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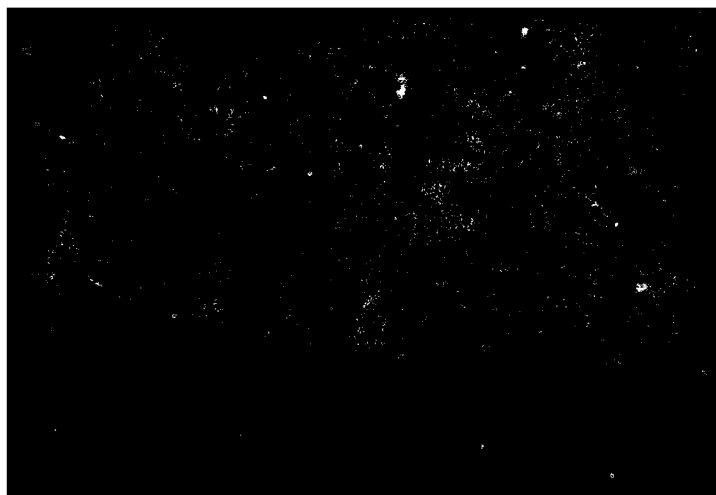
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PLANE TIDAL WAVES GENERATED BY
AN ARRAY OF
SIMULTANEOUS UNDERWATER EXPLOSIONS
THESIS

AFIT/GNE/PH/81M-11 Henry J. Weber
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PLANE TIDAL WAVES GENERATED BY
AN ARRAY OF
SIMULTANEOUS UNDERWATER EXPLOSIONS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Henry J. Weber, B.S.

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Graduate Nuclear Engineering

March 1981

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Henry J. Weber

(This thesis was typed by Sharon A. Gabriel)

Contents

	<u>Page</u>
Acknowledgement-----	ii
List of Figures-----	iv
Abstract-----	v
I. Introduction-----	1
Background-----	1
Problem and Scope-----	1
Assumptions-----	2
Approach and Sequence-----	2
II. Analysis of Problem-----	4
Rationale for Shallow Water Theory-----	4
Parameters Reference Values-----	5
Initial Condition-----	7
III. Computer Program Development-----	10
Derivation of Equations-----	10
Difference Scheme-----	15
Boundary Condition-----	18
Verification-----	18
IV. Discussion and Results-----	20
Yield Sensitivity-----	22
Spacing Sensitivity-----	22
Alignment Sensitivity-----	22
Timing Sensitivity-----	27
Bottom Slope Sensitivity-----	27
Non-Linear Effect-----	30
V. Conclusion-----	33
VI. Recommendation-----	34
Bibliography-----	35
Vita-----	36

List of Figures

<u>Figure</u>		<u>Page</u>
1	Illustration of Coordinate System and Variables-----	11
2	Illustration of Plane Tidal Wave-----	21
3	Yield Sensitivity-----	23
4	Yield Combination Sensitivity-----	24
5	Spacing Sensitivity-----	25
6	Alignment Sensitivity-----	26
7	Timing Sensitivity-----	28
8	Bottom Slope Sensitivity-----	29
9	Non-Linear Contour Printer-Plot-----	31
10	Linear Contour Printer-Plot-----	32

Abstract

This report considers an array of simultaneous underwater nuclear explosions generating plane tidal waves on the continental shelf. Computer simulations are used to study this wave generation process. They are based on non-linear shallow water theory with an initial condition of a stationary raised cone of water. The sensitivity of the wave height to different yields, combination of different yields, spacing, alignment, timing, and bottom slope is determined. The reference values used for this study are a yield of five kilotons TNT, one kilometer spacing, water depth of 100 meters, and no bottom slope. In addition to the sensitivities, the amount of non-linear effect is illustrated by comparison of wave contour printer-plots generated by linear and non-linear shallow water theory. The computer results indicate the wave generation is relatively insensitive to realistic variations in the parameters.

PLANE TIDAL WAVES GENERATED BY
AN ARRAY OF
SIMULTANEOUS UNDERWATER EXPLOSIONS

I. Introduction

Background

This thesis originated from the AFIT GNE-80M class design project which analyzed a shallow underwater mobile (SUM) basing alternative to the land based MX intercontinental ballistic missile weapon system (Ref 1). The SUM system would be deployed over the continental shelf of the United States. This large area of deployment with the location of the SUMs unknown would require a large number of warheads to defeat the weapon system. However, an easy defeat of the SUM based weapon system has been proposed which uses a plane tidal wave generated by an array of simultaneous underwater explosions. The tidal wave would travel across the continental shelf, disabling the SUMs. Computer simulations based on shallow water theory verify the formation of a plane tidal wave. The wave energy resulting from the underwater explosion was approximated with an initial condition of a stationary raised cone of water. Further study of this plane tidal wave generation is the topic of this thesis.

Problem and Scope

The primary purpose of this study is to determine the sensitivity of the plane tidal wave generation to the explosions'

yield, yield combinations, spacing between explosions, alignment of explosions, timing of explosions, and slope of the continental shelf. The reference values used are five kilotons TNT for yield, one kilometer spacing between explosions, and a water depth of 100 meters. The yield is varied from 2 to 8 kilotons. The combination of two yields is varied as a ratio of one yield to the other yield from .25 to 1 with an average yield of 5 kilotons. The spacing is varied $\pm 40\%$ from the reference value of one kilometer. The misalignment is varied from 0.0 to 0.5 kilometer and the timing is varied from 0.0 to 5 seconds. The bottom slope is varied from 0.0 to 0.002. In addition to the sensitivities, the amount that the non-linear effects contribute to the generation of the plane tidal wave is illustrated.

Assumptions

There are two main assumptions. The first is that an explosion on the ocean floor which produces a large enough gas bubble for blowout will generate shallow water waves. The second assumption is part of shallow water theory. It assumes the water's vertical acceleration doesn't affect pressure and the water's vertical velocity doesn't affect the conservation of momentum equation.

Approach and Sequence

The general approach starts by analyzing the problem physically to determine the type of wave generated, realistic

parameter reference values, and modeling of initial condition. Next, the differencing scheme is developed by deriving the shallow water equations and differencing the equations. The scheme is used to simulate the wave generation. The wave heights are evaluated to determine sensitivity to several of the parameters. Finally, the non-linear effects are illustrated.

II. Analysis of Problem

The analysis of this problem focuses on three areas. First is the rationale and basis for using shallow water theory. Next is the determination of the reference values for the independent variables and their range of variation. The last area is the initial condition and its relationship to the explosions' yield.

Rationale for Shallow Water Theory

The rationale for using shallow water theory is primarily based on the type of cavity formed. Shallow water theory can be used if the total depth of the water moves horizontally together. The explosions used for this problem generate a gas bubble with a radius greater than the depth of water. Since the expanding bubble will horizontally excite the total depth of the water, shallow water waves are expected. On this basis, shallow water theory is used.

Research at Los Alamos Scientific Laboratory (LASL) found that shallow water theory was not applicable for a near surface explosion (Ref 6). However, the difference in burst location can account for this. The near surface burst used by LASL formed a hemispherical surface cavity with a radius equal to $1/6$ the depth of the water. The surface cavity would not horizontally excite the total depth of the water at the same time. Therefore, shallow water waves are not expected, which is consistent with shallow water theory not accurately duplicating the experimental data.

Parameters Reference Values

The reference values of the independent parameters and range of variation were selected based on many factors. The water depth and bottom slope are based on the average depth and slope of the continental shelf. The yield was selected based on wave generation efficiency and the minimum yield to ensure blowout. The spacing was limited by the accuracy of the bomb placement. The alignment and timing are assumed perfect for the reference value.

Depth and Slope. The reference water depth is about the average of the continental shelf's depth (0 - 600 ft). The water depth reference value is 100 meters with no variation except when bottom slope is used. The average bottom slope of the continental shelf is .0002 to .0004 based on a width of 300 to 600 miles. The reference value for bottom slope is zero with a variation of 0.0 to .002.

Timing and Alignment. The reference value for timing is zero seconds with a range of variation of 0 to 5 seconds. The reference value for alignment is also zero with a range of variation of 0 to 500 meters. Misalignment distance is the perpendicular distance from the array centerline to the misaligned explosion.

Spacing. The spacing reference value is limited at the lower end by the accuracy of the warhead placement. According to Scientific American, the 1985 estimated circular error

probable (CEP) for Russian missiles is .22 kilometer (Ref 2:54). The reference spacing was limited to a distance of 4*CEP. Based on this, the reference value of one kilometer is used with a variation of .6 to 1.4 kilometer.

