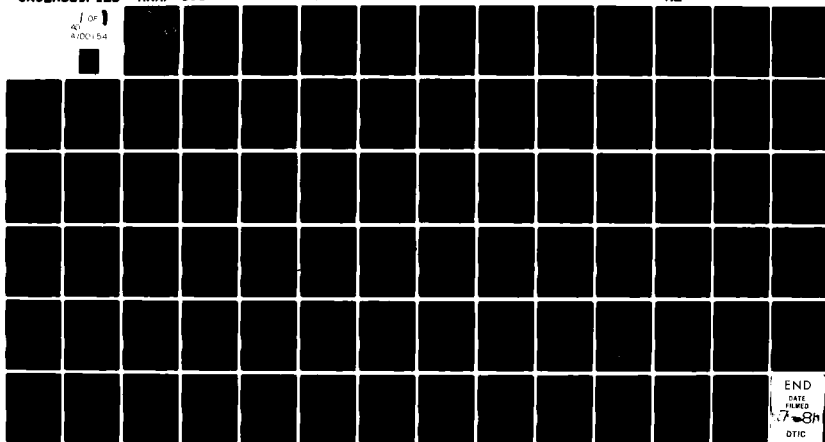


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| Integral Theory of Impact<br>Impact<br>Armor<br>Penetrator<br>Material Properties                                                                                                                                                                                                                                                                                                                                                      |                       |                                                             |  |
| 21. ABSTRACT (Continue on reverse side if necessary and identify by block number)                                                                                                                                                                                                                                                                                                                                                      |                       |                                                             |  |
| The A.R.A.P. Integral Theory of Impact is applied to the problem of modeling long-rod penetrator performance. The rod penetrator is approximated by two cells, one which models the deforming hydrodynamic and plastic region at the leading edge of the rod, and a second cell which models the non-deforming portion of the shaft. Equations of motion are derived based on global conservation of momentum and energy. The material |                       |                                                             |  |

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## I. INTRODUCTION

The purpose of this report is to document the significant results of the second year of our research program under Contract No. DAAD05-76-C-0757. The object of this portion of the program was to apply the A.R.A.P. Integral Theory of Impact to the problem of modeling the behavior of long rod penetrators.

Over the past several years the Integral Theory of Impact has been developed by A.R.A.P. ~~1-5~~ It is already proving to be a useful tool in the design of armor and penetrators. The theory contains the essential physics of the impact process, satisfies all the global conservation equations and is contained in a computer code which is simple and inexpensive to operate. The integral theory requires far less empirical information than some models and avoids the high cost and complexity of multi-element codes. Its simplicity introduces a degree of economy that makes it reasonable to conduct parametric studies so that predicted trends are available, rather than single point predictions. This aspect of the simple theory greatly facilitates the interpretation of observations and the selection of effective designs. The integral theory can, therefore, be used to guide experimental programs and to select those designs which warrant further study using the large codes.

The Integral Theory of Impact assumes that the energy of an incoming projectile is absorbed by the armor in one of three forms:

- 1.) As dissipated energy in the form of plastic work as the target flows around the penetrator, or as fracture energy in newly created fracture surfaces. This nonrecoverable portion of the total energy is characterized by an energy per unit mass of target material,  $E_{*p}$ , and it appears to be roughly independent of velocity for each material. The product of target density times  $E_{*p}$  corresponds to the "adiabatic hardness" of the material, or its hardness measured at the strain rates of impact.

2.) As elastic energy absorbed by the target in its local elastic deformation near the penetrator and in large scale elastic deformation modes. This elastic, or recoverable, portion of the total energy is parameterized by  $E_{*e}$ , the elastic energy per unit target mass.  $E_{*e}$  is a well defined function of  $p/d$  so a single constant defines  $E_{*e}$  for each material over the complete velocity range.

3.) As kinetic energy in the target material as it accelerates and begins to flow around the incoming penetrator. This portion of the energy is expressed by  $\frac{C_D}{2} V^2$ , where  $V$  is the velocity of the penetrator face relative to the target and  $C_D$  is a drag coefficient approximately equal to 1.0 for a penetrator with a spherical front end.

Similarly, when analyzing the dynamics of the deforming penetrator itself, there will be a quantity which measures the dissipated energy per unit mass absorbed by the penetrator as it deforms plastically or fractures. It is the analogue of  $E_{*p}$  for the target material. We will call this quantity  $E_{*d}$ . For ductile materials, the product of penetrator density times  $E_{*d}$  corresponds to the "adiabatic yield strength" of the material, which is the uniaxial yield strength of the material measured at the strain rates of impact.  $E_{*d}$  is assumed to be a constant for each penetrator material.

The kinetic energy in a deforming penetrator is modeled assuming a simple, usually linear, flowfield in the penetrator, and simple shapes such as cubes or cylinders to approximate the deformed shape of the projectile. The elastic energy in penetrators, analogous to  $E_{*e}$  for targets, has been neglected so far because it is relatively small compared to  $E_{*d}$  at the velocities of impact. However, in principle it can be included also.

Once the two parameters  $E_{*p}$  and  $E_{*e}$  are known for a target material, and  $E_{*d}$  is known for the penetrator, the behavior of the armor and penetrator during impact can be

computed from global energy and momentum conservation laws by the A.R.A.P. Integral Theory. Since it is only the sum of  $E_{*p}$  and  $E_{*e}$  which governs target performance, we shall often refer to the sum as  $E_*$ .  $E_{*p}$  and  $E_{*e}$  have been measured in impact tests for a variety of target materials from lead to boron carbide, from salt to Rolled Homogeneous Armor, over a velocity range from 20 ft/sec to 6,400 ft/sec and have been shown to provide an excellent description of armor behavior.<sup>4</sup> Although  $E_*$  for a target is measured in impact tests with nondeforming tungsten carbide balls, the same value of  $E_*$  for the target correctly predict its performance when the impactor is highly deforming, such as lead or soft aluminum spheres,<sup>5</sup> or with a high L/D, such as a long rod penetrator.

Recently, a theory has been developed which related  $E_{*p}$  and  $E_{*e}$  to more fundamental materials properties, such as melting temperature, heat capacity, Young's modulus and Brinell hardness.<sup>4</sup> This makes possible the prediction of armor performance from static tests alone. The theory has been verified experimentally over the same wide range of materials for which impact experiments have been conducted. It accurately predicts  $E_*$  to about  $\pm 15$  percent for all these materials. This formula has enabled A.R.A.P. to conduct parametric studies, using handbook properties of materials, which have pointed up many promising lightweight armor materials, including some which are remarkably economical.

In this paper, we shall report that  $E_{*d}$  for penetrators can also be computed from the same fundamental materials properties, and can therefore be predicted from purely static tests. Thus, all the input parameters required to predict penetration by a deforming penetrator into a target can be obtained from handbook values of materials properties.

## II. INTEGRAL THEORY FOR ROD PENETRATION

The Rod Penetrator Code to be described here, which we refer to as "ROD," uses the Integral Theory approach outlined in the introduction. As a penetrator moves through a target material with some velocity  $V_{\text{face}}$  at the penetrators front face, the Integral Theory for target performance tells us that the pressure at this face must be

$$\text{Pressure} = \rho_t \left( \frac{C_D}{2} V_{\text{face}}^2 + E_{*t} \right), \quad (1)$$

where  $\rho_t$  is the target density and  $\rho_t E_{*t}$  the adiabatic hardness of the target material.  $C_D \cong 1$  for a nondeforming spherical front face, as reported previously. This formula has been verified for rigid sphere penetrators and deforming sphere penetrators over a wide range of projectile and target materials and velocities. Below we set up the equations governing the internal dynamics of a long rod penetrator which, when coupled with Eq. (1), which governs the target dynamics, completely specifies the problem.

It is known from X-ray photographs that the stages of long rod penetration may be roughly characterized as in Fig. 1. As the rod impacts the target the pressures generated at the interface begin to deform the front end of the penetrator, as in Fig. 1b. Simultaneously the target is eroded away by the same pressure, producing a crater, as in Fig. 1c. As penetration continues, material at the leading face of the penetrator is eroded away by the target, forced out laterally from the contact region by the high pressure there and ejected back out of the crater. As material is eroded from the rod face, new material is supplied to this region by the shaft of the rod, which is traveling at a higher velocity than the rod-target interface. At some point, Fig. 1d, the shaft material is used up and the head is decelerated quickly to zero velocity by the target.

We shall model the flowfield of a rod by dividing it into two regions; the head, corresponding to the front region of a

# FLOW FIELD OF ROD AT VARIOUS STAGES OF PENETRATION

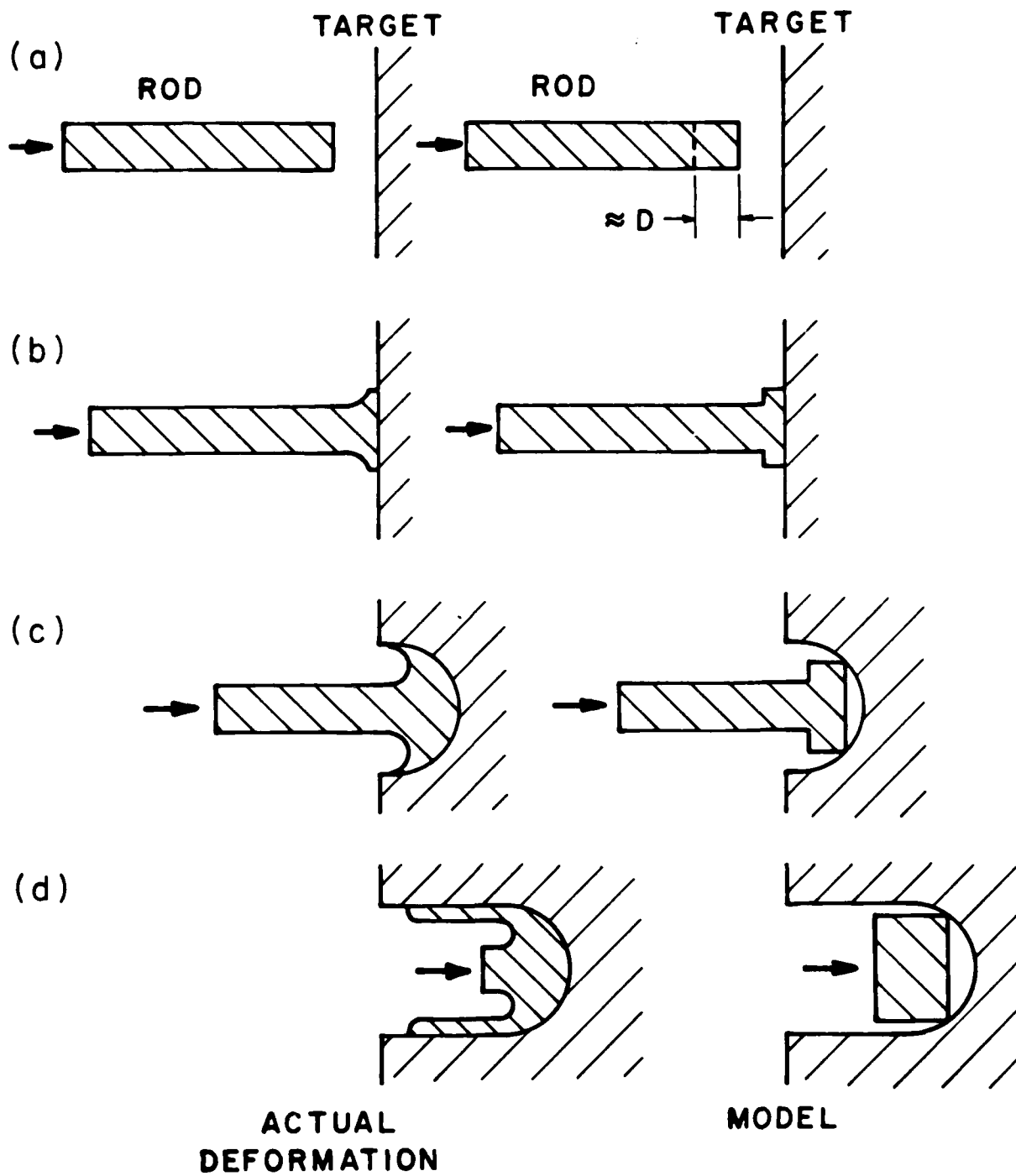
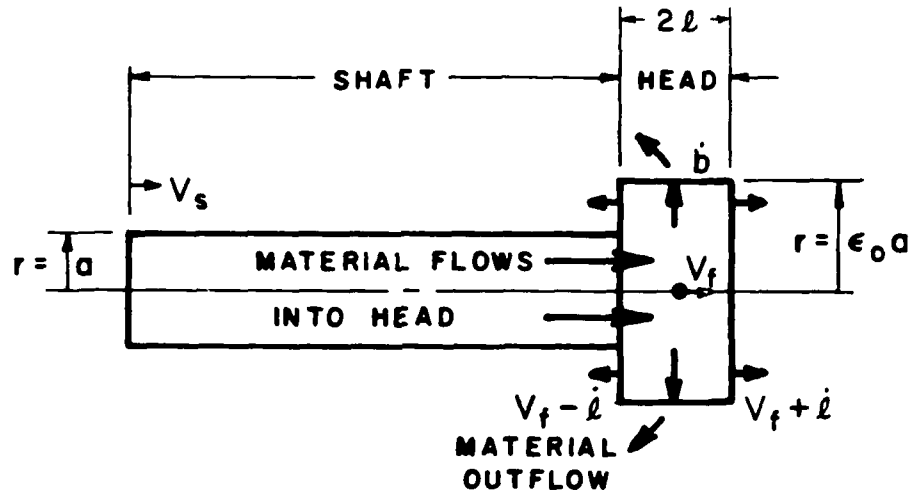


Figure 1

## LONG ROD MODEL



### ASSUMPTIONS:

1. Conservation of energy
2. Conservation of momentum
3. Linear flow field in head
4. Continuity of mass flow across interface
5. Constant yield stress at interface
6. Mass of penetrator erodes from head when radius exceeds  $\epsilon_0 a$
7. The model depends upon two parameters:

The yield stress  $Y_0 \cong \rho E_* d$

and

The shearing radius given by  $\epsilon_0 a$

Figure 2

rod which is undergoing plastic and hydrodynamic strain, and a rear portion of the rod, the shaft, which we assume to be undeformed rod material. In Fig. 2, the assumptions and parameters of the model are summarized. During penetration the head, which is in contact with the target, decelerates and spreads laterally. We assume the mass flowfield in the head is linear, and the head is of cylindrical shape. The motion of the material in the head is characterized by a center of mass velocity  $V_f$ , and by the velocity of its front face  $\dot{l}$  and side face  $\dot{b}$  relative to the center of mass of the head. Conservation of mass across the boundary between the shaft and the head imposes the condition that the rate of flow of material from the shaft into the head is

$$\dot{M}_a = \pi a^2 \rho_p (V_s - V_f + \dot{l}) \quad (2)$$

where  $a$  is the radius of the shaft,  $\rho_p$  the penetrator density, and  $V_s$  the velocity of the shaft.

As penetration proceeds, the head widens as rod material is forced to flow in the lateral direction. At some distance from the axis of the rod, say  $\epsilon_0 a$ , we assume the flow of rod material has been turned or sheared off by the target and no longer can apply decelerating forces to the rod. Thus, when rod material in the head moves beyond a distance  $\epsilon_0 a$ , laterally, it is assumed to be detached from the rod. The dynamics of that material as it is further slowed by the target will not affect the deceleration of the shaft or head. This assumption is justified for some  $\epsilon_0 a$  because the rod material at this point in the flow has been adiabatically heated so much by plastic work that its shear strength is very low, so it is only able to influence the rod through compressive or hydrodynamic forces. However, the axial force component of the compressive hydrodynamic force on the rod shaft will only be significant within one or two rod radii from the central axis. Thus, we shall expect  $\epsilon_0 a \leq 2a$ . Thus,  $\epsilon_0 a$  really characterizes the turning radius

of the rod material in the target or the shape of the flowfield in the head. We further assume for simplicity that the shape of the flowfield in the head region of the rod does not change too much from material to material. Therefore, we may take  $\epsilon_0$  to be the same constant for all rod penetrators, no matter what material.

When  $b$  (the radius of the rod head) reaches  $\epsilon_0 a$ , the cut-off radius, we assume any further increase in the radius of the head simply results in loss of rod material across the boundary at  $\epsilon_0 a$ , as in Fig. 2. The rate of mass loss from the head will be

$$\dot{M}_b = 4\pi b \rho_p \dot{b} \quad (3)$$

where  $\dot{b}$  is the lateral velocity of material in the head of the radius  $b$ . Then the rate of change of mass in the head is

$$\frac{d}{dt} M_f = \dot{M}_a - \dot{M}_b \quad (4)$$

and the rate of change of mass in the shaft is

$$\frac{d}{dt} M_s = -\dot{M}_a \quad (5)$$

The pressure applied to the rod front face by the target during penetration is

$$\rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{l})^2) \quad (6)$$

as described in Eq. (1), where the front face of the rod moves at the velocity  $V_f + \dot{l}$ . This pressure acts across the entire frontal area of the rod head which is in contact with the target. The area is  $\pi b^2$  giving

$$\text{Total force} = \frac{d}{dt} (M_f V_f + M_s V_s) = -\pi b^2 \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{l})^2) - \dot{M}_b V_f \quad (7)$$

from total conservation of momentum. The second term on the right accounts for momentum loss through mass loss out the side of the head. Substituting from (4), we find

$$\dot{M}_a (V_f - V_s) + M_f \dot{V}_f + M_s \dot{V}_s = -\pi b^2 \rho_t \left( (E_{*t} + \frac{C_D}{2} (V_f + \dot{l})^2) \right) \quad (8)$$

This may be separated into two equations for the conservation of momentum of the head and the shaft. The shaft will only experience decelerating forces if it has a nonzero yield strength  $\sigma$ . In this case, we have

$$(\text{Force on Shaft}) = -\pi a^2 \sigma = M_s \dot{V}_s \quad (9)$$

When  $\sigma = 0$ , as in the case of a shaped charge jet, which is liquid, the shaft velocity remains constant throughout penetration. Subtracting this from (8) above, the corresponding equation for the head is obtained:

$$M_f \dot{V}_f = -\pi b^2 \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{l})^2) + \sigma \pi a^2 + \dot{M}_a (V_s - V_f) \quad (10)$$

The first term on the right is the force on the head due to the target pressure, the second term is the acceleration of the head due to the push from behind applied by the yield strength of the shaft, and the third term is the momentum added to the head from the material passing into the head from the shaft.

The equations above account for momentum conservation. Next, we require energy conservation. The total kinetic energy in the rod is given by

$$K = M_s \frac{V_s^2}{2} + \frac{M_f V_f^2}{2} + \frac{M_f}{2\alpha} (\dot{l}^2 + \frac{3}{2} b^2) \quad (11)$$

where the third term on the right accounts for the internal kinetic energy in the flowfield of the head.  $\alpha = 3$  for a cylindrical head. The total work per unit time done on the rod by the target is

$$\dot{U} = \rho_t \pi b^2 (V_f + \dot{l}) (E_{*t} + \frac{C_D}{2} (V_f + \dot{l})^2) \quad (12)$$

This work is converted into either heating of the rod or changing the kinetic energy of the rod. The heating rate is given by the  $\dot{W}$ , the rate at which rod material is converted into the hydrodynamic state, given by

$$\dot{W} = \dot{M}_a E_{*d} \quad (13)$$

where  $E_{*d}$  is the "adiabatic yield strength" of the rod material, and  $\dot{M}_a$  is the rate at which rod material enters the head. The conservation of total energy requires:

$$\dot{W} + \dot{K} + \dot{U} = 0 \quad (14)$$

Lastly, the parameter  $\ell$  for the half-thickness of the head and  $b$  for the radius of the head are related to the mass of the head by

$$2\pi\ell b^2\rho_p = M_f \quad (15)$$

These equations completely specify the problem. We solve them to obtain the following coupled set of differential equations.

$$\dot{M}_f = \dot{M}_a - \dot{M}_b \quad (16)$$

$$\dot{M}_a = \pi a^2 \rho_p (V_s - V_f + \dot{\ell}) \cdot f_1 \quad (17)$$

$$\dot{M}_b = 4\pi\ell b \rho_p \dot{b} \cdot f_2 \quad (18)$$

$$\dot{V}_f = -\frac{1}{M_f} \left( \pi b^2 \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{\ell})^2) + M_a (V_f - V_s) - f_2 \cdot \sigma \pi a^2 \right) \quad (19)$$

$$\dot{V}_s = -\sigma \pi a^2 / M_s \quad (20)$$

$$\dot{K} = -(V_f + \dot{\ell}) \pi b^2 \rho_t \left( \frac{C_D}{2} (V_f + \dot{\ell})^2 + E_{*t} \right) - \dot{M}_a \rho_p E_{*d} - \frac{\dot{M}_b}{2} (V_f^2 + \dot{b}^2) \quad (21)$$

$$\dot{b} = \frac{b}{2} \left( \frac{\dot{M}_a}{M_f} - \frac{\dot{\ell}}{\ell} \right) \quad (22)$$

$$\ddot{b} = K_4 + K_5 \ddot{\ell} \quad (23)$$

where

$$K_4 = (1 - f_2) \frac{\dot{b}^2}{b} + \left( \frac{\dot{\ell}}{\ell} \right)^2 \frac{b}{2} - \frac{b}{2} \left( \frac{\dot{M}_f}{M_f} \right)^2 - \frac{f_1 \pi a^2 \rho_p (\dot{V}_s - \dot{V}_f) + f_2 4\pi\ell b \rho_p \dot{b} \left( \frac{\dot{\ell}}{\ell} \right)}{2M_f} \quad (24)$$

and

$$K_5 = \frac{a^2 f_1}{4 \ell b} - \frac{1}{2} \frac{b}{\ell} \quad (25)$$

$$\ddot{\ell} = \frac{\dot{K} + Q_2 - \frac{3}{2} \frac{M_f}{\alpha} \dot{b} K_4}{\frac{M_f}{\alpha} (\dot{\ell} + \frac{3}{2} \dot{b} K_5)} \quad (26)$$

where

$$Q_2 = - \left\{ \dot{M}_f \left( \frac{\ell^2}{2\alpha} + \frac{\dot{b}^2}{\alpha} + \frac{v_f^2}{2} \right) - \frac{\dot{M}_a v_s^2}{2} + \frac{\dot{M}_b}{2} (\dot{b}^2 + v_f^2) + M_f v_f \dot{v}_f \right\} \quad (27)$$

and penetration rate

$$\dot{p} = v_f + \dot{\ell} \quad (28)$$

$f_1$  and  $f_2$  are integer quantities introduced to allow the numerical integration to proceed smoothly at certain discontinuous transition points in the model.

$f_1$  remains 1 unless the mass  $M_s$  of the shaft becomes zero. This will happen when the shaft has been consumed by erosion. When the mass  $M_s$  becomes zero,  $f_1 = 0$ .

$f_2$  remains zero until  $b$ , the head radius, reaches  $\epsilon_0 a$ . Then  $f_2$  becomes 1, and prevents the radius of the head from increasing beyond  $\epsilon_0 a$ . Thus, mass loss at the head also begins to occur when  $f_2 = 1$ .

This set of equations is incorporated into the computer code ROD, which is reproduced in Appendix I. The input parameters required to operate the code consist only of the length and radius of the rod, the density of the rod material, and its adiabatic hardness  $\rho E_*$ , plus corresponding quantities for the target. The other parameters in the set of equations above are disposed of in the following way: We have learned that  $\sigma$ , the yield strength of the rod, is just its "adiabatic uniaxial flow stress" or

$$\sigma = \rho_p E_* d \quad (29)$$

The cut-off radius  $\epsilon_0$  is assumed to be a constant,  $\epsilon_0 = 1.36$ , for all materials. Furthermore, the penetration depth in the rod program is not very sensitive to the initial assumed value of  $l$ , so we always set  $l = a$ , initially, but this assumption is not critical. The only input parameters needed to operate the code are the physical dimensions, velocities, and densities of the target and penetrators, plus the  $E_*$  values of the materials. The value of the plastic component of  $E_*$  for any material can be derived from the formula<sup>†</sup>

$$E_* = 0.55 C_p T_m \ln \left( \frac{\sigma_F(T_i, \dot{\epsilon})}{.08 \rho C_p T_M} + 1 \right) \quad (30)$$

as derived in our previous interim report,<sup>1</sup> where  $C_p$  is the head capacity,  $T_m$  the melting temperature,  $\rho$  the density and  $\sigma_F$  the strain rate corrected flow stress of the material. Figure 3 displays the value of  $E_*$  predicted by this formula as a function of Brinell hardness for a number of materials of interest. When elastic effects can be neglected as they can be for most armor materials, we simply take  $E_{*t}$  of the target equal to  $E_*$  in Eq. (30).

As we shall show in this report, for penetrators, the corresponding  $E_{*d}$  may be found from

$$E_{*d} = \chi E_* \quad (31)$$

where  $\chi = 0.42$  for the code ROD and  $E_*$  for the rod is computed by substituting the melting temperature, heat capacity, density and flow stress values of the rod material into (30).

<sup>†</sup>The values of the constants in (30) have been modified slightly from those given in Reference 1, as a result of more extensive impact data.

