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FIXED STATE UTILITY SCALING: COMPARISON OF DIFFERENT CRITERIA.(U)

MAR 81 S MAYEKAWA, M R NOVICK

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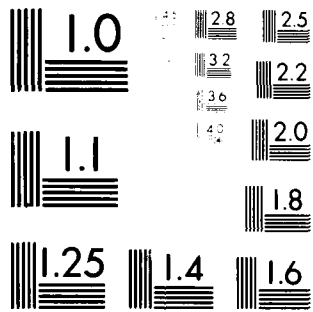
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MARCH, 1981

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FIXED STATE UTILITY SCALING:
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SHIN-ICHI MAYEKAWA
THE UNIVERSITY OF IOWA

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REPORT PREPARED UNDER OFFICE OF NAVAL RESEARCH

CONTRACT N00014-77-C-0428 /

MELVIN R. NOVICK, PRINCIPAL INVESTIGATOR

THE UNIVERSITY OF IOWA

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report 81-3	2. GOVT ACCESSION NO. AD-A098	3. RECIPIENT'S CATALOG NUMBER 227
4. TITLE (and Subtitle) "Fixed State Utility Scaling: Comparison of Different Criteria"		5. TYPE OF REPORT & PERIOD COVERED Technical Report Jan 1, 1980 - Dec. 31, 1980
7. AUTHOR(s) Shin-ichi Mayekawa		6. PERFORMING ORG. REPORT NUMBER Technical Report 81-3 8. CONTRACT OR GRANT NUMBER(s) N00014-77-C-0428
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Iowa Division of Educational Psychology Iowa City, Iowa 52242		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research (Code 458) Arlington, VA 22217		12. REPORT DATE March 20, 1981
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES
		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release, distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) fitting utilities, fixed-state gambles, log-odds transformation, root arc-sine transformation, least squares criteria		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Several methods of fitting utilities using fixed state gambles and least- squares criteria are compared. It was found that similar results are found when the maximization is done in the probability, log-odds, or root arc-sine matrix. cf		

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March 16, 1981

Fixed State Utility Scaling
Comparison of Different Criteria

Shin-ichi Mayekawa

The University of Iowa

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UTILITY SCALING

I. Introduction

In the Bayesian view of decision making, the posterior probability distribution of the variable of interest and its utility function are the essential ingredients of a decision process. The method used to assess the utility function installed in the CADA monitor (Novick, Hamer, Libby, Chen, and Woodworth, 1980) is called the fixed-state utility assessment procedure (Novick and Lindley, 1979) in which the investigator is asked to specify subjective indifference probabilities for gamble pairs. The assessment of the utility function or the scaling of the subjective utility is then done through a least squares procedure based on the log-odds transformed observations (subjective indifference probabilities).

In Novick and Lindley (1979), the following notation is used:

Observations

$$\begin{aligned}P_{ijk}, \quad & i = 0, 1, \dots, N-1 \\& j = 1, 2, \dots, N \\& k = 2, 3, \dots, N+1 \\& i < j < k\end{aligned}$$

where $N+2$ is the number of ordered outcomes. p_{ijk} denotes the numerical values of the subjective probability with which the subject receives the outcome k in a gamble between outcome i and outcome k , which makes the gamble indifferent to receiving the outcome j for sure.

Parameters

If u_i , $i=0, 1, 2, \dots, N+1$, $u_0 = 0$, $u_{N+1} = 1$, is the utility of the

i-th outcome, then the true indifference probabilities are

$$(1) \hat{p}_{ijk} = (u_j - u_i) / (u_k - u_i).$$

Given N observed indifference probabilities the N unspecified utilities can be calculated under certain conditions. However, in most situations the observations contain errors and it is therefore useful to obtain additional observations and to attempt to fit a utility function.

In this paper several different methods of fitting (scaling) will be discussed. Throughout this paper, we assume that the utilities are monotonic to the outcomes: that is, under the appropriate ordering of the outcomes, we assume that if $i < j$ then $u_i < u_j$. We further assume that p_{ijk} 's are measured in the absolute scale, that is, p_{ijk} 's are assumed to have a unique origin and a unique unit.

II. Least Squares Method

The simplest way to estimate the utilities is to solve the least squares problem:

(2) $Q_2 = \sum_{ijk} [(p_{ijk} - \hat{p}_{ijk})^2] \rightarrow \min \text{ w.r.t. } u\text{'s, where } \sum_{ijk} \text{ denotes summation over all the observed } p_{ijk}\text{'s, and } \hat{p}_{ijk} \text{ is the function of the utilities as defined by (1). This solution, although straight forward, lacks the consideration of the comparative magnitude of the deviations or the relative weight of each observation. As indicated by Novick and Lindley (1979), the value of } p_{ijk} \text{ close to } 1/2 \text{ seems to have a relatively large magnitude of deviations, that is, the closer } p_{ijk}\text{'s are to } 1/2, \text{ the less reliable they are. Thus, observed values } p_{ijk} \text{ distant from } 1/2 \text{ would have little influence on the solution.}$

One way to handle this problem is to define the least squares criterion using transformed observations. The transformation which removes the heterogeneity of the deviations of the observations must be chosen. Among the transformations which serve this purpose, we only consider the following two transformations:

$$(3) f_L(x) = \ln(x/(1-x)),$$

and

$$(4) F_A(x) = \sin^{-1}(x^{1/2}).$$

f_L and f_A are called log-odds transformation and arcsine transformation, respectively.