Yield. The reference yield (Y) is based on the efficiency of wave formation and the minimum yield required for a blowout case. The efficiency as a function of yield is indicated by the wave height. The wave height (h) is proportional to $Y^{1/6}$ (Ref 5:133). Therefore, the lowest possible yield will be used to obtain the highest efficiency.

The minimum yield required to produce a maximum hemispherical bubble with a radius equal to the water depth of 100 meters is 1.7 kilotons. This is based on the following approximations and calculations. The energy required to form the hemispherical bubble is approximated by the energy required to form an equal volume spherical bubble with coinciding centroids. Approximately half of the explosion's energy is absorbed by the ocean floor (Ref 5:126). Using these approximations and Eq (1) for the maximum bubble radius (Ref 5:113), the minimum yield is determined as follows. L is the max bubble radius, Y is the yield, D is the depth of burst, R_s is the radius of the spherical bubble, and R_h is the radius of the hemispherical bubble.

$$L(ft) = 13.5 \left(\frac{Y(lbs)}{D(ft) + 33} \right)^{1/3} \quad (1)$$

$$L(m) = 349 \left(\frac{Y(kt)}{D(m) + 10} \right)^{1/3} \quad (2)$$

$$Y(kt) = (D(m) + 10) \left(\frac{L(m)}{349} \right)^3 \quad (3)$$

$$R_h = 100 \text{ meters}$$

$$R_s = \left(\frac{1}{2} R_h^3 \right)^{1/3} = 79.4 \text{ meters}$$

To obtain the minimum yield, use Eq (3) with $L = R_s$,
 D equal to the depth of the hemisphere's centroid, and
 $Y = \frac{1}{2} Y$ to account for the energy absorbed by the bottom.

$$\frac{1}{2} Y(kt) = (62.5 + 10) \left(\frac{79.4}{349} \right)^3$$

$$Y = 1.7 \text{ ktons}$$

Therefore, the yield should be equal to or greater than the minimum yield of 1.7 kilotons. A reference yield of 5 kilotons was selected since it is small and yet more than twice the minimum yield to ensure shallow water waves.

Initial Condition

The initial condition used to represent the wave energy imparted by the explosion is very simplistic. It consists of a raised stationary cone of water. The cone surface has a

constant slope of 0.2 with its volume dependent on the yield.

The relationship between the yield and the potential energy of the cone is based on the scaling laws for the blowout case (Ref 5:132-133). In the blowout case, the wave height is scaled by $Y^{1/6}$. Since the potential energy of the wave is proportional to the wave height squared, it is scaled by $Y^{1/3}$. Therefore, the potential energy (PE) of the cone is equal to some constant times $Y^{1/3}$.

$$PE_{\text{cone}} = \text{Const} * Y^{1/3} \quad (4)$$

The constant is obtained by analyzing the potential energy of the cavity. Starting with the minimum yield of 1.7 kilotons, the maximum bubble radius (L) is 100 meters. Assume the water displaced by the bubble is now located at the surface. The amount of increase in potential energy is determined as follows: where ρ is water density (1000 kg/m³) and g is the acceleration of gravity (9.8 m/s²).

$$\Delta PE = (62.5) \left(\frac{1}{2} \frac{4}{3} \pi L^3 \right) \rho g = 1.28 * 10^{12} \text{ Joules} \quad (5)$$

As the bubble collapses, much of the potential energy is lost in turbulence. One half of the wave energy is assumed to be lost in turbulence. Based on this assumption, the constant is determined as follows:

$$\text{Const} = \frac{1}{Y^{1/3}} = \frac{1}{2} \frac{(1.28 \cdot 10^{12} \text{ J})}{(1.7)^{1/3}} = 5.4 \cdot 10^{11} \quad (6)$$

$$\text{PE}_{\text{cone}}(\text{J}) = 5.4 \cdot 10^{11} Y^{1/3}(\text{kt}) \quad (7)$$

The size of the cone is determined by the amount of wave energy imparted by the explosion. The potential energy of the cone of water is set equal to the energy obtained from Eq (7). Solving for the height (h) of the cone's peak determines the cone's size with a constant slope (s) of 0.2.

$$\text{PE}_{\text{cone}} = \pi g \rho \frac{h^4(\text{m})}{12 s^2} \quad (8)$$

$$h(\text{m}) = \left[\frac{(5.4 \cdot 10^{11}) Y^{1/3} 12 s^2}{\pi g \rho} \right]^{1/4} \quad (9)$$

$$h(\text{m}) = 54 Y^{1/12}(\text{kt}) \quad (10)$$

III. Computer Program Development

The computer program development includes the derivations and differencing of the equations and treatment of the boundary conditions. Both the non-linear and linear equations will be derived and differenced. In addition, a check of the differencing scheme is accomplished.

Derivation of Equations

The shallow water wave equations are derived from the conservation of mass and conservation of momentum equations using the 3-dimensional rectangular coordinate system. The viscosity and compressibility of the water are ignored. The coordinate system is as shown in Figure 1 with the free-standing water surface at $z = 0$. The actual water surface position is described by $h(x,y,t)$ and the bottom is described by $b(x,y)$. The averaged velocity components u , v , and w are in the x , y , and z directions, respectively, with $\bar{S}$ representing the velocity vector (u,v,w) .

The derivation starts with the conservation of mass equation (Ref 4:2,24) and its integration with respect to z .

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \bar{S} = 0 \quad (11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (12)$$

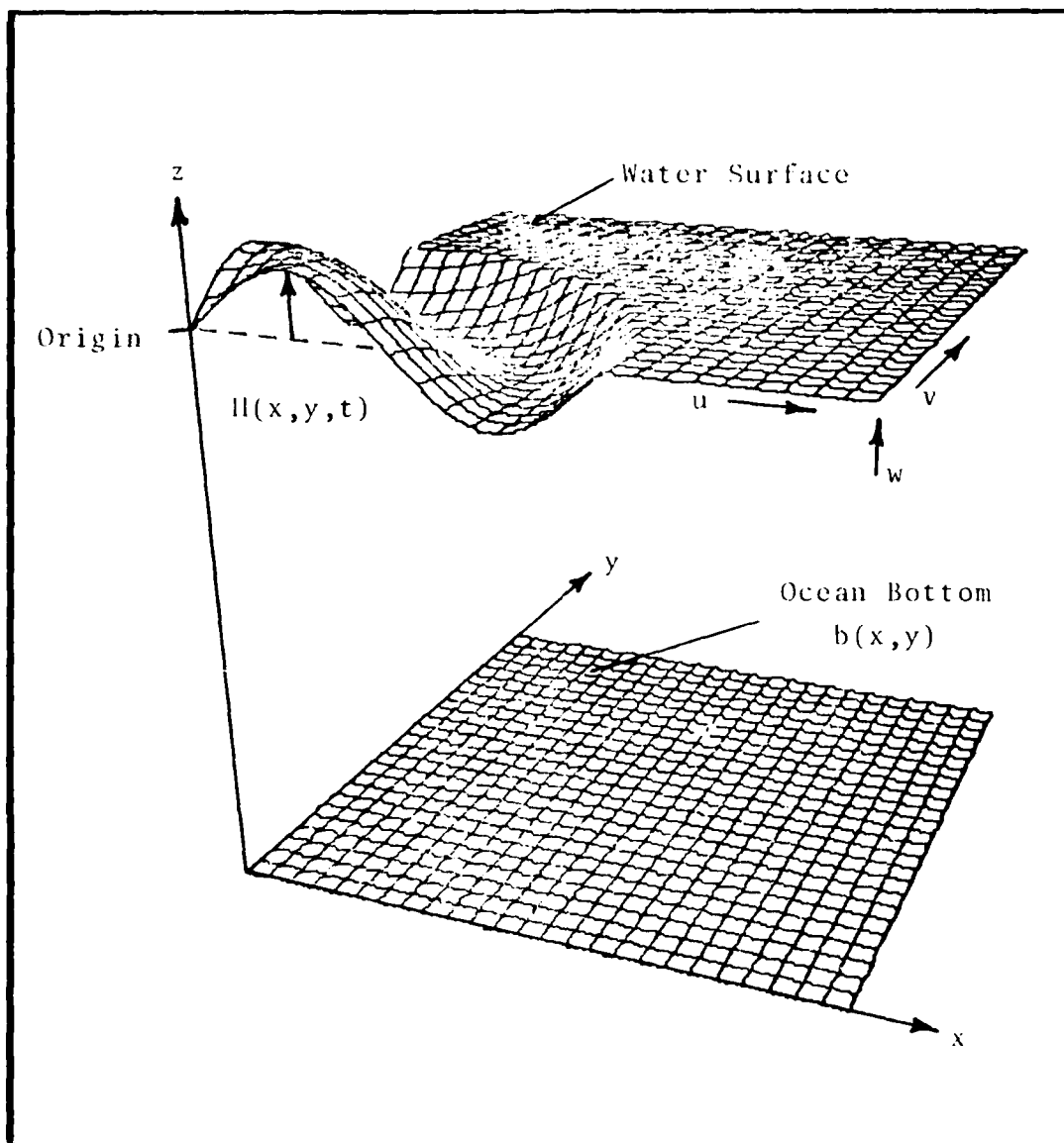


Figure 1. Illustration of Coordinate System and Variables.

