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ON THE NATURE OF BOUNDARY CONDITIONS  
FOR CRACK TIP STRESS

A. Cemal Eringen  
Princeton University

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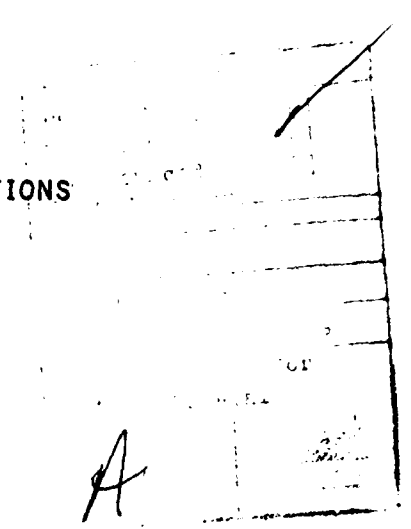
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ON THE NATURE OF BOUNDARY CONDITIONS  
FOR CRACK TIP STRESS\*

A. Cemal Eringen  
Princeton University  
Princeton, NJ 08540  
U.S.A.



ABSTRACT

Our recent calculations on crack tip stresses [1,2,] were scrutinized by Atkinson [3], suggesting that our scheme of calculations is not uniform and the solution of the problem may not actually exist. Here we show that these difficulties arise because of the use of incorrect boundary conditions by Atkinson. Also, we give an exact solution of a problem which refutes the suggested inexistence.

1. INTRODUCTION

Recently, we gave solutions of some crack problems within the context of the theory of nonlocal elasticity. It was shown that [1,2] in nonlocal elasticity, crack tip singularity does not exist and the hoop stress reaches a maximum just outside crack, adjacent to it. By equating this maximum to the cohesive stress, we can find the ultimate stress at which crack begins to propagate. In this way, it was possible to establish a fracture criterion based on maximum stress hypothesis. Cohesive stress calculations and several other findings indicated excellent agreements with the results of the atomic lattice dynamics and Griffith theory.

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Atkinson [3], by analyzing the one-dimensional model, suggested that our scheme of calculation may have a non-uniform character, and the problem posed and solved numerically may not actually possess a solution. Here, we show that this is not the case. Had he used the correct boundary condition in analyzing the hoop stress, these questions do not arise. Basically, Atkinson's difficulty is due to the use of the infinite range  $(-\infty, \infty)$  in calculating the integral for the stress within the crack surface and using nonlocal kernels without support. More precisely, the influence of strains outside crack line must be excluded in calculating the stress within the crack. Otherwise, an incompatibility will exist in the boundary conditions.

We also give the exact solution of a problem which indicates that the solution of the one-dimensional crack with a given load distribution exists.

## 2. NON-LOCAL ELASTICITY

Basic equations of linear, isotropic, nonlocal elasticity [4,5] with vanishing inertia and body forces consist of equations of equilibrium.

$$(2.1) \quad t_{kl,k} = 0 \quad \text{in } V$$

and constitutive equations

$$(2.2) \quad t_{kl} = \int_V \alpha(|\underline{x}' - \underline{x}|) \sigma_{kl}(\underline{x}') dv(\underline{x}')$$

where

$$(2.3) \quad \sigma_{kl} = \lambda u_{r,r} \delta_{kl} + \mu(u_{k,l} + u_{l,k})$$

Here,  $t_{kl}$  is the stress tensor and  $u_k$  is the displacement vector, referred to rectangular coordinates  $x_k$ ,  $\lambda$ , and  $\mu$  are Lamé constants and  $\delta_{kl}$  is the Kronecker delta. The nonlocal modulus  $\alpha(|\underline{x}|)$  is subject to

$$(2.4) \quad \int_V \alpha(|\underline{x}|) dv = 1$$

Since classical elasticity limit is desired,  $\alpha(|\underline{x}|)$  must be a Dirac  $\delta$ -sequence so that in the limit

$$(2.5) \quad \alpha = \delta(\underline{x})$$

nonlocal elasticity reverts to classical elasticity.

Equations (2.1) and (2.2), when combined, gives

$$(2.6) \quad \int_V \alpha(|\underline{x}' - \underline{x}|) \sigma_{kl,k}(\underline{x}') dv(\underline{x}') - \int_{\partial V} \alpha(|\underline{x}' - \underline{x}|) \sigma_{kl}(\underline{x}') da_k(\underline{x}') = 0$$

When (2.3) is substituted into (2.6), we obtain three integro-partial differential equations which must be solved under a set of boundary conditions to determine  $u_k(x)$ .

### 3. ONE-DIMENSIONAL MODEL

The one-dimensional model discussed in [1] is defined by

$$(3.1) \quad \gamma^2 v_{xx} + v_{yy} = 0 \quad , \quad y > 0$$

subject to boundary conditions

$$(3.2) \quad \begin{aligned} t_{yy}(x,0) &= -t_0(x), & |x| < \ell \\ v(x,0) &= 0, & |x| > \ell \\ v &\rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \end{aligned}$$

Here,  $\gamma^2 = \mu/(\lambda+2\mu)$ ,  $v$  is the  $y$ -component of the displacement field,  $t_{yy}$  is the normal stress given by

$$(3.3) \quad t_{yy}(x,y) = \int_{-\infty}^{\infty} \alpha(|x'-x|) \sigma_{yy}(x',y) dx', \quad y > 0$$

where

$$(3.4) \quad \sigma_{yy} = (\lambda + 2\mu) \frac{\partial v(x,y)}{\partial y}$$

It is important to remember that (3.3) is valid for  $y > 0$ . Since at  $y = 0$  there is a crack located at  $|x| < \ell$ ,  $y = 0$ , the influence of strains in  $|x| > \ell$  on the stress in  $|x| < \ell$  are forbidden so that the boundary condition (3.2)<sub>1</sub> reads

$$(3.5) \quad (\lambda+2\mu) \int_{-\ell}^{\ell} \alpha(|x'-x|) \frac{\partial v(x',0)}{\partial y} dx' = -t_0(x), \quad |x| < \ell$$

As we stated in [1], the one-dimensional model cannot be derived rationally from the field equations so that we place no faith in this model. It was treated for computational reasons in support of the two-dimensional

solution discussed there. Since it was considered by Atkinson at length to discuss the questions of non-uniformity in calculations and possible failure of the existence of solution, we consider these questions in the context of correct mathematical and physical considerations.

