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Analysis of
Direct Mechanical Stabilization Platform
for Antenna Stabilization and Control
in a Shipboard Environment

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FOR THE COMMANDER

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LINCOLN LABORATORY

ANALYSIS OF DIRECT MECHANICAL STABILIZATION PLATFORM FOR ANTENNA STABILIZATION AND CONTROL IN A SHIPBOARD ENVIRONMENT

M. TAYLOR

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TECHNICAL REPORT 562

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Abstract

This report presents a rigid body analysis of the dynamics of a shipboard antenna in a three-axis gimbal configuration utilizing Direct Mechanical Stabilization (DMS) for primary control. System parameters are defined and Euler equations applied to obtain a non-linear system model that includes expected torque inputs. A computer simulation is generated with ship input motions due to roll, pitch, turn and flexure and a numerical integration procedure is then employed to determine the motion of the antenna and the resulting pointing error for various scenarios.

The results of the cases studied indicate potential feasibility of this approach for applications requiring pointing accuracy of approximately ±0.15 degree.

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LIST OF ILLUSTRATIONS

1.0 INTRODUCTION

This is the rigid body analysis of a DMS*-controlled antenna on board a surface ship for a three-axis gimbal configuration employing X-Y over train. The combined effects of roll, pitch, turn maneuvers and ship flexure on the pointing accuracy of the antenna are studied.

1.1 Background

The gyroscope has found its most prolific application in modern technology as an angular direction and motion sensor whose electrical output signal is used to activate either a mechanical or computational device. However, throughout the last half century of very active development and refinement of these sensors of shrinking size, there has been a lower level but continuing effort on large gyro wheels for DMS. The most sophisticated effort on DMS in recent years has been associated with the use of control moment gyros for stabilization and attitude control of satellites.

There are two general approaches to the concept of DMS:

1) Mount the stabilized device directly to the frame carrying the gyro rotor. This assembly is then mounted within gimbals and bearings to permit angular isolation from input motion. Examples of this approach are found in both early and more recent literature. Scarborough[1] describes the gyro pendulum; Williams[2] analyzes stabilization of an optical sensor for a shipboard application; Bieser[4] describes a configuration of this type for stabilizing an antenna on board a ship.

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* Direct Mechanical Stabilization

In this general approach the gyro spin axis and the stabilized device (optics, antenna, etc.) move together as the gyro precesses in response to disturbing torques.

2) In the second general approach the gyro rotor spin assembly is mounted to the stabilized frame within a pivot axis that is orthogonal to the spin axis. This is a distinct improvement in stabilizing effectiveness over the first type described in 1) above. Now the primary effect of any disturbing torque on the stabilized device is to cause only the gyro to precess within its pivot axis. The stabilized frame remains essentially undisturbed while the gyro precesses. This method has been used since the beginning of the century on examples such as the Schleck ship stabilizer in 1903[1] and the monorail stabilizer of Scherl and Schilorsky in 1909[1]. More recent examples are Bieser's antenna stabilizer[3], Matthews version of a similar device[5] and an aircraft camera stabilizer development by Westinghouse Canada Limited[6].

The dynamics of the Bieser approach[3] have recently been analyzed[8]. This configuration utilizes the DMS principle to provide a roll-pitch stabilized platform carrying a train-elevation servo-controlled antenna.

The X-Y configuration described in this report applies the second general approach to directly position and stabilize the antenna without an intermediate train-elevation servo.

1.2 General Description

Figure 1 is a pictorial representation of the hardware configuration showing the antenna in a horizontal beam position, mounted to the stabilized structure containing the 'A' and 'B' DMS gyro assemblies. The first gimbal axis supporting the stabilized assembly is the Y axis which is nominally controlled

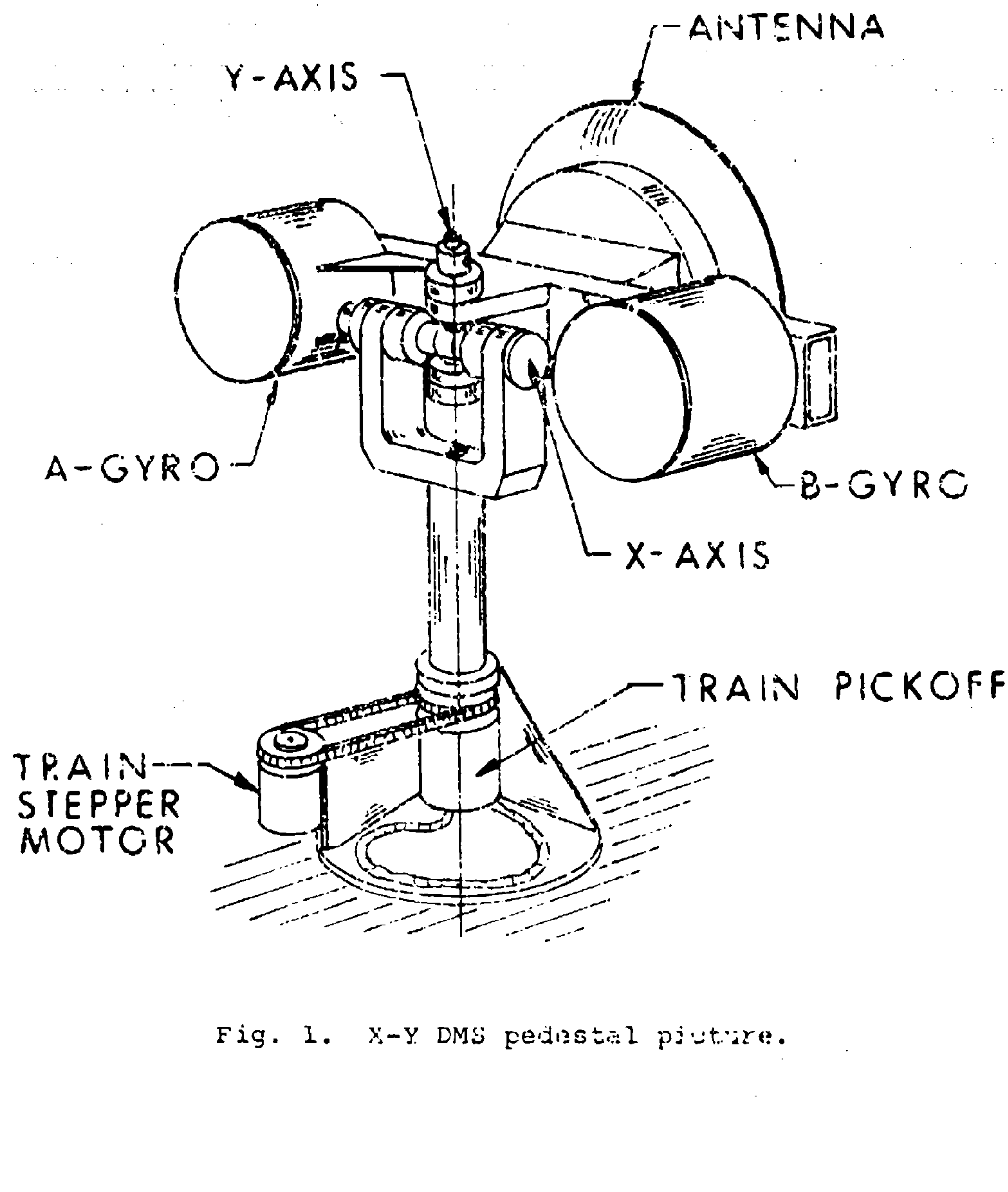


Fig. 1. X-Y DMS pedestal picture.

to be in the vertical plane containing the satellite; the next supporting axis is the X axis which is nominally controlled to be perpendicular to the vertical plane containing the satellite. The total assembly is carried on the train axis which is mounted to the deck of the ship.

Further detail is given in Figure 2 where the antenna is shown in a vertical beam position. The 'A' gyro is free to pivot about an axis parallel to X and the 'B' gyro is free to pivot about an axis parallel to Y. The gyro pivot axes and the X, Y gimbal axes are each equipped with a torquer and angular pickoff. The train axis is positioned by a stepper motor, and has a precise angular pickoff to read its position.

Antenna beam position control is achieved by applying appropriate control torques to the 'A' or 'B' gyro torquer. For example, antenna motion about the X axis is achieved by applying a torque to the 'B' gyro torquer. The antenna is stabilized against the effects of disturbing torques by precession of the gyro assemblies within their respective pivot axes. Narrow bandwidth caging loops are used to maintain the spin axes nominally parallel to the antenna beam axis. For example, the X torquer responds to a pickoff signal from the 'B' gyro to null the 'B' gyro pickoff output.

The details of the control concepts will be explained in the body of this report.

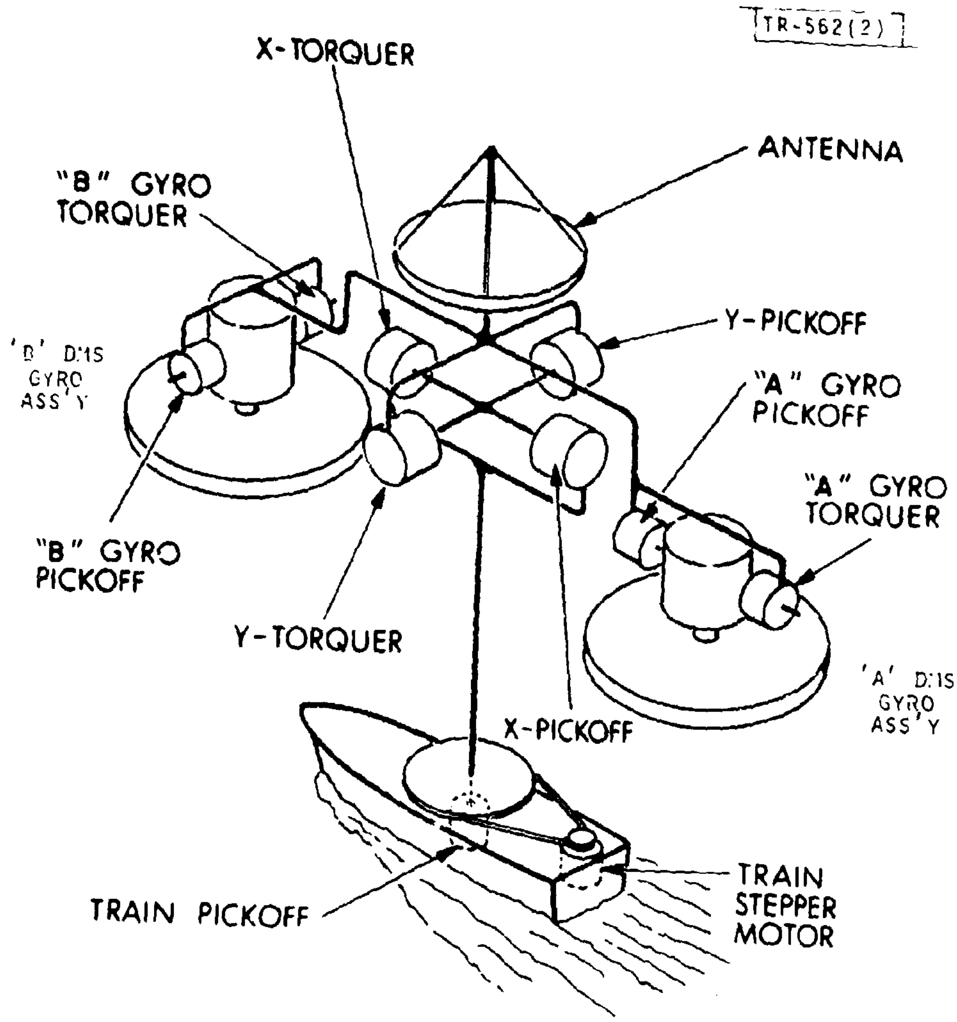


Fig. 2. X-Y DMS components and structure.

2.0 KINEMATIC FRAMES & EULER ROTATIONS

Referring to Figure 3, the following reference frames and their Euler rotations are defined:

$\{\hat{\underline{n}}\}$ = inertial frame

$\{\hat{\underline{f}}\}$ = deck frame

$\{\hat{\underline{p}}\}$ = antenna platform

$\{\hat{\underline{a}}\}$ = 'A' gyro gimbal

$\{\hat{\underline{b}}\}$ = 'B' gyro gimbal

$\{\hat{\underline{e}}\}$ = frame containing satellite position along $\hat{\underline{e}}_2$

$\{\hat{\underline{g}}\}$ = X-Y gimbal frame

Additional intermediate reference frames are defined below in terms of the angles of rotation and the axis of each rotation.

ψ_A = ship azimuth

ψ_P = pitch

ψ_R = roll

ψ_H = heel $(\psi_{RH} = \psi_R + \psi_H)$

θ_T = train

θ_X = X axis

θ_Y = Y axis

γ_a = gyro 'A' gimbal rotation with respect to platform

γ_b = gyro 'B' gimbal rotation with respect to platform

α_A = satellite azimuth position

α_E = satellite elevation position

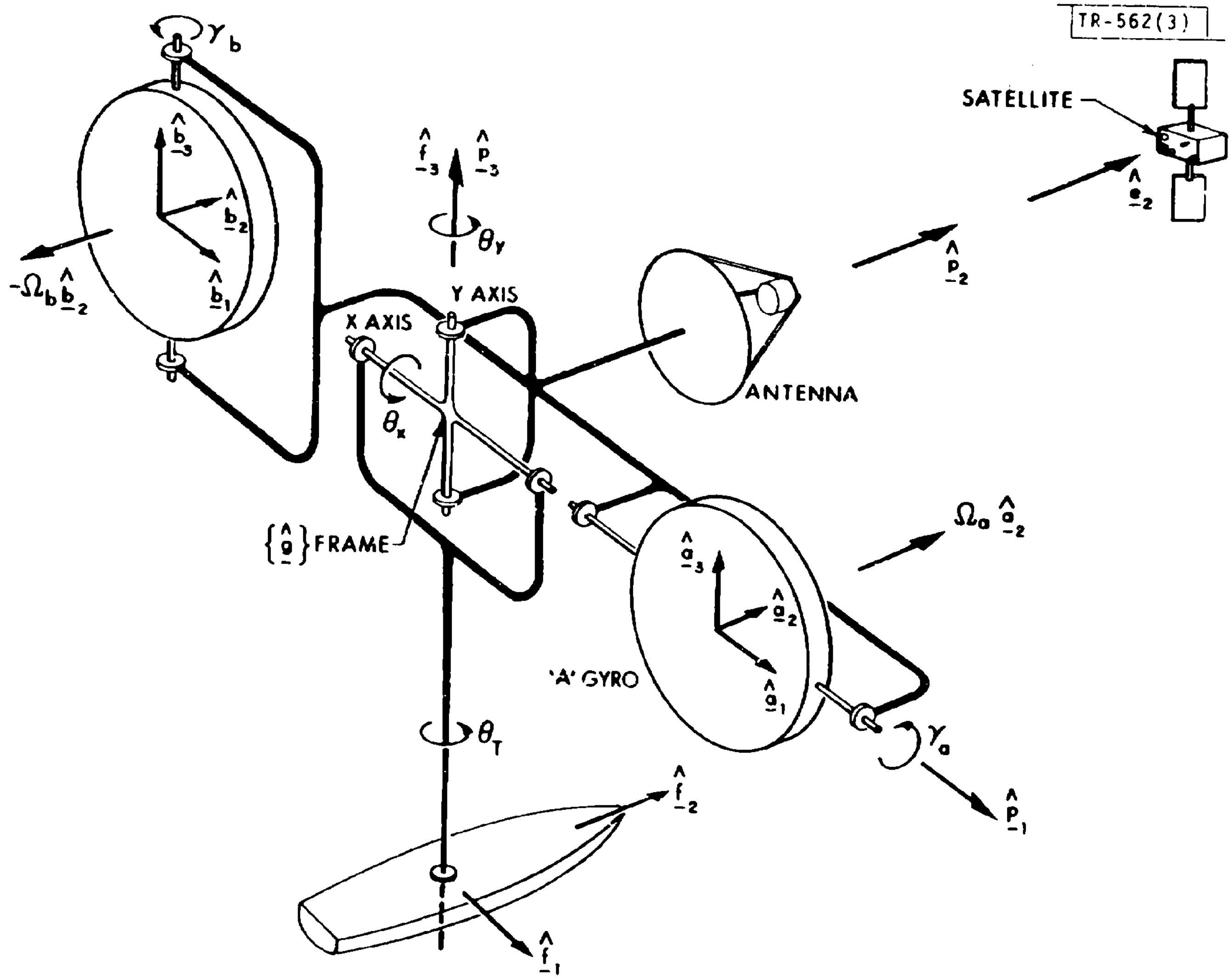


Fig. 3. X-Y DMS kinematic frames.