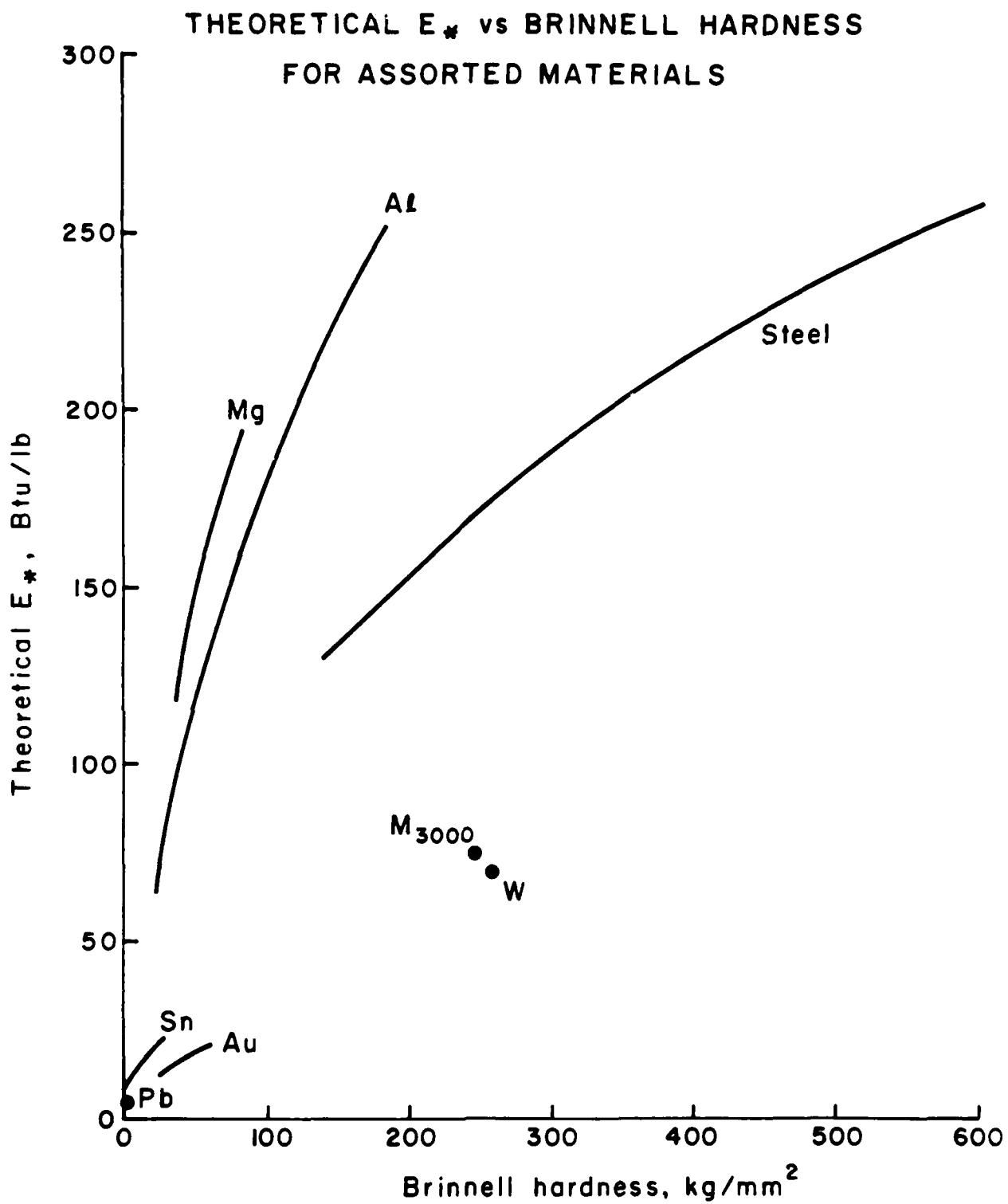


Figure 3

### III. DISCUSSION OF THE SIMPLIFIED CODE "PEN"

Recently a second model for rod penetration has been developed which is conceptually equivalent to the code ROD, but requires considerably less computer running time and is more useful for analytical investigations because of the simplicity of the equations. Most of the equations (16) - (28) are used in calculating the acceleration, energy and momentum in the flowfield of the head. These can be replaced by a term in the equation of momentum conservation (19) which accounts for accelerations in the head. The resulting model and code is named "PEN."

As in the ROD code, a two-element rod is assumed, consisting of a shaft and a head. The cutoff radius  $\epsilon_0 a$  is retained, and again is assumed to have a constant value, independent of rod material. The initial length of the head region is assumed to be  $a$ , and the volume of the head is assumed to be constant as it flattens and widens upon impact. Thus,

$$M_f = \pi a^3 \rho_p . \quad (32)$$

The decelerating force acting on the head is the sum of the decelerating forces provided by the target plus the accelerating force on the head supplied by the yield strength of the shaft:

$$M_f \dot{V}_f = - \pi b^2 \rho_t (E_* t + \frac{C_D}{2} V_f^2) + \pi a^2 \rho_p (E_* d + (V_s - V_f)^2) \quad (33)$$

where the first term on the right is the force exerted on the head by the target and the second term is the force which the shaft exerts on the head. The term  $\rho_p (V_s - V_f)^2$  accounts for the momentum gained by the head from material which has entered the head from the shaft. The front face velocity of the rod is assumed to be equal to the center of mass velocity of the head,  $V_f$ . Substituting Eq. (32) into Eq. (33) produces

$$\dot{V}_f = \frac{1}{a\rho_p} \left( -\rho_t \frac{b^2}{a^2} (E_{*t} + \frac{C_D}{2} V_f^2) + \rho_p ((V_s - V_f)^2 + E_{*d}) \right) \quad (34)$$

The corresponding equation for the velocity of the shaft is Eq. (20), which we rewrite as

$$\dot{V}_s = - \frac{1}{M_s} \pi a^2 \sigma \quad (35)$$

where

$$\sigma = \rho_p E_{*d} .$$

The penetration  $p$  of the rod is given by

$$\dot{p} = V_f , \quad (36)$$

and the erosion of the rod length  $L$  is governed by

$$\dot{L} = V_f - V_s . \quad (37)$$

The radius of the head  $b$  during the early stages of penetration when it is widening, is computed in the following way:

If  $\ell$  is the thickness of the head,

$$\pi b^2 \ell \rho_p = M_f = \rho_p \pi a^3$$

yielding

$$\ell = \frac{a^3}{b^2} .$$

But since the initial thickness of the head is  $a$ , then  $\ell = a - (L_0 - L)$ , for  $b < \epsilon_0 a$ , so

$$b = \begin{cases} \frac{a}{\sqrt{1 - \left( \frac{L_0 - L}{a} \right)}} , & \frac{L_0 - L}{a} \leq 1 - \frac{1}{\epsilon_0^2} \\ \epsilon_0 a , & \frac{L_0 - L}{a} > 1 - \frac{1}{\epsilon_0^2} \end{cases} \quad (38)$$

defines  $b$  and completes the system of equations. This system of equations, Eq. (34) - Eq. (38), defines the numerical code PEN. As with the ROD code, the only input parameters required are the initial velocity of the rod, the values of  $E_*$  for the rod and target, and the dimensions and densities of the materials.

This model can be related to hydrodynamic models of rod penetrators in the following way. During steady-state penetration of a rod,  $V_f$  typically approaches some constant value  $\sim \frac{1}{3} V_s - \frac{1}{2} V_s$ . During this stage of penetration,  $\dot{V}_f$  may be neglected in Eq. (33). Assuming  $b$  has reached its full value of  $\epsilon_0 a$ , Eq. (34) may be approximated as

$$\rho_p (V_s - V_f)^2 = \rho_t \epsilon_0^2 \frac{C_D}{2} V_f^2 + (\rho_t \epsilon_0^2 E_{*t} - \rho_p E_{*d}) \quad (39)$$

This equation may be compared to various models for rod penetration, such as that found in Ref. 6,7

$$\frac{1}{2} \rho_p (V_s - V_f)^2 = \frac{1}{2} \rho_t V_f^2 + (R - Y), \quad (40)$$

where  $R$  is the target strength and  $Y$  the strength of the penetrator. Dividing Eq. (39) by 2 and comparing coefficients, we find

$$\begin{aligned} \frac{1}{2} \epsilon_0^2 C_D &\longleftrightarrow 1 \\ \frac{\rho_t \epsilon_0^2 E_{*t}}{2} &\longleftrightarrow R \\ \frac{\rho_p E_{*d}}{2} &\longleftrightarrow Y \end{aligned} \quad (41)$$

Now the values of  $R$  and  $Y$  which give the best fit to experiment for a number of materials have been deduced in Refs. (6-8). In Table 1, we compile the experimental values<sup>6-8</sup> of  $R$  and  $Y$  as well as our theoretical prediction of them based on Eq. (41) and the theoretical value for  $E_*$ , Eq. (30). The good agreement

TABLE 1

| Material      | $\rho$<br>(kg/m <sup>3</sup> ) | $E_*$ (BTU/lb) | $\gamma$ THEOR.<br>(MPa)<br>( $\rho E_*/2$ ) | $\gamma$ EXP.<br>(MPa) | $R$ THEOR.<br>(MPa)<br>( $\rho E_* \epsilon_o^2/2$ ) | $R$ EXP.<br>(MPa) |
|---------------|--------------------------------|----------------|----------------------------------------------|------------------------|------------------------------------------------------|-------------------|
| Cu            | 3920                           | 38             | 398                                          | 380                    | 1150                                                 |                   |
| 1090 Steel    | 7800                           | 130            | 1195                                         | 1200                   | 3453                                                 |                   |
| Ta            | 16900                          | 61             | 1216                                         | 900                    | 3514                                                 |                   |
| Be            | 1881                           | 407            | 888                                          | 350                    | 2566                                                 |                   |
| U             | 18900                          | 61             | 1337                                         | 1100                   | 3866                                                 |                   |
| Mg(AZ80F)     | 1800                           | 169            | 353                                          | 340                    | 1020                                                 |                   |
| Al(6061)      | 2700                           | 160            | 501                                          | 420                    | 1031                                                 |                   |
| St 37 (B=135) | 7800                           | 135            | 1222                                         |                        | 3531                                                 | 3450              |
| St 52 (B=180) | 7800                           | 145            | 1312                                         |                        | 3791                                                 | 4400              |
| H2B20 (B=300) | 7800                           | 192            | 1737                                         |                        | 5019                                                 | 5175              |
| St (B=230)    | 7800                           | 160            | 1448                                         | 1100                   | 4184                                                 |                   |
| D17 (B=270)   | 17000                          | 80             | 1577                                         | 1550                   | 4557                                                 |                   |
| Al(7075-T6)   | 2700                           | 250            | 783                                          |                        | 2262                                                 | 2100              |

indicated by Table 1 strongly indicates that the parameters  $R$  and  $Y$  employed in the hydrodynamic models are related to the single fundamental material property  $E_*$  by Eq. (41).

Conceptually, then, the rod program PEN is equivalent to a hydrodynamic model with strength in the target and rod, such as Eq. (40), plus an additional term proportional to the deceleration of the rod front face, which takes into account inertial effects at the front of the rod during the early stages of impact before equilibrium of pressures has been established. This inertial term  $M_f \dot{V}_f$  acts as an effective stiffening or strengthening of the rod during this early phase of penetration, and accounts for the observed fact that rods of lower  $L/D$  have a greater penetration vs. rod length,  $P/L$ , than large  $L/D$  rods do. Were it not for this term, and an additional small effect due to the initial spreading of front face, rods of all  $L/D$  would have the same penetration vs. length at the same velocity, as the hydrodynamic theory of Eq. (40) predicts.

The program PEN has been extremely useful as a means of gaining an intuitive understanding of rod penetration, since it can predict rod performance over the same range of materials and velocities as the ROD code, yet has simpler equations which can be dealt with and understood algebraically.

In Section VI, the predictions of PEN are compared to the code ROD and to experimental data for a wide range of materials and velocities. We have found that the value of  $\epsilon_0$  and  $\chi$  which gives the best fit to experiment for PEN is  $\epsilon_0 = 1.7$ ,  $\chi = 1.0$ , and  $C_D = 0.5$ . These values are used in all computations employing the PEN code.

The relationship between the material strengths  $Y$  and  $R$  of Table 1 is reminiscent of the relationship between the uniaxial tensile strength and the Brinell hardness of a material in static tests. It is well known that the Brinell hardness  $B$  for a ductile material, which is just the pressure that a ductile target can sustain when impressed by a rigid ball indenter, may be related to the uniaxial flow stress of a rod of the same material by

$$\sigma \cong (0.3)B$$

Similarly, the effective strength  $R$  of the target, which is analogous to  $B$ , may be related to the uniaxial strength  $Y$  of the rod, which is analogous to  $\sigma$ , by

$$\frac{Y^{\text{THEOR.}}}{R^{\text{THEOR.}}} = \frac{\rho_p E_{*t}/2}{\rho_p E_{*t} \epsilon_0^2/2}$$

$$\longrightarrow Y^{\text{THEOR.}} = .34 R^{\text{THEOR.}}$$

This strongly suggests that the relationship between  $Y$  and  $R$  is just that between a uniaxial tensile test and a Brinell hardness test done at the strain rates of impact. Thus  $\rho E_{*t}$ , which determines both  $R$  and  $Y$ , is a true measure of a materials' strength at impact strain rates. The shear heating process in the deforming material at these strain rates is adiabatic rather than isothermal, since heat is generated locally in regions of shear much faster than it can dissipate by thermal conduction. When the local heating, with the attendant local softening of the material, is considered, as we discussed in Reference 4, Eq. (30) is derived for the effective material strength at impact strain rates. We refer to  $\rho E_{*t}$  as the "adiabatic hardness" of a material, and note that it determines both target and rod strengths at these strain rates.

The quasi-hydrodynamic model of Eq. (39) is useful in another way as a tool for analyzing qualitatively different regimes of penetration. If the target is very hard (high  $E_{*t}$ ) then the rod will not penetrate unless its velocity is sufficiently high to overcome the target strength with kinetic energy. The condition for the lower limit of velocity required for penetration is formed by setting  $V_f = 0$  in (39) and solving for  $V_s$ :

$$(V_s)_0 = \sqrt{\frac{\rho_t}{\rho_p} \epsilon_0^2 E_{*t} - E_{*d}}$$

Penetration will not occur unless the initial rod velocity exceeds  $(V_s)_0$ , according to this model. In reality, some penetration does occur below this velocity but, as in Figs. 31, 32 and 33, there usually is a long straight section of the P/L vs velocity curve which, when extrapolated to zero penetration, intersects the velocity coordinate at a value given approximately by  $(V_s)_0$ . See Ref. 6 for a discussion of this relating to the data in Fig. 31. The discrepancy at velocities below  $(V_s)_0$  occurs because we have neglected the  $\dot{V}_f$  term in Eq. (39).

When the rod is very strong compared to the target, it may not erode at all, and then it behaves like a nondeforming rod. This limit, in which  $V_f = V_s$ , will occur when

$$(V_s)_0 = \sqrt{\frac{\rho_p E_{*d} - \rho_t \epsilon_0^2 E_{*t}}{\rho_t \epsilon_0^2 \frac{C_D}{2}}}.$$

If the quantity under the radical is  $>0$ , there will exist a value  $(V_s)_0$  for which penetration of the rod can occur without erosion. For any initial velocity below  $(V_s)_0$  the rod behaves as a nondeforming projectile, and the A.R.A.P. integral theory for nondeforming projectiles is employed instead of the deforming rod equation. For velocities above  $(V_s)_0$  the deforming rod model applies. Figure 35, discussed in Section VI, affords a striking example of the transition from nondeforming penetration to deforming rod penetration as the striking velocity is increased.

#### IV. BACKFACE EFFECTS

In order to apply the A.R.A.P. integral theory to targets of finite thickness, backface effects must be included. The  $E_*$  concept was originally developed for the flow of target material around a penetrator in a semi-infinite target. The shear work done on the target material in the flow volume defines  $E_{*t}$ , the  $E_*$  for the target. When the projectile has penetrated almost all the way through the target, to within one or two diameters of the backface, the target material can spall or simply bulge on the backside, rather than flowing around the penetrator hydrodynamically. Thus, each small volume of target material absorbs less energy than it would in the semi-infinite case. Thus, the effective  $E_*$  for the target decreases near the backface, and we call this the backface effect.

In order to characterize the backface effect empirically, static Brinell hardness tests were performed on 1100-F aluminum and lexan sheets unsupported at the back, using a .250" diameter WC ball at very shallow and very deep penetrations. At the deep penetrations, backface effects in the target sample affected the hardness measurement. In Fig. 4, the Brinell hardness of a 1/4" thick 1100-F aluminum plate is measured vs. penetration depth of the ball, and plotted as curve (a). The hardness is roughly constant with penetration until the ball is about .150" from the backface, at which point the hardness begins to decrease linearly with further penetration. When the front face of the ball reaches .475" of "penetration," so it has actually passed through the plate, the bulge on the back of the plate fractures and the measured hardness drops to zero. For comparison, the hardness vs. penetration depth for the same WC ball in a semi-infinite 1100-F aluminum plate is plotted as curve (b). The ratio of measured hardness in the .250" thick plate to the measured hardness in the semi-infinite plate is plotted on the same graph as a solid curve, (c). It is clear that as the ball approaches within a diameter or so of the backface, the hardness begins to decrease monotonically with penetration. We should expect that  $\rho E_*$ , which

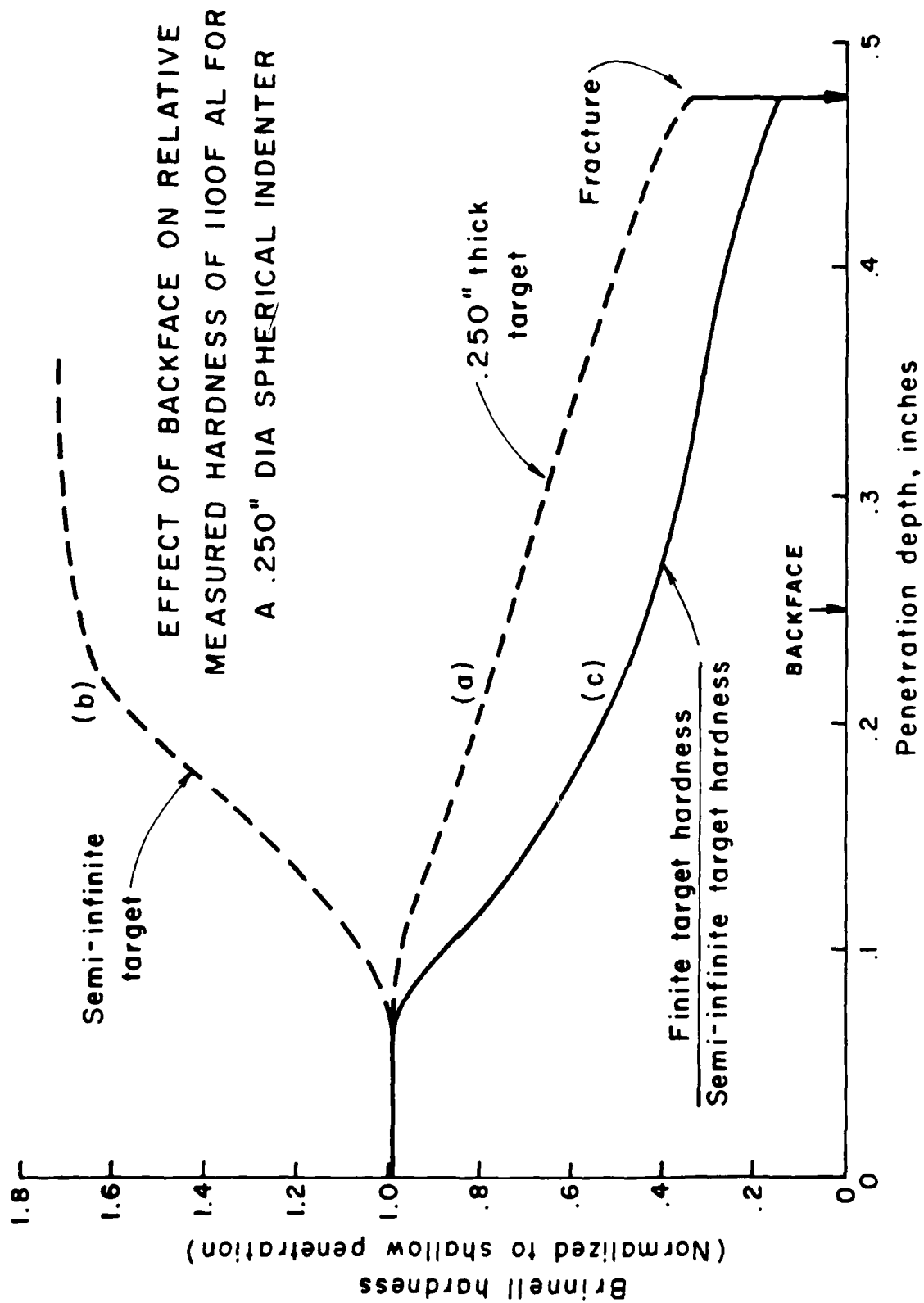


Figure 4

measures the target strength at the strain rates of impact, should decrease in roughly the same way as the measured static hardness does near the backface.

In a second experiment, the Brinell hardness at deep penetration in lexan plates was measured over a range of plate thickness and ball diameters. The results are shown in Fig. 5. It was found that the results could be fit empirically by

$$\text{Brinell hardness } B = \begin{cases} B_0, & \tau - p \geq (\beta - 1)r \\ \frac{\tau - p + r}{\beta r} B_0, & \tau - p < (\beta - 1)r \end{cases} \quad (42)$$

where  $\beta = 4$ ,  $B_0$  is the hardness of a semi-infinite lexan plate,  $\tau$  is the plate thickness,  $p$  the penetration depth of the ball, and  $r$  the contact radius of the depression made by the ball in the target. Obviously,  $r \leq a$ , where  $a$  is the ball radius, and

$$r = \begin{cases} \sqrt{2ap - p^2}, & p \leq a \\ a, & p \geq a \end{cases} \quad (43)$$

In Fig. 6, this expression is compared with the measured value of relative hardness at various penetrations for the aluminum plate discussed in Fig. 4. The agreement is qualitatively good, although there are certainly other expressions which would characterize the hardness near the backface as well.

