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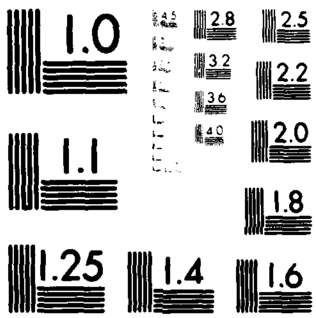

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Substantial results on the use of mixed finite element methods for partial differential eigenvalue problems and for viscous flow problems have been achieved. Both analytical and computational studies have been carried out. The analytical work is concerned with error estimates for the finite element approximations, while the computational efforts consist of implementing these finite element algorithms.

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MIXED FINITE ELEMENT METHODS WITH APPLICATIONS TO FLOW WITH OTHER PROBLEMS

by

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I - ABSTRACT

We report on progress made on USAFOSR Grant No. AFOSR-80-0083. Substantial results on the use of mixed finite element methods for partial differential eigenvalue problems and for viscous flow problems have been achieved. Both analytical and computational studies have been carried out. The analytical work is concerned with error estimates for the finite element approximations, while the computational efforts consist of implementing these finite element algorithms.

II - DESCRIPTION OF WORK

In our proposal, we proposed to consider the approximate solution, via mixed finite element methods, of two physical problems. The first was the energy stability of the incompressible Navier-Stokes equation. This problem has the following variational formulation: seek $(\underline{u}, \phi, \nu^*) \in V \times S \times \mathbb{R}'$ such that

$$\int_\Omega (\underline{u} \cdot D \cdot \underline{w} - \phi \operatorname{div} \underline{w}) = -\nu^* \int_\Omega \nabla\underline{u} \cdot \nabla\underline{w}$$

$$\int_\Omega \psi \operatorname{div} \underline{u} = 0 \tag{2.1}$$

for all $(\underline{w}, \psi) \in V \times S$. Here V and S are appropriately chosen Hilbert spaces and D is the deformation tensor of a given flow. It is easily shown [1] that if the largest eigenvalue $\tilde{\nu}$ of the problem (2.1) is smaller than the kinematic viscosity ν of the given flow, then the given flow is stable in the energy sense. We note that (2.1) is a linear eigenvalue problem which determines stability regions for arbitrary perturbations, i.e. not infinitesimal. The problem (2.1) is, however, not easily solvable. We therefore wish to solve the problem approximately. To this end we choose finite dimensional subspaces $V^h \subset V$ and $S^h \subset S$ and require that (2.1) hold for all $(\underline{w}, \psi) \in V^h \times S^h$. This leads to a linear generalized algebraic eigenvalue problem of the form

$$\begin{pmatrix} A & D^* \\ D & 0 \end{pmatrix} \begin{pmatrix} \vec{W} \\ \vec{\Phi} \end{pmatrix} = \nu_h \begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{W} \\ \vec{\Phi} \end{pmatrix} . \tag{2.2}$$

We choose the spaces V^h and S^h to be finite element spaces.

Under certain conditions on these spaces, conditions which are necessary for the stability of the approximate problem, we have been able to prove that the approximate eigenvalues ν_h converge to the exact eigenvalues ν^* at an optimal rate. In addition, two computer codes have been written which use different finite element spaces which satisfy the above mentioned conditions. The first uses a triangulation of the type illustrated in the figure on the left wherein the vector $\underline{u}$ is approximated by piecewise linear polynomials in each triangle and the

Figure 1

scalar ϕ is approximated by piecewise constant functions in each triangle. The second code uses the element on the right which again uses piecewise linear vectors on triangles, but now uses scalars which are constant throughout the quadrilateral. Our theory predicts that ν^h converges to ν^* quadratically in the grid spacing and, of course, this is verified by our computational results. The codes developed take advantage of the structure and sparsity of the algebraic problem (2.2).

The second physical problem considered is that of acoustic eigenvalue problems such as those described by the equation

$$\Delta\phi = \lambda\phi \quad \text{in} \quad \Omega \tag{2.3}$$

Again this eigenvalue problem can be given the variational characterization: seek $(\underline{u}, \phi, \lambda) \in V \times S \times \mathbb{R}$ such that

$$\begin{aligned} \int_\Omega \psi \operatorname{div} \underline{u} &= \lambda \int_\Omega \psi\phi \\ \int_\Omega (\phi \operatorname{div} \underline{v} + \underline{v} \cdot \underline{u}) &= 0 \end{aligned} \tag{2.4}$$

for all $(\underline{v}, \psi) \in V \times S$, where again V and S are suitable Hilbert spaces. Approximations are defined as before. In this case we have proved optimal error estimates for the eigenvalues λ and eigenfunction pairs $(\underline{u}, \phi)$ and have developed codes to compute these eigenvalues and eigenfunction, again using the two elements described in the figure above.

