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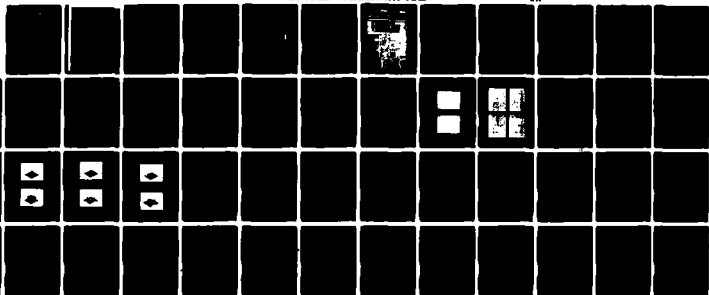
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INTERIM REPORT OF AFOSR

AD A101167

"Investigation of Surface-Scattering Losses of
III-V Compound Semiconductors"

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10 June 1981

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Stages 1 and 2 of the proposed program were completed during the first 18 months of this effort. A satellite chamber for device fabrication was designed, built and incorporated into the main high vacuum chamber. The free surface properties of GaAs (110) and GaP (110) were studied using LEED and AES. Reports on this work were presented at scientific meetings. The problem of a wave propagating with losses in a rectangular waveguide was formulated and solved.		

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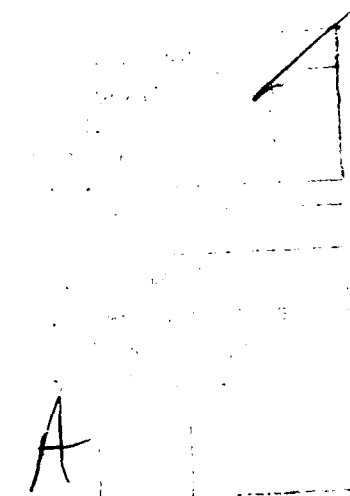
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RESEARCH ACCOMPLISHMENT IN THE FIRST YEAR

The research program was planned in four stages over two calendar years starting September 28, 1979. The first and second stages would encompass both the investigation of free surface properties of the semiconductors in an ultra-high vacuum (10^{-10} torr) system (UHV) equipped with LEED, Auger electron spectroscopy (AES) and ion-sputtering capabilities, and the design and assembly of the satellite chamber for device fabrication. These two stages would be performed at the same time without interference with each other. The third stage of the work would encompass the study of the waveguide surface structure, formulation of surface-scattering theory and characterization of scattering loss parameters in terms of the surface geometry through LEED and SEM analysis in the same ultrahigh vacuum system. The last stage of the work would encompass the fabrication of the oxide-free electro-optical devices, the measuring of the optical propagation losses and the study of the optical characteristics of such waveguides.

The first and the second stages of the planned research were completed. Currently we are conducting the third stage of the research plan. The satellite chamber for device fabrication, as shown in Fig. 1, was designed and assembled to the main system. A schematic diagram of the main system is shown in Fig. 2. The satellite chamber is indicated as subsystem 2 in Fig. 2. Subsystem 1 in Fig. 2 is a plasma reactor designed for oxidation research of GaAs and is in no interference with this research program. A photograph of the completed system is shown in Fig. 3.

The free surface properties of GaAs (110), GaP (110) have been studied by LEED and AES. Results of the studies have been presented in two different scientific conferences. Abstracts, which summarized the results of the research works, are given in Appendices A and B. Abstract of the first paper entitled "Chain Method of LEED/MEED Intensity Calculation for Diatomic Surface" is given in Appendix A

