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GENERAL PURPOSE LEAST-SQUARES ADJUSTMENT OF IPS DATA.(U)
MAY 81 D S ZIMMERMAN, H C HARRIS

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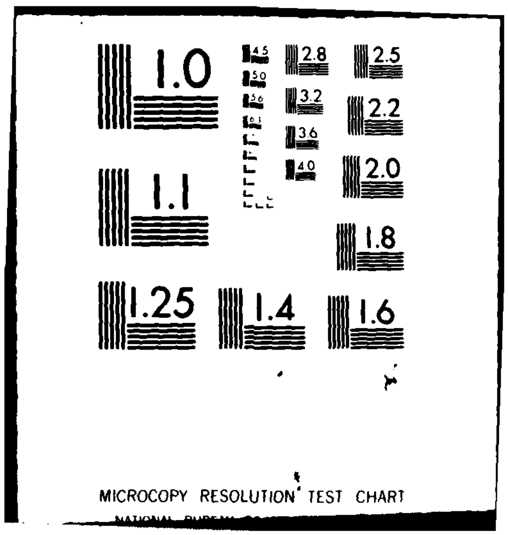
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Many of the IPS survey projects can easily be designed to form an interconnected network which lends itself to adjustment by the least-squares method. While a rigorously modeled IPS adjustment procedure has not yet been developed, DMA has been using a general purpose least-squares adjustment program for several years on IPS networks. The technique and results are discussed.

General Purpose Least-Squares Adjustment

of IPS Data

by

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1. INTRODUCTION

Litton Systems, Inc., delivered inertial positioning system number one (IPS-1) to the U.S. Defense Mapping Agency in 1975. The system became operational a year later. System utilization has been extensively reported by (*Harris, et al 1976*), (*Harris, 1977*), (*Doxey, 1977*), (*Sharp, 1979*), and (*Blackmer, 1981*).

IPS-1 is the Litton Autosurveyor™ with a vertical channel A-1000 accelerometer, as used by the U.S. Bureau of Land Management (BLM), the Canadian Department of Energy, Mines and Resources (EMR), the U.S. Army Engineer Topographic Laboratories (ETL), and SPAN International. The survey data output are raw and smoothed values for station latitude, longitude and height, plus the relative free-air gravity anomalies and output from which the relative deflection of the vertical components may be computed.

The raw and smoothed survey values are stored in a permanent data file of the VAX 11/780 mainframe computer. Using that file, certain post-mission processing is possible. For a suitably designed and executed survey project or area, it is possible to improve the overall results by a least-squares adjustment. A test network designed and executed for another reason is used here to demonstrate the relative accuracies attained using single and double runs without and with least-squares adjustments.

The first part of this paper shall be concerned with the least-squares error model, while the remainder shall discuss the data set and survey results.

2. LEAST-SQUARES PROCEDURE

The adjustment of elevations will be discussed throughout this procedure, but is applicable to the reduction of position, deflection, and gravity components as well. The so-called "SMOOTHED ELEVATIONS" are meaned to obtain an approximate (preliminary) elevation for each station. These values are to be used in the computation of L in the observation equation:

$$(X_i - X_j - L = V_{ij})p \tag{1}$$

p is the observation weight

X_i is the unknown value of the i-th station

X_j is the unknown value of the j-th station

L is the difference between the mean (preliminary) values and the observed values.

$$\text{or } L = [(M_i - M_j)-(O_i - O_j)] \tag{2}$$

Constraint Observation Equation:

$$(X_i - L = V_i)p \tag{3}$$

Given a set of n observations in m unknowns,

$$\begin{bmatrix} A_{11}X_1 + A_{12}X_2 + \cdots\cdots A_{1m}X_m = L_1 \\ A_{21}X_1 + A_{22}X_2 + \cdots\cdots A_{2m}X_m = L_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ A_{n1}X_1 + A_{n2}X_2 + \cdots\cdots A_{nm}X_m = L_n \end{bmatrix}$$

and denoting weights by p_1, p_2, $\cdots\cdot p_n$, the following matrices and vectors are defined as

$$A = \begin{bmatrix} A_{11}A_{12} & \cdot & \cdot & \cdot & A_{1m} \\ A_{21}A_{22} & \cdot & \cdot & \cdot & A_{2m} \\ \cdot\ \ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{n1}A_{n2} & \cdot & \cdot & \cdot & A_{nm} \end{bmatrix} \quad L = \begin{bmatrix} L_1 \\ L_2 \\ \cdot \\ \cdot \\ L_n \end{bmatrix} \quad p = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ p_n \end{bmatrix} \quad X = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_m \end{bmatrix}$$

In this notation the observation equations are given by AX=L.