Several non-local kernels are possible. We mention three:

$$(3.6) \quad \alpha(|x|) = \frac{1}{a} \left(1 - \frac{|x|}{a}\right), \quad |x| < a \\ = 0, \quad |x| > a$$

$$(3.7) \quad \alpha(|x|) = \frac{\beta}{a\sqrt{\pi}} \exp\left[-\left(\frac{\beta}{a}\right)^2 x^2\right]$$

$$(3.8) \quad \alpha(|x|) = \frac{\beta}{2} e^{-\beta|x|}$$

Note that all these kernels are Dirac delta sequence, but only (3.6) has a finite support.

To clarify the physics of the problem, let us examine the nature of the boundary conditions. In Fig. 1 there is shown a perfect lattice with a crack of length  $2\ell$ . It is clear that an arbitrary load can only be applied in the region

$$(3.9) \quad |x| < \ell - a$$

where  $a$  is the atomic distance. Therefore, the support of the kernel  $\alpha(|x|)$  must be finite as in (3.6) and then the range of the applied stress cannot be all the way to  $x = \pm \ell$ . If we pass to the nonlocal continuum limit this implies that the kernel  $\alpha(|x|)$  must be cut off\* at  $x = \pm \ell$  or else an

\*In fact, the absence of this cut-off implies that there is no crack in the perfect lattice unless we introduce an edge dislocation (finite jump in  $v$  for  $|x| < \ell$ ) together with the applied load.

inhomogeneous kernel  $\alpha(x, x')$  must be used within regions

$$(3.10) \quad -l < x < -l + a, \quad l - a < x < l$$

when (3.6) is employed. Kernels (3.7) and (3.8) do not possess finite support.

Therefore, they must be cut off for  $|x| \geq l$ .

From the mathematical point of view, it is clear that if  $t_{yy}(x, 0)$  is calculated by

$$(3.11) \quad t_{yy}(x, 0) = (\lambda + 2\mu) \int_{-\infty}^{\infty} \alpha(|x' - x|) \frac{\partial v(x', 0)}{\partial y} dx' = -t_0(x),$$

in general, boundary conditions (3.2)<sub>1</sub> and (3.2)<sub>2</sub> will be *incompatible*. This is because  $v(x, 0) = 0$  is already specified for  $|x| \geq l$  and that prescription of  $t_{yy}(x, 0)$  for  $|x| < l$  imposes conditions on  $\partial v / \partial y$  within the common region to the support of  $\alpha(|x|)$  and  $|x| > l$ . In the case of the kernel (3.6), this region is given by  $l \leq |x| \leq l + a$ , for kernels (3.7) and (3.8), it is the entire region  $|x| > l$ .

Atkinson investigates the behavior of stress  $t_{yy}$  by using (3.11) in the crack region and finds that the boundary condition is satisfied uniformly for  $|x| < l - a$ , as  $a \rightarrow 0$ , but in the regions  $l - a < |x| < l$  (one atomic distances near the crack tips!),  $t_{yy}$  does not approach to its constant value uniformly as  $a \rightarrow 0$  (in the case of kernels (3.7) and (3.8) as  $\beta \rightarrow 0$ ).\*

In Section 4 below, we will show that: If the correct boundary conditions (3.5) for  $t_{yy}$  is used, then the boundary condition on  $t_{yy}$  is satisfied uniformly.

\* Perhaps what misled him to these conclusions is a missing statement from our work, that the Fourier transform of (3.11) is only an approximation to that of the exact boundary condition (3.5).

#### 4. EVALUATION OF THE STRESS

The Fourier representation of  $v(x,y)$  satisfying (3.1) and (3.2)<sub>3</sub> is given by

$$(4.1) \quad v(x,y) = (2/\pi)^{\frac{1}{2}} \int_0^{\infty} A(k) e^{-\gamma ky} \cos(kx) dk$$

The correct boundary conditions (3.2)<sub>1,2</sub> read

$$(4.2) \quad \begin{aligned} t_{yy}(x,0) &= - (2/\pi)^{\frac{1}{2}} (\lambda+2\mu) \gamma \int_{-l}^l \alpha(|x'-x|) dx' \int_0^{\infty} k A(k) \cos(kx) dk \\ &= - t_0(x), \quad 0 < x < l \end{aligned}$$

$$\int_0^{\infty} A(k) \cos(kx) dk = 0, \quad x > l$$

In [1] for  $A(k)$  we employed

$$(4.3) \quad A(k) = A_c(k) = l t_0 [2\mu(\lambda+2\mu)/\pi]^{-\frac{1}{2}} J_1(kl)/k$$

which is the exact solution of the same problem in classical elasticity.

Clearly (4.3) satisfies (4.2)<sub>2</sub> identically and for (4.2)<sub>1</sub> gives

$$(4.4) \quad t_{yy}(x,0)/t_0 = - \int_{-l}^l \alpha(|x'-x|) dx' \quad 0 < x < l$$

Evaluating this integral with  $\alpha(|x|)$  given by (3.8), we obtain:

$$(4.5) \quad t_{yy}/t_0 = -1 + e^{-\beta l} \cosh \beta x, \quad |x| < l$$

Using the reasoning of Atkinson [3], we set  $\beta x = X$ , then

$$(4.6) \quad P_c(x, \beta) = (t_{yy} + t_0)/t_0 = e^{-\beta l} \cosh X.$$

From this it is clear that as  $\beta \rightarrow \infty$ , for  $x < l$ ,  $P_c(x, \beta)$  tends to zero uniformly in  $\beta$ .

