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FACTORS AFFECTING MEAN POWER RESPONSE TO MULTIPATH RAYS ARRIVIN--ETC(U)  
FEB 80 F HABER, P YEH

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Fred Haber and Paul Yeh, "Factors Affecting Mean Power Response to Multipath Rays Arriving at Different Elevation Angles," and "Multipath Beam Noise Correlation," AFRC QPR No. 32, February 1980.

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# FACTORS AFFECTING MEAN POWER RESPONSE TO MULTIPATH RAYS ARRIVING AT DIFFERENT ELEVATION ANGLES

The power response to a source along the main beam, assumed to be in the y-z plane, was given in QPR No. 31, equation (10), page 30, as

$$|A(\frac{\theta}{2}, \theta_s)|^2 = \sum_{m=1}^M B_m^2 > N \{1 + (N-1) \cdot$$

$$e^{-[c_y^2 k^2 (\sin \theta_m - \sin \theta_s)^2 + c_z^2 k^2 (\cos \theta_m - \cos \theta_s)^2]}\}$$

where it was assumed that elements are distributed normally in dimensions x, y, z, with standard deviation  $c_x$ ,  $c_y$ , and  $c_z$ .  $\theta_m$  are co-latitude angles of arrival of ray m, m=1,2,...,M, and ray magnitude is  $B_m$ . The exponential factors determine the gain variation with element spread and focusing angle  $\theta_s$  (measured with respect to the vertical). To better see how  $c_y$  and  $c_z$  affect the beam width in elevation calculations were made of the two factors.

$$I_y(\theta_m, \theta_s) = e^{-c_y^2 k^2 (\sin \theta_m - \sin \theta_s)^2}$$

and

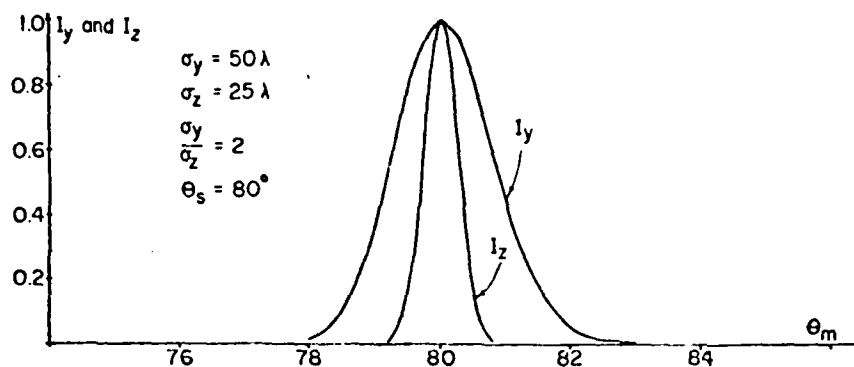
$$I_z(\theta_m, \theta_s) = e^{-c_z^2 k^2 (\cos \theta_m - \cos \theta_s)^2}$$

Figure 3.1 (a),(b),(c),(d),(e),(f) shows some representative results for  $I_y$  and  $I_z$  as a function of  $\theta_m$  with  $\theta_s$  a parameter, and Table I shows the half power beamwidth in elevation, denoted  $2\theta_{0.5}$  corresponding to each function. Note that when  $\theta_s = 90^\circ$ , the array is focused along the y-axis and the mean beamwidth is symmetrical around the y-axis in the y-z plane. When  $\theta_s$  is set to  $80^\circ$  or  $85^\circ$ ,  $I_y$  is a bimodal function of  $\theta_m$  with a local minimum at  $\theta_m = 90^\circ$ . The local minimum is sometimes shallow and sometimes quite deep. When  $I_y$  falls below 0.5 at  $\theta_m = 90^\circ$ , Table I lists the beamwidth of each hump; when  $I_y$  is above 0.5 at  $\theta_m = 90^\circ$ , the beamwidth given is the total range between half power points and runs from a point below the horizontal to a point above the horizontal. For instance, for  $c_y = 20$ ,  $c_z = 4$ ,  $\theta_s = 85^\circ$  the 0.5 power beamwidth determined by  $I_y$  ranges from  $61.75^\circ$  to  $98.25^\circ$ , a range of  $36.5^\circ$ .

