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NONUNIFORM ENERGY TRANSFER IN AXIAL FLOW COMPRESSORS.(U)

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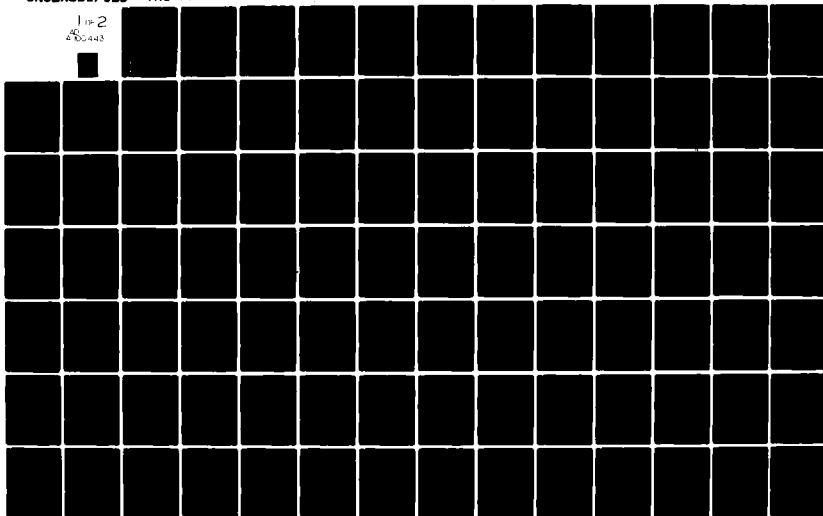
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GRANT AFOSR 80-0119
Final Scientific Report
1980 Jan 15-1981 Jan 14

NONUNIFORM ENERGY TRANSFER
IN AXIAL FLOW COMPRESSORS

G. Holbrook

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. Report Number EOARD/TR-81-4		2. Govt Accession No. AD-A100 443	
3. Recipient's Catalog Number		4. Title (and Subtitle) NONUNIFORM ENERGY TRANSFER IN AXIAL FLOW COMPRESSORS	
5. Type of Report & Period Covered Final Scientific Report - 1980 Jan 15-1981 Jan 14		6. Performing Org. Report Number VKI-PR-1980-16	
7. Author(s) G. HOLBROOK		8. Contract or Grant Number GRANT AFOSR-80-0119	
9. Performing Organization Name and Address von Karman Institute Chaussée de Waterloo 72 B-1640 Rhode Saint Genèse, Belgium		10. Program Element, Project, Task Area & Work Unit Numbers 61102F 2301/D1	
11. Controlling Office Name and Address Air Force Office of Scientific Research /PKN Bolling AFB Washington DC 20332		12. Report Date June 1980	
13. Number of Pages 109		14. Monitoring Agency Name and Address EOARD / LNE PO Box 14 FPO Box 09510	
15. Distribution Statement Approved for public release; distribution unlimited.			
16. Supplementary Notes			
17. Key Words Axial flow compressors			
18. Abstract Current through flow calculation schemes provide the compressor designer with a 2D inviscid flow analysis of a blade row. Due to blade curvature and viscous effects the flow field is quite 3D. A numerical scheme has been developed to calculate the radial migration of secondary flows in a rotating blade row and the influence of these flows on the energy transfer of the rotor. A simple set of equations for the radial equilibrium and resulting trajectory of these fluid particles is derived. A boundary layer calculation was added to correctly model rotor boundary layer fluid. The calculation scheme was then applied to 2D flow field obtained from an existing through flow analysis of a transonic rotor. The results showed the twisting of the free stream streamsheet due to the blade-to-blade velocity gradient, the magnitude of the boundary layer migration and the influence of the radial shift on the enthalpy rise of these particles. The resulting values of radial velocity and temperature rise are similar to experimentally measured values. Thus, this calculation scheme can provide the compressor designer with a quick first approximation of the influence of 3D and viscous effects on the flow field of a rotating blade row.			

This report has been reviewed by the Information Officer (EOARD/CI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

John T. Milton

JOHN T. MILTON
Scientific and Technical
Information Officer

Winston K. Pendleton

WINSTON K. PENDLETON
Lt Colonel, USAF
Chief, Propulsion & Energy

Gordon L. Hermann

GORDON L. HERMANN
Lt Colonel, USAF
Deputy Commander

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ABSTRACT

Current through-flow calculation schemes provide the compressor designer with a 2-dimensional inviscid flow analysis of a blade row. Due to blade curvature and viscous effects the flow field is quite 3-dimensional. A numerical scheme has been developed to calculate the radial migration of secondary flows in a rotating blade row and the influence of these flows on the energy transfer of the rotor. A simple set of equations for the radial equilibrium and resulting trajectory of these fluid particles is derived. A boundary layer calculation was added to correctly model rotor boundary layer fluid. The calculation scheme was then applied to the 2-dimensional flow field obtained from an existing through-flow analysis of a transonic rotor. The results showed the twisting of the freestream stream-sheet due to the blade-to-blade velocity gradient, the magnitude of the boundary layer migration and the influence of the radial shift on the enthalpy rise of these particles. The resulting values of radial velocity and temperature rise are similar to experimentally measured values. Thus this calculation scheme can provide the compressor designer with a quick first approximation of the influence of 3-dimensional and viscous effects on the flow field of a rotating blade row.

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LIST OF SYMBOLS

C_f	skin friction coefficient
F	force
H	stagnation enthalpy
I	calculation station increment
L	Lefoll velocity profile parameter
P	static pressure
R	radial location
t	time
Δt	time increment
u	local velocity
U	wheel speed
U_∞	freestream velocity in Eq. 3.13
V	velocity
y	normal distance to blade
Y	circumferential distance
β	relative flow angle
δ_E	exit boundary layer thickness
η	non-dimensional boundary layer distance, y/δ
ρ	fluid density
σ	velocity ratio factor

Subscripts

FS	freestream
P	particle
R	radial component
u	tangential component
z	axial component

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1. INTRODUCTION

Current compressor analysis tools include through-flow calculation schemes which provide the compressor designer with an inviscid axisymmetric view of the flow field within a blade row. Such results are useful for determining the radial variations in the flow field for a mean blade-to-blade streamline. Such a meridional streamline network is shown in Figure 1 for a transonic rotor. Blade curvatures and viscous effects however cause considerable distortion of this flow field. The blade curvatures produce a velocity gradient in the blade-to-blade plane creating a non-axisymmetric flow field. This velocity gradient will also cause some warpping or twisting of the blade-to-blade streamsheet (see Figure 2.). Viscosity of the fluid will produce hub and tip casing and blade boundary layers and wakes producing a highly three-dimensional flow. Thus while being a useful engineering tool the through-flow calculation needs to be supplimented with a procedure for determining these additional three-dimensional aspects of the flow field.

Due to the fluid viscosity some of the fluid particles in the flow field will have a deficit or defect in velocity relative to the essentially inviscid free-stream flow. These low momentum fluid particles are subjected to the same pressure forces, both circumferen-

tially and radially, as the freestream but due to their lower momentum their path or trajectory through the blade row will be influenced more by these forces. This momentum deficit is thus the source of some of the secondary flows in the blade row. A classical example of such secondary flows is the migration of endwall boundary layer fluid from the pressure surface to the suction surface of a turbine nozzle vane row. The radial migration of rotor blade boundary layer fluid is another example of these flows.

Several experimental investigations have presented complete three-dimensional measurements of a rotor exit flow field (REF 1,2,3,4). These studies have shown that radial velocities in a rotor wake (due to the radial migration of boundary layer fluid) can be quite large. Values of these measured radial velocities range from 10% - 15% of the axial velocity in a low speed rotor (REF 2,3) to reportedly as much as 100% of the axial velocity for a transonic rotor (REF 4). It has also been shown that the magnitude of the radial velocity in the wake is influenced by the blade loading (REF 3).

One aspect of these secondary flows that has not received much attention is how the energy transfer of the rotating blade row is influenced by these flows. It was originally believed that due to the relative motion of the rotating and stationary blade rows circum-

ferential variations in the flow field would be quickly "mixed out". Measurements (REF 5) have however shown that variations in the total temperature as large as 10% of the overall stage temperature rise can still exist at the exit of a stator row downstream of the rotor. These variations in total temperature could only have been caused by the uneven energy addition of the rotor. It is believed that the secondary flows in the rotating blade row are the source of these temperature variations. The magnitude of these variations is significant when attempting to calculate rotor or stage efficiency from measured exit total temperature. The existing temperature gradient can also be the source of thermally induced secondary flows (REF 6).

Smith (REF 7) showed that the energy transfer to a fluid particle in a turbomachine is proportional to the time the particle remains within the rotating blade row. Thus low momentum particle that, due to their velocity defect, remain within the rotating frame longer will acquire a larger enthalpy increase than freestream fluid. These low momentum particles will also have a tendency to move radially outward due to an imbalance of pressure forces and centrifugal forces. This change in radial location will contribute further to the increased energy transfer to the low momentum particles.

With a simple wake model Kerrebrock and

Mikolajczak (REF 8) showed that blade boundary layer fluid due to a slip velocity of the resulting wake will tend to be separated from the freestream fluid as the flow progresses through the successive blade rows. Thus the low momentum but high total temperature wake fluid of the rotor will tend to be collected by the downstream stator and appear in the wake of the stator blade. Thus areas of concentrated high total temperature rotor wake fluid can occur.

Experimental evidence of circumferential total temperature variations in the flow field has been presented by Keenan (REF 5), Leboeuf (REF 9) and Kool (REF 10). Analytical attempts at predicting the influence of secondary flows on the energy transfer and the resulting temperature field are scarce. Before such an analysis can be made it is important to properly predict the secondary flows themselves. Thus a significant portion of this project is devoted to establishing a simple yet physically correct model for the low momentum particles in a rotating blade row. Such a model should include not only the influence of viscosity but also the effects of blade and meridional flow path curvatures.

Throughout this analysis the author took a "particle" approach for formulating and mentally visualizing the resulting secondary flows. This involved assuming an inviscid fluid particle that for some reason

has a velocity different from that of the freestream is allowed to pass through the pressure field of the rotor blade passage. The presence of the freestream fluid is ignored. Thus no mixing or interaction between the particle and the freestream is considered. However with the technique developed such interactions can be introduced through the specification of the velocity and the relative flow angle evolution of the particle. Generally particles that have a velocity defect (as apposed to a velocity excess) are of the most interest, so the analysis was developed in terms of low momentum particles. The analysis does however apply equally well to particles that have a velocity greater than that of the freestream. This should be kept in mind throughout the explanation of the calculation technique. The particle approach helped a great deal in obtaining an intuitive insight into what parameters were important for the calculation of the particle trajectory. The author relied heavily on this intuitive approach to lead to the correct mathematical formulation. The importance of correctly modelling the physical behavior of the particle seemed to dictate such an approach, making mathematical derivations and justifications of secondary importance. Thus few such rigorous mathematical principles will be presented in this analysis.

2. MECHANISMS OF ENERGY TRANSFER

The fundamental equation for the energy transfer in a turbomachine is the Euler pump equation

$$\Delta H = U_2 V_{u2} - U_1 V_{u1} \quad (2.1)$$

The enthalpy addition in a compressor is dependent upon the change in tangential velocity from inlet to exit and on the wheel speed. Basically two different mechanisms for the energy transfer exist. They are the residence time of the particle within the blade row (resulting in tangential velocity changes) and the radial shift of the particle (resulting in changes in the local wheel speed). Due to the associated axial velocity defect of a low momentum particle the time necessary for the particle to pass through the rotor blade passage is greater than that of the freestream fluid. Smith (REF 7) showed that the instantaneous enthalpy addition for an inviscid fluid particle can be expressed as

$$\frac{DH}{Dt} = \frac{1}{\rho} \frac{\partial P}{\partial t} \quad (2.2)$$

where

H : stagnation enthalpy per unit mass

t : time

ρ : density

P : static pressure

In a compressor the right hand side of this equation is positive, so the energy transfer increases as the residence time for a particle becomes greater. This effect can perhaps be seen more clearly by examining the velocity triangles for a low momentum particle and the freestream. Figure 3a is such a velocity triangle for a particle having a defect in the absolute velocity at the entrance of a rotor blade row. Note that the low momentum particle has a smaller absolute tangential velocity than the freestream and also that the relative flow angle of the particle is larger than that of the freestream. Thus the rotor will "see" such low momentum particles as fluid entering the blade row with a high positive incidence. It is assumed throughout this analysis that a particle regardless of the magnitude of the velocity defect or excess will leave the rotor blade row at the same relative flow angle as the mean freestream flow. The rotor exit velocity triangle for the freestream and a low momentum particle is shown in Figure 3b. Here it can be seen that due to associated defect in the axial velocity the low momentum particle has a larger absolute tangential velocity than the freestream. From Eq. 2.1 it can be seen that (assuming equal wheel speeds) the low momentum particle will have a larger enthalpy increase than the freestream. As the magnitude of the velocity defect of the particle becomes greater (i.e. as the axial velocity of the particle is reduced) this difference in the energy transfer for the particle and the freestream will become

larger. The above example was for a particle entering the blade row with an absolute velocity defect. A second possibility exists and that is for a particle entering the blade row with the freestream velocity but due to viscous (or other) effects in the blade passage it leaves the blade row with a defect in the exit relative velocity. The inlet and exit velocity triangles for such a situation are shown in Figure 4. Again due to the velocity defect the particle leaves the blade row with a larger absolute tangential velocity and thus will have a larger enthalpy increase.

Up to this point the influence of changes in the wheel speed has been neglected. From Eq. 2.1 and the velocity triangles of Figure 3. and 4. the influence of the wheel speed can be seen. The wheel speed increases proportionally with the radial location. For two particles with equal inlet and exit tangential velocities the particle with the larger wheel speed will have the larger enthalpy increase. Note that the magnitude of the wheel speed also influences the resulting tangential velocity, thus having a double effect on the enthalpy increase (i.e. enthalpy rise increases with the square of the wheel speed). The influence of changes in wheel speed on the energy transfer is an important aspect of this analysis because the radial pressure field of the blade row will tend to cause low momentum fluid particles to migrate radially outward. Throughout this analysis it is vital

to keep in mind the relationship of the velocity components and the wheel speed to each other in the velocity triangles for the particle and the freestream.

There are two factors that will determine the magnitude of the radial migration of the low momentum fluid particles. First, the magnitude of the imbalance of the pressure forces and the centrifugal forces determines the size of the resultant force on the particle relative to the freestream. The magnitude of this force is dependent upon the difference between the tangential velocity of the particle and the tangential velocity of the freestream at the same radial location. The second parameter influencing the amount of radial shift is the time this net force is applied to the particle. The greater the length of time the force is applied the larger will be the deflection of the particle from a freestream streamline trajectory. This is the so called residence time of the particle and it is directly dependent upon the axial velocity of the particle. Thus the size of the velocity defect effects not only the magnitude of the force imbalance of the particle but also the time this force is applied. The relative flow angle influences the magnitude of the tangential velocity and thus the size of the applied force on the particle.

3. THE COMPUTER PROGRAM

Prior to investing a great deal of time and effort creating an elaborate computational scheme for calculating the trajectory of a low momentum particle a very simple computer program was written to test the feasibility of such a calculation. It was also possible to gain some insight into the assumptions necessary for and the restrictions of this type of calculation from an initial test program.

The calculation of the trajectory of a particle is centered around a radial equilibrium equation. For the initial program the simplest form of the radial equilibrium equation was used

$$\frac{1}{\rho} \frac{\partial p}{\partial R} = \frac{V_u^2}{R} \quad (3.1)$$

This equation is a statement of the assumption that the centrifugal forces generated by the freestream fluid "circling" the compressor axis are in equilibrium with the radial pressure field. Low momentum particles in the flow field are subjected to the same radial pressure field, but due to their lower momentum the trajectory of these particles will be influenced more by the radial pressure field. The low momentum particles will "seek" a radial location where the pressure forces and centrifugal forces are again balanced. For particles with an

excess of tangential velocity (normally associated with low momentum particles) this equilibrium location will occur at a larger radius. This equilibrium equation is the only velocity related equation used in the initial analysis so it is necessary to make two additional assumptions in order to solve for the two remaining velocity components. The first of these assumptions is that the velocity ratio, σ , which is the ratio of the particle axial velocity to the freestream axial velocity is a constant through the entire blade row. The second assumption necessary concerns the evolution of the relative flow angle of the particle. In this initial analysis only defects in entering absolute velocity were considered. From Figure 3.a it can be seen that the inlet relative flow angle, β , is established by the freestream flow direction and the velocity ratio, σ . The resulting relative flow angle for the particle can be considerably larger than the freestream angle at the inlet, but these two angles are assumed to be equal at the blade exit. Thus it is necessary to specify how the relative flow angle of the particle evolves through the blade row. A simple linear variation was used. That is, when the particle is one-half of the way through the blade row the difference between the relative flow angles of the particle and the freestream is assumed to be one-half of what it was at the inlet of the blade row. These two assumptions were made to keep the initial test program as simple as possible. With the equilibrium equation of

3.1 and these assumptions the initial program was written and tested. The freestream velocity field was generated by a Katsanis-McNally through-flow calculation and used as input data for the program. The results of this simplified approach were encouraging and enlightening. There were some discovered limitations of the simple approach but the technique seemed to warrant further improvements and refinements. These improvements would involve the equilibrium equation and the limitations of the velocity and relative flow angle evolution used.

The final program consists of a MAIN deck and four supplementary subroutines. All of the trajectory and energy calculation are done in the MAIN deck. Two interpolation subroutines are used to obtain velocity/space interpolations (LININT) and interpolations of various parameters that are a function of one variable only (INTERPOL). The two remaining subroutines are concerned with a boundary layer thickness calculation (BONLAY) and the resulting boundary layer velocity profile (VELRATIO). Because the principle objective of this project was the development of the calculation technique, the procedure will be explained in some detail. For persons interested only in using the program the input and output are explained in Appendix I and II along with a program listing and examples of input and output data.

3.1 MAIN Deck

The results of the initial program indicated a serious limitation of the equilibrium equation used. When a particle was introduced with a velocity ratio equal to unity (i.e. $\sigma = 1.00$) the trajectory calculated was that of a straight line in the meridional plane. The slope of this trajectory was determined by the initial radial velocity which then remained constant through the blade row. The equilibrium equation did not take into account the radial acceleration of the freestream (i.e. changes in radial velocity along a freestream streamline). This simple equilibrium equation took into account only the acceleration of the particle relative to the freestream (which is zero for $\sigma = 1.00$). The reason for this result can be seen by examining the radial component of the Euler Equations

$$\frac{F_{NET}}{\rho} = -\frac{1}{\rho} \frac{\partial P}{\partial R} + \frac{V_u^2}{R} = \frac{DV_R}{Dt} \quad (3.2)$$

This equation is Newton's 2nd Law applied to a fluid particle, equating the sum of the forces (per unit mass) to the net acceleration of the particle. It can be seen that use of the initial equation for the radial equilibrium (3.1) assumed that no net acceleration of the freestream existed. This is not the case if the radial velocity of the freestream varies along a streamline (due, for example, to meridional curvatures).

It was also realized that for a velocity ratio of unity ($\sigma = 1.00$) the calculation scheme must be able to properly calculate the radial location of a freestream streamline through the blade row. Clearly 3.1 is not capable of doing this because there were large radial variations in the freestream due mainly to the meridional curvatures. Thus a first "test case" for any new calculation scheme will be "can it properly calculate the radial location of a freestream streamline through the blade row".

A brief explanation of the equilibrium equation used in the final program will now be given. The radial equilibrium equation of 3.2 must apply for all particles in the flow field. Writing the equation for a freestream (FS) particle and a low momentum (P) particle yields

$$\left(\frac{DV_R}{Dt} \right)_{FS} = \frac{V_u^2}{R} \Big|_{FS} - \frac{1}{\rho} \frac{\partial P}{\partial R} \Big|_{FS} \quad (3.3)$$

$$\left(\frac{DV_R}{Dt} \right)_P = \frac{V_u^2}{R} \Big|_P - \frac{1}{\rho} \frac{\partial P}{\partial R} \Big|_P \quad (3.4)$$

The radial pressure field of the freestream is imposed upon the low momentum particles. Therefore,

$$-\frac{1}{\rho} \frac{\partial P}{\partial R} \Big|_P = -\frac{1}{\rho} \frac{\partial P}{\partial R} \Big|_{FS} \quad (3.5)$$

Rewriting the freestream equilibrium equation as

$$-\frac{1}{\rho} \left(\frac{\partial p}{\partial R} \right)_{FS} = \left(\frac{DV_R}{Dt} \right)_{FS} - \frac{V_u^2}{R} \Big|_{FS} \quad (3.6)$$

and substituting in for the pressure gradient term in the equilibrium equation of the low momentum particle yields

$$\left(\frac{DV_R}{Dt} \right)_p = \left(\frac{DV_R}{Dt} \right)_{FS} + \frac{V_u^2}{R} \Big|_p - \frac{V_u^2}{R} \Big|_{FS} \quad (3.7)$$

From this equation it can be seen that the total radial acceleration of the particle is equal to that of the freestream plus the acceleration of the particle relative to the freestream due to an imbalance of the centrifugal and pressure forces. If the centrifugal terms are of equal magnitude for the particle and freestream, the net radial acceleration of the particle is equal to that of the freestream and thus should follow the same trajectory (or streamline) as a freestream particle.

It was found that a freestream meridional streamline could be satisfactorily approximated by a Taylor series expansion for the radius, R. That is,

$$R(I+1) = R(I) + \left(\frac{dR}{dt} \right)_I \Delta t + \frac{1}{2} \left(\frac{d^2 R}{dt^2} \right)_I \Delta t^2 \quad (3.8)$$

where "I" and "I+1" indicate axial computational stations. This equation can be simplified and solved by recognizing that the first derivative term of Equation 3.8 is the

radial velocity and the second derivative term is the radial acceleration. Approximating the acceleration term by a forward difference scheme and neglecting all terms of the $O(\Delta t^3)$ and smaller yields,

$$R(I+1) = R(I) + V_R(I) \Delta t + \left[\frac{V_R(I+1) - V_R(I)}{\Delta t} \right] \frac{\Delta t^2}{2} \quad (3.9)$$

which can be further simplified to

$$R(I+1) = R(I) + \left[\frac{V_R(I+1) + V_R(I)}{2} \right] \Delta t \quad (3.10)$$

This trajectory equation states that the new radial location (i.e. at the new axial station) is equal to the old radial location plus the average radial velocity of the particle over the interval times the time the particle is within this interval. The resulting trajectory for a freestream particle (i.e. $\sigma = 1.00$) is in very good agreement with the radial location of the meridional streamline calculated by the through-flow calculation. Solving equation 3.10 involves an iterative solution because $V_R(I+1)$ is a function of the new radial location, $R(I+1)$.

The equilibrium equation is again the only equation that relates the trajectory of the particle to the freestream flow field. Thus it is possible to calculate only one new velocity component. Auxiliary equations or assumptions are necessary for the two

remaining velocity components. As with the initial program it was decided to specify as input the additional pieces of information but remove some of the limiting restrictions of the initial program. The two additional parameters are again the evolution of the velocity ratio, σ ; and the relative flow angle of the particle.

Every effort was made to keep this program as flexible as possible by giving the user complete control over these parameters via an easily changed input. The values of the velocity ratio and relative flow angle can be specified as arbitrary functions of the axial distance through the blade row, or for relative velocity defects (such as surface velocity distributions or boundary layer particles) calculated within the program from the surface velocity data or the boundary layer model.

The equilibrium equation of 3.7 is solved using a forward difference scheme yielding

$$\left[\frac{V_R(I+1) - V_R(I)}{\Delta t} \right]_P = \left[\frac{V_R(I+1) - V_R(I)}{\Delta t} \right]_{FS} + \frac{V_u^2}{R}_P - \frac{V_u^2}{R}_{FS} \quad (3.11)$$

A physical interpretation of this equation is easier when equation 3.11 is written as

$$V_R(I+1) = V_R(I) + \left[\frac{V_R(I+1) - V_R(I)}{\Delta t} \right]_{FS} \frac{\Delta t}_P + \left[\frac{V_u^2}{R}_P - \frac{V_u^2}{R}_{FS} \right] \Delta t_P \quad (3.12)$$

The new radial velocity (at the next axial station) $V_R(I+1)$ is equal to the old radial velocity plus the

change in freestream radial velocity "weighted" by the ratio of residence time for a low momentum particle and the freestream plus an additional term due to the difference in centrifugal and pressure forces. From this equation it is easy to see the influence of the residence time and the tangential velocity difference on the rate of change of the radial velocity of the particle.

Equation 3.12 and 3.10 are used to calculate the velocity and radial location of the particle at a new axial calculation station. From the known freestream velocity values the axial velocity, V_z , and the tangential velocity, V_u , of the particle can be determined using the supplied input data for the velocity ratio and relative flow angle evolution. However, because these values are functions of the new radial location it is necessary again to "guess" a new radial location and solve equation 3.12 and 3.10 iteratively until the guessed radial location and the calculated value are within a specified tolerance.

Once the complete trajectory of the particle is obtained the energy transfer calculations can be made. These calculations are made for the three streamlines shown in Figure 5. Streamline A is the freestream streamline coinciding with the radial location of the particle at the inlet of the blade row. Streamline B is the free-stream streamline coinciding with the radial location of

the particle at the exit of the blade. The temperature difference between streamline A and the particle represents the additional energy transfer due to the different velocity evolution of the particle. The temperature difference between streamline B and the particle indicates the size of the temperature variations in the flow field that will exist at the exit of the rotor.

3.2 Subroutine LININT

This subroutine is a three-dimensional interpolation routine used to obtain velocity values of the freestream at a specified meridional location, from the streamline values obtained from the through-flow calculation. The through-flow calculation provides freestream velocity values along seven meridional streamlines (hub, 10%, 30%, 50%, 70%, 90%, and tip) with 10 to 15 values within the blade row for each streamline. This routine was obtain from an existing program.

3.3 Subroutine INTERPOL

This subroutine is also an interpolation routine. It is used to interpolate between values of surface velocity, relative flow angle, and velocity ratio that are supplied as functions of the axial distance only. This interpolation routine was written by the author.

3.4 Subroutine BONLAY

This routine is a simple boundary layer calculation used to obtain the total boundary layer thickness evolution, the skin friction coefficient, and a velocity profile parameter (Lefoll's L) for the rotor blade. Empirical correlations are used for the conventional shape factor (H_{12}) and the skin friction coefficient (REF 11). An additional empirical correlation was used (see REF 12) to obtain the total boundary layer thickness from the calculated momentum thickness. This value is dependent upon the assumed boundary layer velocity profile (Lefoll). Some difficulty arose in defining a starting condition for the calculation. A simple fundamental equation was derived and used to establish a flat plate type starting condition. The boundary layer thickness calculation was a finite difference scheme (using a central difference) for the integral boundary layer equation. The calculation procedure assumed the boundary layer to be turbulent and no condition was used to detect separation. Thus care must be used when calculating boundary layers where separation may occur.

3.5 Subroutine VELRATIO

Once the total boundary layer thickness evolution is known it is possible to calculate the velocity

defect of a boundary layer particle at any location within the boundary layer for an assumed velocity profile.

