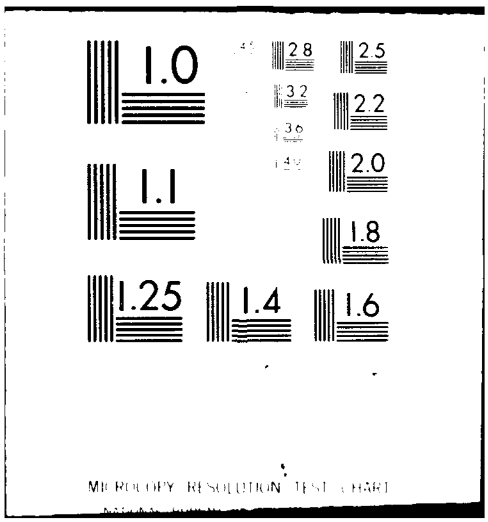
AD-A098 747   PRINCETON UNIV NJ DEPT OF ELECTRICAL ENGINEERING AND--ETC F/G 12/1
AN ALTERNATIVE CONSIDERATION ON SINGULAR LINEAR STATE ESTIMATIO--ETC(U)
1980   Y F HUANG, E FOGEL, J B THOMAS   DAAG29-79-C-0024

UNCLASSIFIED   NL

1 of 1
AD A
098747

END
DATE
FILMED
5-81
DTIC


1.0
2.8
2.5
3.2
2.2
3.6
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

PHOTOGRAPH THIS SHEET

AD A098747

DTIC ACCESSION NUMBER

LEVEL

INVENTORY

1

AN ALTERNATIVE CONSIDERATION ON SINGULAR LINEAR STATE ESTIMATION

DOCUMENT IDENTIFICATION

N00014-75-C-0498

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

DISTRIBUTION STATEMENT

ACCESSION FOR	
NTIS GRA&I	☑
DTIC TAB	☐
UNANNOUNCED	☐
JUSTIFICATION	
BY	
DISTRIBUTION /	
AVAILABILITY CODES	

DIST	AVAIL AND/OR SPECIAL	
A	20	

DISTRIBUTION STAMP

DTIC
ELECTE
MAY 1 1981
S D
B

DATE ACCESSIONED

81 4 27 153

DATE RECEIVED IN DTIC

PHOTOGRAPH THIS SHEET AND RETURN TO DTIC-DDA-2

DTIC FORM OCT 79 70A

DOCUMENT PROCESSING SHEET

AN ALTERNATIVE CONSIDERATION ON SINGULAR LINEAR STATE ESTIMATION

Y.F. Huang
Dept. of Elec. Eng. and Computer Sci.
Princeton Univ.
Princeton, NJ 08544

E.Fogel
Dept. of Elec. Eng.
Univ. of Notre Dame
Notre Dame, IN 46556

J.B. Thomas
Dept. of Elec. Eng. and Computer Sci.
Princeton Univ.
Princeton, NJ 08544

Abstract

The problem of designing an optimal state estimator for a linear, discrete-time system with a singular noise covariance matrix is considered. In this article, this problem is cast as a constrained optimization problem and the approach appears to be more direct. Solution to this optimization problem gives a reduced-order optimal state estimator.

I. INTRODUCTION:

In a linear stochastic system, the output measurement may be only partially noise corrupted. Although, in practice, one may argue that there exist no noise-free measurements, it is quite possible that some of the measurements are noise corrupted while the others are relatively accurate. Under the Gaussian assumption, this implies that the noise covariance matrix has both large and small eigenvalues, which easily leads to numerical difficulties in the implementation of the Kalman filter. It is convenient in this case to model these more accurate measurements as noise-free entities.