The least squares criterion defined on these transformed values are:

$$(5) Q_3 = \sum_{ijk} [(f_L(p_{ijk}) - f_L(\hat{p}_{ijk}))^2] \rightarrow \min,$$

and

$$(6) Q_4 = \sum_{ijk} [(f_A(p_{ijk}) - f_A(\hat{p}_{ijk}))^2] \rightarrow \min.$$

Another way to handle this problem is to define the weighted least squares criterion such that the heterogeneity of the deviations may be absorbed in the weights. Because the observations are assumed to be less reliable around 1/2, we may define the weighted least squares criterion as follows:

$$(7) Q_1 = \sum_{ijk} [w_{ijk}(p_{ijk} - \hat{p}_{ijk})^2] \rightarrow \min,$$

where

$$(8) w_{ijk} = 1/[\hat{p}_{ijk}(1-\hat{p}_{ijk})].$$

In general the least squares criteria can be written as

$$(9) Q_a = \sum_{ijk} [w_{ijk}(f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))^2] \rightarrow \min,$$

where

$$\begin{aligned} w_{aijk} &= 1 && \text{for } a = 2, 3, \text{ and } 4 \\ &= 1/\hat{p}_{ijk} (1 - \hat{p}_{ijk}) && \text{for } a = 1, \end{aligned}$$

and

$$\begin{aligned} f_a(x) &= x && \text{for } a = 1 \text{ and } 2, \\ &= f_L(x) && \text{for } a = 3, \\ &= f_A(x) && \text{for } a = 4. \end{aligned}$$

III. Solution

Rewriting (9) in a matrix form, we have

$$(10) Q_a := [\underline{f}_a(\underline{p}) - \underline{f}_a(\hat{\underline{p}})]' W_a [\underline{f}_a(\underline{p}) - \underline{f}_a(\hat{\underline{p}})] \rightarrow \min,$$

where denoting the number of the observations by K , $\underline{f}_a(\underline{x})$ is the $K \times 1$ vector whose e -th element is $f_a(x_e)$, W_a is the $K \times K$ diagonal matrix whose e -th diagonal element is w_{ae} , and $\underline{p}$ and $\hat{\underline{p}}$ are the $K \times 1$ vectors consisting of p_{ijk} 's and $\hat{p}_{ijk}$'s, respectively. The subscript "e" stands for any ijk triad.

Because u_0 and u_{N+1} are fixed, there are only N free values of u which must be estimated. The solution of the problem is given by solving

$$(11) \frac{\partial Q_a}{\partial \underline{u}} = \left[\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}} \right]' W_a [\underline{f}_a(\hat{\underline{p}}) - \underline{f}_a(\underline{p})]$$

$$= 0$$

where $\underline{u}$ is the $N \times 1$ vector of the utility scale, i.e.

$$\underline{u} = [u_1, u_2, \dots, u_N]',$$

and

$$\left[\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}} \right]$$

is the $K \times n$ matrix whose (e,c) element is

$$\frac{\partial f_a(\hat{p}_e)}{\partial u_c} = \frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c}.$$

[The weight matrix is treated as a constant even when $a = 1$.]

Specifically, in a scalar form $\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c}$ can be written as follows:

(12) For $a = 1$ and 2,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \frac{\partial \hat{p}_{ijk}}{\partial u_c} = \begin{cases} -(u_k - u_j)/(u_k - u_i)^2 & \text{if } c = i \\ 1/(u_k - u_i) & \text{if } c = j \\ -(j_j - u_i)/(u_k - u_i)^2 & \text{if } c = k, \end{cases}$$

for $a = 3$,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \left(\frac{1}{\hat{p}_{ijk}} + \frac{1}{1 - \hat{p}_{ijk}} \right) \frac{\partial \hat{p}_{ijk}}{\partial u_c}$$

and for $a = 4$,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \frac{1}{2\sqrt{\hat{p}_{ijk}(1 - \hat{p}_{ijk})}} \frac{\partial \hat{p}_{ijk}}{\partial u_c}.$$

Among the various numerical methods for solving (11), the Gauss-Newton method seems to be appropriate here because it does not require the evaluation of the second derivatives and yet is as efficient as the Newton-Raphson method. The direction for search in the Gauss-Newton method is defined by

$$(13) \quad \underline{S}(\underline{u}) = - \left[\left(\frac{\partial \underline{f}_a(\hat{p})}{\partial \underline{u}} \right)' W_a \left(\frac{\partial \underline{f}_a(\hat{p})}{\partial \underline{u}} \right) \right]^{-1} \frac{\partial Q_a}{\partial \underline{u}}$$

The Gauss-Newton method with the step-size halving is always convergent. However, we must consider the monotonicity restrictions.

Usually, the restrictions can be enforced by setting the parameters which fall out of the proper region equal to the boundary surface. However, we cannot apply this simple method because our restrictions do not include the equalities: that is, u_i must be strictly less than u_{i+1} . To enforce the monotonicity restrictions, therefore, we modified the step-size halving procedure of the Gauss-Newton method so that all the updated parameters may satisfy the restrictions in each iteration. As long as the initial estimates satisfy the restrictions, this method is guaranteed to converge to the restricted minimum.

For the weighted least squares case ($a = 1$), the W matrix must be successively updated in each iteration by

$$(14) \quad w_{ijk} = 1/\hat{p}_{ijk}(1 - \hat{p}_{ijk})$$

using the latest value of $\hat{p}_{ijk}$.

IV. Maximum Likelihood Method Based on the Normal Distribution

In the previous sections, we used the word "relative magnitudes of the deviations" without introducing the statistical concept. In this section, however, we assume that the observations are the random variables. Under the assumption that $f_a(p_{ijk})$ has the normal distribution with the mean $f_a(\hat{p}_{ijk})$ and the constant variance σ^2 ,

$$(15) \quad f_a(p_{ijk}) \sim N(f_a(\hat{p}_{ijk}), \sigma^2),$$

for $a=2,3$, and 4 ,

or under the assumption that p_{ijk} has the normal distribution with the mean $\hat{p}_{ijk}$ and the variance $c\hat{p}_{ijk}(1-\hat{p}_{ijk})$, where c is the unknown parameter to be estimated,

$$(16) \quad p_{ijk} \sim N(\hat{p}_{ijk}, c\hat{p}_{ijk}(1-\hat{p}_{ijk})),$$

it can be shown that the least squares solutions described in the previous sections coincide with the maximum likelihood solutions. That is, the derivatives of the log likelihood function for the assumption (15),