To keep the boundary analysis as simple as possible it was decided to use the Lefoll (REF 13) boundary layer profile. This velocity profile can be represented by

$$\frac{u}{U_{\infty}} = 1 - \sqrt{\frac{C_F}{2}} \left[\frac{\eta^2 - 1}{2} - \ln(\eta) + L(2\eta^3 - 3\eta^2 + 1) \right] \quad (3.13)$$

From a specified location within the boundary layer it is possible to obtain the local velocity ratio for a particle at that location in the boundary layer. An assumption was necessary concerning the shape of the streamline of a boundary layer particle in the blade-to-blade plane. The assumption made was that a boundary layer particle will travel on a streamline that is a constant distance, y , from the surface of the blade. This distance is specified as a fraction of the exit boundary layer thickness, δ_E . Thus it is possible to choose how "deep" in the boundary layer the particle of interest is. As shown in Figure 6., for $y/\delta_E = 1.00$ the velocity of the particle will be equal to the freestream surface velocity. As y/δ_E becomes smaller the particle enters the boundary layer progressively nearer the blade leading edge and thus the particle will leave the blade row with a larger velocity defect. The velocity defect is not only a function of the specified y/δ_E but also of the evolution of the boundary layer thickness.

3.6 Modes of Operation

The program was developed to handle two distinctly different applications. The specification of the velocity ratio has been separated for absolute velocity ratios and relative velocity ratios (see Figure 3. and Figure 4.). For a defect in the absolute velocity the particle enters the blade row with a relative flow angle different from that of the freestream. This relative flow angle difference does influence the trajectory of the particle because it changes the tangential velocity term in equation 3.12. For this mode of operation it is necessary to supply as input how the relative flow angle progresses from the inlet value to the exit value. In this mode of operation it is possible to model inlet flow nonuniformities such as casing boundary layers and upstream blade wakes entering the blade row and determine the influence of these flows on the energy transfer. Due to a lack of time and the additional complexity of establishing wake or boundary layer decay models this mode of operation was not used during the course of this project. It should be noted however that such entrainment, wake decay and mixing models do exist in the literature and could be incorporated in this calculation procedure. The input format now is such as to allow the user to quickly and easily change the velocity ratio and flow angle evolution data.

The second mode of operation is for velocity ratios in the relative velocity. Here the relative flow direction is not influenced by the velocity ratio, σ . The particle will have the freestream relative flow angle everywhere in the blade row making the specification of the relative flow angle evolution unnecessary. One further simplification was made for this mode of operation. All blade-to-blade streamlines (suction surface to pressure surface) are considered to be the same shape as the mean streamline. That is, the relative flow angle evolution for all blade-to-blade streamlines is assumed equivalent to that of the mean freestream which is essentially the blade camber line. Thus the blades are replaced by their camber lines only as shown in Figure 7. For thin blades away from the leading edge this is a reasonable approximation. The through-flow calculation scheme included a blade-to-blade calculation as well to obtain the blade surface velocity distributions. Because the relative flow angle is assumed equal for all blade-to-blade streamlines the trajectory of a particle becomes dependent only on the relative velocity ratio, σ . This mode of operation is useful for determining the influence of the blade-to-blade velocity gradient and the rotor blade boundary layers on the energy transfer of the blade row. The freestream (out of the boundary layers) blade-to-blade streamline of interest can be specified in the input data (Y/PITCH can range from 0.0 on the suction surface to 1.0 on the pressure surface with the mean streamline

being 0.5). An interpolation routine is used to interpolate between the mean streamline velocities and the surface velocities. This mode of operation was used extensively during the project to obtain freestream and boundary layer particle trajectories and the resulting rotor exit total temperature field variations for a transonic rotor.

4. RESULTS

The flow field of a transonic rotor was used to demonstrate the use and capability of the program developed. The program was used to calculate the influence of the blade-to-blade velocity gradient and the boundary layer migration on the rotor exit flow field. The results will first be presented for the freestream (i.e. fluid outside the blade boundary layers) surface particles and then for the blade boundary layer fluid. Results will be shown at span locations of 10%, 50%, and 90% (based of flow area). The calculated trajectories for these particles and the influence of these trajectories on the energy transfer of the rotor will then be shown. All radial shift and energy transfer comparisons are made with respect to the mean blade-to-blade streamline of the through-flow calculation. The analysis was made for clean inlet flow conditions only. No attempt was made to introduce inlet flow field nonuniformities. Finally, a short summary of how these results compare with existing test data will be given.

4.1 Freestream Surface Velocity Evolution

Due to the curvature of the blade channel the surface velocities of the blade will be different than the mean blade-to-blade streamline velocity. The mag-

nitude of these velocity differences depends upon the blade design and loading. In this analysis the axial velocity and tangential velocity are obtained from the freestream velocity values (generated by the through-flow calculation) and the boundary layer calculation. Only the radial velocity component is actually calculated within the program. In Figure 8.a the axial velocity evolution for the mean, suction surface, and pressure surface streamlines is shown for the 50% span location. The velocity distributions are shown versus a nondimensional axial distance, 0% representing the blade leading edge and 100% the blade trailing edge. The suction surface velocity is everywhere greater than that of the mean streamline velocity producing a velocity ratio, σ , larger than one. The opposite is true of the pressure surface velocity, σ is everywhere less than one. The residence time variations of mean, suction surface, and pressure surface particles are a direct result of these axial velocity differences. From the velocity triangles it can be seen that (recall for this analysis the relative flow angles of all blade-to-blade streamlines are assumed equal to the mean streamline flow angle) the suction surface particles will have a smaller absolute tangential velocity and pressure surface particles a larger absolute tangential velocity than the mean streamline. The absolute tangential velocity evolution through the blade row for each of these streamlines is shown in Figure 8.b, again for 50% span.

Because the differences in tangential velocity is an indication of the magnitude of the force imbalance on the particle it is possible to obtain an idea of the evolution of this force on the particle from this figure. The larger tangential velocity for pressure surface particles will tend (see eq. 3.12) to produce a larger radial acceleration of the particle, while the opposite trend occurs for suction surface particles. The resulting calculated radial velocity evolution is shown in Figure 8.c. As expected due to the larger residence time and tangential velocity the pressure surface radial velocity is larger than that of the mean streamline. The suction surface velocity is smaller than the mean. A few remarks can be made concerning these radial velocity evolutions. From 10% to 40% meridional distance the tangential velocity difference between the pressure surface and the mean streamline is the greatest. The resulting rate of change of the pressure surface radial velocity is also the largest. From 40% to 70% meridional distance the tangential velocity difference (indication of force imbalance) becomes smaller, but still non-zero. However the radial velocity of the pressure surface particle actually decreases in this range. This can be explained by noting the rather large rate of change of the radial velocity for the mean streamline. Recall, the change in radial velocity of a particle is related to the change of radial velocity of the freestream "weighted" by the ratio of residence times. Here the

large rate of change of the freestream radial velocity and the increased residence time of the pressure surface particle combine to overcome the centrifugal force imbalance that tries to increase the particles radial velocity. This is essentially the influence of streamline curvature on the low momentum particle. For the suction surface particles all tendencies are to reduce its radial velocity relative to the mean freestream.

Similar axial, tangential, and resulting calculated radial velocity evolutions are shown in Figure 9. and Figure 10. for the 10% (near the hub) span location and the 90% (near the tip) span location. Figure 9.c shows clearly the influence of the high streamline curvature near the hub on lower and higher momentum particles. Again the centrifugal force imbalance tending to push the pressure surface particles upward (thus increasing the radial velocity) is overcome by the strong change in the mean streamline radial velocity. The suction surface particles because of their greater through flow velocity are less effected by the high streamline curvature and thus have a larger radial velocity than the mean streamline through the entire blade row. The higher wheel speed at the tip (Figure 10.) tends to produce larger tangential velocity differences and thus larger differences between the radial velocity for pressure surface, mean, and suction surface particles. The radial velocity of pressure surface particles is beginning to become

significant at the tip.

4.2 Boundary Layer Velocity Evolution

The boundary layer results are presented for four different boundary layer locations. These locations are specified as a ratio of the distance from the blade surface, y , to the exit boundary layer thickness, δ_E . A value of 1.00 corresponds to the edge of the boundary layer region and thus is identical to the surface velocity evolutions presented above. Values of y/δ_E of 0.25, 0.05, and 0.01 indicate streamlines successively closer to the blade surface and thus "deeper" within the boundary layer.

The influence of the boundary layer location on the axial velocity evolution for the suction surface is shown in Figure 11.a for 50% span. Note that over the first 50% axial distance of the blade most of the boundary layer fluid still has an excess of axial velocity relative to the mean streamline. The tangential velocity evolution for these particles is shown in Figure 11.b, and the calculated radial velocity evolution in Figure 11.c. As expected the radial velocities of the suction surface boundary layer particles are smaller than the mean streamline radial velocity over most of the blade.

Similar pressure surface boundary layer data are shown in Figure 12. Again results are shown for four different boundary layer locations. The freestream surface velocity of the pressure surface is already considerably less than that of the mean stramline. As particles deeper within the boundary layer are considered this velocity defect becomes even larger. From the velocity triangles for the boundary layer particles the tangential velocity evolution is obtained and is shown in Figure 12.b. Recall, the axial velocity ratio is an indication of the residence time and the tangential velocity excess is an indication of the radially outward force on the particle. The particle deepest within the boundary layer have not only the largest residence time but also the greatest force imbalance. Thus the high radial velocities shown in Figure 12.c for these particles is not too surprizing. The largest calculated radial velocity in the boundary layer is approximately 10% of the freestream axial velocity.

4.3 Particle Trajectory and Radial Shift

The meridional trajectory for the freestream surface particles and the boundary layer fluid will be presented in this section. The radial location of the particle is dependent not only on the evolution of the radial velocity of the particle through the blade but also on the time the particle remains within the blade

row. The trajectories for freestream suction surface and pressure surface particles are shown in Figure 13. for the 10%, 50%, and 90% span locations. The twisting of the streamsheet as it progress through the blade row can be seen. The amount of twist varies from about 1 mm at the hub to 2.5 mm at the tip with the pressure surface always higher than the suction surface.

The meridional trajectories for suction surface boundary layer particles are shown in Figure 14., also for the 10%, 50%, and 90% span locations. There is very little variation in the particle paths for the suction surface boundary layer fluid. The radial shift is small and nearly constant over the span.

Similar trajectory results are shown in Figure 15. for the pressure surface boundary layer particles. Because of the larger force imbalance and the greater residence time there is a significant radial migration of the pressure surface boundary layer fluid. The magnitude of this radial shift varies over the blade span, from about 4 mm at the hub to over 12 mm at the tip. The deepest boundary layer particle considered ($y/\delta_E = 0.01$) reached the shroud casing radius before leaving the blade row.

4.4 Energy Transfer

As mentioned in Chapter 2 the radial shift can play an important role in determining the magnitude of the energy transfer to a particle. In figure 16. a circumferential view of the blade-to-blade flow field is shown. The circumferential distance is broken into three small segments showing the suction surface boundary layer, the mean streamline, and the pressure surface boundary layer. Zero, on the horizontal axis, corresponds to the suction surface of the blade and 100% to the pressure surface. Note the different scale used for the suction surface and pressure surface boundary layers. From this view the twisting of the blade-to-blade stream-sheet can clearly be seen. Also the radial migration of pressure surface and suction surface boundary layer particles is shown in the figure. The bar graphs represent the temperature increase these fluid particles receive in addition to that of the mean streamline due to their different velocity evolution. The overall mean free-stream temperature rise through the blade row is approximately 22°C .

Figure 16.a shows the circumferential variations in the flow field at 50% span. It can be seen that freestream suction surface fluid receives the smallest energy addition while pressure surface boundary layer fluid receives the largest. The temperature variation

is shown to be $4^{\circ} - 5^{\circ}$ C. at this span location. Figure 16.b and Figure 16.c show similar data for 10% span and 90% span. Due to the smaller radial shifts and lower wheel speed at 10% span (Figure 16.b) the circumferential temperature variations are very small. At 90% span however the large radial shifts combine with the high wheel speed to produce the largest circumferential temperature variations in the blade row.

4.5 Relation to Test Data

Unfortunately no test data are available for the transonic rotor used in the analysis above. There are however some experimental results available for similar machines. High speed data are necessary for temperature variation comparisons, but rotor wake radial velocity comparisons can be made for both low speed and high speed rotor data.

Several experimenters (REF 2,3) have reported measuring rotor wake radial velocities on the order of 10% to 15% of the axial velocity downstream of a rotor. This is the order of magnitude of radial velocity obtained for particles deep within the pressure surface boundary layer.

Leboeuf (REF 9) and Kool (REF 10) made circumferential temperature field measurements behind a high

speed rotor at VKI. They reported temperature variations from the freestream flow to the blade wake fluid to be $2^{\circ} - 3^{\circ} \text{ C}$ at the blade mid-span. The rotor used for these measurements was not as highly loaded as the rotor used in the above analysis. Kerrebrock and Mikolajczak (REF 8) report circumferential temperature variations at the exit of a stator row downstream of a transonic rotor to be on the order of 10% of the overall stage temperature rise. Again the calculated values obtained from the program compare well, on an order of magnitude basis, with these experimental values.

Although it is difficult to draw too many conclusions from this limited comparison it is felt that the calculation technique developed above does produce results that are realistic and are similar to experimentally determined values.

5. CONCLUSIONS

The derivation and development of a technique to calculate the trajectory of low momentum fluid particles and their influence on the energy transfer of a rotating blade row has been presented. The resulting values of radial velocity for the boundary layer fluid and the associated increased temperature rise for these particles agree well with available test data.

A twisting or warping of a blade-to-blade streamsheet through a rotor blade row was shown to occur due to the velocity gradients in the blade channel. The suction surface freestream fluid tends to go downward while the pressure surface fluid goes radially outward. A quantitative value for this streamsheet twisting was given for a transonic rotor. The amount of twist is greatest at the blade tip.

The radial migration of rotor blade boundary layer fluid was shown to occur. The radial shift of this fluid is more significant in the pressure surface boundary layer than in the suction surface boundary layer. The migration of the boundary layer fluid was shown to be influenced by the wheel speed - increasing the radial shift at the tip and the meridional curvatures - reducing the already small radial shifts near the hub.

The increased residence time and radial shift of the boundary layer particles results in this low momentum fluid obtaining a larger energy transfer from the rotor than the freestream fluid. This nonuniform energy transfer results in circumferential variations in the rotor exit total temperature field. These temperature variations are largest at the tip of the blade due to the influence of the wheel speed on the radial shift and energy transfer.

6. RECOMMENDATIONS FOR FUTURE WORK

Much of this project was devoted to the development of the calculation technique. Only one rotor was analyzed in detail and unfortunately no test data were available for this rotor. The calculation procedure should be applied to a high speed rotor for which reliable velocity and temperature field measurements exist. There is a problem however in comparing the calculation results with measurements. The calculation procedure stops at the blade trailing edge but most measurements are made some distance downstream of the rotor. The calculation should be extended to include some reasonable distance downstream of the rotor. This would entail incorporating a boundary layer - to - wake transition model. This would however make direct comparisons between calculated results and measurement data possible.

The analysis of the transonic rotor was restricted to velocity ratios created within the blade row. The influence of inlet casing boundary layers or upstream blade wakes entering the blade row was not considered. Such an analysis should be made. The program as it exists now is capable of making such an analysis if accompanied by a particle/freestream mixing or interaction model.

Due to the impact the boundary layer model has on the resulting trajectory and energy transfer calculations some thought should be given to incorporating a more elaborate boundary layer calculation.

Finally, the calculation procedure should be extended to include the analysis of centrifugal machines. The radial shift considerations are a little different for this type of application. What might be of particular interest is modelling the inlet casing boundary layers and attempting to determine if there is a relationship between the radial migration of the hub casing boundary layer in the impeller and the separation location.

7. REFERENCES

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APPENDIX I. PROGRAM INPUT

The general input format for the program is given in Table I.1 with sample data shown in Table I.2. The program operation is separated for absolute velocity ratios (low momentum fluid entering the blade row) and relative velocity ratios (low momentum fluid generated within the blade row). The input possibilities for these two modes will be explained separately. Common to all of the input data is the freestream velocity field, the hub the shroud contours, and the radial location of the particle or streamline of interest at the inlet of the blade row. The velocity ratio and flow angle data necessary vary with the cases considered (explained below). The program options are selected through the use of a "trigger" card specifying six triggers, I1, I2, I3, I4, I5, and I6. Trigger I3 determines whether absolute velocity ratios ($I3=0$) or relative velocity ratios ($I3=1$) are considered.

I.1 Absolute Velocity Ratio

For velocity defects in the inlet flow field it is necessary to specify the velocity ratio and the relative flow angle evolution of the particle through the blade row. The velocity ratio can be specified as a constant or as some function of the axial distance through the blade row. The relative flow angle evolution

is assumed to be a linear variation between the inlet and exit if not otherwise specified in the input. Each of these cases will be briefly outlined below.

Case 1. ("trigger" card: 0 0 0 0 0 0)

For this case the velocity ratio is an input parameter and assumed constant through the blade row. The relative flow angle evolution is assumed linear from the inlet angle to the exit angle.

Case 2. (1 0 0 0 0 0)

This is the same as Case 1 except now the velocity ratio is specified as a function of the axial distance. The relative flow angle evolution is still a linear one.

Case 3. (1 1 0 0 0 0)

For this case both the velocity ratio and the relative flow angle evolution are specified as functions of the axial distance. The blade-to-blade trajectory of the particle can now be arbitrarily specified allowing for freestream/particle mixing or interaction models to be incorporated. Note that the inlet relative flow angle is obtained from the freestream flow direction and the velocity ratio. Thus the relative flow angle evolu-

tion should be specified such that it agrees with this value at the inlet.

I.2 Relative Velocity Ratio

One of the initial goals of this project was to predict the radial migration of blade boundary layer fluid. This mode of operation can consider such flows as well as freestream (i.e. out of the boundary layers) velocity variations. This mode of operation is selected by I3 = 1. For relative velocity ratios it is assumed that the relative flow angle evolution for all blade-to-blade streamlines is (unless otherwise specified) identical to that of the mean streamline. Thus only the velocity ratio needs to be known to obtain the particle trajectory.

Case 1. (0 0 1 0 0 0)

The relative velocity ratio is specified in the input and assumed to be constant through the blade row. This case is not of much importance.

Case 2. (1 0 1 0 0 0)

Here the velocity ratio is specified as a function of the axial distance. The relative flow angle evolution is that of the mean freestream streamline.

Case 3. (1 1 1 0 0 0)

The velocity ratio evolution and the relative flow angle evolution are now specified in the input as functions of the axial distance through the blade row, thus allowing for deviations from the mean streamline relative flow angle evolution.

Case 4. (0 0 1 0 0 1)

Up to this time it has been assumed that the particle follows the mean streamline through the blade row. For this case it is possible to specify the blade-to-blade location of the particle of interest. This is done through the specification of Y/PITCH in the input. A value of 0.0 corresponds to the suction surface, 0.5 to the mean streamline, and 1.0 to the pressure surface of the blade (still assumed out of the boundary layers). For intermediate values of Y/PITCH an interpolation is done between the surface velocities and the mean streamline velocity to obtain a velocity ratio relative to the mean streamline velocity.

Case 5. (0 0 1 1 0 0)

For this case it is possible to consider suction surface boundary layer particles. The desired streamline in the boundary layer is specified as a y/δ_E

value ranging from 1.00 for the streamline coinciding with the edge of the boundary layer at the blade exit (identical to the freestream surface velocity) to 0.0 for particles being "deeper" within the boundary layer (see Figure 6.). The velocity defect is obtained from the boundary layer calculation and the velocity profile model and compared to the mean streamline velocity.

Case 6. (0 0 1 0 1 0)

This is identical to Case 5. except for the pressure surface boundary layer. It is also necessary to input the number of points involved in the boundary layer thickness calculation and the fluid dynamic viscosity for these boundary layer cases.

Table I.1 General Input Format

(all reals read in F10.4 FORMAT)

```

card 1: rotational speed (rad/s), pitch (m)
card 2: inlet temperature: particle, strm. A, strm. B
card 3: tip LE radius, axial location (m)
card 4: hub LE radius, axial location (m)
card 5: mid-merid. distance tip radius, axial loc.
card 6: mid-merid. distance hub radius, axial loc.
card 7: tip TE radius, axial location
card 8: hub TE radius, axial location
card 9: num. strmlns - IR, num. pts. - IZ (2I4)
card 10: ----- begin reading freestream -----
card 11: ----- velocity field -----
"          1 - IZ, 1 - IR
"          Read: z,R,U,alpha,beta,wss,wps
"          from Katsanis Output

```

```

then: "trigger" card I1,I2,I3,I4,I5,I6 (6I2)
      : Radial location of entering particle on interest

```

```

then: input varies depending on the option using

```

Table I.2 Input Data

100	826.55	0.095+1	300.	13.77	31.79	225.15	225.15
200	0.015240	0.15073	225.15	11.98	32.19	225.48	225.48
300	0.015241	0.15241	225.48	10.41	32.04	225.93	225.93
400	0.015397	0.15397	226.11	8.98	28.30	226.50	226.50
500	0.0162131	0.162131	226.50	7.81	19.99	226.58	226.58
600	0.016046	0.16046	226.52	6.81	16.02	226.50	226.50
700	0.0160759	0.160759	226.52	6.27	13.14	226.44	226.44
800	0.016351	0.16351	226.52	5.72	10.26	226.11	226.11
900	0.0163517	0.163517	226.52	5.17	7.35	225.51	225.51
1000	0.016240	0.16240	226.52	4.64	4.39	224.78	224.78
1100	0.016397	0.16397	226.52	4.11	1.46	223.61	223.61
1200	0.016530	0.16530	226.52	3.60	-1.60	222.76	222.76
1300	0.016755	0.16755	226.52	3.09	-4.60	222.16	222.16
1400	0.016780	0.16780	226.52	2.59	-8.03	221.72	221.72
1500	0.016854	0.16854	226.52	2.11	-11.86	221.43	221.43
1600	0.016854	0.16854	226.52	1.67	-15.94	221.20	221.20
1700	0.016854	0.16854	226.52	1.25	-19.99	221.00	221.00
1800	0.016854	0.16854	226.52	0.87	-23.99	220.82	220.82
1900	0.016854	0.16854	226.52	0.53	-27.99	220.66	220.66
2000	0.016854	0.16854	226.52	0.23	-31.99	220.52	220.52
2100	0.016854	0.16854	226.52	0.00	-35.99	220.40	220.40
2200	0.016854	0.16854	226.52	-0.23	-39.99	220.30	220.30
2300	0.016854	0.16854	226.52	-0.53	-43.99	220.20	220.20
2400	0.016854	0.16854	226.52	-0.87	-47.99	220.10	220.10
2500	0.016854	0.16854	226.52	-1.25	-51.99	220.00	220.00
2600	0.016854	0.16854	226.52	-1.67	-55.99	219.90	219.90
2700	0.016854	0.16854	226.52	-2.11	-59.99	219.82	219.82
2800	0.016854	0.16854	226.52	-2.59	-63.99	219.76	219.76
2900	0.016854	0.16854	226.52	-3.09	-67.99	219.70	219.70
3000	0.016854	0.16854	226.52	-3.60	-71.99	219.66	219.66
3100	0.016854	0.16854	226.52	-4.11	-75.99	219.62	219.62
3200	0.016854	0.16854	226.52	-4.64	-79.99	219.59	219.59
3300	0.016854	0.16854	226.52	-5.17	-83.99	219.56	219.56
3400	0.016854	0.16854	226.52	-5.72	-87.99	219.52	219.52
3500	0.016854	0.16854	226.52	-6.27	-91.99	219.49	219.49
3600	0.016854	0.16854	226.52	-6.81	-95.99	219.46	219.46
3700	0.016854	0.16854	226.52	-7.35	-99.99	219.43	219.43
3800	0.016854	0.16854	226.52	-7.81	-103.99	219.40	219.40
3900	0.016854	0.16854	226.52	-8.27	-107.99	219.37	219.37
4000	0.016854	0.16854	226.52	-8.72	-111.99	219.34	219.34
4100	0.016854	0.16854	226.52	-9.17	-115.99	219.30	219.30
4200	0.016854	0.16854	226.52	-9.64	-119.99	219.26	219.26
4300	0.016854	0.16854	226.52	-10.11	-123.99	219.22	219.22
4400	0.016854	0.16854	226.52	-10.59	-127.99	219.18	219.18
4500	0.016854	0.16854	226.52	-11.07	-131.99	219.14	219.14
4600	0.016854	0.16854	226.52	-11.55	-135.99	219.10	219.10
4700	0.016854	0.16854	226.52	-12.03	-139.99	219.06	219.06
4800	0.016854	0.16854	226.52	-12.51	-143.99	219.02	219.02
4900	0.016854	0.16854	226.52	-12.99	-147.99	218.98	218.98
5000	0.016854	0.16854	226.52	-13.47	-151.99	218.94	218.94
5100	0.016854	0.16854	226.52	-13.95	-155.99	218.90	218.90
5200	0.016854	0.16854	226.52	-14.43	-159.99	218.86	218.86
5300	0.016854	0.16854	226.52	-14.91	-163.99	218.82	218.82
5400	0.016854	0.16854	226.52	-15.39	-167.99	218.78	218.78
5500	0.016854	0.16854	226.52	-15.87	-171.99	218.74	218.74
5600	0.016854	0.16854	226.52	-16.35	-175.99	218.70	218.70
5700	0.016854	0.16854	226.52	-16.83	-179.99	218.66	218.66
5800	0.016854	0.16854	226.52	-17.31	-183.99	218.62	218.62
5900	0.016854	0.16854	226.52	-17.79	-187.99	218.58	218.58
6000	0.016854	0.16854	226.52	-18.27	-191.99	218.54	218.54
6100	0.016854	0.16854	226.52	-18.75	-195.99	218.50	218.50

Table I.2 (con't)