The study of this problem dates back to the work of Bryson [1], for continuous-time systems, and that of Brammer [2], for discrete-time systems. Kwakernaak [3] and Anderson [4] discussed this problem as a singular linear state estimation problem; however, no explicit solutions were given. Tse and Athans [5] derived a rather complicated "observer-estimator" which is essentially an extension of the Luenberger observer [6]. Later, Yoshikawa [7] gave a simpler derivation for minimum-order optimal state estimators. More recently, Fairman [8] proposed a "hybrid estimator" which features "coordinatization" and achieved a reduced-order optimal estimator.

The main feature of the approach used in this paper is the following: After a proper similarity transformation, the state variables are decomposed into two parts, one to be estimated by a reduced-order filter and the other to be recovered exactly from the noise-free measurements. Then the dynamic equation of the latter part of the state equation is considered a constraint on the optimal estimation of the other part of the states. Hence the state estimation problem in this case is cast as a constrained optimization problem, which leads to a reduced-order optimal state estimator.

2. PROBLEM FORMULATION

A linear, discrete-time stochastic system can be described by the following equations

$$x(k+1)=A(k)\ x(k)+B(k)\ u(k),\quad k=0,1,2,\ldots \quad (1)$$

$$y(k) = C(k)\ x(k) + v(k)\quad ,\ k=1,2,\ldots \quad (2)$$

where $x(\cdot)\in R^n$, $u(\cdot)\in R^p$, and $y(\cdot)\in R^m$. To further specify the problem, the following assumptions are made:

(i) $x(0)$, $u(0)$, $u(1)$, ..., $v(1)$, $v(2)$, ..., are independent random vectors with the following statistics

$$E[x(0)]=x_0 \qquad E[x(0)x^T(0)]=V_{x_0}$$

$$E[u(k)]=\underline{0}\ \forall k \qquad E[u(k)u^T(k-i)] =V_u(k)\,\delta(i)\ \forall k,i$$

$$E[v(k)]=\underline{0}\ \forall k \qquad E[v(k)v^T(k-i)]$$

Presented at 23rd Midwest Symposium on Circuits and Systems, August 1980; to be published in the Proceedings of the Conference

$$E[u(k)v^T(i)] = 0_{p\times m} \ \forall k,i \qquad = V_v(k)\delta(i) \ \forall k,i$$

$$E[u(k)x^T(0)] = 0_{p\times n} \ \forall k$$

$$E[v(k)x^T(0) = 0_{m\times n} \ \forall k$$

where $u^T(\cdot)$ and $v^T(\cdot)$ denotes the transpose of vectors $u(\cdot)$ and $v(\cdot)$, respectively, and $\delta(\cdot)$ deontes the Kronecker delta.

(ii) For any k, the $V_v(k)$ is a non-negative definite matrix with rank m_1, where $m_1 < m$. Under this assumption, implementation of the standard Kalman filter involves inversion of a matrix which may be singular. Tse and Athans [5] proposed an observer-estimator of order $n-m_2$ which performs as well as higher order estimators, where $m_2 \triangleq m-m_1$.

(iii) For any k, the C(k) is of full rank, i.e. every element of the output measurement is independent of the others.

The objective here is to design an optimal state estimator of order $n-m_2$. Without loss of generality, one can assume that

$$v(k) = \begin{bmatrix} v_1(k) \\ \hline 0 \end{bmatrix} \begin{matrix} \}m_1 \\ \}m_2 \end{matrix}$$

where $v_1(k) \in R^{m_1}$, and thus the covariance matrix of v(k) can be written as

$$V_v(k) = \left[\begin{array}{c|c} V_{v_1}(k) & 0_{m_1\times m_2} \\ \hline 0_{m_2\times m_1} & 0_{m_2\times m_2} \end{array}\right]$$

where $V_{v_1}(k)$ is strictly positive definite.

