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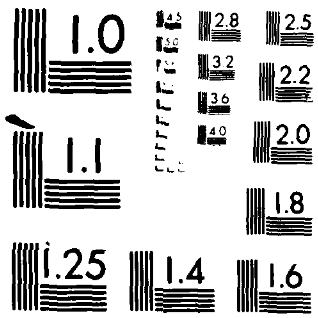

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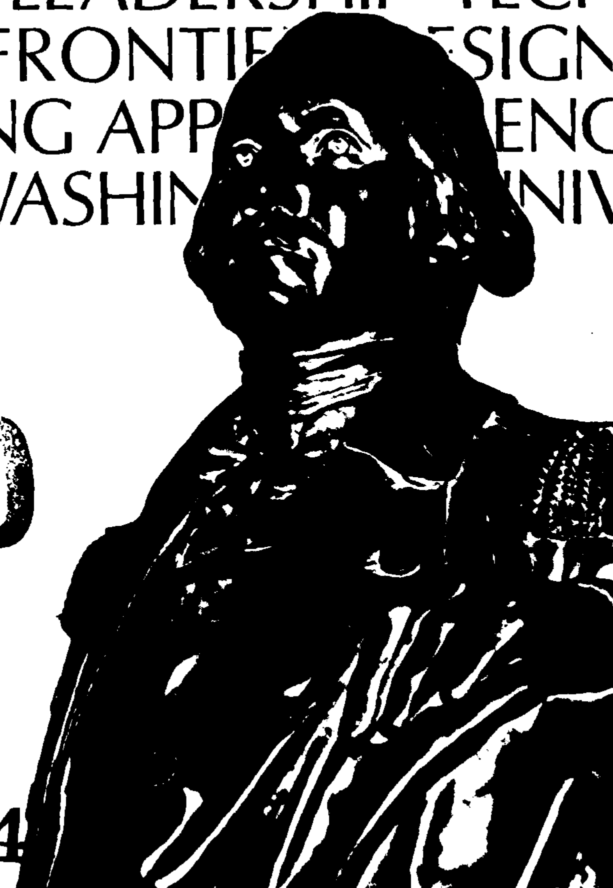

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SCHOOL OF ENGINEERING
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NONPARAMETRIC ESTIMATION AND GOODNESS OF FIT TESTING OF HYPOTHESES FOR DISTRIBUTIONS IN ACCELERATED LIFE TESTING

by

Moshe Shaked*
Nozer D. Singpurwalla†

Serial T-425
~~25 July 1980~~

The George Washington University
School of Engineering and Applied Science
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20. Abstract (continued)

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Moshe Shaked*
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1. INTRODUCTION

It is a common practice to subject long-life items to larger than usual stresses so that failure data can be obtained in a short amount of test time. Such tests are called accelerated life tests, and the goal is to infer the mean time to failure, or more generally, the life distribution of the items at the usual stress level using failure data from accelerated tests.

A parametric approach to this problem [3,5,10] involves making two assumptions: first, the life distribution at each stress level is assumed to come from the same parametric family which is *specified*; second, a relationship called the "time transformation function" is assumed among the parameters of the various distributions. As an example of the former, it is typical to assume that the life distributions at the various stress levels are either exponential, Weibull, lognormal, or an extreme value. The time transformation functions which are commonly assumed are the inverse power law, the Arrhenius law, the Eyring law, or generalizations of these. Whereas the above assumptions may be reasonable in many situations, the possibility does remain that under certain circumstances an analyst may be hesitant or unwilling to entertain any one, or both, of these assumptions.

In this paper, we drop the requirement that the common parametric family of life distributions under all stresses be *specified*, but retain the requirement that the time transformation function be specified. The time transformation function that we consider is a version of the inverse power law. By choosing this form, we do not sacrifice any lack of generality, since the procedures mentioned here can easily be extended to consider the other time transformation functions as well. The inverse power law being commonly used is considered here for purposes of illustration. We indicate how the life distribution and the mean time to failure at use conditions stress can be consistently estimated and how hypotheses about the common parametric family of life distributions can be tested. Thus we are able to test the validity of the distributional assumption traditionally made in accelerated life testing, if the form of the time transformation function can be specified. In another paper [6], both the assumptions mentioned before have been dropped and the problem has been treated completely nonparametrically. However, the statistical precision of the results obtained there is less than that which could be achieved by making one or both of the aforementioned assumptions.