6200	0.0067342	0.24448	246.00	0.93	42.60	275.54	217.69
6300	0.017401	0.24474	247.37	0.44	39.09	304.38	180.50
6400	0.026837	0.24547	248.25	0.40	35.48	304.15	150.89
6500	0.034175	0.24559	220.93	0.31	31.76	268.90	153.13
6600	0.044175	0.24559	213.02	0.11	26.42	264.93	161.23
6700	0.052810	0.24532	208.81	-0.33	26.07	219.50	108.27
6800	0.061420	0.24514	203.55	-0.68	23.69	215.33	103.94
6900	0.070009	0.24500	197.73	-0.99	21.36	245.19	169.45
7000	0.078580	0.24474	195.91	-1.14	19.68	215.79	176.26
7100	0.087131	0.24448	193.45	-1.26	18.25	202.41	164.23
7200	0.095680	0.24413	190.17	-1.39	16.83	199.66	161.52
7300	0.104114	0.24337	186.65	-1.50	16.56	144.64	176.69
7400	0.11258	0.24367	189.74	-1.50	16.54	160.71	167.74
7500	0.12097	0.24342	181.07	-1.40	18.46	161.97	161.07
7600	0.12764	0.24364	161.34	-1.38	16.36	161.34	181.34
7700	0.13437	0.24339	161.65	-1.40	16.26	141.65	161.65
7800	0.0085501	0.24324	255.86	-1.46	40.67	255.86	255.86
7900	0.017401	0.24286	255.00	-1.51	43.91	319.44	203.86
8000	0.026837	0.24298	245.72	-1.51	40.71	317.96	177.21
8100	0.035462	0.24265	232.38	-1.28	37.40	310.18	154.71
8200	0.044119	0.24265	226.66	-1.15	33.96	293.63	157.63
8300	0.052748	0.24231	213.52	-1.23	31.79	298.31	160.95
8400	0.061454	0.24242	207.86	-1.39	29.72	234.94	160.88
8500	0.069438	0.24119	202.15	-1.50	27.76	232.20	172.26
8600	0.076501	0.24149	190.45	-1.50	26.92	212.76	164.40
8700	0.084044	0.24131	196.41	-1.34	26.69	206.06	166.28
8800	0.092583	0.24111	192.41	-1.34	26.31	202.49	162.55
8900	0.10404	0.24105	183.68	-1.43	26.26	193.15	162.41
9000	0.11248	0.24081	183.66	-1.43	26.53	183.66	163.86
9100	0.12089	0.24054	183.64	-1.32	26.49	183.64	183.89
9200	0.12761	0.24037	183.60	-1.33	26.46	183.60	183.60
9300	0.13432	0.24022	183.36	-1.38	26.42	183.36	183.38
9400	0.14100	0.24005	163.16	-1.44	26.36	183.16	183.16
9500	0.1496078	0.24000	267.42	-3.22	99.41	267.42	267.42
9600	0.016666	0.24041	267.62	-2.90	44.19	313.16	221.89
9700	0.027454	0.24010	257.83	-2.59	41.19	330.25	185.95
9800	0.030082	0.24076	241.13	-2.35	41.36	324.77	163.54
9900	0.044873	0.24041	230.56	-2.12	36.31	300.12	161.66
10000	0.053237	0.24066	221.37	-1.95	36.19	271.72	171.07
10100	0.061774	0.24073	214.65	-1.76	34.37	255.37	173.54
10200	0.070292	0.24046	208.38	-1.71	32.31	236.95	181.67
10300	0.078765	0.24020	204.98	-1.48	32.37	217.55	192.48
10400	0.0871259	0.24004	202.51	-1.27	32.03	215.18	169.40
10500	0.095409	0.23984	198.09	-0.99	31.78	212.44	183.69
10600	0.10413	0.24001	193.94	-0.91	31.58	201.01	166.91
10700	0.11254	0.24055	189.96	-0.64	32.23	189.96	189.96
10800	0.12092	0.24050	187.60	-0.74	32.23	189.80	187.60
10900	0.12763	0.24039	183.99	-0.73	32.34	189.89	183.99
11000	0.13434	0.24031	188.30	-0.75	32.44	187.30	187.30
11100	0.14104	0.24022	177.46	-0.94	32.56	187.46	187.46
11200	0.016174	0.24103	273.81	-3.08	50.46	273.81	273.81
11300	0.019241	0.24085	273.69	-3.15	46.36	314.12	273.69
11400	0.026018	0.24099	263.63	-3.11	45.65	337.51	196.15
11500	0.036591	0.24050	249.77	-2.84	42.67	332.54	167.68
11600	0.045130	0.24019	235.70	-2.55	39.97	308.64	163.36
11700	0.053643	0.24014	225.97	-2.28	37.76	276.35	173.69
11800	0.062131	0.24043	216.72	-1.99	36.15	256.92	176.53
11900	0.070546	0.24015	212.57	-1.71	34.86	234.65	197.70
12000	0.079036	0.24072	209.22	-1.42	34.39	224.62	195.41
12100	0.087456	0.24073	205.67	-1.13	34.60	221.30	197.63
12200	0.095856	0.24059	201.92	-0.84	33.72	221.34	182.53

Table I.2 (concluded)

12300	0.10424	0.30748	197.45	-0.61	33.11	197.45	197.45
12400	0.11261	0.30741	193.72	-0.42	33.79	193.72	193.72
12500	0.12097	0.30736	193.47	-0.39	33.82	193.47	193.47
12600	0.12767	0.30733	192.50	-0.23	34.00	192.50	192.50
12700	0.13437	0.30730	191.67	-0.20	34.15	191.67	191.67
12800	0.14107	0.30728	190.64	-0.21	34.35	190.64	190.64
12900	0.14777	0.30725					
13000	0.15447	0.30722					
13100	0.16117	0.30719					
13200	0.16787	0.30716					
13300	0.17457	0.30713					
13400	0.18127	0.30710					
13500	0.18797	0.30707					
13600	0.19467	0.30704					
13700	0.20137	0.30701					
13800	0.20807	0.30698					
13900	0.21477	0.30695					
14000	0.22147	0.30692					

Set 1

12300	0.10424	0.30748	197.45	-0.61	33.11	197.45	197.45
12400	0.11261	0.30741	193.72	-0.42	33.79	193.72	193.72
12500	0.12097	0.30736	193.47	-0.39	33.82	193.47	193.47
12600	0.12767	0.30733	192.50	-0.23	34.00	192.50	192.50
12700	0.13437	0.30730	191.67	-0.20	34.15	191.67	191.67
12800	0.14107	0.30728	190.64	-0.21	34.35	190.64	190.64
12900	0.14777	0.30725					
13000	0.15447	0.30722					
13100	0.16117	0.30719					
13200	0.16787	0.30716					
13300	0.17457	0.30713					

Set 2

12300	0.10424	0.30748	197.45	-0.61	33.11	197.45	197.45
12400	0.11261	0.30741	193.72	-0.42	33.79	193.72	193.72
12500	0.12097	0.30736	193.47	-0.39	33.82	193.47	193.47
12600	0.12767	0.30733	192.50	-0.23	34.00	192.50	192.50
12700	0.13437	0.30730	191.67	-0.20	34.15	191.67	191.67
12800	0.14107	0.30728	190.64	-0.21	34.35	190.64	190.64
12900	0.14777	0.30725					
13000	0.15447	0.30722					
13100	0.16117	0.30719					
13200	0.16787	0.30716					
13300	0.17457	0.30713					

Set 3

12300	0.10424	0.30748	197.45	-0.61	33.11	197.45	197.45
12400	0.11261	0.30741	193.72	-0.42	33.79	193.72	193.72
12500	0.12097	0.30736	193.47	-0.39	33.82	193.47	193.47
12600	0.12767	0.30733	192.50	-0.23	34.00	192.50	192.50
12700	0.13437	0.30730	191.67	-0.20	34.15	191.67	191.67
12800	0.14107	0.30728	190.64	-0.21	34.35	190.64	190.64
12900	0.14777	0.30725					
13000	0.15447	0.30722					
13100	0.16117	0.30719					
13200	0.16787	0.30716					
13300	0.17457	0.30713					

Sample Data Sets

Set 1

card 129: "trigger" card (6I2)
card 130: radial location, % span (I4)
card 131: num. pts. in velocity ratio distribution (I2)
card 132: velocity ratio, axial location
card 133: " " " "
card 134: " " " "
card 135: " " " "
card 136: num. pts. in flow angle distribution (I2)
card 137: relative flow angle, axial distance
card 138: " " " " "
card 139: " " " " "
card 140: " " " " "
card 141: start next data set with new span location

Set 2

card 129: "trigger" card (6I2)
card 130: radial location, % span (I4)
card 131: Y/PITCH
card 132: start next data set with new span location

Set 3

card 129: "trigger" card (6I2)
card 130: radial location, % span (I4)
card 131: num. pts. used in boundary layer calc. (I2)
card 132: fluid dynamic viscosity - SI units
card 133: y/δ_E , b. l. streamline of interest
card 134: start next data set with new span location

APPENDIX II. PROGRAM OUTPUT

Examples of the program output are shown in Table II.1. The headings identify the mode of operation and general inlet conditions. The particles radial location and velocity is calculated at 12 axial locations through the blade row at increments based on the blade axial chord at the specified inlet radius. The axial and radial location of the particle is given at each station, as well as the velocity ratio and the velocity components of the particle and the freestream. The span (here based on the blade height) is also given at each axial location to gain a quick appreciation for the magnitude of radius variations calculated.

The blade inlet conditions are then summarized for three different streamlines, the particle streamline, streamline A, and streamline B (see Figure 5.). The velocity components and flow angles are given for each of these streamlines. The blade exit conditions are likewise given. Lastly the energy transfer for the three different streamlines is summarized. Given is the radial shift of each streamline, the inlet total temperature, the exit total temperature, the temperature rise, and the exit temperature difference between the particle streamline and streamline B.

Two additional parameters are included in the output that have not yet been explained. They are labeled "Q/VZB" and "E/P". These parameters estimate the "cross-flow" of a particle and the resulting displacement in the blade-to-blade plane. Q/VZB is the ratio of the component of the particle velocity that is perpendicular to the relative freestream velocity direction and the freestream axial velocity. E/P is the fraction of the blade pitch the particle will be displaced as it flows through the blade row. Such a displacement will occur only for particles that have at some time a relative flow angle different than that of the freestream. These parameters were included mainly out of the curiosity of the author.

Table II.1 Sample Output.

Results From Data Set 1.

PATH OF PARTICLE INTRODUCED UPSTREAM

AT SPAN = 50% OF MASS FLOW

ATM VELOCITY RATIO, SIGMA = 0.99 IN ABSOLUTE VELOCITY

ARBITRARY SIGMA DISTRIBUTION GIVEN

ARBITRARY BETA DISTRIBUTION GIVEN

BLADE-TO-BLADE POSITION OF THE PARTICLE IS, Y/PITCH = 0.50

***** PARTICLE ***** PRESERVE *****									
COORD	RADIUS	SPAN	Z	SIGMA	VZ	VU	VR	W	WFS
1.000	0.24584	0.58	0.00228	0.994	174.50	33.99	3.05	243.10	175.50
0.900	0.24600	0.58	0.01120	0.972	177.81	38.36	2.01	223.24	182.93
0.800	0.24612	0.58	0.02012	0.950	178.59	44.51	2.04	217.35	187.04
0.700	0.24624	0.58	0.02903	0.927	174.43	51.37	2.64	207.59	186.00
0.600	0.24636	0.58	0.03795	0.905	169.78	103.58	2.79	197.04	187.58
0.500	0.24651	0.58	0.04687	0.883	165.26	114.83	2.28	167.08	187.20
0.400	0.24651	0.58	0.05579	0.861	161.06	125.06	1.22	179.26	187.16
0.300	0.24666	0.58	0.06470	0.838	156.21	134.90	0.53	170.76	186.36
0.200	0.24668	0.58	0.07362	0.816	150.79	144.21	0.25	162.17	184.81
0.100	0.24671	0.58	0.08254	0.794	145.24	152.87	0.65	153.95	183.00
0.000	0.24678	0.58	0.09146	0.771	139.39	161.00	1.71	145.86	180.71
1.000	0.24695	0.58	0.10038	0.749	133.06	168.71	3.34	137.67	177.55

BLADE INLET CONDITIONS

	R	Z	VZ	VU	VR	W	ALPHA	BETA	TUELA
PARTICLE	0.24584	0.00228	174.50	33.99	3.05	243.10	11.02	44.12	1.20
PRESERVE A	0.24584	0.00228	175.50	34.18	3.67	243.68	11.02	43.92	1.20
PRESERVE B	0.24571	0.01128	183.04	39.24	2.38	245.68	12.10	41.83	0.74

BLADE EXIT CONDITIONS

	R	Z	VZ	VU	VR	W	ALPHA	BETA	TUELA
PARTICLE	0.24695	0.10038	133.06	168.71	3.34	137.67	51.75	14.91	1.44
PRESERVE A	0.24632	0.16038	177.75	141.48	-4.47	168.34	38.52	19.26	-1.44
PRESERVE B	0.24675	0.16038	177.55	141.43	-4.46	168.35	38.54	19.45	-1.44

ENERGY TRANSFER

	DEL R	TOT	DEL T	T DIFF	Q/VZB	E/P
PARTICLE	0.00111	300.00	27.39	4.25	0.003	0.003
PRESERVE A	0.00038	300.00	21.75			
PRESERVE B	0.00124	300.00	20.79			

Table II.1 (con't)
Results From Data Set 2.

PATH OF PARTICLE INTRODUCED UPSTREAM

AT SPAN = 50% OF MASS FLOW

WITH VELOCITY RATIO, SIGMA = 1.00 IN RELATIVE VELOCITY

BLADE-TO-BLADE POSITION OF THE PARTICLE IS, Y/PITCH = 0.75

***** PARTICLE *****										***** FREESTREAM *****			
CHORD	RADIUS	SPAN	Z	SIGMA	VZ	VU	VR	W	VZFS	VUS	VRFS	WFS	
0.100	0.24544	0.58	0.00228	1.006	175.50	34.18	3.67	243.68	175.50	34.18	3.67	243.68	
0.200	0.24600	0.58	0.01170	0.924	169.11	51.51	2.40	227.27	162.93	39.14	2.33	235.83	
0.300	0.24610	0.58	0.02012	0.860	162.80	74.18	1.55	207.60	148.05	54.14	1.21	230.09	
0.400	0.24621	0.58	0.02903	0.803	158.62	93.57	2.12	193.00	148.08	73.15	1.12	228.84	
0.500	0.24633	0.58	0.03795	0.755	160.33	107.26	2.42	187.07	167.59	90.88	0.72	218.65	
0.600	0.24645	0.58	0.04687	0.686	165.78	115.99	2.01	187.57	167.21	104.84	-0.18	211.81	
0.700	0.24656	0.58	0.05579	0.633	168.94	123.14	0.95	183.72	186.38	114.43	-1.57	207.79	
0.800	0.24668	0.58	0.06470	0.606	168.87	131.43	0.16	183.69	184.84	132.28	-2.98	199.22	
0.900	0.24679	0.58	0.07362	0.578	171.51	137.77	-0.40	183.69	184.84	137.18	-3.49	194.72	
1.000	0.24693	0.58	0.08254	0.541	175.93	139.77	-0.62	187.36	183.05	139.39	-4.21	191.95	
0.000	0.24649	0.58	0.09146	0.576	170.53	140.91	-0.82	187.36	180.80	139.39	-4.21	191.95	
0.000	0.24643	0.58	0.10036	0.579	173.94	142.79	-1.01	184.30	177.71	141.47	-4.47	188.34	

BLADE INLET CONDITIONS

PARTICLE	R	Z	VZ	VU	VR	W	ALPHA	BETA	DELTA
PARTICLE A	0.24584	0.00228	175.50	34.18	3.67	243.68	11.02	43.92	1.20
FREESTREAM A	0.24584	0.00228	175.50	34.18	3.67	243.68	11.02	43.92	1.20
FREESTREAM B	0.24517	0.01121	183.16	39.14	2.50	245.49	12.07	41.77	0.78

BLADE EXIT CONDITIONS

PARTICLE	R	Z	VZ	VU	VR	W	ALPHA	BETA	DELTA
PARTICLE A	0.24645	0.10638	173.94	142.79	-1.01	184.30	39.38	19.30	-0.30
FREESTREAM A	0.24632	0.10038	177.75	131.48	-4.47	184.34	34.52	19.20	-1.74
FREESTREAM B	0.24645	0.10038	177.71	141.47	-4.47	188.34	36.52	19.30	-1.47

ENERGY TRANSFER

PARTICLE	DEL R	FOI	TOZ	DEL F	F LIF	G/VZB	E/P
PARTICLE A	0.00069	300.00	372.03	22.03	1.25	0.000	0.000
FREESTREAM A	0.00048	300.00	371.75	21.75			
FREESTREAM B	0.00127	300.00	320.78	20.78			

Table II.1 (concluded)
Results From Data Set 3.

PAIR OF PARTICLE INTRODUCED UPSIREAM

AT SPAN = 50% OF MASS FLOW

WITH VELOCITY RATIO, SIGMA = 1.00 IN RELATIVE VELOCITY

PARTICLE IS IN SUCTION SURFACE BOUNDARY LAYER, WITH Y/DEL = 0.250

EXIT BOUNDARY LAYER THICKNESS = 0.005401

***** PARTICLE *****										***** FREESTREAM *****									
CHORD	RADIUS	SPAN	Z	SIGMA	VZ	VU	VR	W	VZFS	VUFS	VRFS	WFS							
0.100	0.24564	0.58	0.00228	1.000	175.50	34.18	3.67	243.68	175.50	34.14	3.67	243.68							
0.000	0.24564	0.58	0.01170	1.152	210.40	14.09	2.30	263.28	162.93	39.11	2.33	245.83							
0.100	0.24609	0.58	0.02012	1.270	236.77	13.90	1.00	304.61	163.05	54.15	1.22	240.07							
0.200	0.24607	0.58	0.02903	1.314	247.22	32.72	0.47	300.70	160.10	73.16	1.14	228.79							
0.300	0.24607	0.58	0.03795	1.291	242.27	58.12	-0.51	260.21	167.63	90.82	0.70	218.76							
0.400	0.24593	0.58	0.04687	1.229	230.19	82.07	-1.92	244.30	167.34	114.40	-0.14	207.30							
0.500	0.24592	0.58	0.05579	1.178	220.71	98.58	-3.08	234.51	156.51	123.95	-1.52	202.72							
0.600	0.24574	0.57	0.06470	1.157	215.84	111.55	-5.15	216.07	146.14	137.24	-3.47	195.14							
0.700	0.24549	0.57	0.07362	1.090	201.84	126.95	-6.23	195.73	143.44	137.24	-4.43	184.78							
0.800	0.24519	0.57	0.08254	1.004	184.26	136.94	-6.69	185.34	141.30	137.24	-4.23	171.34							
0.900	0.24484	0.57	0.09146	0.965	171.97	141.70	-7.13	185.34	141.30	137.24	-4.23	171.34							
1.000	0.24447	0.56	0.10038	0.955	170.24	144.38	-7.25	179.90	141.64	141.64	-4.51	166.34							

BLADE INLET CONDITIONS

	K	Z	VZ	VU	VR	W	ALPHA	DELTA	DELTA
PARTICLE	0.24584	0.00228	175.50	34.18	3.67	243.68	11.02	43.92	1.20
FREESTREAM A	0.24584	0.00228	175.50	34.18	3.67	243.68	11.02	43.92	1.20
FREESTREAM B	0.24304	0.01095	183.20	38.75	3.16	244.80	11.94	41.51	0.99

BLADE EXIT CONDITIONS

	K	Z	VZ	VU	VR	W	ALPHA	DELTA	DELTA
PARTICLE	0.24447	0.10038	170.24	144.38	-7.28	179.90	40.30	18.72	-2.79
FREESTREAM A	0.24447	0.10038	170.24	144.38	-7.28	180.34	38.52	19.26	-1.41
FREESTREAM B	0.24447	0.10038	178.33	141.64	-4.51	188.34	36.46	18.72	-1.45

ENERGY TRANSFER

	DEL K	101	102	DEL F	U DIF	Q/VZ0	F/P
PARTICLE	-0.00137	300.00	322.12	22.12	1.39	0.000	0.000
FREESTREAM A	0.00048	300.00	321.75	21.75			
FREESTREAM B	0.00143	300.00	320.73	20.73			

APPENDIX III. PROGRAM LISTING

A listing of the final program is included for the readers convenience.

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100 THIS PROGRAM WILL CALCULATE THE PATH OF A LOW SURFACIA FLUID
200 PARTICLE THROUGH A KOLOR PLATE FOR TWO GIVE RUTON 1-0-0-1
300 AND FAYT CONDITIONS. GREG HOLBROOK FEBRUARY 1960
400
500 ABSOLUTE VELOCITY RATIO, SIGMA
600 11,12,13,14,15,16 = 0: SIGMA IS CONSTANT, AND BETA DIFFERENCE
700 DECREASES LINEARLY.
800 11 = 1: ARBITRARY SIGMA DISTRIBUTION INPUT
900 12 = 1: ARBITRARY BETA DISTRIBUTION INPUT
1000 13 = 1: RELATIVE VELOCITY RATIO
1100
1200 RELATIVE VELOCITY RATIO, SIGMA
1300 11,12,13,14,15,16 = 0,13 = 1: SIGMA IS CONSTANT, BETA PARTICLE
1400 IS EQUAL TO BETA OF THE FREESTREAM.
1500 11 = 1: ARBITRARY SIGMA DISTRIBUTION INPUT
1600 12 = 1: ARBITRARY BETA DISTRIBUTION INPUT
1700 13 = 1: RELATIVE VELOCITY RATIO
1800 14 = 1: PARTICLE IN SUCTION SURFACE BOUNDARY LAYER
1900 15 = 1: PARTICLE IN PRESSURE SURFACE BOUNDARY LAYER
2000 16 = 1: PARTICLE BETWEEN REAR AND PRESSURE OR SUCTION SURFACES
2100
2200 DIMENSION R(25),Z(25),VZ(25),VR(25),W(25),DELTA(25)
2300 DIMENSION TDELTA(25),CAL(25),AK(25),ABD(25),WFS(25),DELTA(25)
2400 DIMENSION TDELTA(25),VZ(25),VZ(25),DZ(25),VF(25),SIGMA(25)
2500 DIMENSION ZF(25),VZF(25),VZF(25),VF(25),VF(25),VF(25),VF(25)
2600 DIMENSION WZ(25),WZ(25),WZ(25),WZ(25),WZ(25),WZ(25),WZ(25)
2700 DIMENSION VZ(25),VZ(25),VZ(25),VZ(25),VZ(25),VZ(25),VZ(25)
2800 DIMENSION WZ(25),WZ(25),WZ(25),WZ(25),WZ(25),WZ(25),WZ(25)
2900 DIMENSION DELTA(25),DELTA(25),DELTA(25),DELTA(25),DELTA(25),DELTA(25)
3000 DIMENSION WDELTA(25),WDELTA(25),WDELTA(25),WDELTA(25),WDELTA(25)
3100 DIMENSION DELTA(25),DELTA(25),DELTA(25),DELTA(25),DELTA(25)
3200 DIMENSION DELTA(25),DELTA(25),DELTA(25),DELTA(25),DELTA(25)
3300
3400 CF=1005.
3500 FI=3.14159
3600
3700 READ(5,100) OMEGA,F11CH
3800 READ(5,100) F10,F101A,F101B
3900 READ(5,100) F10,F210
4000 READ(5,100) K1L,Z1L
4100 READ(5,100) F500,Z500
4200 READ(5,100) F50L,Z50L
4300 READ(5,100) K20,Z20
4400 READ(5,100) K2L,Z2L
4500 READ(5,101) IR,IZ
4600 FORMAT(8F10.4)
4700 FORMAT(31F)
4800 FORMAT(8F10.4)
4900 FORMAT(612)
5000 FORMAT(2F10.5)
5100
5200 DO 5 J=1,1K
5300 DO 4 I=1,12
5400 READ(5,102) ZF(1,J),VF(1,J),WV(1,J),WV(1,J),WV(1,J)
5500 WDELTA(1,J)=ZF(1,J)/100.*F11
5600 WDELTA(2,J)=VF(1,J)/100.*F11
5700 WDELTA(3,J)=WV(1,J)/100.*F11
5800 WDELTA(4,J)=WV(1,J)/100.*F11
5900 WDELTA(5,J)=WV(1,J)/100.*F11
6000 WDELTA(6,J)=WV(1,J)/100.*F11
6100 WDELTA(7,J)=WV(1,J)/100.*F11

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18400      F(2)=0.5
18500      F(3)=1.00
18600      CALL INTERPOL (P,V,3,3,XOFIT,VAAL)
18700
18800      SIGMA(1)=1.00
18900
19000      72 CONTINUE
19100
19200      CALL LIMIT (ZF,FF,VZF,12,18,20,20,Z(1),R(1),ZFPS(1),126,ORG)
19300      CALL LIMIT (ZF,FF,VZF,12,18,20,20,Z(1),R(1),ZFPS(1),126,ORG)
19400      CALL LIMIT (ZF,FF,VZF,12,18,20,20,Z(1),R(1),ZFPS(1),126,ORG)
19500
19600      VZ(1)=SIGMA(1)+VZPS(1)
19700      VR(1)=SIGMA(1)+VRPS(1)
19800      OMEGA(1)
19900
20000      IF (13.EQ.1) GO TO 74
20100
20200      IF ABSOLUTE VELOCITY RATIO: VU=SIGMA+VUPS
20300
20400      VU(1)=SIGMA(1)+VUPS(1)
20500      GO TO 76
20600
20700      IF RELATIVE VELOCITY RATIO: WU=SIGMA+WUPS
20800
20900      WU(1)=SIGMA(1)+WUPS(1)+W(1)*VUPS(1)
21000
21100      74
21200      76 CONTINUE
21300
21400      WUPS(1)=W-VUPS(1)
21500      WPS(1)=SIGMA(1)+VZPS(1)+VZPS(1)+WUPS(1)+VRPS(1)+VRPS(1)
21600      INEL(1)=ALFA(VZPS(1)/VZPS(1))+180./PI
21700      INEL(1)=ALFA(VZPS(1)/VZPS(1))+180./PI
21800      W(1)=W-VU(1)
21900      W(1)=SIGMA(1)+VZ(1)+VZ(1)+W(1)+VR(1)+VR(1)
22000      DELTA(1)=ALFA(W(1)/W(1))+180./PI
22100      DELTA(1)=ALFA(W(1)/W(1))+180./PI
22200      DELTA(1)=DELTA(1)-DELTA(1)+PI/180.
22300
22400      CA(1)=0.100
22500      KSL(1)=F(1)
22600      FVAA=50+50+Z(1)+C0+Z(1)+Z(1)
22700      KSL=50+50+Z(1)+C0+Z(1)+Z(1)
22800      SPWAA=50+50+Z(1)+C0+Z(1)+Z(1)
22900      VZSL(1)=VZPS(1)
23000      VRSL(1)=VRPS(1)
23100      VRSL(1)=VRPS(1)
23200
23300      230
23400      231
23500      232
23600      233
23700
23800      234
23900      235
24000
24100      236
24200      237
24300      238
24400      239
24500

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41000
41100
41200
41300
41400
41500
41600
41700
41800
41900
42000
42100
42200
42300
42400
42500
42600
42700

14 CALL INTERPOL (ZSIG,SIGMAG,ISIG,25,Z(I+1),SIG*(I+1))
   GO TO 165

15 SIGMA DISTRIBUTION FROM BOUNDARY LAYER MODEL
   CALL INTERPOL (ZBL,WSURF,ABL,60,Z(I+1),ABL)
   CALL INTERPOL (ZBL,BELAS,ABL,60,Z(I+1),BEL)
   SIGMA(I+1)=0.006*(I+1)*REL*CUS(BEL)/VZFS(I+1)
   GO TO 165

16 SIGMA DISTRIBUTION FROM INTERPOLATING IN SHADE-TO-SHADE FLAG
   CALL LBLINT (ZL,PF,WSS,12,IK,Z0,Z0,Z(I+1),TREF,WSSIP1,120,QMG)
   CALL LBLINT (ZL,PF,WPS,12,IK,Z0,Z0,Z(I+1),TREF,WPSIP1,120,QMG)
   IF (WSSIP1*WSSIP1-WPSIP1*WPSIP1*(I+1)+VZFS(I+1)).LT.0.00) GO TO 161
   V(I+1)=SURF(WSSIP1-WPSIP1-VZFS(I+1)+VZFS(I+1))*CUS(BEL)AF(I+1)
   GO TO 162
161 V(I+1)=1.00
162 V(I+1)=VZFS(I+1)
   IF (WPSIP1*WPSIP1-WPSIP1*WPSIP1*(I+1)+VZFS(I+1)).LT.0.00) GO TO 163
   V(I+1)=SURF(WPSIP1-WPSIP1-VZFS(I+1)+VZFS(I+1))*CUS(BEL)AF(I+1)
   GO TO 164
163 V(I+1)=1.00
164 CONTINUE