It is easy to see that there exists a non-singular matrix Q(k), such that the transformation

$$z(k) = Q(k)\ x(k) \tag{3}$$

yields the following state and measurement equations

$$\begin{bmatrix} z_1(k+1) \\ \hline z_2(k+1) \end{bmatrix} = \left[\begin{array}{c|c} \tilde{A}_{11}(k) & \tilde{A}_{12}(k) \\ \hline \tilde{A}_{21}(k) & \tilde{A}_{22}(k) \end{array}\right] \begin{bmatrix} z_1(k) \\ \hline z_2(k) \end{bmatrix} + \begin{bmatrix} \tilde{B}_1(k) \\ \hline \tilde{B}_2(k) \end{bmatrix} u(k) \tag{4}$$

and

$$\begin{bmatrix} y_1(k) \\ \hline y_2(k) \end{bmatrix} = \left[\begin{array}{c|c} \tilde{C}_{11}(k) & \tilde{C}_{12}(k) \\ \hline 0_{m_2\times n_1} & \tilde{C}_{22}(k) \end{array}\right] \begin{bmatrix} z_1(k) \\ \hline z_2(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ \hline 0 \end{bmatrix} \tag{5}$$

where

$$\tilde{A}(k) = Q(k+1)\ A(k)\ Q^{-1}(k)$$

$$\tilde{B}(k) = Q(k+1)\ B(k)$$

and

$$\tilde{C}(k) = C(k)\ Q^{-1}(k)$$

$z_1(\cdot) \in R^{n_1}, z_2(\cdot) \in R^{m_2}, y_1(\cdot) \in R^{m_1}, y_2(\cdot) \in R^{m_2}$, and $n_1 \triangleq n-m_2$. $\tilde{A}(\cdot)$, $\tilde{B}(\cdot)$ and $\tilde{C}(\cdot)$ are partitioned accordingly. Moreover, since C(k) is of full rank, $\tilde{C}_{22}(k)$ is invertible. Hence there exists a one-to-one correspondence between the state $z_2(k)$ and output $y_2(k)$, namely

$$z_2(k) = \tilde{C}_{22}^{-1}(k)\ y_2(k) \tag{6}$$

and thus in the state equation, (4), only $z_1(\cdot)$ must be estimated. The dynamic equations for $z_1(\cdot)$ and $z_2(\cdot)$ are

$$z_1(k+1) = \tilde{A}_{11}(k)z_1(k) + \tilde{A}_{12}(k)z_2(k) + \tilde{B}_1(k)u(k) \tag{7}$$

$$z_2(k+1) = \tilde{A}_{21}(k)z_1(k) + \tilde{A}_{22}(k)z_2(k) + \tilde{B}_2(k)u(k) \tag{8}$$

It is obvious from (7) and (8) that $z_1(k)$ and $z_2(k)$ are mutually dependent; therefore the estimation of $z_1(k)$ does depend on the dynamic behavior of $z_2(k)$. Thus the filtering problem becomes that of finding an optimal $\hat{z}_1(k+1|k+1)$ subject to (7) and constrained by (8), where $\hat{z}_1(k+1|k+1)$ denotes the estimate of $z_1(k+1)$ given measurements up to time k+1. Note, from (6) and (7), that the state $z_2(k)$ can be regarded as a deterministic input in the Kalman filtering problem.

3. THE REDUCED-ORDER OPTIMAL STATE ESTIMATOR

In this section, the optimal estimator for $z_1(k)$ is derived where the performance measure is the trace of the error covariance matrix. Defining the vector s(k) as

$$s(k) \triangleq z_1(k) - P(k)\ z_2(k)\ ;\ P(k) \in R^{n_1 \times m_2} \tag{9}$$