The form of expression, (Eq. (42)), was chosen as our model for the backface because of the following intuitive model for backface effects. We assume that the flowfield of target materials around the penetrator extends for some distance in front of the penetrator. We expect this distance to be proportional to the contact radius  $r$  of the penetrator, and to extend a distance  $\beta r$  in front of the penetrator, from the point of maximum contact width. Thus, for a spherical indenter imbedded less than one radius deep in a target, the flowfield is assumed to extend a distance  $\beta r$  from the surface of the target. Once the ball is fully imbedded, then  $r = a$ , and the flowfield extends a distance  $\beta a$  in front of the ball, as measured from the center

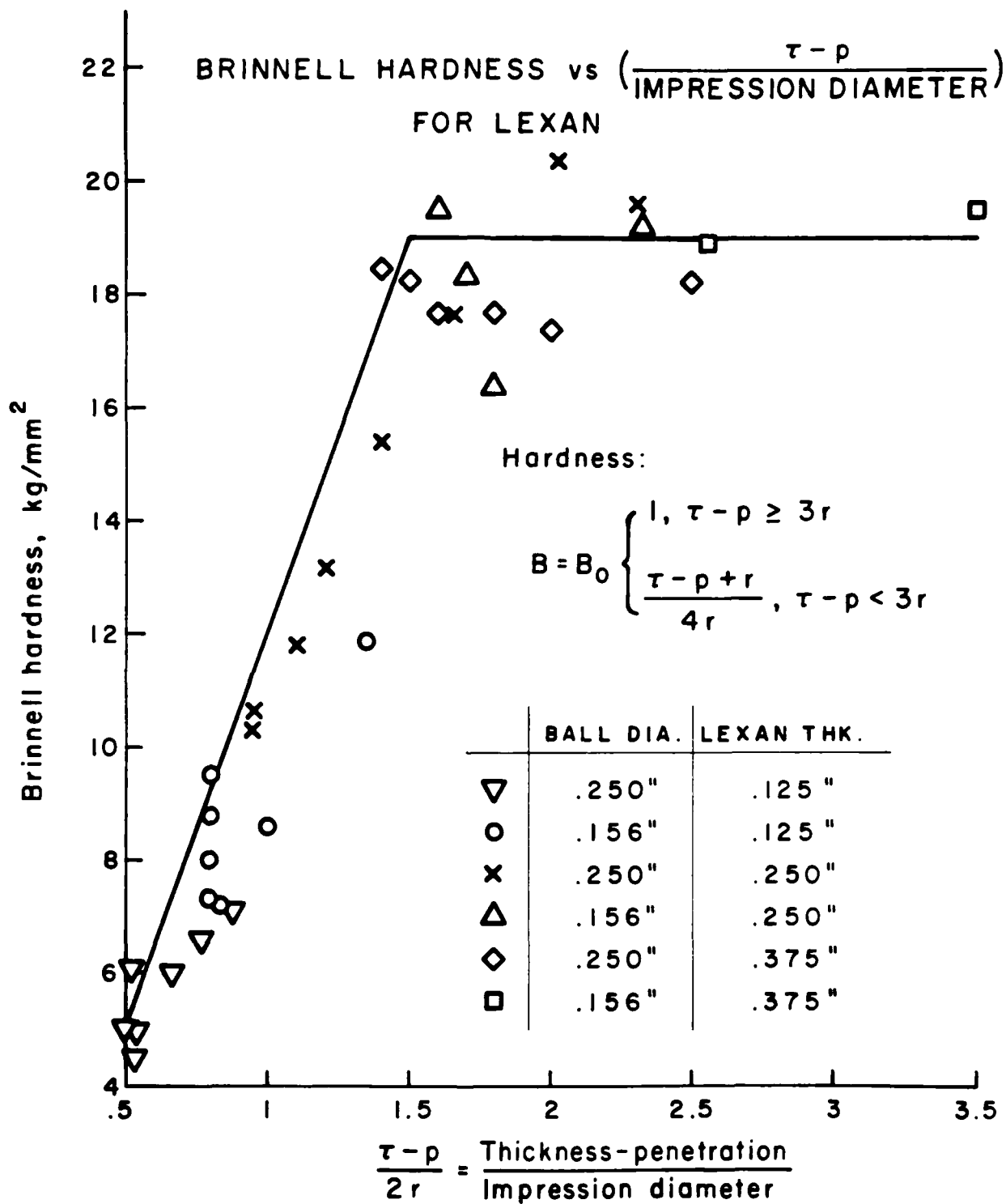


Figure 5

COMPARISON OF BACKFACE MODEL FOR  $E_*$  WITH  
RELATIVE HARDNESS OF ALUMINUM MEAS-  
URED IN STATIC TESTS

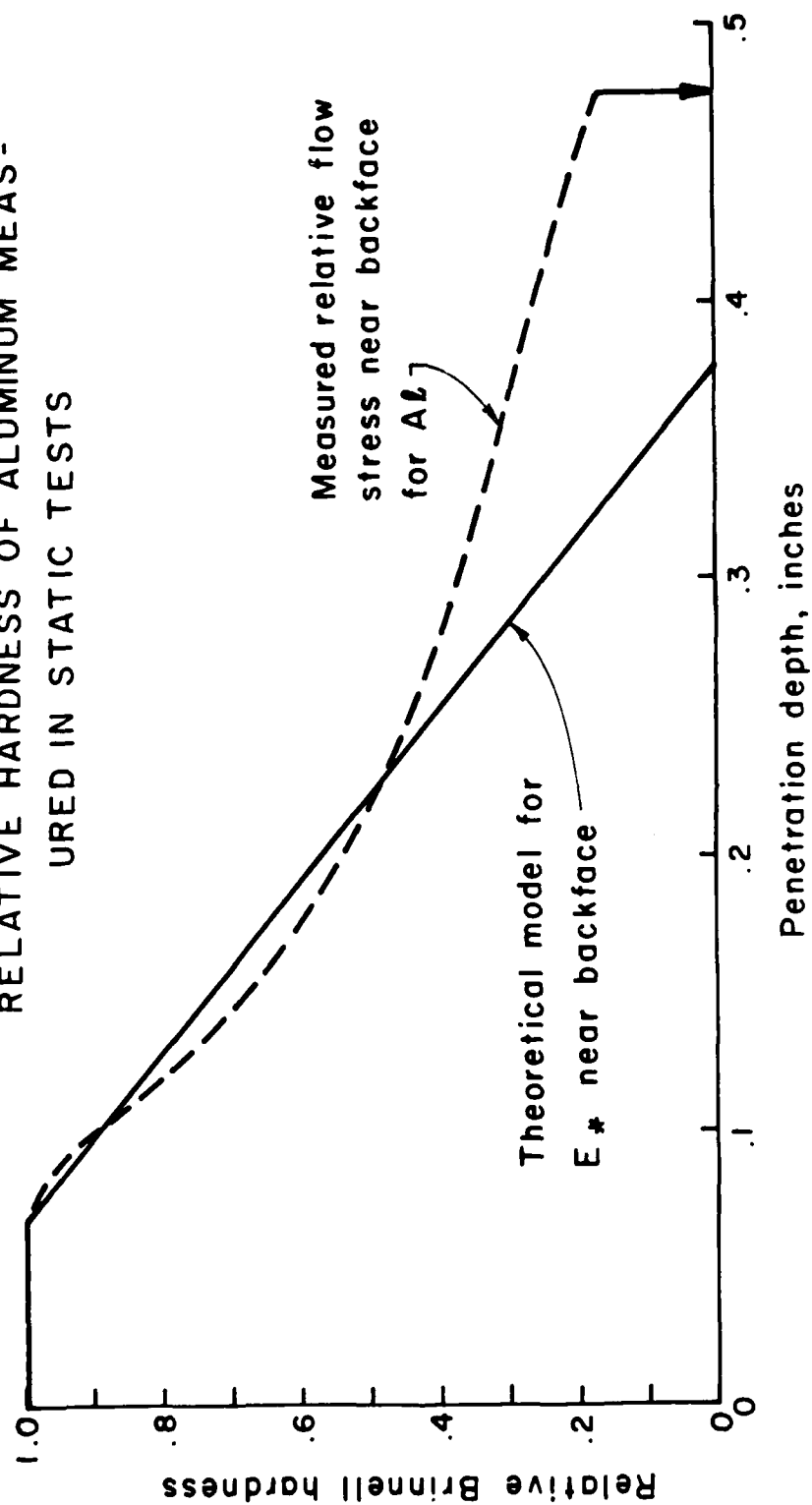


Figure 6

# COMPARISON OF BACKFACE MODEL WITH EXPERIMENT

LEAD → RHA

$E_{*f} = 203 \text{ Btu/lbm}$

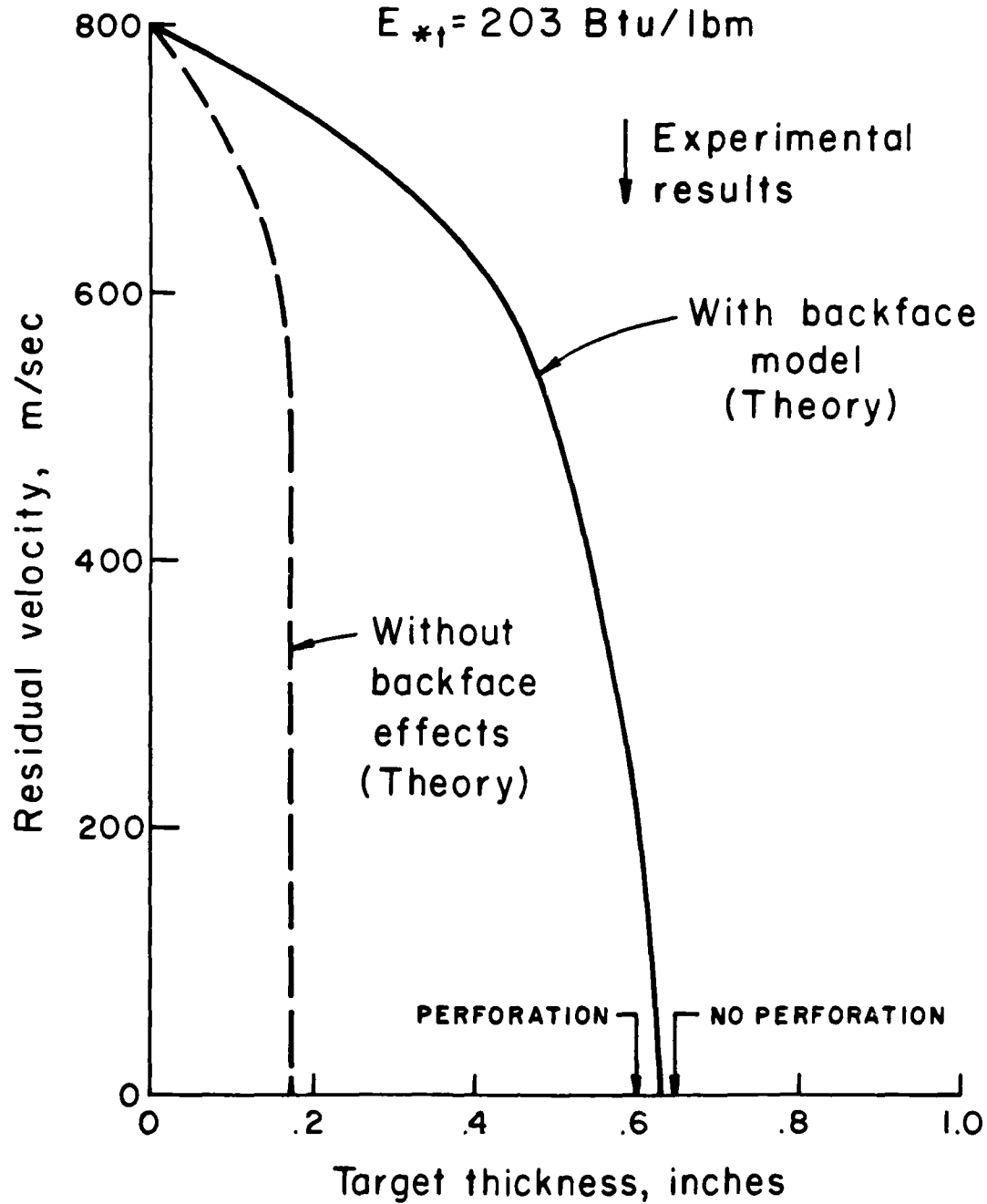


Figure 7

of gravity of the ball. Thus, the backface effect will begin to show up when  $p - a = \tau - \beta a$  for a fully imbedded ball, hence Eq. (42). For a blunt nosed object, such as a cube or cylinder impacting end on into a target, the widest point of the penetrator occurs at the leading face, unlike the sphere. Then the backface effect begins to occur when

$$p = \tau - \beta a ,$$

so we can summarize:

#### Backface Model

For a sphere:

$$E_{*t} = \begin{cases} E_* & , \tau - p \geq (\beta - 1)r \\ E_* \left( \frac{\tau - p + r}{\beta r} \right) & , \tau - p < (\beta - 1)r \end{cases} \quad (44)$$

For a cube or cylinder:

$$E_{*t} = \begin{cases} E_* & , \tau - p \geq \beta r \\ E_* \left( \frac{\tau - p}{\beta r} \right) & , \tau - p < \beta r \end{cases} \quad (45)$$

We have made the assumption here that the dynamic strength of the target,  $E_*$ , decreases near the backface in the same way as the static strength does. We do not *a priori* expect these formulas with the same constants to work well for all materials, since brittle materials will show spall and other backface effects when the penetrator is many diameters from the backface. It is possible that by making  $\beta$  inversely proportional to the failure strain of the target, the formula may be generalized. Such approaches will be considered in subsequent work. For many ductile materials, however, we have found that Eq. (44) and Eq. (45) accurately describe the decrease of  $E_*$  near the backface.

As an example, refer to Figs. 7, 8 and 9 in which nondeforming tungsten carbide balls and highly deforming lead projectiles were fired into rolled homogeneous armor, 5083 aluminum,

# COMPARISON OF BACKFACE MODEL WITH EXPERIMENT

LEAD → TITANIUM

$E_{*1} = 328 \text{ Btu/lbm}$

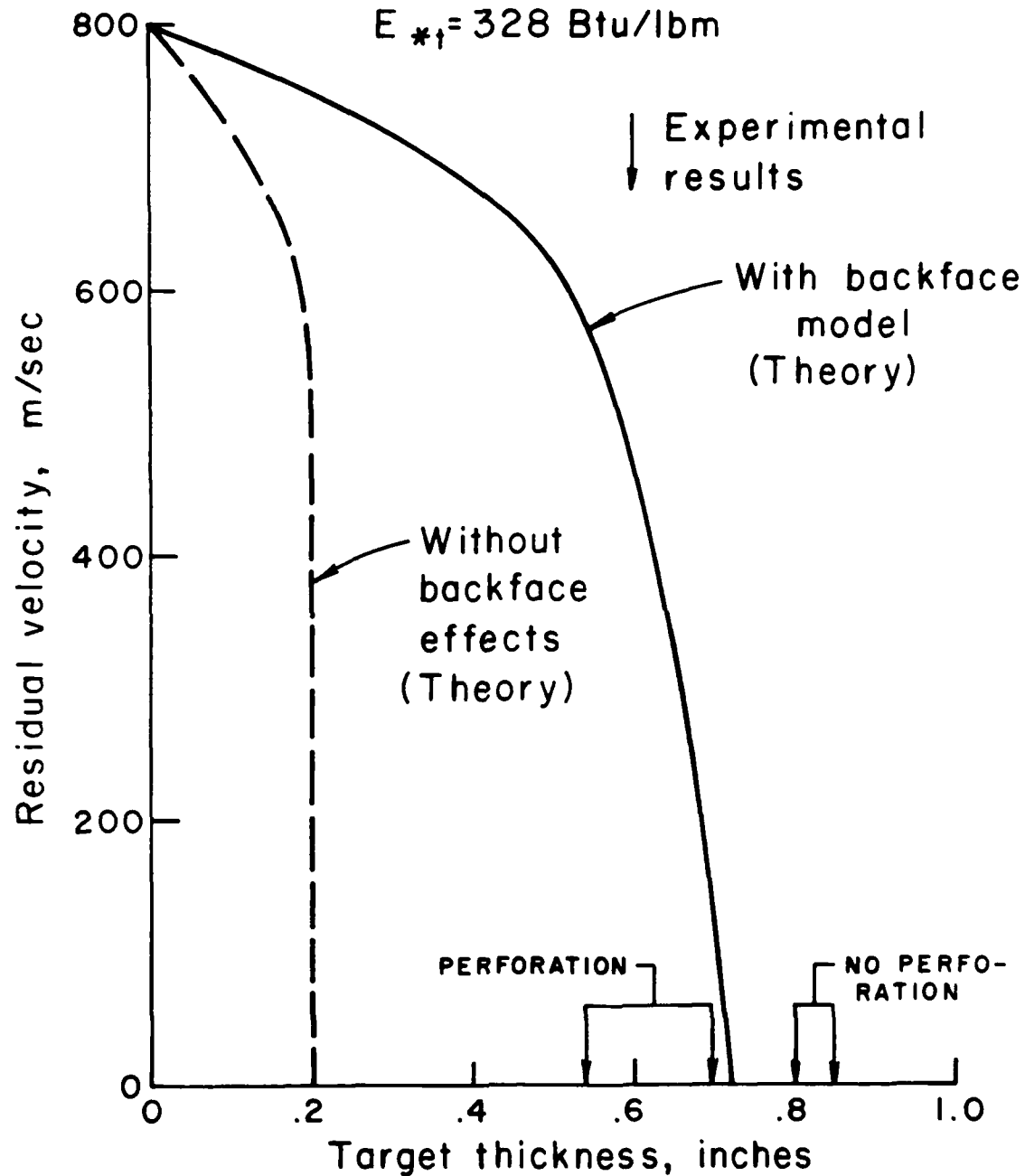


Figure 8

# COMPARISON OF BACKFACE MODEL WITH EXPERIMENT

WC → ALUMINUM

$E_{*1} = 245 \text{ Btu/lbm}$

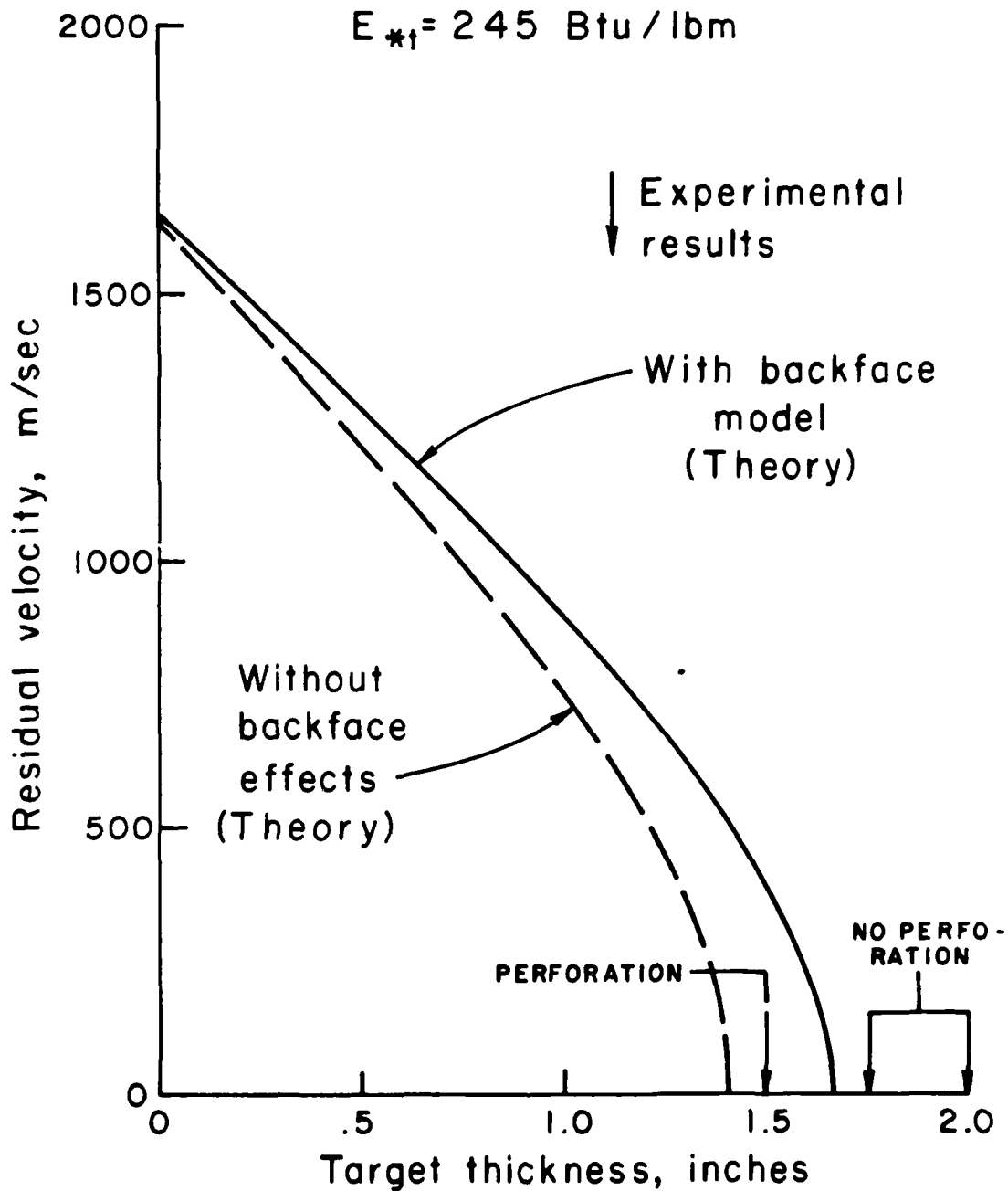


Figure 9

and titanium armor. In each case the theoretical residual velocity of the projectile is plotted vs. target thickness for a given initial projectile velocity, using the A.R.A.P. rigid sphere and deforming cube programs and the backface model of Eqs. (44) and (45). In all cases,  $\beta = 4$ . The theoretical values of  $E_*$ , which have been verified for each material in the semi-infinite case, were used in this calculation. In the same figure are plotted the predicted residual velocities for no backface effect. The value of target thickness at which the residual velocity equals zero measures the stopping thickness required for each projectile at the indicated initial velocity. Arrows on each figure indicate the experimental thickness which stopped or failed to stop the projectile. We conclude that the backface model with  $\beta = 4$  works quite well for RHA, titanium and 5083 aluminum.

It should be pointed out that there are exceptions to this model. Certain composite materials, such as fiberglass and Kevlar woven rovings are better modeled as having no backface effects, or  $\beta \ll 1$ . Similarly, brittle materials act as though  $\beta \gg 4$ . Fortunately, however, a large number of ductile materials including many important armor materials are described by  $\beta = 4$ .

## V. OBLIQUE PENETRATION

When a rod impacts an armor plate at an oblique angle, the forces on the rod will not be axially symmetric. Thus we include in the rod program a lateral force  $F_L$  acting on the head of the rod, as well as the axial force  $F_A$ , which was described in Sections II and III. A bending mode, characterized by  $u_B$ , the lateral displacement of the head relative to the axis of the shaft, and a twisting angle  $\phi$  relative to the direction of rod motion are included. In addition, the trajectory of the rod no longer will follow the initial direction of flight, so instead of one parameter  $p$  for penetration we employ  $p$  as the total length of penetration plus  $\chi$ , the angle of penetration relative to the original velocity direction. All these quantities are defined in Figs. 10a and 10b.

First, consider the dynamics of the rod itself. The total energy of the rod is

$$T = (M_s + M_f) \left( \frac{1}{2} U^2 + \frac{1}{10} \dot{u}_B^2 \right) + \frac{1}{2} I \dot{\phi}^2 \quad (46)$$

where  $I$  is the rod moment of inertia,  $U$  is the center of mass velocity of the rod and  $\dot{u}_B$  the bending velocity. Since bending and other nonaxial effects are generally small corrections to the total penetration, we treat the rod as a single element, not separating it into head and shaft, for the purpose of calculating these effects.

The variables expressing lateral deflection are then determined by

$$\ddot{\phi} = F_L \frac{(L/2)}{I} \quad (47)$$

where  $L$  is the total length of the rod, and

$$\dot{u}_B = \frac{5F_L}{M_s + M_f} \quad (48)$$

The lateral force  $F_L$  is computed in the following way. As the rod impacts a target at an oblique angle, one corner of

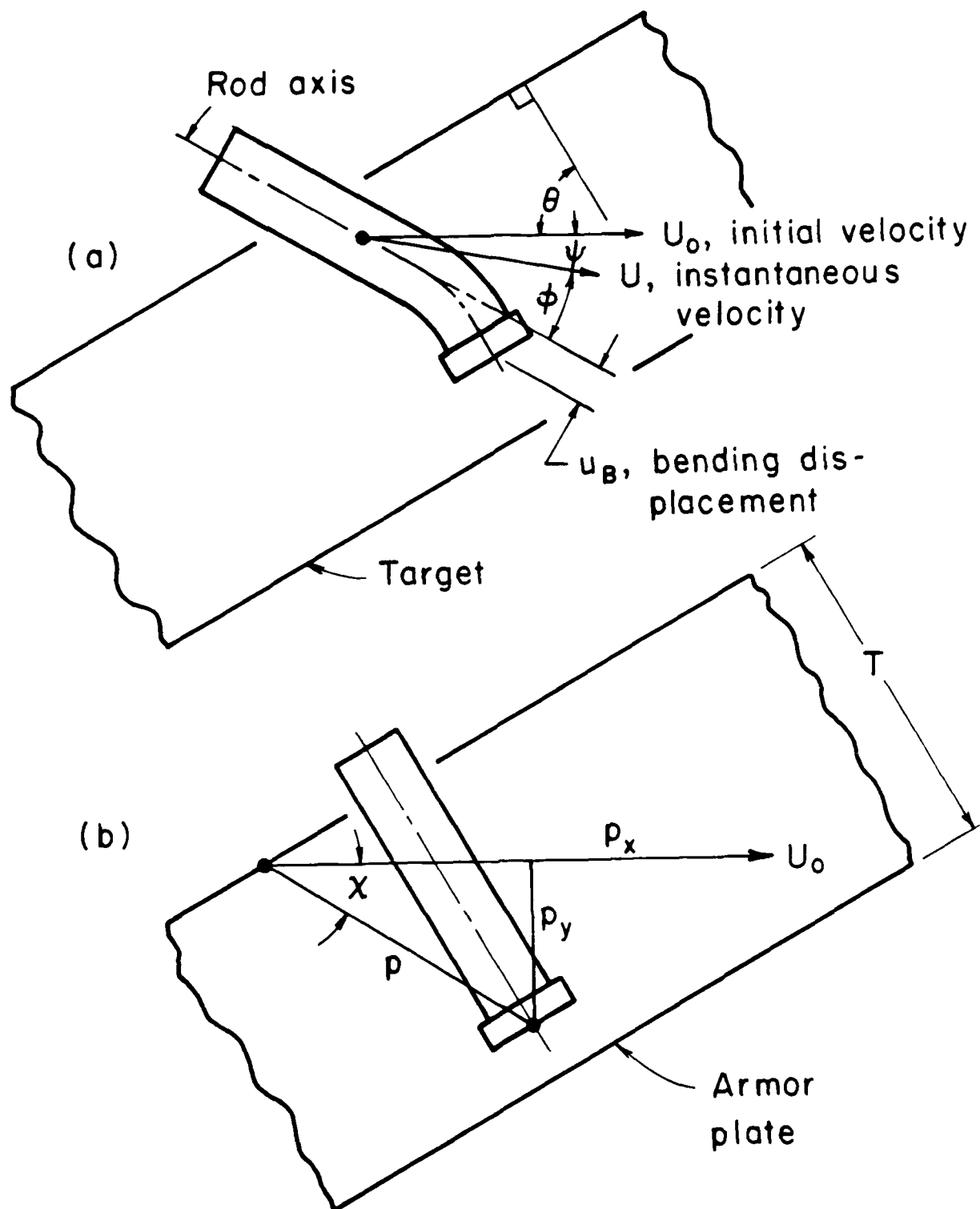


Figure 10

the front end first makes contact with the target when the center of the front face of the rod is still a distance

$$d = a \tan \theta$$

from the target, measured in the direction of the rod velocity. We assume the plastic and drag forces acting on the rod begin to increase from zero at this initial instant of contact and rise linearly to their fully-imbedded values when the face of the rod is fully imbedded in the target. The penetration  $p$  is measured in the program from this initial contact point. At this point the center of the front face of the rod is a distance  $(a \cdot \sin \theta)$  from the target, measured along a normal to the target face. Thus, we treat the target as having an effective thickness  $\tau = T + a \sin \theta$ , where  $T$  is the true target thickness. The total plastic and drag pressure exerted on the head of the rod is computed from (1), and  $F_A$  and  $F_L$  are found by multiplying this pressure by the front face area and lateral area, respectively, in contact with the target. The head of the rod will begin to widen as soon as the decelerating force of the target acts on it. Full embedding of the head (see Fig. 10) occurs when

$$p = p_0 \equiv (a + b) \tan (\theta - \chi) \quad (49)$$

The pressure at the contact interface is

$$p_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{x})^2)$$

and according to our assumption of a linear initial increase in the contact area, the force exerted on the front face of the rod is

$$F_A = \frac{p}{p_0} \pi b^2 \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{x})^2), \quad p \leq p_0 \quad (50)$$

and the force exerted on the side face is

$$F_L = \frac{p}{p_0} 4\pi \ell b \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{x})^2), \quad p \leq p_0 \quad (51)$$

We assume this linear increase in contact area holds true up to the point of full embedding at the penetration  $p = p_0$ . For  $p > p_0$ ,  $F_L$  drops to zero since the lateral forces acting on all sides of the head cancel each other, yielding

$$\left. \begin{aligned} F_A &= \pi b^2 \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{x})^2) \\ F_L &= 0 \end{aligned} \right\} p > p_0 \quad (52)$$

Upon exiting from the backface of the target, the backface model described in the previous section is generalized to oblique exit in the following way.  $E_{*t}$  and the drag coefficient  $C_D$  near the backface are assumed to decrease according to (45), where the distance from the penetrator to the backface is taken as the projected distance measured normal to the backface. The reduced values of  $E_*$  and  $C_D$  are substituted directly into the formula for  $F_A$ . Near the backface, the projected distance from the center of the rod face to the backface is  $\tau - p \cos(\theta - \chi)$ , and the expression for the effective  $E_{*t}$  corresponding to (45) is

$$E_{*t} = \alpha E_* \quad (53)$$

where  $\alpha$  is defined by

$$\alpha = \begin{cases} 1 & , \quad \tau - p \cos(\theta - \chi) \geq \beta r \\ \frac{\tau - p \cos(\theta - \chi)}{4\beta} & , \quad \tau - p \cos(\theta - \chi) < \beta r \end{cases} \quad (54)$$

A similar dependence is assumed for  $C_D$  near the backface.  $\tau$  is the effective plate thickness  $(T + a \sin \theta)$  measured from the point of initial contact. As the axial force decreases near the backface, the lateral force  $F_L$  increases because of the imbalance in the effective  $E_*$  of the target on the sides of the head. Therefore, we assume

$$F_L = (1 - \alpha) (4\pi b l \rho_t (E_{*t} + \frac{C_D}{2} (V_f + \dot{x})^2)) \quad (55)$$

These assumptions completely specify the backface effects for oblique exit from the target.

