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DISCRETE-TIME STOCHASTIC CONTROL OF HYBRID SYSTEMS.(U)
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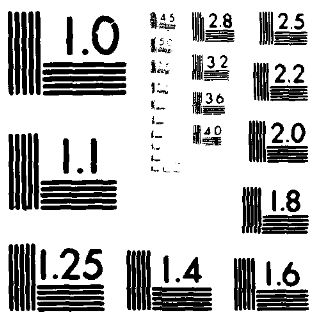
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DISCRETE-TIME STOCHASTIC CONTROL OF HYBRID SYSTEMS

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ABSTRACT

A simple class of discrete-time nonlinear stochastic control problems with quadratic costs for linear systems with randomly-jumping state-dependent parameters are solved using dynamic programming. The resulting controllers exhibit 'active hedging' properties.

1. Introduction

Complex engineering systems occasionally fail to function as desired, regardless of how well they are designed and manufactured. This can result in catastrophically high human and monetary costs. Traditionally, engineering systems have been made reliable by the use of highly reliable components and assembly procedures so that failures are unlikely, and through the use of redundancy in design so that individual failures need not be catastrophic to the entire system. A different approach is to use probabilistic information about component failures as well as observations made during system operation to achieve an adaptive, dynamic form of reliability in which the system anticipates failures and avoids or adapts to them. In this paper we develop a design methodology for feedback controllers with this fault-tolerance.

Fault-prone systems generally experience abrupt changes in structure and state from phenomena such as component and subsystem failures and repairs, changing subsystem interconnections, changes in state equilibrium points and abrupt environmental distrubances. A characterizing attribute of fault-prone systems is their operation in different forms, where each form corresponds to some combination of these events.

We will represent fault-prone systems by discrete-time 'hybrid' continuous-plus-discrete-state models. These model can be used to represent nonlinear systems with multiple equilibria. They can be represented as a set of linearized models (of phase or x dynamics), where the form designates the appropriate operating region of the phase. As long as the phase subsystem state remains within the domain of attraction of its current equilibrium point, the system is in a form associated with that point. When it leaves this region, the form changes. This kind of model should be amenable to detailed analysis since it consists of linear 'pieces'. It has been proposed for the modelling of electric power systems ([10],[11]) because it appears to capture some important aspects of these systems.

For example, an 'initiating event' (component failure or exogenous disturbance) that causes a change in the phase dynamics can lead to large phase transient responses, because the system is not near the equilibrium of the new form. These transients may force voltages or currents (that is, the phase subsystem state) to exceed allowable values, causing new form changes as protective relays switch, components burn-out, etc. These form changes are accompanied by new phase equilibrium values which the phase may be far away from; thus there are extended transients which may cause still more form changes, and so on. Such cascades or 'waves' of system failures have been observed to occur in large scale blackouts (see, for example, [8]).

Continuous time control problems involving hybrid systems have been considered by a number of authors. A survey of these results is included in [7]. Krasovskii and Lidskii [5] obtained most of the results which are available for continuous time models. These problems were also studied by Wonham [12] and Sworder [6]. Discrete time versions of these problems have not been investigated as thoroughly. Some results appear in [1]-[3], and in the (suboptimal) multiple model adaptive algorithm described in [4], [9].

2. Modelling of Fault-Prone Systems

The state of a fault-prone system can be decomposed into two parts: a form process, ρ, which indicates the operational status of the system, and the rest of the state which we call the phase process, x. A logical structure for modelling this kind of system is the hybrid arrangement depicted in Figure 1. It is a feedback connection of two subsystems: a phase subsystem that represents the dynamic evolution of the state between form shifts, and a form subsystem that describes form transitions.

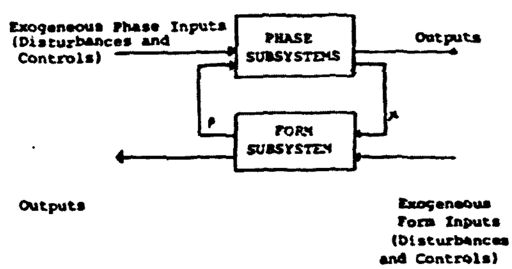

Figure 1: General Hybrid System Structure.

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The phase subsystem has state x (usually taking values in a finite-dimensional vector space). It can be modelled by deterministic or stochastic finite-dimensional vector differential or difference equations. The parameters of these models will depend upon the form, ρ, which feeds into the phase subsystem. In this way the dynamic effects of component failures and other form shifts can be captured in the phase model.