(17)

$$\ln L = -\frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{ijk} (f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))^2$$

for $a = 2, 3$, and 4 ,

with respect to σ^2 and u_c 's result in the equations

$$(18) \quad \sigma^2 = \sum_{ijk} [(f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))^2] / N$$

and

$$(19) \quad \frac{\partial \ln L}{\partial u_c} = \frac{1}{\sigma^2} \sum_{ijk} (f_a(p_{ijk}) - f_a(\hat{p}_{ijk})) \frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c}$$

and for the assumption (16), the derivatives of the log likelihood function

$$(20) \quad \ln L = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \sum_{ijk} \ln c^2 d_{ijk}^2 - \frac{1}{2} \sum_{ijk} \frac{(p_{ijk} - \hat{p}_{ijk})^2}{c^2 d_{ijk}^2}$$

where

$$d_{ijk}^2 = \hat{p}_{ijk}(1-\hat{p}_{ijk}),$$

with respect to c^2 and u_c 's result in the equations

$$(21) \quad c^2 = \frac{1}{N} \sum_{ijk} \frac{(p_{ijk} - \hat{p}_{ijk})^2}{\hat{p}_{ijk}(1-\hat{p}_{ijk})}$$

and

$$(22) \quad \frac{\partial \ln L}{\partial u_c} = \frac{1}{c^2} \sum_{ijk} \frac{(p_{ijk} - \hat{p}_{ijk})}{d_{ijk}^2} \frac{\hat{p}_{ijk}}{\partial u_c}$$

These equations can be written as (11) using the same notation, and the Gauss-Newton method is also appropriate as the numerical method.

One of the merits of the maximum likelihood solution is the inverse matrix of

$$(23) \quad I_a(\underline{u}) = \left(\frac{\partial \underline{f}_a(\underline{p})}{\partial \underline{u}} \right)' W_a \left(\frac{\partial \underline{f}_a(\underline{p})}{\partial \underline{u}} \right),$$

which appeared in (12), can provide the asymptotic variance/covariance matrix of the estimated parameters (see Jennrich and Moor, 1975).

From the Bayesian point of view this is equivalent to saying that with the uniform prior distribution, the posterior distribution of the parameter $\underline{u}$ is approximately multivariate normal with the mean vector $\hat{\underline{u}}$, the maximum likelihood estimates, and the variance/covariance matrix $I_a(\hat{\underline{u}})$ (See Lindley 1965). That is, using the approximate posterior distributions, we can draw rough inferences concerning the imprecision of the estimated utility scale. For example, we can draw the approximate 95 percent HDR of the utility function using the posterior marginal distributions.

V. Maximum Likelihood Method Based on the Beta Distribution

Under the assumption that each observation has the beta distribution with parameters $a\hat{p}_{ijk}+b$ and $a(1-\hat{p}_{ijk})+b$,

$$(24) \quad \hat{p}_{ijk} \sim \beta(a\hat{p}_{ijk}+b, a(1-\hat{p}_{ijk})+b),$$

where a is the unknown parameter related to the variance and b is the constant related to the location, the solution which maximizes

$$(25) \quad \ln L := \sum_{ijk} \ln \left[\frac{p_{ijk}^{a\hat{p}_{ijk}+b} (1-p_{ijk})^{a(1-\hat{p}_{ijk})+b}}{B[a\hat{p}_{ijk}+b, a(1-\hat{p}_{ijk})+b]} \right],$$

where $B[\alpha, \beta]$ denotes the beta function with the arguments α and β , was proposed by Mayekawa (1980). In this formulation, the variance of p_{ijk} can be written as

$$(26) \quad \sigma_{ijk}^2 = \frac{(a\hat{p}_{ijk}+b)(a(1-\hat{p}_{ijk})+b)}{(a+2b)^2(a+2b+1)}$$

which shows that this assumption is compatible with the previous consideration of the errors.

In this section we will consider the two specific cases in which b is set equal to zero and one, respectively. When b is set equal to zero, the mean of the distribution is equal to $\hat{p}_{ijk}$ and when b is set equal to one, the mode of the distribution is equal to $\hat{p}_{ijk}$.

Fischer's information matrix under this assumption is given by

$$(27) \quad \begin{aligned} I_{\ell\ell} &= a^2 \sum_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_\ell} \right)^2 [\psi^V(a\hat{p}_{ijk}+b) + \psi^V(a(1-\hat{p}_{ijk})+b)], \\ I_{\ell m} &= a^2 \sum_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_\ell} \right) \left(\frac{\partial \hat{p}_{ijk}}{\partial u_m} \right) [\psi^V(a\hat{p}_{ijk}+b) + \psi^V(a(1-\hat{p}_{ijk})+b)], \\ I_{N+1, N+1} &= \sum_{ijk} [\hat{p}_{ijk}^2 \psi^V(a\hat{p}_{ijk}+b) + (1-\hat{p}_{ijk}) \psi^V(a(1-\hat{p}_{ijk})+b) \\ &\quad - \psi^V(a+2b)], \end{aligned}$$

and

$$I_{\ell, N+1} = I_{N+1, \ell}$$

$$= a \sum_{ijk} \left[\hat{p}_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_{\ell}} \right) \psi^{\nabla}(a \hat{p}_{ijk} + b) - (1 - \hat{p}_{ijk}) \left(\frac{\partial \hat{p}_{ijk}}{\partial u_{\ell}} \right) \right. \\ \left. \psi^{\nabla}(a(1 - \hat{p}_{ijk}) + b) \right],$$

where

$$\psi^{\nabla}(x) = \frac{d^2 \ln \Gamma(x)}{d x^2}.$$

To estimate the parameters Fisher's scoring method, with the modified step-size halving as before, is used.

VI. Results

Using the nine-point fixed-state method in Component 31 of the CADA monitor, in which $N = 7$, fourteen indifference probabilities were collected for seventeen subjects ($K = 14$). The outcome used in the assessment procedure was the GPA and the subjects were the students of The University of Iowa. For each subject, six utility scales based on the different criteria were estimated. In order to see the differences of the utility scales the interscale correlation coefficients (CC) and the sums of the interscale absolute differences (SAD) were calculated for each subject.

Generally speaking, the differences were very little for all the subjects. Among seventeen subjects, the minimum of the mean correlation coefficients (MCC) over six scales was .96732 and the maximum of the mean of the sums of the absolute differences (MSAD) was .09056, both of which were obtained for the subject #4. However, close

examination shows that while the differences between the scale based on the beta distribution with $b = 1$ and the other scales are fairly large, the differences among the other scales are very small for this subject, which may indicate that the beta scales with $b = 1$ converged at the local minimum for the subject #4.