The equations above, in addition to those described in Section III, are integrated numerically to predict the residual mass, residual velocity, ballistic limit velocity, and other parameters for oblique penetration as well as normal penetration. The oblique model described here has been included in the PEN code, but has not thus far been added to the code ROD. A copy of the code PEN is reproduced in Appendix II. The input parameters which require specification are the geometric dimensions, densities and  $E_*$  values of the target and penetrator materials, and the initial striking velocity and obliquity angle of the penetrator. The output includes ballistic limit velocity or penetration depth, residual mass and residual velocity of the penetrator.

## VI. COMPARISON OF EXPERIMENT WITH THEORY

In order to check the programs ROD and PEN against experiment under controlled conditions, several long rods of  $L/D = 10$  were fired for us into Rolled Homogeneous Armor targets of Brinell hardness  $290 \text{ kg/mm}^2$  by the Ballistics Research Lab, Aberdeen Proving Grounds. In order to avoid back-face effects which might modify the effective  $E_*$  of the target in these initial experiments, very thick targets of thickness greater than twice the total rod penetration were used. The rods were chosen to provide a variety of materials and strengths, from 1018 steel to soft lead to Mallory 3000, a tungsten alloy. The resulting data were compared to predictions from the ROD and PEN programs to select a best fit value of  $\epsilon_0$ , which characterizes the maximum head width, and  $\chi$ , which relates  $E_{*d}$  to the rod strength in Eq. (31). The theoretical values of  $E_*$  for the target and penetrator material were used, based on formula (30) and the melting temperature, hardness and heat capacity of the respective materials. The best fit values for ROD were found to be  $\epsilon_0 = 1.36$ ,  $\chi = .42$ , and for PEN,  $\epsilon_0 = 1.7$  and  $\chi = 1.0$ .

A comparison of the resultant theoretical predictions with experiment for the code ROD is shown in Figs. 11 through 13. The corresponding fit to the data for the code PEN is shown in Fig. 14. The high velocity lead rod deformed upon exit from the gun barrel and had a highly irregular shape and  $L/D \sim 5$  upon impact at the target. We have used  $L/D = 5$  in computing the theoretical penetration for this data point. Agreement with experiment in all cases is within about 15%.

Next, the code was tested for rods against finite thickness targets at normal incidence. The values of the parameters  $\chi$  and  $\epsilon_0$  found above were kept the same. In the numerical code this set of experiments amounts to a test of the accuracy of the backface model, presented in Section IV. The ballistic limit velocity  $V_{BL}$  was determined in the code by incrementally raising the striking velocity of the rod until penetration was

# TANTALUM & MILD STEEL RODS INTO SEMI-INFINITE RHA

$L/D=10$

(ROD PREDICTION)

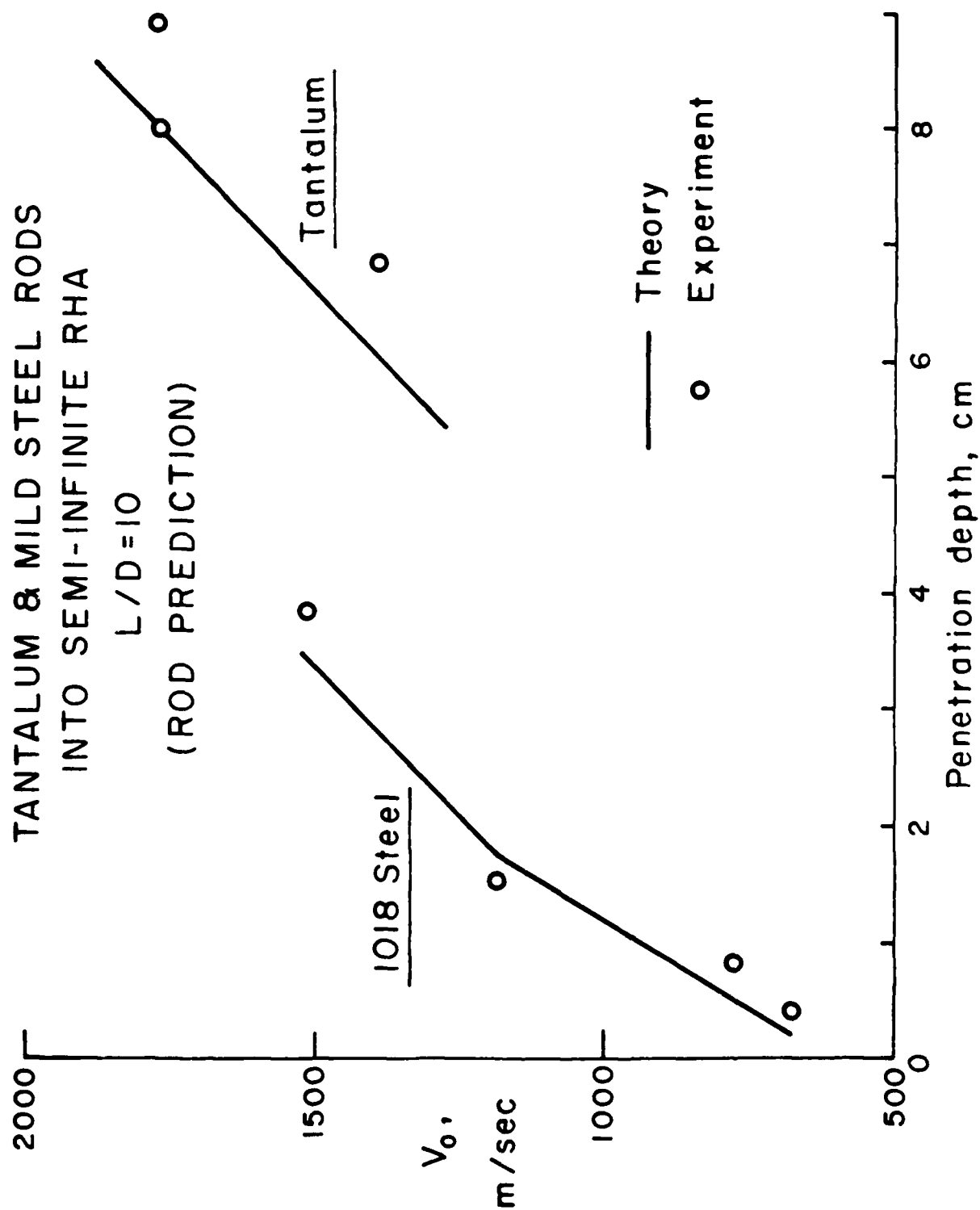


Figure 11

MALLORY 3000 INTO  
SEMI-INFINITE RHA

(ROD CODE)

$L/D = 10$

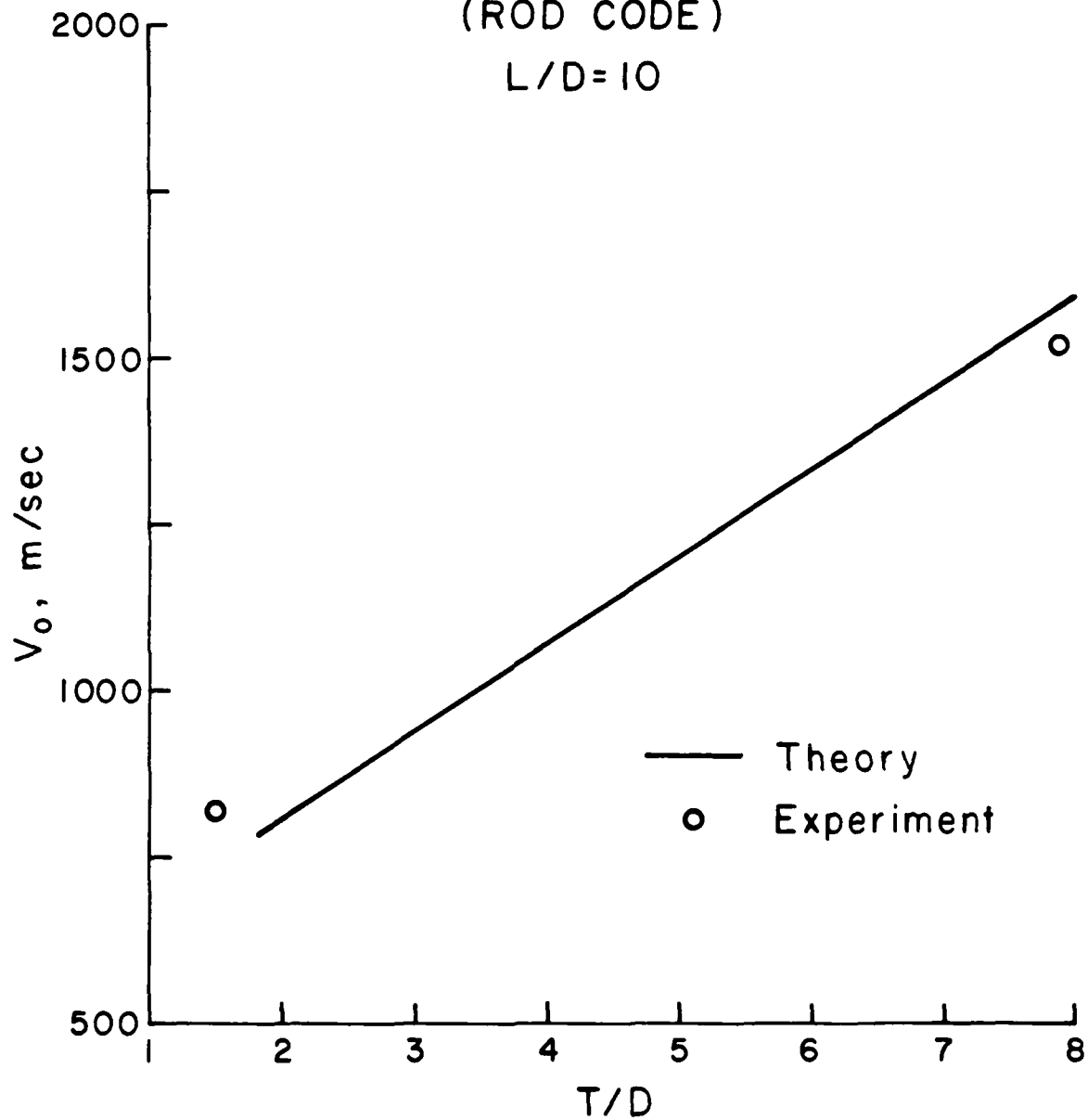


Figure 12

# LEAD INTO SEMI-INFINITE RHA

(ROD CODE)

$L/D=10$

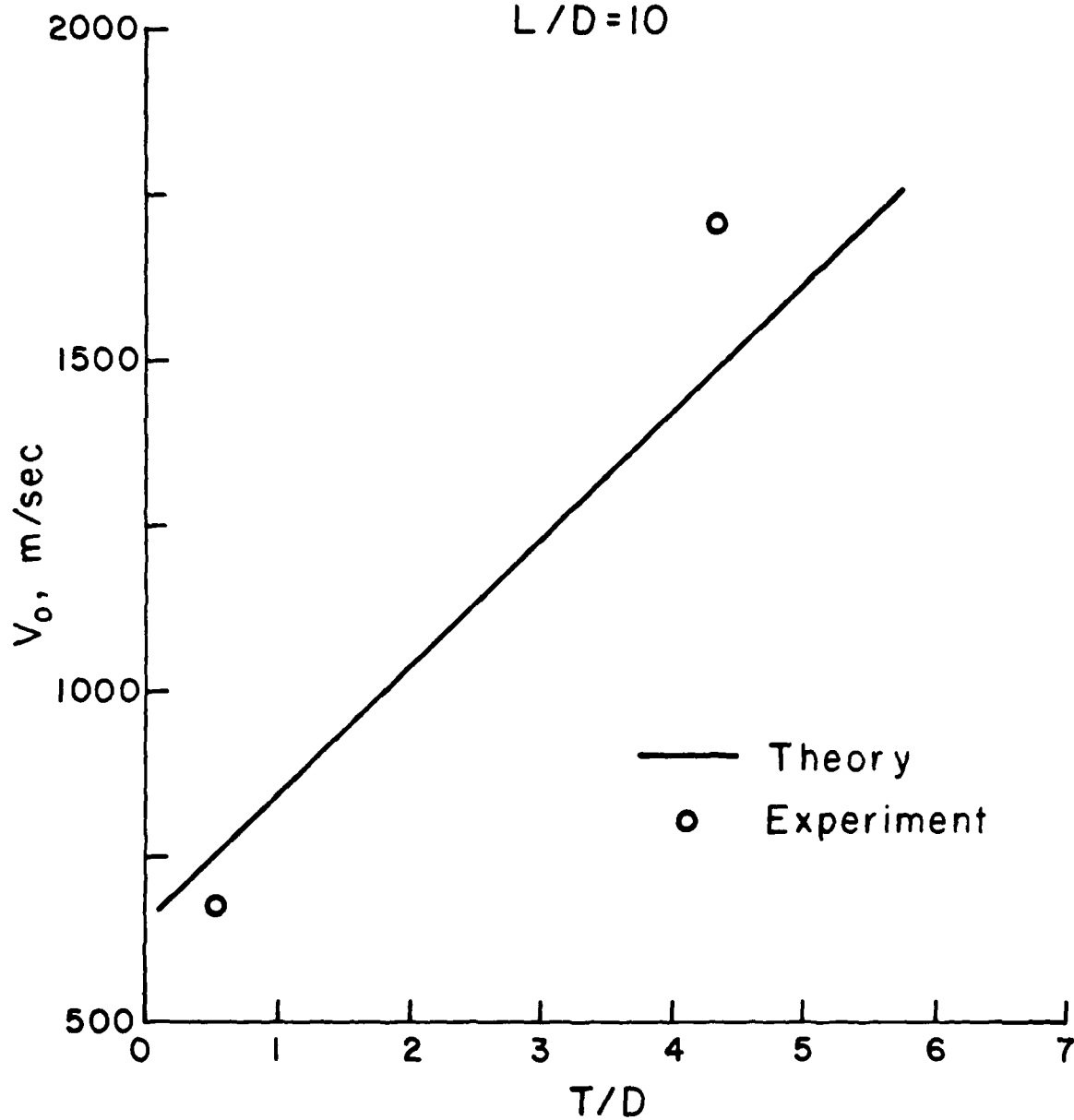


Figure 13

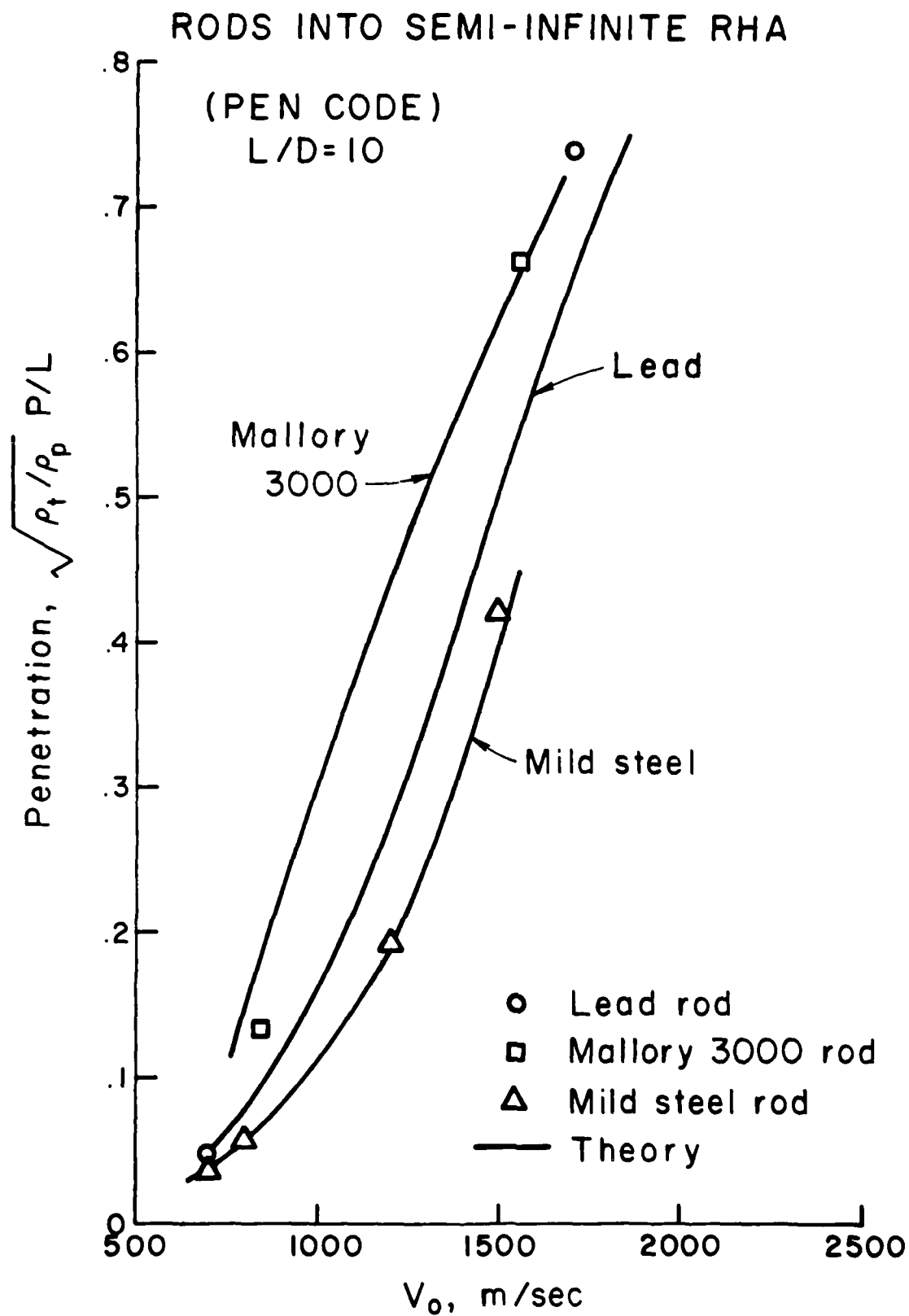


Figure 14

achieved. This theoretical  $V_{BL}$  from the ROD code was compared to one set of experiments (Lambert<sup>9</sup>) with 65 gm Bearcat steel rods of  $L/D = 5$ , 10 and 20 into RHA targets of various hardness from BHN = 260 to 375 kg/mm<sup>2</sup>. These data are presented in Figs. 15 through 17. Agreement is excellent, although there is a slight tendency for the higher  $L/D$  rod to overpenetrate and the lower  $L/D$  rod to underpenetrate. In Figs. 18 through 20 these data of Lambert are plotted in dimensionless form against another set of experiments (Herr<sup>10</sup>) in which 1.94, 3.89, and 7.78 gram Bearcat steel rods were fired into RHA plates which were annealed to a Brinell hardness of 400. In Figs. 18 through 20 the plate thickness is expressed in units of  $T/D$  or (thickness)/(rod diameter), since it is the rod width  $D$  that sets the dimension for backface effects. It is apparent in the figures that the two sets of experimental data do not overlap, particularly for  $L/D = 10$  and 20. This can be shown to be a result of the different Brinell hardnesses of the armor targets used in the two sets of experiments.<sup>11</sup> Note, for example, that the 1.94, 3.89 and 7.78 gram rods do fall on the same ballistic curve. In Herr's experiments, all targets were heat treated to a uniform hardness of BHN = 400. Thus, they all had the same value of  $E_*$ , about 215 Btu/lb. In Lambert's data, the Brinell hardness varies from about 260 to 375 kg/mm<sup>2</sup> with corresponding values of  $E_*$  ranging from 170 to 210 Btu/lb. The theoretical curves corresponding to these two sets of  $E_*$  values are plotted in Figs. 18 through 20, where the average  $E_*$  of Lambert's targets, 190 Btu/lb, is used. The theory clearly shows the same shift in  $V_{BL}$  with hardness that is seen in the experimental data. Thus, the different values of  $E_*$  corresponding to different Brinell hardnesses, as predicted by Eq. (30), account for the apparent failure of scaling in the data. This set of experiments provides a sensitive test of the ability of the Integral Theory to predict the effect of materials properties, such as hardness, on penetration and ballistic limit.