The phase subsystem can also experience abrupt jumps in its state. These jumps might model the instantaneous results of form shifts; in electrical systems, voltage and current jumps that accompany broken connections or sensor failures are examples of this. Phase jumps <u>need not</u> be associated with *form* shifts, however. Aircraft acceleration changes caused by flying into air pockets, transient sensor biases, and voltage jumps in electrical systems caused by lightning bolts are examples.

The form subsystem has a scalar state ρ (usually taking values in a finite set). A crucial consideration in the modelling of stochastic hybrid systems is the realistic representation of form transitions. We wish to consider hybrid systems where the form can change in two kinds of ways:

- <u>independently of the phase</u> (as though *no phase-to-form link in Figure 1*)
- <u>as a (possibly random) function of the phase</u> (that is, with feedback).

Examples of phase-independent form shifts are random 'no wearout' component failures and lightning-induced failures in electrical power distribution systems. Examples of phase-dependent form shifts in electrical power systems include restructuring of the system when generator-protecting relays and circuit-breakers trip, *human operator control actions based* on observation of the phase dynamics (such as switching on auxilary *generators*) and transmission-line failures due to current overloads. Note that both kinds of form shifts can be <u>totally</u> <u>unpredictable</u> (as in random 'no wearout' component failures), <u>totally</u> <u>predictable</u> (as in scheduled direct control actions) or <u>partially</u> <u>predictable</u> (as in the switching of relays precisely (or approximately) when a random quantity reaches a given threshold).

Mathematically, we will describe a hybrid system in discrete time, in terms of a state vector, (x_k, ρ_k) indixed by a time parameter. The form ρ_k belongs to a finite set $\underline{M}=\{1,\ldots,m\}$; the phase x_k belongs to R^n, and the time horizon is $k=0,1,\ldots,N$. The joint stochastic process $\{x_k, \rho_k : k=0,\ldots,N\}$ is generally assumed to be Markovian although neither $\{x_k\}$ nor $\{\rho_k\}$ need be. We adopt the following <u>order</u> of operation convention:

(1) at time k, the system state is (x_k, ρ_k)

(2) control u_k is determined from x_k and ρ_k

(3) during time interval $(k,k+1)$, x_{k+1} is generated from x_k, ρ_k and control u_k

(4) finally, ρ_{k+1} is generated from x_{k+1}, ρ_k and control u_k.

This convention allows a failure or other form shift to be modelled as occurring at the final time k=N.

The form process $\{\rho_k\}$ is modelled by:

$$\text{Prob}\{\rho(k+1)=i \mid \rho(k)=j,\ x(k+1)=x\} = \rho_{k+1}(i,j,x)$$

where $\rho_k(i,j,x) \geq 0$ and $\sum_j \rho_k(i,j,x)=1$.

The phase process $\{x_t\}$ obeys

$$x_{k+1}=F(K,x_k,\rho_k,u_k,\omega_k)+\sum_{J\in M} H(k+1,x_k,\omega_k u_k,\rho_k,\rho_{k+1}=j)\ \Delta N(\rho_k,j,k)$$

$$\Delta N(m,j,k) \overset{\Delta}{=} N(m,j,k) - N(m,j,k-1)$$

where $\{\omega_k\}$ is a sequence of discrete-time vector white noise and the $N(m,j,k)$ are counters recording the number of <u>form jumps</u> from m to j through time k (if $m\neq j$) and the number of <u>phase-only</u> jumps while in form m (if $m=j$).

3. *General Problem Formulation*

A fault-tolerant controller should enable a system to operate acceptably well in many (or all) of its forms. The meaning of 'acceptably well' may vary with the form, reflecting changes in operating costs, constraints and capabilities of the system. There are two types of control actions available for hybrid systems: <u>phase control</u> of x and <u>form control</u> of ρ.