165 CALL INTERPOL (P,V,3,3,YOFL1,VAXIP1)
   SIGMA(I+1)=VAXIP1/VZFS(I+1)
   GO TO 165
165 CONTINUE

VZ(I+1)=SIGMA(I+1)*VZFS(I+1)
IF (12.E0.1) GO TO 17

ASSUMPTION: BETA DECREASES LINEARLY WITH Z
NOTE: IF BETA = 0, BETA FACILCLF = BETA FRESSTFAM

BETA(I+1)=BETAF(I+1)+DBETA*(1.-CA(I+1))
GO TO 16

ELSE: USE THE BETA DISTRIBUTION GIVEN AS INPUT

17 CALL INTERPOL (ZBETA,BFPAIR,BBETA,Z5,Z(I+1),BETA(I+1))
   BETA(I+1)=BFPA(I+1)*PI/180.

18 CONTINUE

BETAF(I+1)=BETAF(I+1)+160./PI
THEFA(I+1)=ATN(VF(I+1)/VZ(I+1))+180./PI
AB(I+1)=VZ(I+1)+1.7*(VF(I+1)/VZ(I+1))
BETA(I+1)=0.7*(BETA(I+1)+160./PI)
V(I+1)=0.7*(V(I+1))
A(I+1)=SURF(VZ(I+1)+VZ(I+1)+V(I+1)+V(I+1)+V(I+1)+V(I+1))
VZ(I+1)=(VZ(I+1)+VZ(I+1))/2.
TP=VZ(I+1)/VZ(I+1)
V(I+1)=(V(I+1)+V(I+1))/2.
AB(I+1)=V(I+1)+V(I+1)/R(I+1)
AB(I+1)=V(I+1)+V(I+1)/R(I+1)

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42800 C ARB(I)=(AR(I+1)+AR(I))/2.
42900 C VR(I+1)=VR(I)+(VRSL(I+1)-VRSL(I))/1SL+IP*(ARB(I)-VRSL(I))*IP
43000 C
43100 C VRB(I)=(VR(I+1)+VR(I))/2.
43200 C
43300 C R(I+1)=R(I)+VRB(I)*TP
43400 C
43500 C IF(ABS(R(I+1)-TEMP).LT.0.00001) GO TO 25
43600 C IF(UT.EQ.25) GO TO 90
43700 C LEAP=K(I+1)
43800 C GO TO 10
43900 C
44000 C
44100 C
44200 C
44300 C IF(R(I+1).GT.RMAX) GO TO 50
44400 C IF(R(I+1).LT.RMIN) GO TO 60
44500 C IF(ABS(R(I+1)-R(I))/R(I).GT.(RMAX+RMIN))
44600 C GO TO 11
44700 C J=1+
44800 C *FITE(6,203) CX(J),R(J),SPARC,2(J),SIGMA(J),VZ(J),VD(J),VR(J),
44900 C 1(J),VZFS(J),VDFS(J),VDFS(J),VDFS(J)
45000 C
45100 C
45200 C
45300 C
45400 C
45500 C
45600 C
45700 C *VZ=VRSGA*P2A-VZ2A
45800 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
45900 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46000 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46100 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46200 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46300 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46400 C *VZ=VRSGA*VZ2A+VZ2A*VR2A+VR2A
46500 C
46600 C
46700 C
46800 C
46900 C
47000 C
47100 C
47200 C
47300 C
47400 C
47500 C
47600 C
47700 C
47800 C
47900 C
48000 C
48100 C
48200 C
48300 C
48400 C
48500 C
48600 C
48700 C
48800 C

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1(1)
207 FOR J=1(13),8PARTICLE,7X,FB,5,PI1.5,4PI0.2,3F9.2)
  WRITE(6,208) F(1),Z(1),VZFS(1),VUFS(1),VRES(1),ALPHA,BETAF
  1(1),REF(1)
208 FOR K=1(13),12HRESSTREAM A,3X,FB,5,PI1.5,4PI0.2,3F9.2)
  *K1EAD,209) K1P,Z10,VZ1P,V010,A10,ALF10,BETAF,TRF10
209 FOR K=1(13),12HRESSTREAM B,3X,FB,5,PI1.5,4PI0.2,3F9.2,///)
  WRITE(6,210)
210 FOR K=1(6),21HBLAKE EXIT CONDITIONS)
  *K1E(6,205)
  *K1E(6,207) K(J),Z(J),VZ(J),VU(J),VR(J),W(J),ALPHA,BETAF(J),THETA
  1(J)
  *K1E(6,208) KZA,ZZA,VZZA,VUZA,VRZA,KZA,ALPHAZA,BETAZA,THETA
  *K1E(6,209) F(J),Z(J),VZ(J),VUFS(J),VRES(J),VRES(J),ALPHA,BE
  THETA(J),REF(J)
  BETA(1)=BETA(1)+PI/180.
  BETAF(1)=BETAF(1)+PI/180.
  CRP=Z(J)-F(1)
  LOZ=LOI*CRP*GA*(K(J)+40(J)-K(1)+VR(1))/CP
  LOIF=CRP*GA*(K(J)+40(J)-K(1)+VR(1))/CP
  1A(G)=1A(G)+BETAF(J)*(CRP+VU1B/R(J)-F(1)+VU(1)/F(J))+VZFS(J)*(1.-SIG
  A(U)=VZZA*VZFS(J)
  1A(Z)=VZZA*VZFS(J)
  1A(Z)=Z(1)-SIG*(1)+2*(SIG*(1)+1A(B*BETA(1))-1A(B*BETAF(1)))*2-
  2*Z(0)/2.71828
  1F(CP*(GOVZ2F).LJ,0.00001) GOVZ2F=0.0
  GOVZ2F=30F(1)GOVZ2F
  ECF=CF*GOVZ2F*(Z.*1AUA/(1AUA+X*F0))/PI*CH
  DEB=FB*Z-R(1)
  LOZ=LOI+GOVZ2F*(KZA+VUZA-K(1)+VUFS(1))/CP
  1A=102A-101A
  CRP=CF(J)-F1B
  1A(Z)=101B+GOVZ2F*(F(J)+VUFS(J)-F1B+VU1B)/CF
  PIF=102B-101B
  DEB=102B-101
  WRITE(6,221)
221 FOR J=1(6),15HENERGY TRANSFER)
  *K1E(6,222)
222 FOR K=1(30),5HDEL 1,6X,30101,7X,30102,6X,5HDEL 1,4X,5HJ PIF,4X,5HJ/
  1ZP,5X,30LZF,/)
  *K1E(6,223) PIF,101,102,DEL,101F,GOVZ2F,FUP
223 FOR K=1(13),12HRESSTREAM A,3X,FB,5,PI1.5,4PI0.2,3F9.2,3)
  *K1E(6,224) F(J),Z(J),VZ(J),VU(J),VRES(J),ALPHA,BETA,THETA
224 FOR K=1(13),12HRESSTREAM B,3X,FB,5,PI1.5,4PI0.2,3F9.2)
  *K1E(6,225) F(J),Z(J),VZ(J),VU(J),VRES(J),ALPHA,BETA,THETA
225 FOR K=1(13),12HRESSTREAM C,3X,FB,5,PI1.5,4PI0.2,3F9.2)
  GO TO 1
50 *K1E(6,240)
240 FOR K=1(10X,/,10X,4HRADIUS OF PARTICLE IS GREATER THAN SHROUD RAD)
  1GO TO 1
50 *K1E(6,241)
241 FOR K=1(10X,/,10X,4HRADIUS OF PARTICLE IS LESS THAN TWO RADII)
  GO TO 1
70 *K1E(6,242)
242 FOR K=1(10X,/,10X,4HRADIUS OF PARTICLE IS GREATER AFTER 25 ITERATIONS)
  GO TO 1

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55000 90 WRITE(6,244)
55100 244 FOR I=1(10X,/,10X,45HR(1+1) LOOP DID NOT CONVERGE AFTER 25 ITERATIO
55200 165)
55300 GO TO 1
55400 C 11 CONFIDENCE
55500 WRITE(6,260)
55600 200 FOR I=1(10X,/,10X,45HR(1+1)
55700 END
55800 C
55900 C
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6200 C--CHECK TO SEE IF QUADRATIC EQUATION IS CLOSE TO LINEAR
6300 IF (ABS(4.*JA+CC).DE.GN+2+.01) GO TO 80
6400 FA = -GB/2./GA
6500 FB = SQR(DISCN)/2./GA
6600 F1 = FA+FB
6700 F2 = FA-FB
6800 C--CHECK TO DETERMINE WHETHER F1 OR F2 IS THE PROPER SOLUTION
6900 CASE = -1
7000 IF (CEATP(IJEX)) 40,50,60
7100 C--EXTRAPOLATION BELOW OR TO LEFT (FF LESS THAN 0.)
7200 40 IF (F1.LT..01) CASE = CASE+1.
7300 IF (F2.LT..01) CASE = CASE+2.
7400 IF (CASE.LI.1.5) GO TO 70
7500 CASE = CASE-1.
7600 IF (F2.LI.F1) CASE = CASE-1.
7700 GO TO 70
7800 C--NO EXTRAPOLATION
7900 50 IF (ABS(F1-.5).LI..51) CASE = CASE+1.
8000 IF (ABS(F2-.5).LI..51) CASE = CASE+2.
8100 GO TO 70
8200 C--EXTRAPOLATION ABOVE OR TO RIGHT (FF GREATER THAN 1.)
8300 60 IF (F1.GT..99) CASE = CASE+1.
8400 IF (F2.GT..99) CASE = CASE+2.
8500 IF (CASE.LI.1.5) GO TO 70
8600 CASE = CASE-1.
8700 IF (F1.LI.F2) CASE = CASE-1.
8800 IF (ABS(CASE-.5).GI..5) GO TO 110
8900 FF = (1.-CASE)*F1+CASE*F2
9000 GO TO 90
9100 C--IF QUADRATIC EQUATION IS NEAR LINEAR, USE BINOMIAL EXPANSION FOR FF
9200 80 ACBZ = GA/GB+GC/GB
9300 IF (ABS(CBZ).LI.1.F-8) ACBZ=0.
9400 FF = -GC/GB+(1.+ACBZ+2.*ACBZ**2)
9500 IF (IJEAF=0.2) GO TO 100
9600 IJEA = IJEA+1
9700 FX = FF
9800 C--INTERCHANGE CORNER POINTS TO GET FX
9900 Y13 = Y(I,J)-Y(1+1,J)
10000 X13 = X(I,J)-X(1+1,J)
10100 Y42 = Y(I+1,J+1)-Y(I,J+1)
10200 X42 = X(I+1,J+1)-X(I,J+1)
10300 Y04 = Y(I,J)-Y(I,J+1)
10400 X04 = X(I,J)-X(I,J+1)
10500 Y21 = Y(I,J+1)-Y(1,J)
10600 X21 = X(I,J+1)-X(1,J)
10700 GO TO 30
10800 C--CALCULATE INTERPOLATED VALUE
10900 100 FX = FF
11000 100 FF = Z(I,J)*(1.-F2)*(1.-F1)+Z(I+1,J)*FX+(1.-F1)+Z(I,J+1)*(1.-FX)
11100 100 FF = F1+Z(I+1,J)*FX+FY
11200 RETURN
11300 C--PRINT ERROR MESSAGE IF THERE IS A PROBLEM IN OBTAINING A SOLUTION
11400 110 Z0 = 0.
11500 WRITE(6,1000) 1,J
11600 RETURN
11700 1000 RETURN (SUBROUTINE CANNOT FIND INTERPOLATED VALUE/48 I =,16,48 J =,
11800 110)
11900 END
12000 C-----
12100 C SUBROUTINE INTERPOL (X,I,GP,EDIC,X0,Y0)
12200

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12300 C
12400 C THIS IS AN INTERPOLATION ROUTINE USING A SECOND ORDER FIT OF
12500 C THE FUNCTION Y=F(X). IT WILL ALSO EXTRAPOLATE TO THE LEFT
12600 C AND THE RIGHT A DISTANCE NOT GREATER THAN THE FIRST (TO THE
12700 C LEFT) OR THE LAST (TO THE RIGHT) VALID X INCREMENT.
12800 C
12900 C DIMENSION X(NDIM),Y(NDIM)
13000 C
13100 C IF(XO.LE.X(1)) GO TO 40
13200 C IF(XO.GE.X(NP)) GO TO 50
13300 C
13400 C INTERPOLATION
13500 C
13600 C I=0
13700 C
13800 C 10 CONTINUE
13900 C I=I+1
14000 C IF(X(1).GT.XO) GO TO 20
14100 C IF(I.GT.NP) GO TO 70
14200 C GO TO 10
14300 C 20 CONTINUE
14400 C IF(XO-X(I-1)).LT.(X(1)-XO)) I=I-1
14500 C
14600 C IF(I.EQ.1) I=2
14700 C IF(I.EQ.NP) I=NP-1
14800 C
14900 C C=((Y(I+1)-Y(I))/(X(I+1)-X(I))-(Y(I)-Y(I-1))/(X(I)-X(I-1)))/(X(I+1
15000 C I)-X(I-1))
15100 C W=((Y(I)-Y(I-1))-C*(X(I)-X(I-1)+X(I-1)*X(I-1)))/(X(I)-X(I-1))
15200 C A=Y(I)-C*X(I)-C*X(I)*X(I)
15300 C Y0=A+C*XO+C*XO*XO
15400 C RETURN
15500 C
15600 C EXTRAPOLATION TO THE LEFT
15700 C
15800 C 40 CONTINUE
15900 C XA=X(2)-X(1)
16000 C XM=X(1)-DX
16100 C IF(XO.M1.XA) GO TO 80
16200 C
16300 C C=((Y(I+1)-Y(I))/(X(I+1)-X(I))-(Y(I)-Y(I-1))/(X(I)-X(I-1)))/(X(I+1
16400 C I)-X(I-1))
16500 C W=((Y(I)-Y(I-1))-C*(X(I)-X(I-1)+X(I-1)*X(I-1)))/(X(I)-X(I-1))
16600 C A=Y(I)-C*X(I)-C*X(I)*X(I)
16700 C Y0=A+C*XO+C*XO*XO
16800 C RETURN
16900 C
17000 C EXTRAPOLATION TO THE RIGHT
17100 C
17200 C 50 CONTINUE
17300 C J=NP-1
17400 C DX=X(J)-X(J)
17500 C XN=X(J)+DX
17600 C IF(XO.GT.XN) GO TO 90
17700 C
17800 C C=((Y(J+1)-Y(J))/(X(J+1)-X(J))-(Y(J)-Y(J-1))/(X(J)-X(J-1)))/(X(J+1
17900 C I)-X(J-1))
18000 C W=((Y(J)-Y(J-1))-C*(X(J)-X(J-1)+X(J-1)*X(J-1)))/(X(J)-X(J-1))
18100 C A=Y(J)-C*X(J)-C*X(J)*X(J)
18200 C Y0=A+C*XO+C*XO*XO
18300 C RETURN

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24500 BETA(2)=BETA(1)+(CF(1))*(ZBL(2)-ZBL(1))/COS((BETA(2)+BETA(1))/2.)
24600 1/2.*(2.+H12)*H12*(H12+1)*(H12+1)/H12
24700 H12=1.63-0.07/5+ALOG10(RED2)
24800 CF(2)=0.24*EXP(-1.561*H12)+(RED2)*+(-0.268)
24900 G=(H12-1.)/H12/SORT(CF(2)/2.)
25000
25100
25200 A=0.3714
25300 H=1.15-G*0.41/2.
25400 C=1.2443-2.*G+0.41/3.
25500
25600 GASP=(-B*SORT(B*B-4.*A*C))/2./A
25700 GASP=(-B*SORT(B*B-4.*A*C))/2./A
25800
25900 GASP(2)=GASP
26000
26100 H1=1.535*(H12-0.70)*+(-2.715)+3.3
26200 ZBL(2)=BETA(2)*(H1+H12)
26300
26400 H12=1.63-0.07/5+ALOG10(RED2)
26500
26600
26700
26800
26900
27000
27100
27200
27300
27400
27500
27600
27700
27800
27900
28000
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29100
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29400
29500
29600
29700
29800
29900
30000
30100
30200
30300
30400
30500

      LOOP TO CALCULATE BOUNDARY LAYER PARAMETERS
      DO 10 I=2,NBLN1
        BETA(1)=BETA(1+1)+BETA(1)/2.
        BETA(1+1)=BETA(1+1)+(CF(1))*(ZBL(1+1)-ZBL(1))/COS((BETA(1)+BETA(1+1))/2.+H12)
        1/BETA(1+1)*(H12-1)-BETA(1+1)/H12
        RED2=(1+1)*A(1+1)/VIS
        H12=1.63-0.07/5+ALOG10(RED2)
        CF(1+1)=0.24*EXP(-1.561*H12)+(RED2)*+(-0.268)
        G=(H12-1.)/H12/ (CF(1+1)/2.)
        A=0.3714
        H=1.15-G*0.41/2.
        C=1.2443-2.*G+0.41/3.
        GASP=(-B*SORT(B*B-4.*A*C))/2./A
        GASP=(-B*SORT(B*B-4.*A*C))/2./A
        GASP(2)=GASP
        H1=1.535*(H12-0.70)*+(-2.715)+3.3
        ZBL(1+1)=BETA(1+1)*(H1+H12)
      10 CONTINUE
      RETURN
30 WHILE (0.200) L
300 FORK(167,348)SOMETHING IS WRONG IN BOUNDARY, L = ,13)
200 RETURN
END
-----
SUBROUTINE VELEATIO (CF,GASP,ZBL,ZBL,Y,ZBL,NO,MODE)
C
C THIS SUBROUTINE WILL CALCULATE THE BOUNDARY LAYER VELOCITY
C PROFILE USING DEFOUL'S METHOD.
C

```