from (5), (7) and (8), one obtains

$$s(k+1)=F(k)s(k)+G(k)z_2(k)+M(k)u(k) \tag{10}$$

and

$$y_1(k)=H(k)s(k)+N(k)z_2(k)+v_1(k) \tag{11}$$

where

$$F(k) = \tilde{A}_{11}(k) - P(k+1)\ \tilde{A}_{21}(k) \tag{10.a}$$

$$G(k) = \tilde{A}_{12}(k) - P(k+1)\ \tilde{A}_{22}(k) + F(k)\ P(k) \tag{10.b}$$

$$M(k) = \tilde{B}_1(k) - P(k+1)\ \tilde{B}_2(k) \tag{10.c}$$

$$H(K) = \tilde{C}_{11}(k) \tag{11.a}$$

and

$$N(k) = \tilde{C}_{12}(k) + \tilde{C}_{11}(k)\ P(k) \tag{11.b}$$

Notice that $P(k)$, as defined in the above equations, can be viewed as the Lagrange multiplier in the standard constrained optimization problem. Now, the problem of estimating $z_1(k)$ is replaced by that of estimating $s(k)$ given the measurements $\{y_1(1), y_1(2), \ldots, y_1(k)\}$ and states $\{z_2(1), z_2(2), \ldots, z_2(k)\}$. From (9), it is obvious that

$$\hat{s}(k|k) = \hat{z}_1(k|k) - P(k)\ z_2(k) \tag{12}$$

and

$$V_{\tilde{s}}(k|k) = V_{\tilde{z}_1}(k|k) \tag{13}$$

where $\hat{s}(k|k)$ denotes the estimate of $s(k)$ conditioned on input-output measurements up to time k, and

$$\tilde{s}(k|k) \triangleq \hat{s}(k|k) - s(k)$$

$$\tilde{z}_1(k|k) \triangleq \hat{z}_1(k|k) - z_1(k)$$

$$V_{\tilde{s}}(k|k) \triangleq E[\tilde{s}(k|k)\tilde{s}^T(k|k)]$$

The unbiased linear estimator of $s(k)$ is given by the following n_1th-order filter

$$\begin{aligned}\hat{s}(k+1|k+1) = {} & [I-K(k+1)H(k+1)]F(k)\hat{s}(k|k) \\ & +K(k+1)[y_1(k+1)-N(k+1)\ z_2(k+1) \\ & -H(k+1)G(k)z_2(k)]\end{aligned} \tag{14}$$

Hence the error quantity $\tilde{s}(k|k)$ propagates as

$$\begin{aligned}\tilde{s}(k+1|k+1) = {} & [I-K(k+1)H(k+1)]F(k)\tilde{s}(k|k) \\ & +[I-K(k+1)H(k+1)]M(k)u(k) \\ & -K(k+1)v_1(k+1)\end{aligned} \tag{15}$$

and the error covariance matrix $V_{\tilde{s}}(\cdot|\cdot)$ is given by

$$\begin{aligned}V_{\tilde{s}}(k+1|k+1) = {} & [I-K(k+1)H(k+1)]\Gamma(k+1) \\ & [I-K(k+1)H(k+1)]^T \\ & +K(k+1)V_{v_1}(k+1)K^T(k+1)\end{aligned} \tag{16}$$

where

$$\Gamma(k+1) \triangleq F(k)V_{\tilde{s}}(k|k)F^T(k)+M(k)V_u(k)M^T(k) \tag{17}$$

Observe that $\Gamma(k+1)$ is essentially the one-step prediction error covariance matrix [9]. An optimal estimator is taken to be an estimator which minimizes the trace of the error covariance matrix. Therefore, it is left to minimize $Tr[V_{\tilde{s}}(k+1|k+1)]$ with respect to $[K(k+1) \vdots P(k+1)]$. Notice that here $K(k+1)$ plays the role of standard Kalman gain while $P(k+1)$ is the Langrangian of the optimization problem. Minimizing $Tr[V_{\tilde{s}}(k+1|k+1)]$ with respect to $K(k+1)$ yields