In general, both kinds of control will be feedback controls depending upon possibly noisy observations of current (or past) values of the hybrid system state (x,ρ). If these quantities are not perfectly observable then the design of phase and form estimators is an integral part of the overall control problem. In this paper we consider only the <u>phase</u> control, u, which is found by minimizing the expected value of a cost functional

$$J(u)=E \sum_{k=0}^{N-1} L(K,x_k,\rho_k,u_k)+\sum_{k=0}^{N-1}\sum_{j\in M} J(k+1,x_{k+1},\rho_k,j,u_k)\Delta N(\rho_k,j,k)+Q[x_N,\rho_N]$$

over control sequences $\{u_0,u_1,\ldots,u_{N-1}\}$ where the

- $L(K,x_k,\rho_k,u_k)$ are <u>operating costs</u> that penalize control energy expenditure and system performance differently in each form.
- $J(K+1,x_{k+1},\rho_k,\rho_{k+1},u_k)$ are <u>jump costs</u> that are charged if a . when the form changes. These might represent start-up or shut-down costs of equipment, or undesirable transient phenomena; load shedding costs in electric power systems are examples.
- $Q[x_N,\rho_N]$ are <u>terminal costs</u> dependent upon the final state (including form) of the system.

In this research we propose extensions of the well-known Linear Quadratic (LQ) problem to systems having randomly jumping parameters, as appropriate formulations for fault-tolerant control problems. Our approach differs from the usual methods of reliability engineering, in that we are seeking a <u>feedback</u>, <u>on-line</u> method to obtain system reliability. These <u>jump linear quadratic</u> (JLQ) control problems involve stochastic hybrid systems modelled by linear difference equations (in u and x) and the costs are all (at most) quadratic in (x,u).

These *JLQ* control problems will be solved via dynamic programming. There are two principle factors that determine the difficulty of these problems:

- Form Dependence on the Phase

  Case A: The form is independent of the phase
  Case B: The form depends upon current (or past) values of the phase (ie: with feedback).

- Availability of Form Observations

  Case I: The form is observed perfectly (without noise). The phase is either perfectly observed or observed in the presence of noise.
  Case II: The form is not observed perfectly; thus it must be estimated.

Most of the results in the literature have focused on Case IA. In this paper, we examine a class of problems corresponding to case IB above, illustrating the differences and additional complexity of control problems of this kind. The problems corresponding to Case II are correspondingly more complex, and will be the subject of future research.

4. The Scalar, Noiseless, Piecewise-Constant Problem

Consider the following scalar, noiseless, two-form, one-transition discrete-time JLQ phase control problem with perfect (x_k,ρ_k) observations:

$$x_{k+1} = a_k(\rho_k)x_k + b_k(\rho_k)u_k \qquad \rho\in\{1,2\} \tag{1}$$

$$\min_{u_0,\dots,u_{N-1}} E \sum_{k=0}^{N-1} \{x_k^2 Q_k(\rho_k)+u_k^2 R_k(\rho_k)\}+x_N^2 Q_N(\rho_N) \tag{2}$$

where the transition probabilities are piecewise-constant in x (with finitely many pieces):

$$\begin{pmatrix} p_k(1,1) & p_k(1,2) \\ p_k(2,1) & p_k(2,2)\end{pmatrix} = \begin{pmatrix} 1-\lambda_k(i) & \lambda_k(i) \\ 0 & 1\end{pmatrix} \tag{3}$$

for

$$\nu_k(i-1) < x_k \le \nu_k(i)$$

where at time k, $i=1,2,\dots,\bar{\nu}_k$ and

$$-\infty \triangleq \nu_k(0) < \nu_k(1) < \dots < \nu_k(\bar{\nu}_k) \triangleq \infty. \tag{4}$$

That is, at time k the transition probabilities in (3) have $\bar{\nu}_k$ constant (in x) pieces. A wide variety of phase dependencies can be modelled by piecewise-constant approximations like (3)-(4).

Once the system enters form 2, it stays there. Thus the usual LQ solution yields:

$$V_k[x_k,\rho_u=2] = x_k^2 K_k(2) \qquad k=0,1,\dots,N \tag{5}$$

$$u_k^*(x_k,\rho_k=2) = -\left[\frac{b_k(2)K_{k+1}(2)a_k(2)}{R_k(2)+b_k^2(2)K_{k+1}(2)}\right]x_k \qquad k=0,1,\dots,N-1 \tag{6}$$

where

$$K_k(2)=Q_k(2)+a_k^2(2)\ K_{k+1}(2)-\frac{b_k^2(2)K_{k+1}^2(2)}{R_k(2)+b_k^2(2)K_{k+1}(2)},$$

$$K_N(2)=Q_N(2) \tag{7}$$

It is clear that

$$V_N[x_N,\rho_N=1] = x_N^2 Q_N(1) \tag{8}$$

The following recursive algorithm must be solved to obtain $V_k(x_k,\rho_k=1)$ and $u_k^*(x_k,\rho_k=1)$ for $k=N-1,\dots,0$.