```

30600 DIMENSION CF(NDIM),GAM(NDIM),DEL(NDIM),ZBL(NDIM)
30700 DIMENSION UOUE(NDIM)
30800 C
30900 C
31000 C
31100 C
31200 C
31300 C
31400 C
31500 CALL INTERPOL (ZBL,CF,-BL,NDIM,Z,CFVR)
31600 CALL INTERPOL (ZBL,GAM,NBL,NDIM,Z,GAMVR)
31700 CALL INTERPOL (ZBL,DEL,NBL,NDIM,Z,DELVR)
31800 C
31900 C
32000 C
32100 C
32200 C
32300 C
32400 C
32500 C
32600 C
32700 C
32800 C
32900 C
33000 C
33100 C
33200 C
33300 C
33400 C
33500 C

```

UOUE(1)=1.00
 GO TO 10
 5 UOUE(1)=1.00
 10 CONTINUE
 RETURN
 END

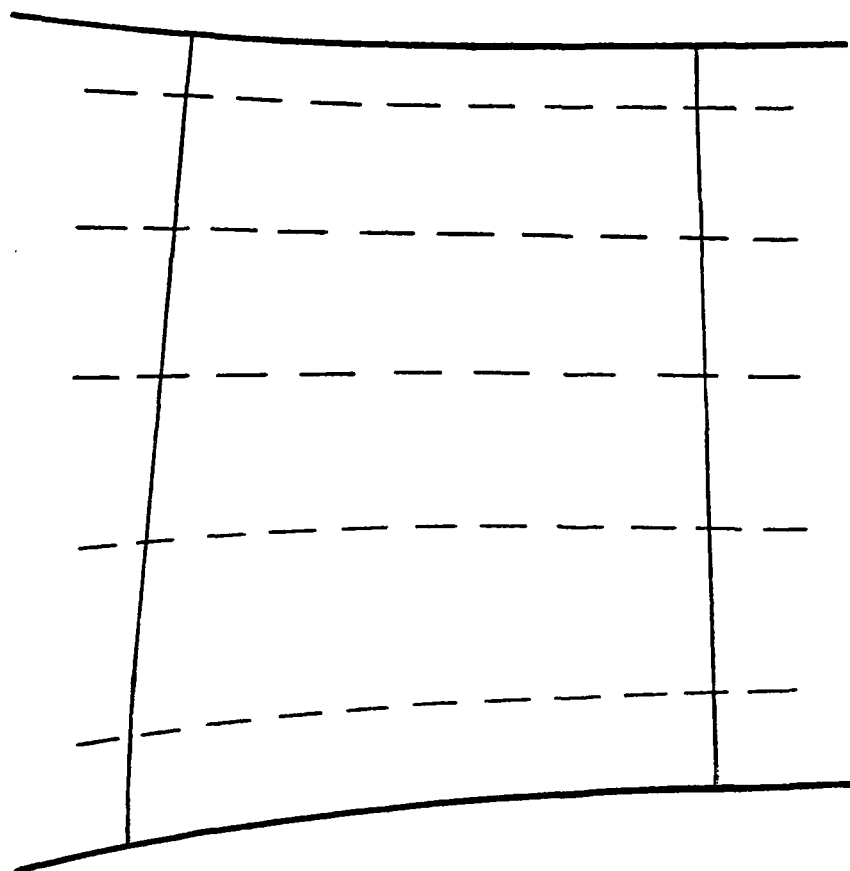


Figure 1. Meridional Streamline Network.

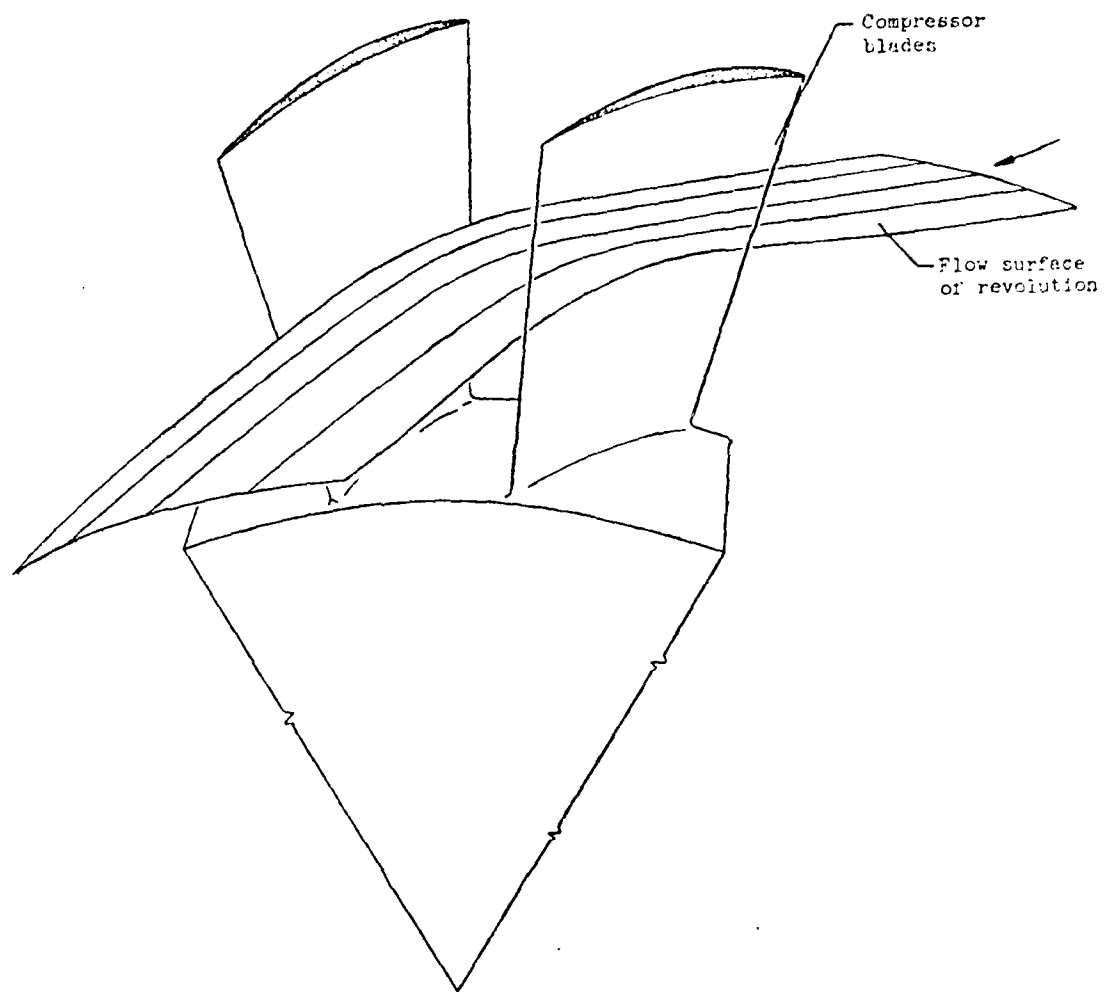
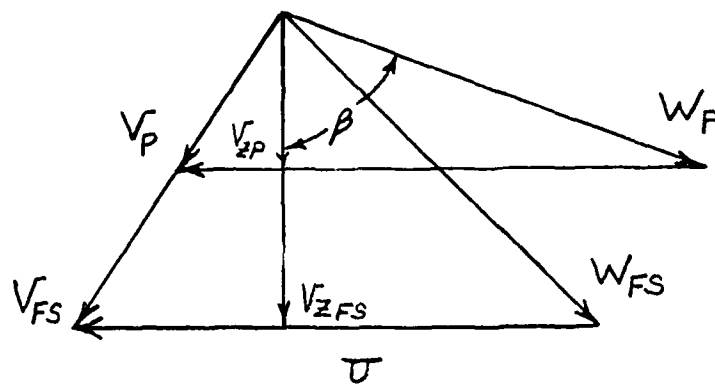
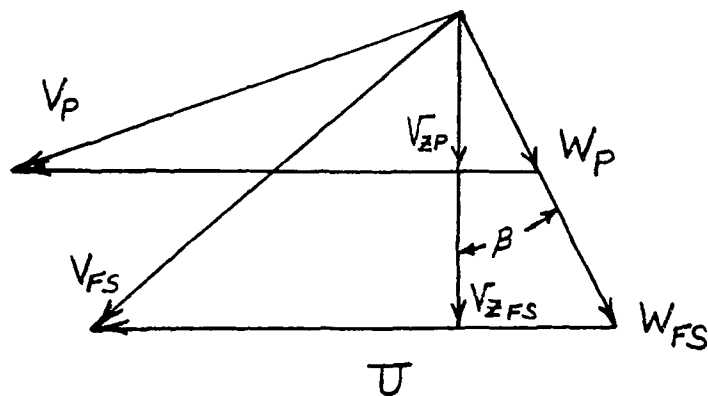


Figure 2. Inviscid Through-Flow Streamsheet.

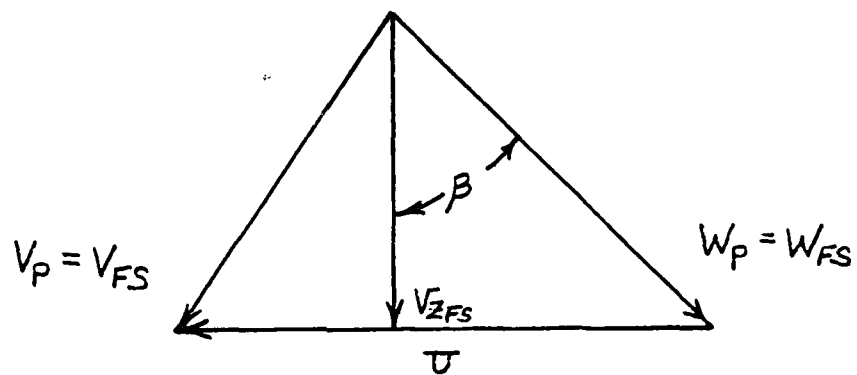


a). Rotor Inlet.

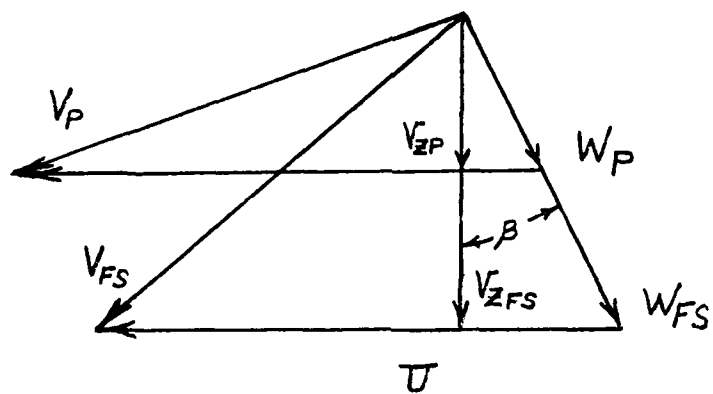


b). Rotor Exit.

Figure 3. Rotor Velocity Triangles For Absolute Velocity Defect.



a). Rotor Inlet.



b). Rotor Exit.

Figure 4. Rotor Velocity Triangles For Relative Velocity Defect.

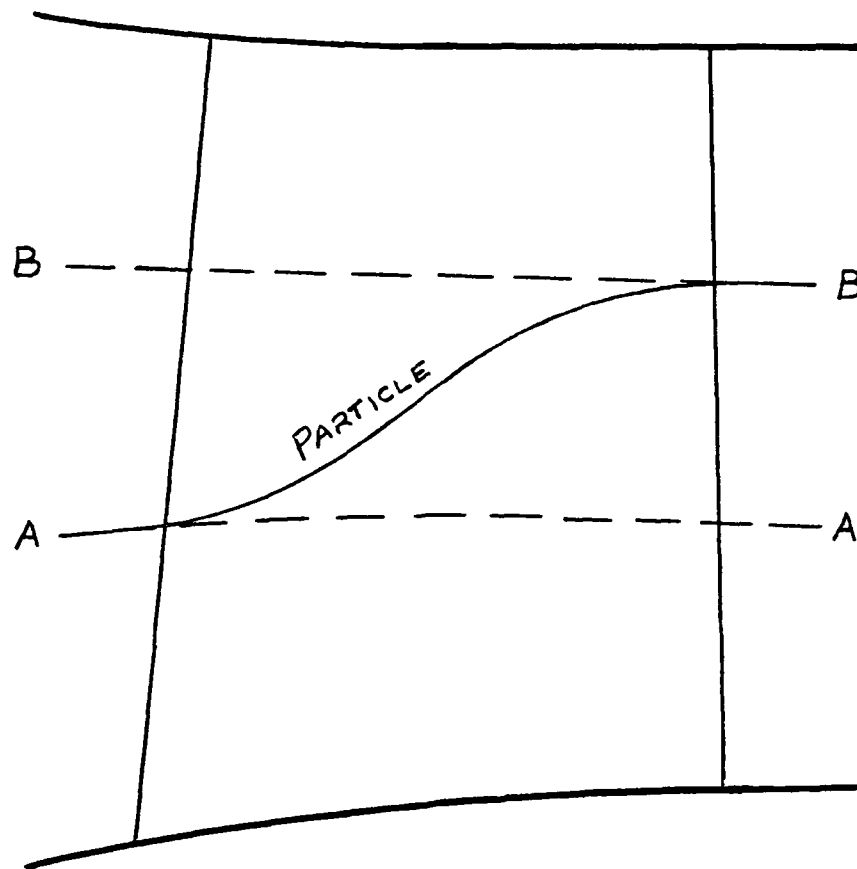