At the crack tips,  $x = \pm l$ , which is not included in the region of validity of (4.5), we have

$$(4.7) \quad P_c(l, \beta) \rightarrow \frac{1}{2} \quad \text{as} \quad \beta \rightarrow \infty$$

A similar situation is valid for the kernel (3.7):

$$(4.8) \quad t_{yy}/t_0 = -\frac{1}{2} \phi[\beta(l+x)/a] - \frac{1}{2} \phi[\beta(l-x)/a]$$

where  $\phi(z)$  is the Fresnel integral defined by

$$(4.9) \quad \phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$$

For the kernel (3.6), we find

$$(4.10) \quad \begin{aligned} t_{yy}/t_0 &= -1, & 0 < x \leq l-a \\ t_{yy}/t_0 &= -\frac{1}{2} - \frac{l-x}{a} \left(1 - \frac{l-x}{2a}\right), & l-a < x < l \end{aligned}$$

which shows that the boundary conditions on stress is satisfied all the way up to one atomic distance away from the crack tip. Beyond this,  $t_{yy}$  goes from  $-t_0$  at  $|x| = l-a$  to  $-t_0/2$  at  $|x| = l$ .\*

The stress field outside the crack is given by

$$(4.11) \quad t_{yy}/t_0 = l^2 \int_l^\infty \frac{\alpha(|x'-x|) dx'}{(x'^2 - l^2)^{\frac{1}{2}} [x' + (x'^2 - l^2)^{\frac{1}{2}}]}, \quad x > l$$

If we let

$$(4.12) \quad x' = l \cosh y$$

This integral becomes

$$(4.13) \quad t_{yy}/t_0 = l \int_0^\infty e^{-y} \alpha(|l \cosh y - x|) dy$$

which is bounded for all  $x > l$ .

If we use the kernel (3.8) and carry out the integration in (4.13) for  $x = l$ , we obtain

$$(4.14) \quad t_{yy}/t_0 = \frac{1}{2} \beta l e^{\beta l} K_1(\beta l) - \frac{1}{2}$$

where  $K_1(x)$  is the Bessel's function. For large  $\beta l$ , this gives

$$(4.15) \quad C = (\beta l)^{-\frac{1}{2}} P(\beta l) = \frac{1}{2} (\pi/2)^{\frac{1}{2}} = 0.8$$

\* Note that in this region,  $\alpha(|x|)$  is not normalized to unity. If we normalize it by dividing it with its area (which is  $1/2$  at  $|x| = l$ ), we would obtain  $t_{yy} = -t_0$  at  $|x| = l$ .

where

$$(4.16) \quad P(\ell) \equiv [t_{yy}(\ell,0)/t_0] + 1 .$$

For the kernel (3.6), the integration of (4.13) gives

$$(4.17) \quad t_{yy}(x,0)/t_0 = \beta f(x) ,$$

where

$$(4.18) \quad f(x) = 1 - \frac{x}{a} + \frac{\ell}{a} \left(\frac{x}{\ell}\right)^2 - 2 \frac{x}{a} [(x/\ell)^2 - 1]^{\frac{1}{2}} - \frac{2x}{\ell} - \frac{a}{\ell} \\ + \left(1 + \frac{x}{a}\right) \left[\left(\frac{x+a}{\ell}\right)^2 - 1\right]^{\frac{1}{2}} - \frac{\ell}{a} (x/\ell)^2 + \frac{\ell}{2a} + \frac{x}{a} [(x/\ell)^2 - 1]^{\frac{1}{2}} \\ + \frac{\ell}{a} \text{Arch}(x/\ell) - \frac{\ell}{2a} \text{Arch}\left(\frac{x+a}{\ell}\right) + \frac{\ell}{2a} \left(\frac{x+a}{\ell}\right)^2 \\ - \frac{\ell}{2a} \left(\frac{x+a}{\ell}\right) \left[\left(\frac{x+a}{\ell}\right)^2 - 1\right]^{\frac{1}{2}} \quad \ell \leq x < \ell + a$$

For  $x = \ell$ , we obtain the hoop stress at the crack tip.

$$(4.19) \quad t_{yy}(\ell,0)/t_0 = -\frac{1}{2} + (2\ell/a)^{\frac{1}{2}} \frac{1+(a/\ell)}{2} \left(1 + \frac{a}{2\ell}\right)^{\frac{1}{2}} - \frac{1}{2} (\ell/a)^2 \text{Arch}\left(1 + \frac{a}{\ell}\right)$$

exactly. For large  $\ell/a$ , this gives

$$(4.20) \quad t_{yy}(\ell,0)/t_0 \approx -\frac{1}{2} + \frac{2}{3} (2\ell/a)^{\frac{1}{2}}$$

Consequently,

$$(4.21) \quad C = (a/2\ell)^{\frac{1}{2}} P(\ell) = \frac{2}{3} .$$

These results compare well with our previous computer results given in [1].

We notice that the hoop stress experiences a jump discontinuity across crack tips (Figure 2). This is as expected, since the region of influence of strains on stresses within and outside the crack were terminated at crack tips. By employing an inhomogeneous kernel  $\alpha(x,x')$  that vanishes at crack tips, continuity in the stress field can be maintained across the tips. However, in this case, the mathematical problem becomes much more difficult to tackle.