We should remark that the dual problems of estimation and testing for the goodness of fit at use conditions stress have already been considered in two separate papers by Shaked, Zimmer, and Ball [9], and by Sethuraman and Singpurwalla [8]. These papers emphasize technical details pertaining to a formal justification of the methodology, the development of the appropriate formulae, and the performance of the procedures. Thus, a practitioner interested in using the methodologies may find it difficult to extract that information which is pertinent to his goals. Furthermore, the results of the above two papers complement each other and can therefore be combined to constitute a useful package for a nonparametric approach to inference from accelerated life tests.

Our goal in writing this paper is to combine the techniques of [8] and [9] in a manner than is comprehensive and user-oriented so that this nonparametric approach to accelerated life testing is accessible to the reliability analyst. We demonstrate the utility of this approach by applying it to some data from a realistic situation.

2. A MODEL FOR ACCELERATED LIFE TESTING

Let $V_1, V_2, \ldots, V_k$ and V_0 denote the accelerated and the use conditions stresses, respectively, and let $F_1, \ldots, F_k$ and F_0 denote the corresponding cumulative distribution functions. We shall assume that $F_0, \ldots, F_k$ belong to a common but unknown family of distributions, and that for some distribution F, which also belongs to this family,

$$F_i(t) = F(AV_i^{\alpha} t) , \quad i=0,\ldots,k , \tag{2.1}$$

where $A > 0$ and $\alpha > 0$ are unknown constants.

This assumption means that a change of stress does not change the shape of the life distribution, but only changes its scale. This model is a generalization of the familiar power law model which is traditionally applied to the scale parameter of the failure distribution [4, 5, 11].

Our goal is to estimate F_0, the life distribution under use conditions stress, and to perform some goodness of fit tests for hypotheses about the general family of distributions to which F, and $F_0, \ldots, F_k$, belong.

3. ESTIMATION OF F_0

Typically, data from an accelerated life test consists of the set of observations $T_{i\ell}$, $\ell=1,\ldots,n_i$, $i=1,\ldots,k$, where $T_{i\ell}$ is the time to failure of the ℓth item in a sample of size n_i that is run under the constant application of stress V_i , $i=1,\ldots,k$. We assume that $V_1,\ldots,V_k$, k , and $n_1,\ldots,n_k$ are all fixed in advance. If data at the nonaccelerated stress level V_0 are also available, then the procedure to be described can still be used, but by augmenting a 0 to the range of indices i and j below.

3.1 Background

Denote the scale factor between F_i and F_j by θ_{ij} , where

$$\theta_{ij} = AV_j^{\alpha}/AV_i^{\alpha} = (V_j/V_i)^{\alpha} , \quad i \neq j . \tag{3.1}$$

The first step in the procedure for estimating F_0 is the estimation of α . A method for estimating α is suggested by (3.1), since

$$\alpha = \log(\theta_{ij})/(\log V_j - \log V_i) .$$

Let $\bar{T}_i = n_i^{-1} \sum_{\ell=1}^{n_i} T_{i\ell}$, $i=1,\ldots,k$, be the sample means of the $T_{i\ell}$'s , $\ell=1,\ldots,n_i$. Then, an estimator of θ_{ij} is

$$\hat{\theta}_{ij} = \bar{T}_i/\bar{T}_j , \quad i \neq j ,$$

from which we can obtain an estimator of α as

$$\hat{\alpha}_{ij} = \log(\hat{\theta}_{ij})/(\log V_j - \log V_i) , \quad i \neq j .$$

The $\frac{1}{2}k(k-1)$ estimators of α can be used to form a weighted average of the $\hat{\alpha}_{ij}$'s , which gives us the final estimate of α , as

$$\hat{\alpha} = \frac{\sum_{i=1}^{k} \sum_{j=i+1}^{k} \left(\log(V_j/V_i)\right)\left(\log(\bar{T}_i/\bar{T}_j)\right)}{\sum_{i=1}^{k} \sum_{j=i+1}^{k} \left(\log(V_j/V_i)\right)^2} . \tag{3.2}$$

If α were known, we could, in the light of relationship (2.1), transform (rescale) the times to failure from any stress level to correspond to the times to failure from any other stress level. Thus, observing $T_{i\ell}$, $\ell=1,\ldots,n_i$ under stress V_i is, under our model, equivalent to observing $(V_i/V_j)^{\alpha}T_{i\ell}$, $\ell=1,\ldots,n_i$ under stress V_j. Since α is not known, we shall use its estimate $\hat{\alpha}$ to rescale our variables. This is the key notion which enables us to obtain an estimate of F_0.