$$K^*(k+1) = \Gamma(k+1)\ H^T(k+1)\ R^{-1}(k+1) \tag{18}$$

where $R(\cdot)$ is the positive-definite symmetric matrix given by

$$R(k+1) = H(k+1)\Gamma(k+1)H^T(k+1)+V_{v_1}(k+1) \tag{18.a}$$

Observe that (15)-(18) are identical to the formulation of the standard Kalman filter [9]. However, in this case, it is further required to optimize the state estimator with respect to the choice of *P(k+1)*; i.e. minimize $Tr[V_{\tilde{s}}(k+1|k+1)]$ with respect to $P(k+1)$. Let the optimal $P(k+1)$ which minimizes $Tr[V_{\tilde{s}}(k+1|k+1)]$ be denoted by $P^*(k+1)$. Then it can be shown that

$$P^*(k+1) \in \Sigma^*(k+1) \tag{19}$$

where $\Sigma^*(k+1)$ is the set given by

$$\Sigma^*(k+1) = \{P:\ P\ \Lambda_1(k)=\Lambda_2(k),\ P \in R^{n_1 \times m_2}\} \tag{19.a}$$

$$\Lambda_1(k) = \tilde{A}_{21}(k)V_{\tilde{s}}(k|k)\tilde{A}_{21}^T(k)+\tilde{B}_2(k)V_u(k)\tilde{B}_2^T(k) \tag{19.b}$$

$$\Lambda_2(k) = \tilde{A}_{11}(k)V_{\tilde{s}}(k|k)\tilde{A}_{21}^T(k)+\tilde{B}_1(k)V_u(k)\tilde{B}_2^T(k) \tag{19.c}$$

Notice that, when $\Lambda_1(k)$ is non-singular, $P^*(k+1)$ is given by

$$P^*(k+1) = \Lambda_2(k)\ \Lambda_1^{-1}(k) \tag{19.d}$$

The set $\Sigma^*(k+1)$ will be discussed in the next section.

Once $P^*(k+1)$ is found, the matrices $F(k)$, $G(k)$, and $M(k)$ can be specified and denoted by $F^*(k)$, $G^*(k)$, $M^*(k)$, respectively, by substituting $P^*(k+1)$ and $P^*(k)$ into (10). Similarly, $N^*(k+1)$, $\Gamma^*(k+1)$ and $R^*(k+1)$ can be obtained from (11.b), (17), and (18.a).

All in all, the reduced-order optimal

state estimator is formulated by the following equations

$$\hat{s}(k+1|k+1) = [I-K^*(k+1)H(k+1)]F^*(k)\hat{s}(k|k) + K^*(k+1)[y_1(k+1) - N^*(k+1) z_2(k+1) - H(k+1)G\theta(k)z_2(k)] \quad (20.a)$$

$$\hat{z}_1(k+1|k+1) = \hat{s}(k+1|k+1) + P^*(k+1)\, z_2(k+1) \quad (20.b)$$

$$z_2(k+1) = \tilde{C}_{22}^{-1}(k+1)\, y_2(k+1) \quad (20.c)$$

$$V_{\tilde{z}}(k+1|k+1) = [I-K^*(k+1)H(k+1)]\Gamma^*(k+1) \quad (20.d)$$

$$K^*(k+1) = \Gamma^*(k+1)H^T(k+1)[H(k+1)\Gamma^*(k+1) H^T(k+1) + V_{v_1}(k+1)]^{-1} \quad (20.e)$$

$$\Gamma^*(k+1) = F^*(k)V_{\tilde{z}}(k|k)F^{*T}(k) + M^*(k)V_u(k)$$

$$M^{*T}(k) = [I:-P^*(k+1)][\tilde{A}(k) V_{\tilde{z}}(k|k)\tilde{A}^T(k) + \tilde{B}(k)V_u(k)\tilde{B}^T(k)] [I:-P^*(k+1)]^T \quad (20.f)$$

where

$$V^*_{\tilde{z}}(k|k) = \left[\begin{array}{c|c} V^*_{z_1}(k|k) & 0_{m_1 \times m_2} \\ \hline 0_{m_2 \times m_1} & 0_{m_2 \times m_2} \end{array}\right]$$

due to the fact that $z_1(k)$ is exactly measurable for every k. Also, the error covariance matrix $V^*_{\tilde{z}}(k+1|k+1)$ is given by

$$V^*_{\tilde{z}}(k+1|k+1) = [(\Gamma^*(k+1))^{-1} + H^T(k+1) V_{v_1}(k+1)\, H(k+1)]^{-1} \quad (20.g)$$

and $P^*(k+1)$ is specified by Eqs. (19).