Proposition:

At time k, suppose that $V_k(x_k,\rho_k)$ is piecewise-quadratic and $u_k^*(x_k,\rho_k)$ is piecewise-linear (in x), each with m_k pieces (thus $m_N=1$):

$$V_k[x_k,\rho_k=1]=x_k^2 K_k(1,i)+x_k\bar{K}_k(1,i)+\bar{\bar{K}}_k(1,i) \tag{9}$$

$$u_k^*[x_k,\rho_k=1] = L_k(1,i)x_k + \bar{L}_k(1,i) \tag{10}$$

for

$$\mu_k(i-1) < x_k a_k(1) \le \mu_k(i) \tag{11}$$

where at time k, $i=1,2,\dots,m_k$ and

$$-\infty \triangleq \mu_k(0) < \mu_k(1) < \dots < \mu_k(m_k) \triangleq \infty.$$

Then $V_{k-1}(x_{k-1},\rho_{k-1}=1)$ and $u_{k-1}^*(x_{k-1},\rho_{k-1})$ are also piecewise-quadratic and piecewise-linear in x, respectively:

$$V_{k-1}[x_{k-1},\rho_{k-1}=1]=x_{k-1}^2 K_{k-1}(1,i)+x_{k-1}\bar{K}_{k-1}(1,i)+\bar{\bar{K}}_{k-1}(1,i) \tag{12}$$

$$u_{k-1}^*[x_{k-1},\rho_{k-1}=1]=L_{k-1}(1,i)x_{k-1}+\bar{L}_{k-1}(1,i) \tag{13}$$

for

$$\mu_{k-1}(i-1) < x_{k-1}a_{k-1}(1) \le \mu_{k-1}(i) \tag{14}$$

$(i=1,2,\dots,m_{k-1})$.

The number of pieces, m_{k-1} obeys

$$m_{k-1} \le 3\psi_k \le 3[m_k+\bar{\nu}_k-1] \tag{15}$$

Proof: The number ψ_k is obtain as follows: Arrange the $(1+m_k)$ numbers $\{\mu_k(0),\dots,\mu_k(m_k)\}$ in (11) and the $(\bar{\nu}_k+1)$ numbers $\{a_k(1)\nu_k(0),\dots,a_k(1)\nu_k(\bar{\nu}_k)\}$ on the real line (for $\nu_k(i)$ as in (3)). We then have an ordered set

$$-\infty \triangleq \gamma_k(0) < \gamma_k(1) < \dots < \gamma_k(\psi_k) \triangleq +\infty \tag{16}$$

of $(1+\psi_k)$ distinct values. Thus

$$\psi_k \le m_k + \bar{\nu}_k - 1 \;, \tag{17}$$

since $-\infty$ and $+\infty$ are each present twice.

Now define the ψ_k different x_k regions $\{\Delta_k(i):\ i=1,\dots,\psi_k\}$ by

$$\Delta_k(i) \triangleq \{x_k : \gamma(i-1) < a_k(1)x_k \le \gamma_k(i)\} \quad i=1,\dots,\psi_k \tag{18}$$

for $\gamma(i)$ as in (16). Note: the equalities in (18) are arbitrarily chosen to be on the right-they could be on the left instead; the same is true in (3). In each $\Delta_k(i)$ region, only one of the $\bar{\nu}_k$ different probability values $\lambda_k(j)$ is valid. Let $\ell_k(i)$ denote this valid index. That is,

for $x_k \in \Delta_k(i)$, if $p_k(1,2) = \lambda_k(j)$ then $\ell_k(i)=j$.

In each $\Delta_k(i)$ region, only one of the m_k different cost-to-go $V_k(x_k,1)$ pieces is valid. Let $\xi_k(i)$ denote the index of the valid piece. That is,

for $x_k \in \Delta_k(i)$, if $\mu_k(j-1) < a_k(1)x_k \le \mu_k(j)$ then $\xi_k(i)=j$.