The solution in the sense of least squares is $X=(A'pA)^{-1}\,(A'pL)$. (4)

Given two legs of observations from the IPS

IPS STA.	SMOOTH ELEV.
1001	459.10
1002	449.22
1003	399.78
1004	401.09
1005	429.66
1010	423.70

End of Leg

IPS STA.	SMOOTH ELEV.
1010	422.89
1005	429.33
1004	401.88
1003	401.42
1002	450.29
1001	458.30

End of Leg

UNKNOWN NUMBER	IPS STA.	MEAN ELEV.	CONSTRAINT (SIGMA)
1	1001	458.70	.005
2	1002	449.78	
3	1003	400.83	
4	1004	401.48	
5	1005	429.49	
6	1010	422.29	

Observations equations are formed thusly:

To express it in equation notation and eliminating the zero elements

Observ. Equation	$X_i - X_j - [(M_i - M_j) - (O_i - O_j)] = V_{ij}$ (5)
1.	1.(1) 0. = V(1)
2.	1.(1) - 1.(2) - [(458.70-449.78)-(459.10-449.22)] = V(1,2)
3.	1.(2) - 1.(3) - [(449.78-400.83)-(449.22-399.78)] = V(2,3)
4.	1.(3) - 1.(4) - [(400.83-401.48)-(399.78-401.09)] = V(3,4)
5.	1.(4) - 1.(5) - [(401.48-429.49)-(401.09-429.66)] = V(4,5)
6.	1.(5) - 1.(6) - [(429.49-422.29)-(429.66-423.70)] = V(5,6)
7.	1.(6) - 1.(5) - [(422.29-429.49)-(422.80-429.33)] = V(6,5)
8.	1.(5) - 1.(4) - [(429.49-401.48)-(429.33-401.88)] = V(5,4)
9.	1.(4) - 1.(3) - [(401.48-400.83)-(401.88-401.42)] = V(4,3)
10.	1.(3) - 1.(2) - [(400.83-449.78)-(401.42-450.29)] = V(3,2)
11.	1.(2) - 1.(1) - [(450.29-458.70)-(450.29-458.30)] = V(2,1)

To express it in matrix form

$$A=\begin{bmatrix} 1. & 0. & 0. & 0. & 0. & 0. \\ 1. & -1. & 0. & 0. & 0. & 0. \\ 0. & 1. & -1. & 0. & 0. & 0. \\ 0. & 0. & 1. & -1. & 0. & 0. \\ 0. & 0. & 0. & 1. & -1. & 0. \\ 0. & 0. & 0. & 0. & 1. & -1. \\ 0. & 0. & 0. & 0. & -1. & 1. \\ 0. & 0. & 0. & -1. & 1. & 0. \\ 0. & 0. & -1. & 1. & 0. & 0. \\ 0. & -1. & 1. & 0. & 0. & 0. \\ -1. & 1. & 0. & 0. & 0. & 0. \end{bmatrix} \qquad p=\begin{bmatrix} 40000. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \\ 1. \end{bmatrix}$$

(Constraint Equation) $1/p^2$

$$L=\begin{bmatrix} 0.0000D+00 \\ -0.9600D+00 \\ -0.4900D+00 \\ 0.6600D+00 \\ 0.5600D+00 \\ 0.1240D+01 \\ -0.7600D+00 \\ 0.5600D+00 \\ 0.1900D+00 \\ -0.8000D-01 \\ -0.9100D+00 \end{bmatrix} \qquad A'pL = \begin{bmatrix} 0.50000D-01 \\ 0.36000D+00 \\ -0.88000D+00 \\ 0.47000D+00 \\ -0.20000D+01 \\ 0.20000D+01 \end{bmatrix}$$