The format for showing the rotation sequences are:

$$\left\{\begin{matrix}\text{starting}\\ \text{frame}\end{matrix}\right\} \xrightarrow[\text{Rotation Axis}]{\text{Rotation Angle}} \left\{\begin{matrix}\text{frame after}\\ \text{rotation}\end{matrix}\right\}$$

Applying this format and the previously defined frames and rotation angles the rotation sequences are as follows:

<u>Deck of Ship</u> $\{\hat{\underline{f}}\}$

$$\{\hat{\underline{n}}\} \xrightarrow[\hat{\underline{n}}_3]{\psi_A} \{\hat{\underline{n}}'\} \xrightarrow[\hat{\underline{n}}_1']{\psi_P} \{\hat{\underline{n}}''\} \xrightarrow[\hat{\underline{n}}_2'']{\psi_{RH}} \{\hat{\underline{f}}\}$$

<u>Antenna Platform</u> $\{\hat{\underline{p}}\}$

$$\{\hat{\underline{f}}\} \xrightarrow[\hat{\underline{f}}_3]{\theta_T} \{\hat{\underline{f}}'\} \xrightarrow[\hat{\underline{f}}_1']{\theta_X} \{\hat{\underline{g}}\} \xrightarrow[\hat{\underline{g}}_3]{\theta_Y} \{\hat{\underline{p}}\}$$

<u>'A' Gyro</u> $\{\hat{\underline{a}}\}$

$$\{\hat{\underline{p}}\} \xrightarrow[\hat{\underline{p}}_1]{\gamma_a} \{\hat{\underline{a}}\}$$

'B' Gyro $\{\hat{\underline{b}}\}$

$$\{\hat{\underline{p}}\} \xrightarrow[\hat{\underline{p}}_3]{\gamma_b} \{\hat{\underline{b}}\}$$

Satellite Position Vector Along $\{\hat{\underline{e}}_2\}$

$$\{\hat{\underline{n}}\} \xrightarrow[\hat{\underline{n}}_3]{\alpha_A} \{\hat{\underline{e}}'\} \xrightarrow[\hat{\underline{e}}'_1]{\alpha_E} \{\hat{\underline{e}}\}$$

3.0 ANGULAR VELOCITIES

The following angular velocities are defined:

Deck of Ship $\{\hat{\underline{f}}\}$

$$\underline{\omega}_f = \omega_{f1}\,\hat{\underline{f}}_1 + \omega_{f2}\,\hat{\underline{f}}_2 + \omega_{f3}\,\hat{\underline{f}}_3 \tag{3-1}$$

$$\underline{\omega}_f = \dot{\psi}_A\,\hat{\underline{n}}'_3 + \dot{\psi}_P\,\hat{\underline{n}}''_1 + \dot{\psi}_R\,\hat{\underline{f}}_2 \tag{3-2}$$

Antenna Platform $\{\hat{\underline{p}}\}$

$$\underline{\omega}_p = \omega_{p1}\,\hat{\underline{p}}_1 + \omega_{p2}\,\hat{\underline{p}}_2 + \omega_{p3}\,\hat{\underline{p}}_3 \tag{3-3}$$

$$\underline{\omega}_p = \underline{\omega}_f + \dot{\theta}_T\,\hat{\underline{f}}'_3 + \dot{\theta}_x\,\hat{\underline{g}}_1 + \dot{\theta}_y\,\hat{\underline{p}}_3 \tag{3-4}$$

'A' Gyro $\{\hat{\underline{a}}\}$

Gimbal: $$\underline{\omega}_a = \omega_{a1}\,\hat{\underline{a}}_1 + \omega_{a2}\,\hat{\underline{a}}_2 + \omega_{a3}\,\hat{\underline{a}}_3 \tag{3-5}$$

Rotor: $$\underline{\Omega}_a = \omega_{a1}\,\hat{\underline{a}}_1 + \Omega_a\,\hat{\underline{a}}_2 + \omega_{a3}\,\hat{\underline{a}}_3 \tag{3-6}$$

where Ω_a = constant wheel spin speed

Note that $\dot{\gamma}_a = \omega_{a1} - \omega_{p1}$

'B' Gyro $\{\hat{\underline{b}}\}$

$$\text{Gimbal:} \quad \underline{\omega}_b = \omega_{b1}\,\hat{\underline{b}}_1 + \omega_{b2}\,\hat{\underline{b}}_2 + \omega_{b3}\,\hat{\underline{b}}_3 \tag{3-7}$$

$$\text{Rotor:} \quad \underline{\Omega}_b = \omega_{b1}\,\hat{\underline{b}}_1 - \Omega_b\,\hat{\underline{b}}_2 + \omega_{b3}\,\hat{\underline{b}}_3 \tag{3-8}$$

Note that the polarity of $\Omega_b\,\hat{\underline{b}}_2$ is negative. This is essential to assure that both rotors precess in the same direction with the antenna when a control torque is applied to either rotor. For example the torquer used to control X axis motion applies a torque to the 'B' gyro about $\hat{\underline{b}}_3$ and, via reaction, an opposite polarity torque to the 'A' gyro about $\hat{\underline{a}}_3$. Both rotors then precess in the same direction about $\hat{\underline{b}}_1$ and $\hat{\underline{a}}_1$ respectively while the antenna moves about $\hat{\underline{p}}_1$.

Note that $\dot{\gamma}_b = \omega_{b3} - \omega_{p3}$.

4.0 STATE VARIABLES

The following state variables are defined for the system equations that will be developed:

$$X_1 = \theta_x$$
$$X_2 = \theta_y$$
$$X_3 = \omega_{a1}$$
$$X_4 = \omega_{b3}$$
$$X_5 = \omega_{p3}$$
$$X_6 = \omega_{p1}$$
$$X_7 = \omega_{p2}$$
$$X_8 = \gamma_a$$
$$X_9 = \gamma_b$$
$$X_{10} = \int \gamma_a dt$$
$$X_{11} = \int \gamma_b dt$$
$$X_{12} = \theta_T$$
$$X_{13} = \dot{\theta}_T$$

5.0 GYRO EULER EQUATIONS

The Euler equations for the gyros are developed in Appendix A and result in the following relations:

$$\dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \tag{5-1}$$

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

$$\dot{\omega}_{b1} = J_{b1} + T_{b1}/I_{g1} \tag{5-3}$$

$$\dot{\omega}_{b3} = J_{b3} + T_{b3}/I_{g3} \tag{5-4}$$

where

$$J_{a1} = \omega_{a3}\Omega_a I_{g2}/I_{g1} - \omega_{a3}\omega_{a2}$$

$$J_{a3} = -\omega_{a1}\Omega_a I_{g2}/I_{g1} + \omega_{a1}\omega_{a2}$$

$$J_{b1} = -\omega_{b3}\Omega_b I_{g2}/I_{g1} - \omega_{b3}\omega_{b2}$$

$$J_{b3} = \omega_{b1}\Omega_b I_{g2}/I_{g1} + \omega_{b1}\omega_{b2}$$

I_{g1}, I_{g2} = gyro assembly inertia about 1 and 2 axes of $\{\hat{\underline{a}}\}$ or $\{\hat{\underline{b}}\}$

T_{a1}, T_{a3}, T_{b1}, T_{b3} = torque applied to gyro about subscript axis (see Eq. (9-6) and (9-12))

6.0 PLATFORM EULER EQUATIONS

The platform Euler equations are developed in Appendix A and result in the following relations:

$$\dot{\omega}_{p1} = T_{p1}/I_{p1} \tag{6-1}$$

$$\dot{\omega}_{p2} = T_{p2}/I_{p2} \tag{6-2}$$

$$\dot{\omega}_{p3} = T_{p3}/I_{p3} \tag{6-3}$$

where

T_{pj} = torque applied to platform about j axis (see Eq. (9-20))

7.0 CONSTRAINTS

Mechanical constraints exist due to the gimballing configuration. The angular velocity relations that reserve these constraints are developed in Appendix B with the following results:

$$\omega_{a2} = \omega_{p2} \cos \gamma_a + \omega_{p3} \sin \gamma_a \tag{7-1}$$

$$\omega_{a3} = -\omega_{p2} \sin \gamma_a + \omega_{p3} \cos \gamma_a \tag{7-2}$$

$$\omega_{b1} = \omega_{p1} \cos \gamma_b + \omega_{p2} \sin \gamma_b \tag{7-3}$$

$$\omega_{b2} = -\omega_{p1} \sin \gamma_b + \omega_{p2} \cos \gamma_b \tag{7-4}$$

8.0 TRANSFORMATIONS AND CONTROL OF θ_x, θ_y, θ_T

8.1 Transformation for θ_x, and θ_y

The antenna beam lies along axis $\hat{\underline{p}}_2$; the satellite position vector lies along $\hat{\underline{e}}_2$. We desire to orient $\hat{\underline{p}}_2$ coincident with $\hat{\underline{e}}_2$ using the rotations defined in Section 2.0 to determine appropriate input command angles θ_{xc} and θ_{yc} as functions of α_A, α_E, ψ_A, ψ_p, ψ_{RH}, θ_T. The computation process determines the $\{\hat{\underline{n}}\}$ components of $\hat{\underline{p}}_2$ and $\hat{\underline{e}}_2$, then equates these components to solve for θ_{xc} and θ_{yc}.

The direction cosine matrices for the rotations in Section 2.0 are defined using the symbols,

$[C_i(\zeta)]$ = Direction cosine matrix for rotation about i^{th} axis of starting frame through the angle ζ.

Using this notation the relations between various reference frames are defined as follows:

$$\left.\begin{aligned}\{\hat{\underline{f}}'\} &= [C_3(\theta_T)][C_2(\psi_{RH})][C_1(\psi_p)][C_3(\psi_A)]\{\hat{\underline{n}}\}\\ \{\hat{\underline{f}}'\} &= [B_1]\{\hat{\underline{n}}\}\end{aligned}\right\} \tag{8-1}$$

$$\left.\begin{aligned}\{\hat{\underline{p}}\} &= [C_3(\theta_{yc})][C_1(\theta_{xc})]\{\hat{\underline{f}}'\}\\ \{\hat{\underline{p}}\} &= [B_2]\{\hat{\underline{f}}'\}\end{aligned}\right\} \tag{8-2}$$

$$\{\hat{\underline{a}}\} = [C_1(\gamma_a)]\{\hat{\underline{p}}\} \tag{8-3}$$

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)]\{\hat{\underline{p}}\} \tag{8-4}$$

$$\{\hat{\underline{e}}\} = [C_1(\alpha_E)][C_3(\alpha_A)]\{\hat{\underline{n}}\} \tag{8-5}$$

$$\{\hat{\underline{e}}\} = [B_3]\{\hat{\underline{n}}\}$$

From (8-1) and (8-2)

$$\{\hat{\underline{p}}\} = [B_2 B_1]\{\hat{\underline{n}}\} \tag{8-6}$$

$$\{\hat{\underline{n}}\} = B_1^T\ B_2^T\ \{\hat{\underline{p}}\}$$

From (8-5)

$$\{\hat{\underline{n}}\} = B_3^T\ \{\hat{\underline{e}}\} \tag{8-7}$$

The $\hat{\underline{n}}$ components of $\hat{\underline{p}}_2$ and $\hat{\underline{e}}_2$ are now equated using (8-6) and (8-7)

$$B_1^T\ B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \tag{8-8}$$

The expansion and solution of Eq. (8-8) for the desired angles is performed in Appendix C with the following results:

$$\theta_{yc} = \sin^{-1}(-t_1) \tag{8-9}$$

$$\theta_{xc} = \sin^{-1}(t_3/\cos\theta_{yc}) \tag{8-10}$$

where

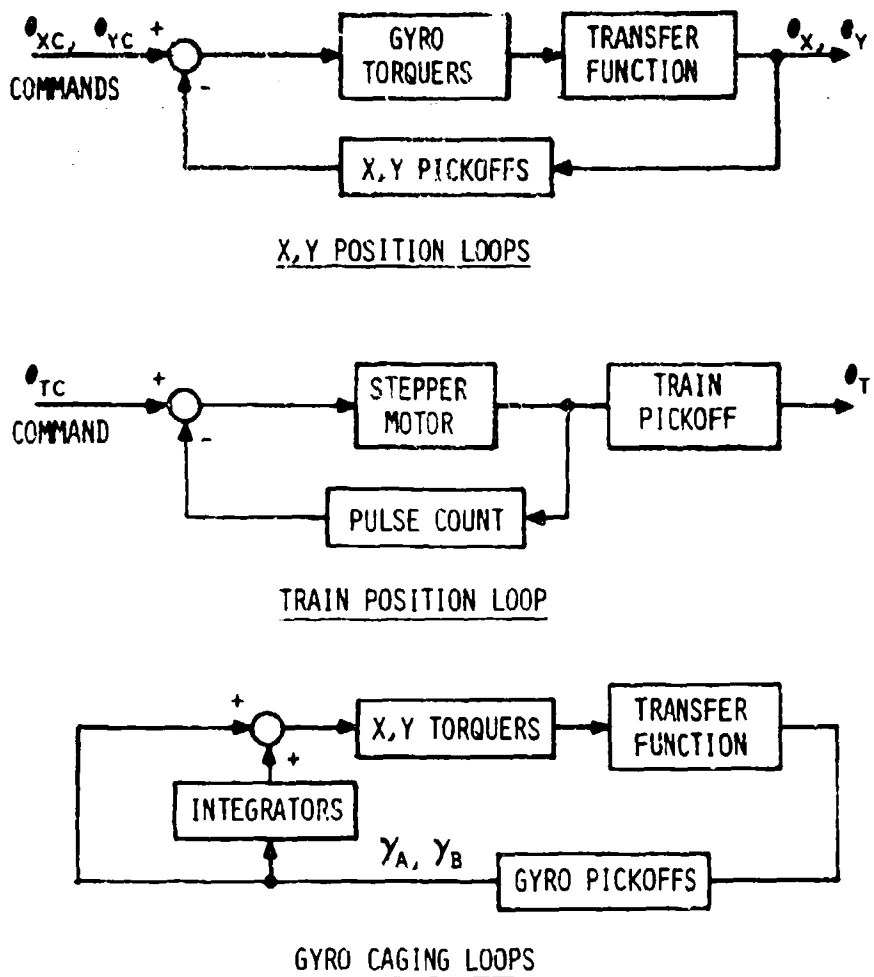
t_1, t_3 = results of matrix equation (C-3) in Appendix C.

8.2 System Control

The control approach that will be used to achieve the desired values θ_{xc}, θ_{yc} is shown in Figures 4 and 5.

Referring to Figure 4, the command values θ_{xc}, θ_{yc} are compared to actual values measured by the X, Y pickoffs and the error signals drive the appropriate gyro torquers. A train command based on ship's heading and satellite position (see Figure 5 and Section 8.3) drives the train stepper motor. The bottom loop shown in Figure 4 is a 'housekeeping' loop that keeps the gyro spin axis nominally parallel to the antenna beam axis.

Figure 5 shows the flow of information used to generate the control commands. The ship's master compass and heading reference provide roll, pitch and heading information used in the coordinate transformations. The heading output of the compass is also utilized directly to position the train servo.



NOTE: THERE IS NO ACTIVE STABILIZATION LOOP. THE GYROS PROVIDE THIS FUNCTION.

Fig. 4. X-Y DMS control outline.

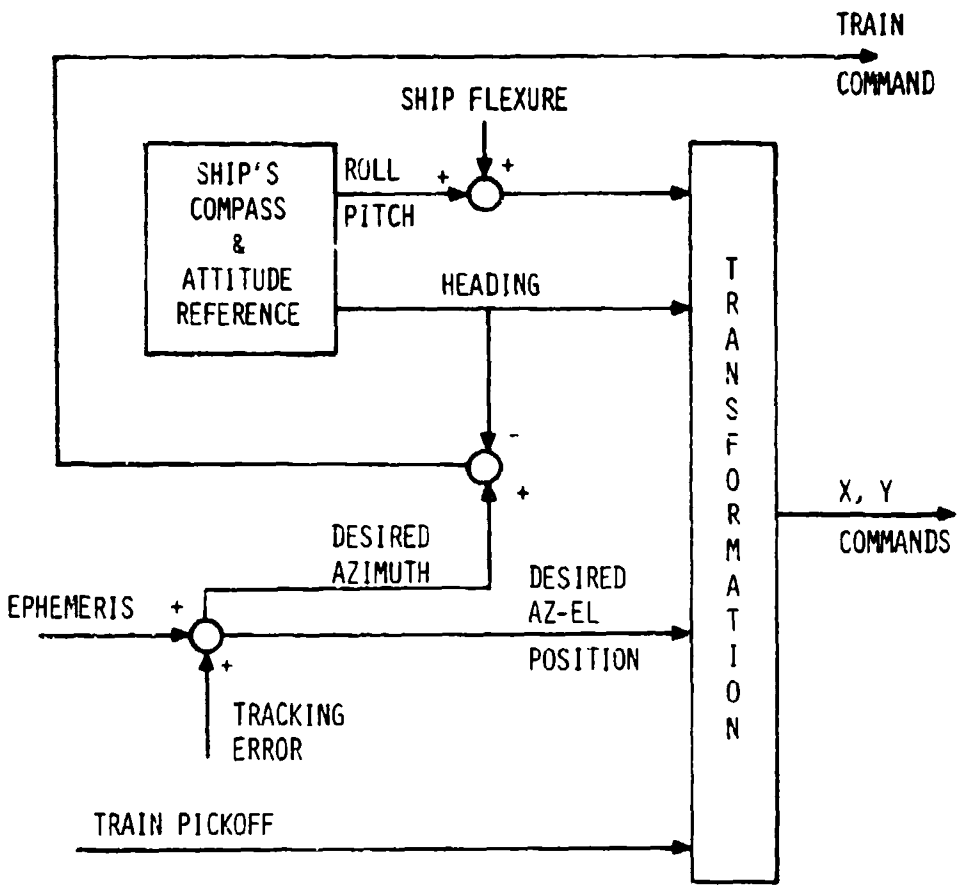


Fig. 5. X-Y DMS commands.

The satellite ephemeris data is updated by down-link tracking error signals to provide the desired antenna beam position to the satellite. The coordinate transformation computation generates the desired values θ_{xc}, θ_{yc} based on the inputs shown.

The basic stability of the DMS gyros permits a narrow bandwidth control from the command signals to the gyro torquers. This attenuates or filters the effect of high frequency ship flexure on the controlled output. Long term flexure due to thermal effects or loading are corrected by the down-link tracking error.

8.3 Train Axis Control of θ_T

The purpose of the train axis is to nominally position the Y axis in the vertical plane containing the satellite. This is not a critical requirement and so it is expected that an inexpensive stepper motor will be used in train. For the purpose of modelling, the following damped second order system has been assumed in train:

$$\ddot{\beta} + 2\omega_n \zeta \dot{\beta} + \omega_n^2 \beta = \omega_n^2 \alpha_A \tag{8-11}$$

where

$\beta = \psi_A + \theta_T$

ω_n = natural frequency of train servo

ζ = damping factor

α_A = azimuth position of satellite

Making the substitution for β as indicated above leads to the defining equation for θ_T.

$$\ddot{\theta}_T + 2\omega_n\zeta\,\dot{\theta}_T + \omega_n^2\theta_T = F \tag{8-12}$$

where

$$F = -\ddot{\psi}_A - 2\omega_n\zeta\dot{\psi}_A + \omega_n^2(\alpha_A - \psi_A)$$

9.0 TORQUE EQUATIONS

In this section the torque contributors that produce the system response are defined for computational purposes:

9.1 Gyros 'A' and 'B' Torques

Spring

Friction

Unbalance

Bias

Control

Bearing Reaction

Total

Platform Torques

Spring

Friction

Unbalance

Bias

Gyro Reaction

Bearing Reaction

Control

Total

9.2 Gyro 'A' Torques

9.2.1 Spring

An elastic spring torque due to rotation of the 'A' gyro about its pivot axis, $\hat{\underline{a}}_1$ is modelled.

$$\underline{T}_{as} = -K_{sG}\gamma_a\hat{\underline{a}}_1 \tag{9-1}$$

where

K_{sG} = gyro spring gradient

9.2.2 Friction

Friction about the pivot axis is modelled as a constant magnitude torque with a polarity that opposes the relative angular velocity of the 'A' gyro with respect to the platform.

$$\underline{T}_{aF} = -K_{FG}[\dot{\gamma}_a/|\dot{\gamma}_a|]\,\hat{\underline{a}}_1 \tag{9-2}$$

where

K_{FG} = gyro friction torque amplitude

9.2.3 Unbalance

Torque about the pivot axis due to unbalance has been modelled as a displacement along the rotor spin axis, $\hat{\underline{a}}_2$, coupled with transverse acceleration components. Unbalance torque due to radial displacement along $\hat{\underline{a}}_3$ has been assumed to be insignificant in recognition of the fact that radial stability of balance is much better than axial stability. See Appendix D (Eq. (D-14)) for development of this expression.

$$\underline{T}_{au} = -m\ell_g[-A_2\sin\gamma_a + A_3\cos\gamma_a]\,\hat{\underline{a}}_1 \tag{9-3}$$

where m = gyro mass

ℓ_g = displacement of gyro mass center from center of support along $\hat{\underline{a}}_2$

A_2, A_3 = acceleration components along $\hat{\underline{p}}_2$ and $\hat{\underline{p}}_3$ respectively

9.2.4 Bias

Bias torque has been modelled about the pivot axis.

$$\underline{T}_{aB} = T_{aB}\,\hat{\underline{a}}_1 \tag{9-4}$$

where

T_{aB} = bias torque value

9.2.5 Control

Referring to Figure 3, the control torque on the 'A' gyro is exerted about the pivot axis $\hat{\underline{a}}_1$ to correct for Y axis error as indicated by the difference between the command value, θ_{yc}, and the actual value, θ_y. Recall from Figure 5 that the command angle computation is degraded by ship flexure between the master attitude reference and the deck at the antenna. A conservative worst-case situation is modelled with the full ship flexure amplitude introduced into the control equation.

$$\underline{T}_{ca} = -K_a(\theta_{yc} - \theta_y + \theta_F)\ \hat{\underline{a}}_1 \tag{9-5}$$

where

K_a = control gain (in-lb/rad)

θ_F = ship flexure amplitude (rad)

9.2.6 Bearing Reaction

Radial forces at the pivot bearings introduce torque about $\hat{\underline{a}}_2$ and $\hat{\underline{a}}_3$. The component about $\hat{\underline{a}}_2$ has no significant effect since $\hat{\underline{a}}_2$ is along the spin axis of the rotor. Accordingly, the torque due to bearing reaction is defined as:

$$\underline{T}_{Ra} = T_{a3}\ \hat{\underline{a}}_3 \tag{9-6}$$

where

T_{a3} = magnitude of bearing reaction torque (in-lb)

9.2.7 <u>Total Torque on 'A' Gyro</u>

The total torque on the 'A' gyro is

$$\underline{T}_a = T_{a1}\,\hat{\underline{a}}_1 + T_{a2}\,\hat{\underline{a}}_2 + T_{a3}\,\hat{\underline{a}}_3 \tag{9-7}$$

where

$$T_{a1}\,\hat{\underline{a}}_1 = \underline{T}_{as} + \underline{T}_{aF} + \underline{T}_{au} + \underline{T}_{aB} + \underline{T}_{cA}$$

$$T_{a2} = 0$$

9.3 <u>Gyro 'B' Torques</u>

The torques acting on the 'B' gyro are similar to those on the 'A' except for different axes and a geometric consideration applied to the control torque.