$$\int_b^H \frac{\partial u}{\partial x} dz + \int_b^H \frac{\partial v}{\partial y} dz + w \Big|_b^H = 0 \quad (13)$$

The boundary conditions are used to determine w for $z = H$ and $z = b$. The values of w at these points are used to expand the integral of the conservation of mass Eq (13).

$$w_{z=H} = \frac{\partial H}{\partial t} + u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \quad (14)$$

$$w_{z=b} = u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} \quad (15)$$

$$\begin{aligned} \int_b^H \frac{\partial u}{\partial x} dz + \int_b^H \frac{\partial v}{\partial y} dz + \frac{\partial H}{\partial t} + u \frac{\partial H}{\partial x} \\ + v \frac{\partial H}{\partial y} - u \frac{\partial b}{\partial x} - v \frac{\partial b}{\partial y} = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\partial H}{\partial t} + \int_b^H \frac{\partial u}{\partial x} dz + \int_b^H \frac{\partial v}{\partial y} dz + u \frac{\partial (H-b)}{\partial x} \\ + v \frac{\partial (H-b)}{\partial y} = 0 \end{aligned} \quad (17)$$

Now the first approximation must be made for the shallow water theory. The vertical acceleration of the water is assumed not to affect the pressure (P) (Ref 8:23-24). This results in hydrostatic pressure, $P = \rho g(H-z)$ where $b \leq z \leq H$. Hydrostatic pressure provides a horizontal

pressure gradient which is independent of z . Therefore, the horizontal accelerations and velocities are independent of z . With this approximation, the integration of the conservation of mass equation (17) is completed.

$$\begin{aligned} \frac{\partial H}{\partial t} + (H-b) \frac{\partial u}{\partial x} + (H-b) \frac{\partial v}{\partial y} \\ + u \frac{\partial (H-b)}{\partial x} + v \frac{\partial (H-b)}{\partial y} = 0 \end{aligned} \quad (18)$$

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} [u(H-b)] + \frac{\partial}{\partial y} [v(H-b)] = 0 \quad (19)$$

The conservation of momentum equation (Ref 4:3,24) is used to derive the velocity equations. The previous approximation is expanded to accomplish this. The vertical velocity w is assumed to have a negligible effect on the conservation of momentum. Therefore, the velocity vector $\bar{S}$ becomes (u,v) .

$$\rho \frac{\partial u}{\partial t} + \rho \nabla \cdot u \bar{S} + \frac{\partial P}{\partial x} = 0 \quad (20)$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + g \frac{\partial H}{\partial x} = 0 \quad (21)$$

The equation for v is obtained similarly.

$$\frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial y} + \frac{\partial uv}{\partial x} + g \frac{\partial H}{\partial y} = 0 \quad (22)$$

Equations (19), (21) and (22) are the non-linear shallow water equations. The linear shallow water equations are obtained by assuming u , v , H , and their derivatives are small enough so their squares and products can be neglected (Ref 8:24). This results in linear equations, where $-b$ is simply the depth of the water.

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} [u(-b)] + \frac{\partial}{\partial y} [v(-b)] = 0 \quad (23)$$

$$\frac{\partial u}{\partial t} + g \frac{\partial H}{\partial x} = 0 \quad (24)$$

$$\frac{\partial v}{\partial t} + g \frac{\partial H}{\partial y} = 0 \quad (25)$$

If the water depth is assumed constant, the variables u and v can be eliminated as follows to obtain the linear wave Eq (30) (Ref 8:25).

$$\frac{\partial H}{\partial t} - b \frac{\partial u}{\partial x} - b \frac{\partial v}{\partial y} = 0 \quad (26)$$

$$\frac{\partial^2 H}{\partial t^2} - b \frac{\partial^2 u}{\partial x \partial t} - b \frac{\partial^2 v}{\partial y \partial t} = 0 \quad (27)$$

$$\frac{\partial^2 u}{\partial x \partial t} = -g \frac{\partial^2 H}{\partial x^2} \quad (28)$$

$$\frac{\partial^2 v}{\partial y \partial t} = -g \frac{\partial^2 H}{\partial y^2} \quad (29)$$

$$\frac{\partial^2 H}{\partial t^2} + gb \left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} \right] = 0 \quad (30)$$

This shows that the characteristic wave velocity is $\sqrt{g(-b)}$.

Difference Scheme

The second order explicit difference scheme is three levels in time and space, conservative, and has an amplification factor of unity. Correct space and time centering provide an amplification factor of unity if the Courant-Friedrich-Lewy criterion is satisfied (Ref 7:288). This criterion is $|u \frac{\Delta t}{\Delta x}| + |v \frac{\Delta t}{\Delta y}| \leq 1$. Using the characteristic velocity equally distributed between u and v , the criterion value is 0.44 for $\Delta t = .5$ sec and $\Delta x = \Delta y = 50$ meters. The difference equations are based on the integral conservation of mass Eq (19) and the conservation of momentum Eqs (21) and (22).

$$\frac{\partial H}{\partial t} = - \frac{\partial}{\partial x} \{u(H-b)\} - \frac{\partial}{\partial y} \{v(H-b)\} \quad (31)$$

$$\frac{\partial u}{\partial t} = - \frac{\partial u^2}{\partial x} - \frac{\partial uv}{\partial y} - g \frac{\partial H}{\partial x} \quad (32)$$

$$\frac{\partial v}{\partial t} = - \frac{\partial v^2}{\partial y} - \frac{\partial uv}{\partial x} - g \frac{\partial H}{\partial y} \quad (33)$$

Kinematic viscosity can be added to the last two equations to account for viscosity or to help prevent instability. The kinematic viscosity is incorporated by

adding to the right side of Eqs (52) and (53) a kinematic viscosity coefficient, multiplied by the Laplacian of u and v , respectively. A slight diffusion coupling might also be added to keep the independent conservative flows in step. This is done by adding the product of a coupling coefficient and the Laplacian of H to the right side of Eq (31).

The difference equations use the indices i , j , and n for the variables x , y , and t , respectively. The equations are second order in time and space with no kinematic viscosity or diffusion coupling as shown below.

$$\begin{aligned}
 H_{i,j}^{n+1} = & H_{i,j}^{n-1} - \frac{\Delta t}{\Delta x} \left[u_{i+1,j}^n \left(H_{i+1,j}^n - b_{i+1,j}^n \right) \right. \\
 & \left. - u_{i-1,j}^n \left(H_{i-1,j}^n - b_{i-1,j}^n \right) \right] - \frac{\Delta t}{\Delta y} \left[v_{i,j+1}^n \left(H_{i,j+1}^n - b_{i,j+1}^n \right) \right. \\
 & \left. - v_{i,j-1}^n \left(H_{i,j-1}^n - b_{i,j-1}^n \right) \right] \quad (54)
 \end{aligned}$$

$$\begin{aligned}
 u_{i,j}^{n+1} = & u_{i,j}^{n-1} - \frac{\Delta t}{\Delta x} \left[u_{i+1,j}^n{}^2 - u_{i-1,j}^n{}^2 \right. \\
 & \left. + g \left(H_{i+1,j}^n - H_{i-1,j}^n \right) \right] - \frac{\Delta t}{\Delta y} \left[u_{i,j+1}^n v_{i,j+1}^n - u_{i,j-1}^n v_{i,j-1}^n \right] \quad (55)
 \end{aligned}$$

$$\begin{aligned}
v_{i,j}^{n+1} = & v_{i,j}^{n-1} - \frac{\Delta t}{\Delta y} \left[v_{i,j+1}^n - v_{i,j-1}^n \right. \\
& \left. + g \left(\eta_{i,j+1}^n - \eta_{i,j-1}^n \right) \right] \\
& - \frac{\Delta t}{\Delta x} \left[u_{i+1,j}^n v_{i+1,j}^n - u_{i-1,j}^n v_{i-1,j}^n \right] \quad (36)
\end{aligned}$$

The linear difference equation is based on the linear wave Eq (30). It is second order with space and time centering.

$$\frac{\partial^2 \eta}{\partial t^2} = -gb \left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right] \quad (37)$$

$$\begin{aligned}
\eta_{i,j}^{n+1} = & 2 \eta_{i,j}^n - \eta_{i,j}^{n-1} \\
& - gb \frac{\Delta t^2}{\Delta x^2} \left[\eta_{i+1,j}^n - 2 \eta_{i,j}^n + \eta_{i-1,j}^n \right] \\
& - gb \frac{\Delta t^2}{\Delta y^2} \left[\eta_{i,j+1}^n - 2 \eta_{i,j}^n + \eta_{i,j-1}^n \right] \quad (38)
\end{aligned}$$

The difference scheme used a standard rectangular mesh with all the point values calculated for each time step. Symmetry was used as much as possible with the aid of reflecting boundaries.