$$A'pA=\begin{bmatrix} 0.4000D+05 & -0.2000D+01 & 0. & 0. & 0. & 0. \\ -0.2000D+01 & 0.4000D+01 & -0.2000D+01 & 0. & 0. & 0. \\ 0. & -0.2000D+01 & 0.4000D+01 & -0.2000D+01 & 0. & 0. \\ 0. & 0. & -0.2000D+01 & 0.4000D+01 & -0.2000D+01 & 0. \\ 0. & 0. & 0. & -0.2000D+01 & 0.4000D+01 & -0.2000D+01 \\ 0. & 0. & 0. & 0. & -0.2000D+01 & 0.2000D+01 \end{bmatrix}$$

$$(A'pL)^{-1} = \begin{bmatrix} 0.2500D-04 & 0.2500D-04 & 0.2500D-04 & 0.2500D-04 & 0.2500D-04 & 0.2500D-04 \\ 0.2500D-04 & 0.5000D+00 & 0.5000D+00 & 0.5000D+00 & 0.5000D+00 & 0.5000D+00 \\ 0.2500D-04 & 0.5000D+00 & 0.1000D+01 & 0.1000D+01 & 0.1000D+01 & 0.1000D+01 \\ 0.2500D-04 & 0.5000D+00 & 0.1000D+01 & 0.1500D+01 & 0.1500D+01 & 0.1500D+01 \\ 0.2500D-04 & 0.5000D+00 & 0.1000D+01 & 0.1500D+01 & 0.2000D+01 & 0.2000D+01 \\ 0.2500D-04 & 0.5000D+00 & 0.1000D+01 & 0.1500D+01 & 0.2000D+01 & 0.2500D+01 \end{bmatrix}$$

$$\text{Solution} = (A'pA)^{-1}(A'pL) \qquad X = \begin{bmatrix} -0.8470D-21 \\ -0.2500D-01 \\ -0.2300D+00 \\ 0.5002D-02 \\ 0.5002D-02 \\ 0.1005D+01 \end{bmatrix}$$

Results of least-squares adjustment showing the adjusted elevations, error terms and residuals.

NR OF STATIONS	6
NR OF OBSERVATIONS	11
NR OF UNKNOWNS	6
REJECTION LIMIT (MTRS)	2
NR OF OBSERVATIONS REJECTED	0
NR OF ADJUSTMENTS	1
STANDARD ERROR OF UNIT WEIGHT	0.776

STATION	OBS/REJ	ELEVATION CORRECTION	ADJUSTED ELEVATION	SIGMA (METERS)	CONSTRAINTS (METERS)
1001	2/ 0	0.00	458.70	0.00	0.0050
1002	4/ 0	-0.03	449.75	0.55	
1003	4/ 0	-0.23	400.60	0.78	
1004	4/ 0	0.01	401.49	0.95	
1005	4/ 0	0.01	429.50	1.10	
1006	4/ 0	1.01	423.30	1.23	

STA.	TO	STA.	RESIDUALS IN METERS HEIGHT
1001		1002	0.935
1002		1003	0.285
1003		1004	-0.425
1004		1005	-0.560
1005		1010	-0.240
END OF LEG		1	
1010		1005	-0.240
1005		1004	-0.560
1004		1003	-0.425
1003		1002	0.285
1002		1001	0.935
END OF LEG		2	

3. SAMPLE DATA SET

In 1980, the U.S. Army Engineer Topographic Laboratories observed a test network at the White Sands Missile Range (WSMR) as a cooperative effort with the Defense Mapping Agency. The network and techniques are described in detail in (*Todd, 1981a and 1981b*). The unique feature of this data set is that it was observed between known survey stations in something approaching a network fashion, which permits a reduction by least-squares methods, and gives a reference for comparison against very well established values.

The network itself is shown in Figure 1. There are 11 separate test lines containing a total of 79 unique survey points. Latitude, longitude and elevation are known at all 79 stations, gravity at 78 and astrogeodetic deflection of the vertical at 47 of the stations. Eighteen of the 79 stations were treated as known points and the remaining 61 stations as new survey points. Thirty of the stations are at the junction of two or more traverses. Each of the 11 test lines was run once each way (direct and reverse) with the system in a vehicle and once each way in a helicopter. Only the vehicle data set is used in this illustration of least-squares results.