BEARCAT ROD INTO RHA

(ROD CODE)

$L/D = 5$

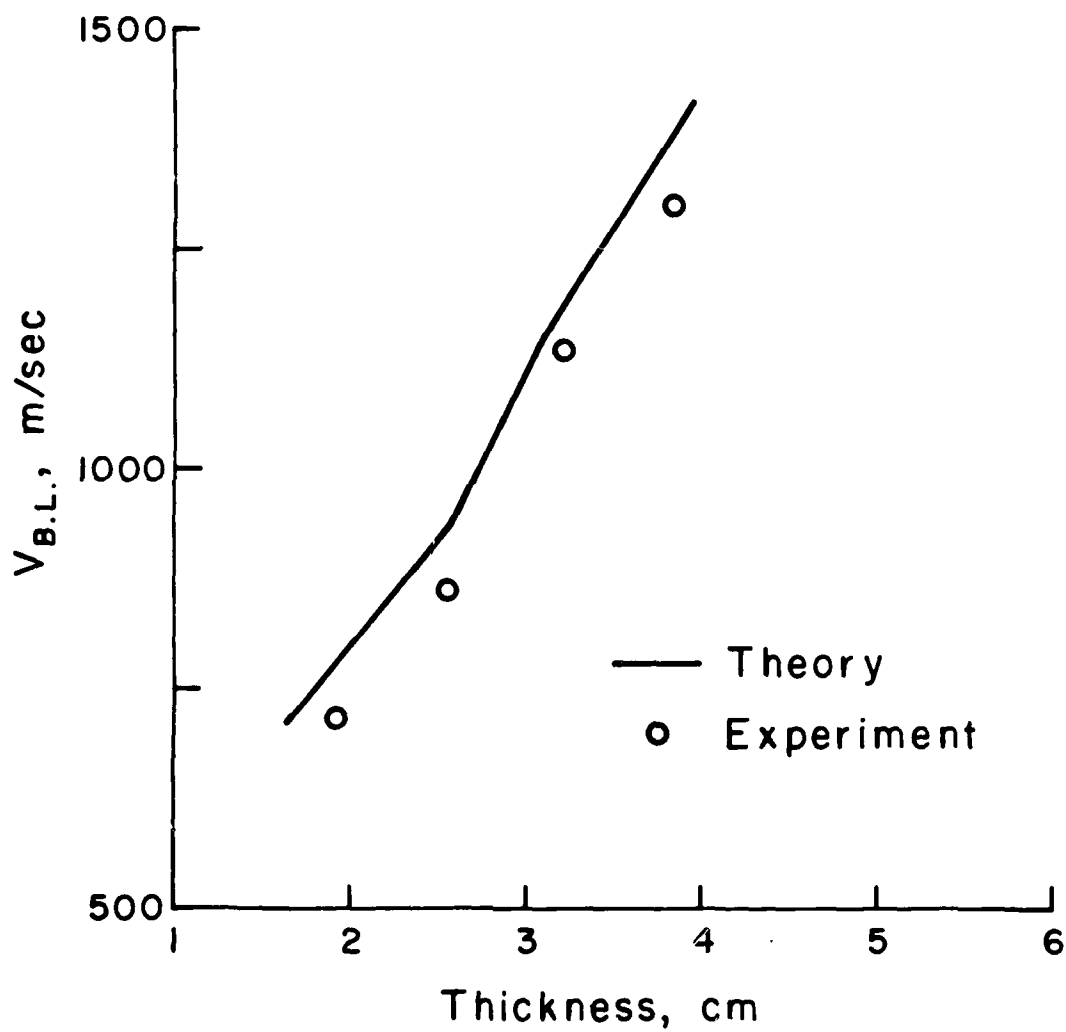


Figure 15

BEARCAT ROD INTO RHA  
(ROD CODE)  
 $L/D = 10$

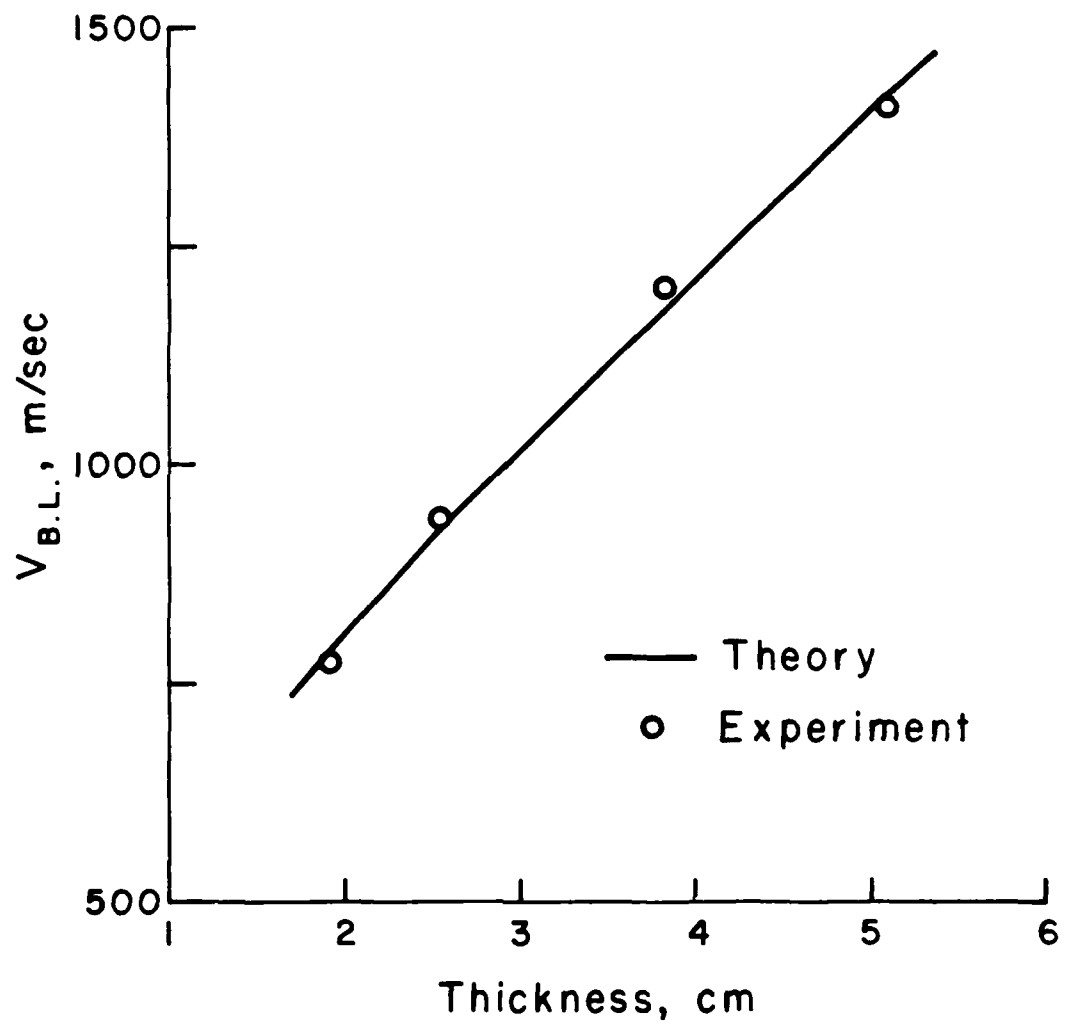


Figure 16

BEARCAT ROD INTO RHA  
(ROD CODE)  
 $L/D = 20$

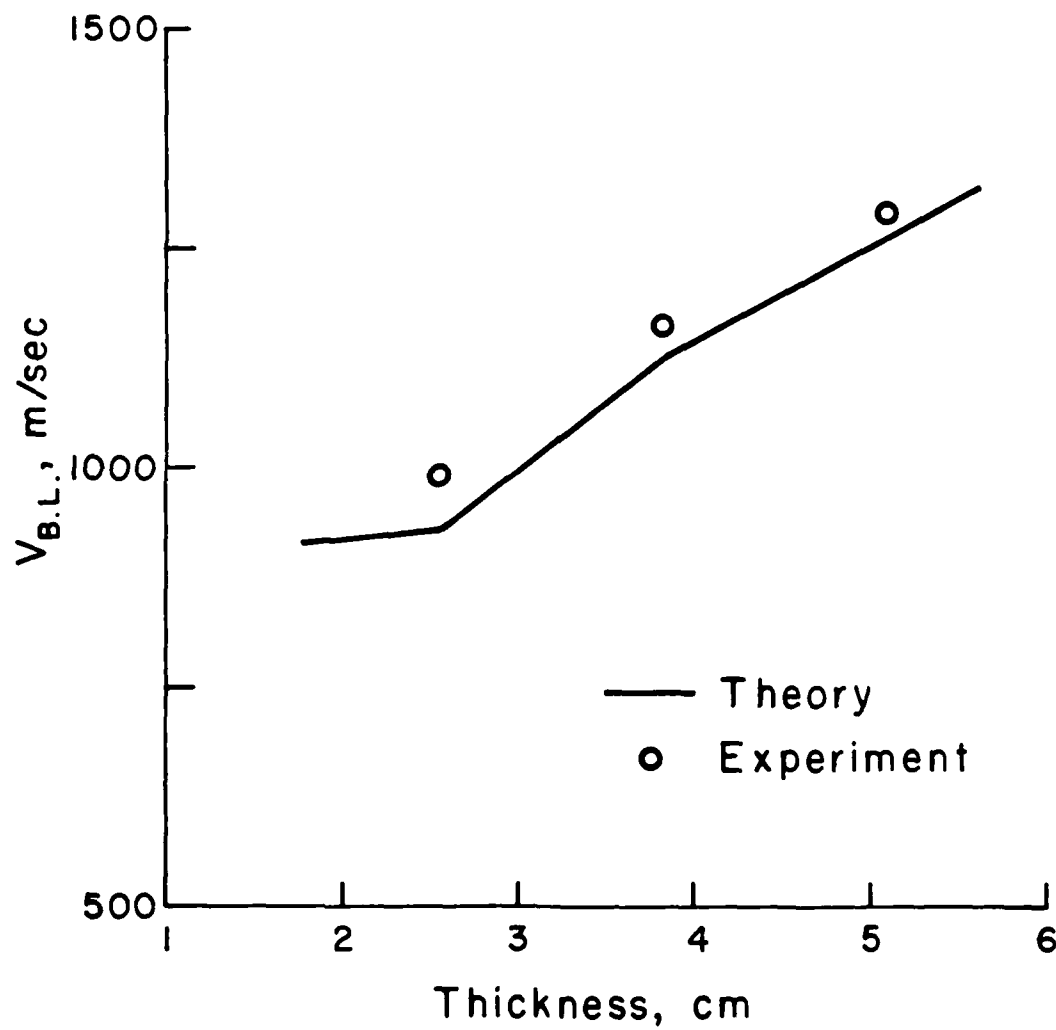


Figure 17

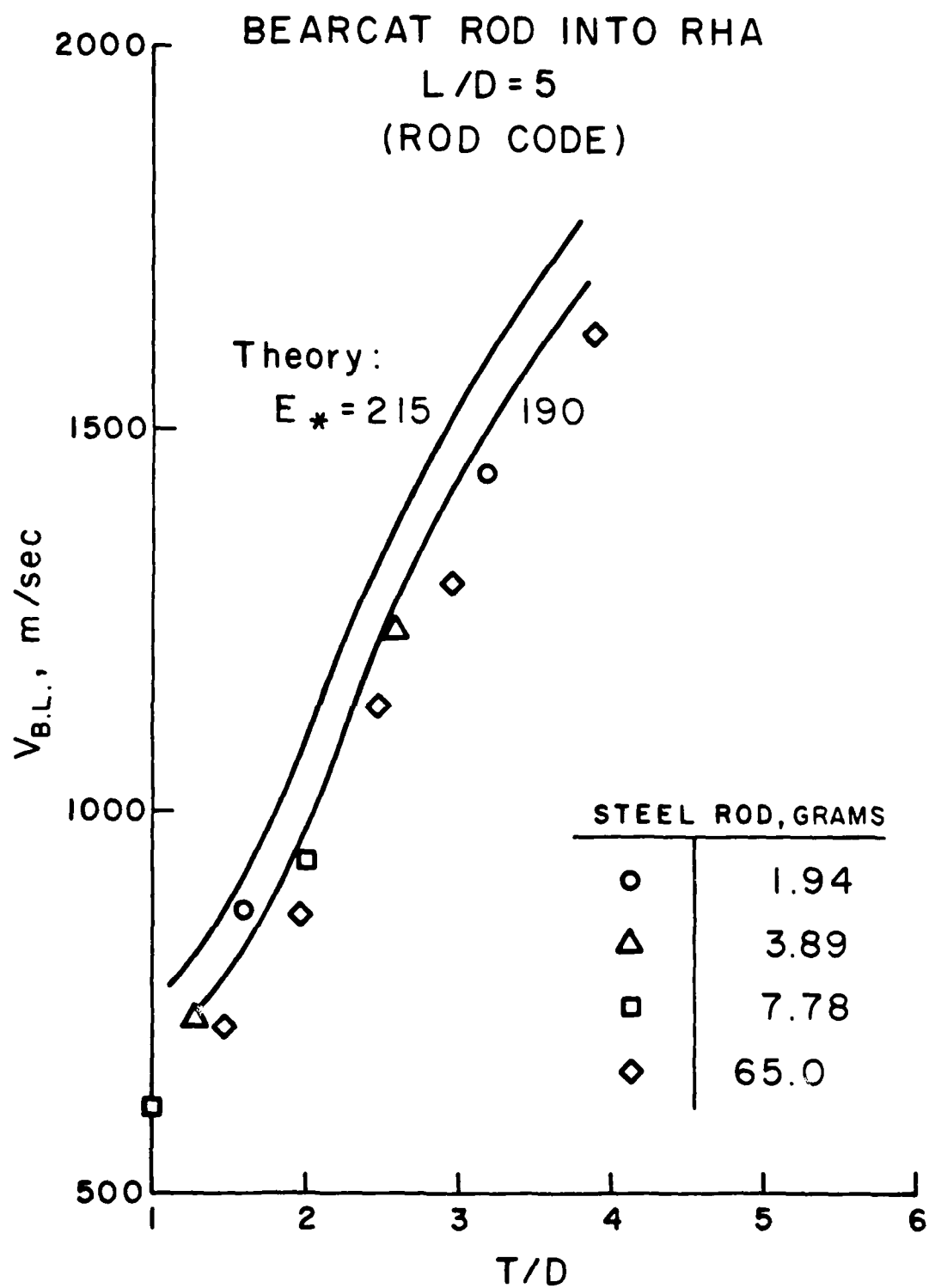


Figure 18

# BEARCAT ROD INTO RHA

L/D=10  
(ROD CODE)

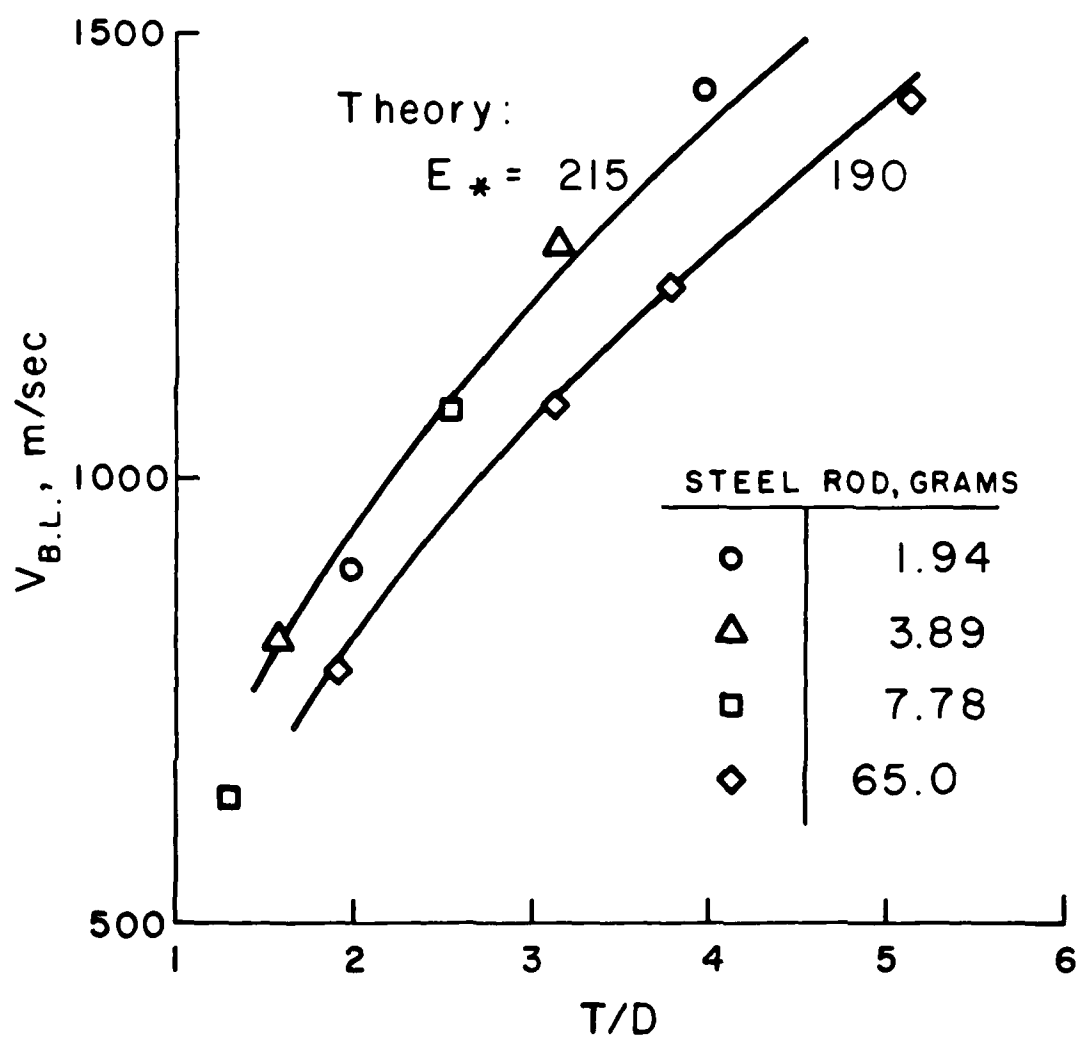


Figure 19

# BEARCAT ROD INTO RHA

L/D=20  
(ROD CODE)

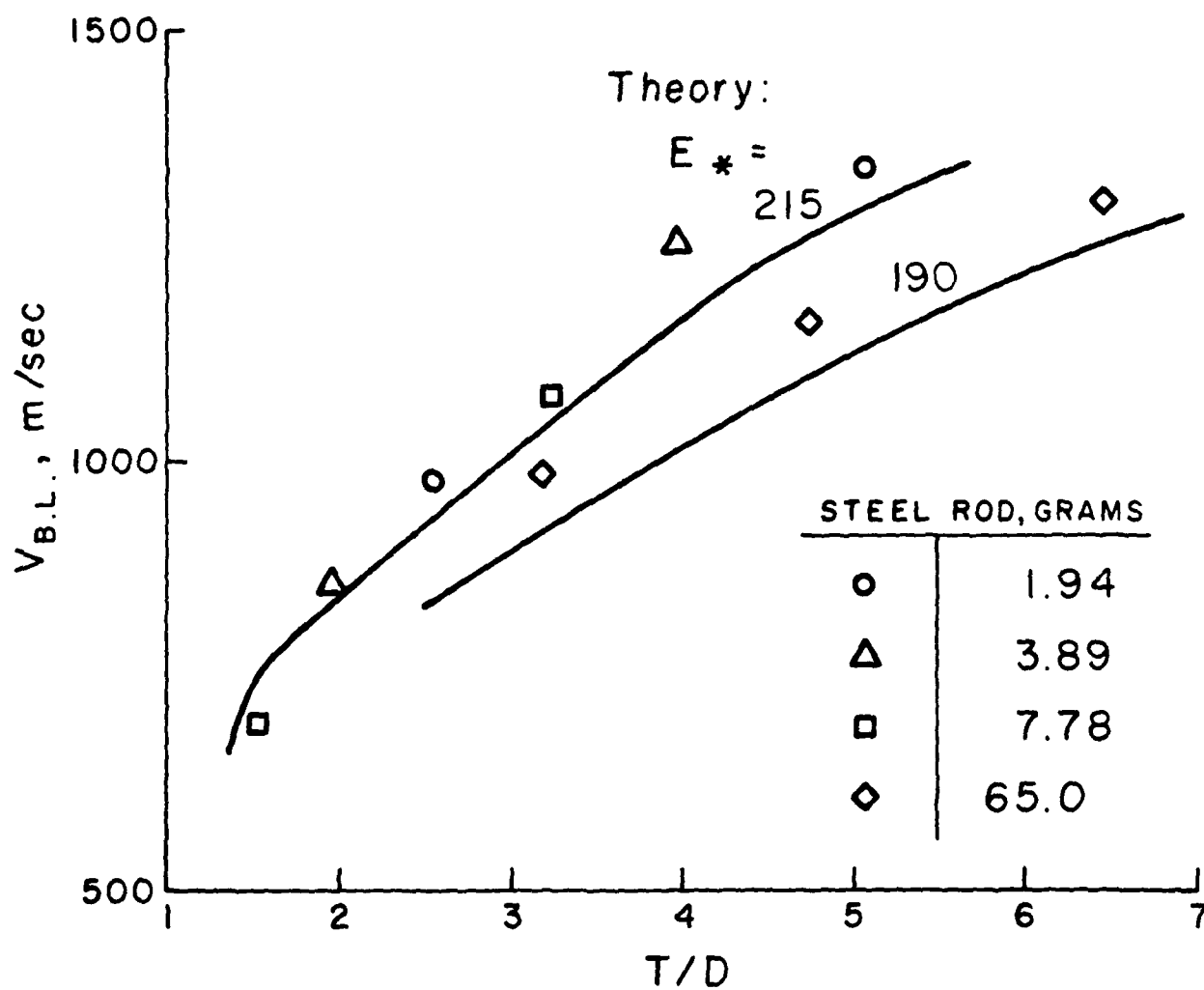


Figure 20

In Figs. 21 through 24 the theoretical value of the residual mass  $M_R$  versus striking velocity is compared to experiment<sup>9,10</sup> for Bearcat steel and Mallory 3000 rods. The residual mass computed in the program is the sum of the mass of the head and shaft of the rod at the instant the target backface is reached. Although there is a fair amount of scatter in the data, good qualitative agreement is attained over the range of velocities and target thicknesses in the experiment. In Figs. 25 through 29 the predicted residual velocity of the rod fragment is compared with experiment for various thickness targets and rods of various  $L/D$ , for Mallory 3000 as well as steel. These comparisons are typical. The agreement is good between the code and experiment to about  $\pm 15\%$ .

In Figs. 30 and 31 typical oblique rod shots<sup>9</sup> are compared to the PEN code prediction. The ballistic limit is plotted versus striking velocity for Bearcat rods into RHA at  $60^\circ$  incidence. Again, the agreement between theory and experiment is good to better than 10%.

In order to provide further confirmation of the numerical codes, especially over a wider range of materials, published data were obtained for long rods (wires) of gold, tin, aluminum, and magnesium fired into 7075-T6 Aluminum semi-infinite targets.<sup>6</sup> The values of  $E_*$  for the targets and penetrators were obtained from handbook data on the materials involved, and substituted into formula (30). The resulting values of  $E_*$  are displayed in Fig. 32, together with the experimental and theoretical curves of penetration versus velocity for the four rod materials. The penetration is normalized to allow presentation of all curves on the same graph. The very good agreement over a range of rod density from 1.8 to 19 gm/cc confirms that density variations are taken into account correctly in the code.

As yet another test of the influence of the material hardness on penetration, published data<sup>7</sup> for steel and Densimet 17 rods fired into various types of semi-infinite steel targets