3.2 Estimation of F_0 and the Mean Time to Failure under V_0

Using $\hat{\alpha}$ given by (3.2), we define the $N = \sum_{i=1}^{k} n_i$ rescaled variables

$$\tilde{T}_{i\ell} = \left(\frac{V_i}{V_0}\right)^{\hat{\alpha}} T_{i\ell} , \quad \ell=1,\ldots,n_i , \quad i=1,\ldots,k , \tag{3.3}$$

and estimate F_0 by the empirical distribution function of the $\tilde{T}_{i\ell}$'s. Thus, $\hat{F}_0$, an estimator of the failure distribution at use conditions stress V_0, is given by

$$\hat{F}_0(t) = \frac{1}{N}\{\text{number of observations among the } \tilde{T}_{i\ell}\text{'s} \leq t\} ,$$
$$\text{for} \quad 0 \leq t < \infty . \tag{3.4}$$

An estimator of the mean time to failure at use conditions stress, say μ, is given by

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{k}\sum_{\ell=1}^{n_i}\tilde{T}_{i\ell} = \sum_{i=1}^{k}\frac{n_i}{N}\left(\frac{V_i}{V_0}\right)^{\hat{\alpha}}\bar{T}_i . \tag{3.5}$$

The estimators $\hat{F}_0$ and $\hat{\mu}$ are s-consistent [9]. A method for obtaining the approximate s-uniform confidence limits for F_0 using $\hat{F}_0$ is discussed in Section 5.

4. GOODNESS OF FIT TESTING OF HYPOTHESES FOR F_0

Having estimated F_0 by $\hat{F}_0$ given by (3.4), our next goal is to see if F_0 is a member of a specified family of distribution functions. This type of information may be of interest in its own right, or it can be used to obtain s-uniform confidence bounds for F_0. Testing to see if F_0 is a member of a specified family is equivalent to testing whether $F_1,\ldots,F_k$ are also members of the same family. For reasons given in [8], it is convenient for us to consider the random variables $X_{i\ell} = \log T_{i\ell}$, $\ell=1,\ldots,n_i$, $i=1,\ldots,k$, and test for the underlying distribution of the $X_{i\ell}$'s, say H_i, $i=1,\ldots,k$. For example, if we wish to test the hypothesis that the F_i's are members of the family of Weibull distributions, then this is equivalent to testing whether the H_i's are members of the family of extreme value distributions [3]. Similarly, if we wish to test the hypothesis that the F_i's are members of the lognormal family of distributions, then this is equivalent to testing whether the H_i's are normal [3].

There are two limitations to our testing procedure. The first one is that we can only work with complete (uncensored) samples, and the second one is that the H_i's can only be tested for being members of the s-location-scale family of distributions. The commonly used members of the s-location-scale family are the normal, the exponential, and the extreme-value distributions. Since H_i is an exponential distribution whenever F_i is a Pareto distribution, and since a Pareto distribution is rarely used to describe life lengths, the desire for testing whether H_i is an exponential is not very common.

The procedure for testing depends upon the particular family of distributions that is being considered; this will be made clear in the following section. Furthermore, the procedure is valid when each of the n_i's is large.

4.1 Testing Whether F_0 is a Weibull

Suppose that we wish to test the hypothesis that the common underlying family of distributions is a Weibull distribution. This is equivalent to testing whether the H_i's are extreme value distributions

$$H_i(x) = 1 - \exp\left\{-\exp\left(\frac{x-\alpha_i}{\beta_i}\right)\right\}, \quad i=1,\ldots,k ,$$

where α_i and β_i are the s-location and the s-scale parameters, respectively. The test procedure entails the following steps.

(1) Using $X_{i\ell}$, $\ell=1,\ldots,n_i$, we first obtain the s-maximum likelihood estimates of α_i and β_i , say $\hat{\alpha}_i$ and $\hat{\beta}_i$, respectively, using the above specified H_i . Methods for obtaining these estimates are given in [3]; they can also be obtained using the computer program discussed in Section 4.3.

(2) For $0 < t \leq 1$, we compute

$$W_i(t) = \frac{1}{n_i}\sum_{\ell=1}^{n_i} I\left(H_i\left(\frac{X_{i\ell}-\hat{\alpha}_i}{\hat{\beta}_i}\right) \leq t\right),$$

where

$$H_i\left(\frac{X_{i\ell}-\hat{\alpha}_i}{\hat{\beta}_i}\right) = 1 - \exp\left\{-\exp\left(\frac{X_{i\ell}-\hat{\alpha}_i}{\hat{\beta}_i}\right)\right\},$$

and $I\{E\}$ denotes the indicator of the event E . Verify that $W_i(t)$ is merely the empirical distribution function of the $H_i\left((X_{i\ell}-\hat{\alpha}_i)/\hat{\beta}_i\right)$, $\ell=1,\ldots,n_i$.