4. COMMENTS ON THE REDUCED-ORDER OPTIMAL ESTIMATOR

The formulation for the optimal state estimator derived in last section, (19)-(20), is identical to that of the "hybrid estimator" given in [8], except that in [8] a deterministic input is inserted to the system dynamics. However, the approach here is more straightforward and it is clearer here that the choice of $P(k+1)$ is crucial to the optimality of the estimator. It can be seen that the general coordinate transformation discussed in [8] is split into two coordinate transformations: one which depends on the system output matrix H only and one which depends on the matrices P_{11} and P_{12}. It can be seen that the similarity transformation $Q(\cdot)$ of (3) is equivalent to $J^{-1}(\cdot)$ defined in (14) of [8], and thus the state variable z defined in (3) can be regarded as equivalent to ζ defined in (14) of [8]. Furthermore, the variable s defined in (9) can be similarly related to the variable ξ_1 in (20) of [8].

The relations in (19) which govern the choice of $P^*(k+1)$ are vital to the understanding of the optimal state estimator, and thus deserves some detailed discussion. First, notice that

$$\mathfrak{h}(\Lambda_1(k)) \subset \mathfrak{h}(\Lambda_2(k))$$

where $\mathfrak{h}(\Lambda_1(k))$ and $\mathfrak{h}(\Lambda_2(k))$ denote the null spaces of $\Lambda_1(k)$ and $\Lambda_2(k)$, respectively. The following observation is thus made:

<u>Observation</u> The set $\Sigma^*(k+1)$ given in (19.a) is a non-empty set. Moreover, if $\Lambda_1(k)$ is singular, any member $P(k+1) \in \Sigma^*(k+1)$ yields the same estimator performance.

Now, according to the value of $\Lambda_1(k)$, the following special cases are of interest:

<u>Case 1</u>: $\Lambda_1(k) = 0_{m_2 \times m_2}$. In this case, $\Lambda_2(k) = 0_{n_1 \times m_2}$ and thus

$$\Sigma^*(k+1) = R^{n_1 \times m_2}$$

This case is possible if (8) does not contain any information pertaining to the estimation of $z_1(k)$; for example, if $\tilde{A}_{21}(k) = 0_{m_2 \times n_1}$ and $\tilde{B}_2(k) = 0_{m_2 \times p}$. An extreme example for this case is that $m_2 = 0$, i.e. all measurements are noise corrupted. In this condition, the estimator presented in Section 3 is identical to the standard full-order Kalman filter whose performance is independent of the choice of $P(k+1)$.

<u>Case 2</u>: $\Lambda_1(k) \geq 0_{m_2 \times m_2}$; i.e. $\Lambda_1(k)$ is a singular non-zero matrix. In this case, only some components of $z_2(k)$ contain information about $(u(k), z_1(k))$. Thus the similarity transformation discussed in Section 2 can be redefined so as to isolate only those elements of $z_2(k)$ which constitute a constraint on $(u(k), z_1(k))$. Hence the Lagrange multiplier $P(k+1)$ that should be considered is an element in $R^{(n-r)\times r}$, where $r < m_2$. Alternatively, any member in $\Sigma^*(k+1)$ can be used in the filter realization.

<u>Case 3</u>: $\Lambda_1(k)$ is positive-definite. This condition can be fulfilled when $V_u(k)$ is positive-definite for any $k = 0,1,2,\ldots$. In this case, $\Sigma^*(k+1)$ contains one and

only one element $P^*(k+1)$, which is given by (19.d).