Boundary Conditions

Reflecting boundary conditions are used in the program to take advantage of symmetry (Ref 7:288-291). With reflecting boundaries, the boundary conditions are set up to ensure no mass flow across the boundary.

First, the velocity components perpendicular to the boundary are set to zero at the boundary and both velocity components are zero at the corners. The perpendicular velocity components just outside the boundary are set equal to the negative of the perpendicular components just inside the boundary. The parallel velocity components just outside the boundary are set equal to the parallel velocity components just inside the boundary. The surface height just outside the boundary is set equal to the surface height just inside the boundary. The corners are treated similarly with the corner point being the center of symmetry (e.g., if corner point is (2,2) , the $V(1,2) = -V(3,2)$, $U(1,2) = 0$, and $U(1,1) = -U(3,3)$).

Verification

The program based on these difference equations and boundary conditions was verified by duplicating the results obtained by Mader using his SWAN code (Ref 6). The problem solved by Mader consisted of a .5 meter radius hemispherical surface cavity in water three meters deep. The cavity was the result of a near surface burst. This program used the

same time step of 0.001 seconds and cell size of .06 meters square. The results of this program agreed very well with the results obtained by Mader.

IV. Discussion and Results

Computer simulations based on the difference equations, initial conditions, and boundary conditions were used to determine the sensitivity of the tidal wave generation. Figure 2 illustrates the general shape of the tidal wave generated by the computer simulations. The wave velocity is 33(m/s), which is very close to the characteristic wave velocity of 31.3(m/s). The initial condition used produced a wave with a period of about 50 seconds which is nearly three times the expected period of 18 seconds based on the relationship $\text{Period} \propto 14.1 Y^{0.144}$ (Ref 3:272). This indicates the wavelength of 1600 meters produced by this initial condition is also about three times too long. The height of the initial wave above the surface was selected as the indicator of the wave generation sensitivity. The wave height is measured at a nearly constant distance of 8 kilometers from the explosions. The wave height was obtained from the printer-plots which expressed the wave height within increments of .25 meters. The error bars on the graphs indicate the variation in wave height along the plane tidal wave. Both the linear and non-linear difference schemes used the same time step of 0.5 seconds and cell size of 50 meters square.

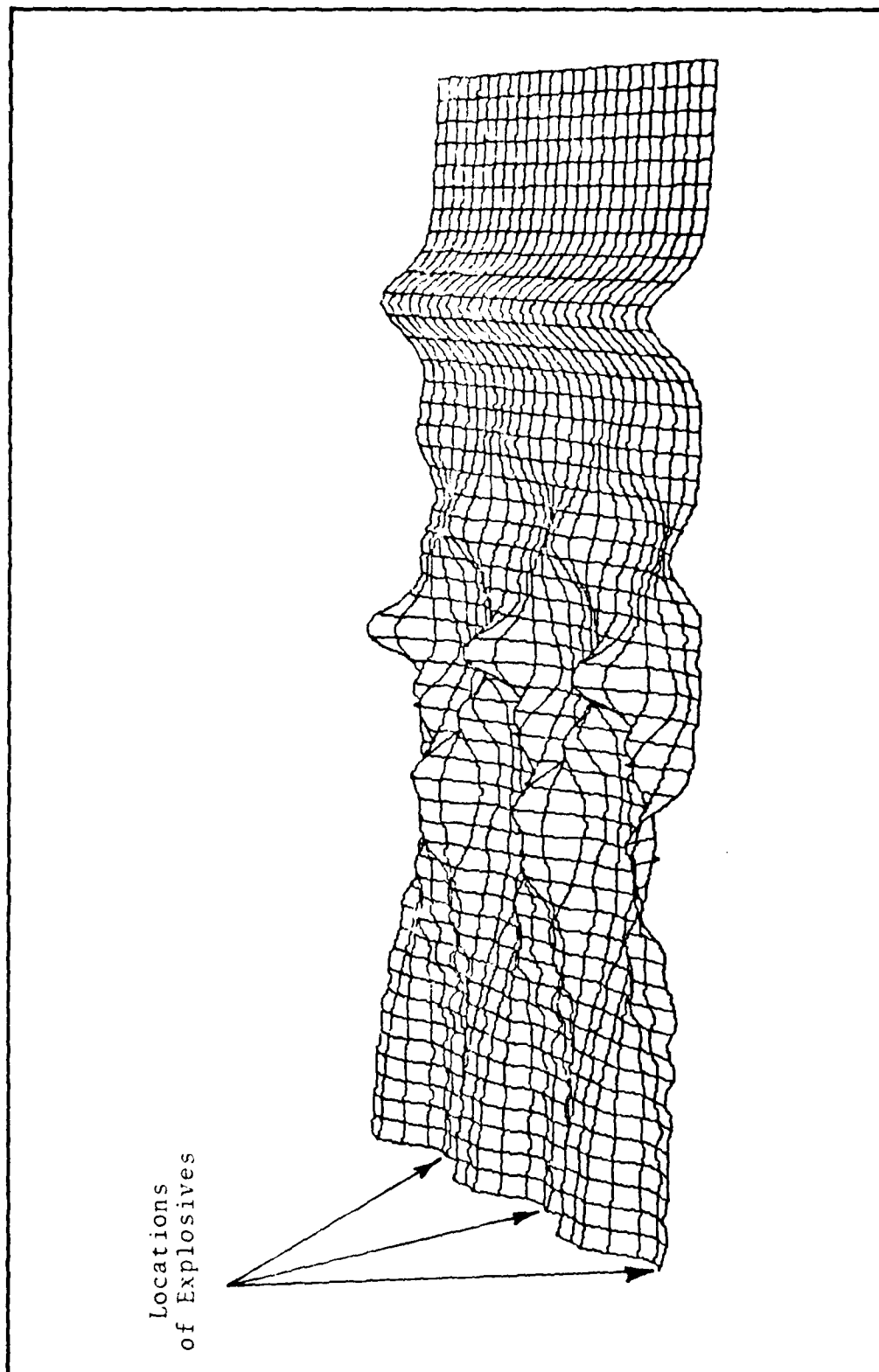


Figure 2. Illustration of Plane Tidal Wave

Yield Sensitivity

The wave height sensitivity to yield is proportional to $Y^{1/3}$. This does not agree with the scaling of wave height as $Y^{1/6}$ used for the initial condition (Ref 5:133). However, it does come close to the scaling of wave height as $Y^{.54}$ for deep water and $Y^{1/4}$ for shallow water used by Glasstone (Ref 3:272-273). The sensitivity to yield over the range of 2 - 8 kilotons is $+4.5(m/(kt)^{1/3})$.

The wave sensitivity to combinations of two yields whose average is 5 kilotons is small. The sensitivity is determined relative to the ratio of the small yield over the larger yield. As shown in Figure 4, the wave height is not very sensitive to combinations of yields over a practical range of 0.5 to 1.

Spacing Sensitivity

The spacing sensitivity was determined over the range of .6 to 1.4 kilometers. The wave height did not change linearly with spacing as shown in figure 5. Over the range of 0.8 - 1.2 kilometers spacing, the approximate sensitivity is $-11.25(m/km)$ spacing.