(3) We then compute, for $0 \leq t \leq 1$,

$$\tilde{V}_i(t) = \sqrt{n_i}\,\{W_i(t)-t\},$$

and repeat the Steps (1), (2), and (3) for all values of i , $i=1,\ldots,k$.

(4) Our test statistic D is the s-Kolmogorov-Smirnov statistic obtained by pooling the $\tilde{V}_i(t)$'s in a manner shown below. Let

$$A = n_i \Big/ \sum_{i=1}^{k} n_i \text{ , and let}$$

$$Z(t) = \sum_{i=1}^{k} A_i \tilde{V}_i(t) \text{ , } 0 \leq t \leq 1 \text{ .}$$

Then $D = \max(D^+, -D^-)$, where

$$D^+ = \max_{0 \leq t < 1} \left(\sum_{i=1}^{k} A_i^2 \right)^{-\frac{1}{2}} Z(t) \text{ , and}$$

$$D^- = \min_{0 \leq t < 1} \left(\sum_{i=1}^{k} A_i^2 \right)^{-\frac{1}{2}} Z(t) \text{ .}$$

(5) We will accept the hypothesis that the common underlying family of distributions is a Weibull (i.e., the H_i's are extreme value distributions) at a desired level of significance, say α , if the value D is less than the critical values given below. These values have been abstracted from [2].

Level of Significance, α	.10	.05	.025	.01
Critical Values of D, D*	.800	.870	.940	1.000

4.2 Testing Whether F_0 is Lognormal

Testing to see if the common underlying family of distributions is a lognormal is equivalent to testing whether the H_i's are normal distributions,

$$H_i(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\beta_i} \exp\left(-\frac{1}{2}\left(\frac{u-\alpha_i}{\beta_i}\right)^2\right) du ,$$

$i=1,\ldots,k$, where the α_i and β_i are the s-location (mean) and s-scale (standard deviation) parameters, respectively. The test procedure entails the following steps.

(1) Using $X_{i\ell}$, $\ell=1,\ldots,n_i$, we obtain

$$\hat{\alpha}_i = \sum_{\ell=1}^{n_i} X_{i\ell} / n_i \quad \text{and} \quad \hat{\beta}_i^2 = \sum_{\ell=1}^{n_i} (X_{i\ell}-\hat{\alpha}_i)^2 / (n_i-1) ,$$

the s-maximum likelihood estimators of α_i and β_i , respectively.

(2) We repeat steps (2), (3), and (4) of Section 4.1, except that now

$$H_i\left(\frac{X_{i\ell}-\hat{\alpha}_i}{\hat{\beta}_i}\right) = \int_{-\infty}^{X_{i\ell}} \frac{1}{\sqrt{2\pi}\hat{\beta}_i} \exp\left(-\frac{1}{2}\left(\frac{u-\hat{\alpha}_i}{\hat{\beta}_i}\right)^2\right) du ,$$

$\ell=1,2,\ldots,n_i$, and to test for the significance of D , we use the following critical values abstracted from [7].

Level of Significance, α	.10	.05	.025	.01
Critical Values of D, D*	.752	.835	.910	.991

4.3 Some Comments on the Test Procedure

The power of the above tests against various alternatives is discussed in [8]. The main point here it that it is difficult to distinguish between the Weibull and the lognormal alternatives, so that the final choice of the family of distributions to which F_0 belongs must be based on criteria other than a formal goodness of fit test. For example, if the hazard rate is known to be increasing in time, then the distribution cannot be lognormal and the Weibull assumption should be chosen.

To aid in the computation of D, and also the s-maximum likelihood estimates $\hat{\alpha}_i$ and $\hat{\beta}_i$ for the various hypotheses that we wish to entertain, we have written a computer program, described in [1]. This program can be made available to the interested reader.

5. UNIFORM CONFIDENCE BOUNDS FOR F_0

The approximate *s-uniform* $(1-\alpha)\%$ *confidence* bounds for F_0 can be obtained once a test of hypothesis results in the acceptance of the hypothesis. The approximate confidence bounds will be based on $\hat{F}_0$ given by (3.4) and the critical values D^* given in Section 4.1 and 4.2. (The confidence bounds are approximate because of the finiteness of the sample sizes, and because they are based on $\hat{F}_0$ and not on a complicated transformation of $\{Z(t), t \geq 0\}$ of Section 4.) These bounds can also be used to obtain an approximate $(1-\alpha)\%$ s-confidence bound on $\tilde{\mu}$, the median of the time to failure distribution under V_0.