9.3.1 <u>Spring</u>

$$\underline{T}_{bs} = -K_{sG}\gamma_b\,\hat{\underline{b}}_3 \tag{9-8}$$

9.3.2 <u>Friction</u>

$$\underline{T}_{bF} = -K_{FG}[\dot{\gamma}_b/|\dot{\gamma}_b|]\,\hat{\underline{b}}_3 \tag{9-9}$$

9.3.3 Unbalance

$$\underline{T}_{bu} = -m\ell_g[A_1\cos\gamma_b + A_2\sin\gamma_b] \tag{9-10}$$

See Eq. (D-19) in Appendix D.

9.3.4 Bias

$$\underline{T}_{bB} = T_{bB}\ \hat{\underline{b}}_3 \tag{9-11}$$

9.3.5 Control

$$\underline{T}_{cb} = -K_b(\theta_{xc} - \theta_x + \theta_F)\cos\theta_y\ \hat{\underline{b}}_3 \tag{9-12}$$

Note that the control torque is responsive to the antenna error signal about the X axis with ship flexure again introduced as a worst-case effect. Any displacement in Y serves to increase the X rate response to a given torque input. This is offset by introducing $\cos\theta_y$ into Eq. (9-12).

9.3.6 Bearing Reaction

Following similar arguments to that in 9.2.6, we write

$$\underline{T}_{Rb} = T_{b1}\ \hat{\underline{b}}_1 \tag{9-13}$$

9.3.7 Total Torque on 'B' Gyro

The total torque on the 'B' gyro is

$$\underline{T}_b = T_b \hat{\underline{b}}_1 + T_{b3} \hat{\underline{b}}_3 \tag{9-14}$$

where

$$T_{b3} \hat{\underline{b}}_3 = \underline{T}_{bs} + \underline{T}_{bF} + \underline{T}_{bu} + \underline{T}_{bB} + \underline{T}_{cb}$$

9.4 Platform Torques

9.4.1 Spring

Spring torque acting on the platform is a function of θ_x and θ_y displacement. Spring torque due to θ_T is cancelled out by the stepper motor.

$$\underline{T}_{ps} = -K_{sp}[\theta_x \hat{\underline{g}}_1 + \theta_y \hat{\underline{p}}_3] \tag{9-15}$$

where K_{sp} = spring gradient

9.4.2 Friction

Friction is modelled as a constant amplitude torque with a polarity in opposition to velocities $\dot{\theta}_x$ and $\dot{\theta}_y$.

$$\underline{T}_{pF} = -K_{Fp}[\{\dot{\theta}_x/|\dot{\theta}_x|\} \hat{\underline{g}}_1 + \{\dot{\theta}_y/|\dot{\theta}_y|\} \hat{\underline{p}}_3] \tag{9-16}$$

9.4.3 Unbalance

The platform unbalance torque is defined in Section D-10 of Appendix D with the result

$$\underline{T}_{pu} = T_{u1}\,\hat{\underline{p}}_1 + T_{u2}\,\hat{\underline{p}}_2 + T_{u3}\,\hat{\underline{p}}_3 \tag{9-17}$$

where

$T_{u1} = M(A_2\ell_3 - A_3\ell_2)$

$T_{u2} = M(A_3\ell_1 - A_1\ell_3)$

$T_{u3} = M(A_1\ell_2 - A_2\ell_1)$

M = platform mass

ℓ_i = displacement of mass center along $\hat{\underline{p}}_i$ axis

9.4.4 Bias

$$\underline{T}_{pB} = T_{Bx}\,\hat{\underline{q}}_1 + T_{By}\,\hat{\underline{p}}_3 \tag{9-18}$$

where T_{Bx}, T_{By} = bias torque about X and Y axes

9.4.5 Gyro Bearing Reaction

The total torques acting on the 'A' and 'B' gyros are defined as $\underline{T}_a$ and $\underline{T}_b$ in Eqs. (9-7) and (9-14) respectively. These are primarily reaction torques between the gyros and the platform except for the gyro unbalance torques which are small.

Accordingly, the gyro reaction torque acting on the platform is modelled by:

$$\underline{T}_{Rp} = -\underline{T}_a - \underline{T}_b \tag{9-19}$$

9.4.6 Control

Torquers on the X and Y axes are used to cage the gyro spin axes by nulling γ_a and γ_b using a proportional plus integral control loop.

$$\underline{T}_{pc} = -K_{pc}(\gamma_b\ \hat{\underline{g}}_1 + \gamma_a\ \hat{\underline{p}}_3)\ -K_{pI}(\int\gamma_b dt\ \hat{\underline{g}}_1 + \int\gamma_a dt\ \hat{\underline{p}}_3) \tag{9-20}$$

where K_{pc} = direct gain

K_{pI} = integrator gain

Note that:

$$\int\gamma_a dt = X_{10} \quad \text{and} \quad \int\gamma_b dt = X_{11}$$

9.4.7 Bearing Reaction

Radial forces at the X and Y axis bearings generate a torque about axis $\hat{\underline{g}}_2$. We define this term as

$$\underline{T}_{pG} = T_{pG}\ \hat{\underline{g}}_2 \tag{9-21}$$

9.4.8 Total Platform Torque

Sum the previously defined torque contributions to determine the total platform torque:

$$\underline{T}_p = \underline{T}_{ps} + \underline{T}_{pF} + \underline{T}_{pu} + \underline{T}_{pB} + \underline{T}_{Rp} + \underline{T}_{pc} + \underline{T}_{pG} \tag{9-22}$$

or in terms of $\hat{\underline{p}}$ frame components,

$$\underline{T}_p = T_{p1}\,\hat{\underline{p}}_1 + T_{p2}\,\hat{\underline{p}}_2 + T_{p3}\,\hat{\underline{p}}_3 \tag{9-23}$$

10.0 SYSTEM EQUATIONS

The system equations are generated in the Appendices and are summarized below followed by a description of their development process.

$$\dot{X}_1 = \dot{\theta}_x = (\omega_{p1} - D_1\cos\theta_y - Y_2\sin\theta_y)/\cos\theta_y \tag{10-1}$$

where $D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$

$$Y_2 = D_2\cos\theta_x + (\dot{\theta}_T + \omega_{f3})\sin\theta_x$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T$$

$$\dot{X}_2 = \dot{\theta}_y = \omega_{p3} + D_2\sin\theta_x - (\dot{\theta}_T + \omega_{f3})\cos\theta_x \tag{10-2}$$

$$\dot{x}_3 = \dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \qquad (10\text{-}3)$$

where $J_{a1} = \omega_{a3}\Omega_a(I_{g2}/I_{g1}) - \omega_{a3}\omega_{a2}$

$$\dot{x}_4 = \dot{\omega}_{b3} = J_{b3} + T_{b3}/I_{g1} \qquad (10\text{-}4)$$

where $J_{b3} = \omega_{b1}\Omega_b(I_{g2}/I_{g1}) + \omega_{b1}\omega_{b2}$

$$\dot{x}_5 = \dot{\omega}_{p3} = \frac{d_{20}\dot{\omega}_{p2} + d_{22}}{1-d_{21}} \qquad (10\text{-}5)$$

where d_i terms are defined in Appendix F

$$\dot{x}_6 = \dot{\omega}_{p1} = d_{11}/d_{12} \qquad (10\text{-}6)$$

$$\dot{x}_7 = \dot{\omega}_{p2} = -\frac{d_{11}}{d_{12}}\tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad (10\text{-}7)$$

$$\dot{x}_8 = \dot{\gamma}_A = \omega_{a1} - \omega_{p3} \qquad (10\text{-}8)$$

$$\dot{x}_9 = \dot{\gamma}_b = \omega_{b3} - \omega_{p3} \qquad (10\text{-}9)$$

$$\dot{x}_{10} = \gamma_a \tag{10-10}$$

$$\dot{x}_{11} = \gamma_b \tag{10-11}$$

$$\dot{x}_{12} = \dot{\theta}_T \tag{10-12}$$

$$\dot{x}_{13} = \ddot{\theta}_T = -2\omega_n\zeta(\dot{\theta}_T + \dot{\psi}_A) + \omega_n^2(\alpha_A - \theta_T - \psi_A) \tag{10-13}$$

The following table shows the source of development of the system equations:

Equation(s)	Source
10-1, 10-2	Appendix E
10-3, 10-4	Sect. 5.0 & Appendix A
10-5, 10-6, 10-7	Appendix F
10-8, 10-9	Section 3.0
10-10, 10-11	Equation (9-20)
10-12, 10-13	Equation (8-12)

11.0 COMPUTER SIMULATION

The computer program based on the preceding analysis is described and listed in Appendix H.

11.1 Parametric Values

Parameter values are shown below for two cases. The variable designation in front of each corresponds to that used in the computer simulation.

Case 1: Ship rolling and flexing

Case 2: Ship rolling, pitching and turning at high speed. No flexing included.

			Case 1	Case 2
D(1)	=	Desired Azim (Deg)	0	0
D(2)	=	Desired Elev (Deg)	57.3	57.3
D(3)	=	Roll Amplit (Deg)	20.0	20.0
D(4)	=	Pitch Amplit (Deg)	0	10.0
D(5)	=	Flex Amplit (Deg)	.5	0
D(6)	=	Roll Period (Sec)	10.0	10.0
D(7)	=	Pitch Period (Sec)	-	15.0
D(8)	=	Flex Period (Sec)	1.0	-
D(9)	=	Roll Height (In)	1200.0	1200.0
D(10)	=	Velocity (Knots)	0	20.0
D(11)	=	Turn Rate (Deg/Sec)	0	3.0
D(12)	=	Heel Angle (Deg)	0	0
D(13)	=	Cant Angle (Deg)	0	0

D(14)	=	WN Train Servo (Rad/Sec)	3.	3.
D(15)	=	Damping Ratio Train Servo	.7	.7

The values below were used in both cases:

E(1)	=	Platform Mass (Lb-Sec2/In)	0.25
E(2)	=	Gyro Mass (Lb-Sec^2/In)	0.06
E(3)	=	Gyro Mass Offset (In)	0.003
E(4)	=	Plat 1 Mass Offset (In)	0.003
E(5)	=	Plat 2 Mass Offset (In)	0.003
E(6)	=	Plat 3 Mass Offset (In)	0.003
E(7)	=	Gyro Friction (In-Lb)	0.04
E(8)	=	Gyro Spring (In-Lb/Rad)	0.20
E(9)	=	Gyro A Control Gain (In-Lb/Rad)	108.0
E(10)	=	Gyro B Control Gain (In-Lb/Rad)	108.0
E(11)	=	Platform Friction (In-Lb)	0.5
E(12)	=	Platform Spring (In-Lb/Rad)	1.5
E(13)	=	Gyro Inertia 1 and 3 (Lb-In-Sec2)	0.5
E(14)	=	Gyro Inertia 2	1.0
E(15)	=	Platform Inertia 1	40.0
E(16)	=	Platform Inertia 2	30.0
E(17)	=	Platform Inertia 3	40.0
E(18)	=	Gyro Spin Speed (Rad/Sec)	361.0

E(19)	=	Platf Contr Gain (In-Lb/Rad)	50.0
E(20)	=	Platf Torq Integr Gain (In-Lb/Rad-Sec)	5.0
E(21)	=	Gyro A Torque Bias (In-Lb)	0
E(22)	=	Gyro B Torque Bias (In-Lb)	0
E(23)	=	Plat Elev Bias Torque (In-Lb)	0
E(24)	=	Plat Cross Elev Bias (In-Lb)	0
E(25)	=	Limit on Gyro Torquer (In-Lb)	5.0
E(26)	=	Gyro Speed Mismatch	0

11.2 Initial State Values

In both cases the system is initialized with the antenna pointing at the satellite ($X_1(0) = \theta_x(0) = 1.$ radian; $D(2) = \alpha_A = 57.3^o$) and with a body rate, ω_{p2}, necessary to match the $\hat{p}_2$ component of the ship's roll rate.

Results

Case 1

Figure 6 shows a plot of the X and Y axis error over a period of 75 seconds. The error is the difference between the computed command angle and the actual angle determined by the integration process. Figure 7 shows the motion of the 'A' and 'B' gyros about their pivot axes during the same period. There are two aspects of the response to be examined.

The ±0.5 degree ship flexure input is attenuated to approximately ±0.02 degree of X-Y error by the narrow bandwidth transfer function of the overall system.

The roll input with a 10-second period causes oscillations at that period in the X-Y error and the 'A' gyro motion. The 'B' gyro shows a double frequency response with a 5-second period.

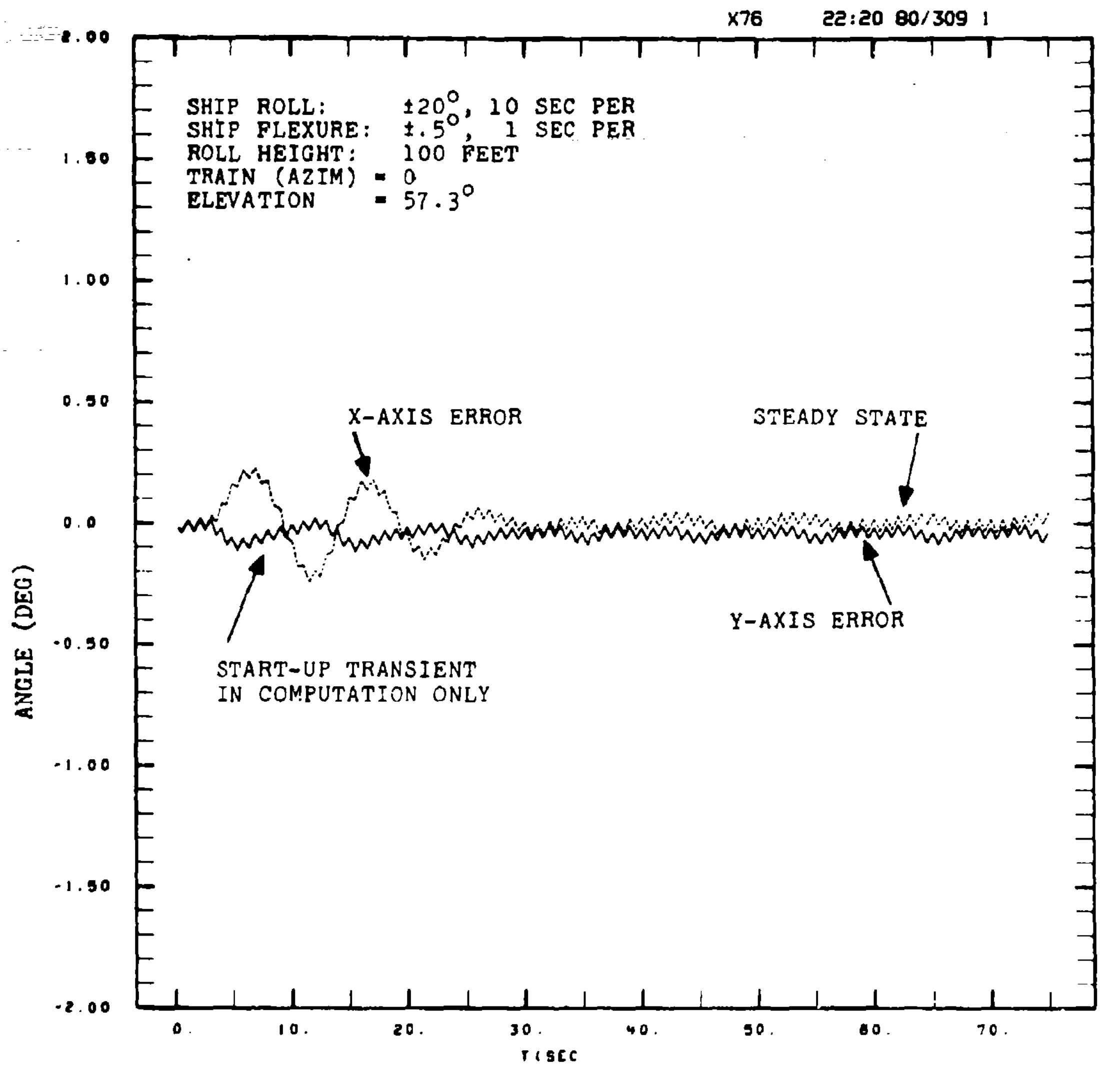


Fig. 6. X-Y DMS error (rolling).