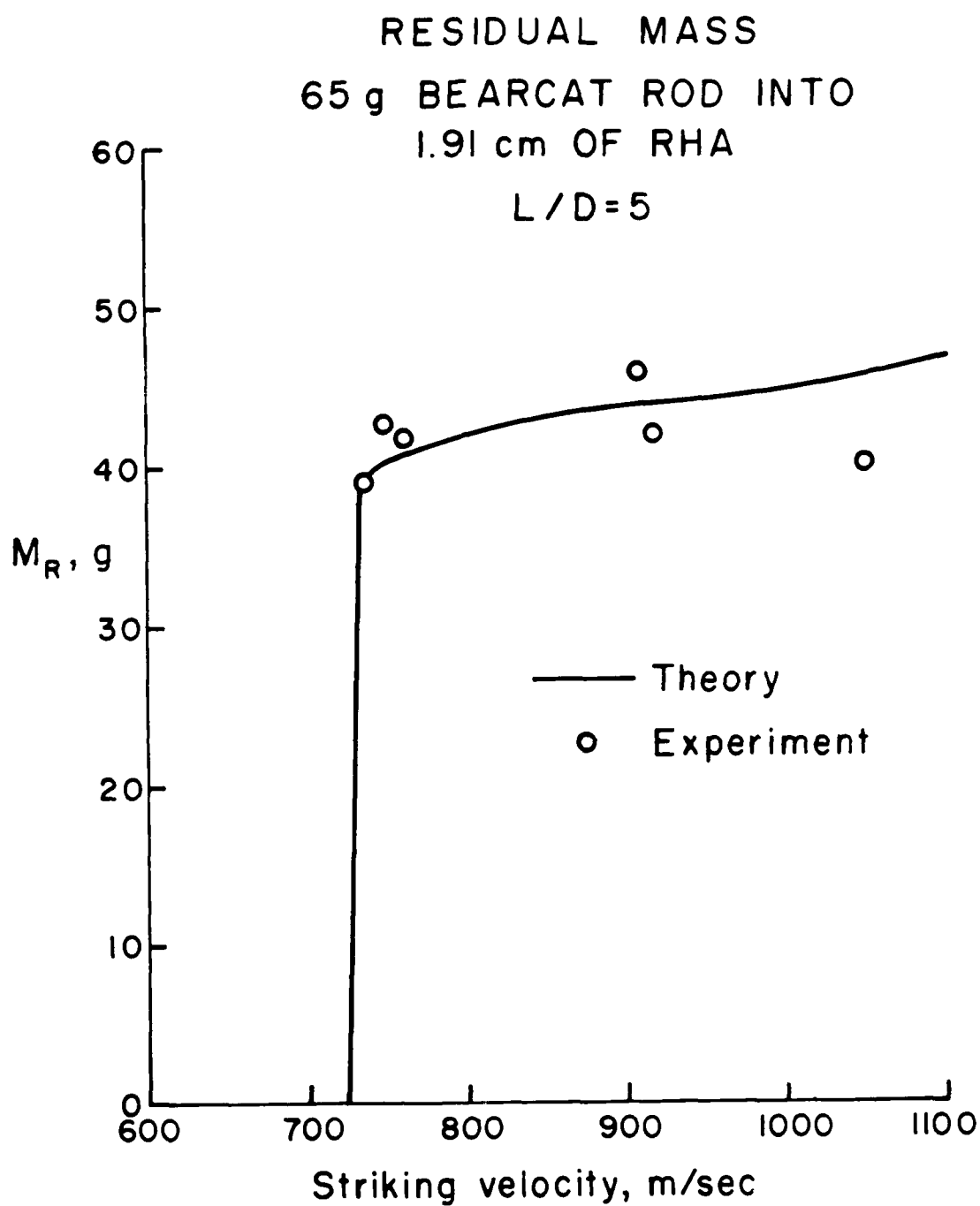


Figure 21

RESIDUAL MASS  
65 g BEARCAT ROD INTO  
3.18 cm OF RHA  
 $L/D=5$

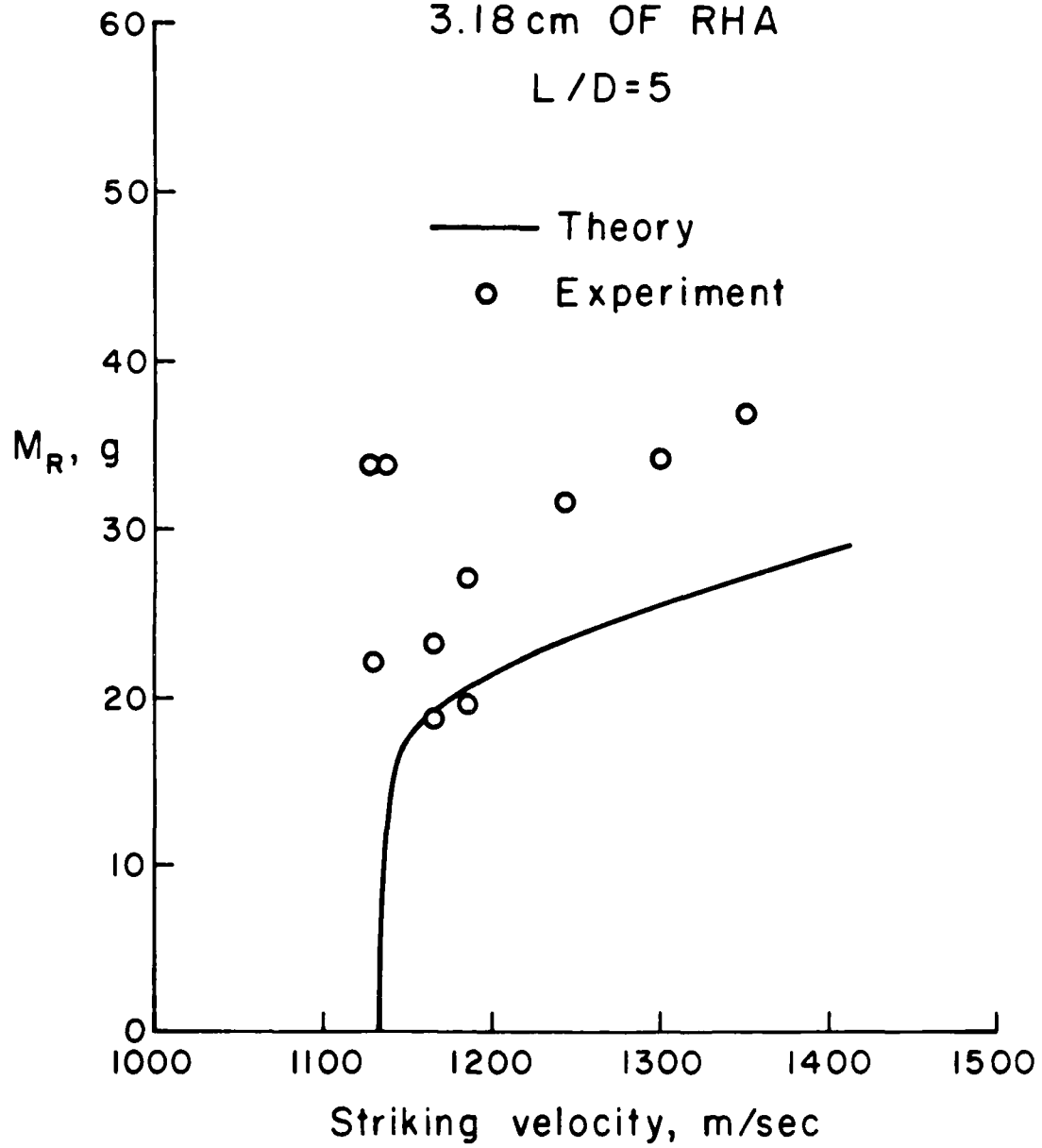


Figure 22

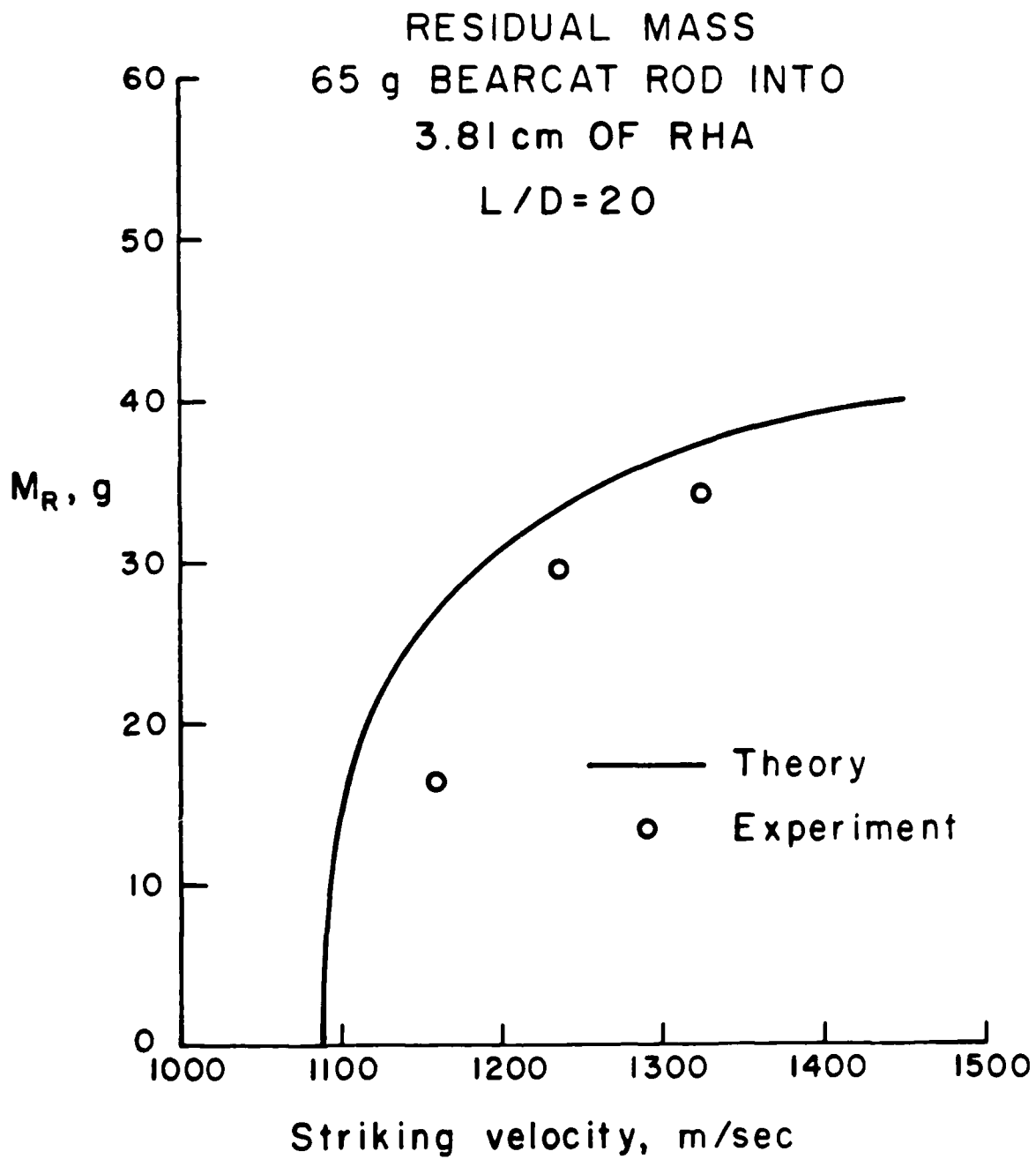


Figure 23

RESIDUAL MASS - MALLORY 3000 RODS INTO  
12.7 mm OF RHA

$L/D=5$

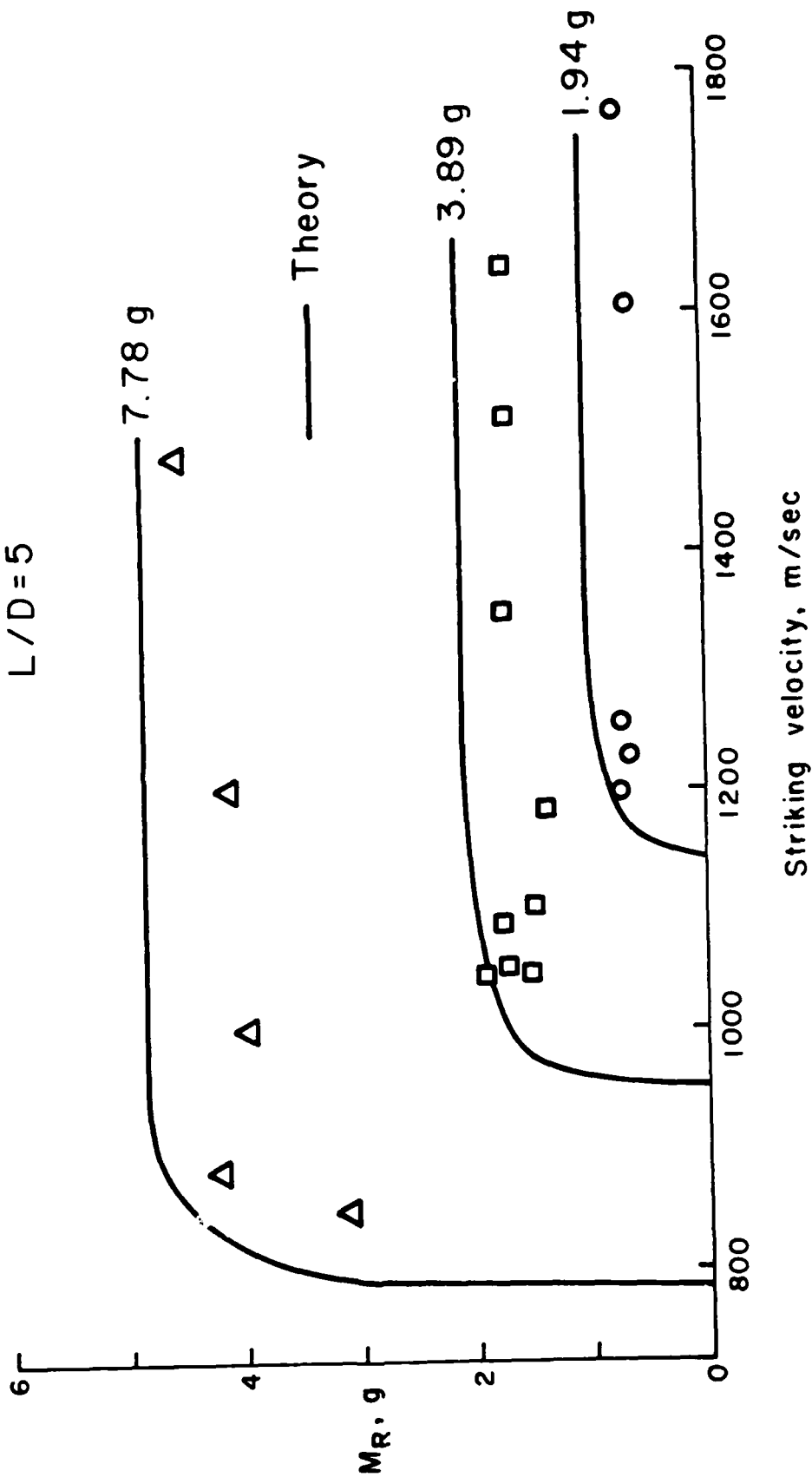


Figure 24

# BEARCAT ROD INTO RHA

$L/D=10$

$D=1.02$  cm

$T=5.08$  cm RHA

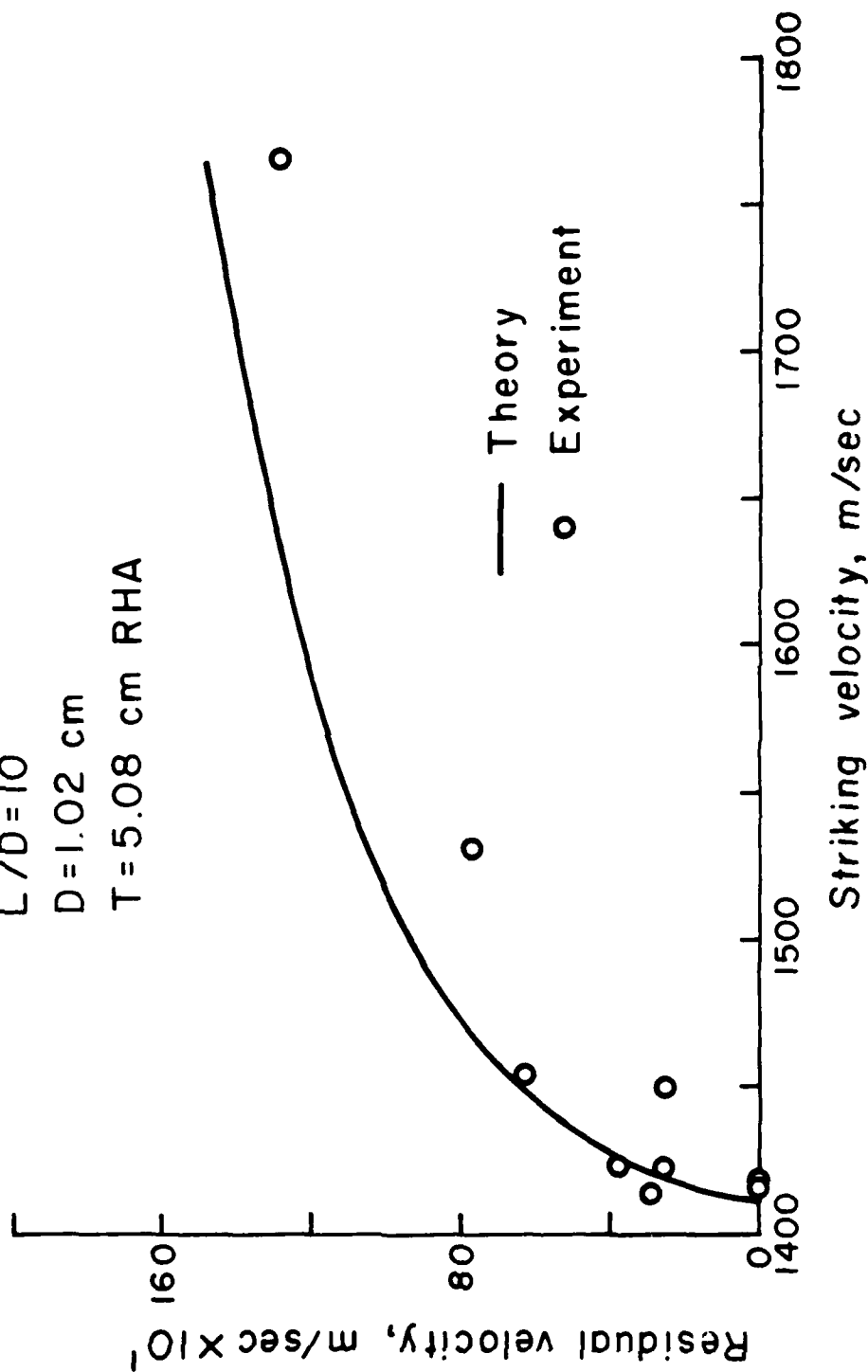


Figure 25

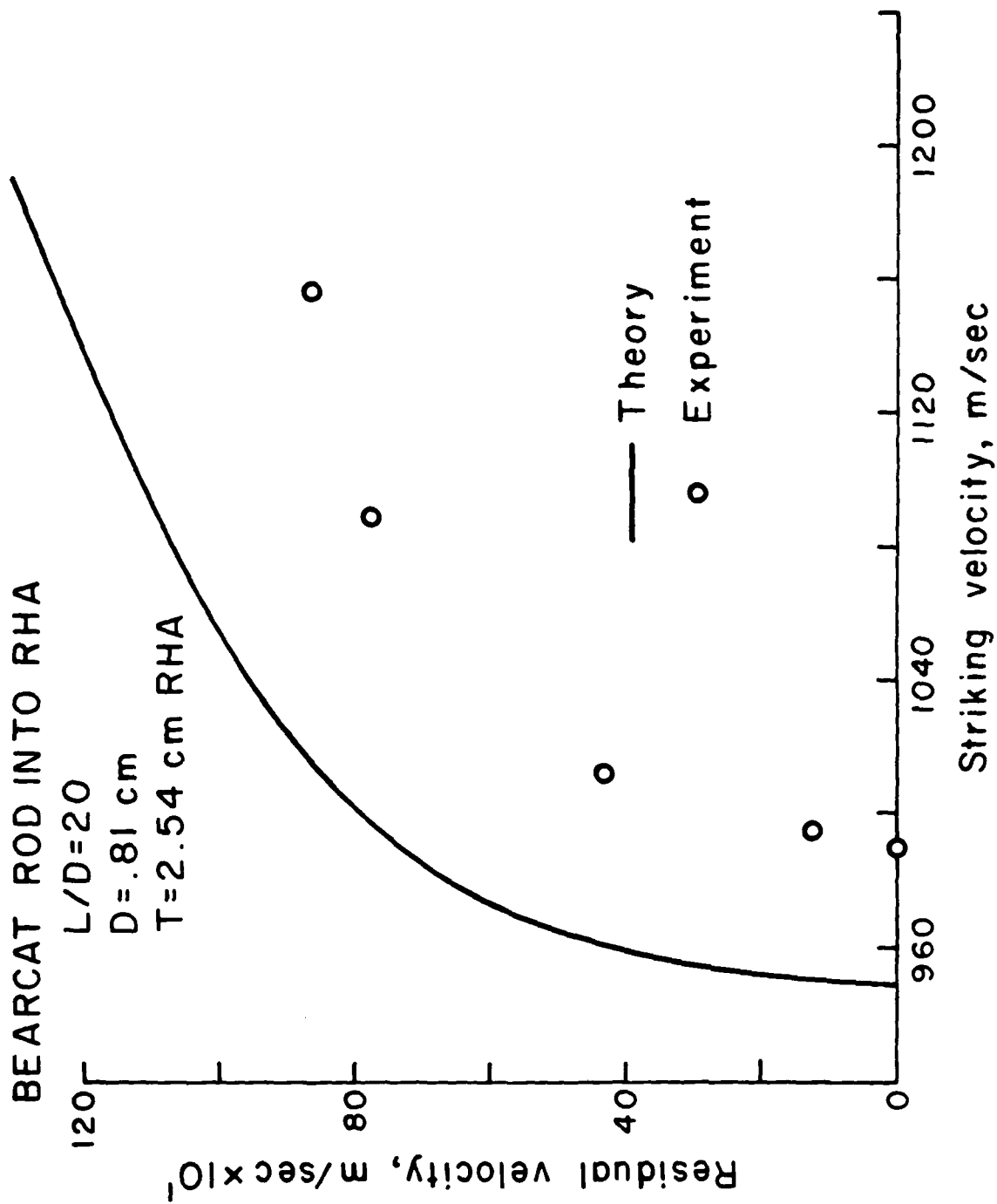


Figure 26

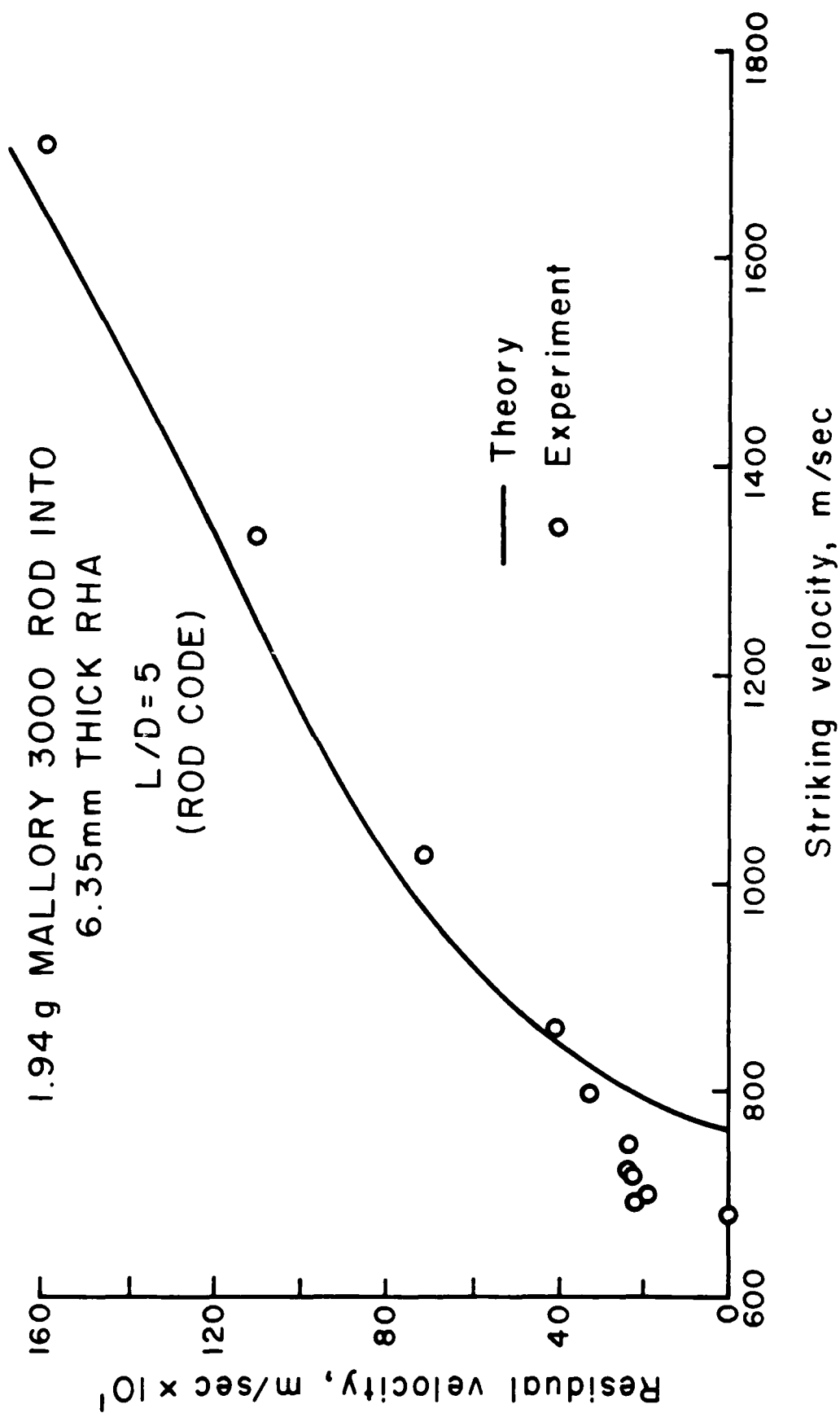


Figure 27

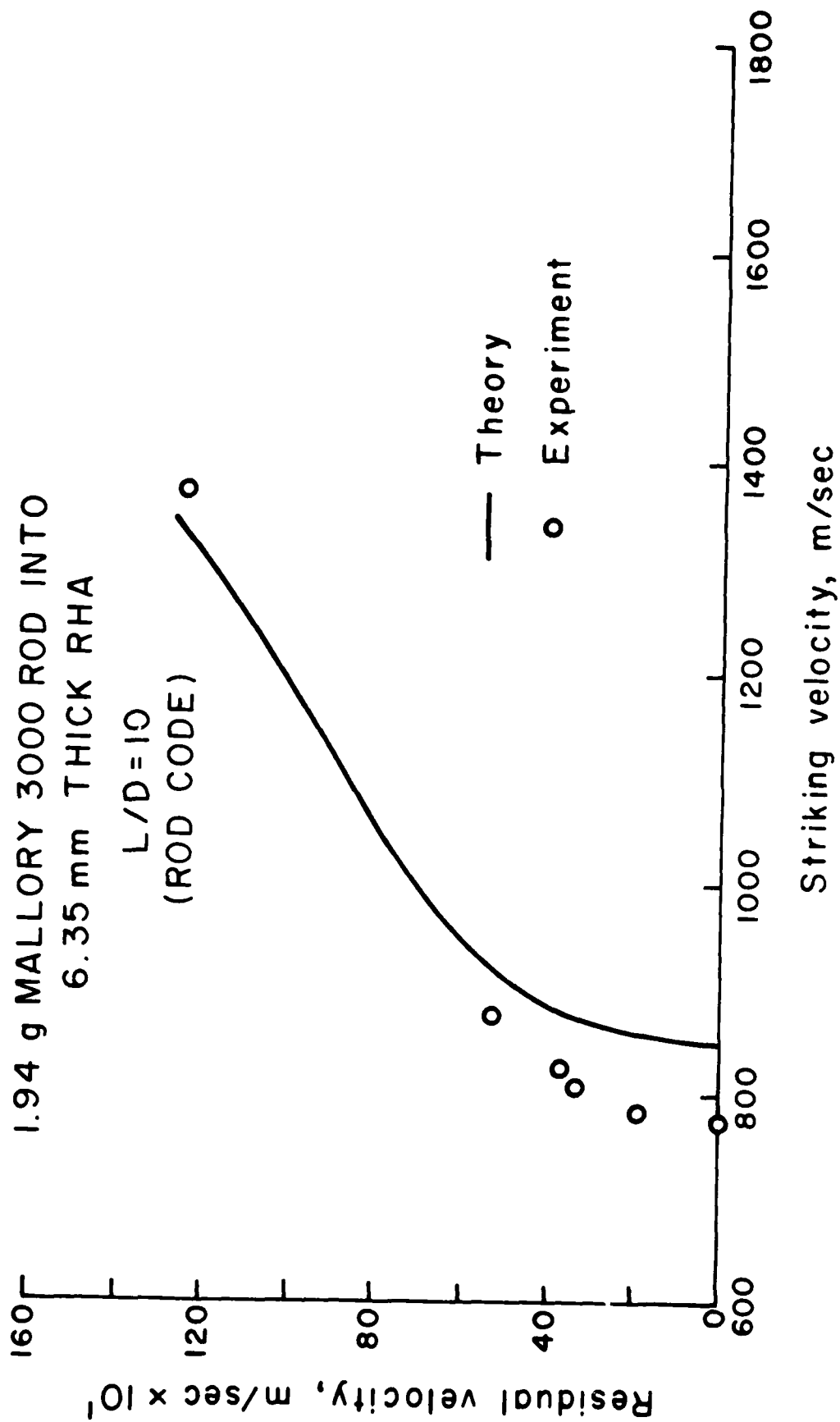


Figure 28

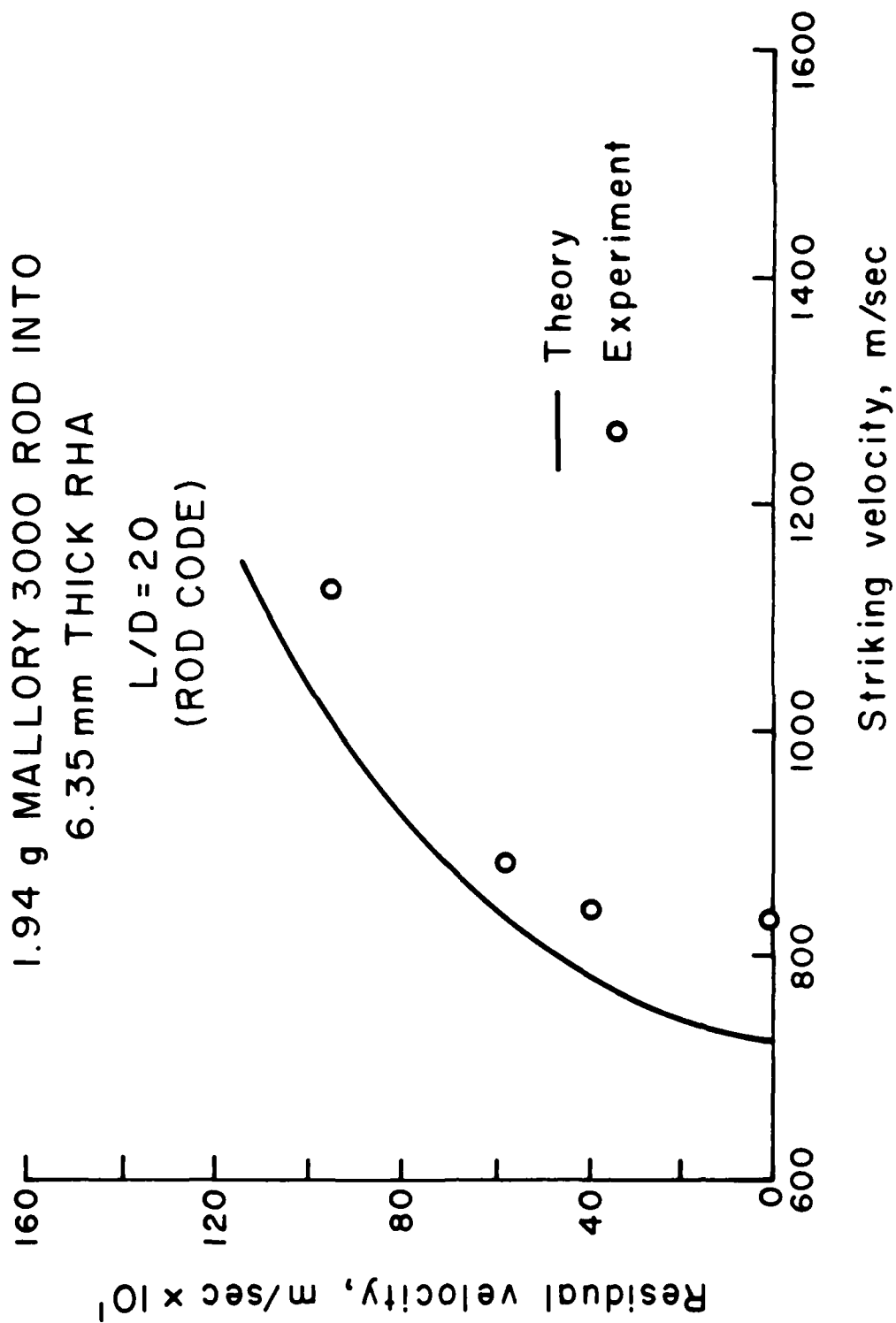


Figure 29

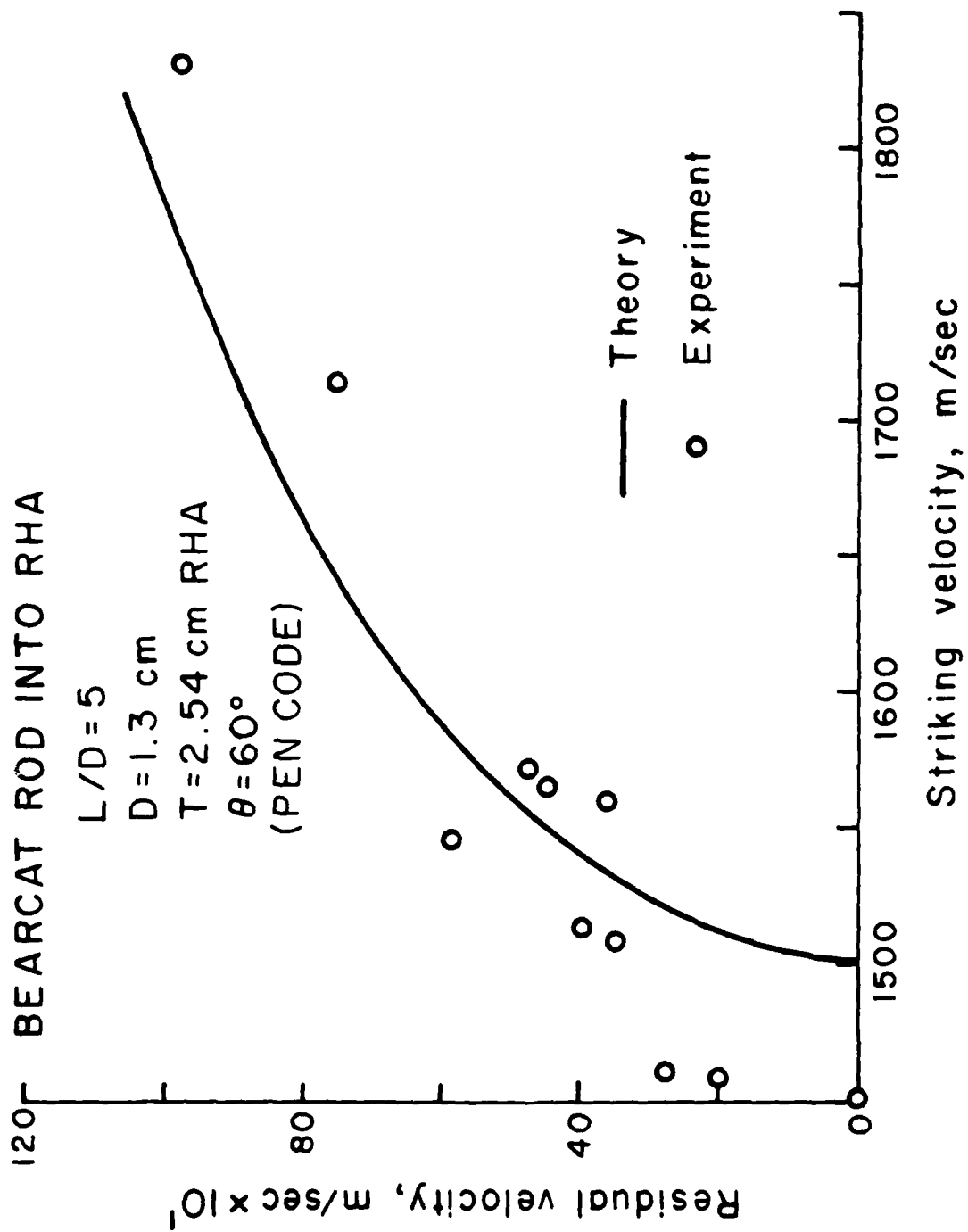


Figure 30

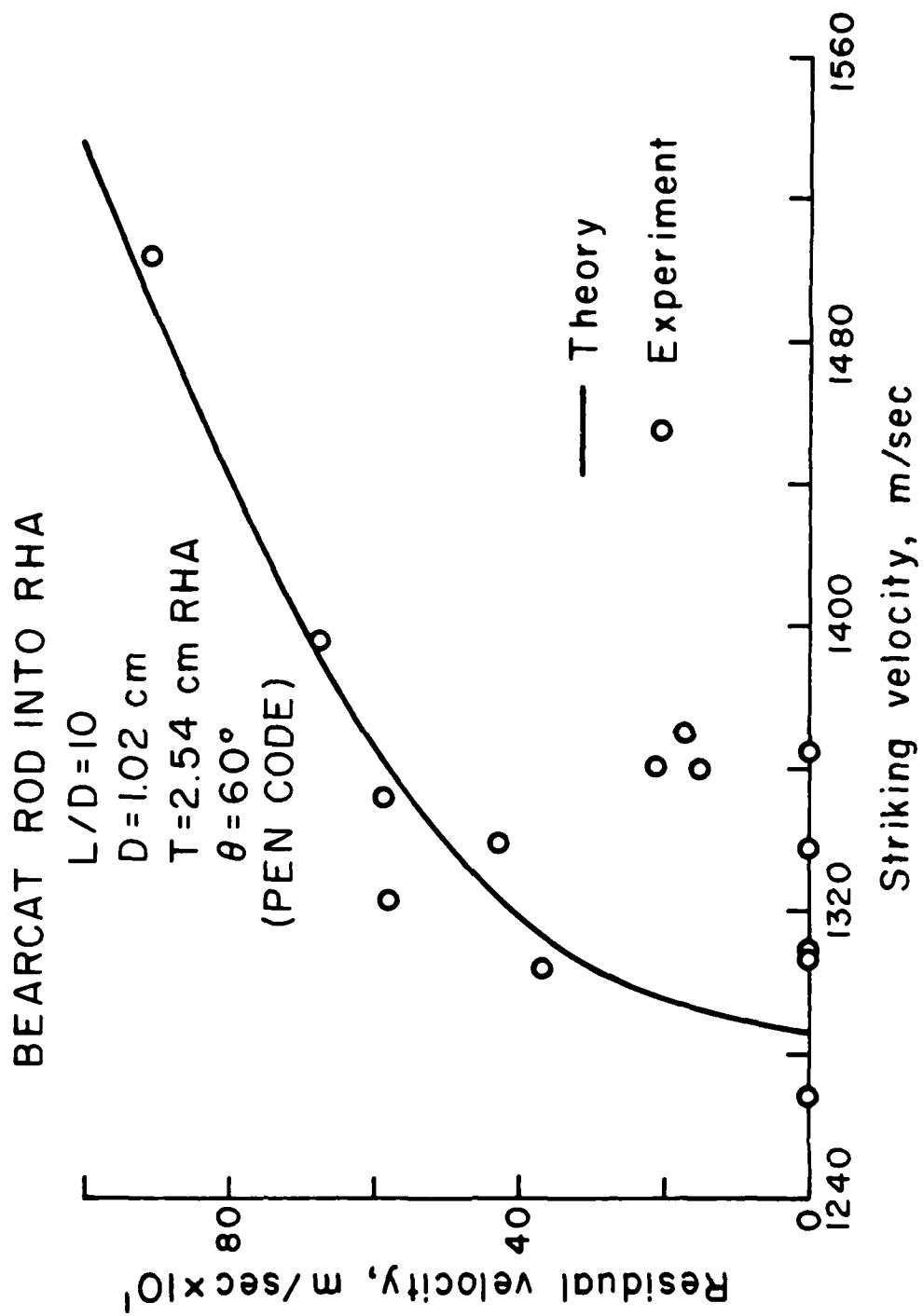