## 5. QUESTION OF EXISTENCE

In order to understand the correct behavior of the hoop stress at the crack tips,  $x = \pm \ell$ , we note that for  $\alpha(|x|)$  given by (3.8) by differentiation from (3.5), we get

$$(5.1) \quad \sigma_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} = t_{yy} - \frac{1}{\beta^2} \frac{\partial^2 t_{yy}}{\partial x^2}$$

which is valid also for (3.3). At first sight, it appears that  $t_{yy} = -t_0 = \text{const.}$  in  $|x| \leq \ell$  gives  $\sigma_{yy} = -t_0 = \text{const.}$ , so that the classical elasticity solution for  $v$  is in fact the solution of the nonlocal problem. The fact that this is not the case can be seen if we set  $\sigma_{yy} = -t_0 = \text{const.}$ , in (3.5) we get (4.5). This is because

$$(5.2) \quad t_{yy} = -t_0 + A e^{-\beta x} + B e^{\beta x}$$

where  $A$  and  $B$  are constants, also gives  $\sigma_{yy} = -t_0 = \text{const.}$ . Clearly, Eq. (5.1) is not equivalent to Eq. (3.5) unless we adjoin to it two appropriate boundary conditions at  $x = \pm \ell$ . In the case of (3.3), these conditions are  $t_{yy} = 0$  for  $x = \pm \infty$ . In the case of (3.5), the region  $|x| \geq \ell$  is forbidden for the calculation of  $t_{yy}$  in the crack region  $|x| < \ell$ . Similarly, for the evaluation of the stress,  $t_{yy}$  outside the crack, the region  $|x| < \ell$  is forbidden.

In Section 3 of [3], Atkinson employs an integral equation (his Eq. (3.5)) to demonstrate that the solution of the problem (using still another kernel) may not exist. Under the correct boundary conditions (4.2), (his

Eq. (3.5)) will have to be modified to eliminate the effect of strains in  $|x| \geq \ell$  on the stress in  $|x| < \ell$ . On the basis of the correct integral equation that replaces his Eq. (3.5), it can be seen that all of his arguments regarding inexistence are no longer valid.

For the stress boundary condition (4.5), in fact the boundary conditions are satisfied exactly.  $v(x,y)$  is identical to the classical solution, the hoop stress  $t_{yy}$  for  $x > \ell$  is given by (4.13) which has no singularity.

We have therefore found an exact solution of the problem. This solution also uniformly approximates the case of constant loading for large  $\beta\ell$ , to a very high degree of accuracy in  $|x| < \ell$ . For example, for steel  $a = 2.48 \times 10^{-8}$  cm. Even for a microcrack of length  $2\ell = 6 \times 10^{-4}$  cm. we have  $\beta\ell = 10^4$  so that the error in  $t_{yy}$  is

$$(5.3) \quad P_c = \frac{1}{2} \exp[-10^4(1 - x/\ell)]$$

If we accept an error  $P_c < 10^{-3}$ ,  $t_{yy}$  will be nearly constant all along the crack up to the point

$$(5.4) \quad x/\ell = 1 - 3 \times 10^{-4} \ln(20)$$

for kernels having finite support such as (3.6). The error in the satisfaction of the boundary condition on stress begins only in intervals within one atomic distances from the crack tips.

## 6. CONCLUSION

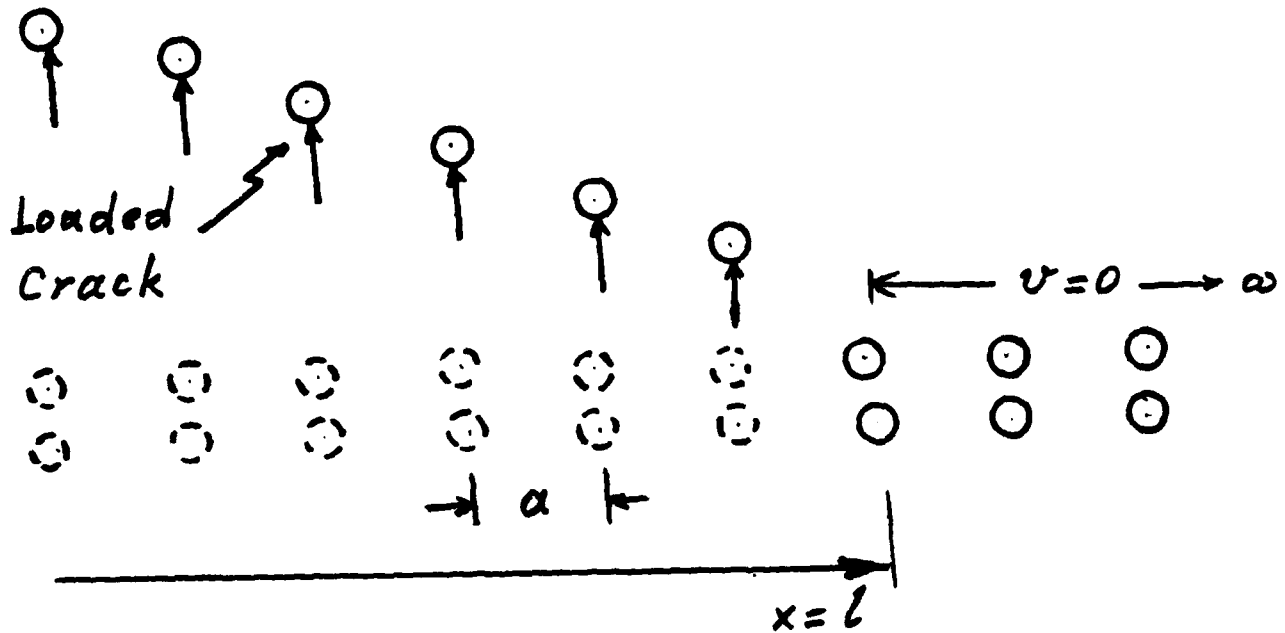
The non-uniformity in the satisfaction of the stress boundary conditions and possible failure of existence of solution noted by Atkinson [3] is due to his use of the infinite range for the integral for analyzing the stress near crack tips. In nonlocal elasticity, the stress at a point is influenced by strains at all other points within the support of the kernel. If the kernel has no support, this brings the influence of strains at points outside the crack to the stress specified within the crack. If the support of the kernel is finite, then only the set of points within the intersection of the support and the complement of the set of points in the crack region influences the stress. This is not compatible with the specification of the displacement field in this common region which is outside the crack. To restore the compatibility to the mixed boundary conditions, we must exclude the influence of strains on the stress in the common regions, to the support of the kernel and the outside crack. This can be done either by limiting the range of integration to the crack length or by using an inhomogeneous kernel (in regions near the crack tips) whose support is confined to the crack line, varying with distance from the crack tip. This latter approach, while correct mathematically, is cumbersome and presents major analytical difficulties. Nevertheless, it indicates that a solid with surfaces and cuts is an inhomogeneous body and the use of homogeneous kernels represents an approximation.