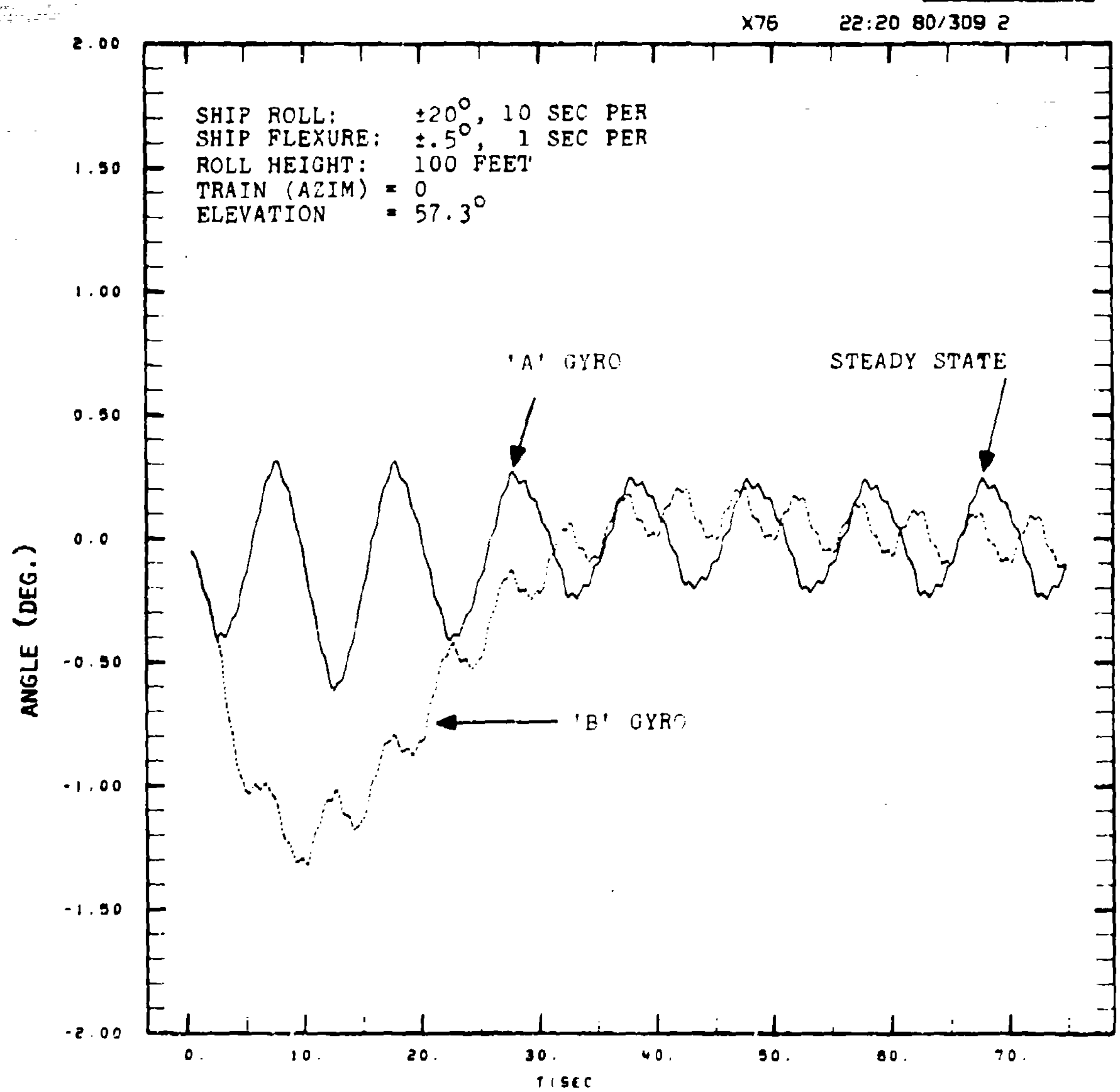


Fig. 7. X-Y DMS gyro motion (rolling).

After an initial transient a steady state response is achieved in approximately 30 seconds with an X-Y error averaging less than 0.05°.

Case 2

Figures 8 and 9 show the X-Y errors and the gyro motion over a 90-second period. This is a more confused dynamic environment than in Case 1 and, accordingly, we see a more confused response in Figures 8 and 9.

After an initial transient a general steady state band of response is achieved in approximately 60 seconds with an average X-Y error of less than 0.1 degree.

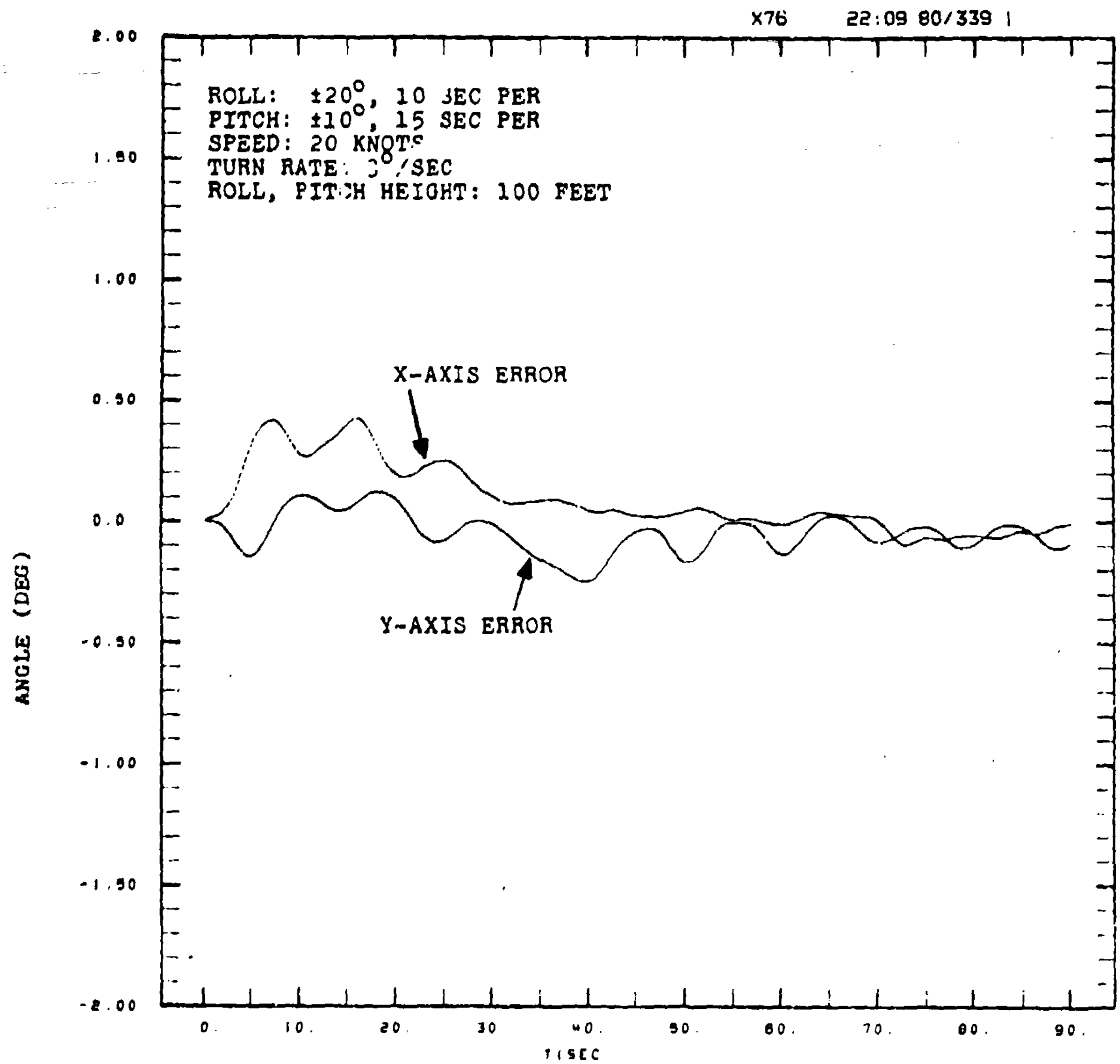


Fig. 8. X-Y DMS error (roll, pitch, turn).

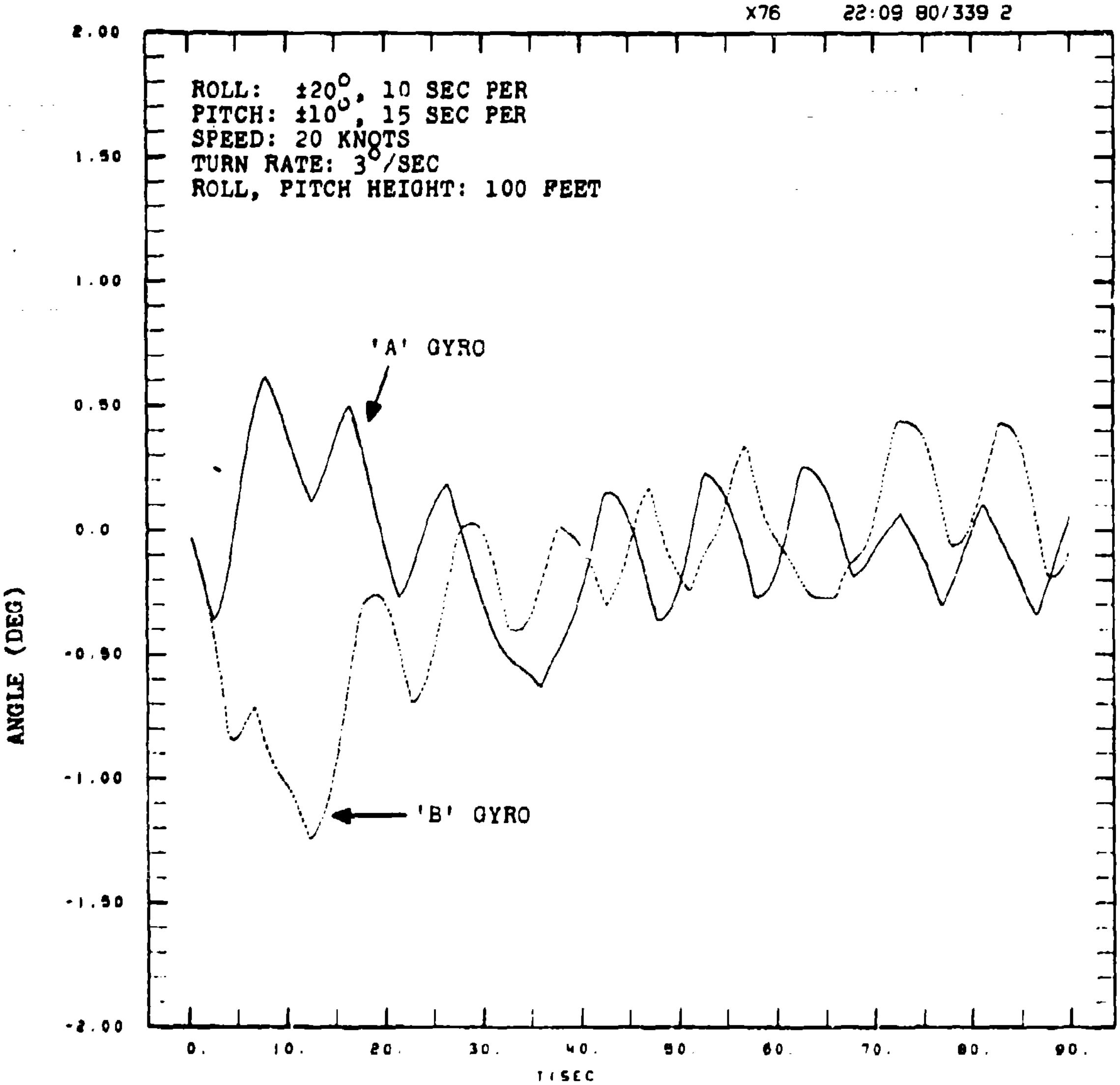


Fig. 9. X-Y DMS gyro motion (roll, pitch, turn).

APPENDIX A

EULER EQUATIONS FOR GYROS AND PLATFORM

1. Gyro 'A'

The Euler equations* for gyro 'A' are written in tensor form as

$$\hat{\underline{a}} \; \frac{d}{dt} (\underline{\Omega}_a) = -\underline{I}_g^{-1} [\tilde{\omega}_a] \, \underline{I}_g \underline{\Omega}_a + \underline{I}_g^{-1} \, \underline{T}_a \qquad \text{(A-1)}$$

where $$\underline{\Omega}_a = \omega_{a1} \, \hat{\underline{a}}_1 + \Omega_a \, \hat{\underline{a}}_2 + \omega_{a3} \, \hat{\underline{a}}_3 \qquad \text{(3-6)}$$

Ω_a = constant rotor spin speed

$$\underline{I}_g = \begin{bmatrix} I_{g1} & 0 & 0 \\ 0 & I_{g2} & 0 \\ 0 & 0 & I_{g1} \end{bmatrix}$$

$$[\tilde{\omega}_a] = \begin{bmatrix} 0 & -\omega_{a3} & \omega_{a2} \\ \omega_{a3} & 0 & -\omega_{a1} \\ -\omega_{a2} & \omega_{a1} & 0 \end{bmatrix}$$

$$\underline{T}_a = T_{a1} \, \hat{\underline{a}}_1 + T_{a2} \, \hat{\underline{a}}_2 + T_{a3} \, \hat{\underline{a}}_3 \qquad \text{(9-7)}$$

Differentiating Eq. (3-6) gives

* See Ref. (7)

$$\frac{{}^{\hat{\underline{a}}}d}{dt}(\underline{\Omega}_a) = \dot{\omega}_{a1}\,\hat{\underline{a}}_1 + \dot{\omega}_{a3}\,\hat{\underline{a}}_3 \tag{A-2}$$

Combine (A-1), (A-2) and the terms defined above to get

$$\begin{Bmatrix} \dot{\omega}_{a1} \\ 0 \\ \dot{\omega}_{a3} \end{Bmatrix} = - \begin{bmatrix} 1/I_{g1} & & 0 \\ 0 & 1/I_{g2} & 0 \\ 0 & 0 & 1/I_{g1} \end{bmatrix} \begin{bmatrix} 0 & -\omega_{a3} & \omega_{a2} \\ \omega_{a3} & 0 & -\omega_{a1} \\ -\omega_{a2} & \omega_{a1} & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{g1} & 0 & 0 \\ 0 & I_{g2} & 0 \\ 0 & 0 & I_{g1} \end{bmatrix} \begin{Bmatrix} \omega_{a1} \\ \Omega_a \\ \omega_{a3} \end{Bmatrix} +$$

$$\begin{bmatrix} 1/I_{g1} & 0 & 0 \\ 0 & 1/I_{g2} & 0 \\ 0 & 0 & 1/I_{g1} \end{bmatrix} \begin{Bmatrix} T_{a1} \\ T_{a2} \\ T_{a3} \end{Bmatrix} \tag{A-3}$$

After some algebra the second of the (A-3) equations vanishes leaving:

$$\dot{\omega}_{a1} = 1/I_{g1}[\omega_{a3}\Omega_a I_{g2} - \omega_{a2}\omega_{a3}I_{g1} + T_{a1}] \tag{A-4}$$

$$\dot{\omega}_{a3} = 1/I_{g1}[\omega_{a2}\omega_{a1}I_{g1} - \omega_{a1}\Omega_a I_{g2} + T_{a3}] \tag{A-5}$$

which can then be put into the form of Eqs. (5-1) and (5-2)

$$\dot{\omega}_{a1} = J_{a1} + T_{a1}/I_{g1} \tag{5-1}$$

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

where

$$J_{a1} = \omega_{a3}(\Omega_a I_{g2} - \omega_{a2} I_{g1})/I_{g1}$$

$$J_{a3} = \omega_{a1}(-\Omega_a I_{g2} + \omega_{a2} I_{g1})/I_{g1}$$

2. Gyro 'B'

The development of Eqs. (5-3) and (5-4) for the 'B' gyro is similar starting with

$$\hat{\underline{b}}\ \frac{d}{dt}\ (\underline{\Omega}_b) = -\underline{\underline{I}}_g^{-1}[\tilde{\omega}_b]\underline{\underline{I}}_g\underline{\Omega}_b + \underline{\underline{I}}_g^{-1}\ \underline{T}_b \tag{A-6}$$

3. Platform

The Euler equations for the platform are

$$\dot{\underline{\omega}}_p = -\underline{\underline{I}}_p^{-1}[\tilde{\omega}_p]\underline{\underline{I}}_p\underline{\omega}_p + \underline{\underline{I}}_g^{-1}\ \underline{T}_p \tag{A-7}$$

where $$\underline{\omega}_p = \omega_{p1}\,\hat{\underline{p}}_1 + \omega_{p2}\,\hat{\underline{p}}_2 + \omega_{p3}\,\hat{\underline{p}}_3 \tag{3-3}$$

$$\underline{I}_p = \begin{bmatrix} I_{p1} & 0 & 0 \\ 0 & I_{p2} & 0 \\ 0 & 0 & I_{p3} \end{bmatrix} \text{ in } \hat{\underline{p}} \text{ frame}$$

$$\tilde{\omega}_p = \begin{bmatrix} 0 & -\omega_{p3} & \omega_{p2} \\ \omega_{p3} & 0 & -\omega_{p1} \\ -\omega_{p2} & \omega_{p1} & 0 \end{bmatrix}$$

$$\underline{T}_p = T_{p1}\,\hat{\underline{p}}_1 + T_{p2}\,\hat{\underline{p}}_2 + T_{p3}\,\hat{\underline{p}}_3 \tag{9-23}$$

There are no gyroscopic terms in (A-7) since the platform motions are relatively slow, therefore expansion of (A-7) leads to

$$\dot{\underline{\omega}}_p = \dot{\omega}_{p1}\,\hat{\underline{p}}_1 + \dot{\omega}_{p2}\,\hat{\underline{p}}_2 + \dot{\omega}_{p3}\,\hat{\underline{p}}_3 \tag{A-8}$$

where $$\left.\begin{aligned} \dot{\omega}_{p1} &= T_{p1}/I_{p1} \\ \dot{\omega}_{p2} &= T_{p2}/I_{p2} \\ \dot{\omega}_{p3} &= T_{p3}/I_{p3} \end{aligned}\right\} \tag{A-9}$$

Equations (A-9) are Eqs. (6-1), (6-2), (6-3).

APPENDIX B

CONSTRAINTS

The angular velocities of the $\{\underline{\hat{a}}\}$ and $\{\underline{\hat{b}}\}$ frames are constrained to that of the $\{\underline{\hat{p}}\}$ frame by the gimbal arrangement shown in Figure 3.