The typical time between ZUPTs (zero-velocity updates) was 3.5 minutes, and the average length of a one-way traverse between updates was 2.18 hours.

The ETL inertial system used to perform the observations is identical to the Defense Mapping Agency IPS-1. The software used, however, is an IPS program modified specifically to improve the accuracy of the interpolation of the deflection of the vertical. That on-board software is referred to as RGSS for Rapid Geodetic Survey System. Position and gravity accuracy of the RGSS software is the same as that for the standard IPS program, while RGSS elevations are expected to be slightly inferior unless post-mission processing is employed.

4. PRELIMINARY COMPUTATIONS

The RGSS program adjusts the latitude, longitude and elevation of each intermediate survey station between the end, update points of each individual traverse. The adjustment is a distribution of the error of closure of each of these survey parameters at the terminal update station. In that on-line smoothing the known values of the end points are held fixed.

The on-line program yields output from which the anomalous gravity vector can be computed. The gravity anomaly is given as the Kalman Z accelerometer bias DZ. The deflection of the vertical components are normally computed for the IPS software by:

$$\xi_i = -(SUME + DN/4.85) \tag{6}$$

$$\eta_i = -(SUMN - DE/4.85) \tag{7}$$

where, SUMN and SUME are the Kalman torque about north and east, respectively, and DN and DE are Kalman north and east accelerometer biases.

However, in using the RGSS software the deflections are computed by:

$$\xi_i = -DN/4.85 \tag{8}$$

$$\eta_i = DE/4.85 \tag{9}$$

The relative gravity anomaly and deflection of the vertical must be corrected to a datum based on a known value. In this case the known station was the beginning point of the traverse. The datum shift is given by:

$$F_i = f_i + C \tag{10}$$

Where, F_i = the observed value corrected to the datum.

f_i = the observed value at the ith station.

C = known value$_1$ - f_1

The gravity anomalies and deflection of the vertical components are then adjusted as a function of time:

$$P_i = F_i + D_j\left(\frac{t_i - t_1}{t_n - t_1}\right) \tag{11}$$

where P_i = the preliminary value of the gravity anomaly or deflection component for the least-squares adjustment,

D_j = the error of closure on the terminal station for the j th traverse.

t_1 = the time of observation of the initial update station

t_i = the time of observation of the ith station

t_n = the time of observation of the terminal update station

F_i = the observation at the ith station, corrected to the datum in question based on the known value at the initial update station.

The statistical term used in all computations is the standard deviation, one sigma level, as given by the equation:

$$S = \sqrt{\frac{\Sigma vv}{n-k}} \tag{12}$$

where, v = the residual formed by the difference between the observed and known values.

n = the number of observations

k = the number of unknowns

5. THE ADJUSTMENTS

The general least squares adjustment described in this paper has been in use for some time. There are two basic options available; a position adjustment or an adjustment of elevation. It is routine procedure to store the position and elevation data from an IPS survey in a permanent automated data file. The extraction and use of the anomalous gravity vector data was a special procedure for this test project. The elevation adjustment option was used for separate adjustments of the gravity anomalies and the deflection of the vertical components with only slight modifications to handle the formats for input and output.

The data sets were established as follows:

Single Run - All residuals from 22 runs were pooled. The residuals were formed by differencing the individual observed and smoothed values and the known values.

Single Run Least-Squares - The same observations used above were introduced into a least-squares adjustment as preliminary values. The residuals were formed by differencing the adjusted and known values.

Double Run - The observations from the 22 runs were sorted into 11 double run traverses. The smoothed direct and reverse observations were meaned to yield a single value at each station for each traverse. The residuals were formed by differencing these smoothed mean values and the known values.

Double Run Least-Squares - The double run mean values were introduced into the least-squares adjustment as preliminary values. The adjusted and known values were differenced for the residuals.

In the least-squares adjustment the 18 known stations were estimated to have a standard deviation of ±0.08 meters in latitude and longitude, ±0.03 meters in elevation, ±0.3 seconds of arc for each deflection component, and ±0.1 mgal for the gravity anomaly.