Suppose that the procedure of Section 4.1 results in our accepting the hypothesis that the H_i are extreme value distributions. This means that F_0 is a Weibull distribution, so that the approximate uniform $(1-\alpha)\%$ confidence limits for F_0 are given by

$$\hat{F}_0(t) \pm D^*/\sqrt{N} ,$$

where D^* is the critical value of D corresponding to α , and $N = \sum_{i=1}^{k} n_i$. For example, if we choose $\alpha = .05$, then the approximate 95% uniform upper and lower confidence limits for F_0 are

$$g(t) = \hat{F}_0(t) + .870/\sqrt{N} ,$$

and

$$h(t) = \hat{F}_0(t) - .870/\sqrt{N} ,$$

respectively. In computing the above, we should make sure to confine both $g(t)$ and $h(t)$ to be between 0 and 1.

In a similar manner, if the procedure of Section 4.2 results in our acceptance of the hypothesis that F_0 is a lognormal distribution, then the 95% uniform confidence bounds for $F_0(t)$ will be $\hat{F}_0(t) \pm .835/\sqrt{N}$.

The uniform confidence bounds $g(t)$ and $h(t)$ can be used to obtain an approximate s-confidence bound for $\tilde{\mu}$ by taking their inverses. Thus, the approximate s-confidence bounds for $\tilde{\mu}$ are $g^{-1}(.5)$ and $h^{-1}(.5)$, where $g^{-1}(.5) < h^{-1}(.5)$.

6. EXAMPLE

We apply the methodology of this paper to some accelerated life test data given in [4] and [5]. These data represent the times to breakdown of an insulating fluid subjected to seven elevated voltage levels. Since the methodology of Section 4 is valid for large sample sizes, and since three of the stress levels of [5] contain too few failures, we delete them from our consideration, and consider an abstracted version of the data. These data are given in Table 1, and have also been considered in [6]. The use conditions stress corresponds to $V_0 = 20$ kilovolts.

TABLE 1

TIMES TO BREAKDOWN OF AN INSULATING FLUID (IN MINUTES) UNDER VARIOUS VALUES OF THE STRESS

36 kV	34 kV	32 kV	30 kV
.35	.19	.27	7.74
.59	.78	.40	17.05
.96	.96	.69	20.46
.99	1.31	.79	21.02
1.69	2.78	2.75	22.66
1.97	3.16	3.91	43.40
2.07	4.15	9.88	47.30
2.58	4.67	13.95	139.07
2.71	4.85	15.93	144.12
2.90	6.50	27.80	175.88
3.67	7.35	53.24	194.90
3.99	8.01	82.85	$N_4 = 11$
5.35	8.27	89.29	
13.77	12.06	100.58	
25.50	31.75	215.10	
$N_1 = 15$	32.52	$N_3 = 15$	
	33.91		
	36.71		
	72.89		
	$N_2 = 19$		

6.1 Estimation of F_0

Using the failure times given in Table 1, we obtain via (3.2) an estimate of α , $\hat{\alpha} = 16.9844$; note that this estimate is different from the estimate obtained in [9], since there, all the data of [5] were used. We then rescale the data in Table 1 using (3.3), with $\hat{\alpha} = 16.9844$ and $V_0 = 20$, and use these to compute the empirical distribution function $\hat{F}_0$, given by (3.4). This is our estimate of F_0 , and is given in Figure 1. The median of these rescaled variables is easily verified as being 45572 minutes, and an estimate of μ, obtained from (3.5), is $\hat{\mu} = 105998$ minutes. Other s-percentiles of F_0 can also be estimated using $\hat{F}_0$.

6.2 Testing Hypotheses

A plot of the logarithms of the 60 rescaled observations mentioned in Section 6.1 on normal probability paper, Figure 2, suggests that the logarithms of the failure times could be well described by a normal distribution.

The plot of Figure 2 suggests that we test for normality by taking H_i, $i=1,2,3,4$, to be normal distributions. Our estimates $(\hat{\alpha}_i, \hat{\beta}_i)$ turn out to be (.902, 1.109), (1.786, 1.525), (2.228, 2.198), and (3.821, 1.111); D is computed as .4023. From the table of critical values given in Section 4.2 we see that for $\alpha = .05$, $D^* = .835$, and since $D < D^*$, we have no reason to reject the null hypothesis that the failure times of Table 1 could be described by a lognormal distribution.