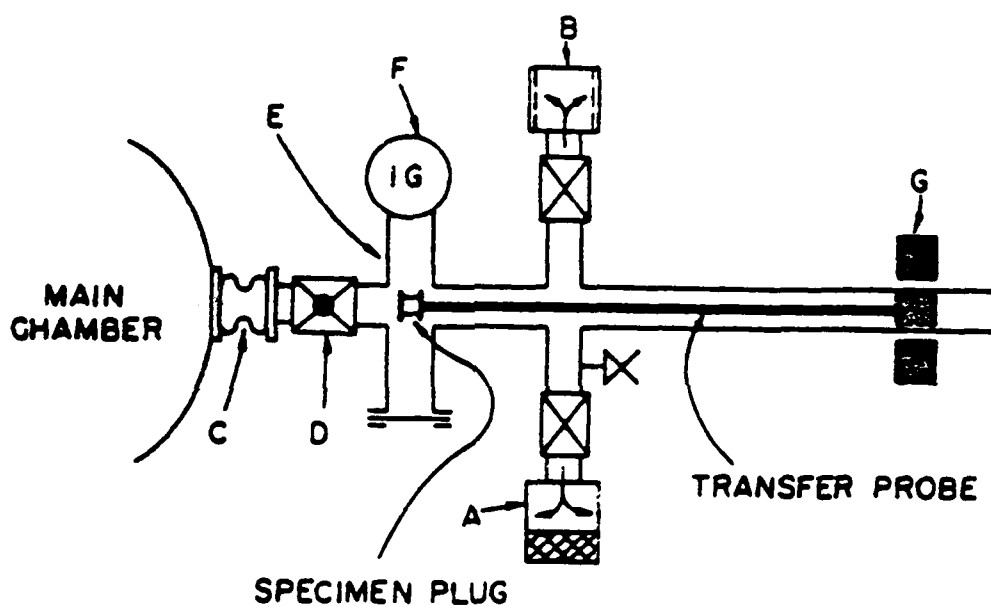


Fig. 1. Top-view schematic of the satellite system attached to the main chamber (left). A: sorption forepump; B: Vacion UHV pump; C: bellows-coupled flange; D: isolation valve; E: six-armed cross; F: ionization gauge; G: transfer probe magnet actuator. In the schematic the transfer probe is shown completely retracted, with the specimen plug located at the center of the six-armed cross.

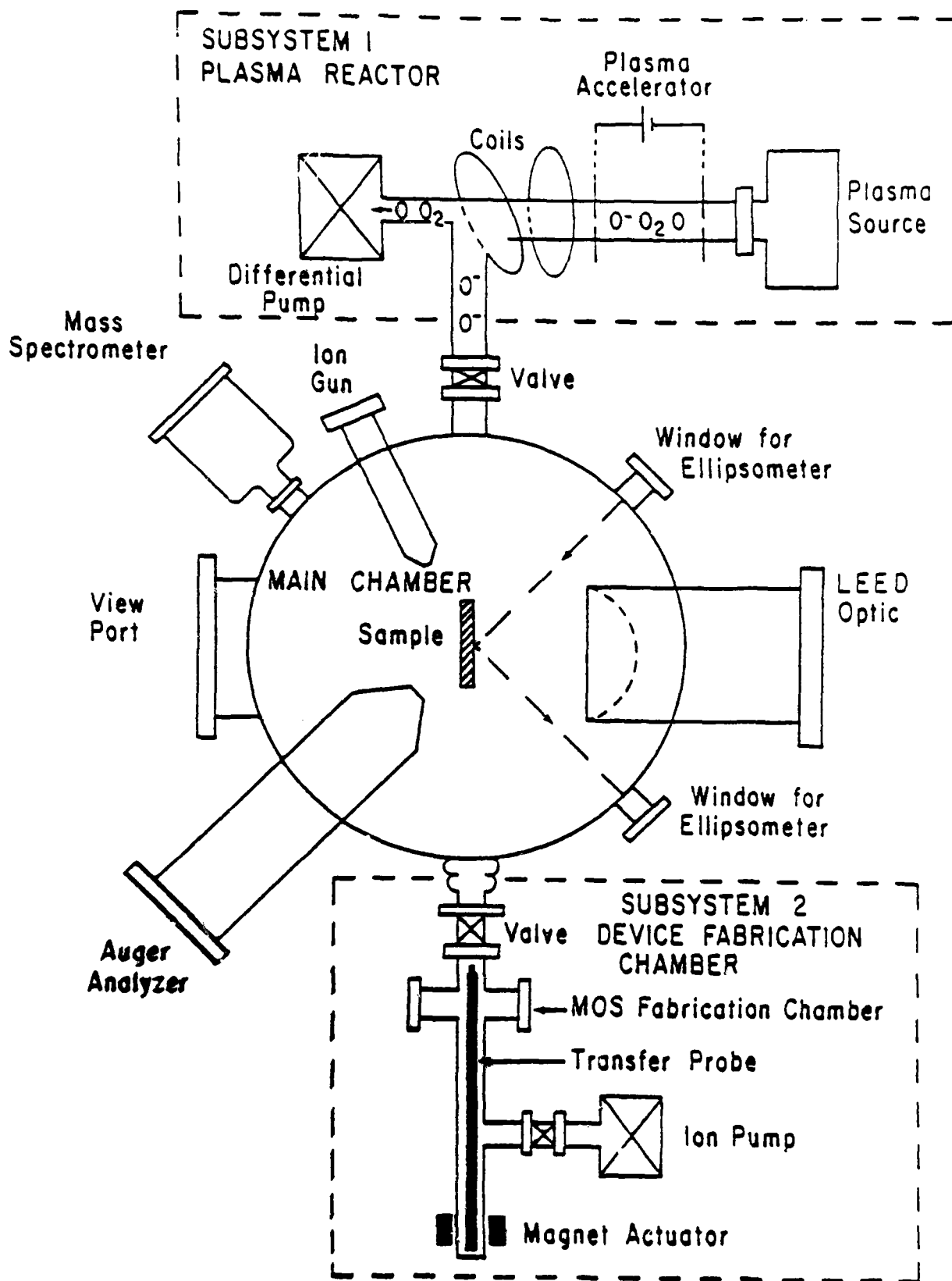


Fig. 2. Schematic diagram of the UHV system equipped with LEED, AES, mass spectrometer, ion sputtering gun and ellipsometer for surface analysis. Subsystem 1 is the plasma reactor which is currently under construction. Subsystem 2 is the MOS device fabrication chamber.