Alignment Sensitivity

The sensitivity to misalignment was determined for misalignments up to .5 kilometer as indicated in Figure 6. The misalignment situation was set up by having every other explosion offset a given distance from the array. The

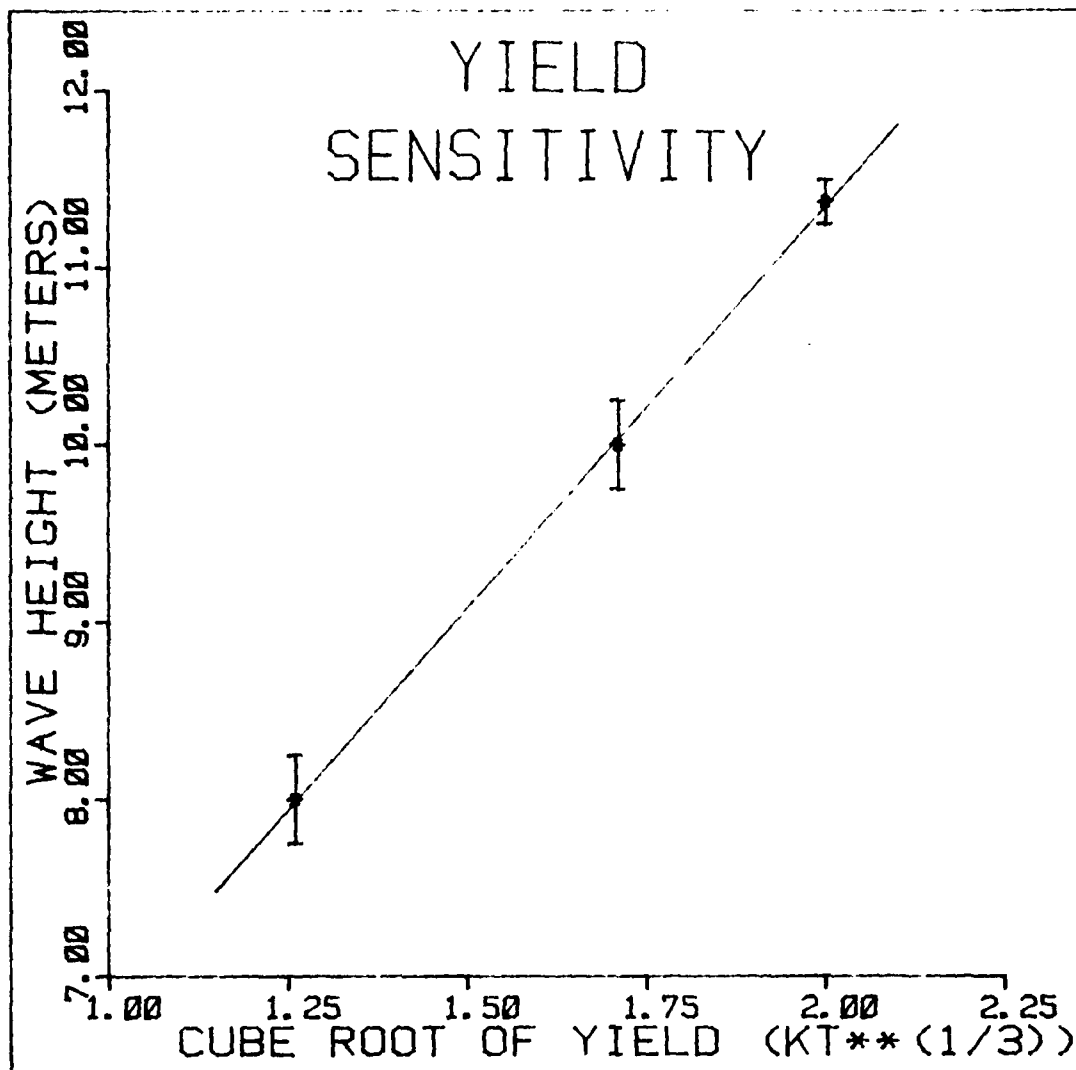


Figure 3. Yield Sensitivity

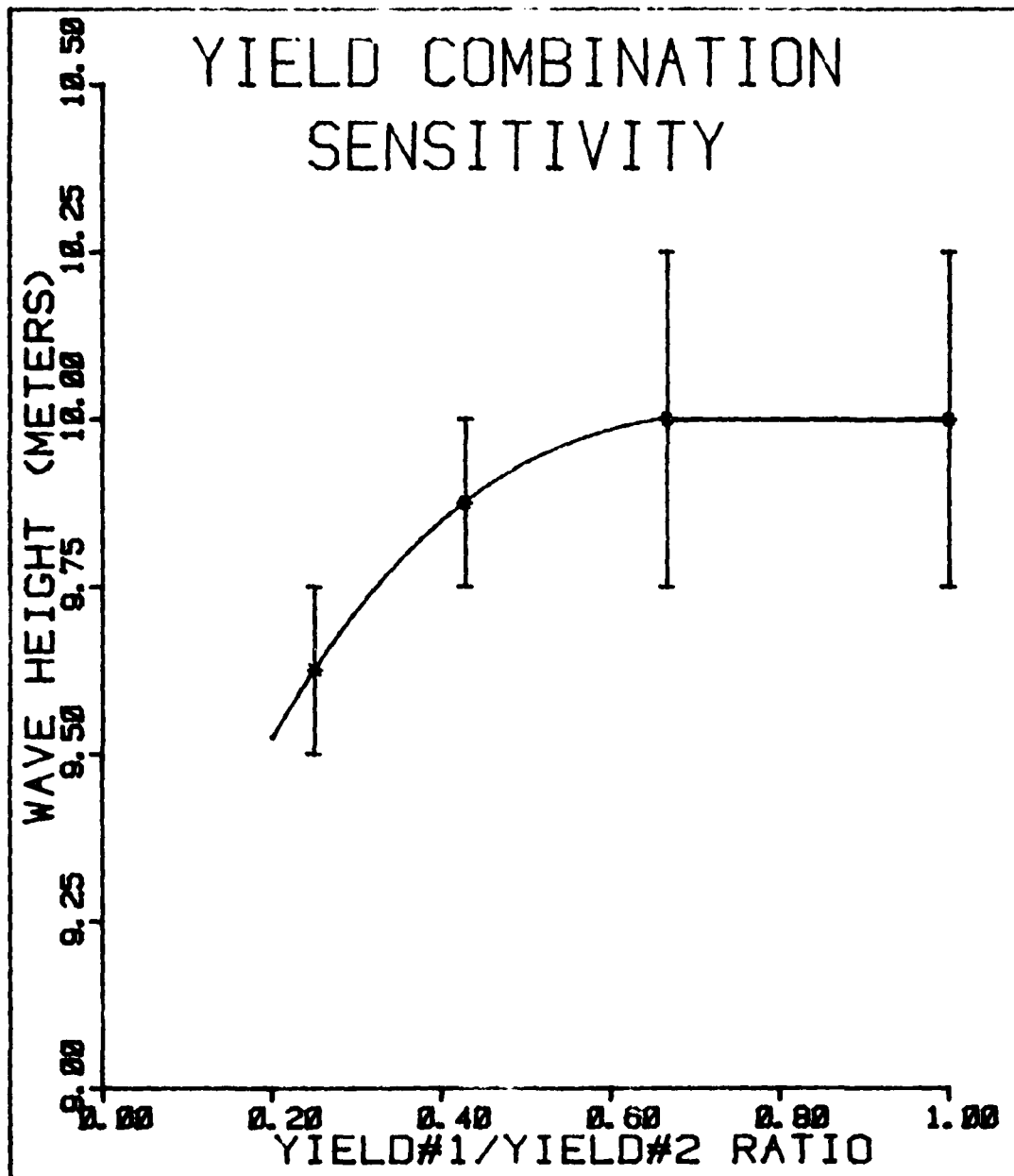


Figure 4. Yield Combination Sensitivity

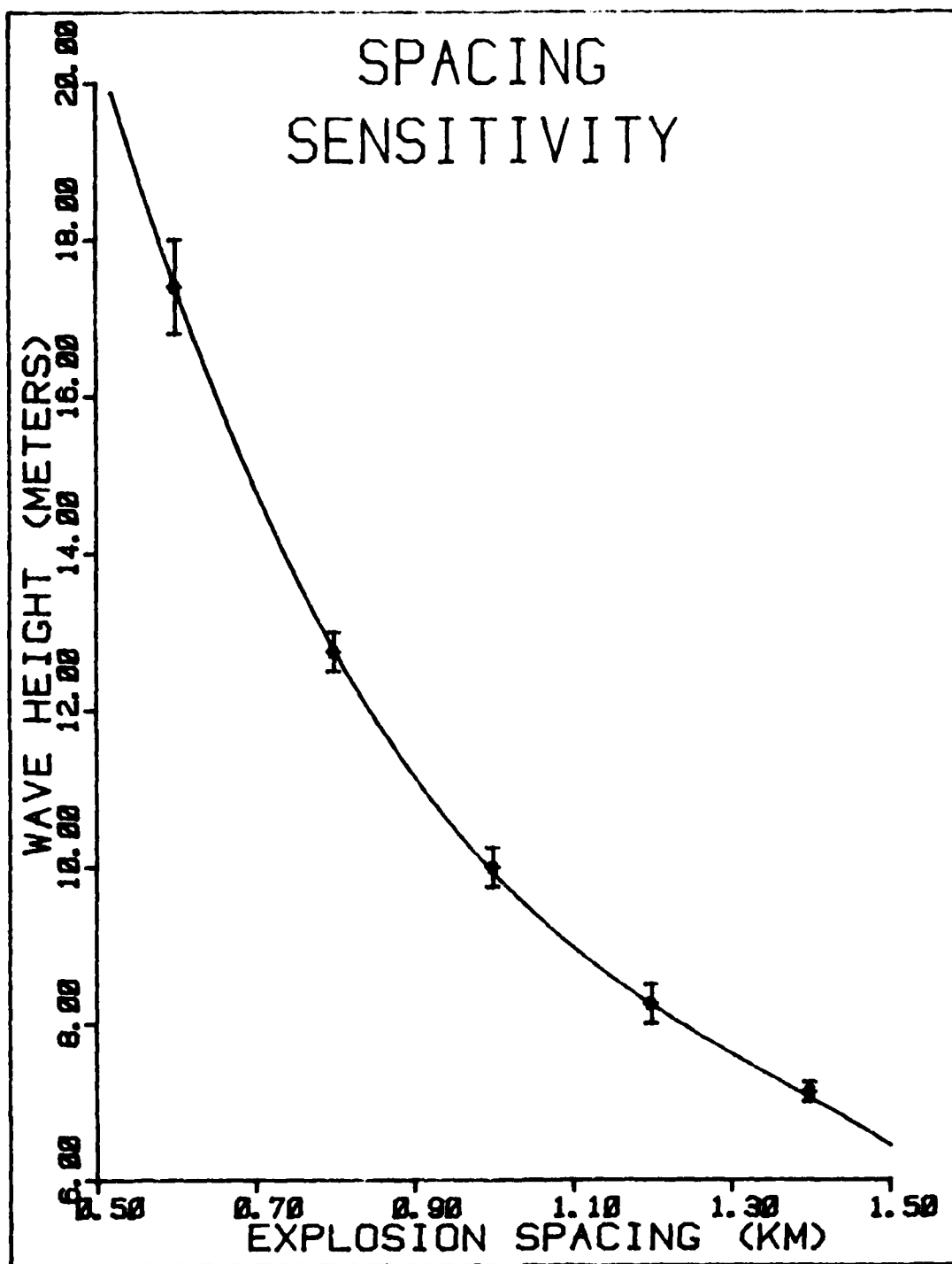


Figure 5. Spacing Sensitivity

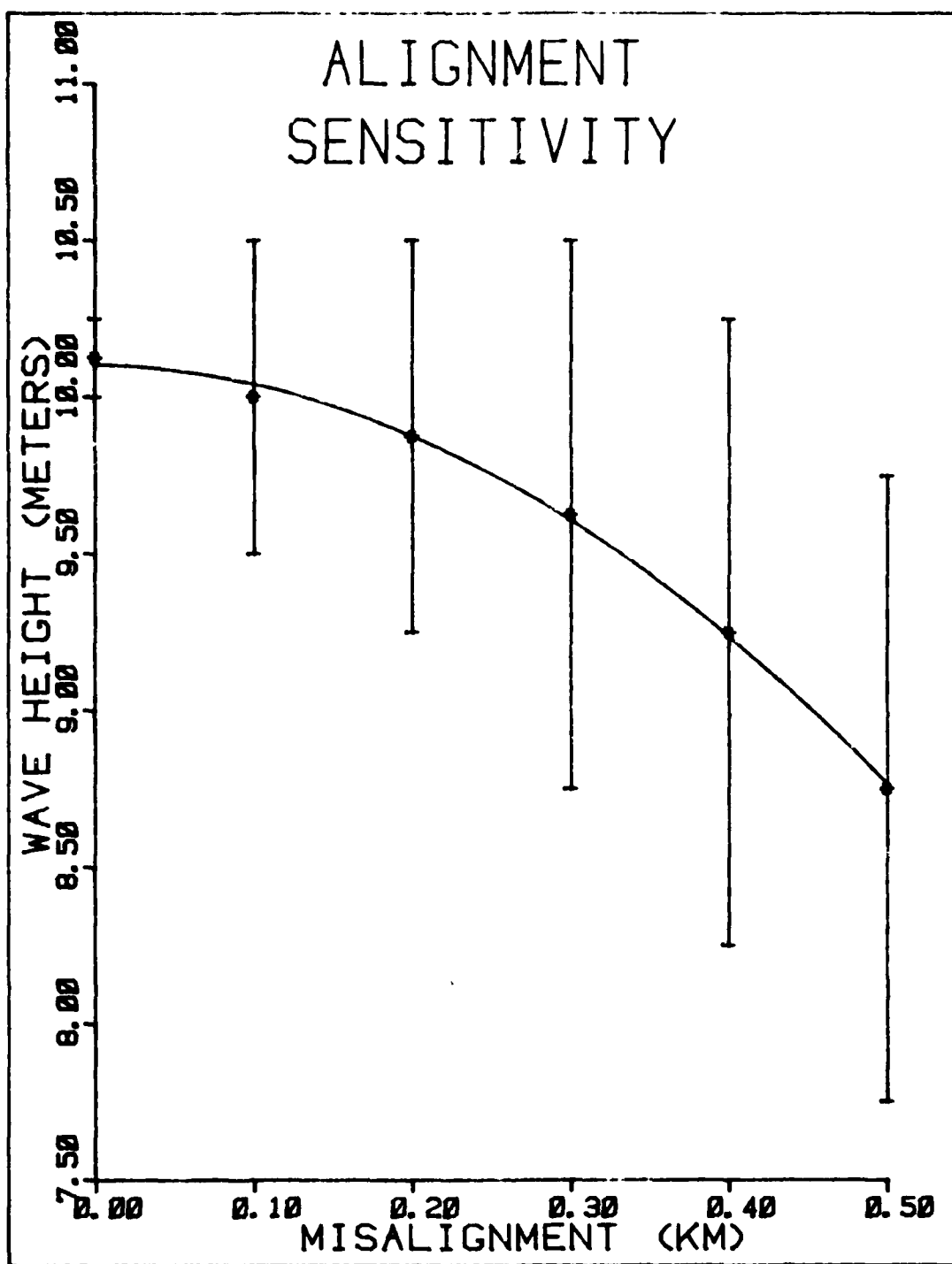


Figure 6. Alignment Sensitivity

misalignment distance is defined as this offset distance. The approximate sensitivity over the range of 0 to .5 kilometer is $-2.7(\text{m/km})$ misalignment. The misalignments did increase the wave height variation along the wave front as indicated by the error bars.

Timing Sensitivity

The wave height sensitivity to timing delays of 0 - 5 seconds is very small, as shown in Figure 7. The time delay situations are modeled by having every other explosion delayed the same amount of time. In the range of 0 - 2.5 seconds delay, the sensitivity is only $-.026(\text{m/sec})$ time delay.

Bottom Slope Sensitivity

The average slope of the continental shelf had a negligible effect on the wave height in 8 kilometers of travel. The bottom was treated as being smooth and with constant slope. The slope was varied from 0.0 to 0.002 as indicated by Figure 8. The wave height sensitivity at 8 kilometers from the explosions is $+182(\text{m})$ per the slope. Assuming the effect was constant over the 8 kilometers, the sensitivity was $+23(\text{m/km})$ per kilometer traveled by the wave per the slope. For example, using the averaged continental shelf slope of 0.0003, the expected increase in wave height in 100 kilometers is 0.7 meters.