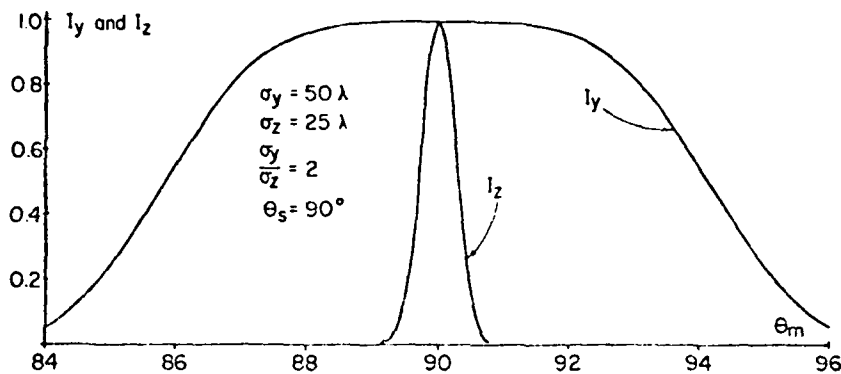
As one might expect  $I_y$  does not usually strongly influence the beamwidth

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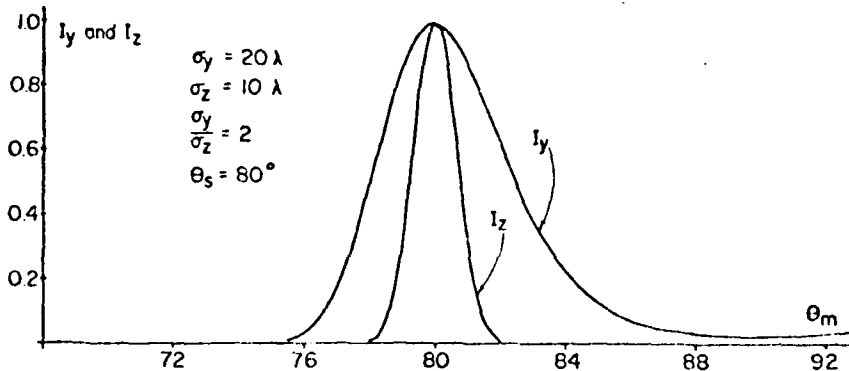
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(a)



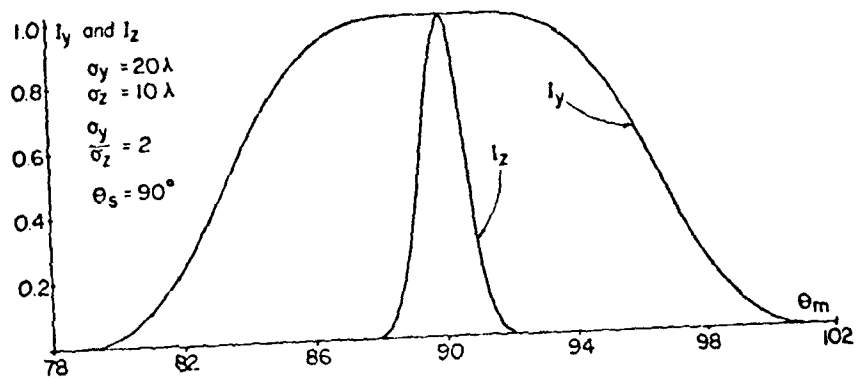
(b)



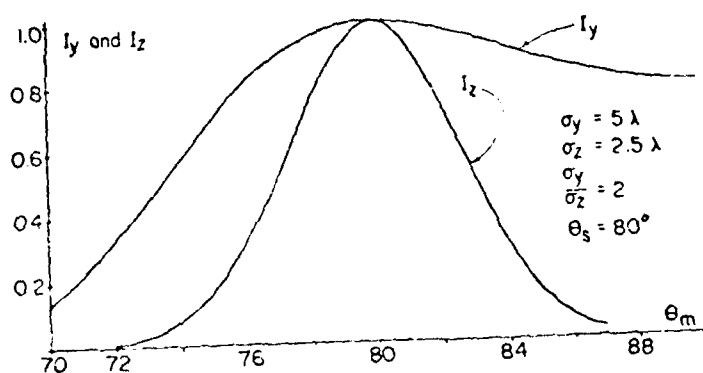
(c)