Then the pieces of $V_{k-1}(x_{k-1},\rho_{k-1}=1)$ and $u^*_{k-1}(x_{k-1},\rho_{k-1}=1)$ in (12)-(13) are obtained by solving following minimization problem for each x_{k-1} value:

$$V_{k-1}[x_{k-1},\rho_{k-1}=1] = \min_{i=1,2,\dots,\psi_k} \{V_{k-1}[x_{k-1},1|i]\}$$

where i denotes $x_k \in \Delta_k(i)$. (19)

This requires the solving of the ψ_k separate problems, and comparing their costs for each x_{k-1}, to determine the optimal at that value. Fortunately the ψ_k problems each have optimal costs-to-go that are piecewise-quadratic in x with, at most, <u>3 pieces</u> (each valid over different ranges of x_{k-1} values). Thus in the very worst case, when each of these three pieces is valid for some x_{k-1}, there are <u>at most $3\psi_k$ pieces</u> to $V_{k-1}(x_{k-1},\rho_{k-1}=1)$. Solving (19) we get

$$V_{k-1}[x_{k-1},1|i] = \begin{cases} x_{k-1}^2\,{}^{L}K_{k-1}(1,\ell_i,\xi_i)+x_{k-1}\,{}^{L}\bar{K}_{k-1}(1,\ell_i,\xi_i)+{}^{L}\bar{\bar{K}}_{k-1}(1,\ell_i,\xi_i) & \text{if } a_{k-1}(1)x_{k-1} \le \theta_{k-1}(i) \\ x_{k-1}^2 K_{k-1}(1,\ell_i,\xi_i)+x_{k-1}\bar{K}_{k-1}(1,\ell_i,\xi_i)+\bar{\bar{K}}_{k-1}(1,\ell_i,\xi_i) & \text{if } \theta_{k-1}(i) < a_{k-1}(1)x_{k-1} \le \Theta_{k-1}(i) \\ x_{k-1}^2\,{}^{R}K_{k-1}(1,\ell_i,\xi_i)+x_{k-1}\,{}^{R}\bar{K}_{k-1}(1,\ell_i,\xi_i)+{}^{R}\bar{\bar{K}}_{k-1}(1,\ell_i,\xi_i) & \text{if } \Theta_{k-1}(i) < a_{k-1}(1)x_{k-1} \end{cases} \tag{20}$$

with corresponding control laws

$$u^*_{k-1}[x_{k-1},1|i] = \begin{cases} {}^{L}L_{k-1}(1,\ell_i,\xi_i)x_{k-1}+{}^{L}\bar{L}_{k-1}(1,\ell_i,\xi_i) & \text{if } a_{k-1}(1)x_{k-1} \le \theta_{k-1}(i) \\ L_{k-1}(1,\ell_i,\xi_i)x_{k-1}+\bar{L}_{k-1}(1,\ell_i,\xi_i) & \text{if } \theta_{k-1}(i) < a_{k-1}(1)x_{k-1} \le \Theta_{k-1}(i) \\ {}^{R}L_{k-1}(1,\ell_i,\xi_i)x_{k-1}+{}^{R}\bar{L}_{k-1}(1,\ell_i,\xi_i) & \text{if } \Theta_{k-1}(i) < a_{k-1}(1)x_{k-1} \end{cases} \tag{21}$$

(the superscripts L and R are for left and right), where

$$\hat{K}_k(1,\ell,\xi) = [1-\lambda_k(\ell)]K_k(1,\xi)+\lambda_k(\ell)K_k(2) \tag{22}$$

$$\hat{\bar{K}}_k(1,\ell,\xi) = [1-\lambda_k(\ell)]\bar{K}_k(1,\xi)+\lambda_k(\ell)\bar{K}_k(2) \tag{23}$$

$$\hat{\bar{\bar{K}}}_k(1,\ell,\xi) \triangleq [1-\lambda_k(\ell)]\bar{\bar{K}}_k(1,\xi)+\lambda_k(\ell)\bar{\bar{K}}_k(2) \tag{24}$$

and for $i=1,\dots,\psi_k$:

$$\theta_{k-1}(i) \triangleq \frac{\gamma_k(i-1)}{a_k(1)R_{k-1}(1)}[R_{k-1}(1)+b^2_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)] + \frac{b^2_{k-1}(1)\hat{\bar{K}}_k(1,\ell_i,\xi_i)}{2R_{k-1}(1)} \tag{25}$$