Figure 31

# PENETRATION OF LONG WIRES OF VARIOUS MATERIALS INTO 7075-T6 ALUMINUM

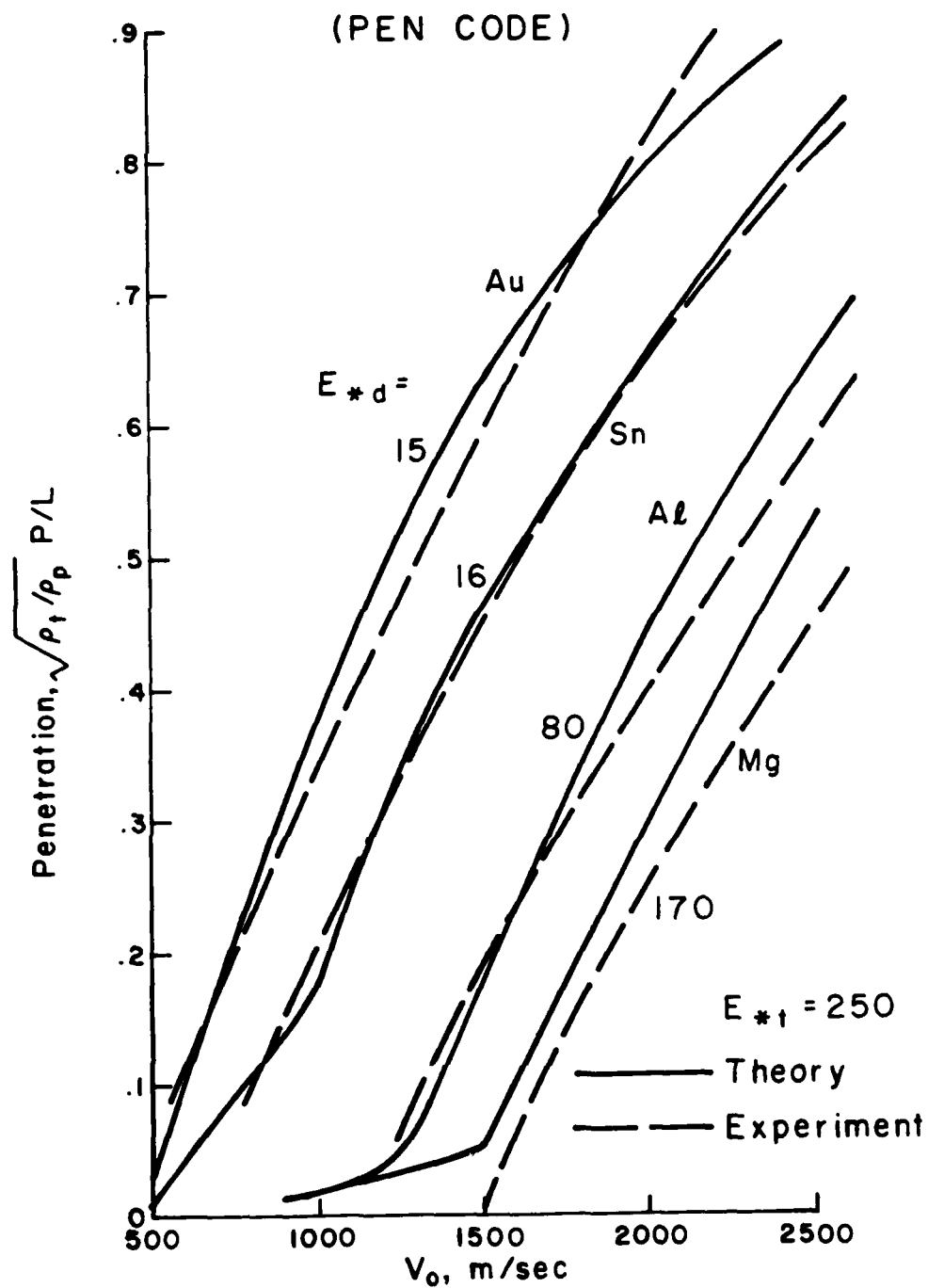


Figure 32

DENSIMET 17 RODS INTO  
SEMI-INFINITE RHA

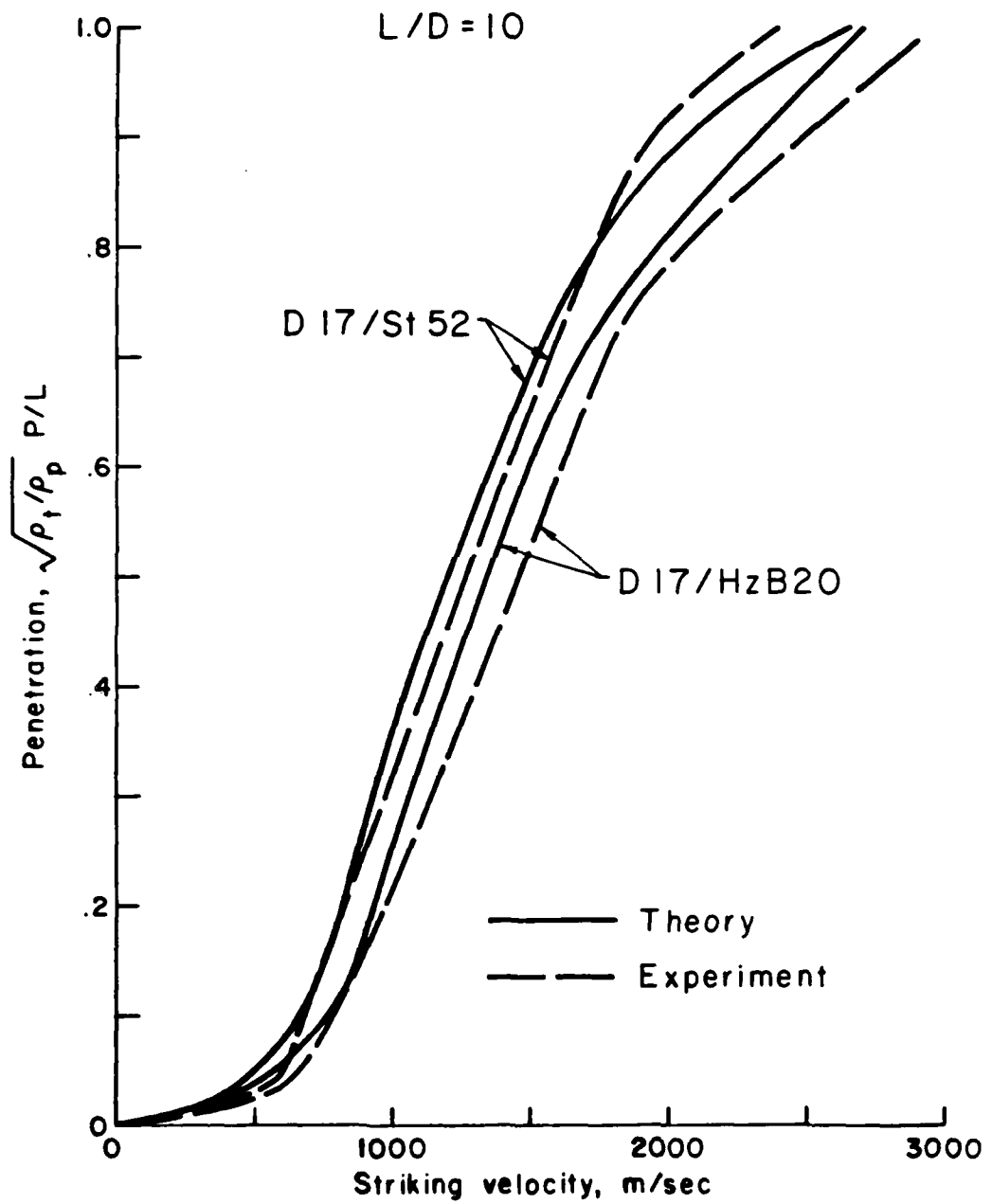


Figure 33

# MILD STEEL RODS INTO SEMI-INFINITE RHA

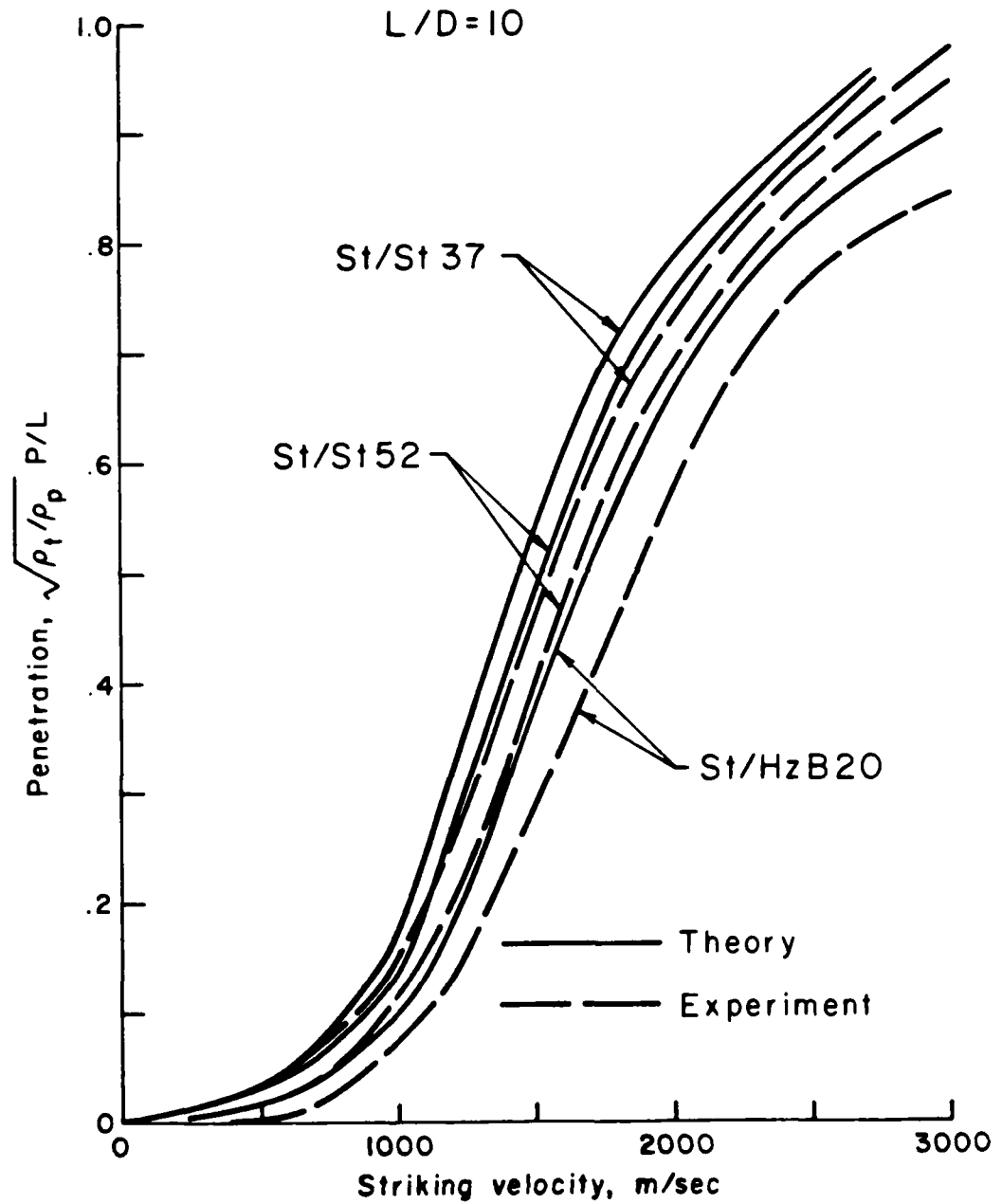


Figure 34

$\left( \frac{\text{PENETRATION DEPTH}}{\text{PROJECTILE LENGTH}} \right)$  vs IMPACT VELOCITY

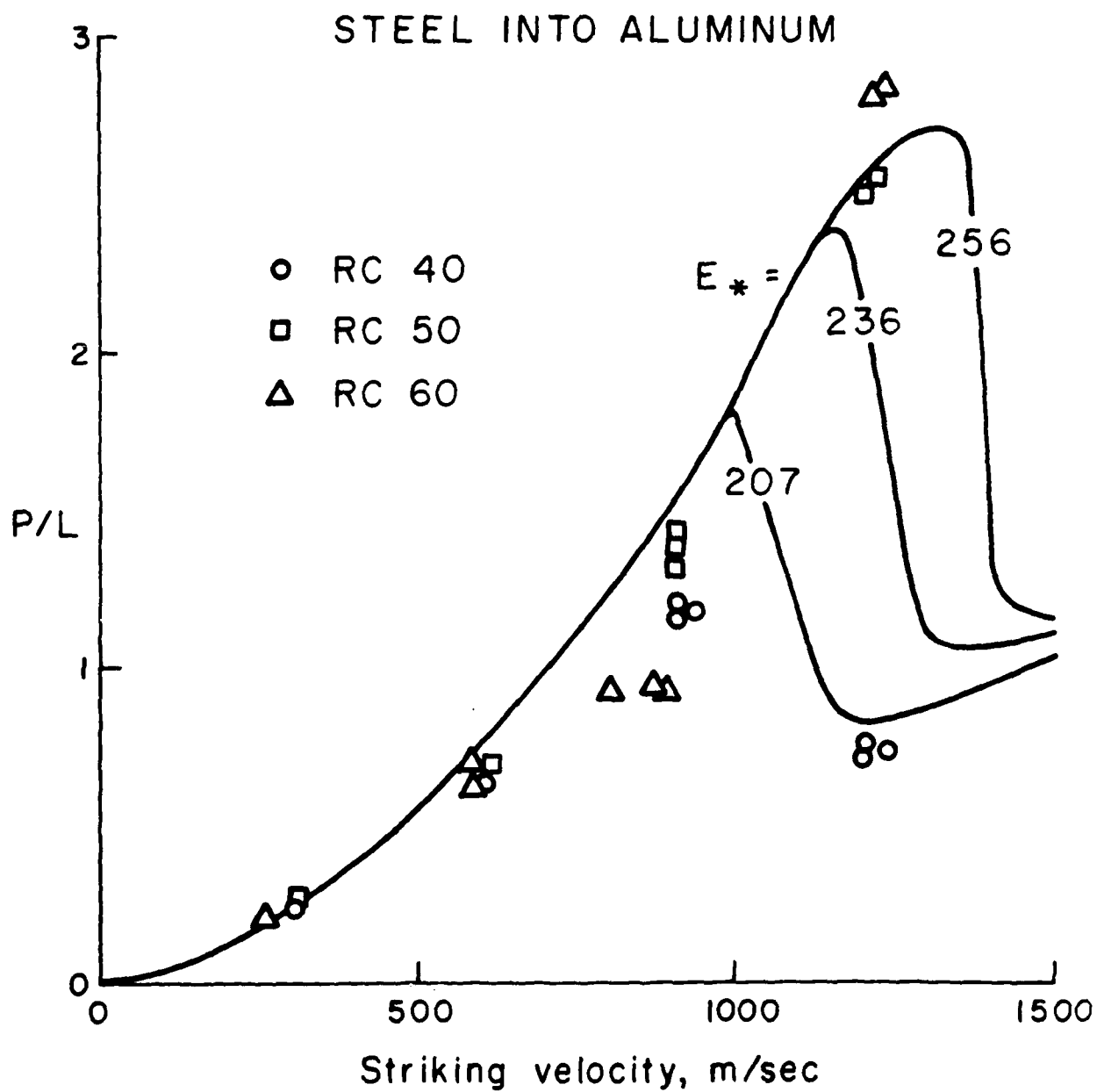


Figure 35

were compared to the PEN numerical code. For these data, the maximum velocity is 3000 m/sec, considerably higher than for the previous data for steel rods in Fig. 11. As Figs. 33,34 show agreement is good over the entire range of velocity for all of the materials.