6. RESULTS

The overall results from the analysis of the 4 data sets previously described are given in Table 1. In order of improved accuracy, the data sets are single run, double run, single run with least squares adjustment, and double run with least squares adjustment.

The efficiency quotient, EQ, is computed for each method by:

$$EQ = I/L \tag{13}$$

where, I = improvement compared to single runs

L = level of effort compared to single runs

The level of effort required for each technique varies considerably depending on many factors such as size of project, availability of good control, spacing of stations, etc. On a project of 100 or more stations, the levels of effort, compared to single runs, are about 2 to 1, 1.2 to 1 and 2.4 to 1 for double run, single run network, and double run network respectively, where 20% of the stations are cross-connected in the network.

7. SUMMARY

While post-mission adjustments of inertial positioning data have been done for some time, several problems have been recognized. Most people agree that we should be using "raw" inertial data, or at least "raw" survey data in the adjustments, but rigorous error modeling and adjustment techniques for these data have not been available. So we now adjust "smoothed" survey data using general least-squares adjustment programs.

There has been a general acceptance of the fact that the inertial survey systems can produce sub-meter position and elevation accuracies relative to the local control, but the anomalous gravity vector has been generally neglected except by the U.S. Army Engineer Topographic Laboratories with encouragement from the Defense Mapping Agency.

There has not been a test data network available for analysis where good survey control was available at all stations. It has been theorized that post-mission adjustment must improve the quality of the network surveys, but the magnitude of that improvement has not been measured.

The joint DMA/ETL effort to collect and analyze data from a pseudo network observed in a high quality test area will not solve the problem of lack of rigorous error modeling and adjustment techniques. Efforts to produce such rigorous techniques specifically for inertial systems must be given more emphasis and encouragement.

The test data do show the quality of survey product which can be obtained, including amazingly good recovery of the anomalous gravity vector. This aspect has been ably reported by ETL.

The purpose of this paper is to prove that the post-mission adjustments do very significantly improve the accuracy of the inertial network data. This proof is given in the table of results. It is not claimed that these techniques can remedy the problems of poor quality basic survey control, although if proper relative weighting can be determined then some improvement would be expected. With good basic control, one can expect about a 25% improvement in overall accuracy of double run data, with twice the survey effort, compared to single run. That yields an effectiveness quotient of 0.125. On the other hand, a 34% improvement in overall accuracy is to be achieved by observing a single run network, at a level of effort 1.2 times the single run simple traverses, followed by post-mission least-squares adjustment. This effectiveness quotient of 0.28 is clearly superior to the simple double run method in those cases where both techniques are adaptable to the survey situation.

The best accuracy is obtained by the double run, least-squares adjusted network data, with an overall 44% improvement in accuracy. The level of effort is 2.4 times that required for simple traverses of single runs. The effectiveness quotient of 0.18 is better than that of the simple double run data, but not as good as the 0.28 of the single run, least-squares adjusted data.

For those relatively large projects which are adaptable to network type observations, the most cost effective survey method is single run, inter-connected traverses with post-mission adjustments.