When $P^*(\cdot)$ is uniquely specified (Case 3), one can compare the error covariance matrix given by (20.g) with that obtained for arbitrary $P(\cdot)$ and observe the same expression for $V_{\tilde{z}}(k+1|k+1)$.

The difference is that $\Gamma^*(k+1)$ of (20.f) has the following property

$$\mathrm{Tr}[\Gamma^*(k+1)] < \mathrm{Tr}[\Gamma(k+1)]$$

where $\Gamma(\cdot)$ is obtained from non-optimal $P(\cdot)$.

Finally, the implementation of this estimator should be initiated as follows:

$$\hat{x}(0|0) = E[x(0)] = x_0$$

i.e.

$$\hat{z}_1(0|0) = E[z_1(0)]$$

therefore

$$V_{\tilde{z}}(0|0) = \left[\begin{array}{c|c} V_{z_1}(0) & 0_{m_1 \times m_2} \\ \hline 0_{m_2 \times m_1} & 0_{m_2 \times m_2} \end{array}\right] = Q(0) V_{x_0} Q^T(0)$$

and

$$P^*(0) = 0_{n_1 \times m_2}$$

5. CONCLUSION

A *reduced-order optimal state* estimator for a linear, discrete-time system associated with a singular noise covariance matrix has been derived in this paper. The main idea in this derivation is to cast this singular state estimation problem as a constrained optimization problem. The estimator derived here is fundamentally the same as that derived by Fairman [8]. The major differences are: the approach here is more straightforward, the optimality of the estimator is more explicitly exposed and, furthermore, the possibility of nonuniqueness of $P^*(\cdot)$ is discussed here.

It is worth mentioning that the estimator given here requires lower order matrix inversion than the standard full-order Kalman filter does in the singular case; thus the computational efficiency is improved. This estimation procedure can be applied similarly to smoothing and predicting problems or systems with colored noise.

REFERENCES

[1] A.E. Bryson, Jr. and D.E. Johansen, "Linear filtering for time-varying systems using measurements containing colored noise," IEEE Trans. Automat. Contr., vol. AC-10, pp.4-10, Jan. 1965.

[2] K.G. Brammer, "Lower order optimal linear filtering of non-stationary random sequence," IEEE Trans. Automat. Contr., vol. AC-13, pp.198-199, April 1968.

[3] H. Kwakernaak and R. Sivan, Linear Optimal Control Systems. Wiley Interscience, 1972.

[4] B.D.O. Anderson and J.B. Moore, Optimal Filtering. Prentice-Hall, 1979.

[5] E. Tse and M. Athans, "Optimal minimal-order observer-estimators for discrete linear time varying systems," IEEE Trans. Automat. Contr., vol. AC-15, pp.416-426, Aug. 1970.

[6] D.G. Luenberger, "Observing the state of a linear system," IEEE Trans. Mil. Elec., vol. MIL-8, pp.74-80, April, 1964.

[7] T. Yoshikawa, "Minimal-order optimal filters for discrete-time linear stochastic system," Int. J. of Contr., vol. 21, pp.1-19, Jan. 1975.

[8] F.W. Fairman, "Hybrid estimators for discrete-time stochastic systems," IEEE Trans. Sys. Man., and Cyber., vol. SMC-8, pp.849-854, Dec. 1978.

[9] J.S. Meditch, Stochastic Optimal Linear Estimation and Control. New York: McGraw-Hill Book Co., Inc., 1969.

ACKNOWLEDGEMENT:

The authors wish to express their appreciation to Dr. R.W. Liu for his valuable comments during the course of this work. This research is supported in part by the U.S. Army Research Office under contract DAAG 29-79-C-0024, and in part by the Office of Naval Research under contract N00014-78-C-0444.

N00014-75-C-0498