## ACKNOWLEDGEMENT

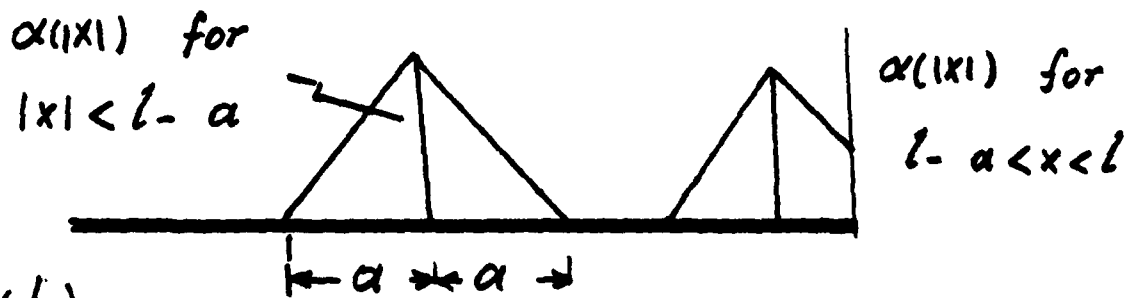
Dr. Atkinson was kind enough to send me his manuscript before its publication. Unfortunately, I could not devote sufficient time to his work to supply him with a detailed answer earlier. I acknowledge, with thanks, various discussions I have had with my students, Nasit Ari, A. Suresh and V. Chen and with Professors Speziale and Srivastav, on this and other related questions.

Post Script

After this work was completed, it was brought to my attention that Prof. Atkinson published another paper on the same questions related to the plane shear, plane stress and antiplane shear problems (Arch. Mech., 32, 4, pp. 597-614, Warszawa, 1980). In this paper, he uses the same type of wrong boundary conditions discussed above. Hence the present conclusions are valid for this paper as well.



(a)



(b)

Figure 1: a) Perfect lattice with loaded crack.  
 b) Influence kernel  $\alpha(|x|)$  for nonlocal continuum.

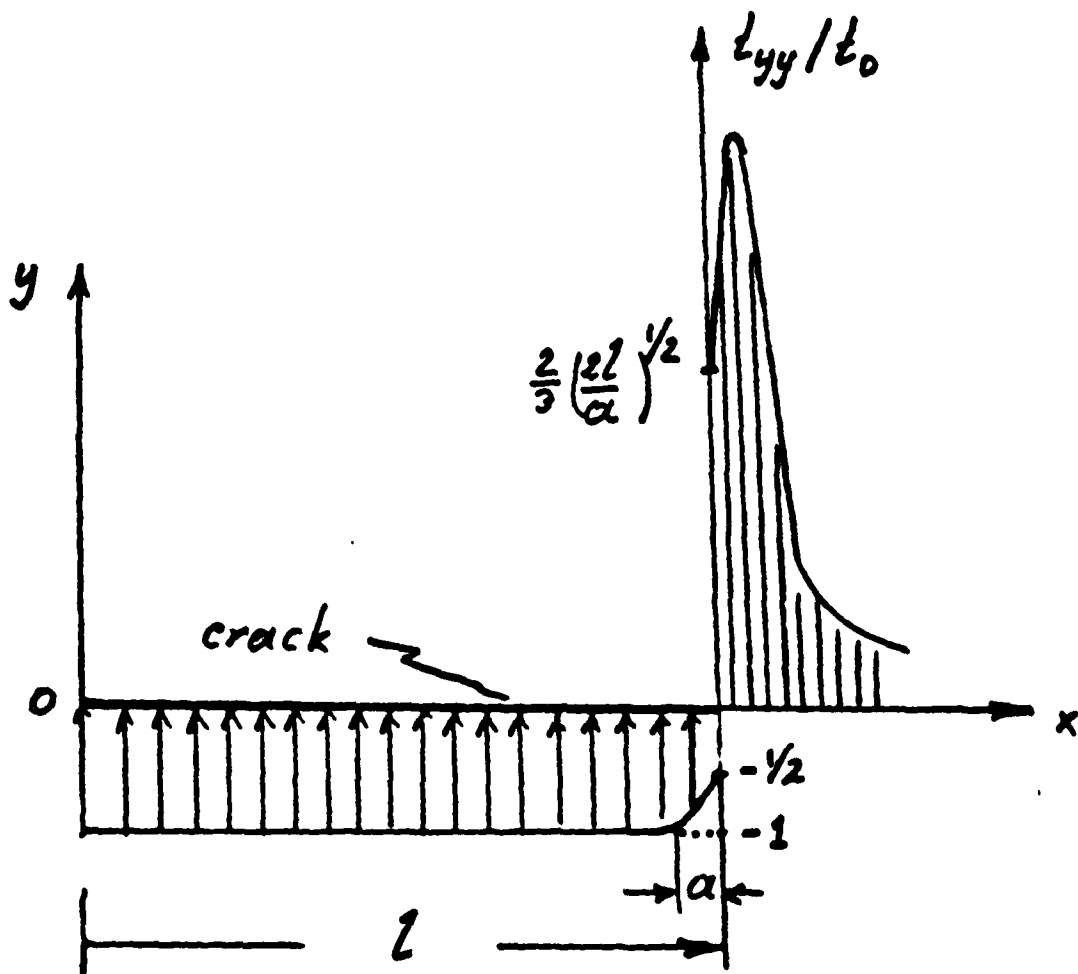


FIGURE 2

Stress Distribution along crack line.