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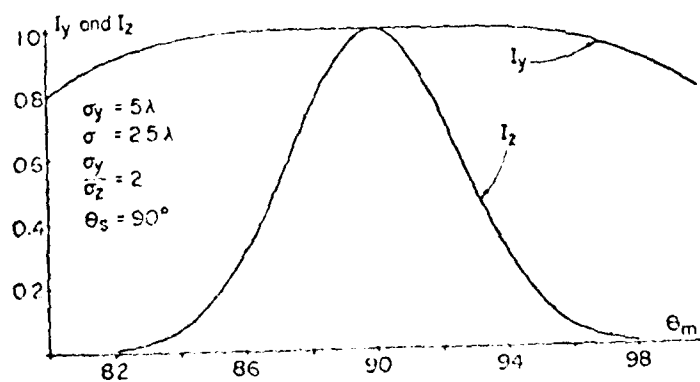
FIGURE 3.1. FACTORS  $I_y$  AND  $I_z$  DETERMINING VERTICAL BEAMWIDTH.  
(a,b,c)



(d)



(e)



(f)

FIGURE 3.1. FACTORS  $I_y$  AND  $I_z$  DETERMINING VERTICAL BEAMWIDTH.  
 (d, e, f)

|                                                  | $\theta_s$                                                                      | $\Delta\theta_m$ on $I_z$                                             | $\Delta\theta_m$ on $I_y$                                                   |
|--------------------------------------------------|---------------------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------------|
| $\sigma_y = 5\lambda$<br>$\sigma_z = 2.5\lambda$ | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 6.1^\circ \\ 6.1^\circ \\ 6.1^\circ \end{array}$    | $\begin{array}{l} > 20.0^\circ \\ 28.2^\circ \\ 33.2^\circ \end{array}$     |
| $\sigma_y = 20$<br>$\sigma_z = 10\lambda$        | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 1.6^\circ \\ 1.5^\circ \\ 1.5^\circ \end{array}$    | $\begin{array}{l} 13.2^\circ \\ 16.6^\circ \\ 4.5^\circ \end{array}$        |
| $\sigma_y = 50$<br>$\sigma_z = 25$               | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 0.6^\circ \\ 0.6^\circ \\ 0.6^\circ \end{array}$    | $\begin{array}{l} 8.4^\circ \\ 3.75^\circ \\ 1.75^\circ \end{array}$        |
| $\sigma_y = 5\lambda$<br>$\sigma_z = \lambda$    | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 15.2^\circ \\ 15.3^\circ \\ 14.2^\circ \end{array}$ | $\begin{array}{l} > 20.0^\circ \\ > 20.0^\circ \\ > 20.0^\circ \end{array}$ |
| $\sigma_y = 20\lambda$<br>$\sigma_z = 4\lambda$  | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 3.8^\circ \\ 3.8^\circ \\ 3.8^\circ \end{array}$    | $\begin{array}{l} 13.0^\circ \\ 16.5^\circ \\ 4.5^\circ \end{array}$        |
| $\sigma_y = 50\lambda$<br>$\sigma_z = 10\lambda$ | $\left\{ \begin{array}{l} 90^\circ \\ 85^\circ \\ 80^\circ \end{array} \right.$ | $\begin{array}{l} 1.5^\circ \\ 1.5^\circ \\ 1.5^\circ \end{array}$    | $\begin{array}{l} 8.4^\circ \\ 3.75^\circ \\ 1.75^\circ \end{array}$        |

TABLE I MEAN 1/2-POWER BEAMWIDTH,  $\Delta\theta_m$ , WITH NORMALLY DISTRIBUTED ELEMENT POSITIONS.

in elevation, though for  $\sigma_y = 50\lambda$ ,  $\sigma_z = 10\lambda$ , and  $\theta_z = 80^\circ$  the two functions are nearly equal. As a rule with  $\sigma_y/\sigma_z = 2$ ,  $I_z$  established the pattern in elevation and it is sufficient to consider it and to let  $I_y = 1$ .

In the work discussed below on the noise correlation we consider elements uniformly dispersed in depth where here we have assumed normally distributed elements. Extraordinary differences between these two cases are not expected though. Similar computations of the uniformly distributed case were therefore not carried out.

Paul Yeh  
Fred Haber

#### MULTIBEAM NOISE CORRELATION

Consider the system in Figure 3.2, QPR No. 31. The branch signal level, when the array is focused in the vertical plane containing the Y-axis and at an

angle  $\theta_{si}$  to the vertical, is given by (2) of QPR 31. The equation is rewritten here

$$A(\theta_{si}) = \sum_{m=1}^M \sum_{n=1}^N B_m e^{j\{ky_n(\sin \theta_m - \sin \theta_{si}) + kz_n(\cos \theta_m - \cos \theta_{si}) + \phi_m\}} \quad (1)$$