Consider the $\{\underline{\hat{a}}\}$ frame which pivots about $\underline{\hat{a}}_1$. The component ω_{a1} is not constrained; however, ω_{a2} and ω_{a3} are by the equality of the vector sums:

$$\omega_{a2}\, \underline{\hat{a}}_2 + \omega_{a3}\, \underline{\hat{a}}_3 = \omega_{p2}\, \underline{\hat{p}}_2 + \omega_{p3}\, \underline{\hat{p}}_3 \tag{B-1}$$

Using the relations defined in Section 2.0 we can write

$$\{\underline{\hat{a}}\} = [C_1(\gamma_a)]\ \{\underline{\hat{p}}\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & \sin\gamma_a \\ 0 & -\sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{Bmatrix} 0 \\ \omega_{p2} \\ \omega_{p3} \end{Bmatrix}_{\underline{\hat{p}}} = \begin{Bmatrix} 0 \\ \omega_{p2}\cos\gamma_a + \omega_{p3}\sin\gamma_a \\ -\omega_{p2}\sin\gamma_a + \omega_{p2}\cos\gamma_a \end{Bmatrix}_{\underline{\hat{a}}} \tag{B-2}$$

Combining (B-1) and (B-2) we get the equations

$$\omega_{a2} = \omega_{p2}\cos\gamma_a + \omega_{p3}\sin\gamma_a \tag{B-3}$$

$$\omega_{a3} = -\omega_{p2}\sin\gamma_a + \omega_{p3}\cos\gamma_a \tag{B-4}$$

Equations (B-3) and (B-4) are Eqs. (7-1) and (7-2). Similarly for frame $\{\hat{\underline{b}}\}$ which has a single degree of freedom with respect to $\{\hat{\underline{p}}\}$ about axis $\underline{b}_3$ we get

$$\omega_{b1}\,\hat{\underline{b}}_1 + \omega_{b2}\,\hat{\underline{b}}_2 = \omega_{p1}\,\hat{\underline{p}}_1 + \omega_{p2}\,\hat{\underline{p}}_2 \tag{B-5}$$

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)]\ \{\hat{\underline{p}}\}$$

$$\begin{bmatrix} \cos\gamma_b & \sin\gamma_b & 0 \\ -\sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \omega_{p1} \\ \omega_{p2} \\ 0 \end{Bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \\ -\omega_{p1}\sin\gamma_b + \omega_{p2}\cos\gamma_b \\ 0 \end{Bmatrix}_{\hat{\underline{b}}} \tag{B-6}$$

$$\omega_{b1} = \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \tag{B-7}$$

$$\omega_{b2} = -\omega_{p1}\sin\gamma_b + \omega_{p2}\cos\gamma_b \tag{B-8}$$

Equations (B-7) and (B-8) are Eqs. (7-3) and (7-4).

APPENDIX C

TRANSFORMATIONS FOR θ_{xc} AND θ_{yc}

In this appendix we expand Eq. (8-8) and solve the results for θ_{xc} and θ_{yc}.

$$B_1^T\ B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \tag{8-8}$$

The unknowns θ_{xc} and θ_{yc} are contained in B_2 and so we premultiply by B_1 to get

$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = B_1 B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \tag{C-1}$$

Expand the left side

$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = [C_1(\theta_{xc})]^T [C_3(\theta_{yc})]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$B_2^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{xc} & -\sin\theta_{xc} \\ 0 & \sin\theta_{xc} & \cos\theta_{xc} \end{bmatrix} \begin{bmatrix} \cos\theta_{yc} & -\sin\theta_{yc} & 0 \\ \sin\theta_{yc} & \cos\theta_{yc} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$= \begin{bmatrix} -\sin\theta_{yc} \\ \cos\theta_{xc} \quad \cos\theta_{yc} \\ \sin\theta_{xc} \quad \cos\theta_{yc} \end{bmatrix} \tag{C-2}$$

Expand the right side of (C-1)

$$B_1 B_3^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = [C_3(\theta_T)][C_2(\psi_{RH})][C_1(\psi_p)][C_3(\psi_A)][C_1(\alpha_A)]^T[C_1(\alpha_E)]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \end{Bmatrix} \tag{C-3}$$

The t_i terms will be determined by numerical computation of (C-3). Then equate the first and third components of (C-2) and (C-3).

$$t_1 = -\sin\theta_{yc}$$

$$t_3 = \sin\theta_{xc}\cos\theta_{yc}$$

Solve for θ_{yc} and θ_{xc}

$$\left.\begin{aligned} \theta_{yc} &= \sin^{-1}(-t_i) \\ \theta_{xc} &= \sin^{-1}(t_3/\cos\theta_{yc}) \end{aligned}\right\} \qquad \text{(C-4)}$$

APPENDIX D

ACCELERATION VECTOR & TORQUE DUE TO UNBALANCE

We consider accelerations due to the following sources:

a) Roll (horizontal component along $\hat{\underline{n}}_1'$)

b) Pitch (horizontal component along $\hat{\underline{n}}_2'$)

c) Gravity (vertical along $\hat{\underline{n}}_3'$)

d) Turn (horizontal component along $\hat{\underline{n}}_1'$)

1. Roll Acceleration (horizontal component along $\hat{\underline{n}}_1'$)

$$\underline{A}_R = H \ddot{\psi}_R \hat{\underline{n}}_1' \tag{D-1}$$

where $\ddot{\psi}_R = -\omega_R^2 \psi_{RO} \sin\omega_R t$

H = roll height

ω_R = roll frequency

ψ_{RO} = roll amplitude

2. Pitch Acceleration (horizontal component along $\underline{n}_2'$)

$$\underline{A}_p = H \ddot{\psi}_p \hat{\underline{n}}_2' \tag{D-2}$$

where $\ddot{\psi}_p = -\omega_p^2 \psi_{p0} \sin\omega_p t$

ω_p = pitch frequency

ψ_{p0} = pitch amplitude

3. Gravity (along $\hat{\underline{n}}_3'$)

$$\underline{A}_g = 386 \text{ in/sec}^2 \cdot \hat{\underline{n}}_3' \tag{D-3}$$

Note on Polarity of $\underline{A}_g$:

Force due to gravity acts down at the center of mass. Its corresponding support force at the pivot axis is positive in the upward direction. Torque about the mass center results when the line of the positive support force does not pass through the mass center.

4. Turn (horizontal component along $\underline{n}_1'$)

$$\underline{A}_T = -V\, \omega_T\, \hat{\underline{n}}_1' \tag{D-4}$$

where

V = velocity

ω_T = turn rate

5. Total Acceleration in $\{\hat{\underline{n}}'\}$

We add components above to obtain

$$\underline{A} = A_1' \hat{\underline{n}}_1' + A_2' \hat{\underline{n}}_2' + A_3' \hat{\underline{n}}_3' \tag{D-5}$$

where
$$A_1' = H \ddot{\psi}_R - V \omega_T$$

$$A_2' = H \ddot{\psi}_P$$

$$A_3' = 386$$

6. Transform Acceleration Vector to $\{\hat{\underline{p}}\}$

We transform from $\{\hat{\underline{n}}'\}$ to $\{\hat{\underline{p}}\}$ using

$$\{\hat{\underline{p}}\} = [C_3(\theta_y)][C_1(\theta_x)][C_3(\theta_T)][C_2(\psi_R)][C_1(\psi_P)] \{\hat{\underline{n}}'\} \tag{D-6}$$

and define the resulting components in $\{\hat{\underline{p}}\}$ by

$$\underline{A} = A_1 \hat{\underline{p}}_1 + A_2 \hat{\underline{p}}_2 + A_3 \hat{\underline{p}}_3 \tag{D-7}$$

7. Unbalance Torque

Unbalance torque due to the combined effect of acceleration and displacement berween the centers of support and mass is defined by the cross product:

$$\underline{L} = \underline{F} \times \underline{R} \tag{D-8}$$

where $\underline{L}$ = unbalance torque vector

$\underline{F}$ = force vector at support point

$\underline{R}$ = position vector from support point to mass center

The force vector, $\underline{F}$, is due to the effect of gravity and acceleration acting on the mass.

$$\underline{F} = m\,\underline{A} \tag{D-9}$$

8. 'A' Gyro Unbalance Torque

Transform $\underline{A}$ in Eq. (D-7) from $\{\hat{\underline{p}}\}$ to $\{\hat{\underline{a}}\}$ coordinates using

$$\{\hat{\underline{a}}\} = [C_1(\gamma_a)]\ \{\hat{\underline{p}}\} \tag{D-10}$$

$$\{\underline{A}\}_{\hat{\underline{a}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & \sin\gamma_a \\ 0 & -\sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} A_1 \\ A_2\cos\gamma_a + A_3\sin\gamma_a \\ -A_2\sin\gamma_a + A_3\cos\gamma_a \end{Bmatrix}_{\hat{\underline{a}}} \tag{D-11}$$

Write the result of (D-11) as

$$\underline{A} = A_1\ \hat{\underline{a}}_1 + a_2\ \hat{\underline{a}}_2 + a_3\ \hat{\underline{a}}_3 \tag{D-12}$$

where $a_2 = A_2\cos\gamma_a + A_3\sin\gamma_a$

$a_3 = -A_2\sin\gamma_a + A_3\cos\gamma_a$

Now consider

$$\underline{R} = \ell_g \, \hat{\underline{a}}_2$$

$$\underline{F} = m \, \underline{A} = m(A_1 \, \hat{\underline{a}}_1 + a_2 \, \hat{\underline{a}}_2 + a_3 \, \hat{\underline{a}}_3)$$

where ℓ_g = displacement of mass center from support point

m = gyro mass

then $$\underline{T}_{au} = \underline{F} \times \underline{R} = m\ell_g(A_1 \, \hat{\underline{a}}_3 - a_3 \, \hat{\underline{a}}_1) \tag{D-13}$$

The $\hat{\underline{a}}_3$ component in (D-13) is an insignificant bearing reaction torque which can be neglected. Therefore using the definition of a_3 in (D-12) we can write

$$\underline{T}_{au} = -m\ell_g(-A_2\sin\gamma_a + A_3\cos\gamma_a) \tag{D-14}$$

9. 'B' Gyro Unbalance Torque

Transform $\underline{A}$ in Eq. (D-7) from $\{\hat{\underline{p}}\}$ to $\{\hat{\underline{b}}\}$ using

$$\{\hat{\underline{b}}\} = [C_3(\gamma_b)] \; \{\hat{\underline{p}}\} \tag{D-15}$$

$$\{\underline{A}\}_{\hat{\underline{b}}} = \begin{bmatrix} \cos\gamma_b & \sin\gamma_b & 0 \\ -\sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}_{\hat{\underline{p}}} = \begin{Bmatrix} A_a\cos\gamma_b + A_2\sin\gamma_b \\ -A_1\sin\gamma_b - A_2\cos\gamma_b \\ A_3 \end{Bmatrix}_{\hat{\underline{b}}} \qquad \text{(D-16)}$$

Write the result of (D-16) in $\{\hat{\underline{b}}\}$ coordinates as

$$\underline{A} = b_1\,\hat{\underline{b}}_1 + b_2\,\hat{\underline{b}}_2 + A_3\,\hat{\underline{b}}_3 \qquad \text{(D-17)}$$

Now consider

$$\underline{R} = \ell_g\,\hat{\underline{b}}_2$$

$$\underline{F} = m\,\underline{A} = m(b_1\,\hat{\underline{b}}_1 + b_2\,\hat{\underline{b}}_2 + A_3\,\hat{\underline{b}}_3)$$

Then $$\underline{T}_{bu} = \underline{F} \times \underline{R} = m\ell_g(-b_1\,\hat{\underline{b}}_3 + A_3\,\hat{\underline{b}}_1) \qquad \text{(D-18)}$$

We neglect the $\hat{\underline{b}}_1$ component following the same logic that preceded Eq. (D-14). Then use the definition of b_1 in Eq. (D-17) to obtain

$$\underline{T}_{bu} = -m\ell_g(A_1\cos\gamma_b + A_2\sin\gamma_b) \qquad \text{(D-19)}$$

10. Platform Unbalance Torque

Here we model unbalance about all three axes by defining

$$\underline{R} = \ell_1 \hat{\underline{P}}_1 + \ell_2 \hat{\underline{P}}_2 + \ell_3 \hat{\underline{P}}_3$$

now

$$\underline{T}_{pu} = \underline{F} \times \underline{R} = M(A_1\hat{\underline{P}}_1 + A_2\hat{\underline{P}}_2 + A_3\hat{\underline{P}}_3) \times (\ell_1\hat{\underline{P}}_1 + \ell_2\hat{\underline{P}}_2 + \ell_3\hat{\underline{P}}_3)$$

$$\underline{T}_{pu} = T_{u1} \hat{\underline{P}}_1 + T_{u2} \hat{\underline{P}}_2 + T_{u3} \hat{\underline{P}}_3 \qquad \text{(D-20)}$$

where

$$T_{u1} = M(A_2\ell_3 - A_3\ell_2)$$

$$T_{u2} = M(A_3\ell_1 - A_1\ell_3)$$

$$T_{u3} = M(A_1\ell_2 - A_2\ell_1)$$

M = platform mass

APPENDIX E

GENERATION OF SYSTEM EQUATIONS (10-1) AND (10-2) FOR $\dot{\theta}_x$ AND $\dot{\theta}_y$

1. Approach

System equations (10-1) and (10-2) are integrated to provide θ_x and θ_y position data. In the real system θ_x and θ_y would be the output of the synchros on those axes.

The solution for $\dot{\theta}_x$ and $\dot{\theta}_y$ is obtained by using two of the three scalar equations in the vector equation (3-4)

$$\underline{\omega}_p = \underline{\omega}_f + \dot{\theta}_T \, \hat{\underline{f}}'_3 + \dot{\theta}_x \, \hat{\underline{g}}_1 + \dot{\theta}_y \, \hat{\underline{p}}_3 \tag{3-4}$$

The process is one of defining $\underline{\omega}_f$, putting all components of (3-4) into a common frame $\{\hat{\underline{p}}\}$ and solving for $\dot{\theta}_x$ and $\dot{\theta}_y$ as functions of ω_{pi}, ω_{fi}, $\dot{\theta}_T$, θ_x and θ_y.

2. Generate ω_{fi} Components

First generate the ω_{fi} components in Eq. (3-1)

$$\underline{\omega}_f = \omega_{f1} \, \hat{\underline{f}}_1 + \omega_{f2} \, \hat{\underline{f}}_2 + \omega_{f3} \, \hat{\underline{f}}_3 \tag{3-1}$$

by making use of Eq. (3-2)

$$\underline{\omega}_f = \dot{\psi}_A \, \hat{\underline{n}}'_3 + \dot{\psi}_p \, \hat{\underline{n}}''_1 + \dot{\psi}_R \, \hat{\underline{f}}_2 \tag{3-2}$$

Transform $\dot{\psi}_A\ \hat{\underline{n}}'_3$ into $\{\hat{\underline{n}}''\}$ using from Section 2.0

$$\{\hat{\underline{n}}''\} = [C_1(\psi_p)]\ \{\hat{\underline{n}}'\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_p & \sin\psi_p \\ 0 & -\sin\psi_p & \cos\psi_p \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{\psi}_A \end{Bmatrix}_{\hat{\underline{n}}'} = \begin{Bmatrix} 0 \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_A \cos\psi_p \end{Bmatrix}_{\hat{\underline{n}}''} \tag{E-1}$$

Next add $\dot{\psi}_p\ \hat{\underline{n}}''_1$ to (E-1) and transform to $\{\hat{\underline{f}}\}$ using from Section 2.0

$$\{\hat{\underline{f}}\} = [C_2(\psi_R)]\ \{\hat{\underline{n}}''\}$$

$$\begin{bmatrix} \cos\psi_R & 0 & -\sin\psi_R \\ 0 & 1 & 0 \\ \sin\psi_R & 0 & \cos\psi_R \end{bmatrix} \begin{Bmatrix} \dot{\psi}_p \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_A \cos\psi_p \end{Bmatrix}_{\hat{\underline{n}}''} = \begin{Bmatrix} \dot{\psi}_p \cos\psi_R - \dot{\psi}_A \sin\psi_R \cos\psi_p \\ \dot{\psi}_A \sin\psi_p \\ \dot{\psi}_p \sin\psi_R + \dot{\psi}_A \cos\psi_R \cos\psi_p \end{Bmatrix}_{\hat{\underline{f}}} \tag{E-2}$$

Finally add $\dot{\psi}_R\ \hat{\underline{f}}_2$ to the result of (E-2) to complete the determination of the $\{\hat{\underline{f}}\}$ components of $\underline{\omega}_f$

$$\omega_{f1} = \dot{\psi}_p \cos\psi_R - \dot{\psi}_A \sin\psi_R \cos\psi_p$$

$$\omega_{f2} = \dot{\psi}_R + \dot{\psi}_A \sin\psi_p \qquad \text{(E-3)}$$

$$\omega_{f3} = \dot{\psi}_p \sin\psi_R + \dot{\psi}_A \cos\psi_R \cos\psi_p$$

3. Transform (3-4) into $\hat{\underline{p}}$ Coordinates

Now proceed to transform all components of Eq. (3-4) into the $\{\hat{\underline{p}}\}$ frame.

Transform ω_{fi} in Eq. (E-3) into $\{\hat{\underline{f}}'\}$ using from Section 2.0

$$\{\hat{\underline{f}}'\} = [C_3(\theta_T)]\ \{\hat{\underline{f}}\}$$

$$\begin{bmatrix} \cos\theta_T & \sin\theta_T & 0 \\ -\sin\theta_T & \cos\theta_T & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \omega_{f1} \\ \omega_{f2} \\ \omega_{f3} \end{Bmatrix}_{\hat{\underline{f}}} = \begin{Bmatrix} \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T \\ -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T \\ \omega_{f3} \end{Bmatrix}_{\hat{\underline{f}}'} \qquad \text{(E-4)}$$

Next add $\dot{\theta}_T\ \hat{\underline{f}}'_3$ to the result of Eq. (E-4) and transform into $\{\hat{\underline{g}}\}$ using from Section 2.0

$$\{\hat{\underline{g}}\} = [C_1(\theta_x)]\ \{\hat{\underline{f}}'\}$$

We facilitate this by defining the first two terms in Eq. (E-4)

$$D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T \qquad \text{(E-5)}$$

Then

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ \omega_{f3}+\dot\theta_T \end{Bmatrix}_{\underline{\hat{f}}'} = \begin{Bmatrix} D_1 \\ D_2\cos\theta_x+(\dot\theta_T+\omega_{f3})\sin\theta_x \\ -D_2\sin\theta_x+(\dot\theta_T+\omega_{f3})\cos\theta_x \end{Bmatrix}_{\underline{\hat{g}}} \qquad \text{(E-6)}$$

Next add $\dot\theta_x\, \underline{\hat{g}}_1$ to the result of Eq. (E-6) and transform into $\{\underline{\hat{p}}\}$ using from Section 2.0

$$\{\underline{\hat{p}}\} = [C_3(\theta_y)]\ \{\underline{\hat{g}}\}$$

First redefine the last two terms in (E-6) as

$$Y_2 = D_2\cos\theta_x + (\dot\theta_T + \omega_{f3})\sin\theta_x$$

$$Y_3 = -D_2\sin\theta_x + (\dot\theta_T + \omega_{f3})\cos\theta_x \qquad \text{(E-7)}$$

Then

$$\begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} D_1+\dot{\theta}_x \\ y_2 \\ y_3 \end{Bmatrix}_{\hat{\underline{q}}} = \begin{Bmatrix} (D_1+\dot{\theta}_x)\cos\theta_y+y_2\sin\theta_y \\ -(D_1+\dot{\theta}_x)\sin\theta_y+y_2\cos\theta_y \\ y_3 \end{Bmatrix}_{\hat{\underline{p}}} \quad \text{(E-8)}$$

Finally add $\dot{\theta}_y \hat{\underline{p}}_3$ to the result of (E-8) to get

$$\left.\begin{aligned} \omega_{p1} &= (D_1+\dot{\theta}_x)\cos\theta_y + y_2\sin\theta_y \\ \omega_{p2} &= -(D_1+\dot{\theta}_x)\sin\theta_y + y_2\cos\theta_y \\ \omega_{p3} &= y_3+\dot{\theta}_y \end{aligned}\right\} \quad \text{(E-9)}$$

4. Solution for $\dot{\theta}_x$, $\dot{\theta}_y$

Now solve directly for $\dot{\theta}_x$ and $\dot{\theta}_y$ using the third and first of Eq. (E-9)

$$\left.\begin{aligned} \dot{\theta}_y &= \omega_{p3} - y_3 \\ \dot{\theta}_x &= (\omega_{p1} - D_1\cos\theta_y - y_2\sin\theta_y)/\cos\theta_y \end{aligned}\right\} \quad \text{(E-10)}$$

These are system equations (10-1) and (10-2). It is significant to note that these equations for $\dot{\theta}_y$ and $\dot{\theta}_x$ are functions of ω_{p1}, ω_{p3}, $\dot{\theta}_T$, ω_{f1}, ω_{f2}, ω_{f3}, θ_x and θ_T. Notice that ω_{p2} does not enter these calculations. We make use of this fact in Appendix G.