The approximate 95% uniform confidence limits for F_0 are obtained as $\hat{F}_0(t) \pm .835/\sqrt{60}$, and these are shown by the dotted lines of Figure 1. The 95% confidence interval for $\tilde{\mu}$ discussed in Section 5 is obtained by reading the ordinates of the horizontal line at .5 in Figure 1; these turn out to be 32,320 and 64,000 minutes.

Since a Weibull distribution for times to failure has been assumed in [5], it is appropriate that we test for this distribution as well. We thus take H_i, $i=1,\ldots,4$ to be an extreme value distribution and compute $(\hat{\alpha}_i, \hat{\beta}_i)$ as (1.473, 1.154), (2.531, 1.353), (3.310, 1.898), and (4.373, .987); we obtain D as .88. From the table of

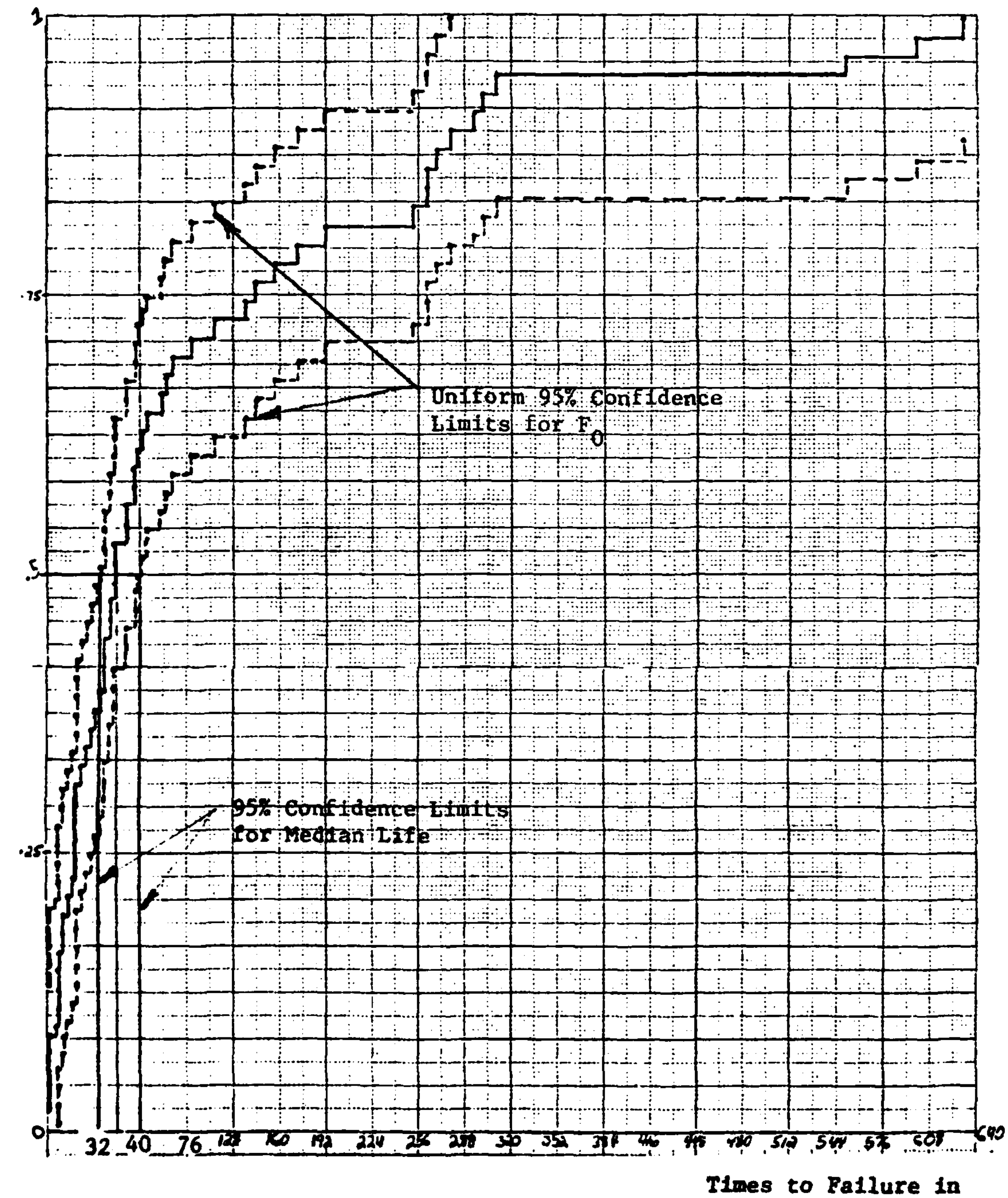

Figure 1. Empirical distribution function $\hat{F}_0$ of the 60 rescaled observations of Table 1, the approximate 95% uniform confidence limits for F_0, and a confidence interval for $\tilde{\mu}$.