The complex weight

$$W_i = A_i e^{-j\psi_i \text{sgnf}} \quad (2)$$

shown in the figure referred to is given by

$$W_i = A^*(\theta_{si}) \quad (3)$$

when noise levels into all the branches have equal mean square value and are independent from branch to branch. The total system signal output is, in this case,

$$W = \sum_{i=1}^I A(\theta_{si}) W_i = \sum_{i=1}^I |A^2(\theta_{si})| = \sum_{i=1}^I A_i^2 \quad (4)$$

and the total noise output has a mean square value given by the second expression p. 35 of QPR 31, namely,

$$\langle N^2(t) \rangle = \sum_{n=1}^N \langle n^2(t) \rangle = \sum_{i=1}^I \sum_{j=1}^I \langle A_i A_j \cos(\psi_{ni} - \psi_{nj} + \psi_i - \psi_j) \rangle \quad (5)$$

where the  $A_i$  are defined by (2). The angles in (5) are given by the following

$$\psi_{ni} = -k[y_n \sin \theta_{si} + z_n \cos \theta_{si}] \quad (6)$$

$$\psi_i = \tan^{-1} \frac{\sum_{m=1}^M \sum_{n=1}^N B_n \sin[\psi_{ni} + \psi_{nm} + \psi_m]}{\sum_{m=1}^M \sum_{n=1}^N B_n \cos[\psi_{ni} + \psi_{nm} + \psi_m]} \quad (7)$$

where we have used (1) and (6) in writing (7) and where we have denoted

$$\psi_{nm} = k(y_n \sin \theta_m + z_n \cos \theta_m) \quad (8)$$

If the noise processes in the different branches were uncorrelated the output mean square noise level would be

$$\langle N^2(t) \rangle = \sum_{n=1}^N \langle n_n^2(t) \rangle \sum_{i=1}^I \langle A_i^2 \rangle \quad (9)$$

The output signal-to-noise ratio (SNR) is here defined by

$$\gamma = \frac{\langle W^2 \rangle}{2 \langle N^2(t) \rangle} = \frac{\left( \sum_{i=1}^I A_i^2 \right)^2}{2N \langle n_n^2(t) \rangle \sum_{i=1}^I \langle A_i^2 \rangle} \quad (10)$$

where we have assumed mean noise power at all elements to be equal and where we have used (2), (3), (4) and (9) to write (10). If the  $A_i$  were not random variables (or if they were to have small variance),  $\gamma$  would be the sum of SNR's of the branches, a result well known for maximal ratio combining of diversity branches in communications. In our situation the branch noises may not be uncorrelated and there is the possibility of larger noise levels. We examine conditions which will give uncorrelated noises. (5) can be expanded to give

$$\begin{aligned} \langle N^2(t) \rangle = & \sum_{n=1}^N \langle n_n^2(t) \rangle \sum_{i=1}^I \sum_{j=1}^I \langle A_i A_j \cos(\phi_{ni} - \phi_{nj}) \cos(\psi_i - \psi_j) \\ & - A_i A_j \sin(\phi_{ni} - \phi_{nj}) \sin(\psi_i - \psi_j) \rangle \end{aligned} \quad (11)$$

$A_i$  and  $\psi_i$  are the amplitude and phase of the complex amplitude on branch  $i$  as given by (1). These random variables are not determined by the position random variables  $y_n$  and  $z_n$  because of the presence of  $\phi_{ni}$  in the exponent of (1) which is uniformly distributed in  $(0, 2\pi)$ . On the other hand, the  $\phi_{ni}$  are determined only by the position random variables  $y_n$  and  $z_n$ . Thus the pair  $(A_i, \psi_i)$  are independent of  $\phi_{ni}$  for all  $i$ . Furthermore from (6) we see that  $\phi_{ni}$  is a linear function of the random variables  $y_n$  and  $z_n$ . We assume the latter to be symmetrically distributed around zero so that  $\langle \sin(\phi_{ni} - \phi_{nj}) \rangle = 0$ . (11) thus reduces to

$$N^2(t) > = \sum_{n=1}^N \langle n_n^2(t) \rangle > \sum_{i=1}^I \sum_{j=1}^I \langle A_i A_j \cos(\psi_i - \psi_j) \rangle \langle \cos(\phi_{ni} - \phi_{nj}) \rangle \quad (12)$$