APPENDIX F

GENERATION OF SYSTEM EQUATIONS (10-5), (10-6) AND (10-7) FOR $\dot{\omega}_{p3}$, $\dot{\omega}_{p1}$, $\dot{\omega}_{p2}$

1. Overview of Process

Considerable algebra is involved in obtaining Eqs. (10-5) and (10-6) due to the need to eliminate unknown bearing reaction torques T_{a3}, T_{b1}, T_{pG}. The Euler equations (6-2) and (6-3) are the starting points with additional relations coming from the constraint equations in Section 7.0. A number of transformations are required to obtain scalar equations in a common frame.

Specific steps in the process are:

a) Differentiate constraint Eqs. (7-2) and (7-3) to obtain new equations for $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$.

b) Combine results of step a) with gyro Euler Eqs. (5-2) and (5-3) to define reaction torques T_{a3} and T_{b1}.

c) Perform transformations to define $\hat{\underline{p}}$ components of total torque $\underline{T}_p$ in Eq. (9-22).

d) Introduce T_{pi} components from step c) into platform Euler equations in Section 6.0, then use results of step b) to eliminate T_{a3} and T_{b1}.

e) Use $\dot{\omega}_{p2}$ as defined from velocity constraints in Appendix G to eliminate T_{pG} leaving equations for $\dot{\omega}_{p3}$ and $\dot{\omega}_{p1}$.

2. Differentiate Constraint Equations

Differentiate constraint Eqs. (7-2) and (7-3) to obtain new equations for $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$.

$$\omega_{a3} = -\omega_{p2}\sin\gamma_a + \omega_{p3}\cos\gamma_a \tag{7-2}$$

$$\omega_{b1} = \omega_{p1}\cos\gamma_b + \omega_{p2}\sin\gamma_b \tag{7-3}$$

Then

$$\dot{\omega}_{a3} = -\dot{\omega}_{p2}\sin\gamma_a + \dot{\omega}_{p3}\cos\gamma_a + U_{a3} \tag{F-1}$$

$$\dot{\omega}_{b1} = \dot{\omega}_{p1}\cos\gamma_b + \dot{\omega}_{p2}\sin\gamma_b + U_{b1} \tag{F-2}$$

where

$$U_{a3} = -\omega_{p2}\dot{\gamma}_a\cos\gamma_a - \omega_{p3}\dot{\gamma}_a\sin\gamma_a$$

$$U_{b1} = -\omega_{p1}\dot{\gamma}_b\sin\gamma_b + \omega_{p2}\dot{\gamma}_b\cos\gamma_b$$

?. Define T_{a3} and T_{b1}

From the Euler equations in Section 5.0

$$\dot{\omega}_{a3} = J_{a3} + T_{a3}/I_{g3} \tag{5-2}$$

$$\dot{\omega}_{b1} = J_{b1} + T_{b1}/I_{g1} \tag{5-3}$$

Use (F-1), (F-2), (5-2), (5-3) to eliminate $\dot{\omega}_{a3}$ and $\dot{\omega}_{b1}$ and define T_{a3} and T_{b1}

$$T_{a3} = I_{g3}[-\dot{\omega}_{p2}\sin\gamma_a + \dot{\omega}_{p3}\cos\gamma_a + U_{a3} - J_{a3}] \tag{F-3}$$

$$T_{b1} = I_{g1}[\ \dot{\omega}_{p1}\cos\gamma_b + \dot{\omega}_{p2}\sin\gamma_b + U_{b1} - J_{b1}] \tag{F-4}$$

4. Platform Torque in $\{\hat{\underline{p}}\}$ coordinates

Next express

$$\underline{T}_p = T_{p1}\ \hat{\underline{p}}_1 + T_{p2}\ \hat{\underline{p}}_2 + T_{p3}\ \hat{\underline{p}}_3 \tag{F-5}$$

for use in the platform Euler equations, Section 6.0.

Start with Eq. (9-22) for $\underline{T}_p$ and identify the components in that equation from Section 9.0.

$$\underline{T}_p = \underline{T}_{ps} + \underline{T}_{pF} + \underline{T}_{pu} + \underline{T}_{pB} + \underline{T}_{Rp} + \underline{T}_{pc} + \underline{T}_{pG} \tag{9-22}$$

$$\underline{T}_{ps} = -K_{sp}[\theta_x\ \hat{\underline{g}}_1 + \theta_y\ \hat{\underline{p}}_3] \tag{9-15}$$

$$\underline{T}_{pF} = -K_{Fp}[\{\dot{\theta}_x/|\dot{\theta}_x|\}\ \hat{\underline{g}}_1 + \{\dot{\theta}_y/|\dot{\theta}_y|\}\ \hat{\underline{p}}_3] \tag{9-16}$$

$$\underline{T}_{pu} = T_{u1}\ \hat{\underline{p}}_1 + T_{u2}\ \hat{\underline{p}}_2 + T_{u3}\ \hat{\underline{p}}_3 \tag{9-17}$$

$$\underline{T}_{pB} = T_{Bx}\ \hat{\underline{g}}_1 + T_{By}\ \hat{\underline{p}}_3 \tag{9-18}$$

$$\underline{T}_{Rp} = -\underline{T}_a - \underline{T}_b \tag{9-19}$$

$$\underline{T}_{pc} = -K_{pc}(\gamma_b \hat{\underline{q}}_1 + \gamma_a \hat{\underline{p}}_3) - K_{pI}(\int\gamma_b dt\ \hat{\underline{q}}_1 + \int\gamma_a dt\ \hat{\underline{p}}_3) \tag{9-20}$$

$$\underline{T}_{pG} = T_{pG} \hat{\underline{q}}_2 \tag{9-21}$$

Rewrite Eq. (9-22) as

$$\underline{T}_p = T_1 \hat{\underline{q}}_1 + T_3 \hat{\underline{p}}_3 - \underline{T}_a - \underline{T}_b + T_{pG} \hat{\underline{q}}_2 + \underline{T}_{pu} \tag{F-6}$$

where

$$T_1 = -K_{sp}\theta_x - K_{Fp}\{\dot{\theta}_x/|\dot{\theta}_x|\} + T_{Bx} - K_{pc}\gamma_b - K_{pI}\int\gamma_b dt$$

$$T_3 = -K_{sp}\theta_y - K_{Fp}\{\dot{\theta}_y/|\dot{\theta}_y|\} + T_{By} - K_{pc}\gamma_a - K_{pI}\int\gamma_a dt$$

$$\underline{T}_a = T_{a1} \hat{\underline{a}}_1 + T_{a3} \hat{\underline{a}}_3 \quad \text{(see Eq. (9-7))}$$

$$\underline{T}_b = T_{b1} \hat{\underline{b}}_1 + T_{b3} \hat{\underline{b}}_3 \quad \text{(see Eq. (9-14))}$$

Now start a series of transformations to put all components of Eq. (F-6) into the $\{\hat{\underline{p}}\}$ frame.

First transform $\underline{T}_a = -T_{a1} \hat{\underline{a}}_1 - T_{a3} \hat{\underline{a}}_3$ into $\{\hat{\underline{p}}\}$ using, from Section 2.0

$$\{\hat{\underline{p}}\} = [C_1(\gamma_a)]^T\{\hat{\underline{a}}\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_a & -\sin\gamma_a \\ 0 & \sin\gamma_a & \cos\gamma_a \end{bmatrix} \begin{Bmatrix} -T_{a1} \\ 0 \\ -T_{a3} \end{Bmatrix}_{\hat{\underline{a}}} = \begin{Bmatrix} -T_{a1} \\ T_{a3}\sin\gamma_a \\ -T_{a3}\cos\gamma_a \end{Bmatrix}_{\hat{\underline{p}}} \tag{F-7}$$

Next transform $\underline{T}_b = -T_{b1}\ \hat{\underline{b}}_1 - T_{b3}\ \hat{\underline{b}}_3$ into $\{\hat{\underline{p}}\}$ using from Section 2.0

$$\{\hat{\underline{p}}\} = [C_3(\gamma_b)]^T\{\hat{\underline{b}}\}$$

$$\begin{bmatrix} \cos\gamma_b & -\sin\gamma_b & 0 \\ \sin\gamma_b & \cos\gamma_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -T_{b1} \\ 0 \\ -T_{b3} \end{Bmatrix}_{\hat{\underline{b}}} = \begin{Bmatrix} -T_{b1}\cos\gamma_b \\ -T_{b1}\sin\gamma_b \\ -T_{b3} \end{Bmatrix}_{\hat{\underline{p}}} \tag{F-8}$$

Next transform $T_1\ \hat{\underline{g}}_1 + T_{pG}\ \hat{\underline{g}}_2$ into $\{\hat{\underline{p}}\}$ using from Section 2.0

$$\{\hat{\underline{p}}\} = [C_3(\theta_y)]\ \{\hat{\underline{g}}\}$$

$$
\begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_{pG} \\ 0 \end{Bmatrix}_{\hat{\underline{q}}} = \begin{Bmatrix} T_1\cos\theta_y + T_{pG}\sin\theta_y \\ -T_1\sin\theta_y + T_{pG}\cos\theta_y \\ 0 \end{Bmatrix}_{\hat{\underline{p}}} \qquad \text{(F-9)}
$$

Now sum all of the $\{\hat{\underline{p}}\}$ components in (F-7), (F-8), (F-9) with $T_3\,\hat{\underline{p}}_3$ in (F-6) and $\underline{T}_{pu}$ as defined in (9-17) to obtain the T_{pi} components in (F-5).

$$
\left.\begin{aligned}
T_{p1} &= -T_{a1} - T_{b1}\cos\gamma_b + T_1\cos\theta_y + T_{pG}\sin\theta_y + T_{u1} \\
T_{p2} &= T_{a3}\sin\gamma_a - T_{b1}\sin\gamma_b - T_1\sin\theta_y + T_{pG}\cos\theta_y + T_{u2} \\
T_{p3} &= -T_{a3}\cos\gamma_a - T_{b3} + T_3 + T_{u3}
\end{aligned}\right\} \qquad \text{(F-10)}
$$

5. Put Terms into Euler Equations

At this point combine the definitions of T_{a3} and T_{b1} in (F-3) and (F-4) with the $\{\hat{\underline{p}}\}$ coordinates of $\underline{T}_p$ in (F-10) into the platform Euler equations in Section 6.0.

The Euler equations are:

$$\dot{\omega}_{p1} = T_{p1}/I_{p1} \qquad \text{(6-1)}$$

$$\dot{\omega}_{p2} = T_{p2}/I_{p2} \qquad \text{(6-2)}$$

$$\dot{\omega}_{p3} = T_{p3}/I_{p3} \qquad \text{(6-3)}$$

Using Eqs. (F-3) and (F-4) write T_{a3} and T_{b1} as

$$T_{a3} = r_1\dot{\omega}_{p2} + r_2\dot{\omega}_{p3} + r_3 \tag{F-11}$$

$$T_{b1} = s_1\dot{\omega}_{p1} + s_2\dot{\omega}_{p2} + s_3 \tag{F-12}$$

where

$$r_1 = -I_{g3}\sin\gamma_a \qquad s_1 = I_{g1}\cos\gamma_b$$

$$r_2 = I_{g3}\cos\gamma_a \qquad s_2 = I_{g1}\sin\gamma_b$$

$$r_3 = I_{g3}(U_{a3} - J_{a3}) \qquad s_3 = I_{g1}(U_{b1} - J_{b1})$$

Use (F-11) and (F-12) to eliminate T_{a3} and T_{b1} in Eq. (F-10) and substitute the result into Eqs. (6-1), (6-2), (6-3).

$$\begin{aligned} I_{p1}\dot{\omega}_{p1} &= -T_{a1} - \cos\gamma_b(s_1\dot{\omega}_{p1} + s_2\dot{\omega}_{p2} + s_3) \\ &\quad + T_1\cos\theta_y + T_{u1} + T_{pG}\sin\theta_y \end{aligned} \tag{F-13}$$

$$\begin{aligned} I_{p2}\dot{\omega}_{p2} &= \sin\gamma_a(r_1\dot{\omega}_{p2} + r_2\dot{\omega}_{p3} + r_3) \\ &\quad - \sin\gamma_b(s_1\dot{\omega}_{p1} + s_2\dot{\omega}_{p2} + s_3) \\ &\quad - T_1\sin\theta_y + T_{u2} + T_{pG}\cos\theta_y \end{aligned} \tag{F-14}$$

$$I_{p3}\dot{\omega}_{p3} = -\cos\gamma_a(r_1\dot{\omega}_{p2} + r_2\dot{\omega}_{p3} + r_3) - T_{b3} + T_3 + T_{u3} \qquad \text{(F-15)}$$

6. Elimination of T_{pG} and Solutions

Eliminate T_{pG} by combining Eqs. (F-13) and (F-14) to obtain:

$$\dot{\omega}_{p1}d_5 + \dot{\omega}_{p2}d_6 + \dot{\omega}_{p3}d_7 = d_8 \qquad \text{(F-16)}$$

where

$$d_5 = s_1\sin\gamma_b \tan\theta_y - I_{p1} - s_1\cos\gamma_b$$

$$d_6 = (s_2\sin\gamma_b - r_1\sin\gamma_a + I_{p2}) \tan\theta_y - s_2\cos\gamma_b$$

$$d_7 = -r_2\sin\gamma_a \tan\theta_y$$

$$d_8 = T_{a1} + s_3\cos\gamma_b - T_1\cos\theta_y - T_{u1} + (r_3\sin\gamma_a - s_3\sin\gamma_b - T_1\sin\theta_y + T_{u2}) \tan\theta_y$$

Write Eq. (F-15) as:

$$\dot{\omega}_{p3} = d_{20}\dot{\omega}_{p2} + d_{21}\dot{\omega}_{p3} + d_{22} \qquad \text{(F-17)}$$

where

$$d_{20} = -(r_1\cos\gamma_a)/I_{p3}$$

$$d_{21} = -(r_2\cos\gamma_a)/I_{p3}$$

$$d_{22} = (-r_3\cos\gamma_a - T_{b3} + T_3 + T_{u3})/I_{p3}$$

This leaves two equations ((F-16) and (F-17)) in three unknowns, $\dot{\omega}_{p1}$, $\dot{\omega}_{p2}$, $\dot{\omega}_{p3}$. Obtain a third equation from the velocity constraints due to the gimballing that is determined in Appendix G. The result is:

$$\dot{\omega}_{p2} = -\dot{\omega}_{p1} \tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad \text{(G-3)}$$

where d_{10} and d_{13} are defined in Appendix G

The solution of this set of equations ((F-16), (F-17)), (G-3) is:

$$\dot{\omega}_{p1} = \frac{d_{11}}{d_{12}} \qquad \text{(F-18)}$$

$$\dot{\omega}_{p2} = -\frac{d_{11}}{d_{12}} \tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad \text{(F-19)}$$

$$\dot{\omega}_{p3} = \frac{d_{20}\dot{\omega}_{p2} + d_{22}}{1 - d_{21}} \qquad \text{(F-20)}$$

where $d_{11} = d_8 + (d_{13} - d_{10}\sin\theta_y)(-d_6 - \frac{d_7 d_{20}}{1-d_{21}}) - (\frac{d_7 d_{22}}{1-d_{21}})$

$$d_{12} = d_5 + \tan\theta_y(-d_6 - \frac{d_7 d_{20}}{1-d_{21}})$$

APPENDIX G

GENERATION OF $\dot{\omega}_{p2}$ FROM VELOCITY CONSTRAINTS

This appendix develops $\dot{\omega}_{p2}$ from velocity constraints imposed by the gimballing configuration for use in Appendix F as the third of the three platform system equations.