The minimum of MCC and the maximum of MSAD among the sixteen subjects excluding the subject #4 was .99325 for the subject #9 and .04912 for the subject #3 respectively. The estimated utility scales and the 95 percent approximate HDR for the subject #3 are shown in Fig. 1 and Fig. 2. As indicated above, the differences of the six utility scales are the largest for this subject except subject #4. For all the other fifteen subjects the utility scales and their HDR's were very similar to each other. One of the typical results (subject #8) are shown in Fig. 3 and Fig. 4.

While there seems to be no consistent qualitative differences among the different scales, it is clear that the fit of the model and the interscale differences are negatively correlated. That is, if the fit is good (lower sum of the residuals for the normal models, or higher likelihood for the beta models), the differences among the scales become smaller. In Table 1 the correlation coefficients among the least squares criterions (Q_1 through Q_4) minimized, the log likelihoods (with $b = 0$ and $b = 1$), CC's, and SAD's over seventeen subjects are shown. It is also clear that the fit and the size of the HDR's for the beta scales with $b = 1$ seem to be very sensitive to the fit and always resulted in the largest intervals when the fit was poor. (The HDR for the subject #3 in Fig. 2 lies out of the range of the graph.)

VII. Discussion

As shown in the previous sections, for almost all the subjects, the differences among the utility scales based on the different criteria or distributional assumptions are very small. The fact that the least square solution under the assumption that p_{ijk} 's have the constant variances all over the range showing no qualitative differences tells us that the least squares solutions are very robust. While the other scales seem to be theoretically more sound, the least squares solution based on the probability itself might be recommended as the simplest method. However, as for the computations, there are no great differences among the least squares solutions, which indicates the choice is very arbitrary.

On the contrary, the maximum likelihood solutions based on the beta distributions require a great deal of effort to evaluate the beta function, psy function, and the derivative of the psy function, which may affect the computational time on small computers seriously.

Finally, we may mention the reliability of the utility scales. Unlike the area of the mental test theory, little attention has been paid to the reliability concept in psychological scaling.

The reasons are:

- 1) In the mental test theory, the observations and the estimated scales are essentially the same quantity, which leads to the variance-ratio definition of the reliability coefficient.

2) In the psychological scaling, usually, the observations and the scale have a functional relation. For example, we have to estimate the utility scale through the subjective probabilities. This fact makes it difficult to calculate the observed variance.

If we have the repeated observations, it is possible to calculate the interscale correlation coefficient among the two scales based on the different sets of observations like split-half methods. On the contrary, it is possible to define the reliability of the scale as

$$(28) \quad p = \frac{\text{Var}(\hat{p}_{ijk})}{\text{Var}(p_{ijk})}$$

which can be calculated without repeated observations. However, because this quantity is defined on the probability, rather than the utility, it may not be appropriate to call this the reliability of the utility scale. Further study of the reliability seems to be necessary.

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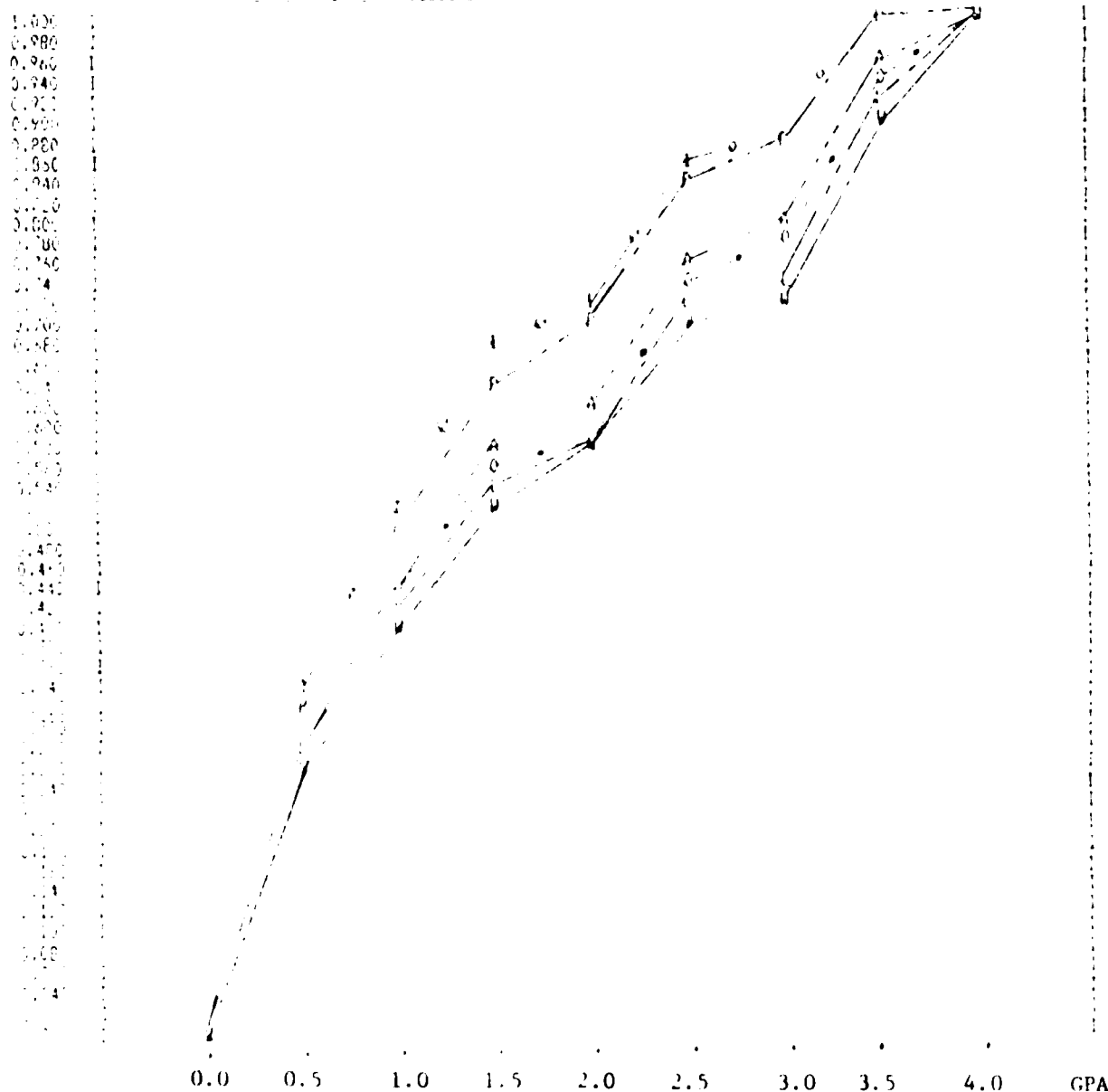


