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The Nonlinear Behavior of Elastic Slender Straight Beams Undergoing Small Strains and Moderate Rotations²

Dewey H. Hodges.² The subject paper deals with mathematical modeling of the nonlinear behavior of beams. A set of equations is derived and used to investigate the static behavior of a slender cantilever beam loaded transversely at the free end. Since the loading is not necessarily along the principal axes, the principal bending deflections and torsion are coupled elastically. In a strictly linear theory there would be no torsion; thus the presence of torsion is, itself, a manifestation of nonlinear behavior. The agreement obtained with experimental data achieved in the subject paper is excellent and confirms that one may apply the geometric nonlinear theory of elasticity to beams with confidence provided an adequate degree of nonlinearity is retained in the mathematical model to account for large deflections.

The same problem was treated by Dowell, et al., in [1]. The analytical results of [1] and those of the subject paper agree quite well as long as w , the bending deflection in the plane of greatest flexural flexibility, remains relatively small compared to the beam length. In fact, analytical results of [1] depart from those of the subject paper only when w ceases to be a small fraction of the beam length. An examination of the w equation (33b) in the subject paper reveals that in addition to linear and second-degree terms, several terms of third degree in the deflections appear. These terms are retained in the authors' original derivation of these equations [2] for the special case when I_{22}/I_{33} is large compared to unity. Although not stated in the subject paper, these third-degree terms constitute the only difference between the equations of [1] and the subject paper. Thus the improved agreement reported in the subject paper must be due to the presence of these third-degree terms and not to the reasons indicated in the paper. The two main reasons given for the improved agreement shown in the subject paper are now examined.

The first reason is mentioned on Page 162, Column 1, where the authors attribute the accuracy of their results to "a more careful and consistent" derivation than in [1]. The original derivation of the authors' equations [2] concludes with a set of nonlinear equations of second degree in bending and torsion deflections. These equations are intended for applications to rotating blades and are based on the assumption that bending and torsion rigidities are of the same order of magnitude. For the special case where I_{22}/I_{33} and EI_{22}/GJ are large compared to unity, the authors endeavored to retain third-degree terms whenever they are multiplied by a large coefficient in the equations. It is not clear that appropriate measures were taken,

however, to ensure that all third-degree terms multiplying the large coefficients were retained. In fact, only terms through second degree were retained in the bending curvature expressions, and it was never demonstrated that third-degree terms from these expressions would not appear in the final equations. Moreover, the final equations in the subject paper, because of the particular third-degree terms retained, do not have a self-adjoint structural operator. While it is acknowledged that the authors deserve credit for adding appropriate higher-degree terms to the equations to improve the correlation with experimental data, this, in itself, does not necessarily imply a higher degree of care or consistency.

The second reason given by the authors appears in the Concluding Remarks section: "The superior agreement obtained with the present equations is due to differences between the final equations of equilibrium used here and those given in [3]. These differences have been discussed with considerable detail in [2]." (Reference numbers refer to those of this discussion). The first sentence, by itself, appears to be referring to the third-degree terms retained in equation (33b) because the ratio I_{22}/I_{33} is large compared to unity for the particular beam considered. This is not the intended meaning according to the second sentence, however, because differences related to these third degree terms are not discussed in [2]. The differences between the equations of [2, 3] that are discussed in [2] relate to the second-degree equations only and have been recently clarified in [4, 5]. The final second-degree nonlinear equations of [2, 3], while differing slightly for pretwisted beams [4], are actually equivalent for the present case of a nonrotating beam without pretwist [5]. Therefore, the improved agreement reported in the subject paper does not prove that the equations for rotating blade applications derived in [2] are more "reliable" nor that they "can be used with confidence" any more than those of [3]. The results simply prove that the equations of the subject paper are more accurate than those of [1] when applied to problems in which the bending deflection w may exceed a small fraction of the blade length and the ratio I_{22}/I_{33} is large compared to unity. These cases are not treated in [1] in which the squares of bending slopes were assumed to be negligible with respect to unity and the equations were taken directly from [3] where I_{22} and I_{33} were assumed to be the same order of magnitude [3, pp. 8, 9].

References

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3. Hodges, D. H., and Dowell, E. H., "Nonlinear Equations of Motion for the Elastic Bending and Torsion of Twisted Nonuniform Rotor Blades," NASA TN D-7818, Dec. 1974.
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¹ By A. Rosen and P. Friedmann, and published in the March, 1979, issue of the *ASME JOURNAL OF APPLIED MECHANICS*, Vol. 46, pp. 161-168.

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