The process here is to use Eq. (E-9) for ω_{p2} and differentiate. Additional relations are obtained from Appendix E, noting that θ_x and θ_y are independent of ω_{p2} as indicated by Eq. (E-10).

$$\omega_{p2} = -(D_1 + \dot{\theta}_x)\sin\theta_y + y_2\cos\theta_y \qquad \text{(E-9)}$$

where

$$D_1 = \omega_{f1}\cos\theta_T + \omega_{f2}\sin\theta_T$$

$$D_2 = -\omega_{f1}\sin\theta_T + \omega_{f2}\cos\theta_T$$

$$y_2 = D_2\cos\theta_x + (\dot{\theta}_T + \omega_{f3})\sin\theta_x$$

Differentiating Eq. (E-9) produces

$$\omega_{p2} = -(\dot{D}_1 + \ddot{\theta}_x)\sin\theta_y - \dot{\theta}_y(D_1 + \dot{\theta}_x)\cos\theta_y$$

$$+ \dot{y}_2\cos\theta_y - y_2\dot{\theta}_y\sin\theta_y \qquad \text{(G-1)}$$

We now require relations for $\ddot{\theta}_x$, $\dot{D}_1$, $\dot{y}_2$. From Appendix E, Eq. (E-10)

$$\dot{\theta}_x = (\omega_{p1} - D_1\cos\theta_y - y_2\sin\theta_y)/\cos\theta_y \qquad \text{(E-10)}$$

Differentiating Eq. (E-12) produces

$$\ddot{\theta}_x = \frac{\dot{\omega}_{p1}}{\cos\theta_y} + d_{10} \qquad \text{(G-2)}$$

where

$$d_{10} = (-\dot{D}_1\cos\theta_y + D_1\dot{\theta}_y \sin\dot{\theta}_y - \dot{y}_2\sin\theta_y - y_2\dot{\theta}_y\cos\theta_y)$$
$$(\cos^{-1}\theta_y) + \dot{\theta}_y(\omega_{p1} - D_1\cos\theta_y - y_2\sin\theta_y)$$
$$\cos^{-2}\theta_y\sin\theta_y$$

Substitute Eq. (G-2) into (G-1) to obtain the solution

$$\dot{\omega}_{p2} = -\dot{\omega}_{p1}\tan\theta_y - d_{10}\sin\theta_y + d_{13} \qquad \text{(G-3)}$$

where $$d_{13} = -\dot{D}_1\sin\theta_y - \dot{\theta}_y(D_1 + \dot{\theta}_x)\cos\theta_y$$
$$+ \dot{y}_2\cos\theta_y - y_2\dot{\theta}_y\sin\theta_y$$

We define $\dot{D}_1$, $\dot{D}_2$ and $\dot{y}_2$ by differentiating the equations under (E-9) above:

$$\dot{D}_1 = \dot{\omega}_{f1}\cos\theta_T - \omega_{f1}\dot{\theta}_T\sin\theta_T + \dot{\omega}_{f2}\sin\theta_T + \omega_{f2}\dot{\theta}_T\cos\theta_T \tag{G-4}$$

$$\dot{D}_2 = -\dot{\omega}_{f1}\sin\theta_T - \omega_{f1}\dot{\theta}_1\cos\theta_T + \dot{\omega}_{f2}\cos\theta_T - \omega_{f2}\dot{\theta}_T\sin\theta_T \tag{G-5}$$

$$\begin{aligned}\dot{y}_2 = {} & \dot{D}_2\cos\theta_x - D_2\dot{\theta}_x\sin\theta_x \\ & + (\ddot{\theta}_T + \dot{\omega}_{f3})\sin\theta_x + \dot{\theta}_x(\dot{\theta}_T + \omega_{f3})\cos\theta_x\end{aligned} \tag{G-6}$$

We define the $\dot{\omega}_{fi}$ terms by differentiating Eqs. (E-3) for ω_{fi} in Appendix E.

$$\begin{aligned}\dot{\omega}_{f1} = {} & \ddot{\psi}_p\cos\psi_R - \dot{\psi}_p\dot{\psi}_R\sin\psi_R - \ddot{\psi}_A\sin\psi_R\cos\psi_p - \dot{\psi}_A\dot{\psi}_R\cos\psi_R\cos\psi_p \\ & + \dot{\psi}_A\dot{\psi}_p\sin\psi_R\sin\psi_p\end{aligned} \tag{G-7}$$

$$\dot{\omega}_{f2} = \ddot{\psi}_R + \ddot{\psi}_A\sin\psi_p + \dot{\psi}_A\dot{\psi}_p\cos\psi_p \tag{G-8}$$

$$\begin{aligned}\dot{\omega}_{f3} = {} & \ddot{\psi}_p\sin\psi_R + \dot{\psi}_p\dot{\psi}_R\cos\psi_R + \ddot{\psi}_A\cos\psi_R\cos\psi_p \\ & - \dot{\psi}_A\dot{\psi}_R\sin\psi_R\cos\psi_p - \dot{\psi}_A\dot{\psi}_p\cos\psi_R\sin\psi_p\end{aligned} \tag{G-9}$$

APPENDIX H

COMPUTER PROGRAMS

INTRODUCTION

The Fortran computer programs contain the inputs, torques and system equations defined in this report and computes the system trajectory by use of Runge Kutta integration.

PROGRAMS AND SUBROUTINES

MT21D:	Main program
MTRK1:	4-cycle Runge Kutta integration
MT22:	Ship motion and coordinate transformation for command inputs
MT19B:	Torques
MT20B:	State equations
PLOT:	Library plotting routine

Figure H-1 is an overall block representation of the program. The listing of the program is attached.

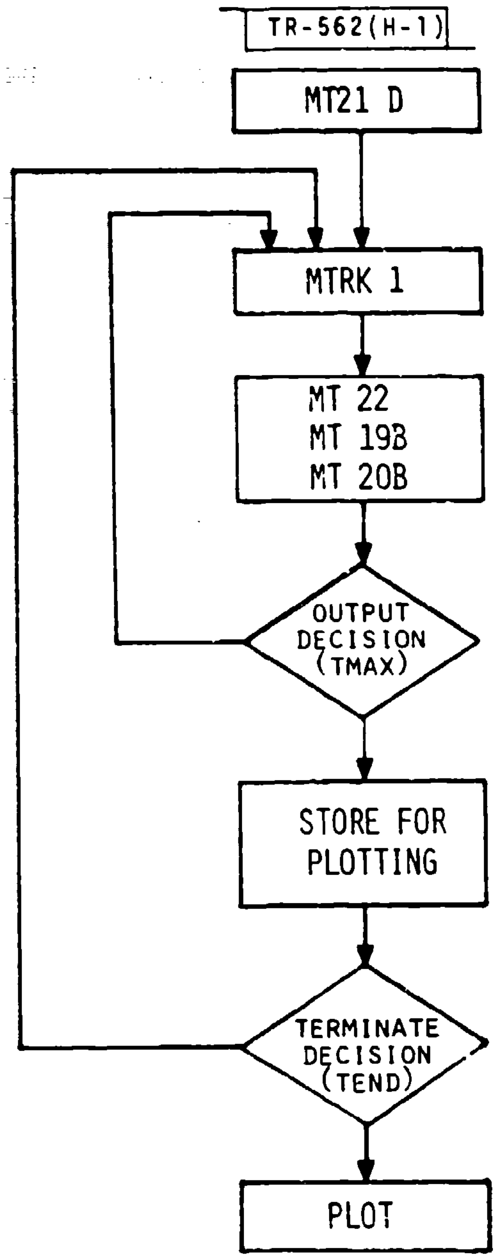


Fig. H-1. Computer flow diagram.

MT21D

```
C  3 AXIS DMS PLATFORM
C TRAINABLE X-Y PLATFORM
C  MT19D CONTAINS ,DYNAMICS, TORQUES
C  MT20D CONTAINS SYSTEM EQUATIONS
C MT22 CONTAINS GEOMETRY,SHIP MOTION
C
C  SYSTEM VARIABLES
C
C  XI(1)-THETA ELEVATION OUT      XI(5)-WP3
C  XI(2)-THETA CROSS ELEV OUT     XI(6)-WP1
C  XI(3)-WA1                      XI(7)-WP2
C  XI(4)-WB3                      XI(8)-GAMMA A
C                                 XI(9)-GAMMA B
C  XI(10)-GAMMA A INTEGR          XI(11)-GAMMA B INTEGR
C  XI(12)-THETA TRAIN             XI(13)-THETA DOT TRAIN
C
C  D(1)-DESIRED AZIM(DEG)         D(8)-FLEX PERIOD(SEC)
C  D(2)-DESIRED ELEV(DEG)         D(9)-ROLL HEIGHT(IN)
C  D(3)-ROLL AMPLIT(DEG)          D(10)-VELOCITY(KNOTS)
C  D(4)-PITCH AMPLIT(DEG)         D(11)-TURN RATE(DEG/SEC)
C  D(5)-FLEX AMPLIT(DEG)          D(12)-HEEL ANGLE(DEG)
C  D(6)-ROLL PERIOD(SEC)          D(13)-CANT ANGLE(DEG)
C  D(7)-PITCH PERIOD(SEC)         D(14)-WN TRAIN SERVO(RAD/SEC)
C                                 D(15)-DAMPING RATIO TRAIN SERVO
C
C  E(1)-PLATFORM MASS(LB-SEC2/IN)
C  E(2)-GYRO MASS
C  E(3)-GYRO MASS OFFSET(IN)
C  E(4)- PLAT 1 MASS OFFSET(IN)
C  E(5)- PLATFORM 2 MASS OFFSET(IN)
C  E(6)- PLATFORM 3 MASS OFFSET(IN)
C  E(7)-GYRO FRICT.(IN-LB)
C  E(8)-GYRO SPRING(IN-LB/RAD)
C  E(9)-GYRO A CONTROL GAIN(IN-LB/RAD)
C  E(10)-GYRO B CONTROL GAIN(IN-LB/RAD)
C  E(11)-PLATF FRICT(IN-LB)
C  E(12)-PLATFORM SPRING(IN-LB/RAD)
C  E(13)-GYRO INERTIA 1 AND 3(LB-IN-SEC2)
C  E(14)-GYRO INERTIA 2
C  E(15)- PLATFORM INERTIA 1
C  E(16)-PLATFORM INERTIA 2
C  E(17)-PLATFORM INERTIA 3
C  E(18)-GYRO SPIN SPEED(RAD/SEC)
C  E(19)-PLATF CONTR GAIN(IN-LB/RAD)
C  E(20)-PLATF TORQ INTEGR GAIN(IN-LB/RAD-SEC)
C  E(21)-GYRO A TORQUE BIAS(IN-LB)
C  E(22)-GYRO B TORQUE BIAS(IN-LB)
C  E(23)-PLAT ELEV BIAS TORQUE(IN-LB)
C  E(24)-PLAT CROSS ELEV BIAS(IN-LB)
C  E(25)-LIMIT ON GYRO TORQUER(IN-LB)
C E(26)-GYRO SPEED MISMATCH
C    SPEED OF GYRO A=E(18)*(1.D0+.5D0*E(26))
C    SPEED OF GYRO B=E(18)*(1.D0-.5D0*E(26))
C
C
C  Q(1)-TPCE               Q(8)-WF2
C  Q(2)-TPE                Q(9)-WF3
C  Q(3)-TB3                Q(10)-TU1
C  Q(4)-TA1                Q(11)-TU2
C  Q(5)-TEI                Q(12)-TU3
C  Q(6)-TCEI               Q(13)-WFD1
C  Q(7)-WF1                Q(14)-WFD2
C  Q(15)-WFD3
C
      IMPLICIT REAL*8 (A-H,O-Z)
      NAMELIST/INPUTS/TEND,TMAX,DT,XI,DXDT,D,E,TI,TITRN,STPLT
      DIMENSION XI(13),XW(13),DXDT(13),CDUM(65),D(15),
     1  NP(3),B(3),CR(3,3),CP(3,3),CA(3,3),BV(3),
     1    S(3),CE(3,3),CT(3,3),C1(3,3),C2(3,3),
     1  A(3),Q(20),E(26),RUNNAM(10)
C
      REAL*8 NP
      DATA XI/13*0.D0/
      DATA DXDT/13*0.D0/
      DATA D/0.D0, 0.D0, 15.D0, 0.D0, 0.D0, 10.D0, 15.D0, 1.D0,
     1     1200.D0, 0.D0, 0.D0, 0.D0, 0.D0,3.D0,0.7D0/
      DATA E/.25D0, .06D0, .003D0, .003D0, .003D0, .003D0,
     1       .04D0, .2D0, 108.D0, 108.D0, .5D0, 1.5D0, .5D0, 1.D0,
     1     40.D0, 30.D0, 40.D0, 361.D0,50.D0,5.D0,0.D0,0.D0,0.D0,0.
D0,5
     2.D0,0.D0/
C
      TEND=90.D0
      DT=.005D0
      STPLT=0.D0
      TITRN=200.D0
      TI=0.D0
      TCAL=0.D0
      TMAX=.25D0
C
C READ RUN NAM AND PARAMETERS TO CHANGE
      READ(1,5555) (RUNNAM(K),K=1,10)
 5555 FORMAT(10A4)
      READ(1,INPUTS)
      ADD=.25D0
      WRITE(6,5556) (RUNNAM(K),K=1,10)
 5556 FORMAT(1X,10A4/)
C
C
      WRITE(6,INPUTS)
C  WRITE RUN NAME TO PLOT FILE
      WRITE(16,5557) (RUNNAM(K),K=1,10)
 5557 FORMAT('%ANM DOT ',10A4)
C
      WRITE(16,66)
  66   FORMAT('%COL'/1X,'T(SEC     CER(DEG     EER(DEG     GA(DEG
     C
     1G(DEG      TB3(IL     TA1(IL     WP1(D,SEC     .1XWP2(D,SEC
WP3(D
     2,SEC')
C
  1   L=1
      IF(TI.GT.TITRN)D(11)=3.D0
  2   CALL MTRK1(65,L,13,DXDT,XI,TI,DT,XW,TW,CDUM)
      IF(L-6)3,4,3
  3   CONTINUE
      CALL MT22(D,TI,XI,DXDT,NP,B,CR,CP,CA,BV,S,
     1     CE,CT,C1,C2,A,G,E)
C
      CALL MT19D(D,TI,XI,DXDT,A,Q,E,SUR,DT,TAOLD,TBOLD)
      CALL MT20D(XW,DXDT,Q,E,D,TW)
      GO TO 2
  4   CONTINUE
      IF(TMAX-TI)5,5,1
   5   CONTINUE
C
```

```
      CALL MTBB(B,TI,XI,DXDT,MP,B,CR,CP,CA,BU,S,
     1   CE,CT,C1,C2,A,G,5)
C
      V1=(G(8)-XI(8))*57.29578D0
      V2=(G(6)-XI(11))*57.29578D0
      V3=XI(8)*57.29578D0
      V4=XI(8)*57.29578D0
      V5=XI(6)*57.29578D0
      V6=.1D0*XI(7)*57.29578D0
      V7=XI(5)*57.29578D0
      V8=G(3)
      V9=G(4)
C
      IF(TI.LT.STPLT)GO TO 671
C
C  STORE FOR PLOTTING ......................
C
      WRITE(16,67)TI,V1,V2,V3,V4,V8,V9,V5,V6,V7
 67    FORMAT(1X,10(1X,D10.4))
C
C
 671  CONTINUE
      TMAX=TMAX+ADD
      IF(TEND-TI)6,6,1
 6    CONTINUE
C
C  COMPUTATION COMPLETED.....PLOT STORED RESULTS
      WRITE(15,688)
 688 FORMAT('$YRANGE -3. +2.'/'$SELECT CER(DEG'/
    1'$SELECT EER(DEG'/'$PLOT')
C
      WRITE(16,699)
 699  FORMAT('$YRANGE -2. +2.'/'$SELECT GA(DEG'/
    1'$SELECT GB(DEG'/'$PLOT')
C
      WRITE(16,701)
 701  FORMAT('$YRANGE -2. +2.'/'$SELECT TB3(IL'/'$SELECT TA1(IL
    1'/'$PLOT')
C
      WRITE(16,700)
 700 FORMAT('$YRANGE -1. +1.'/'$SELECT UP1(D,SEC'/'$SELECT .1XUP2(D,SEC
    1'/'$SELECT UP3(D,SEC'/'$PLOT')
      STOP
      END
      SUBROUTINE MTRK1(MS,L,N,DXDT,XI,TI,DT,XU,TU,C)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C  4- CYCLE RUNGE-KUTTA INTEGRATION.
C
      DIMENSION C(MS),XI(N),XU(N),DXDT(N)
C
 33   GO TO (1,2,3,4,5),L
C
 1    DO 20 I=1,N
 20   XU(I)=XI(I)
      TU=TI
      L=2
      RETURN
C
 2    DO 6 J=1,N
      C(J)=DXDT(J)*DT
 6    XU(J)=XI(J)+C(J)*.5D0
      TU=TI+DT*.5D0
      L=3
      RETURN
C
 3    DO 7 J=1,N
      I=N+J
      C(I)=DXDT(J)*DT
 7    XU(J)=XI(J)+C(I)*.5D0
      L=4
      RETURN
C
 4    DO 8 J=1,N
      I=2*N+J
      C(I)=DXDT(J)*DT
 8    XU(J)=XI(J)+C(I)
      TU=TI+DT
      L=5
      RETURN
C
 5    DO 34 J=1,N
      I=3*N+J
 34   C(I)=DXDT(J)*DT
C
C
C
C COMPUTE ANSWER
 21   DO 17 J=1,N
      M2=N+J
      M3=2*N+J
      M4=3*N+J
 17   XI(J)=XI(J)+(C(J)+2.D0*(C(M2)+C(M3))+C(M4))/6.D0
      TI=TI+DT
C
 19   L=6
      RETURN
      END
      SUBROUTINE MT22(D,TU,XU,DXDT,MP,B,CR,CP,
     1   CA,BU,S,CE,CT,C1,C2,A,G,E)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C SHIP MOTION,TRANSFORMATIONS,ACCELERATION,TORQUES
C FOR 2 AXIS DMS STABILIZATION
      DIMENSION D(15),MP(3),B(3),CR(3,3),CP(3,3),
     .   CA(3,3),BU(3),S(3),Y(13),CE(3,3),CT(3,3),
     1   C1(3,3),C2(3,3),A(3),DYDT(13),G(20),E(26)
      REAL*8 MP
C
C
C ROLL POSITION(ANGULAR)
      WR=6.28D0/D(6)
      SR=(D(3)/57.29578D0)*DSIN(WR*TU)
      SRC=SP+(D(13)+D(12))/57.29578D0
C ROLL ACCELERATION IN M1 PRIME
      SRD=-WR*WR*SR
      AR=D(9)*SRD
C  ROLL VELOCITY(ANGULAR)
      SUR=WR*(D(3)/57.29578D0)*DCOS(WR*TU)
C PITCH POSITION(ANGULAR)
      WP=6.28D0/D(7)
      SP=(D(4)/57.29578D0)*DSIN(WP*TU)
C PITCH ACCELERATION IN M2 PRIME
      SPD=-WP*WP*SP
      AP=D(9)*SPD
C  PITCH VELOCITY(ANGULAR)
```

```
C ACCELERATION IN M1 PRIME
      AT=-D(10)*80.863D0*D(11)/57.29578D0
C TOTAL ACCEL VECTOR IN M PRIME
      MP(1)=AR + AT
      MP(2)=AP
      MP(3)=386.D0
C
C AZIMUTH POSITION
      SA=D(11)*TU/57 29578D0
C
C SET UP FOR TRANSFORMATIONS
C GENERATE DESIRED CROSS ELEV-ELEV TO ACHEIVE AZ-ELEV VECTOR
      B(1)=-DSIN(D(1)/57.29578D0)*DCOS(D(2)/57.29578D0)
      B(2)= DCOS(D(1)/57.29578D0)*DCOS(D(2)/57.29578D0)
      B(3)= DSIN(D(2)/57.29578D0)
      CALL DCMAT(3,XU(12),CT
      CALL DCMAT(2,SRC,CR)
      CALL DCMAT(1,SP,CP)
      CALL DCMAT(3,SA,CA)
C
      CALL MTRY3(CA,B,BU,3,3,1)
      CALL MTRY3(CP,BU,B,3,3,1)
      CALL MTRY3(CR,B,BU,3,3,1)
      CALL MTRY3(CT,BU,S,3,3,1)
C THIS THE DESIRED CROSS ELEV - ELEV
      TCEI = DARSIN(-S(1))
      TEI=DARSIN(S(3)/DCOS(TCE`))
C
C TRANSFORM ACCEL VECTOR FROM MPRIME TO P
      CALL DCMAT(1,XU(1),CE)
      CALL DCMAT(3,XU(2),C1)
      CALL MTRY3(C1,CE,C2,3,3,3)
      CALL MTRY3(C2,CT,C1,?,3,3)
      CALL MTRY3(C1,CR,C2,.,3,3)
      CALL MTRY3(C2,CP,C1,3,3,3)
C THE VECTOR A IS ACCEL IN P FRAME
      CALL MTRY3(C1,MP,A,3,3,1)
C
C  FORM UF ELEMENTS, DECK VELOCITY COMPONENTS
      UF1=SUP*DCOS(SRC) -(D(11)/57.29578D0)*DCOS(SP)*DSIN(SRC)
      UF2=SUR + (D(11)/57.29578D0)*DSIN(SP)
      UF3=SUP*DSIN(SRC) +(D(11)/57.29578D0)*DCOS(SP)*DCOS(SRC)
C
C
C DECK ANGULAR ACCEL.  UFD COMPNENTS
      UFD1 = SPD*DCOS(SRC) - SUP* SUR * DSIN(SRC)
     1       -(D(11)/57.29578D0) * SUR * DCOS(SRC) * DCOS(SP)
     2       +(D(11)/57.29578D0) * SUP * DSIN(SRC) *DSIN(SP)
C
      UFD2 = SRD + (D(11)/57.29578D0) * SUP * DCOS(SP)
      UFD3=SPD*DSIN(SRC) + SUP*SUR*DCOS(SRC)
     1    -(D(11)/57.29578D0)*(SUR*DSIN(SRC)*DCOS(SP)
     2    +SUP*DCOS(SRC)*DSIN(SP))
C  FORM OUTPUT VECTOR
      G(5)=TEI
      G(6)=TCEI
      G(7)=UF1
      G(8)=UF2
      G(9)=UF3
      G(13)=UFD1
      G(14)=UFD2
      G(15)=UFD3
C
C G(16) FOR USE IN 106
      G(16)=SUR
      VAL1=DTAN(D(2)/57.29578D0)*DCOS(SRC)
C
      ELSP=DATAN(VAL1)
      VAL2=DCOS(D(2)/57.29578D0)/DCOS(ELSP)
      ELSP=DATAN(VAL1)
      CESP=DARCOS(VAL2)
      G(19)=ELSP
      G(20)=CESP
      RETURN
      END
      SUBROUTINE MT19D(D,TU,XU,DXDT,
     1   A,G,E,SUR,DT,TAOLD,TBOLD)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C FOR 2 AXIS DMS STABILIZATION
      DIMENSION D(15),A(3),DXDT(13),XU(13),G(20),E(26)
      REAL*8 MP
C
C  SHIP FLEXURE
      SF=(D(5)/57.29578D0)*DSIN(6.28D0*TU/D(8))
C
C TORQUE EQUATIONS
C
C GYRO 'A' CONTROL,BIAS,FRICTION,SPRING,UNBAL,TOTAL
      TCA=-E(9)*(G(6)-XU(2)+SF)
C     IF(DABS(TCA-TAOLD).LT.3.)GO TO 20
C     TCA=TAOLD+(DT/5.)*(TCA-TAOLD)
      IF(DABS(TCA).GT.E(25))TCA=E(26)*(TCA/DABS(TCA))
C     TAOLD=TCA
      GDA=XU(3)-XU(6)
      IF(GDA.EQ.0.D0)GO TO 10
      TFA=-E(7)*(GDA/DABS(GDA))
      GO TO 11
 10   TFA=0.D0
 11   CONTINUE
      TSA=-E(8)*XU(8)
      TUA=-E(2)*E(3)*(-A(2)*DSIN(XU(8))
     1         +A(3)*D^^CS(XU(8)))
      TA1=TCA+TFA+TSA+TUA+E(21)
C
C GYPO 'B' CONTROL,FRICT,SPRING,UNBAL,BIAS,TOTAL
      TCB=-E(10)*(G(5)-XU(1)+SF)*DCOS(XU(2))
C     IF(DABS(TCB-TBOLD).LT.3.)GO TO 21
C     TCB=TBOLD + (DT/5.)*(TCB - TBOLD)
      IF(DABS(TCB).GT.E(25))TCB=E(26)*(TCB/DABS(TCB))
      TBOLD=TCB
      GDB= XU(4)-XU(5)
      IF(GDB.EQ.0.D0)GO TO 12
      TFB=-F(7)*(GDB/DABS(GDB))
      C   U 13
 12   ?  0.D0
 13   C  INUE
          -E(8)*XU(9)
      TU =-E(2)*E(3)*(A(1)*DCOS(XU(B))+A(2)*DSIN(XU(9)))
      TB3= TCB+TFB+TGB+TUB
C
C PLATFORM UNBALANCE IN P FRAME
      TU1=E(1)*(A(2)*E(6)-A(3)*E(5))
      TU2=E(1)*(A(3)*E(4)-A(1)*E(6))
      TU3=E(1)*(A(1)*E(5)-A(2)*E(4))
C PLATFORM ELEVATION _ FRICTION,SPRING, CONTROL,BIAS, TOTAL
```

```
     .EQ.0.D0)GO TO  16
      TFE=-E(11)*(DXDT(1)/DABS(DXDT(1)))
      GO TO 17
   16 TFE=0.D0
   17 CONTINUE
      TELCON=-E(19)*XU(9)-E(20)*XU(11)
      TSE=-E(12)*XU(1)
      TPE=TFE+TSE+TELCON+E(23)
C PLATFORM CROSS ELEV _ FRICTION,SPRING,CONTROL,BIAS,TOTAL
      IF(DXDT(2).EQ.0.D0)GO TO 14
      TFCE=-E(11)*(DXDT(2)/DABS(DXDT(2)))
      GO TO 15
   14 TFCE=0.D0
   15 CONTINUE
      TSCE=-E(12)*XU(2)
      TCECON=-E(19)*XU(8) - E(20)*XU(10)
      TPCE=TFCE+TSCE+TCECON+E(24)
C
C FORM OUTPUT VECTOR
      G(1)=TPCE
      G(2)=TPE
      G(3)=TA3
      G(4)=TA1
      G(10)=TU1
      G(11)=TU2
      G(12)=TU3
C
      RETURN
      END
      SUBROUTINE MT20B(XU,DXDT,G,E,D,TW)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XU(13),DXDT(13),G(28),E(26),D(15)
C     STATE EQUATIONS FOR INTEGRATION
C     TRAINABLE X-Y SYSTEM
      SX1=DSIN(XU(1))
      CX1=DCOS(XU(1))
      SX2=DSIN(XU(2))
      CX2=DCOS(XU(2))
      SX8=DSIN(XU(8))
      CX8=DCOS(XU(8))
      SX9=DSIN(XU(9))
      CX9=DCOS(XU(9))
      SX12=DSIN(XU(12))
      CX12=DCOS(XU(12))
      C1=G(7)*CX12+G(8)*SX12
      C2=-G(7)*SX12+G(8)*CX12
      B2=C2*CX1+(XU(13)+G(9))*SX1
C
C TRAIN SERVO CONTROL
      SA=D(11)*TW/57.2957795
      DXDT(12)=XU(13)
      DXDT(13)=-2.D0*D(14)*D(15)*(XU(13)+D(11)/57.2957795D0)+D(14)**2*
     1    (D(1)/57.2957795D0-XU(12)-SA)
C  GYROS A AND B
      W=E(14)/E(13)
      UA3=-XU(7)*SX8+XU(6)*CX8
      UA2=XU(7)*CX8+XU(6)*SX8
      UB2=-XU(8)*SX9+XU(7)*CX9
      UB1=XU(8)*CX9+XU(7)*SX9
      DXDT(3)=UA3*E(18)*(1.D0+.5D0*E(26))*W-UA3*UA2+G(4)/E(13)
      DXDT(4)=UB1*E(18)*(1.D0-.5D0*E(26))*W+UB1*UB2+G(3)/E(13)
C
      GDA=XU(3)-XU(8)
      GDB=XU(4)-XU(5)
      UA3=-XU(7)*GDA*CX8-XU(5)*GDA*SX8
      UB1=-XU(6)*GDB*SX9+XU(7)*GDB*CX9
      AJ3=-XU(3)*E(18)*(1.D0+.5D0*E(26))*W+XU(3)*UA2
      BJ1=-XU(4)*E(18)*(1.D0-.5D0*E(26))*W-XU(4)*UB2
C
      S1=E(13)*CX9
      S2=E(13)*SX9
      S3=E(13)*(UB1-BJ1)
C
      CD1=G(13)*CX12-G(7)*XU(13)*SX12+G(14)*SX12
     1    +G(8)*XU(13)*CX12
      CD2=-G(13)*SX12-G(7)*XU(13)*CX12+G(14)*CX12
     1    -G(8)*XU(13)*SX12
      DXDT(1)=(XU(6)-C1*CX2-D2*SX2)/CX2
      XU(7)=-(C1*DXDT(1))*SX2 + D2*CX2
      YD2=CD2*CX1-C2*DXDT(1)*SX1
     1    +(DXDT(13)+G(15))*SX1
     2    +DXDT(1)*(XU(13)+G(9))*CX1
      Y3=-C2*SX1+(XU(13)+G(9))*CX1
      DXDT(2)=XU(5)-Y3
C
      R1=-E(13)*SX8
      R2=E(13)*CX8
      R3=E(13)*(UA3-AJ3)
C
      D5=S1*SX9*DTAN(XU(2))-E(15) - S1*CX9
      D6=DTAN(XU(2))*(E(16)-R1*SX8+S2*SX9)
     1    -S2*CX9
      D7=-R2*SX8*DTAN(XU(2))
      D8=G(4)+S3*CX9-G(2)*CX2-G(10)-DTAN(XU(2))
     1    *(-R3*SX8+S3*SX9+G(2)*SX2-G(11))
C
      D10=-CD1 - D2*DXDT(2) + DTAN(XU(2))*
     1      (C1*DXDT(2) - YD2) + (XU(6) - C1*CX2 - D2 *SX2)
     2      *DXDT(2)*SX2/(CX2**2)
      D20=-R1*CX8/E(17)
      D21=-R2*CX8/E(17)
      D67= D6 + D7*D20/(1.D0-D21)
      D12= D5 - D67*DTAN(XU(2))
      D13=SX2*(-CD1-D2*DXDT(2))+CX2*
     1    (YD2-DXDT(2)*(C1+DXDT(1)))
C
      D22=(-R3*CX8-G(3)+G(1)+G(12))/E(17)
      D11=D8 - (D13 - D10*SX2)*D67 - D7*D22/(1.D0-D21)
      DXDT(6) = D11/D12
      DXDT(7) = -DXDT(6)*DTAN(XU(2)) + D13 - D10*SX2
      DXDT(5) = (D20*DXDT(7) + D22)/(1.D0 - D21)
      DXDT(8)=GDA
      DXDT(9)=GDB
      DXDT(10)=XU(8)
      DXDT(11)=XU(9)
C SIMPLE CALCULATION FOR DXDT(1) AND DXDT(2) IN 104 B
      CEDOT=G(16)*SX1
      ELDOT=-G(16)*CX1*DTAN(XU(2))
      G(17)=CEDOT
      G(18)=ELDOT
      RETURN
      END
      SUBROUTINE MTRY3(A,B,C,I,K,J)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(I,K),B(K,J), C(I,J)
      DO 10 IW=1,I
```

```
      DO 80 JU=1,J
      CP=0.D0
      DO 30 KU=1,K
      C(IU,JU)=A(IU,KU)*B(KU,JU)+CP
   30 CP=C(IU,JU)
   80 CONTINUE
   10 CONTINUE
       RETURN
      END
C
C
      SUBROUTINE DCMAT(N,TH,C)
      IMPLICIT REAL*8 (A-H,O-Z)
C  THIS FILLS IN THE DIRECTION COSSINE MATRIX
C  N=AXIS OF ROTATION,  TH=ANGLE OF ROTATION, C=MATRIX
C
      DIMENSION C(3,3)
C
      DO 1 J=1,3
      C(J,J)=DCOS(TH)
      C(N,J)=0.D0
      C(J,N)=0.D0
    1 CONTINUE
      C(N,N)=1.D0
      IF(N.EQ.1) C(2,3)=DSIN(TH)
      IF(N.EQ.1) C(3,2)=-DSIN(TH)
C
      IF(N.EQ.2) C(1,3)=-DSIN(TH)
      IF(N.EQ.2) C(3,1)=DSIN(TH)
C
      IF(N.EQ.3) C(2,1)=-DSIN(TH)
      IF(N.EQ.3) C(1,2)=DSIN(TH)
      RETURN
      END
C
C

R; T=0.10/0.35 09:53:14

CMS
```