Figure 5. Meridional Streamline Nomenclature.

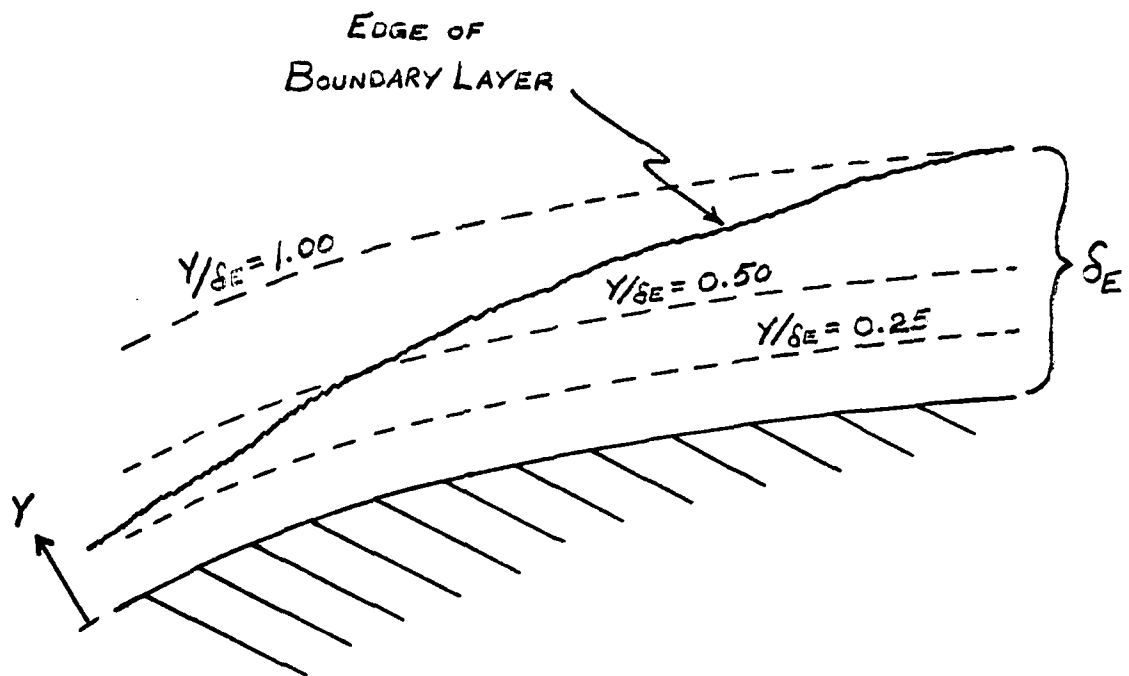


Figure 6. Boundary Layer Streamline Specification.

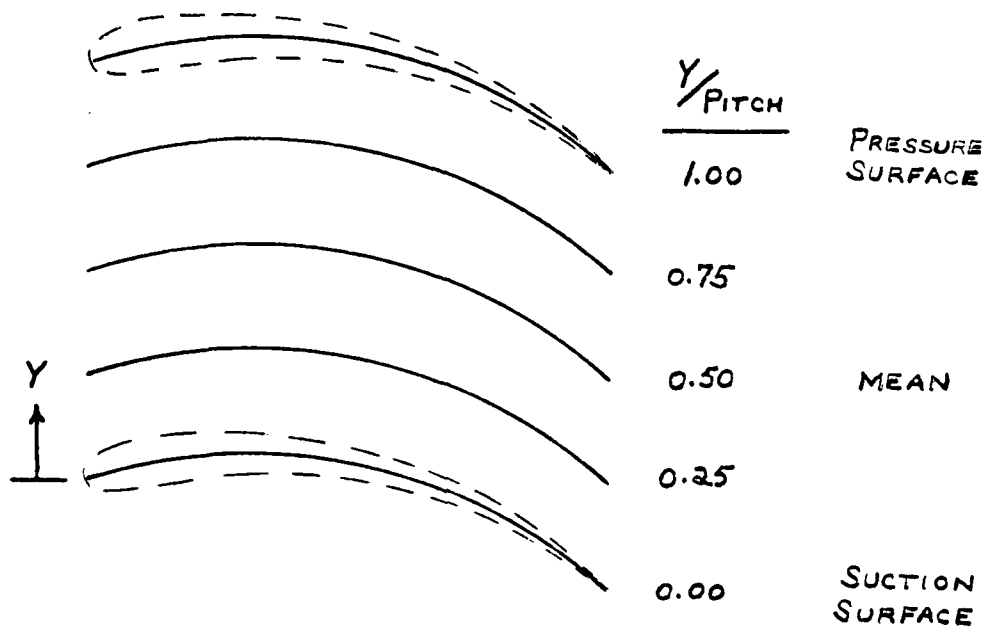


Figure 7. Blade-to-Blade Streamline Approximation and Notation.

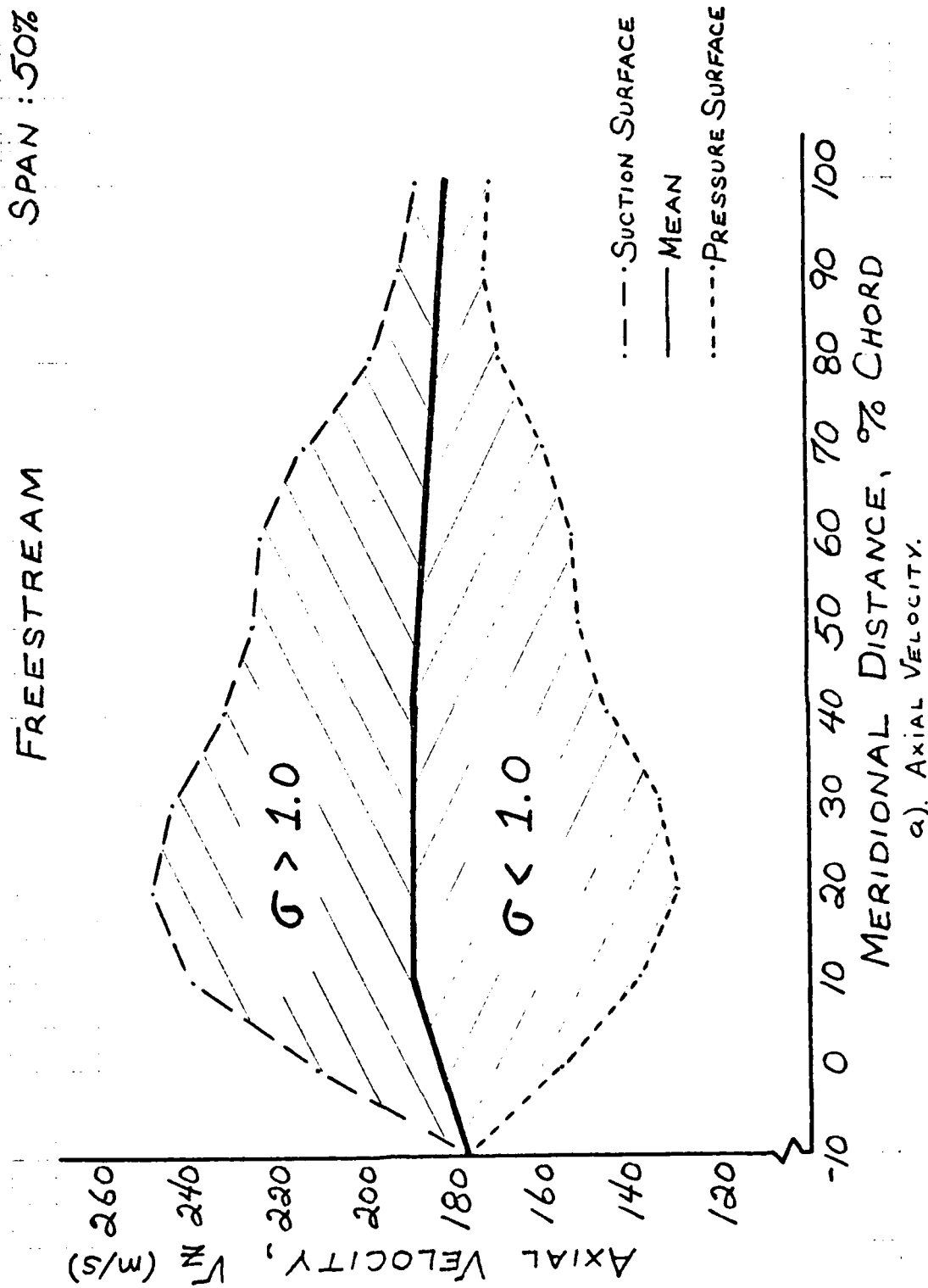


FIGURE 8. FREESTREAM VELOCITY EVOLUTION, 50% SPAN.

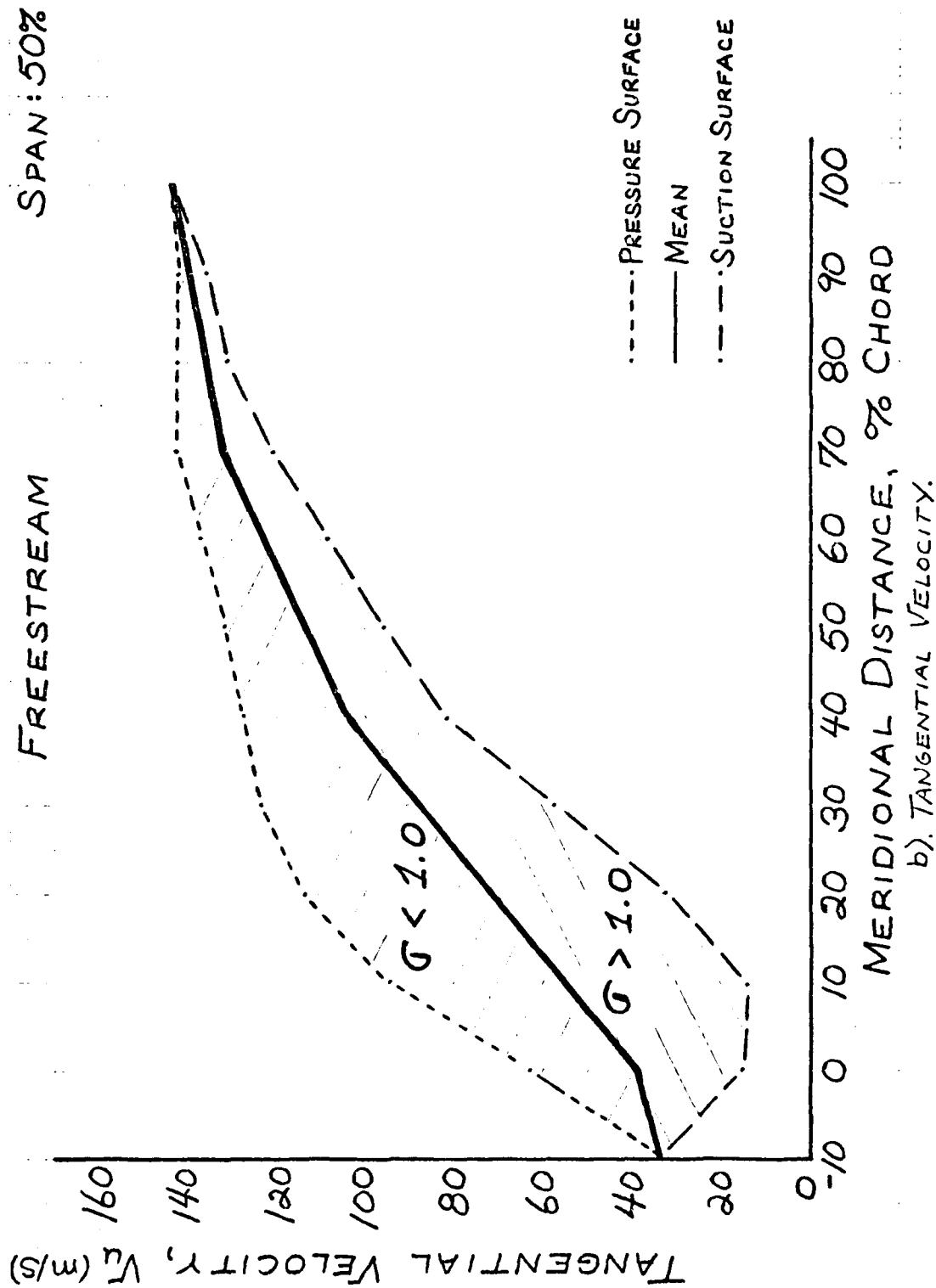


FIGURE 8. FREESTREAM VELOCITY EVOLUTION, 50% SPAN.

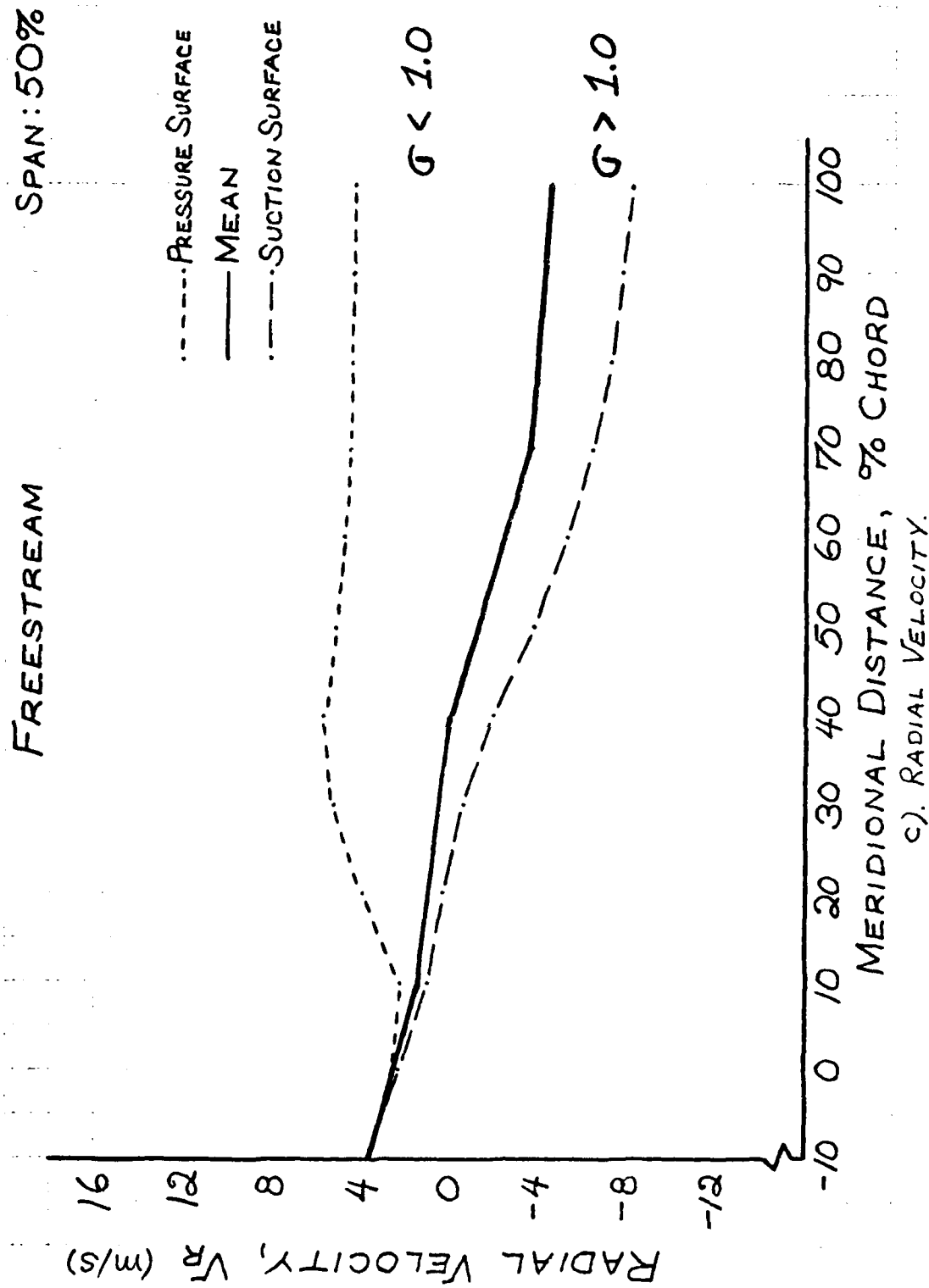


FIGURE 8. FREESTREAM VELOCITY EVOLUTION, 50% SPAN.

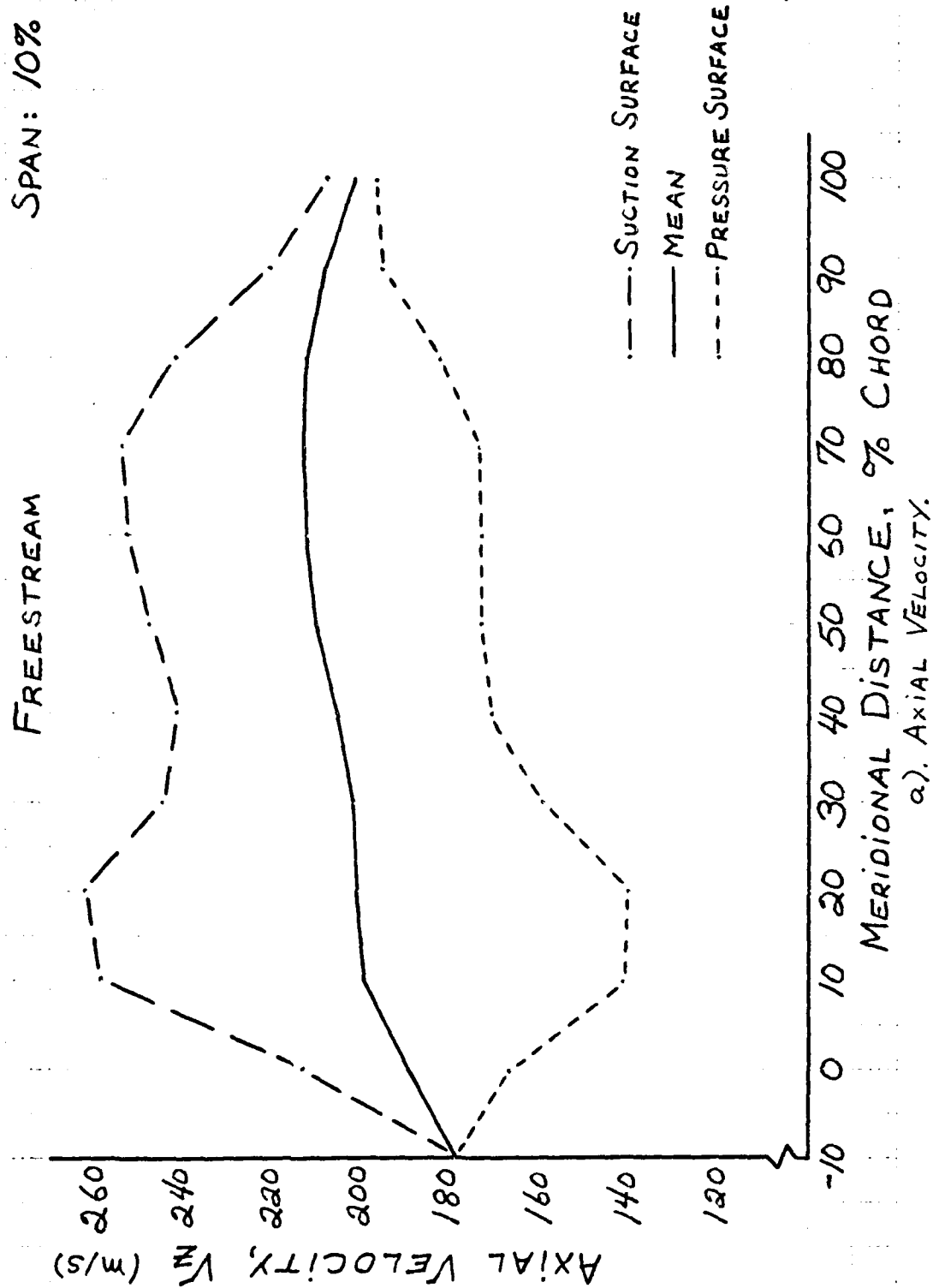


FIGURE 9. FREESTREAM VELOCITY EVOLUTION, 10% SPAN.

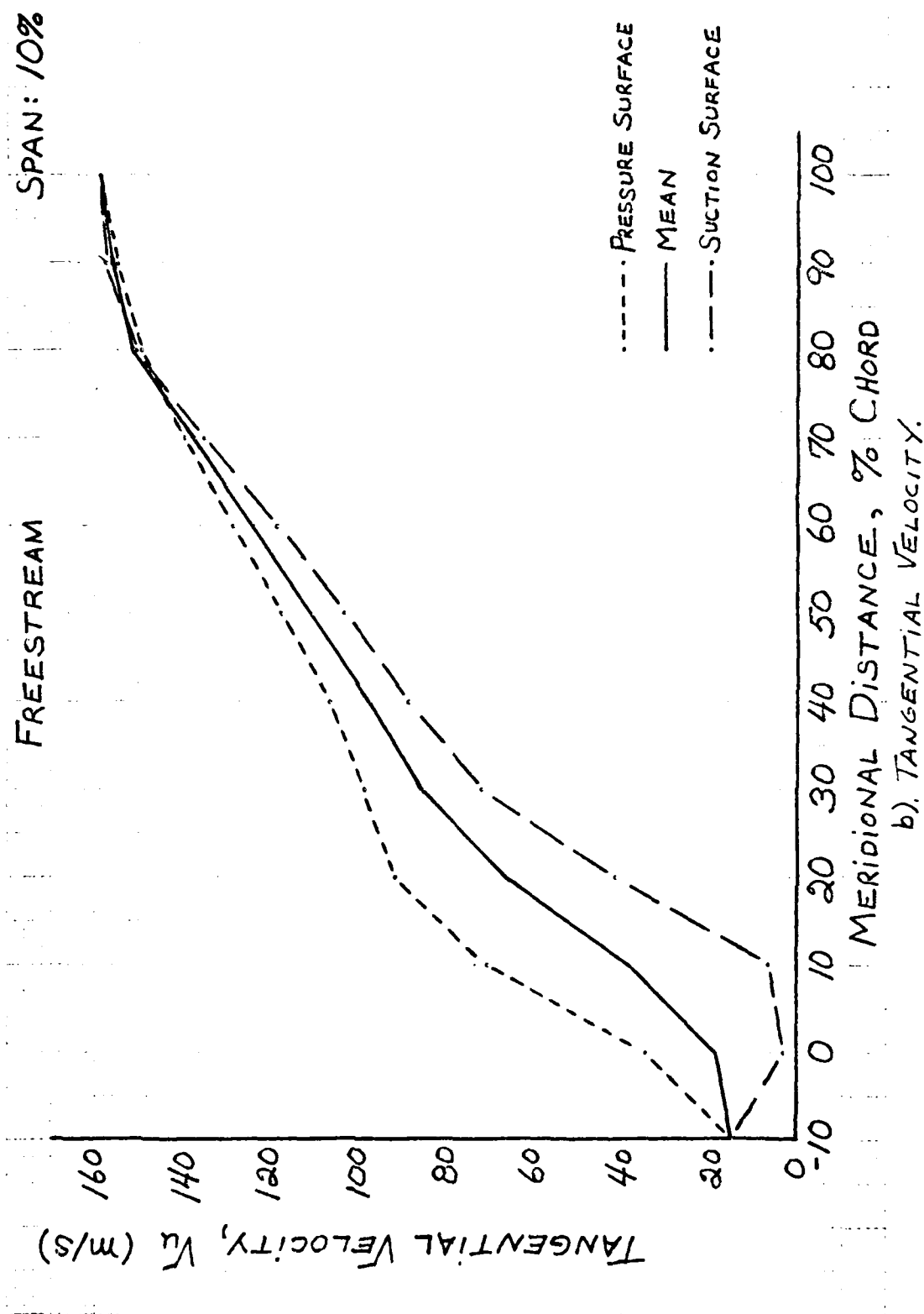


FIGURE 9. FREESTREAM VELOCITY EVOLUTION, 10% SPAN.

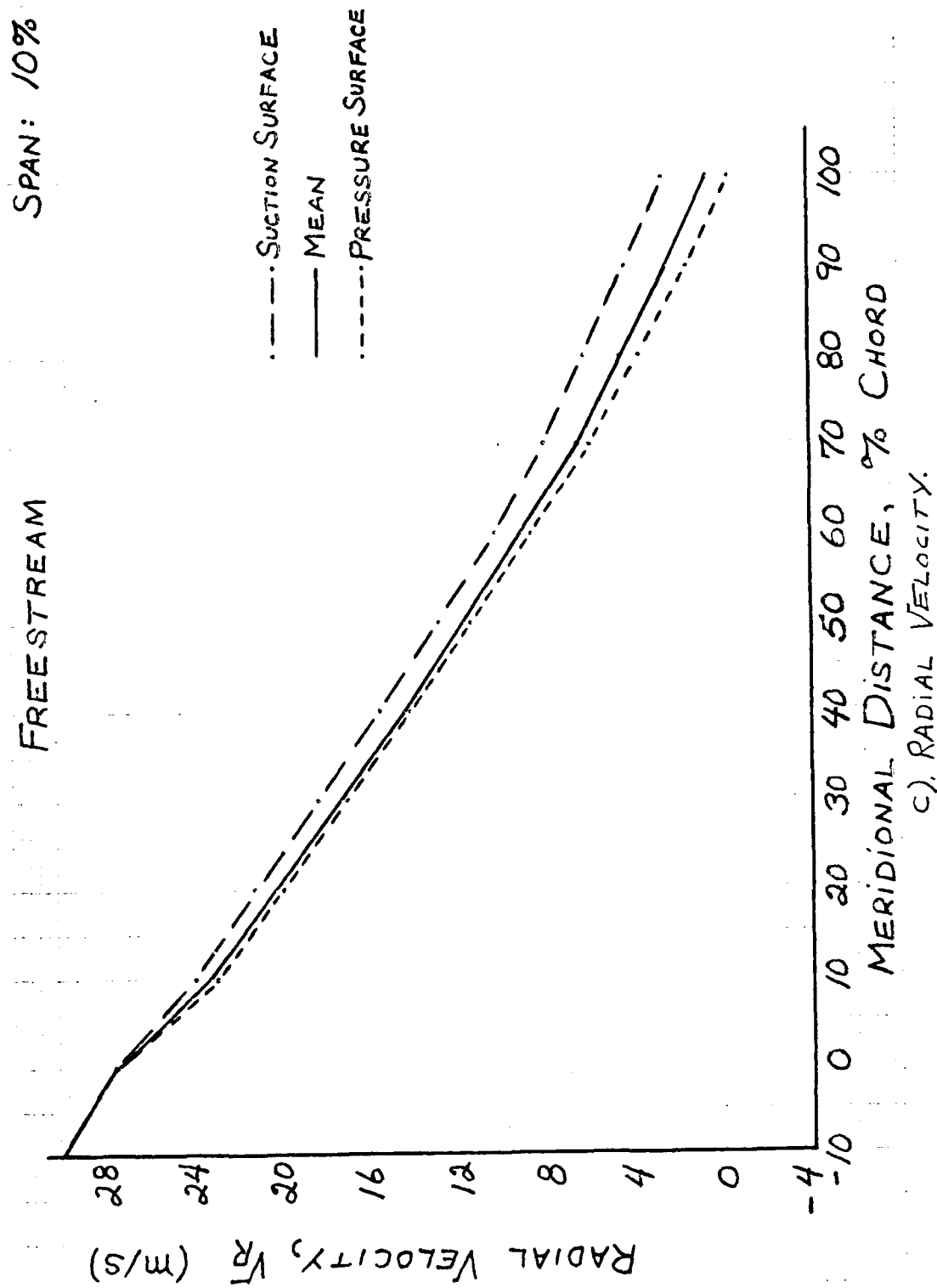


FIGURE 9. FREESTREAM VELOCITY EVOLUTION, 10% SPAN.

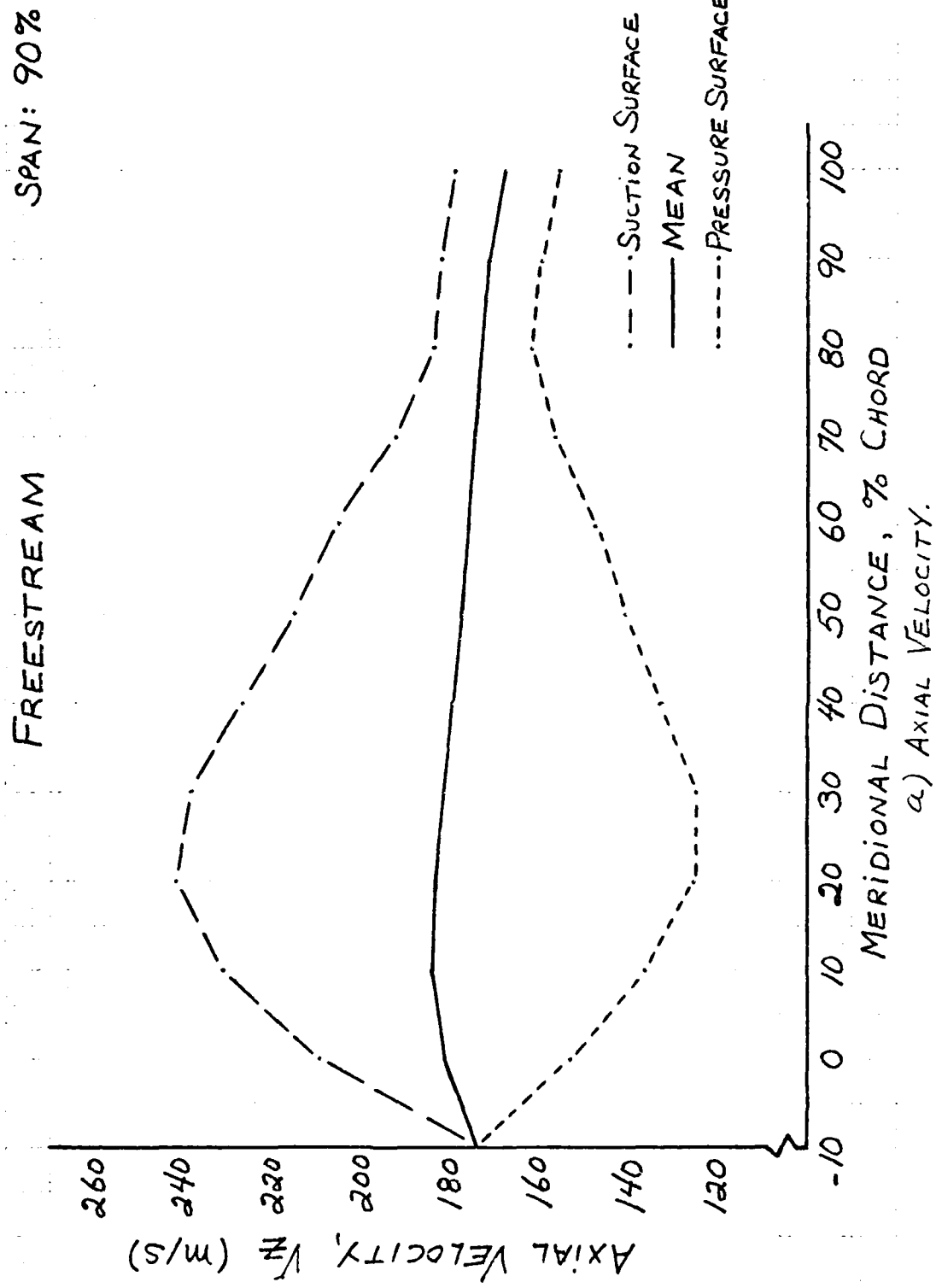


FIGURE 10. FREESTREAM VELOCITY EVOLUTION, 90% SPAN.

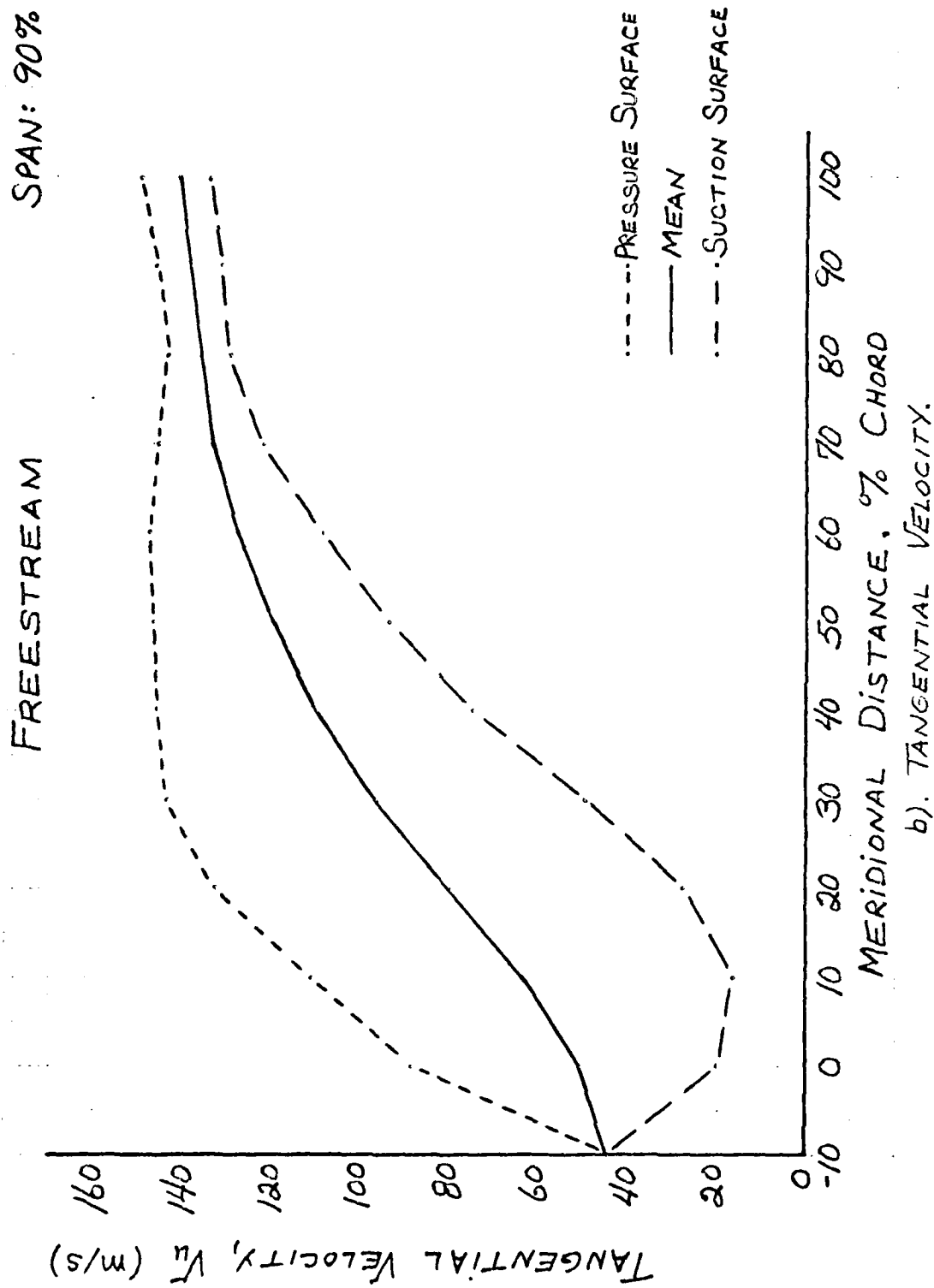


FIGURE 10. FREESTREAM VELOCITY EVOLUTION, 90% SPAN.

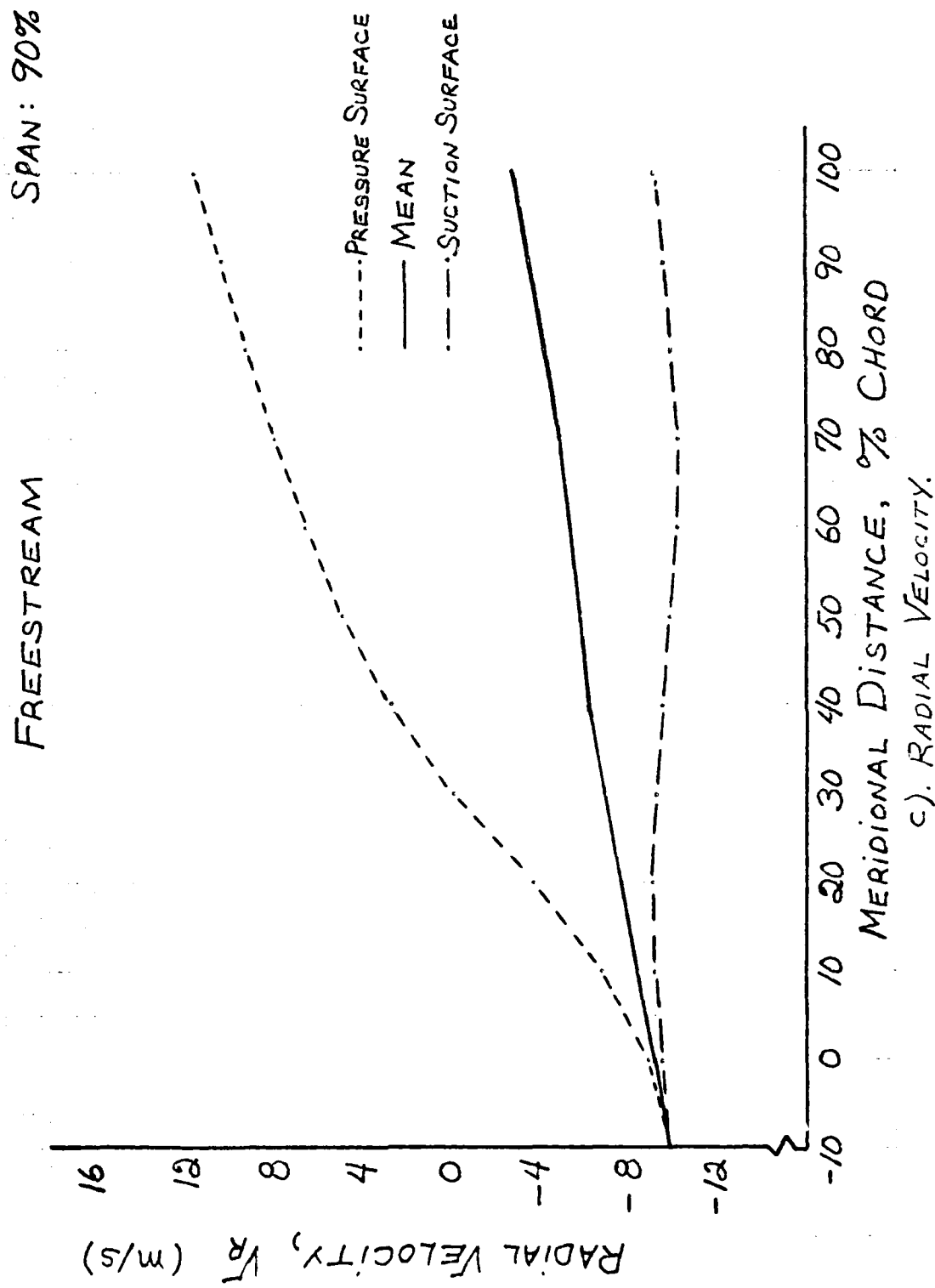