$l$  = half crack length

$a$  = Atomic distance.

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Naval Ocean Systems Center  
San Diego, California 92152

Supervisor of Shipbuilding  
U.S. Navy  
Newport News, Virginia 23607

Navy Underwater Sound  
Reference Division  
Naval Research Laboratory  
P.O. Box 8337  
Orlando, Florida 32806

Chief of Naval Operations  
Department of the Navy  
Washington, D.C. 20350  
Attn: Code OP-098

Navy (Con't.)

Strategic Systems Project Office  
Department of the Navy  
Washington, D.C. 20376  
Attn: NSP-200

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Department of the Navy  
Washington, D.C. 20361  
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Naval Air Development Center  
Warminster, Pennsylvania 18974  
Attn: Aerospace Mechanics  
Code 606

U.S. Naval Academy  
Engineering Department  
Annapolis, Maryland 21402

Naval Facilities Engineering Command  
200 Stovall Street  
Alexandria, Virginia 22332  
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Department of the Navy  
Washington, D.C. 20362  
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David W. Taylor Naval Ship  
Research and Development Center  
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Attn: Code 042

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Naval Underwater Systems Center  
Newport, Rhode Island 02840  
Attn: Bruce Sandman, Code 3634

Naval Surface Weapons Center  
Dahlgren Laboratory  
Dahlgren, Virginia 22448  
Attn: Code G04  
G20

Technical Director  
Mare Island Naval Shipyard  
Vallejo, California 94592

U.S. Naval Postgraduate School  
Library  
Code 0384  
Monterey, California 93940

Webb Institute of Naval Architecture  
Attn: Librarian  
Crescent Beach Road, Glen Cove  
Long Island, New York 11542

Army

Commanding Officer (2)  
U.S. Army Research Office  
P.O. Box 12211  
Research Triangle Park, NC 27709  
Attn: Mr. J. J. Murray, CRD-AA-IP

Army (Con't.)

Watervliet Arsenal  
MAGGS Research Center  
Watervliet, New York 12189  
Attn: Director of Research

U.S. Army Materials and Mechanics  
Research Center  
Watertown, Massachusetts 02172  
Attn: Dr. R. Shea, DRXMR-T

U.S. Army Missile Research and  
Development Center  
Redstone Scientific Information  
Center  
Chief, Document Section  
Redstone Arsenal, Alabama 35809

Army Research and Development  
Center  
Fort Belvoir, Virginia 22060

NASA

National Aeronautics and Space  
Administration  
Structures Research Division  
Langley Research Center  
Langley Station  
Hampton, Virginia 23365

National Aeronautics and Space  
Administration  
Associate Administrator for Advanced  
Research and Technology  
Washington, D.C. 20546

Air Force

Wright-Patterson Air Force Base  
Dayton, Ohio 45433  
Attn: AFFDL (FB)  
(FBE)  
(FBE)  
(FBS)  
AFML (MEM)

Chief Applied Mechanics Group  
U.S. Air Force Institute of Technology  
Wright-Patterson Air Force Base  
Dayton, Ohio 45433

Air Force (Con't.)

Chief, Civil Engineering Branch  
WLRC, Research Division  
Air Force Weapons Laboratory  
Kirtland Air Force Base  
Albuquerque, New Mexico 87117

Air Force Office of Scientific Research  
Bolling Air Force Base  
Washington, D.C. 20332  
Attn: Mechanics Division

Department of the Air Force  
Air University Library  
Maxwell Air Force Base  
Montgomery, Alabama 36112

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Commandant  
Chief, Testing and Development Division  
U.S. Coast Guard  
1300 E Street, NW.  
Washington, D.C. 20226

Technical Director  
Marine Corps Development  
and Education Command  
Quantico, Virginia 22134

Director Defense Research  
and Engineering  
Technical Library  
Room 3C128  
The Pentagon  
Washington, D.C. 20301

Dr. M. Gaus  
National Science Foundation  
Environmental Research Division  
Washington, D.C. 20550

Library of Congress  
Science and Technology Division  
Washington, D.C. 20540

Director  
Defense Nuclear Agency  
Washington, D.C. 20305  
Attn: SPSS

Other Government Activities (Con't)

Mr. Jerome Perah  
Staff Specialist for Materials  
and Structures  
OUSDR&E, The Pentagon  
Room 3D1089  
Washington, D.C. 20301

Chief, Airframe and Equipment Branch  
FS-120  
Office of Flight Standards  
Federal Aviation Agency  
Washington, D.C. 20553

National Academy of Sciences  
National Research Council  
Ship Hull Research Committee  
2101 Constitution Avenue  
Washington, D.C. 20418  
Attn: Mr. A. R. Lytle

National Science Foundation  
Engineering Mechanics Section  
Division of Engineering  
Washington, D.C. 20550

Picatinny Arsenal  
Plastics Technical Evaluation Center  
Attn: Technical Information Section  
Dover, New Jersey 07801

Maritime Administration  
Office of Maritime Technology  
14th and Constitution Avenue, NW.  
Washington, D.C. 20230

PART 2 - Contractors and Other Technical  
Collaborators

Universities

Dr. J. Tinsley Oden  
University of Texas at Austin  
345 Engineering Science Building  
Austin, Texas 78712

Professor Julius Miklowitz  
California Institute of Technology  
Division of Engineering  
and Applied Sciences  
Pasadena, California 91109

Universities (Con't)

Dr. Harold Liebowitz, Dean  
School of Engineering and  
Applied Science  
George Washington University  
Washington, D.C. 20052