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NOMENCLATURE*

$\underline{A}$ = total linear acceleration vector (App. D)

A_i = $\hat{\underline{p}}_i$ components of $\underline{A}$

$\underline{A}_G$ = acceleration term due to gravity

$\underline{A}_P$ = fore-aft acceleration

$\underline{A}_R$ = athwartship acceleration

$\underline{A}_T$ = centripetal acceleration due to turn

$\underline{F}$ = force applied to platform at support point (App. D)

H = height above center of roll

$\underline{\underline{I}}_g$ = gyro inertia diadic

I_{gi} = $\hat{\underline{a}}_i$ and $\hat{\underline{b}}_i$ principal inertias of $\underline{\underline{I}}_g$

$\underline{\underline{I}}_p$ = platform inertia diadic

I_{pi} = $\hat{\underline{p}}_i$ principal inertias of $\underline{\underline{I}}_p$

K_a = control gain on 'A' gyro for Y axis error

K_b = control gain on 'B' gyro for X axis error

K_{FG} = gyro friction torque magnitude

K_{FP} = platform friction torque magnitude

K_{pc} = controller gain for X and Y axis torquers

K_{pI} = integrator control gain for X and Y axis torquers

K_{SG} = gyro spring gradient

K_{SP} = platform spring gradient

$\underline{L}$ = torque vector due to unbalance

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* (_) denotes a vector

subscript i applies to all integers except for d_i

ℓ_g	=	displacement between center of support and mass for gyro
ℓ_i	=	displacement between center of support and mass for platform along $\hat{\underline{p}}_i$
M	=	mass of stabilized platform assembly
m	=	mass of gyro rotor and motor assembly
$\underline{R}$	=	position vector from support point to mass center
t	=	time
$\underline{T}_a$	=	total torque vector action on 'A' gyro
T_{ai}	=	$\hat{\underline{a}}_i$ components of $\underline{T}_a$
$\underline{T}_{aB}$	=	bias torque vector acting on 'A' gyro
T_{aB}	=	magnitude of $\underline{T}_{aB}$
$\underline{T}_{aF}$	=	friction torque acting on 'A' gyro
$\underline{T}_{aS}$	=	spring torque acting on 'A' gyro
$\underline{T}_{au}$	=	unbalance torque acting on 'A' gyro
$\underline{T}_b$	=	total torque vector acting on 'B' gyro
T_{bi}	=	$\hat{\underline{b}}_i$ components of $\underline{T}_b$
$\underline{T}_{bB}$	=	bias torque acting on 'B' gyro
$\underline{T}_{bF}$	=	friction torque acting on 'B' gyro
$\underline{T}_{bS}$	=	spring torque acting on 'B' gyro
$\underline{T}_{bu}$	=	unbalance torque acting on 'B' gyro
T_{Bx}	=	x component of bias torque acting on platform
T_{By}	=	y component of bias torque acting on platform
$\underline{T}_{ca}$	=	control torque applied to 'A' gyro
$\underline{T}_{cb}$	=	control torque applied to 'B' gyro

$\underline{T}_p$ = torque vector acting on platform

T_{pi} = $\hat{\underline{p}}_i$ components of $\underline{T}_p$

$\underline{T}_{pB}$ = bias torque vector acting on platform

$\underline{T}_{pc}$ = control torque acting on platform

$\underline{T}_{pF}$ = friction torque acting on platform

$\underline{T}_{pG}$ = gimbal bearing reaction torque on platform

T_{pG} = magnitude of $\underline{T}_{pG}$

$\underline{T}_{pS}$ = spring torque acting on platform

$\underline{T}_{pu}$ = unbalance torque acting on platform

$\underline{T}_{Ra}$ = pivot bearing reaction torque on 'A' gyro

$\underline{T}_{Rb}$ = pivot bearing reaction torque on 'B' gyro

T_{ui} = $\hat{\underline{p}}_i$ components of $\underline{T}_{pu}$

V = ship's speed

GREEK LETTERS

α_A = satellite azimuth ephemeris

α_E = satellite elevation ephemeris

γ_a = $\{\hat{\underline{a}}\}$ frame rotation angle with respect to $\{\hat{\underline{p}}\}$

γ_b = $\{\hat{\underline{b}}\}$ frame rotation angle with respect to $\{\hat{\underline{p}}\}$

ψ_A = ship's azimuth

ψ_H = heel angle of deck

ψ_p = pitch angle of deck

ψ_{po} = amplitude of harmonic pitch motion

ψ_R = roll angle of deck

ψ_{RH} = $\psi_R + \psi_H$

ψ_{Ro} = amplitude of harmonic roll motion

θ_F = ship flexure angle

θ_T = train axis angular displacement

θ_x = X axis angular displacement

θ_{xc} = command value for θ_x

θ_y = Y axis angular displacement

θ_{yc} = command value for θ_y

$\underline{\omega}_a$ = angular velocity of $\{\hat{\underline{a}}\}$ frame

ω_{ai} = $\hat{\underline{a}}_i$ components of $\underline{\omega}_a$

$\underline{\omega}_b$ = angular velocity of $\{\hat{\underline{b}}\}$ frame

ω_{bi} = $\hat{\underline{b}}_i$ components of $\underline{\omega}_b$

$\underline{\omega}_f$ = angular velocity of local deck frame $\{\hat{\underline{f}}\}$

ω_{fi} = $\hat{\underline{f}}_i$ components of $\underline{\omega}_f$

ω_n = natural frequency of train axis controller

$\underline{\omega}_p$ = angular velocity of $\{\hat{\underline{p}}\}$

ω_p = frequency of ship's pitching motion

ω_{pi} = $\hat{\underline{p}}_i$ components of $\underline{\omega}_p$

ω_R = frequency of ship's rolling motion

ω_T = $\dot{\psi}_A$ = rate of change of ship's heading

$\underline{\Omega}_a$ = angular velocity of 'A' rotor

Ω_a = constant 'A' rotor spin speed

$\underline{\Omega}_b$ = angular velocity of 'B' rotor

Ω_b = constant 'B' rotor spin speed

ζ = train axis control damping factor

TERMS IN SOLUTIONS

TERM	WHERE DEFINED
d_i (i = 5,6,7,8,11,12,20,21,22) . . .	Appendix F.6
d_i (i = 10,13)	Appendix G
D_i	Appendix E
J_{ai}	Section 5.0
J_{bi}	Section 5.0
r_i	Appendix 5.F
S_i	Appendix F.5
t_i	Appendix C
U_{a3}	Appendix F.2
U_{b1}	Appendix F.2
y_i	Appendix E

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This report presents a rigid body analysis of the dynamics of a shipboard antenna in a three-axis gimbal configuration utilizing Direct Mechanical Stabilization (DMS) for primary control. System parameters are defined and Euler equations applied to obtain a non-linear system model that includes expected torque inputs. A computer simulation is generated with ship input motions due to roll, pitch, turn and flexure and a numerical integration procedure is then employed to determine the motion of the antenna and the resulting pointing error for various scenarios.

The results of the cases studied indicate potential feasibility of this approach for applications requiring pointing accuracy of approximately ±0.15 degree.

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