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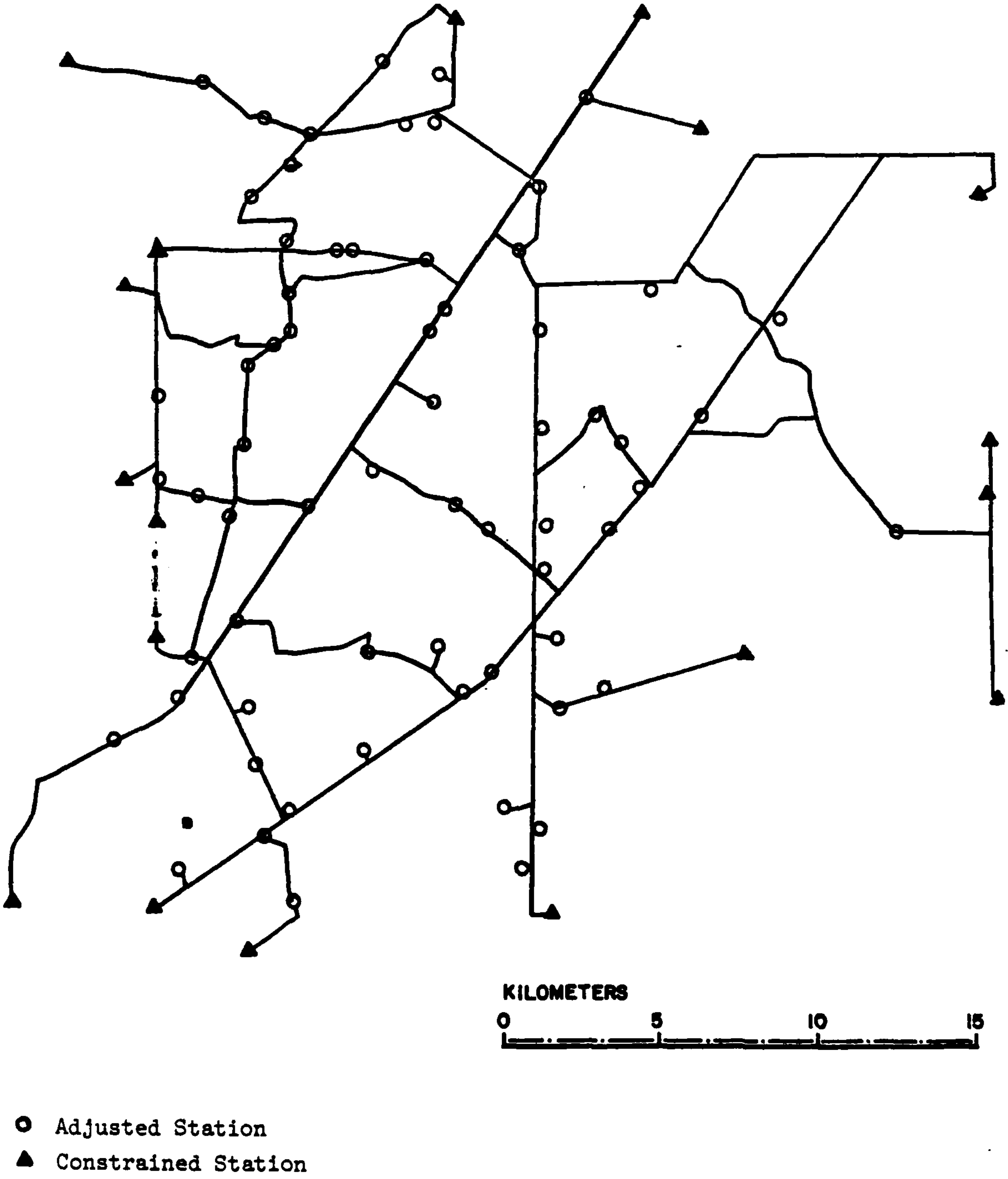

Figure 1. WSMR Test Network

1980 RGSS TEST RESULTS IN VEHICLE

DATA SET	Lat(cm)	Lon(cm)	h(cm)	N-S(sec)	E-W(sec)	Δg(mgal)
	STANDARD DEVIATIONS, ONE SIGMA					
Single Run	45	42	28	1.6	1.4	2.0
Double Run	32	32	21	1.1	0.9	1.7
Single Run L. S.	19	26	19	1.0	0.9	1.6
Double Run L. S.	13	22	18	0.8	0.8	1.4

EQ		PERCENT IMPROVEMENT		EQ
0.0	38.3cm	Single Run	1.5sec	0.0
	44% 26%		37% 33%	
0.13	28.3cm	Double Run	1.0sec	0.165
	54% 3-D POSITION		47% DEFLECTION	
0.37	21.3cm	Single Run L. S.	0.95sec	0.31
0.225	17.7cm	Double Run L. S.	0.8sec	0.196

EQ			EQ
0.0	2.0mgal	.	0.0
	20% 15%	34% 25%	
0.075	1.7mgal	.	0.125
	30% GRAVITY ANOMALY	44% OVERALL	
0.17	1.6mgal	.	0.28
0.125	1.4mgal	.	0.18

Table 1. Test Results

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