$$\Theta_{k-1}(i) \triangleq \frac{\gamma_k(i)}{a_k(1)R_{k-1}(1)}[R_{k-1}(1)+b^2_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)] + \frac{b^2_{k-1}(1)\hat{\bar{K}}_k(1,\ell_i,\xi_i)}{2R_{k-1}(1)} \tag{26}$$

$${}^{R}L_{k-1}(1,\ell_i,\xi_i) \equiv {}^{L}L_{k-1}(1,\ell_i,\xi_i) \triangleq -a_{k-1}(1)/b_{k-1}(1) \tag{27}$$

$${}^{R}\bar{L}_{k-1}(1,\ell_i,\xi_i) \triangleq \gamma_k(i)/a_k(1)b_{k-1}(1) \tag{28}$$

$${}^{L}\bar{L}_{k-1}(1,\ell_i,\xi_i) \triangleq \gamma_k(i-1)/a_k(1)b_{k-1}(1) \tag{29}$$

$${}^{R}K_{k-1}(1,\ell_i,\xi_i) \equiv {}^{L}K_{k-1}(1,\ell_i,\xi_i) \triangleq Q_{k-1}(1) + \frac{a^2_{k-1}(1)R_{k-1}(1)}{b^2_{k-1}(1)} \tag{30}$$

$${}^{R}\bar{K}_{k-1}(1,\ell_i,\xi_i) \triangleq -2\gamma_k(i)R_{k-1}(1)a_{k-1}(1)/b^2_{k-1}(1)a_k(1) \tag{31}$$

$${}^{L}\bar{K}_{k-1}(1,\ell_i,\xi_i) \triangleq -2\gamma_k(i-1)R_{k-1}(1)a_{k-1}(1)/b^2_{k-1}(1)a_k(1) \tag{32}$$

$${}^{R}\bar{\bar{K}}_{k-1}(1,\ell_i,\xi_i) \triangleq \hat{\bar{\bar{K}}}_k(1,\ell_i,\xi_i) + \frac{\gamma_k(i)\hat{\bar{K}}_k(1,\ell_i,\xi_i)}{a_k(1)} + \frac{\gamma_k^2(i)}{a_k^2(1)}\left[\hat{K}_k(1,\ell_i,\xi_i) + \frac{R_{k-1}(1)}{b^2_{k-1}(1)}\right] \tag{33}$$

$${}^{L}\bar{\bar{K}}_{k-1}(1,\ell_i,\xi_i) \triangleq \hat{\bar{\bar{K}}}_k(1,\ell_i,\xi_i) + \frac{\gamma_k(i-1)\hat{\bar{K}}_k(1,\ell_i,\xi_i)}{a_k(1)} + \frac{\gamma_k^2(i-1)}{a_k^2(1)}\left[\hat{K}_k(1,\ell_i,\xi_i) + \frac{R_{k-1}(1)}{b^2_{k-1}(1)}\right] \tag{34}$$

$$L_{k-1}(1,\ell_i,\xi_i) = \frac{-b_{k-1}(1)a_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)}{R_{k-1}(1)+b^2_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)} \tag{35}$$

$$\bar{L}_{k-1}(1,\ell_i,\xi_i) \triangleq \frac{-b_{k-1}(1)\hat{\bar{K}}_k(1,\ell_i,\xi_i)}{2[R_{k-1}(1)+b^2_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)]} \tag{36}$$

$$K_{k-1}(1,\ell_i,\xi_i) = Q_{k-1}(1) + a^2_{k-1}(1)\cdot\left[\hat{K}_k(1,\ell_i,\xi_i) - \frac{b^2_{k-1}(1)\hat{K}^2_k(1,\ell_i,\xi_i)}{R_{k-1}(1)+b^2_{k-1}(1)\hat{K}_k(1,\ell_i,\xi_i)}\right] \tag{37}$$

$$\bar{K}_{k-1}(1,\ell_i,\xi_i)=a_{k-1}(1)\hat{\bar{K}}_k(1,\ell_i,\xi_i)\cdot$$
$$\cdot\left[1-\frac{b_{k-1}^2(1)\hat{K}_k(1,\ell_i,\xi_i)}{R_{k-1}(1)+b_{k-1}^2(1)\hat{K}_k(1,\ell_i,\xi_i)}\right] \tag{38}$$

$$K_{k-1}(1,\ell_i,\xi_i)=\hat{\bar{\bar{K}}}_k(1,\ell_i,\xi_i) - \frac{\hat{\bar{K}}_k^2(1,\ell_i,\xi_i)b_{k-1}^2(1)}{4[R_{k-1}(1)+b_{k-1}^2(1)\hat{K}_k(1,\ell_i,\xi_i)]} \tag{39}$$

$$\bar{K}_N(1,\ell_i,\xi_i)=0,\ K_N(1,\ell_i,\xi_i)=Q_N(1),\ \bar{\bar{K}}_N(1,\ell_i,\xi_i)=0$$