The variational problems (2.1) and (2.4) are special cases of the abstract mathematical problem: seek $(\underline{u}, \phi, \lambda) \in V_1 \times S_1 \times \mathbb{C}$ such that

$$\begin{aligned} a(\underline{u}, \underline{v}) + b_1(\underline{v}, \phi) &= \lambda c(\underline{u}, \underline{v}) \\ b_2(\underline{u}, \psi) \qquad &= \lambda d(\psi, \lambda) \end{aligned} \tag{2.5}$$

for all $(\underline{v}, \psi) \in V_2 \times S_2$. Here a, b_1, b_2, c and d are sesquilinear forms defined on the appropriate Hilbert spaces. We have been able to prove optimal error estimates for eigenvalue problems of the type (2.5) for a variety of combinations of forms a, b_1, etc. under reasonable hypothesis on these forms. For example, the cases $b_1 = b_2$ with c or $d = 0$, $b_1 = b_2$ with both c and $d \neq 0$, and $b_1 \neq b_2$, all with $a(\underline{u}, \underline{v})$ coercive or weakly coercive have been successfully analyzed and illustrative computer programs have been implemented in each case. The various cases which have considered describe a variety of physical examples including some generalized non-self- adjoint acoustic eigenvalue problems, eigenvalue problems emanating from transmission line theory, etc.

Most of the work described above concerning the use of mixed finite element methods for eigenvalue problems constitutes the Ph.D. dissertation of Ms. Janet Peterson, whose studies have been supported by the AFOSR grant. This work was reported on at the SIAM Fall 1980 meeting in Houston and will be the subject of a series of papers now in preparation.

In our proposal, we also proposed to develop mixed finite element discretizations which satisfy certain conditions (described in our previous proposal) which insure the stability and optimal accuracy of the discretizations. We have carried out this study, concentrating on second order L_2 approximations to the velocity vector in viscous flow problems. So far our study has considered only the linear Stokes or Oseen equations. The elements considered are those of Figures 1 and 2.

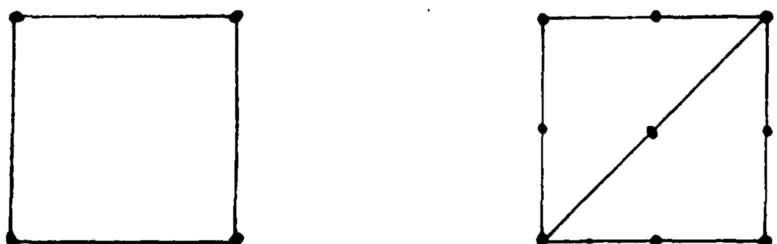

Figure 2

In Figure 2, on the left, the velocities and pressures are approximated on each quadrilateral, by bilinear and constant function, respectively, while for the element on the right, the velocities and pressures are approximated, on each triangle, by piecewise quadratic and piecewise constant functions. This last element appears often in the literature [2]. For all four elements the velocity approximations are second order (in the grid spacing) accurate in L_2 while the pressure approximations are first order accurate. However, the elements pictured on the right of Figure 1 and on the left of Figure 2 achieve this rate of accuracy with considerably fewer unknowns than the other two elements. Computer codes implementing all four elements have been written and are presently being used to generate information which will be used to compare the elements.

This work was also reported on at the SIAM Fall meeting and in various papers (1, 2, 3, 4 of Section 3).

[1] Serrin, J.: "Mathematical principles of classical fluid mechanics", Handbuch der Physick, 8(1959), Springer.

[2] Girault, V. and P. Raviart: Finite Element Approximations of the Navier-Stokes Equation, (1979), Springer.

III - REPORTING ACTIVITIES

PAPERS PREPARED UNDER GRANT SPONSORSHIP

1. "The computational accuracy of some finite element methods for incompressible viscous flow problems"; submitted to SIAM J. Scientific and Statistical Computing. (with R.A. Nicolaides).
2. "New results in the finite element solutions of steady viscous flows"; to appear in the Proceedings of MAFELAP 1981. (with R.A. Nicolaides).
3. "On conforming finite element methods for incompressible viscous flow problems"; in preparation. (with R.A. Nicolaides and J.S. Peterson).
4. "On mixed finite element methods for first order elliptic systems"; to appear in Num. Math.. (with G. Fix and R.A. Nicolaides).

Ph.D. THESIS PARTIALLY SUPPORTED BY GRANT

J.S. Peterson, "On mixed finite element methods for eigenvalue problems"; Ph.D., U. of Tennessee, Dec. 1980.

TALKS PRESENTED UNDER GRANT SPONSORSHIP

J.S. Peterson, "On mixed finite element methods for the stability of the Navier-Stokes equations"; SIAM Fall Meeting, Houston, 1980.

M.D. Gunzburger and R.A. Nicoliades, "Mixed finite element methods for viscous flow problems"; SIAM Fall Meeting, Houston, 1980.

In the next few months, additional papers and talks will be prepared based on work performed under grant sponsorship.

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