Fig. 1 Utilities (subject #3)

w	Q_1 (weighted L.S.)	K (# of gambles) = 14
P	Q_2 (probability)	
L	Q_3 (log-odds)	
A	Q_4 (arc sin)	
β	Beta ($b=0$)	
β	Beta ($b=1$)	

The goodness of fit for each model (the values of Q_i and the likelihoods) is: .6737, .2772, 1.4458, .3144, -2.6456, and 1.2851, respectively.

Utility

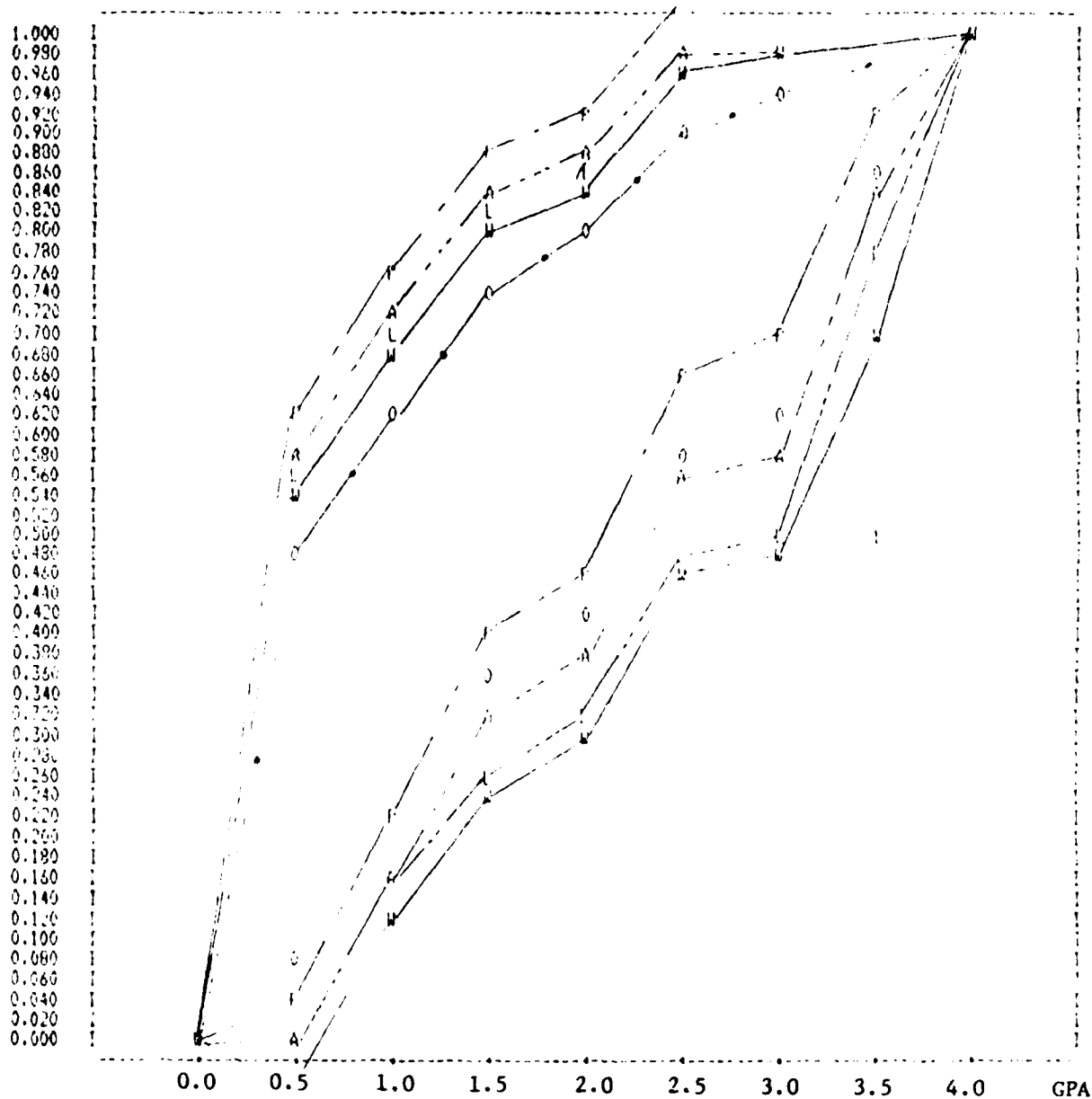


Fig. 2 Approximate 95% HDR's

HDR for Beta (b=1) model lies out of the range of this graph.