Professor Eli Sternberg  
California Institute of Technology  
Division of Engineering and  
Applied Sciences  
Pasadena, California 91109

Professor Paul M. Naghdi  
University of California  
Department of Mechanical Engineering  
Berkeley, California 94720

Professor A. J. Durelli  
Oakland University  
School of Engineering  
Rochester, Missouri 48063

Professor F. L. DiMaggio  
Columbia University  
Department of Civil Engineering  
New York, New York 10027

Professor Norman Jones  
The University of Liverpool  
Department of Mechanical Engineering  
P. O. Box 147  
Brownlow Hill  
Liverpool L69 3BX  
England

Professor E. J. Skudrzyk  
Pennsylvania State University  
Applied Research Laboratory  
Department of Physics  
State College, Pennsylvania 16801

Professor J. Klosner  
Polytechnic Institute of New York  
Department of Mechanical and  
Aerospace Engineering  
333 Jay Street  
Brooklyn, New York 11201

Professor R. A. Schapery  
Texas A&M University  
Department of Civil Engineering  
College Station, Texas 77843

Universities (Con't.)

Professor Walter D. Pilkey  
University of Virginia  
Research Laboratories for the  
Engineering Sciences and  
Applied Sciences  
Charlottesville, Virginia 22901

Professor K. D. Willmert  
Clarkson College of Technology  
Department of Mechanical Engineering  
Potsdam, New York 13676

Dr. Walter E. Haisler  
Texas A&M University  
Aerospace Engineering Department  
College Station, Texas 77843

Dr. Hussein A. Kamel  
University of Arizona  
Department of Aerospace and  
Mechanical Engineering  
Tucson, Arizona 85721

Dr. S. J. Fenves  
Carnegie-Mellon University  
Department of Civil Engineering  
Schenley Park  
Pittsburgh, Pennsylvania 15213

Dr. Ronald L. Huston  
Department of Engineering Analysis  
University of Cincinnati  
Cincinnati, Ohio 45221

Professor G. C. M. Sih  
Lehigh University  
Institute of Fracture and  
Solid Mechanics  
Bethlehem, Pennsylvania 18015

Professor Albert S. Kobayashi  
University of Washington  
Department of Mechanical Engineering  
Seattle, Washington 98105

Professor Daniel Frederick  
Virginia Polytechnic Institute and  
State University  
Department of Engineering Mechanics  
Blacksburg, Virginia 24061

Universities (Con't)

Professor A. C. Eringen  
Princeton University  
Department of Aerospace and  
Mechanical Sciences  
Princeton, New Jersey 08540

Professor E. H. Lee  
Stanford University  
Division of Engineering Mechanics  
Stanford, California 94305

Professor Albert I. King  
Wayne State University  
Biomechanics Research Center  
Detroit, Michigan 48202

Dr. V. R. Hodgson  
Wayne State University  
School of Medicine  
Detroit, Michigan 48202

Dean B. A. Boley  
Northwestern University  
Department of Civil Engineering  
Evanston, Illinois 60201

Professor P. G. Hodge, Jr.  
University of Minnesota  
Department of Aerospace Engineering  
and Mechanics  
Minneapolis, Minnesota 55455

Dr. D. C. Drucker  
University of Illinois  
Dean of Engineering  
Urbana, Illinois 61801

Professor N. M. Newmark  
University of Illinois  
Department of Civil Engineering  
Urbana, Illinois 61803

Professor E. Reissner  
University of California, San Diego  
Department of Applied Mechanics  
La Jolla, California 92037

Professor William A. Nash  
University of Massachusetts  
Department of Mechanics and  
Aerospace Engineering  
Amherst, Massachusetts 01002

Universities (Con't)

Professor G. Herrmann  
Stanford University  
Department of Applied Mechanics  
Stanford, California 94305

Professor J. D. Achenbach  
Northwest University  
Department of Civil Engineering  
Evanston, Illinois 60201

Professor S. B. Dong  
University of California  
Department of Mechanics  
Los Angeles, California 90024

Professor Burt Paul  
University of Pennsylvania  
Towne School of Civil and  
Mechanical Engineering  
Philadelphia, Pennsylvania 19104

Professor H. W. Liu  
Syracuse University  
Department of Chemical Engineering  
and Metallurgy  
Syracuse, New York 13210

Professor S. Bodner  
Technion R&D Foundation  
Haifa, Israel

Professor Werner Goldsmith  
University of California  
Department of Mechanical Engineering  
Berkeley, California 94720

Professor R. S. Rivlin  
Lehigh University  
Center for the Application  
of Mathematics  
Bethlehem, Pennsylvania 18015

Professor F. A. Cozzarelli  
State University of New York at  
Buffalo  
Division of Interdisciplinary Studies  
Karr Parker Engineering Building  
Chemistry Road  
Buffalo, New York 14214

Universities (Con't)

Professor Joseph L. Rose  
Drexel University  
Department of Mechanical Engineering  
and Mechanics  
Philadelphia, Pennsylvania 19104

Professor B. K. Donaldson  
University of Maryland  
Aerospace Engineering Department  
College Park, Maryland 20742

Professor Joseph A. Clark  
Catholic University of America  
Department of Mechanical Engineering  
Washington, D.C. 20064

Dr. Samuel B. Batdorf  
University of California  
School of Engineering  
and Applied Science  
Los Angeles, California 90024

Professor Isaac Fried  
Boston University  
Department of Mathematics  
Boston, Massachusetts 02215

Professor E. Krempl  
Rensselaer Polytechnic Institute  
Division of Engineering  
Engineering Mechanics  
Troy, New York 12181

Dr. Jack R. Vinson  
University of Delaware  
Department of Mechanical and Aerospace  
Engineering and the Center for  
Composite Materials  
Newark, Delaware 19711

Dr. J. Duffy  
Brown University  
Division of Engineering  
Providence, Rhode Island 02912