The minimization in (19) can be computed by first determining $\theta_{k-1}(i)$ and $\Theta_{k-1}(i)$, for each $i=1,..,\psi_k$ using (25)-(26). Then the three costs for each $V_{k-1}(x_{k-1},\rho_{k-1}=1|x_k\in\Delta_k(i))$ can be computed via (20) for each i. All these $3\psi_k$ quadratic pieces are then compared over the x_{k-1} values where they hold, so as to determine the optimal cost.

5. Discussion and Research Plans

In general, the algorithm will involve a large number of calculations even for relatively small N. The comparisons in (19) become quite tedious to do by hand. Fortunately there exist symbolic mathematical problem-solving computer programming languages which can be used to implement the algorithm.

At each time k, the phase subsystem state space is broken up into m_k regions. In some of these regions, the optimal control law does not influence the transition probabilities of the next form through control of x; these are the non-superscripted control laws in (21) (ie; where $\theta_{k-1}(i)<a_{k-1}(i)x_{k-1}\leq\Theta_{k-1}(i)$). For other regions, the phase control is used to alter transition probabilities; that is, to actively hedge. These are the leftwards (superscript L; $a_{k-1}(1)x_{k-1}\leq\theta_{k-1}(i)$) or rightwards (superscript R; $a_{k-1}(1)x_{k-1}>\Theta_{k-1}(i)$) control laws in (21).

The number of comparisons required in (19) depends upon how the intervals $[\theta_{k-1}(i),\Theta_{k-1}(i)]$ overlap and how they fill the real line. At most, if every possible cost in (20) is optimal for some x_{k-1} value, there will be $3\psi_k$ pieces in the optimal expected costs-to-go and control laws.

For some problems it may be possible to bound the regions in which the controller actively hedges. That is, there may exist some b such that, for $|x|>b$, the controller does not actively hedge. For such problems we might solve the non-hedging control problems that are valid outside of this region and then interpolate these costs-to-go over the (bounded) hedging region. In this way a computationally efficient approximation to the optimal hedging controller might be obtained.

The extension of these results to problems with driving noise is currently being examined. When the phase subsystem is subject to input noise, the piecewise-quadratic nature of the optimal cost-to-go is lost due to the smoothing effects of the noise. Process noise alters form transition probabilities because of their phase dependence. Driving noise also makes it impossible to drive the phase with certainty to the region boundaries, as in the noiseless case.

For finite time-horizon problems, if we assume that the driving noise has bounded magnitude then there are large portions of the phase space (R) in which the quadratic nature of the cost is maintained. These are regions where no possible noise is big enough to force the phase into a different x-region.

If the phase process is not perfectly observed because of additive observation noise, the situation is more complicated than in the phase-independent case. Even if the form is perfectly observed, uncertainty about x is a source of uncertainty about form transitions that must be reflected in the optimal control law. It is not clear whether separation or certainty-equivalence results will hold for such problems.

The scalar case addressed above is, of course, only of academic interest. It is the vector cases (where $x\in R^n$) that are the important ones. In the scalar case, the boundaries between x-regions are trivial-- they are points. In n-dimensional spaces, they will be (n-1)-dimensional surfaces. Active hedging appears to involve driving the phase to 'best' place on the region boundary. This is a simple fixed end-point optimization in the scalar case, in higher dimensions we must optimize over a surface. A further complication arise in problems that include phase discontinuities. These jumps in x will complicate the problem because, as with driving noise, it will not be possible to drive the phase into x-regions with certainty.

In conclusion, we have presented a design methodology for feedback control of a class of simple fault-prone systems based on optimization of suitable cost functionals. The solutions exhibit hedging properties desirable in these feedback controllers, and provide insight into the structure of solutions to more complex problems. However, much work remains to be done in extending the concepts used in this work towards the understanding of the feedback control of general hybrid systems.

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