Utility

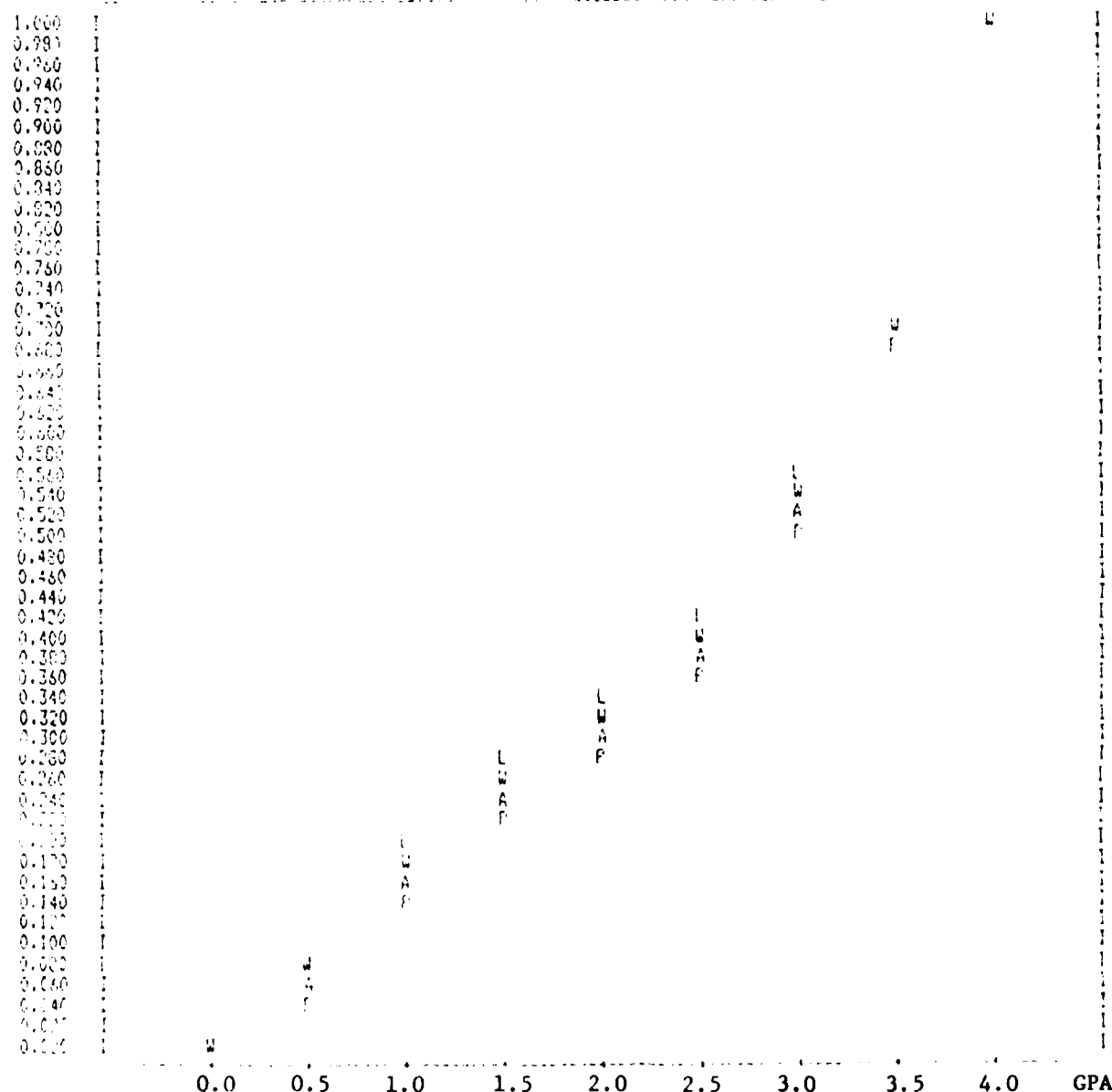


Fig. 3 Utilities (subject #8)

The goodness of fit for each model is: .2894, .0941, .5744, .1175, 14.0572, 16.8665, respectively. Beta models are close to the weighted L.S. Model.