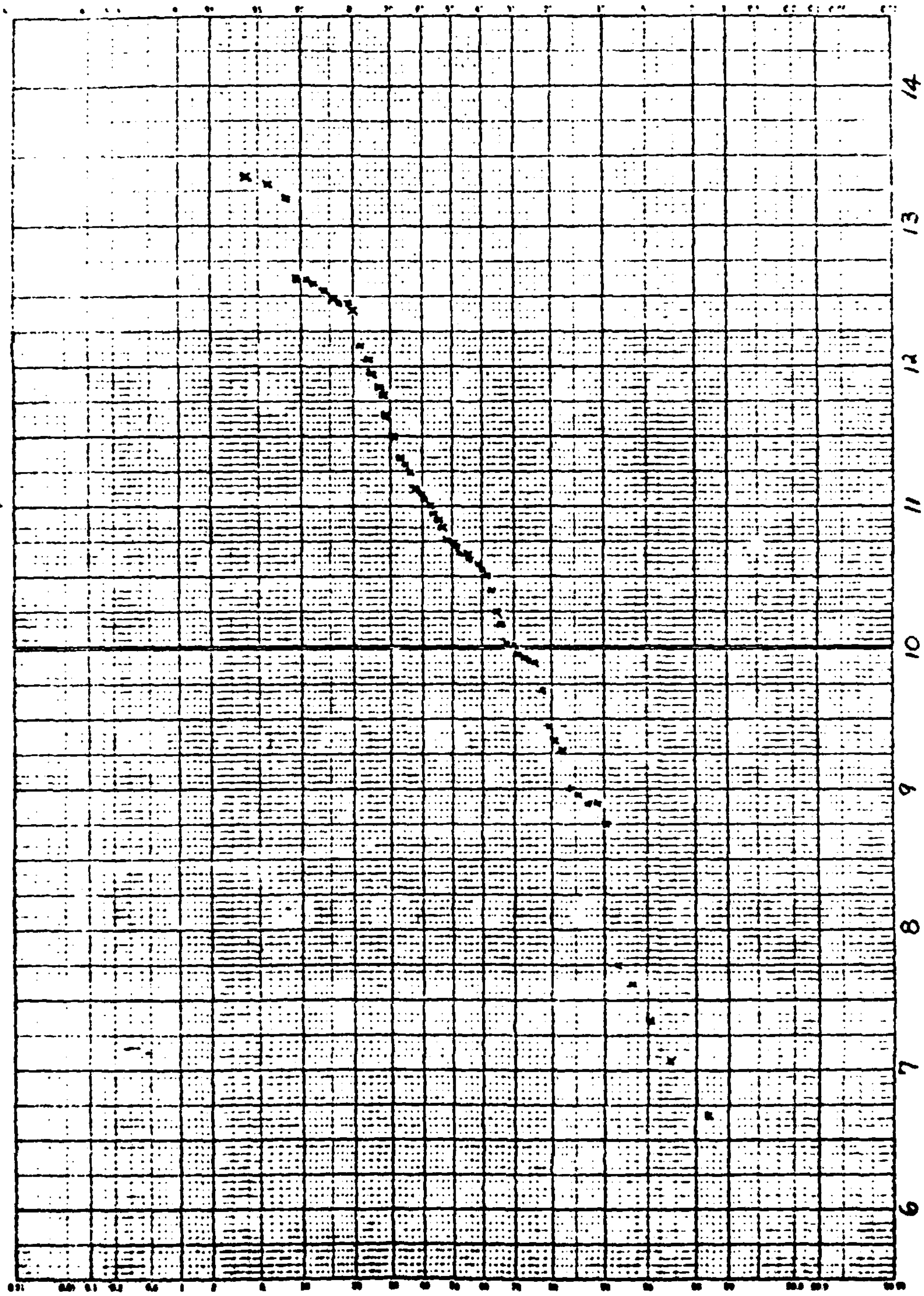

Figure 2. A plot of the logarithms of the 60 rescaled observations on normal probability paper.

critical values given in Section 4.3 we see that for $\alpha = .05$, $D^* = .87$. Thus D is significant at the 95% level of significance, but barely so.