FIGURE 10. FREESTREAM VELOCITY EVOLUTION, 90% SPAN.

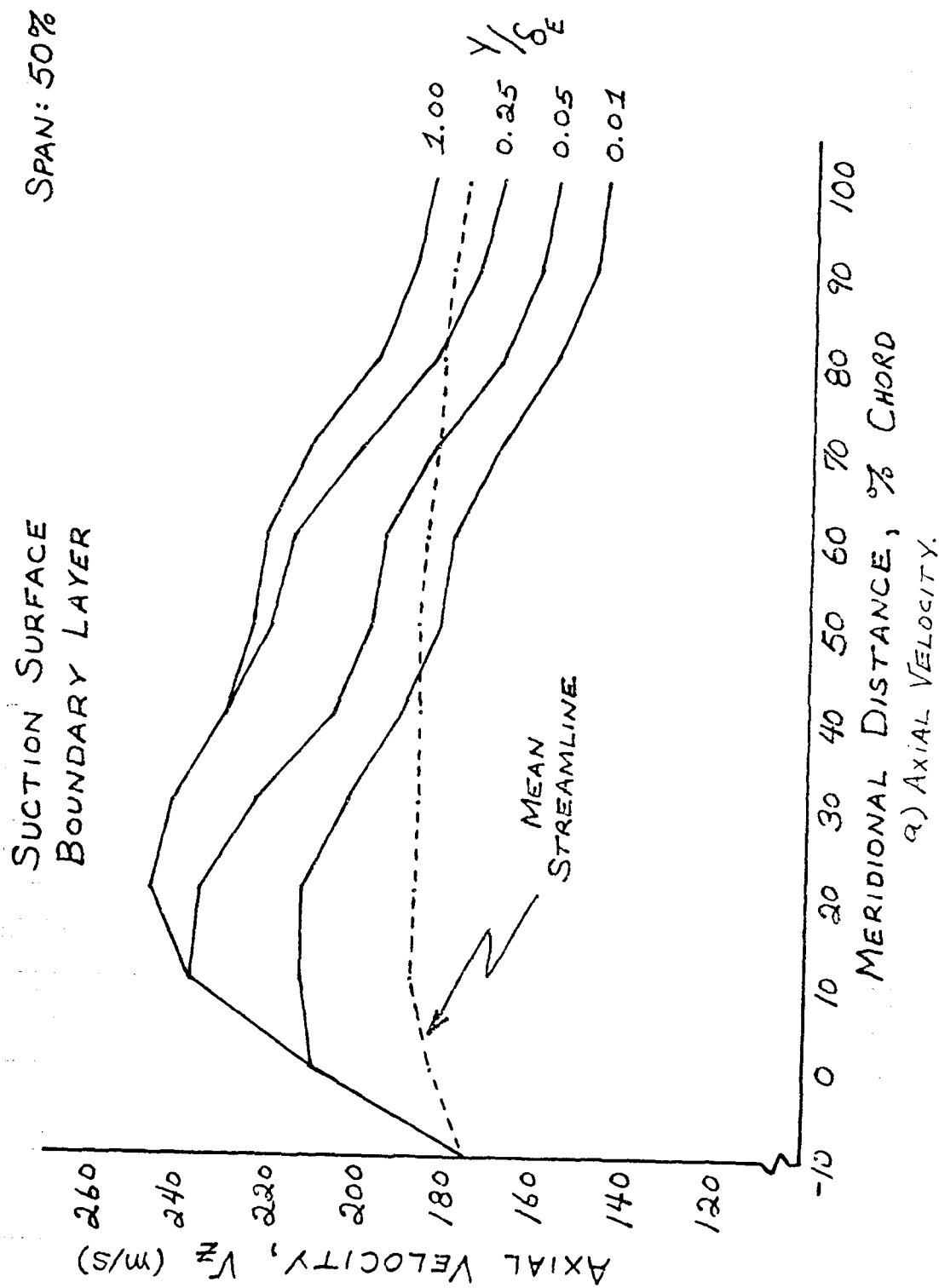


FIGURE 11. SUCTION SURFACE BOUNDARY LAYER VELOCITY
EVOLUTION, 50% SPAN.

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VON KARMAN INST FOR FLUID DYNAMICS RHODE-SAINT-GENESE--ETC F/6 20/4
NONUNIFORM ENERGY TRANSFER IN AXIAL FLOW COMPRESSORS.(U)

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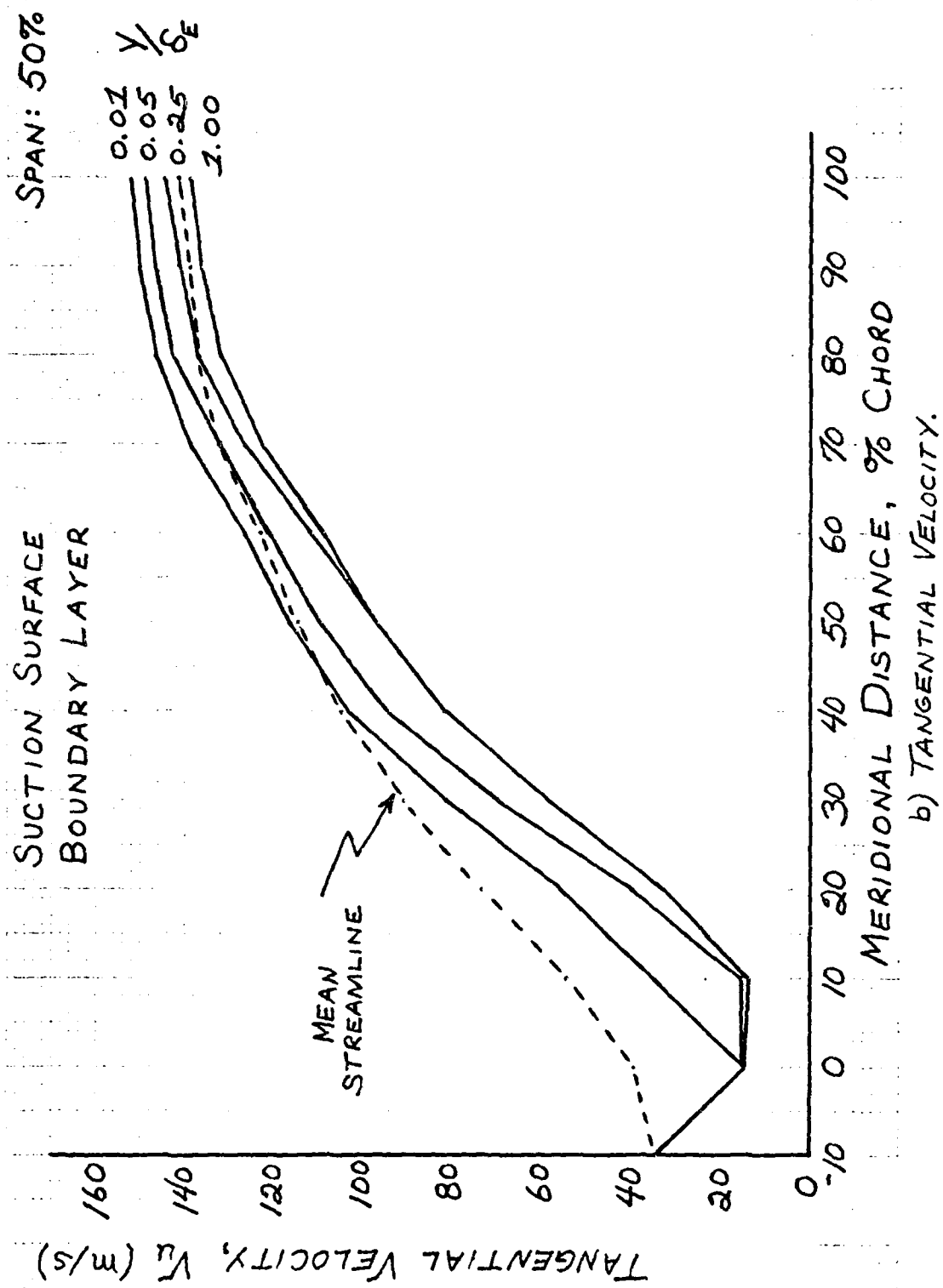
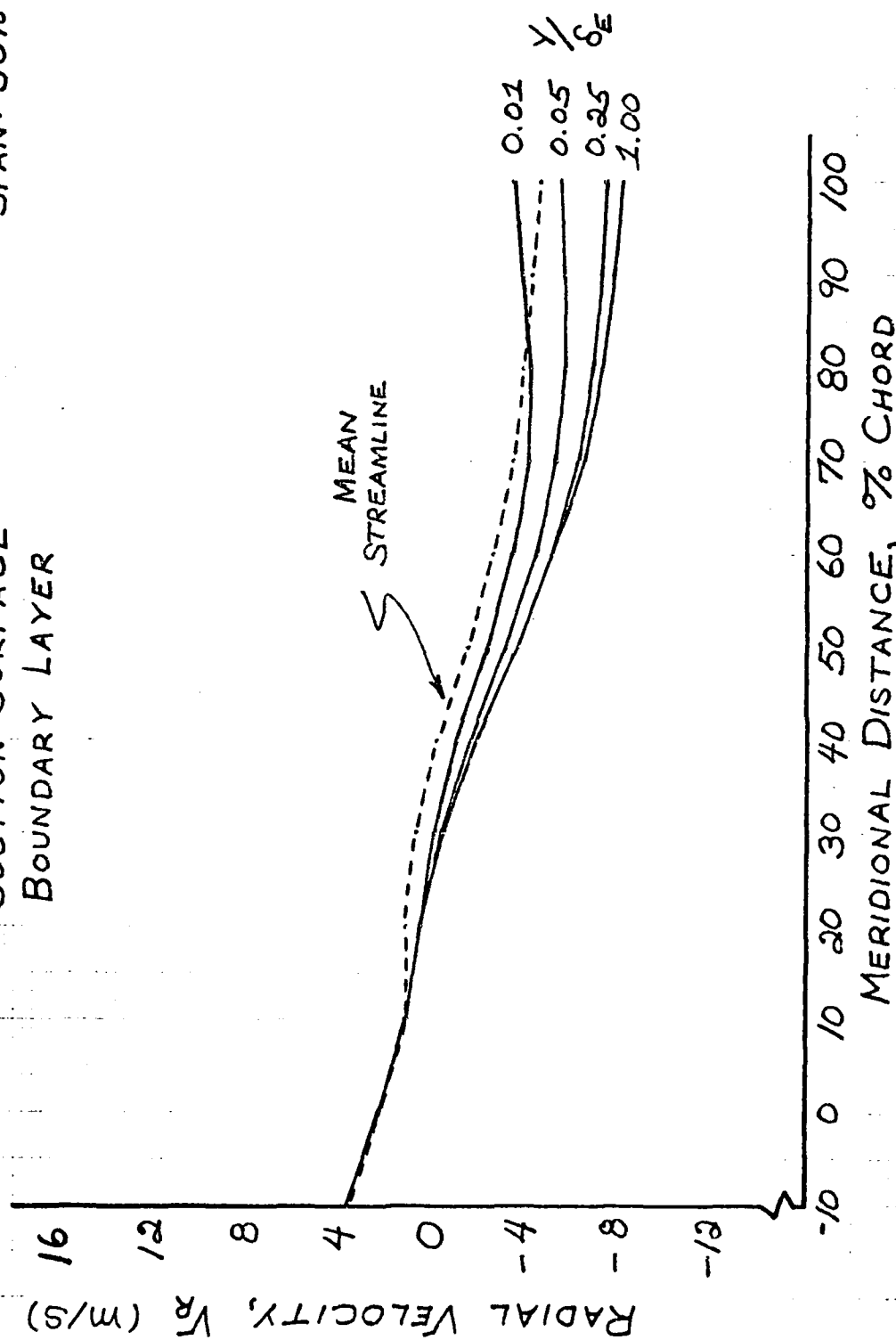


FIGURE 11. SUCTION SURFACE BOUNDARY LAYER VELOCITY

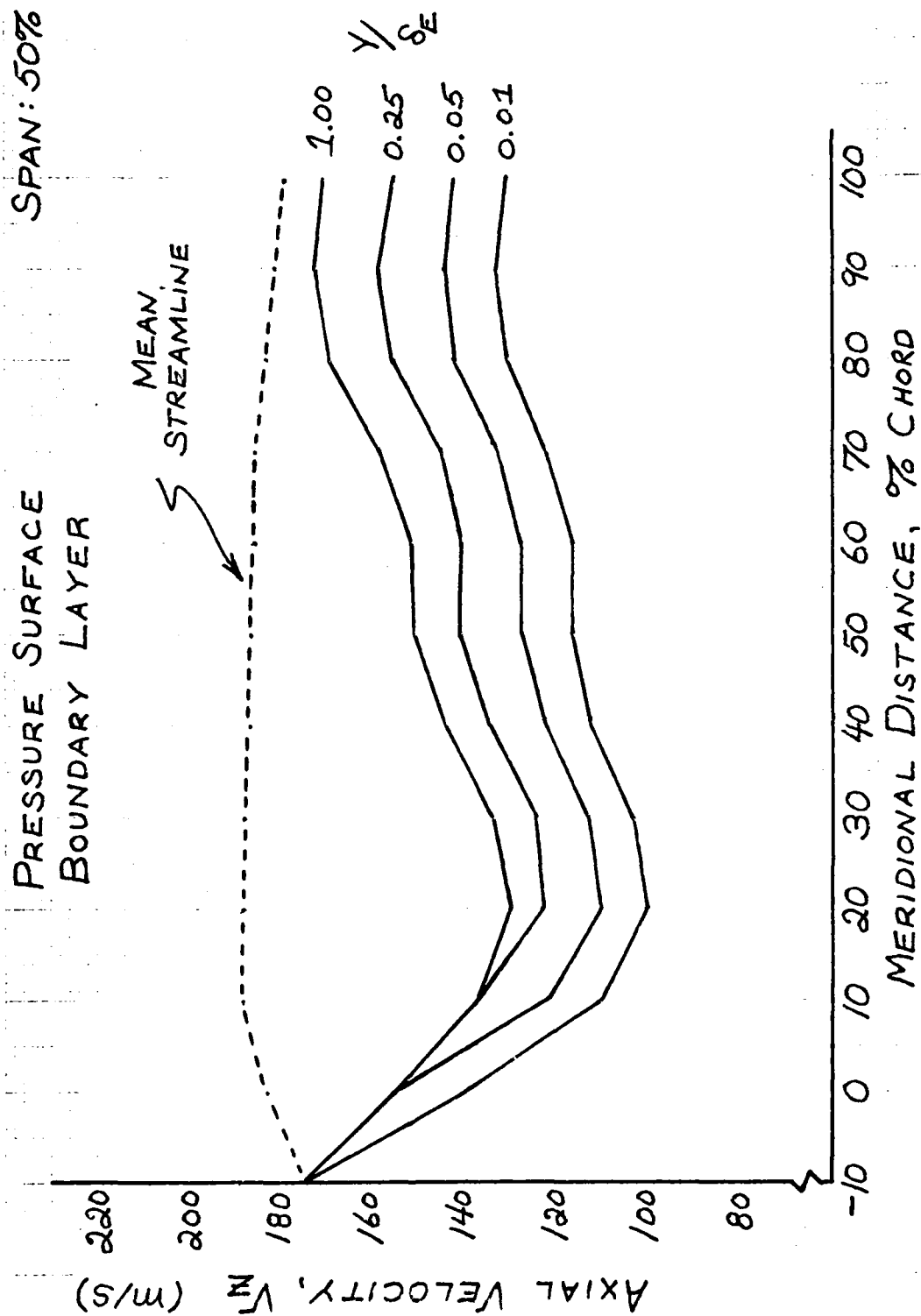
SUCTION SURFACE
BOUNDARY LAYER

SPAN: 50%



c). RADIAL VELOCITY.

FIGURE 11. SUCTION SURFACE BOUNDARY LAYER VELOCITY
EVOLUTION, 50% SPAN.



a) AXIAL VELOCITY.

FIGURE 12. PRESSURE SURFACE BOUNDARY LAYER VELOCITY

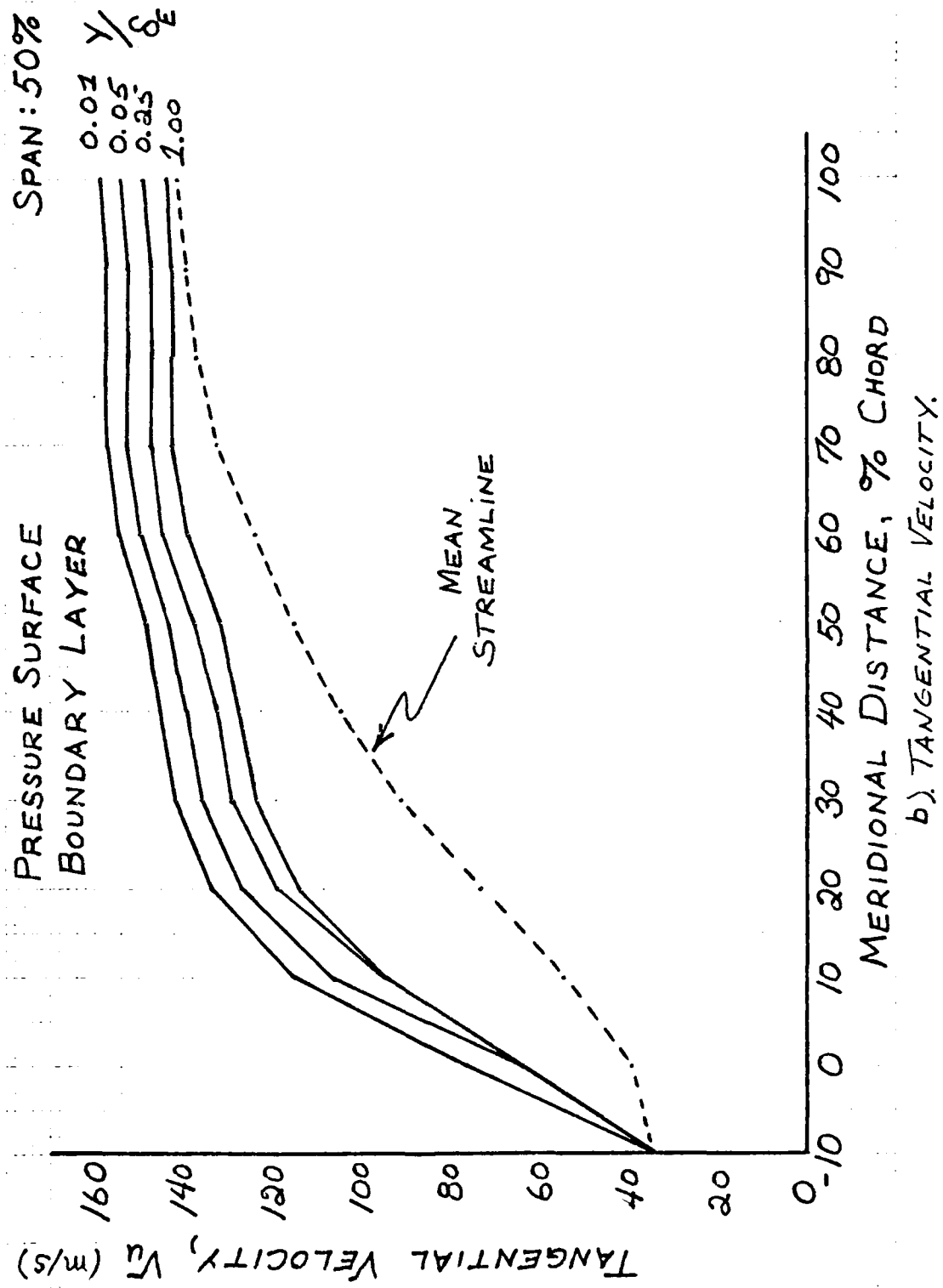
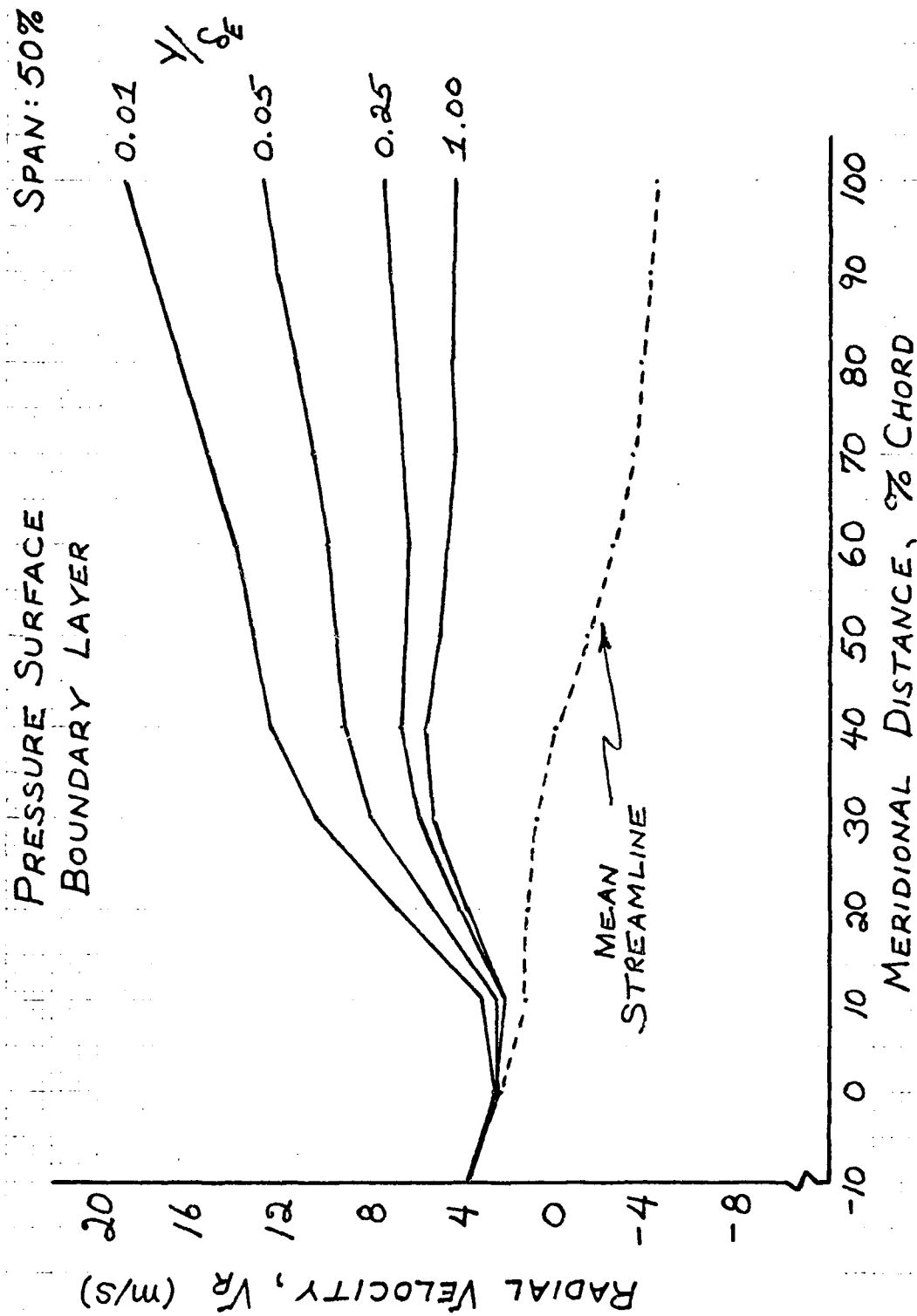


FIGURE 12. PRESSURE SURFACE BOUNDARY LAYER VELOCITY
EVOLUTION, 50% SPAN.



c). RADIAL VELOCITY.

FIGURE 12. PRESSURE SURFACE BOUNDARY LAYER VELOCITY

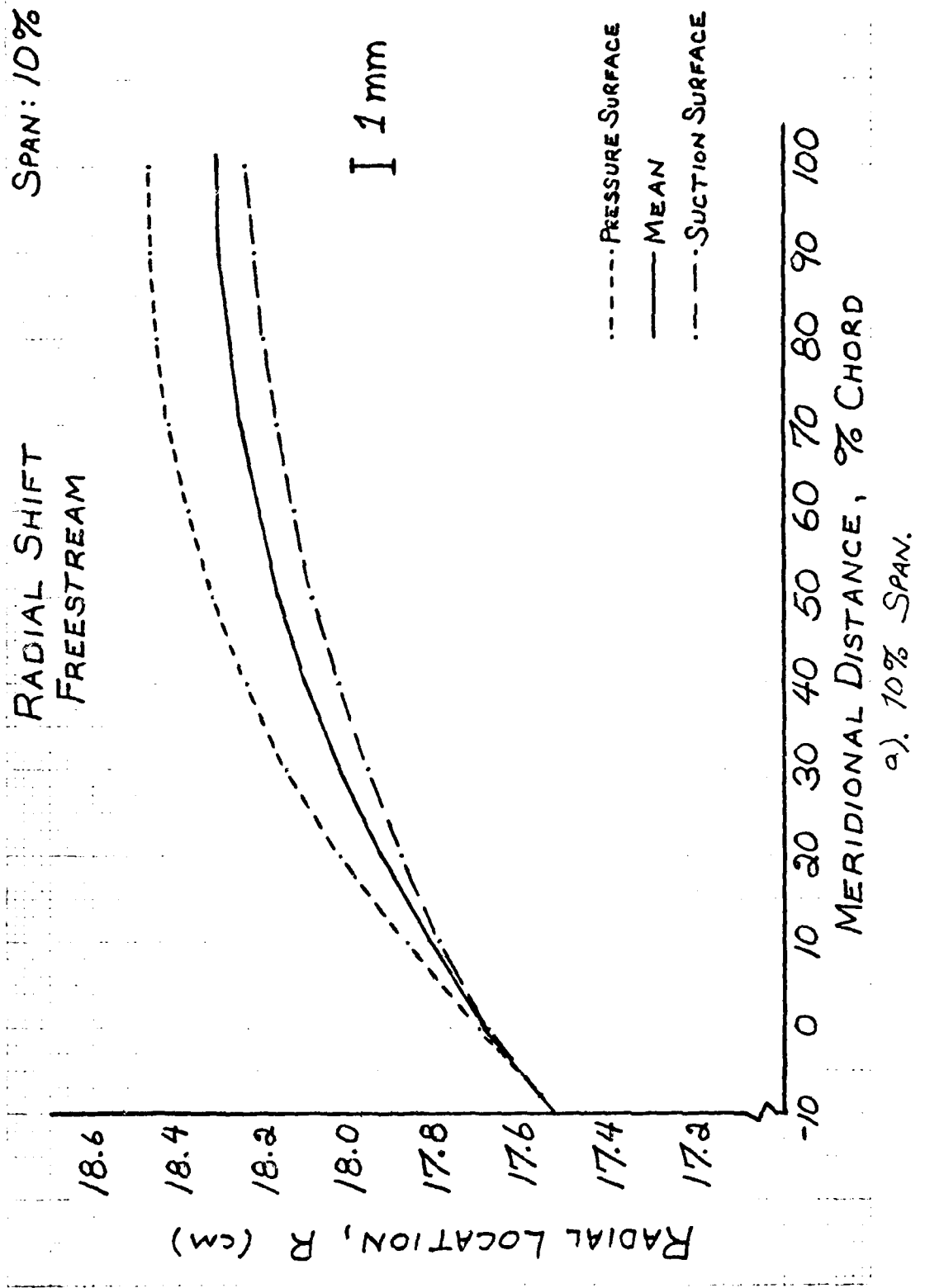