Utility

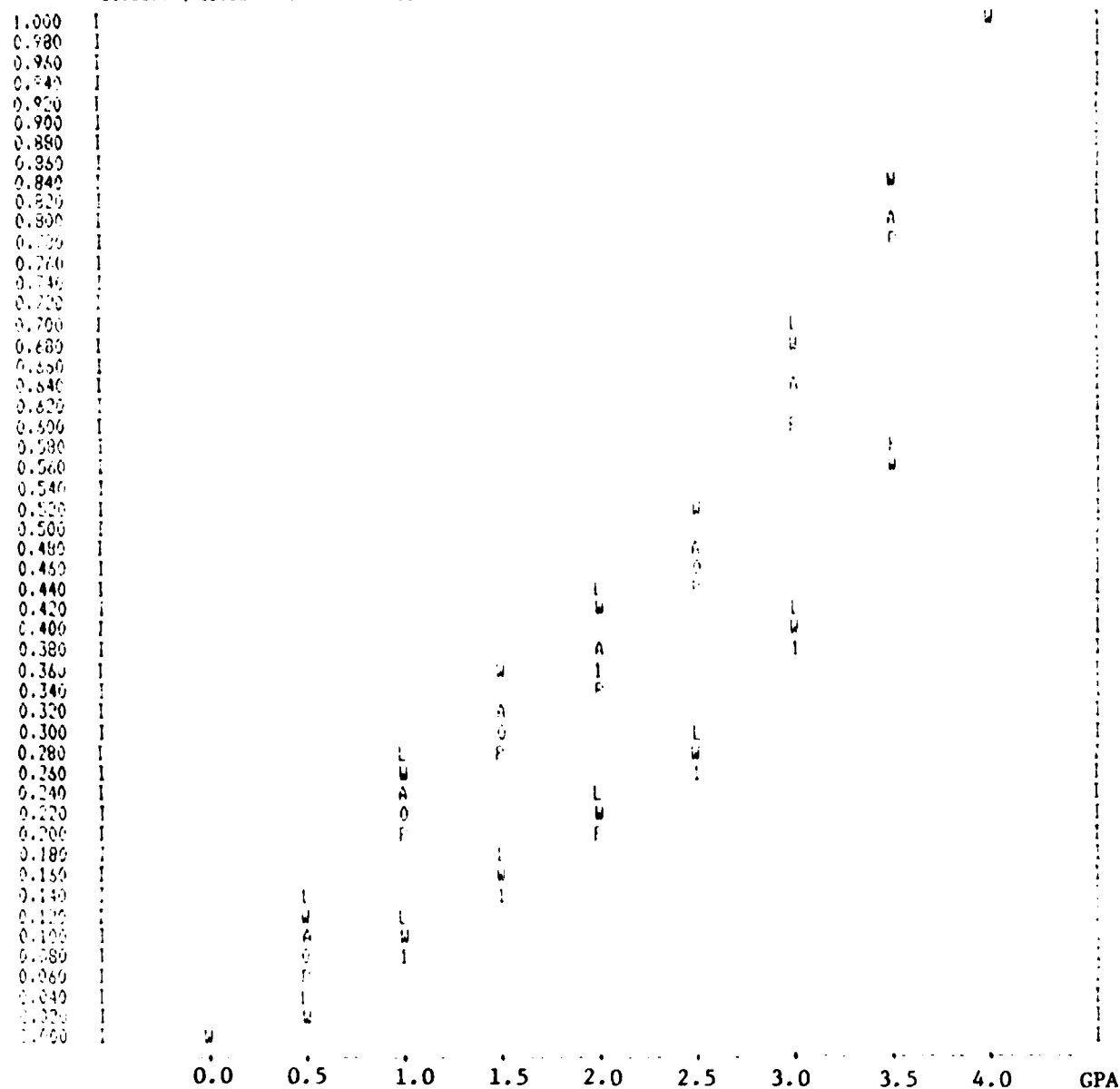


Fig. 4 Approximate 95% of HDR's

	L.S. criteria				log likelihoods			
	Q_1	Q_2	Q_3	Q_4	$b = 0$	$b = 1$	MSAD	MCC
Q_1	1.0000	0.9366	0.9547	0.9396	0.7119	0.7314	0.7017	0.7337
Q_2	0.9366	1.0000	0.9571	0.9769	0.8047	0.7771	0.7857	0.7617
Q_3	0.9547	0.9571	1.0000	0.9403	0.8137	0.7673	0.7451	0.7696
Q_4	0.9396	0.9769	0.9403	1.0000	0.7706	0.7917	0.7371	0.7576
$b=0$	0.7119	0.8047	0.8137	0.7706	1.0000	0.9777	0.9771	0.9777
$b=1$	0.7314	0.7771	0.7673	0.7917	0.9777	1.0000	0.9777	0.9777
MSAD	0.7017	0.7857	0.7451	0.7371	0.9771	0.9777	1.0000	0.9777
MCC	0.7337	0.7617	0.7696	0.7576	0.9777	0.9777	0.9777	1.0000

Table 1. Correlation coefficients among the fits
and the differences

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Navy Personnel R&D Center
San Diego, CA 92152
- 1 Dr. John Ford
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Dr. Henry M. Halff
Department of Psychology, C-009
University of California at San Diego
La Jolla, CA 92093
- 1 Dr. Patrick R. Harrison
Psychology Course Director
LEADERSHIP & LAW DEPT. (7b)
DIV. OF PROFESSIONAL DEVELOPMENT
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402

Navy

- 1 CDR Robert S. Kennedy
Head, Human Performance Sciences
Naval Aerospace Medical Research Lab
Box 29407
New Orleans, LA 70189
- 1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054
- 1 Dr. William L. Maloy
Principal Civilian Advisor for
Education and Training
Naval Training Command, Code 00A
Pensacola, FL 32508
- 1 Dr. Kneale Marshall
Scientific Advisor to DCNO(MPT)
OP01T
Washington DC 20370
- 1 CAPT Richard L. Martin, USN
Prospective Commanding Officer
USS Carl Vinson (CVN-70)
Newport News Shipbuilding and Drydock Co
Newport News, VA 23607
- 1 Dr. James McBride
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Ted M. I. Yellen
Technical Information Office, Code 201
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152
- 1 Library, Code P201L
Navy Personnel R&D Center
San Diego, CA 92152
- 6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

Navy

- 1 Psychologist
ONR Branch Office
Bldg 114, Section D
666 Summer Street
Boston, MA 02210
- 1 Psychologist
ONR Branch Office
536 S. Clark Street
Chicago, IL 60605
- 1 Office of Naval Research
Code 437
800 N. Quincy SStreet
Arlington, VA 22217
- 5 Personnel & Training Research Programs
(Code 458)
Office of Naval Research
Arlington, VA 22217
- 1 Psychologist
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101
- 1 Office of the Chief of Naval Operations
Research Development & Studies Branch
(OP-115)
Washington, DC 20350
- 1 LT Frank C. Petho, MSC, USN (Ph.D)
Code L51
Naval Aerospace Medical Research Laborat
Pensacola, FL 32508
- 1 Dr. Bernard Rimland (03B)
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Dr. Worth Scanland
Chief of Naval Education and Training
Code N-5
NAS, Pensacola, FL 32508
- 1 Dr. Robert G. Smith
Office of Chief of Naval Operations
OP-987H
Washington, DC 20350

Navy

- 1 Dr. Alfred F. Smode
Training Analysis & Evaluation Group
(TAEG)
Dept. of the Navy
Orlando, FL 32813
- 1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Dr. Ronald Weitzman
Code 54 WZ
Department of Administrative Sciences
U. S. Naval Postgraduate School
Monterey, CA 93940
- 1 Dr. Robert Wisher
Code 309
Navy Personnel R&D Center
San Diego, CA 92152
- 1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

Army

- 1 Technical Director
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Myron Fischl
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Dexter Fletcher
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Michael Kaplan
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
- 1 Dr. Milton S. Katz
Training Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Harold F. O'Neil, Jr.
Attn: PERI-OK
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Mr. Robert Ross
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Robert Sasmor
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

Army

- 1 Commandant
US Army Institute of Administration
Attn: Dr. Sherrill
FT Benjamin Harrison, IN 46256
- 1 Dr. Frederick Steinheiser
U. S. Army Reserch Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Joseph Ward
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

- 1 Air Force Human Resources Lab
AFHRL/MPD
Brooks AFB, TX 78235
- 1 Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235
- 1 Research and Measurement Division
Research Branch, AFMPC/MPCYPR
Randolph AFB, TX 78148
- 1 Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235
- 1 Dr. Marty Rockway
Technical Director
AFHRL(OT)
Williams AFB, AZ 58224

Marines

- 1 H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134
- 1 Director, Office of Manpower Utilization
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134
- 1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

CoastGuard

- 1 Mr. Thomas A. Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Other DoD

- 12 Defense Technical Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
- 1 Dr. William Graham
Testing Directorate
MEPCOM/MEPCT-P
Ft. Sheridan, IL 60037
- 1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defense
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301
- 1 MAJOR Wayne Sellman, USAF
Office of the Assistant Secretary
of Defense (MRA&L)
3B930 The Pentagon
Washington, DC 20301
- 1 DARPA
1400 Wilson Blvd.
Arlington, VA 22209