We point out that if either  $\langle \cos(\phi_{ni} - \phi_{nj}) \rangle$  or  $\langle A_i A_j \cos(\psi_i - \psi_j) \rangle$  is zero for all  $i \neq j$  (12) reduces to (9) that is, the uncorrelated branch noise case is obtained. We investigate

$$\langle \cos(\phi_{ni} - \phi_{nj}) \rangle = \langle \cos k[y_n(\sin \theta_{si} - \sin \theta_{sj}) + z_n(\cos \theta_{si} - \cos \theta_{sj})] \rangle \quad (13)$$

since it is much less complicated than  $\langle \cos(\psi_i - \psi_j) \rangle$ . The angles  $\theta_{si}$  of interest to us are in the range of about  $50^\circ$  to  $100^\circ$ . It can be demonstrated that  $(\sin \theta_{si} - \sin \theta_{sj}) \ll (\cos \theta_{si} - \cos \theta_{sj})$  for angles in this range. Also, if we are to have high vertical resolution, the variance of  $z_n$  will have to be of the same order as the variance of  $y_n$ . Thus we can argue that (13) can be approximated by

$$\langle \cos(\phi_{ni} - \phi_{nj}) \rangle \approx \langle \cos k z_n (\cos \theta_{si} - \cos \theta_{sj}) \rangle \quad (14)$$

so that (12) becomes

$$\begin{aligned} \langle N^2(t) \rangle &= \sum_{n=1}^N \langle n_n^2 \rangle > \sum_{i=1}^I \sum_{j=1}^I \langle A_i A_j \cos(\psi_i - \psi_j) \rangle \\ &\quad \cdot \langle \cos k z_n (\cos \theta_{si} - \cos \theta_{sj}) \rangle \end{aligned} \quad (15)$$

If the rv  $z_n$  is here assumed uniformly distributed in a range  $(-h, h)$  then the rv  $(\phi_{ni} - \phi_{nj})$  will be correspondingly uniformly distributed in a range  $(-a_{ij}, a_{ij})$ . We therefore have

$$\langle \cos(\phi_{ni} - \phi_{nj}) \rangle = \frac{\sin a_{ij}}{a_{ij}} \quad (16)$$

where

$$a_{ij} = kh(\cos \theta_{si} - \cos \theta_{sj}) \quad (17)$$



(16) suggests that to make  $\langle \cos(\phi_{ni} - \phi_{nj}) \rangle = 0$  for  $i \neq j$ , branch foci be placed so that  $a_{ij} = m\pi$ ,  $m$  an integer.

The  $\theta_{si}$  will all be concentrated around  $90^\circ$ , generally in the range  $(80^\circ, 100^\circ)$ . Thus we are led to write

$$\hat{\theta}_{si} = \theta_{si} - 90^\circ$$

so that

$$\cos \theta_{si} = -\sin \hat{\theta}_{si} \doteq -\hat{\theta}_{si}$$

and using (17)

$$a_{ij} \doteq kh (\theta_{sj} - \theta_{si}) \quad (18)$$

The angular separation between beams to get uncorrelated noise outputs at the different branches should therefore be

$$\theta_{sj} - \theta_{si} = \frac{m\pi}{kh}$$

If, for instance,  $h = 25\lambda$  (meaning that the array size in depth is 50 wavelengths which at Hz implies a depth of about 750 meters) adjacent beams ought to be spaced

$$\Delta\theta_s = \frac{\pi}{kh} = 1/50 \text{ rad} \doteq 1.15^\circ$$

in order for the noise variables entering into the final summer to be uncorrelated.

The next step to be taken will be to determine the array power response with array output processed as described above. If the multipath rays were to come in on the center lines of the vertically spread beams the overall array response would be maximized. Some beams are expected however to see no incoming rays, others may see more than one ray. In the latter case the multipath is not resolved by the array processor and the combined rays add non-coherently. The corresponding diversity branch will see a fluctuating level depending on the relative ray phases. While the processor cannot improve the signal level in this case the weighting circuit will take account of this fluctuation to maximize the signal to noise ratio by suppressing the branch output if the signal component is small and amplifying the branch output if the signal component is large.

The calculation of these effects is time consuming. We plan to carry out a simulation of this next step by assuming a fixed number of rays arriving, each uniformly distributed over a range of latitude angles about  $\pm 10^\circ$  relative to the horizontal. Independent noise at each sensor will be assumed. Beams will be spaced as specified above covering the same range of latitudes and the statistical properties of the array output SNR will be found. This calculation will also serve as a test of assumptions made earlier to simplify calculations of noise output.

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 --10A2 - PRIMARY PROJECT NUMBER: F11121  
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 --10A2A - PRIMARY PROJECT AGENCY AND PROGRAM: F11121  
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