FIGURE 13. FREESTREAM MERIDIONAL TRAJECTORY.

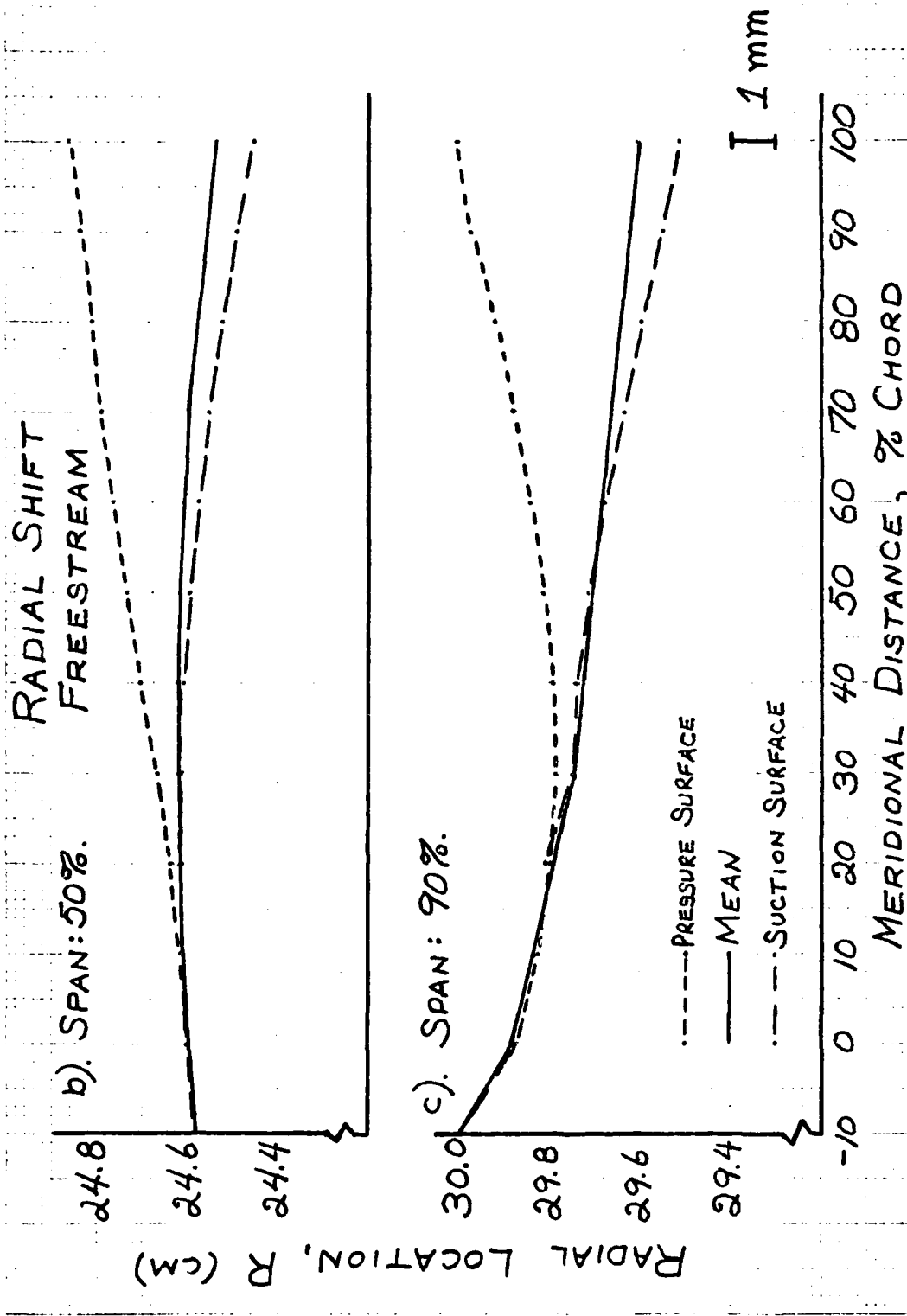


FIGURE 13. FREESTREAM MERIDIONAL TRAJECTORY.

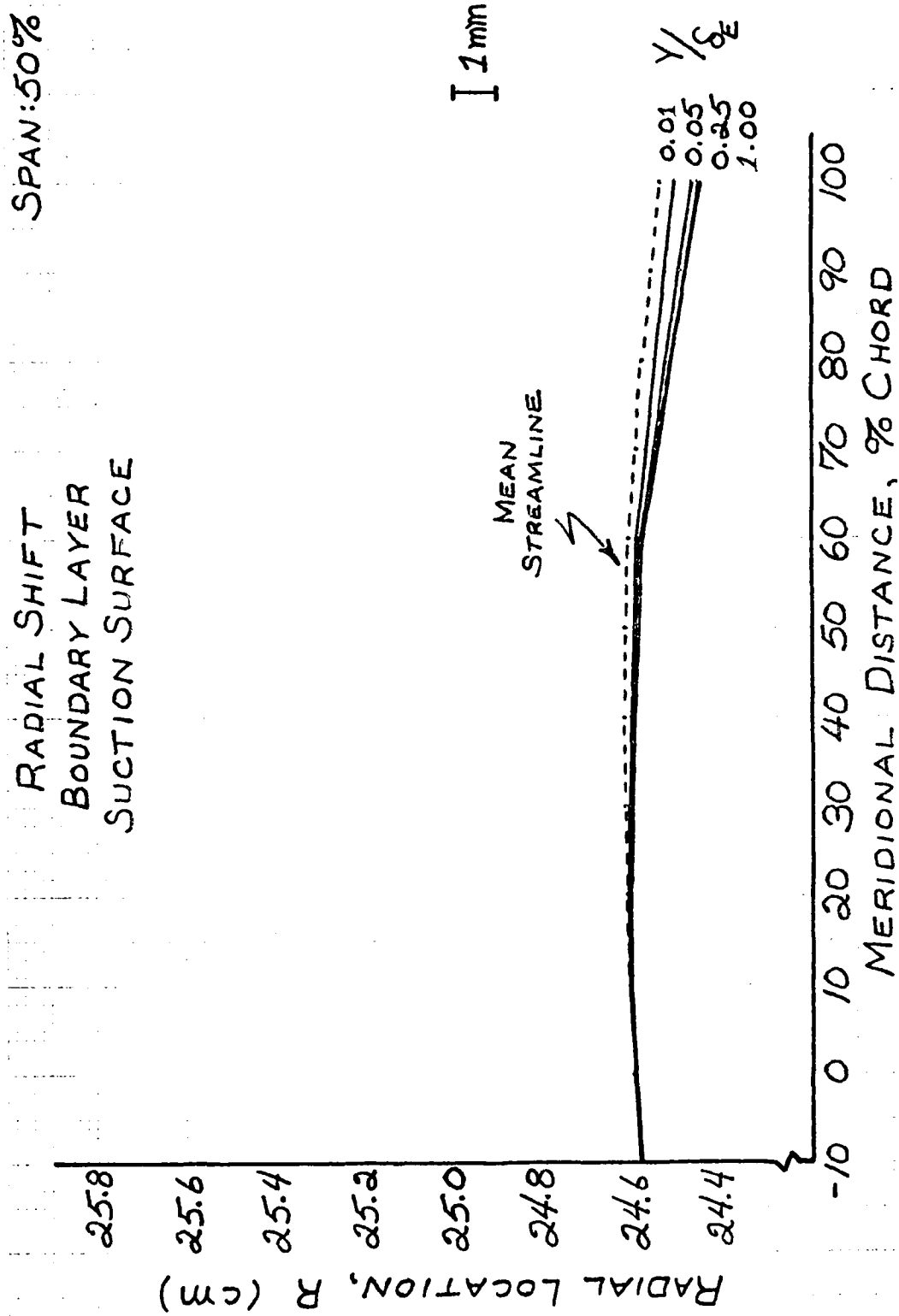


FIGURE 74. SUCTION SURFACE BOUNDARY LAYER
MERIDIONAL TRAJECTORY.

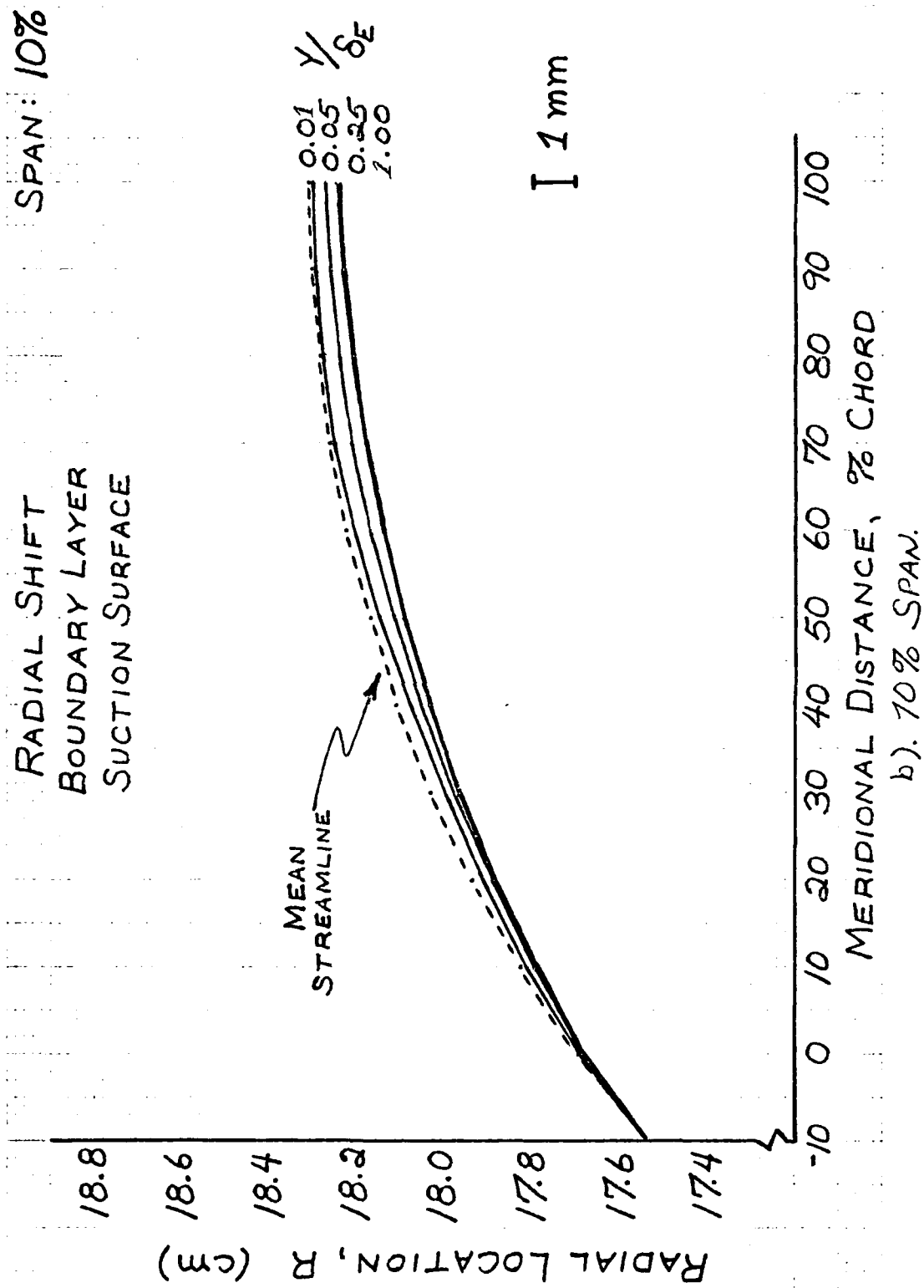


FIGURE 14. SUCTION SURFACE BOUNDARY LAYER

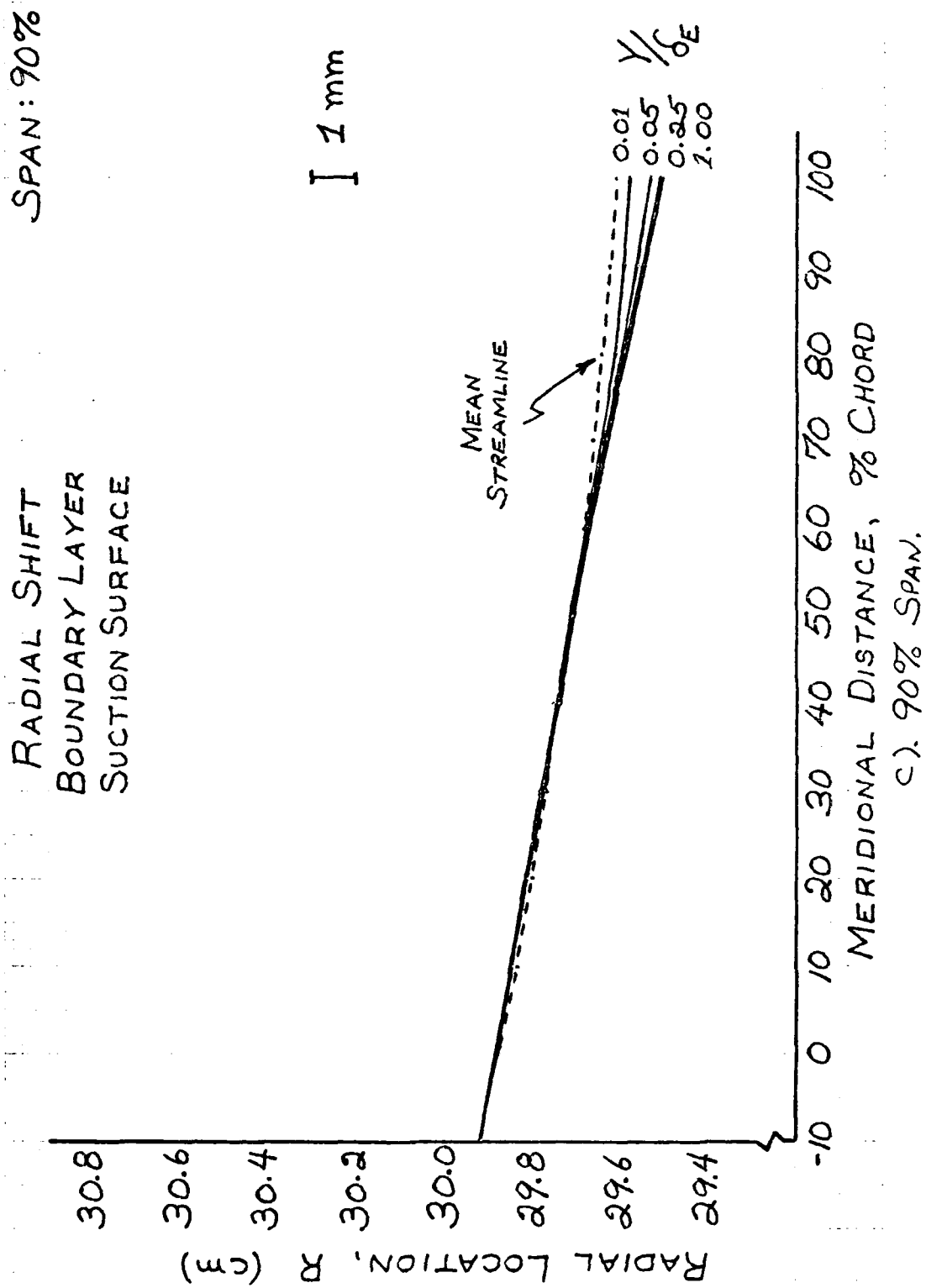
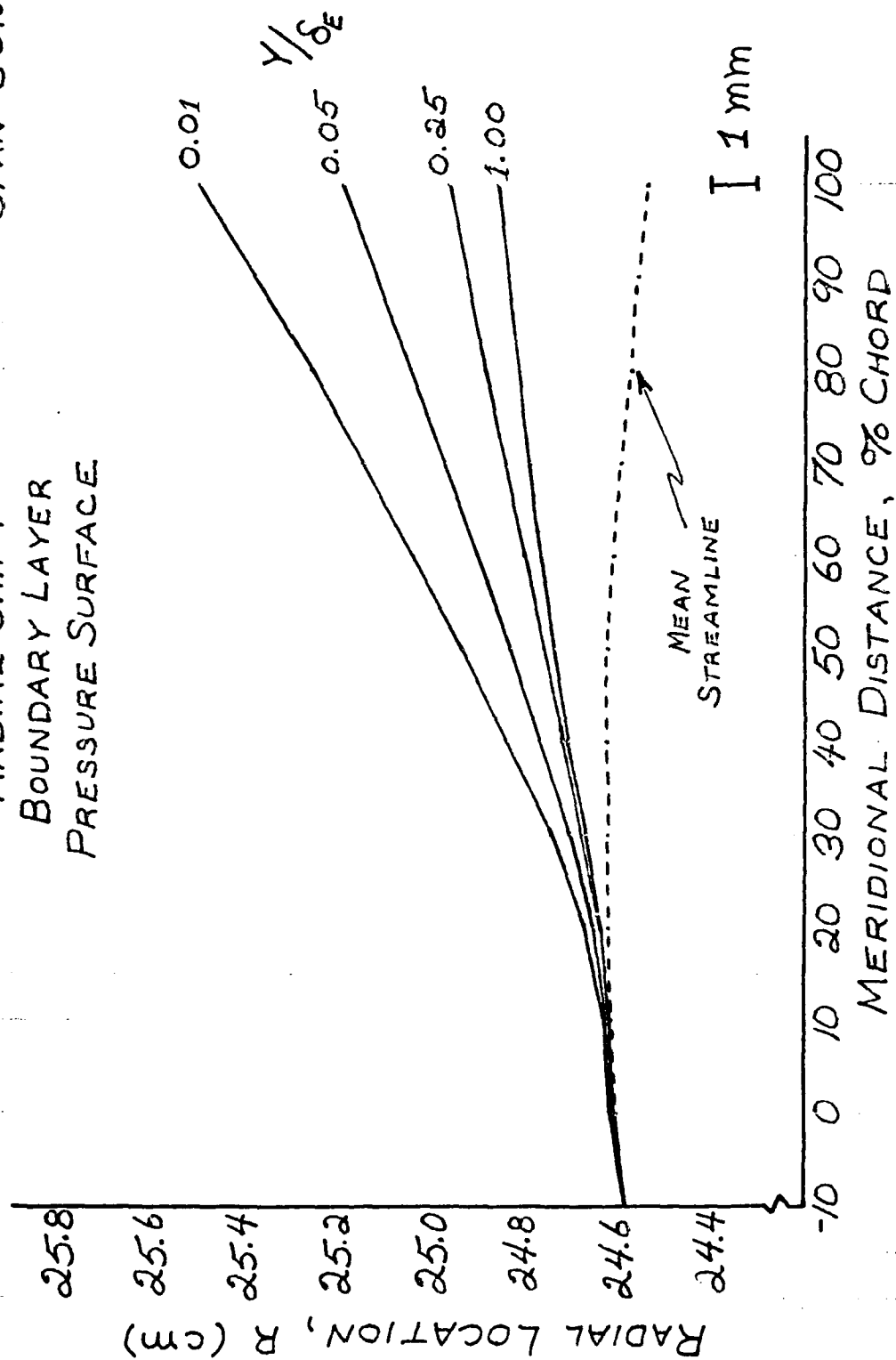


FIGURE 74. SUCTION SURFACE BOUNDARY LAYER
MERIDIONAL TRAJECTORY.

RADIAL SHIFT
BOUNDARY LAYER
PRESSURE SURFACE

SPAN: 50%



a). 50% SPAN.

FIGURE 15. PRESSURE SURFACE BOUNDARY LAYER

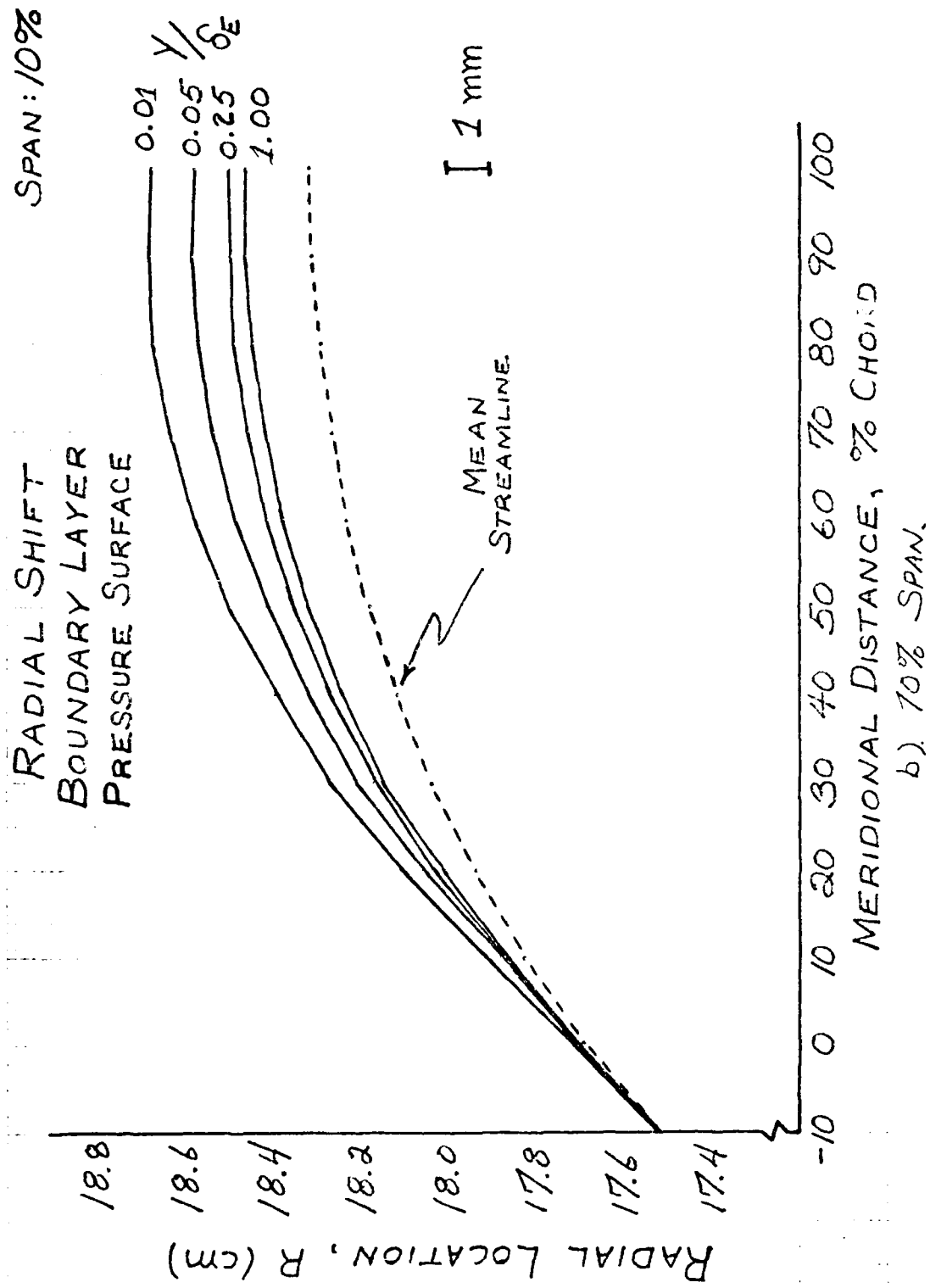


FIGURE 15. PRESSURE SURFACE BOUNDARY LAYER
MERIDIONAL TRAJECTORY.

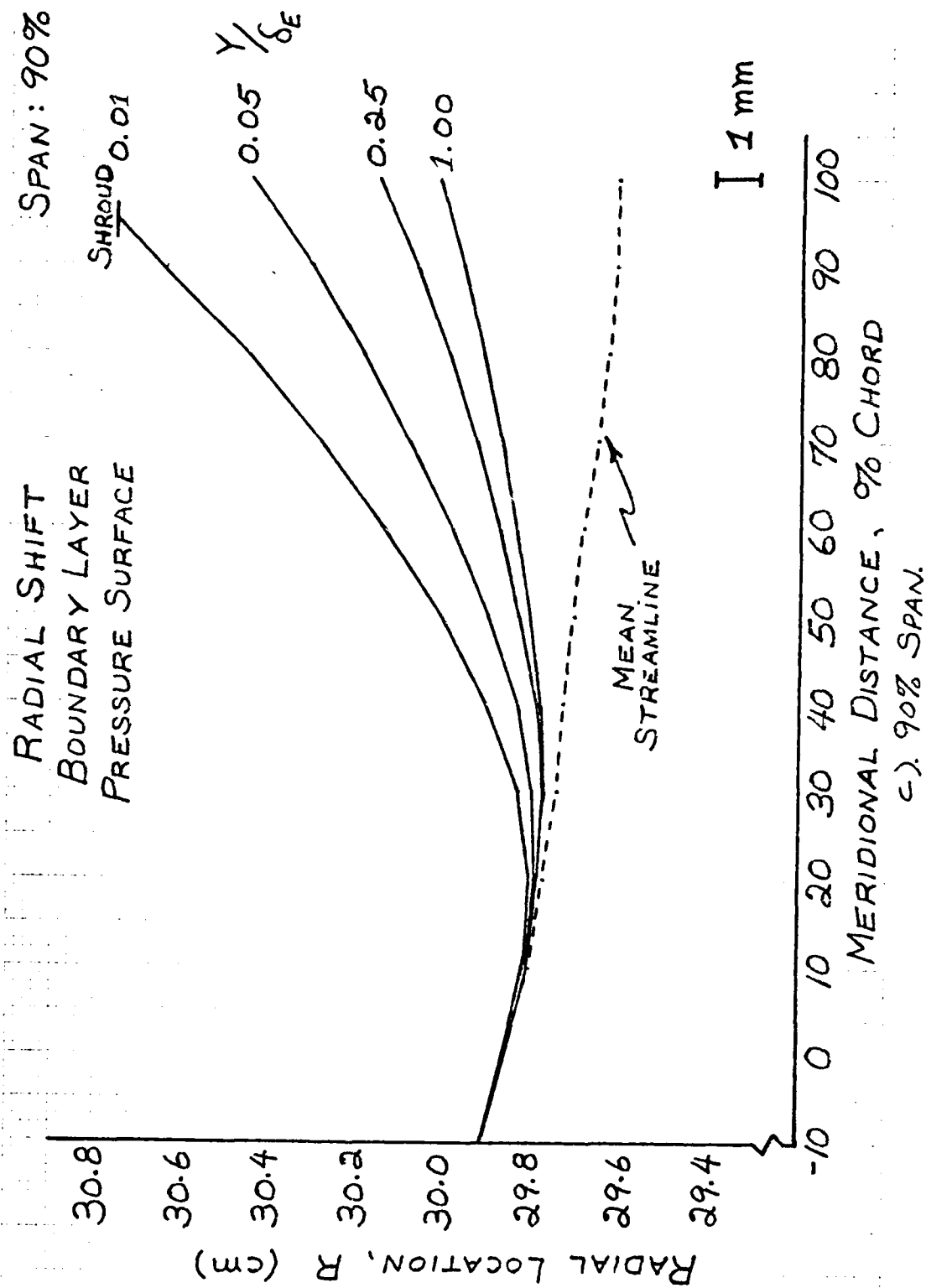
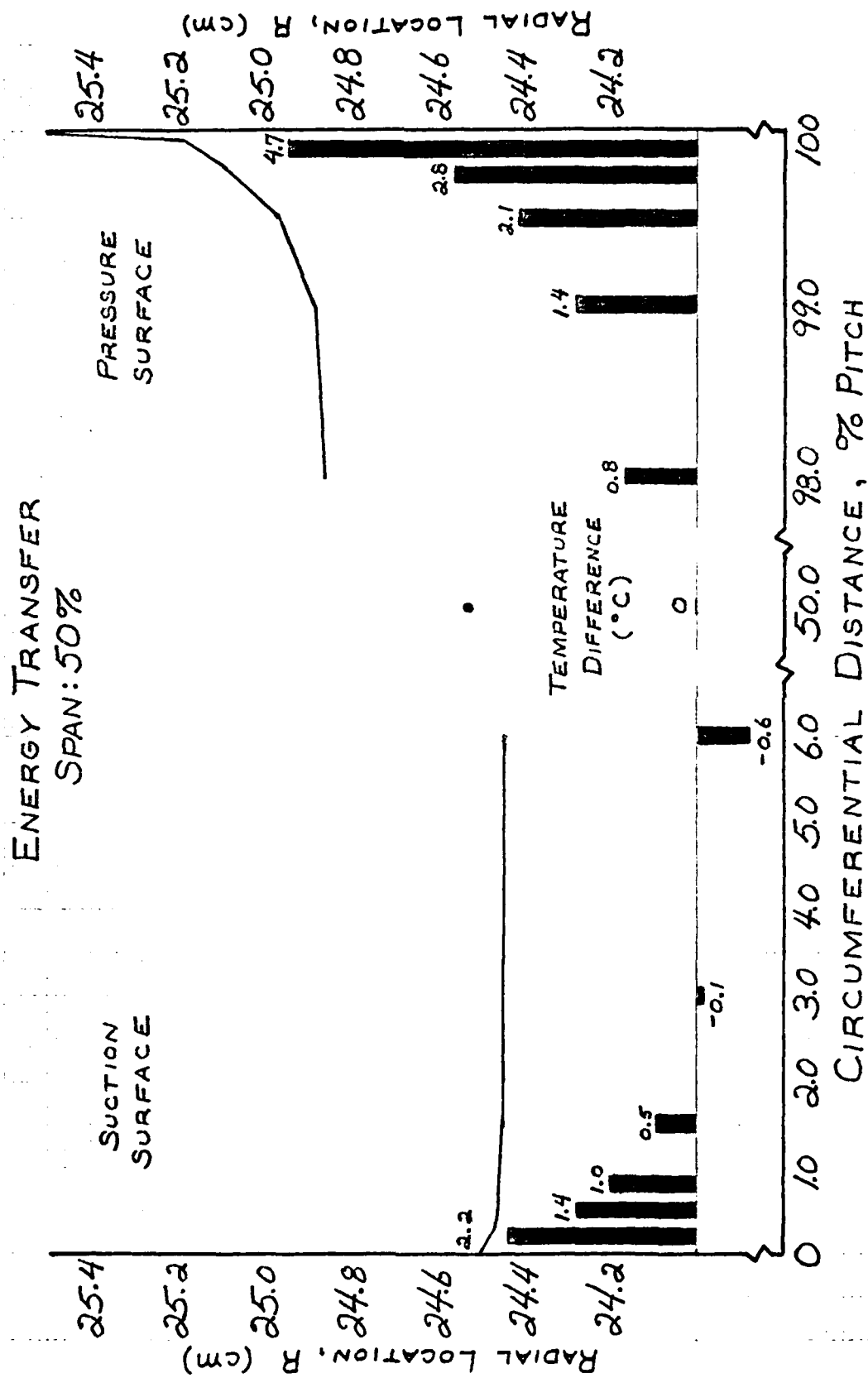


FIGURE 15. PRESSURE SURFACE BOUNDARY LAYER



a). 50% SPAN.

FIGURE 16. CIRCUMFERENTIAL TOTAL TEMPERATURE VARIATIONS.

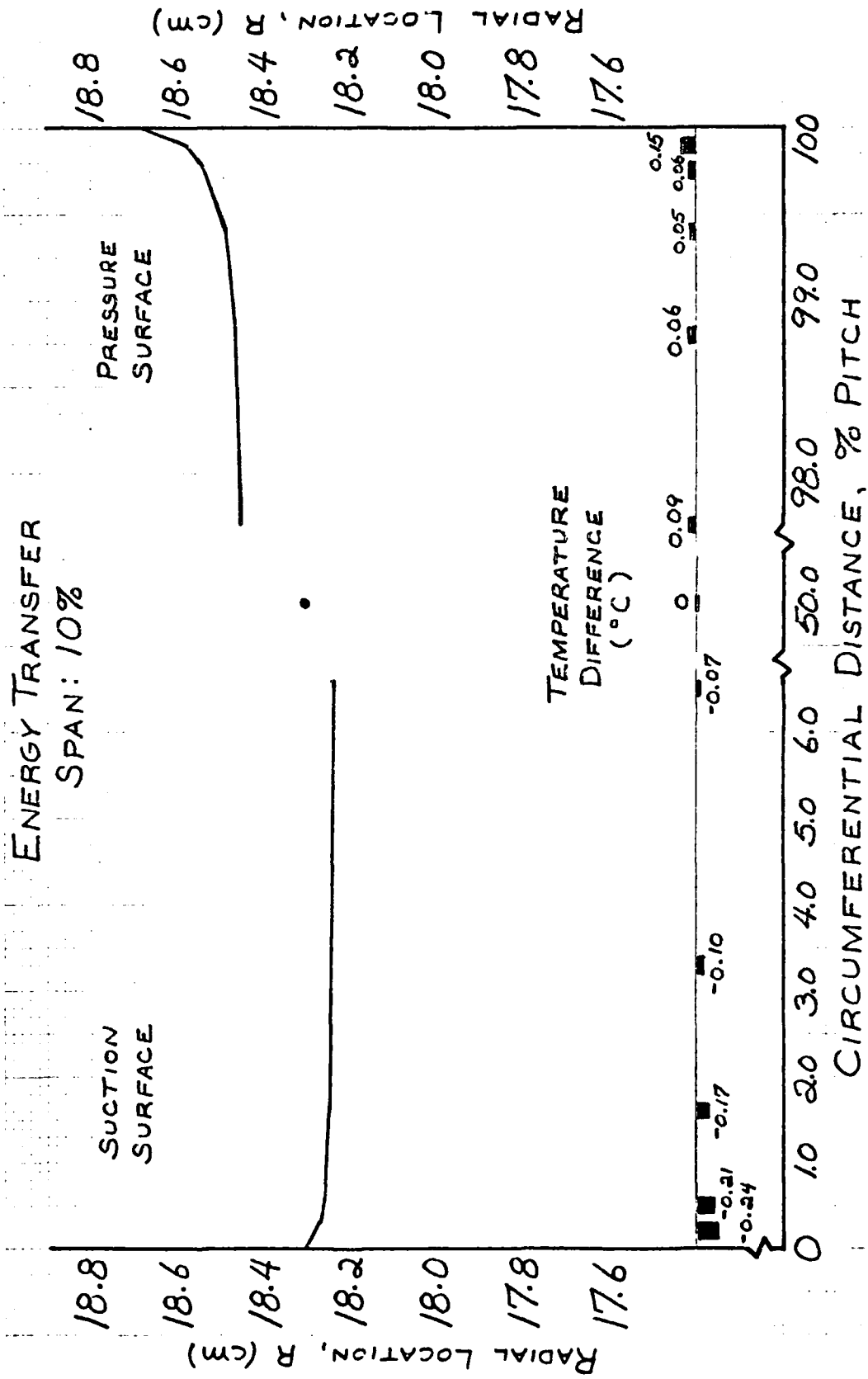


FIGURE 16. CIRCUMFERENTIAL TOTAL TEMPERATURE

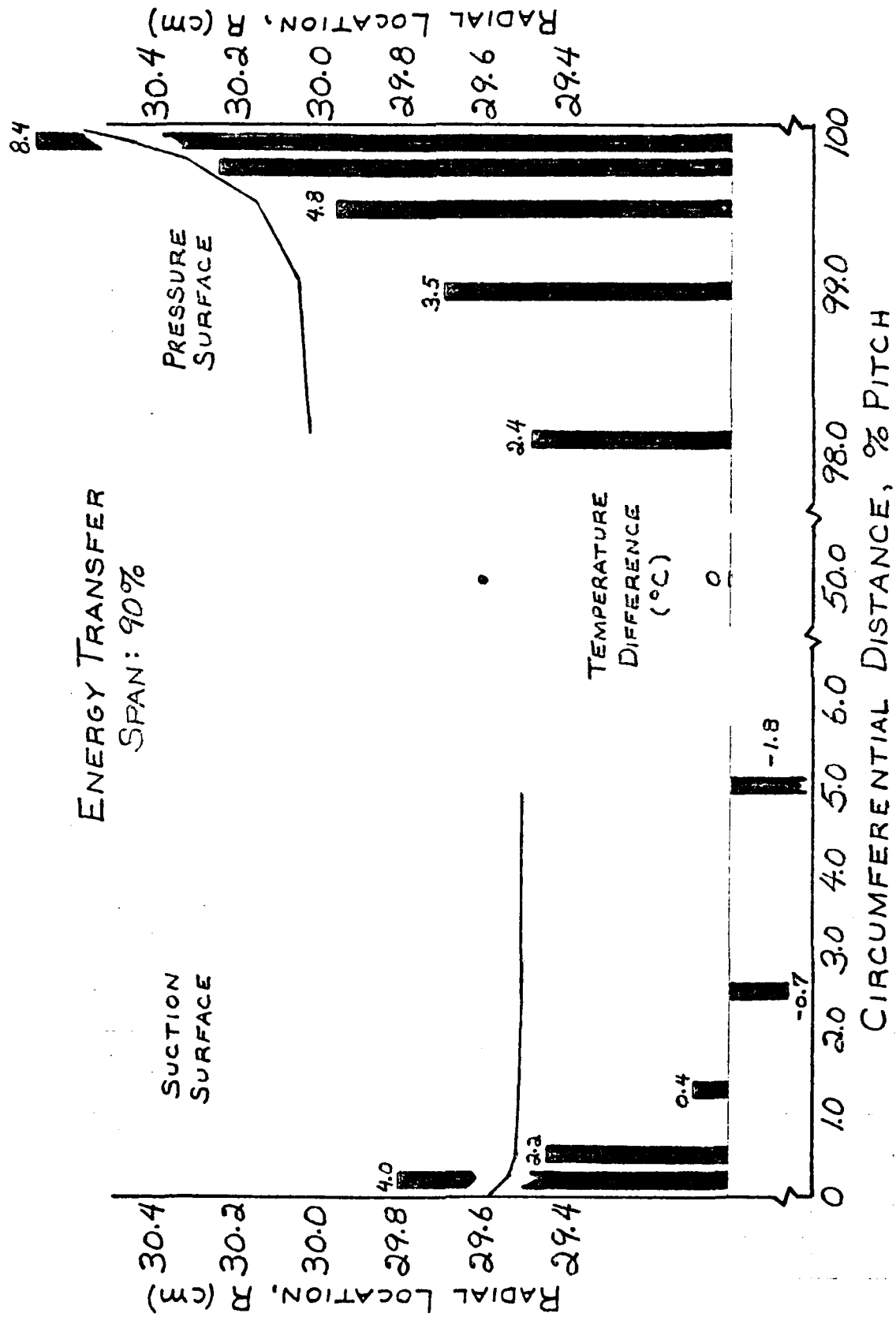


FIGURE 16. CIRCUMFERENTIAL TOTAL TEMPERATURE VARIATIONS.

