{\sigma 2} - \theta) = \text{sgn} \alpha$ with $\theta_{\sigma 2}$ given in Eq. (24). The induced inflow velocity may be calculated from Eq. (22) with

$$\bar{v}_{i0} + \bar{V} = \frac{\frac{1}{2} \bar{V} - \sigma (\frac{1}{2} c_{l1} + c_{l2} \theta) + \bar{Z}}{1 - (\sigma c_{l2} / \bar{r})} \quad (25)$$

It is not necessary to explain here, in detail, all possible behavior of the solutions of these equations. It is noted that results from the above formulas may be infinite or complex for certain values of the parameters. Depending on the nature of specific cases, however, such problems can usually be circumvented with little difficulty.

As is evident, the present results reduce to the classical formula as given in textbooks, for example Gessow and Myers (Ref. 1), when c_l is linear, c_d is ignored, and the sign of $\bar{Z}$ is taken positive. The choice of the sign of $\bar{Z}$, however, is not adequately discussed in Ref. 1. One possible extension of the present work is incorporation of the wake rotation component induced by lift and drag.

References

- Gessow, A. and Myers, Jr., G.D., *Aerodynamics of the Helicopter*, Frederick Ungar Publishing Co., New York, 1967.
- Ormiston, R.A. and Bousman, W.G., "A Study of the Stall-Induced Flap-Lag Instability of Hingeless Rotors," Presented at the 29th Annual Forum of the American Helicopter Society, Washington, D.C., May 1973.
- Hodges, D.H. and Ormiston, R.A., "Stability of Elastic Bending and Torsion of Uniform Cantilever Rotor Blades in Hover with Variable Structural Coupling," NASA TN D-8192, 1976.

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