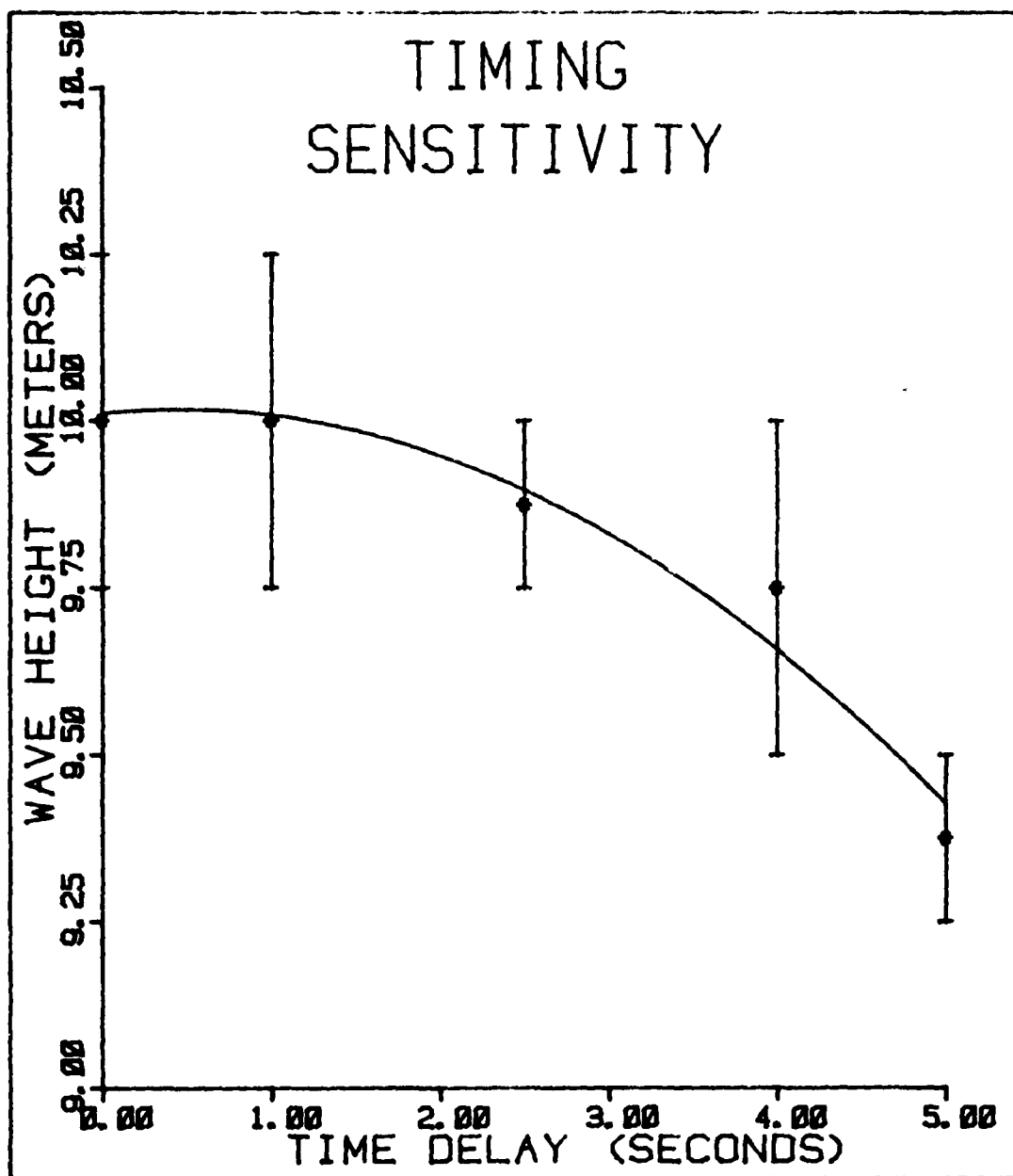


Figure 7. Timing Sensitivity

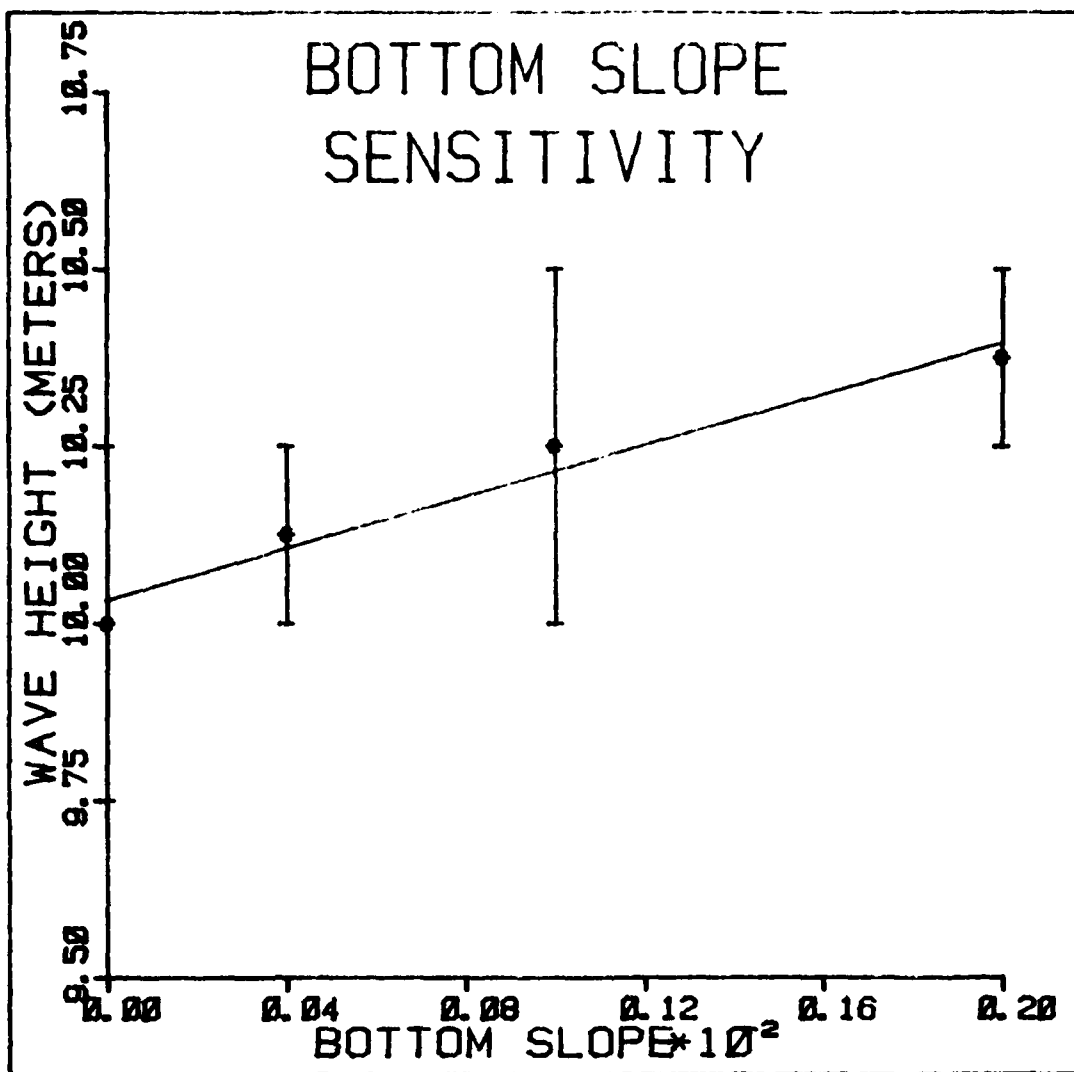


Figure 8. Bottom Slope Sensitivity

Non-Linear Effects

The non-linear effects are illustrated by comparing the wave formation of a linear and a non-linear computer simulation. Wave contour printer-plots of Figures 9 and 10 show the non-linear and linear initial waves, respectively, at four kilometers from the explosions. Each line of the contour plots indicates a change in height of two meters. The troughs or areas of the water surface below the normal free surface are not represented by contour lines, but left blank. The plots show how the non-linear effects produce a slightly smoother and larger wave in the same distance. However, the linear simulation provides a very good approximation for the initial conditions of this study.