Civil Govt

- 1 Dr. Andrew R. Molnar
Science Education Dev.
and Research
National Science Foundation
Washington, DC 20550
- 1 Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E Street NW
Washington, DC 20415
- 1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Non Govt

- 1 Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK
- 1 1 psychological research unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600, Australia
- 1 Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450
- 1 Dr. Werner Birke
DezWPs im Streitkrafteamt
Postfach 20 50 03
D-5300 Bonn 2
WEST GERMANY
- 1 Dr. Nicholas A. Bond
Dept. of Psychology
Sacramento State College
600 Jay Street
Sacramento, CA 95819
- 1 Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52240
- 1 DR. C. VICTOR BUNDERSON
WICAT INC.
UNIVERSITY PLAZA, SUITE 10
1160 SO. STATE ST.
OREM, UT 84057
- 1 Dr. John B. Carroll
Psychometric Lab
Univ. of No. Carolina
Davie Hall 013A
Chapel Hill, NC 27514

Non Govt

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Livingstone House
Livingstone Road
Stratford
London E15 2LJ
ENGLAND
- 1 Dr. Kenneth E. Clark
College of Arts & Sciences
University of Rochester
River Campus Station
Rochester, NY 14627
- 1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007
- 1 Dr. William E. Coffman
Director, Iowa Testing Programs
334 Lindquist Center
University of Iowa
Iowa City, IA 52242
- 1 Dr. Meredith P. Crawford
American Psychological Association
1200 17th Street, N.W.
Washington, DC 20036
- 1 Dr. Leonard Feldt
Lindquist Center for Measurement
University of Iowa
Iowa City, IA 52242
- 1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240
- 1 Dr. Victor Fields
Dept. of Psychology
Montgomery College
Rockville, MD 20850
- 1 Univ. Prof. Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Non Govt

- 1 Professor Donald Fitzgerald
University of New England
Armidale, New South Wales 2351
AUSTRALIA
- 1 Dr. Edwin A. Fleishman
Advanced Research Resources Organ.
Suite 900
4330 East West Highway
Washington, DC 20014
- 1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138
- 1 DR. ROBERT GLASER
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213
- 1 Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002
- 1 Dr. Chester Harris
School of Education
University of California
Santa Barbara, CA 93106
- 1 Dr. Lloyd Humphreys
Department of Psychology
University of Illinois
Champaign, IL 61820
- 1 Library
HUMRRO/Western Division
27857 Berwick Drive
Carmel, CA 93821
- 1 Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Non Govt

- 1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105
- 1 Dr. Huynh Huynh
College of Education
University of South Carolina
Columbia, SC 29208
- 1 Professor John A. Keats
University of Newcastle
AUSTRALIA 2308
- 1 Mr. Marlin Kroger
1117 Via Goleta
Palos Verdes Estates, CA 90274
- 1 Dr. Michael Levine
Department of Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801
- 1 Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat
Groningen
NETHERLANDS
- 1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801
- 1 Dr. Frederick M. Lord
Educational Testing Service
Princeton, NJ 08540
- 1 Dr. Gary Marco
Educational Testing Service
Princeton, NJ 08450
- 1 Dr. Scott Maxwell
Department of Psychology
University of Houston
Houston, TX 77004

Non Govt

- 1 Dr. Samuel T. Mayo
Loyola University of Chicago
820 North Michigan Avenue
Chicago, IL 60611
- 1 Dr. Jesse Orlansky
Institute for Defense Analyses
400 Army Navy Drive
Arlington, VA 22202
- 1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207
- 1 MR. LUIGI PETRULLO
2431 N. EDGEWOOD STREET
ARLINGTON, VA 22207
- 1 DR. DIANE M. RAMSEY-KLEE
R-K RESEARCH & SYSTEM DESIGN
3947 RIDGEMONT DRIVE
MALIBU, CA 90265
- 1 MINRAT M. L. RAUCH
P II 4
BUNDESMINISTERIUM DER VERTEIDIGUNG
POSTFACH 1328
D-53 BONN 1, GERMANY
- 1 Dr. Mark D. Reckase
Educational Psychology Dept.
University of Missouri-Columbia
4 Hill Hall
Columbia, MO 65211
- 1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007
- 1 Dr. Leonard L. Rosenbaum, Chairman
Department of Psychology
Montgomery College
Rockville, MD 20850

Non Govt

- 1 Dr. Ernst Z. Rothkopf
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974
- 1 Dr. Lawrence Rudner
403 Elm Avenue
Takoma Park, MD 20012
- 1 Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208
- 1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916
- 1 DR. ROBERT J. SEIDEL
INSTRUCTIONAL TECHNOLOGY GROUP
HUMRRO
300 N. WASHINGTON ST.
ALEXANDRIA, VA 22314
- 1 Dr. Kazuo Shigemasu
University of Tohoku
Department of Educational Psychology
Kawauchi, Sendai 980
JAPAN
- 1 Dr. Edwin Shirkey
Department of Psychology
University of Central Florida
Orlando, FL 32816
- 1 Dr. Robert Smith
Department of Computer Science
Rutgers University
New Brunswick, NJ 08903
- 1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305

Non Govt

- 1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520
- 1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN
THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305
- 1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003
- 1 Dr. Brad Sympson
Psychometric Research Group
Educational Testing Service
Princeton, NJ 08541
- 1 Dr. Kikumi Tatsuoka
Computer Based Education Research
Laboratory
252 Engineering Research Laboratory
University of Illinois
Urbana, IL 61801
- 1 Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044
- 1 Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201
- 1 Dr. J. Uhlaner
Perceptrons, Inc.
6271 Variel Avenue
Woodland Hills, CA 91364

Non Govt

- 1 Dr. Howard Wainer
 Bureau of Social Science Research
 1990 M Street, N. W.
 Washington, DC 20036
- 1 Dr. Phyllis Weaver
 Graduate School of Education
 Harvard University
 200 Larsen Hall, Appian Way
 Cambridge, MA 02138
- 1 DR. SUSAN E. WHITELY
 PSYCHOLOGY DEPARTMENT
 UNIVERSITY OF KANSAS
 LAWRENCE, KANSAS 66044
- 1 Wolfgang Wildgrube
 Streitkraefteamt
 Box 20 50 03
 D-5300 Bonn 2

ATE
LMED
-8