Thus it appears that for this abstracted set of data, either the lognormal or the Weibull distribution would provide an adequate description, with the lognormal having a slight edge over the Weibull. In choosing between these two, one should remember that even though the formal goodness of fit tests favor the lognormal, the Weibull with its monotone failure rate may be a more appropriate distribution for describing the times to failure. The failure rate of a lognormal distribution first increases and then decreases, and this may not be realistic for the situation considered here.

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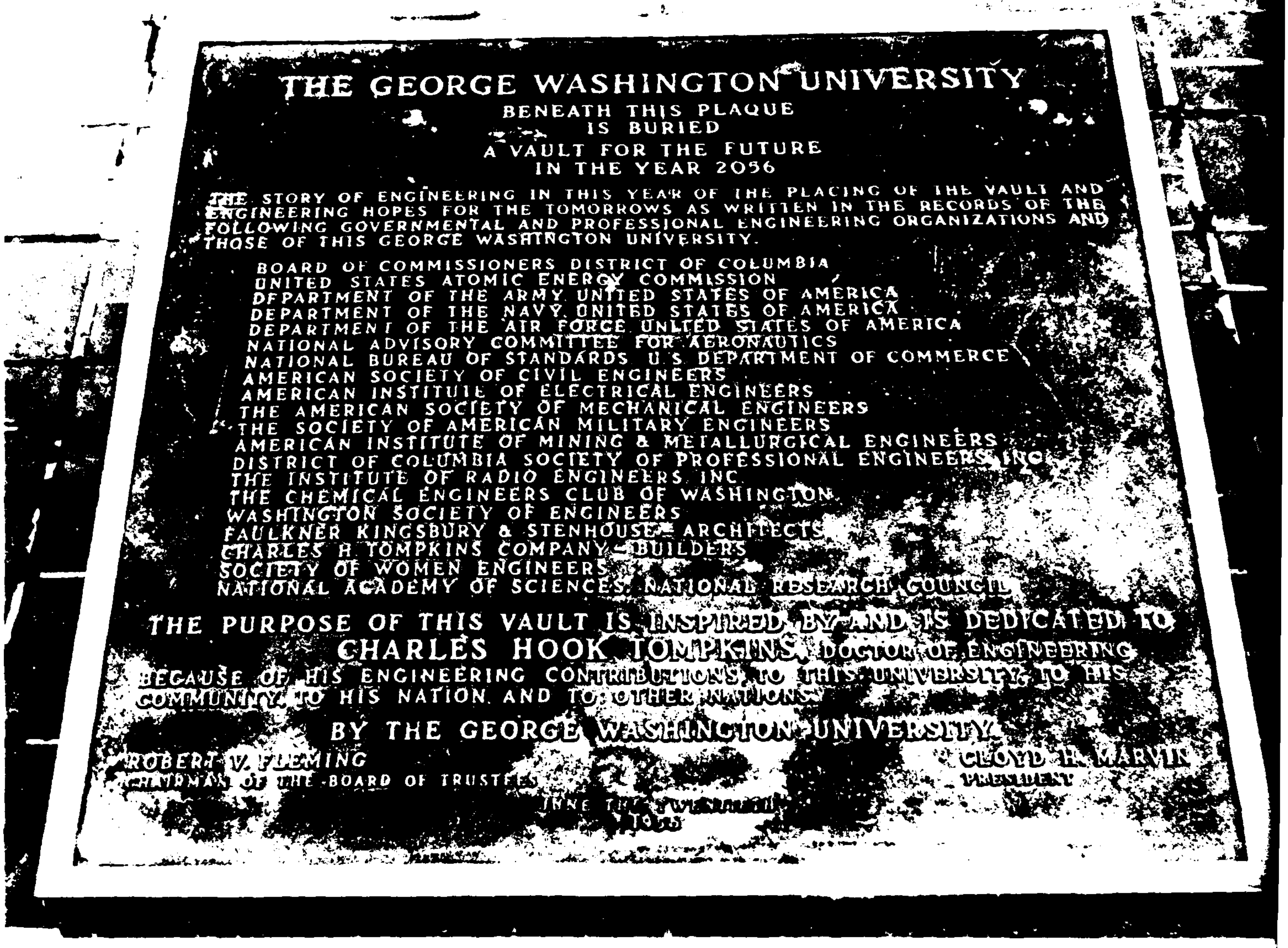

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