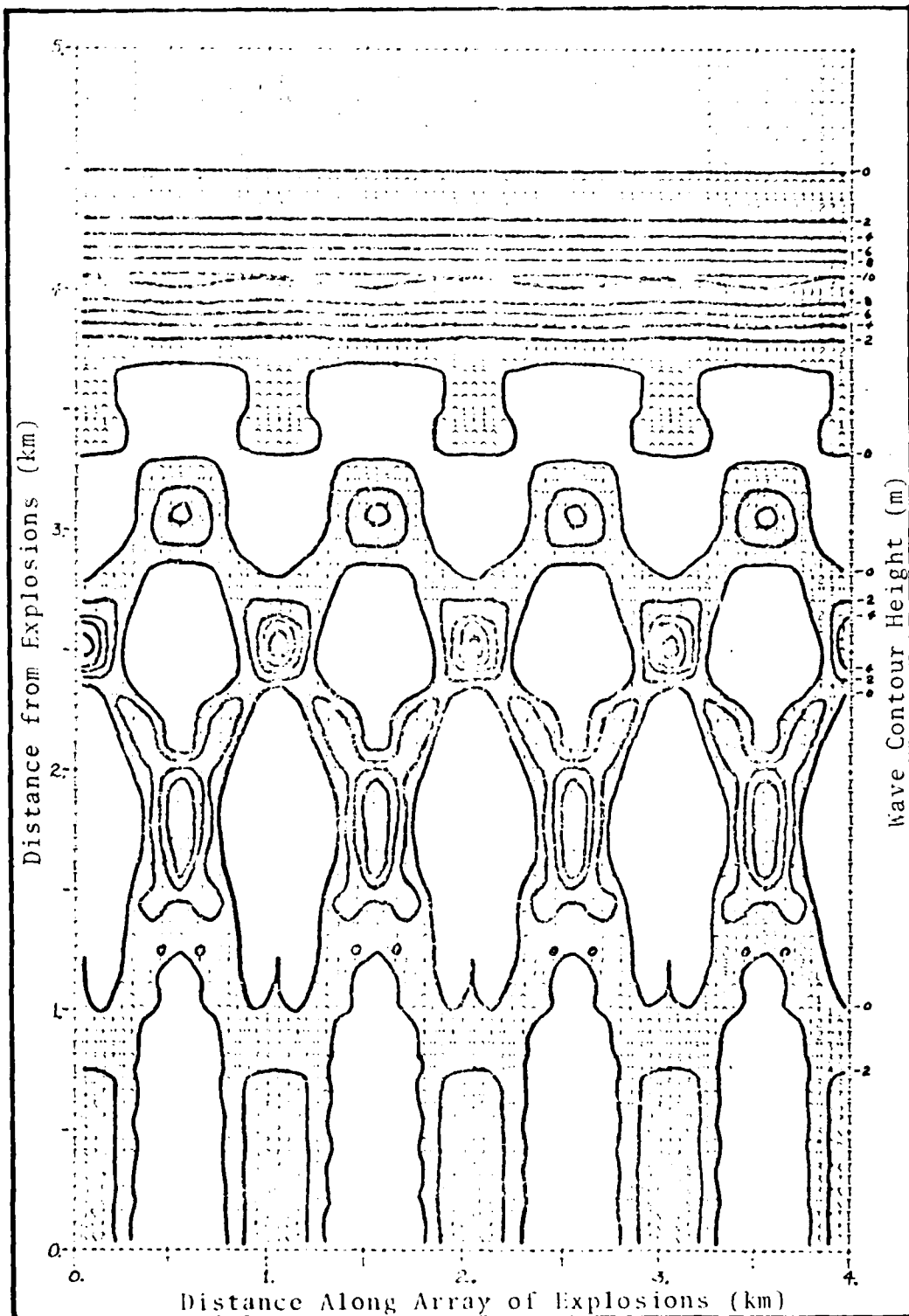


Figure 9. Non-Linear Contour Printer-Plot

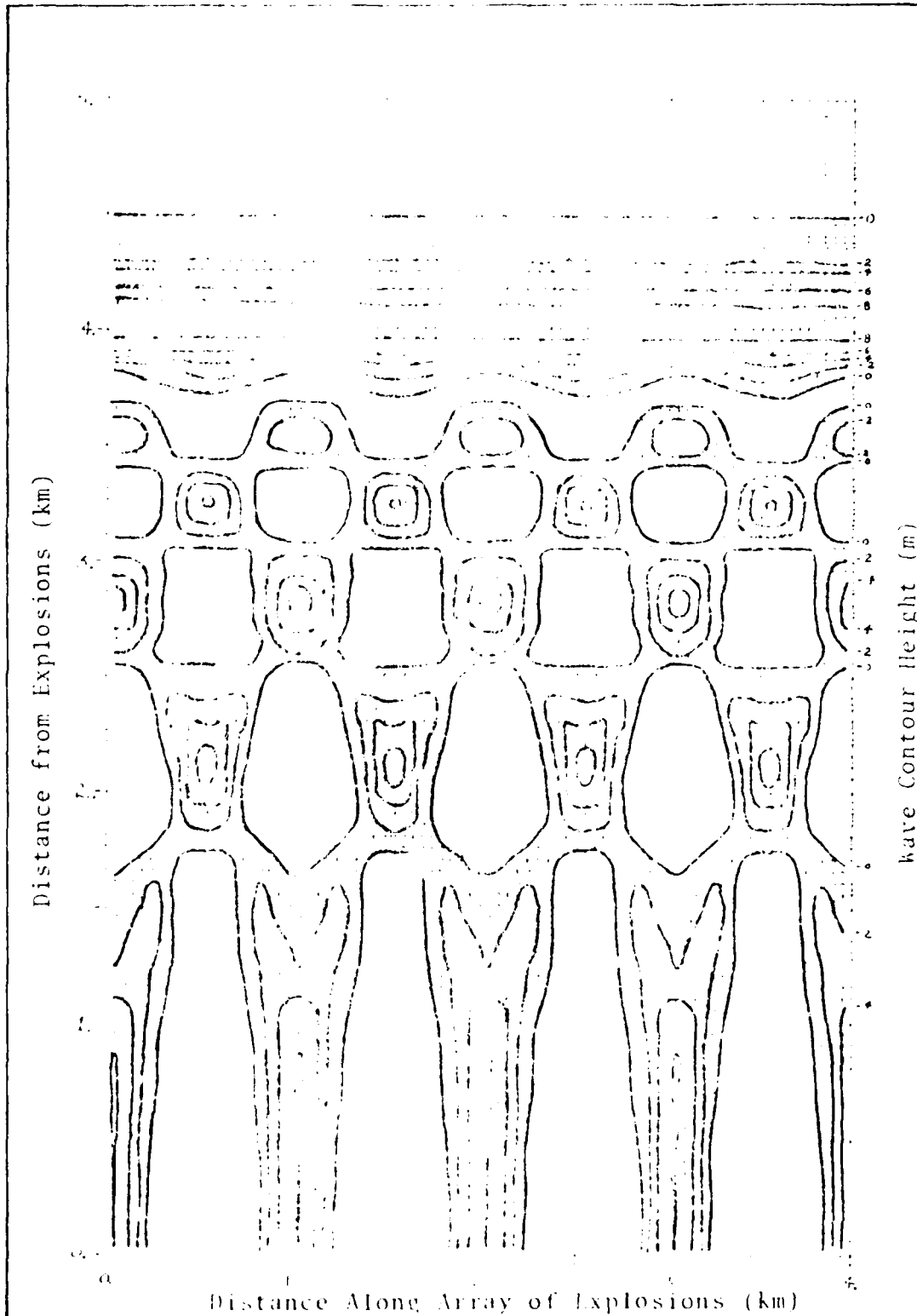


Figure 10. Linear Contour Printer-Plot

V. Conclusion

An array of simultaneous underwater explosions is very effective in generating a plane tidal wave. This conclusion is based on shallow water theory with the explosion represented by a raised cone of water which produces waves with a wavelength of about 1600 meters.

The sensitivity of the wave generation process was determined by the height of the initial wave. The wave height was not very sensitive to misalignments less than 200 meters, time delays less than 5 seconds, bottom slope less than 0.0004, and ratios of yields in the range of 0.5 to 1.0. The wave height sensitivity to spacings of $1.0 \pm .2$ kilometers is approximately -11.25 meters per kilometers spacing. The yield sensitivity, which depends on the derived initial condition, is 4.55 meters per cube root of the yield in kilotons. The non-linear effects increase the size of the tidal wave.

VI. Recommendation

More research on the initial condition is recommended. The waves produced by the raised cone of water have a wavelength about three times too long and the initial wave is not preceded by a trough as normally expected (Ref 3:273). In addition, the cone does not conserve the volume of water. A new initial condition should be developed which conserves the water volume and produces waves with a shorter wavelength. It is recommended that a cylindrical cavity with its depth equal to its radius (100 meters) and with a lip of water surrounding it be used as a new initial condition.

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This report considers an array of simultaneous underwater nuclear explosions generating plane tidal waves on the continental shelf. Computer simulations are used to study this wave generation process. They are based on non-linear shallow water theory with an initial condition of a stationary raised cone of water. The sensitivity of the wave height to different yields, combination of different yields, spacing, alignment, timing, and bottom (Continued on Reverse)		

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slope is determined. The reference values used for this study are a yield of five kilotons TNT, one kilometer spacing, water depth of 100 meters, and no bottom slope. In addition to the sensitivities, the amount of non-linear effect is illustrated by comparison of wave contour printer-plots generated by linear and non-linear shallow water theory. The computer results indicate the wave generation is relatively insensitive to realistic variations in the parameters.

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