Finally, the rod code was tested for the interesting case of a dense, strong rod into a low density target, in this case Bearcat steel rods into a 6061-T6 Al target.<sup>12</sup> The steel rod, if sufficiently strong, will not deform at low velocities and will follow the ballistic curve of a nondeforming projectile. At high enough striking velocity, the front face pressure on the rod exceeds the yield strength, and the rod begins to erode rapidly and behaves like a deforming rod. In Fig. 35, data are presented which display this transition for rods of various strengths. The theoretical curves corresponding to rods of Rockwell hardness  $R_c = 40$ , 50 and 60 ( $E_* = 207, 236$  and 256 Btu/lb) clearly show that the stronger rod begins to deform at higher velocity. These data provide a critical test of the way the rod strength enters the code, since the transition from nondeforming to deforming rod is so sharp. Experiments such as this one provide a sensitive means for empirically determining the uniaxial adiabatic yield strength of a rod.

## VII. CONCLUSIONS

The Integral Theory of Impact has been applied to the problem of modeling the behavior of long-rod penetrators. A two-cell model for the deforming rod is employed, assuming conservation of energy and momentum, and using the A.R.A.P. concept of adiabatic hardness,  $\rho E_*$ , to account for material strengths. A numerical code based on these assumptions has been developed which can predict the performance of rod penetrators impacting finite thickness targets. The input parameters required to operate the code consist only of the physical dimensions of the target and penetrator, and standard handbook properties of the materials, such as density, heat capacity and Brinell hardness, which is needed to compute  $\rho E_*$ .

The code predictions of ballistic limit velocity, penetration depth, residual mass and residual velocity are in good agreement (typically  $\pm 15\%$ ) with experiment over a wide range of materials and velocities. The code accurately predicts the relative improvement in performance of a rod when its strength is increased, or when the target hardness is changed, and also predicts the approximate velocity at which a rod transitions from nondeforming to deforming penetration.

The current treatment of oblique impact does not attempt to handle fracture of the rod shaft or jetting of the rod front end during impact. However, where shaft fracture is not a problem, the code predictions of ballistic limit are in good agreement with experiment.

## REFERENCES

1. Donaldson, Coleman duP. and McDonough, Thomas B.: A Simple Integral Theory for Compact Cratering by High Speed Particles. DNA 3234F, Aeronautical Research Associates of Princeton, Inc., Princeton, NJ, 08540, 4 December 1973.
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9. Lambert, J.P., Misey, J.J., Morfogenis, P.G., and Zukar, J.A.: Behind Armor Data for Long Rod Penetrators. B.R.L. Interim Memorandum Report No. 430, USA Ballistic Research Laboratories, Aberdeen Proving Ground, MD, September 1975.
10. Herr, Louis, and Grabanek, Chester: Ballistic Performance and Beyond Armor Data for Rods Impacting Steel Armor Plates. B.R.L. Memorandum Report No. 2575, USA Ballistic Research Laboratories, Aberdeen Proving Ground, MD, January 1976.
11. Private Communication, Konrad Frank, USA Ballistic Research Laboratories, Aberdeen Proving Ground, MD.
12. Private Communication, John H. Suckling, USA Ballistic Research Laboratories, Aberdeen Proving Ground, MD.

APPENDIX 1

LISTING OF ROD CODE

```

C      MOD WITH SHEAR AND STRENGTH
C      VERSION 2
      DOUBLE PRECISION EPST(12), ASAVE(36),ASCALE(36)
      DOUBLE PRECISION ALEN(36),VSAVE(36),ES(6),RTAR(6), THIK(12)
      DOUBLE PRECISION ESDIN
      DOUBLE PRECISION Q2,P,DP,L2,DL2,K,MA,DMA,MD,DMD,K4,K5,LOVERD,N
      DOUBLE PRECISION MPRIM,PTARG,P2,MP,MO,B2,DB2, DMF,VU
      DOUBLE PRECISION TOVERD
      DOUBLE PRECISION EOFAIL,ESTIN,YIELD
      DOUBLE PRECISION ESTART,RHUTAR,EPSTAR,THICK,COPEN,TELL,RHUP
      DOUBLE PRECISION A,LENGTH
      DOUBLE PRECISION ESTFAC
      DOUBLE PRECISION VF,DVF,DVO,DDL2,I
      DOUBLE PRECISION POVERL, PRL
      PI=3.14159

```

```

L
      ESDIN=1.0
      FLAT=0.0
      FLAT=1.0
      L1=1.0
      L2=0.0
      READ(5,888) NVELUC,NTARG,NPEN,NHUTAR,NV2,NV3 ,NSLAR
888      FORMAT(7I10)
      DO 15 I=1,NVELUC
      READ(5,12) LENGTH,A,VU
12      FORMAT(3D10.0)
      VSAVE(1)=VU
      ALEN(1)=LENGTH
      ASAVE(1)=A
      ASCALE(1)=0.005/A
15      CONTINUE
      DO 16 I=1,NTARG
      READ(5,11) ESTART,RHUTAR,EPSTAR,THICK
11      FORMAT(4D10.0)
      ESTART=ESTART*1055.0*2.2
      ES(1)=ESTART
      RTAR(1)=RHUTAR
      THIK(1)=THICK
      EPST(1)=EPSTAR
16      CONTINUE
      DO 10 J1=1,NPEN
      READ(5,10) COPEN,TELL,RHUP,YIELD,EOFAIL,ESTIN
10      FORMAT(6D10.0)
      ESTIN=ESTIN*ESDIN
      YIELD=YIELD*1.076
      YIELD=1055.0*2.2*ESTIN*RHUP
      YIELD=0.42*YIELD
      YELDP=YIELD
      DO 102 J5=1,NTARG
      ESTART=ES(J5)
      ESTAR0=ES(J5)
      RHUTAR=RTAR(J5)
      EPSTAR=EPST(J5)

```

```

C
C      INITIALIZE PARAMETERS
      DO 103 J6=1,NVELUC
      SCALE=ASCALE(J6)
      V0=VSAVE(J6)
      A=ASAVE(J6)*SCALE
      THICK=THIK(J5)*SCALE
      LENGTH=ALEN(J6)*SCALE
      DO 96 NV1=NV2,NV3,1

```

```

IF (NSCAN,LI, 0.5) GO TO 47
V0=FLUAI(NV1)*100.0
47 CONTINUE
B2=A
EPSFAL=0.0
MA=0.0
UMA=0.0
MB=0.0
UMB=0.0
STR=0.0
I=0.0
P=0.0
P2=0.0
L2=LENGTH/2.0
L2=A/6.0
DL2=-1.0
ALPHA1=0.4
TEMP0=300.
TEMP=TEMP0
DMF=0.0
DB2=-0.5*(DL2*B2/L2)
M0=P1*A*A*LENGTH*KHUP
MUM0=M0*V0
MUM0=0.05*MUM0
MF=2.0*L2*KHUP*A*A*P1
MPK1M=M0-MF
DV0=0.0
VF=V0
VF=VF-DL2
ALF=0.0
K=0.5*M0*V0*V0
ENERG0=0.01*K
VB=V0
ESTAR0=YIELD/KHUP
LOVER0=LENGTH*0.5/A
DT=1.0E-8
DT=DT*DT1A*SQRT(KHUP/KHUTAR)
EPFAIL=1.30
WRITE(5,609) KHUP,LENGTH,A
609 FORMAT(3D16.6)
ESTLOR=ESTAR0/(2.2*1055)
WRITE(5,610) ESTLOR
610 FORMAT(D16.6)
VIGIT=V0
DIST0=0.0
SPHER=0.0
WRITE(5,991) ESTAR1
991 FORMAT(D16.6,'=ESTAR1')
WRITE(5,993) EUFAIL,ESTIN
993 FORMAT(D16.6,'=EUFAIL', D16.6,'=ESTIN')
DO 100 I=1,1000
DO 99 J=1,100
CD=1.0
C
C MODIFICATION OF EU IN HEAD
ADEN=KHUTAR*A*(ESTAR1+0.5*VF*VF)
EPS0=EPFAIL*C1+KHUP*2.0*L2*DB2*DM2*L2/ADEN
FAC1=1.0
FAC3=1.0
FAC5=1.0
FAC7=1.0
FAC8=1.0
FAC9=0.0

```

```

L2=L2+DL2*DT
DL2=DL2+DOL2*DT
C KEEPS TRACK OF TAIL MOTION
DISTB=DISTB+V0*DT
V0=V0+DV0*DT
FLAC=1.0
IF(P,G1,P2) FLAC=0.0
DP=VFALZ*DT
C POSITION OF FACE
P2=P2+DP
C POSITION OF CRATER BOTTOM
P=P+DMAX1(DP,V,DU)*FLAC
VF=VF+DVF*DT
MA=MA+DMA*DT
MB=MB+DMB*DT
B2=B2+DB2*DT*(1.0-FLAC)
IF(B2,G1,EPS0*A) B2=A*EPS0
MF=2.0*P1*L2*B2*B2*RHUP
MPR1M=MO=MF=MB
I=I+DT
C
C SUPERFLUOUS VARIABLES
DTEMP=FAC5*3.0*P1*B2*B2*DL2*YIELDP/(RHUP*LLPEN)
DTEMP=ABS(DTEMP)
TEMP=TEMP+DTEMP*DT
C
C CONDITION FOR HALT OF PENETRATION
ENERGY=0.5*(MO-MF-MB)*V0*V0+0.5*MF*VF*VF
IF(ENERGY.LT.ENERG0) GO TO 1
MOM=(MO-MF-MB)*V0+MF*VF
IF(MOM.LT.MOM0) GO TO 1
99 CONTINUE
C
C INTERMEDIATE OPTIONAL VARIABLE PRINTOUT
IF(NWRITE.LT.0.5) GO TO 895
WRITE(5,51)
51 FORMAT('0')
WRITE(5,50) P,VF,L2,B2,MF,FAC1,FAC9,FAL3,V0
WRITE(5,50) I,VFACE,DL2,DB2,MO,DVF,R1,ENERGY,R1
WRITE(5,50) MA,DMA,MB,DMB,DMF,P2,DOL2,TEMP
50 FORMAT(9D12.4)
WRITE(5,55) EPS0,FYIELD
55 FORMAT(' EPS0=',D16.6,' FYIELD=',D16.6)
895 CONTINUE
100 CONTINUE
C
C FINAL VARIABLES PRINTOUT
222 CONTINUE
THICK=THICK/SCALE
WRITE(5,223) THICK,EPSTAR
223 FORMAT('0','PENETRATION OCCURS', ' THICK=',E12.4,' EPSTAR=',E12.4)
1 CONTINUE
POVERL=P/LENGTH
PRL=POVERL*SQRT(RHUTAK/RHUP)
A=A/SCALE
LENGTH=LENGTH/SCALE
WRITE(5,998) A,V0,RHUP,LENGTH
WRITE(5,999) ESTART,RHUTAK
998 FORMAT('0','INITIAL: A=',D12.4,' V0=',D12.4,' RHUP=',D12.4,
* ' LENGTH=',D12.4)
999 FORMAT(' ESTART=',D12.4,' RHUTAK=',D12.4)
WRITE(5,59)
59 FORMAT('0','END OF RUN. FINAL PARAMETERS:')

```

```

WRITE(5,50) P,VF,L2,B2,MF,FAC1,FAC9,FAC3
WRITE(5,50) T,VFAC,DL2,DB2,M0,DVF,K1,ENERGY
PTARG=P+2.0*B2
P=P/SCALE
PTARG=PTARG/SCALE
WRITE(5,996) VINI,P,PTARG
996 FORMAT('0',' VINI=',D12.4,' P=',D12.4,' PTARG=',D12.4)
WRITE(5,801) POVERL,PRL
801 FORMAT(' POVERL=',D12.4,' PRL=',D12.4)
TOVERD=PTARG/(2.0*A)
WRITE(5,843) TOVERD
843 FORMAT(' TOVERD=',D12.4)
MR=MO-MR
VRRESID=(VO*MRIM+VF*MF)/MR
MR=MR/SCALE**3
WRITE(5,224) LOVERD,VRRESID,MR
224 FORMAT(' LOVERD=',D12.4,' VRRESID=',D12.4,' MRRESID=',D12.4)
96 CONTINUE
103 CONTINUE
102 CONTINUE
101 CONTINUE
CALL EXIT
END

```

APPENDIX 2

LISTING OF PEN CODE

```

REAL L,ESTIN,EUFAL,P,E0,MUN,MPRIN,MUVO
REAL LENGTH
REAL LOVERD
REAL PEAK,PKMAX
REAL INUT, MPRIN,H,PFERP,PPAR
REAL MINIT, LALPH
REAL MKDAX
PI=3.14159

```

```

C
886 READ(5,886) NVELUC,NTARG,NPEN,NPRINT
      FORMAT(4I10)
      READ(5,41) IV0,IV1,IOV0
      41  FORMAT(3I10)
C
C      THETA=STRIKING ANGLE, RADIANS
C      THICK=TARGET THICKNESS IN METERS.
C      ESTARS=ESTAR OF TARGET
      42  READ(5,42) THETA,THICK,ESTARS
            FORMAT(3E10,0)
C
C      NESTAR: IF ZERO,ESTAR GIVEN BY TART INUT, IF ONE, THEN
C              ESTAR GIVEN BY ESTARS.
C      NVREPL: ALLOWS SEQUENCE OF VELOCITY INCREMENTATION. IF
C              EQUAL TO ONE,THE VALUES OF IV0,IV1,AND IOV0 DETERMINE
C              THE INITIAL,FINAL, AND INCREMENT VELOCITIES.
      887  READ(5,887) NESTAR,NVREPL
            FORMAT(2I10)
      ISAVE=THICK
      THETA=THETA*PI/180.0
      DO 15 I=1,NVELUC
C
C      LENGTH=PENETRATOR LENGTH IN METERS.
C      A=PENETRATOR DIAMETER IN METERS.
C      VO=STRIKING VELOCITY.
      12  READ(5,12) LENGTH,A,VO
            FORMAT(3E10,0)
      A=A/2.0
C      ESTAR=ESTAR OF TARGET IN BTU/LB.
C      RHOTAR=DENSITY OF TARGET IN KG/MS.
C      EPSTAR=NOT USED.
      11  READ(5,11) ESTAR1,RHOTAR,EPSTAR,THICK
            FORMAT(4E10,0)
C
      IF(NESTAR.LE.0.5) GO TO 995
      ESTAR=ESTARS
      995  CONTINUE
      ESTAR0=ESTAR1*1055.0*2.2
      THICK=ISAVE
C
C      RHUP=PENETRATION DENSITY IN KG/MS.
C      ENREV=NOT USED.
C      ESTIN=ESTAR OF PENETRATOR MATERIAL.
      10  READ(5,10) RHUP,ENREV,ESTIN
            FORMAT(3E10,0)
      LOVERD=LENGTH/(2.0*A)
      IF(NVREPL.LE.0.5) IV0=1
      IF(NVREPL.LE.0.5) IV1=1
      IF(NVREPL.LE.0.5) IOV0=1
      DO 999 IV=IV0,IV1,IOV0
      IF(NVREPL.LE.0.5) GO TO 996
      VO=FLOAT(IV)
      996  CONTINUE

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C      EUPAIL=MAX WIDTH IF HYDRODYNAMIC HEAD
      EUPAIL=1.7
C      BETA=SPREADIN OF FRONT FACE
      BETA=2.0
C      GAMMA=ACCELERATION EFFECT IN FRONTFACE
      GAMMA=1.0
      EU=EUPAIL**2
      ELT=EU
      CU=0.25
      CUD=C0
      YU=1055.0*2.2*FSI1*RHOF
      U=VU
      L=ELT*V01H
      L*LENGU=0.5*L*P1**4**RHOF*U*U
      MUMOBEL**4**P1*RHOF*U
      D1=L*0.001/VU
      P=0.0
      WRITE(6,41) CDFIX,YOFIX,BETA,GAMMA
41    FORMAT(4E12.4)
      WRITE(6,24) ENERGO,MUMU,LENGTH,A,VU,ESTART
      WRITE(6,24) RMUTAN,EPSTAR,THICK,RHOF,EUPAIL,ESTIN
      VF=0
      MPR1=PI**4**L*RHOF
      M111=MPR1*H
      WRITE(6,24) ENERGO,MUMU,MPR1*U,VF,P
24    FORMAT(6E12.4)
      WRITE(6,302) EUPAIL,CU,NESTAR,MREFL,BETA,GAMMA
302   FORMAT(2E12.5,2I12,2E12.5)
      WRITE(6,300) LENGTH,A,RHOF
300   FORMAT('  LENGTH=',E12.5,'  A=',E12.5,'  RHOF=',E12.5)
      WRITE(6,304) VU,THETA,ESTIN
304   FORMAT('  VU=',E12.5,'  THETA=',E12.5,'  FSTIN=',E12.5)
      WRITE(6,301) RMUTAN,FSTART,THICK
301   FORMAT('  RMUTAN=',E12.5,'  ESTART=',E12.5,'  THICK=',E12.5)
      IF(.PRINF.LE.0.5) GO TO 490
      WRITE(6,47)
47    FORMAT('U', '  PHI          PSI          MPR1          U          VF          '
* , ' , ' , '
* UVT          F          PRISAR          VL          UL          BEND          EF          L*U)
490   DO 1110E
      RHYDRU=1
      RHYDRU=0
      INITIALIZE
      IMU=MPR1*M*L**2*(0.005)
      EU=1.0
      FL=0.0
      BEVD=0.0
      DBEND=0.0
      DDBEND=0.0
      PH1=0.0
      DPH1=0.0
      DDPH1=0.0
      PFAR=0.0
      PPEMP=0.0
      CM1=0.0
      UL=0.0
      DUL=0.0
      PSI=0.0
      OPS1=0.0
      ER=0.0
      EF=0.0
      VL=0.0

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SIG1=0.0
EFAL=0.1
EFFAC=1.0
CDFAL=1.0
DO 100 J=1,1000
DO 19 J1=1,100

C
C   COMPUTE TRIG FUNCTIONS
SIN1=SIN(THETA)
SIN2=SIN(THETA-PHI-PSI=4.0*BEND/L)
COS2=COS(THETA-PHI-PSI=4.0*BEND/L)
SIN3=SIN(PHI)
COS3=COS(PHI)
SIN4=SIN(PSI+PHI)
COS4=COS(PSI+PHI)
COS5=COS(THETA-CHI)

C
C   COMPUTE FRONT FACE EFFECTS
TAU=THICK+A*SIN1
SWE0=SGN1(E0)
EF=AMIN1((P*COS5)/(A*(SIN1+SWE0*SIN2+0.0001)),1.0)
DEL=0.0
IF(EF.GE.0.9) DEL=0.25
IF(EF.GE.1.0) EFFAC=0.0
EFA=EF*COS2
EFA=EFA*EFFAC
EFL=EF*SIN2
EFL=EFL*EFFAC

C
C   BACKFACE EFFECTS
C
C   COMPUTE H, DISTANCE TO BACKFACE
PPAR=PPAR+VF*DT*COS4-VL*DT*SIN4
PPERP=PPERP+VF*DT*SIN4+VL*DT*COS4
P=SGN1(PPAR*PPAR+PPERP*PPERP)
CHI=ATAN(PPERP/(PPAR+A*0.001))
HPRIM=TAU-P*COS5
IF(HPRIM.LE.0.0) CDFAC=0.0
H=HPRIM

C
C   CALCULATE EK, BACKFACE RELEASE FORCE
EK=(1.0-H/(4.0*(SWE0*EF+0.001)*A))-DEL
IF(EK.GE. 1.0) WRITE(6,50)
50  FORMAT('0','PERFORATION ACHIEVED')
IF(EK.GE. 1.0) GO TO 2
IF(EK.LE.0.0) EK=0.0
EKL=EK*SIN2
EKA=EK*COS2

C
C   COMPUTE FL, LATERAL FORCE ON FACE
VL=0.5*L*DPHI-U*SIN3+DBEND
IF(VL.GT.0.0) SGNVL=1.0
IF(VL.LT.0.0) SGNVL=-1.0
IF(VL.EQ.0.0) SGNVL=0.0
SGNVL=1.0
FL=(P1*A*A*E0)*(EFL-EKL)*(ESTAKU+LDU*VF*VF)

C
C   COMPUTE DDPHI
DDPHI=0.5*FL*L/IRUT
PHI=PHI+DPHI*DT
UPHI=DPHI+DDPHI*DT

C
C   BENDING EQUATION

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SIGBEN=PI*AA*AMIN1( BEND,EFAIL*A)*SIG1
DBBEND=5.0*(FL-SIGBEN)/MPRIM
BEND=BEND+DBBEND*DT
DBBEND=DBBEND+DBBEND*DT
BEND=BEND+DL*SIN(4.0*BEND/L)

C
C   COMPUTE DUL AND DPSI
DUL=FL/MPRIM
DL=DUL+DUL*DT
DPSI=DUL/0
PSI=PSI+DPSI*DT

C
C   COMPUTE VF
C
C   VF AT EARLY TIMES
EU=ELF
B=BETA*A
IF((LENGTH-L)/B+1.0,LT,ELF) EU=1.0+(LENGTH-L)/0
LI=EU*CUFAC*EF
DVF=((LI*KHUTAK*VF*VF*EU*CUS3**2-KHUP*(VF-U)**2)+KHUTAK*EU
*   *ESTAR0*(1.0-EKA-EFA) -Y0)
DVF=DVF*(-1.0)*GAMMA/(KHUP*A)
VF=VF+DVF*DT
IF(VF,LT,0.0) VF=0.0
UFAC=1.0
IF(VF,GE,0) GO TO 14
GO TO 20

14  CONTINUE
DU=KHUTAK*EU*(CUS3**2*VF**2*LI+ESTAR0*(1.0-EKA-EFA))
DU=-DU/MPRIM
DU=DU*PI*AA*A
U=U+DU*DT
VF=U
UFAC=0.0

20  CONTINUE
GO TO 1

30  CONTINUE

C
C   VF DURING HYDRODYNAMIC PENETRATION
C3=LI*CUS3*CUS3
ESTAR3=ESTAR0*(1.0-EKA-EFA)
IF(Y0/(EU*KHUTAK*C3)-ESTAR3/C3 .GE. 0*0) VF=0
IF(Y0/(EU*KHUTAK*C3)-ESTAR3/C3 .GE. 0*0) GO TO 1
FAC=U*U*KHUP/(EU*KHUTAK*C3)
IF(Y0/(EU*KHUTAK*C3)-ESTAR3/C3+ FAC,LE,0.0) VF=0.0
IF(Y0/(EU*KHUTAK*C3)-ESTAR3/C3+ FAC,LE,0.0) GO TO 1
DEL=4.0*KHUP**2*U**2-4.0*(EU*KHUTAK*C3-KHUP)*(EU*KHUTAK*ESTAR3
*   *KHUP*U-U=Y0)
IF(DEL,LT,0.0) WRITE(5,21)
21  FORMAT('DEL LESS THAN ZERO')
VF=-2.0*KHUP*U+SQRT(DEL)
VF=VF*0.5/ (EU*KHUTAK*C3-KHUP)
DVF=0.0

1  CONTINUE

C
C   COMPUTE OTHER VARIABLES
DU=-Y0*DT/(KHUP*L)
U=U+DU*UFAC
DL=(-U+VF)*DT
IF(L,LT,0.02*LENGTH) GO TO 2
L=L+DL
MPRIM=L*PI*AA*AKHUP
ENERGY=0.5*MPRIM*U*U

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MOM=L*PI*A*A*RHUP*U
IF(MOM/MOMO .LT.0.05) GO TO 2
IF(ENERGY/ENEMGO.LE.0.01) GO TO 2
14 CONTINUE
PBAR=P/LENGTH
MRBAR=SQRT(RHDTAK/RHUP)*PBAR
IF(MPRIM.LE.0.5) GO TO 991
WRITE(6,4) PHI,PSI,MPRIM,U,VF,DVF,P,PBAR,VL,UL,BEND,EF,ER
4 FORMAT(11E10.3,2E9.2)
991 CONTINUE
100 CONTINUE
2 CONTINUE
LALPHA=L-A
IF(LALPHA.LE.0.0) LALPHA=0.0
VRESID=(A*VF+LALPHA*U)/L
MRBAR=MPRIM/MIN1
TOVERD=THICK/(2.0*A)
THET1=THETA*180.0/PI
WRITE(6,44) THET1,THICK,TOVERD
44 FORMAT(' THETA=',E12.5,' THICK=',E12.5,' TOVERD=',E12.5)
WRITE(6,45) MRBAR,VRESID
45 FORMAT(' MRBAR=',E12.5,' VRESID=',E12.5)
WRITE(6,7) LOVERD,VO,MRBAR
7 FORMAT(' LOVERD=',E12.5,' VO=',E12.5,' MRBAR=',E12.5)
WRITE(6,25)
25 FORMAT('0')
994 CONTINUE
15 CONTINUE
CALL EXIT
END

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