Dr. J. L. Swedlow  
Carnegie-Mellon University  
Department of Mechanical Engineering  
Pittsburgh, Pennsylvania 15213

Universities (Con't)

Dr. V. K. Varadan  
Ohio State University Research Foundation  
Department of Engineering Mechanics  
Columbus, Ohio 43210

Dr. Z. Hashin  
University of Pennsylvania  
Department of Metallurgy and  
Materials Science  
College of Engineering and  
Applied Science  
Philadelphia, Pennsylvania 19104

Dr. Jackson C. S. Yang  
University of Maryland  
Department of Mechanical Engineering  
College Park, Maryland 20742

Professor T. Y. Chang  
University of Akron  
Department of Civil Engineering  
Akron, Ohio 44325

Professor Charles W. Bert  
University of Oklahoma  
School of Aerospace, Mechanical,  
and Nuclear Engineering  
Norman, Oklahoma 73019

Professor Satya N. Atluri  
Georgia Institute of Technology  
School of Engineering and  
Mechanics  
Atlanta, Georgia 30332

Professor Graham F. Carey  
University of Texas at Austin  
Department of Aerospace Engineering  
and Engineering Mechanics  
Austin, Texas 78712

Dr. S. S. Wang  
University of Illinois  
Department of Theoretical and  
Applied Mechanics  
Urbana, Illinois 61801

Professor J. F. Abel  
Cornell University  
Department of Theoretical  
and Applied Mechanics  
Ithaca, New York 14853

Universities (Con't)

Professor V. H. Neubert  
Pennsylvania State University  
Department of Engineering Science  
and Mechanics  
University Park, Pennsylvania 16802

Professor A. W. Leissa  
Ohio State University  
Department of Engineering Mechanics  
Columbus, Ohio 43212

Professor C. A. Brebbia  
University of California, Irvine  
Department of Civil Engineering  
School of Engineering  
Irvine, California 92717

Dr. George T. Hahn  
Vanderbilt University  
Mechanical Engineering and  
Materials Science  
Nashville, Tennessee 37235

Dean Richard H. Gallagher  
University of Arizona  
College of Engineering  
Tucson, Arizona 85721

Professor E. F. Rybicki  
The University of Tulsa  
Department of Mechanical Engineering  
Tulsa, Oklahoma 74104

Dr. R. Haftka  
Illinois Institute of Technology  
Department of Mechanics and Mechanical  
and Aerospace Engineering  
Chicago, Illinois 60616

Professor J. G. de Oliveira  
Massachusetts Institute of Technology  
Department of Ocean Engineering  
77 Massachusetts Avenue  
Cambridge, Massachusetts 02139

Dr. Bernard W. Shaffer  
Polytechnic Institute of New York  
Route 110  
Farmingdale, New York 11735

Industry and Research Institutes

Dr. Norman Hobbs  
Kaman Avidyne  
Division of Kaman  
Sciences Corporation  
Burlington, Massachusetts 01803

Argonne National Laboratory  
Library Services Department  
9700 South Cass Avenue  
Argonne, Illinois 60440

Dr. M. C. Junger  
Cambridge Acoustical Associates  
54 Rindge Avenue Extension  
Cambridge, Massachusetts 02140

Mr. J. H. Torrance  
General Dynamics Corporation  
Electric Boat Division  
Groton, Connecticut 06340

Dr. J. E. Greenspon  
J. G. Engineering Research Associates  
3831 Menlo Drive  
Baltimore, Maryland 21215

Newport News Shipbuilding and  
Dry Dock Company  
Library  
Newport News, Virginia 23607

Dr. W. F. Bozich  
McDonnell Douglas Corporation  
5301 Bolsa Avenue  
Huntington Beach, California 92647

Dr. H. N. Abramson  
Southwest Research Institute  
8500 Culebra Road  
San Antonio, Texas 78284

Dr. R. C. DeHart  
Southwest Research Institute  
8500 Culebra Road  
San Antonio, Texas 78284

Dr. M. L. Baron  
Weidlinger Associates  
110 East 59th Street  
New York, New York 10022

Industry and Research Institutes (Con't)

Dr. T. L. Geers  
Lockheed Missiles and Space Company  
3251 Hanover Street  
Palo Alto, California 94304

Mr. William Caywood  
Applied Physics Laboratory  
Johns Hopkins Road  
Laurel, Maryland 20810

Dr. Robert E. Dunham  
Pacifica Technology  
P.O. Box 148  
Del Mar, California 92014

Dr. M. F. Kanninen  
Battelle Columbus Laboratories  
505 King Avenue  
Columbus, Ohio 43201

Dr. A. A. Hochrein  
Daedalean Associates, Inc.  
Springlake Research Road  
15110 Frederick Road  
Woodbine, Maryland 21797

Dr. James W. Jones  
Swanson Service Corporation  
P.O. Box 5415  
Huntington Beach, California 92646

Dr. Robert E. Nickell  
Applied Science and Technology  
3344 North Torrey Pines Court  
Suite 220  
La Jolla, California 92037

Dr. Kevin Thomas  
Westinghouse Electric Corp.  
Advanced Reactors Division  
P. O. Box 158  
Madison, Pennsylvania 15663

Dr. H. D. Ribbitt  
Ribbitt & Karlsson, Inc.  
132 George M. Cohan Boulevard  
Providence, Rhode Island 02903

Dr. R. D. Mindlin  
89 Deer Hill Drive  
Ridgefield, Connecticut 06877

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Industry and Research Institutes (Con't)

Dr. Richard E. Dame  
Mega Engineering  
11961 Tech Road  
Silver Spring, Maryland 20904

Mr. G. M. Stanley  
Lockheed Palo Alto Research  
Laboratory  
3251 Hanover Street  
Palo Alto, California 94304

Mr. R. L. Cloud  
Robert L. Cloud Associates, Inc.  
2972 Adeline Street  
Berkeley, California 94703

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