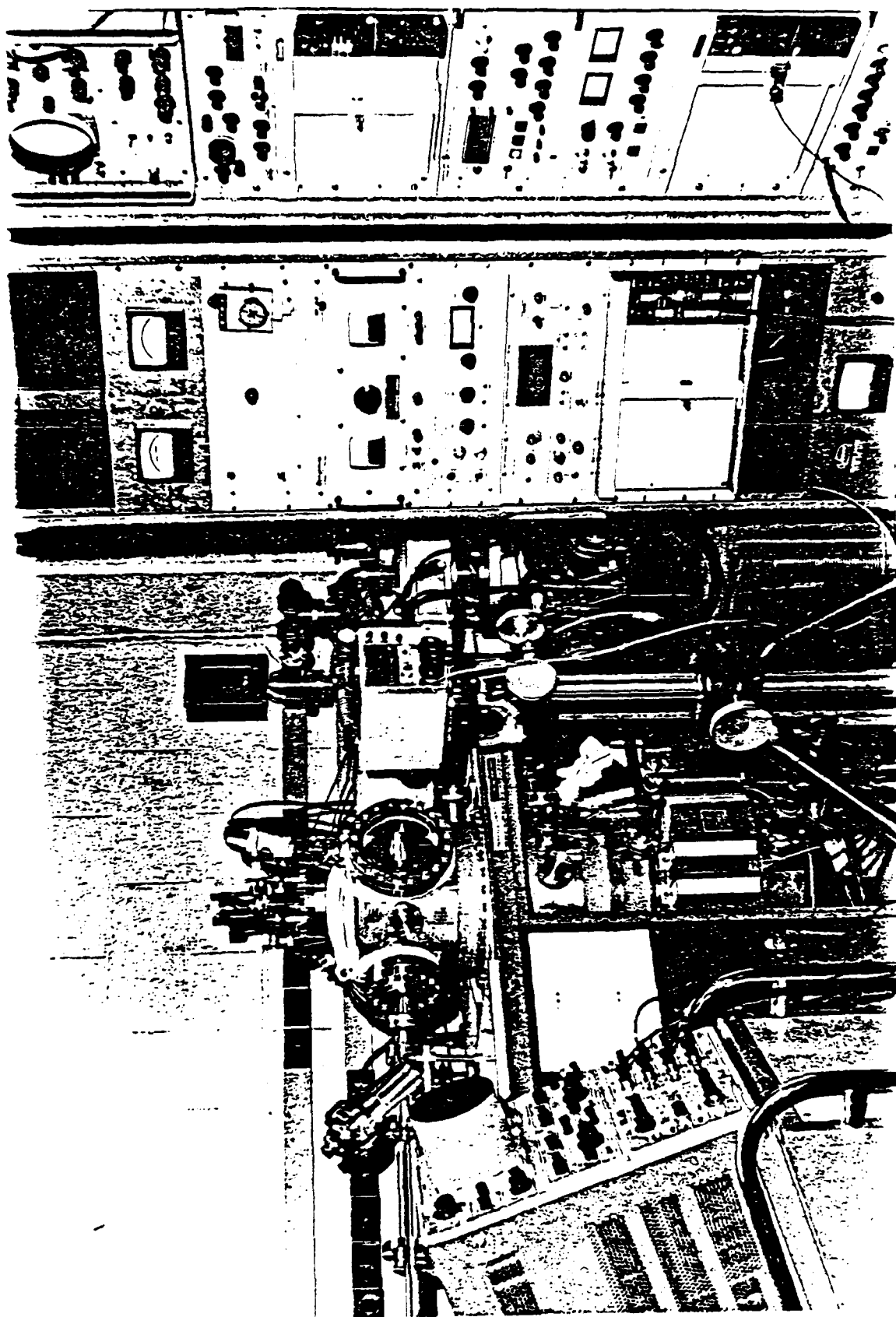


Fig. 3. LEED/AES system for investigation of surface and interface properties of semiconductors and ceramic materials. The capital equipment is acquired through the NSF equipment grant.

and the second paper entitled "Atomic Structure of GaAs (110) Face" is given in Appendix B. Both papers are being prepared for publication.

The third stage of the research work is currently ongoing. The theoretical formulation of surface scattering theory and characterization of the scattering loss parameters are currently being developed. The problem of a wave propagating along z direction in a rectangular dielectric waveguide with dimensions a and b as shown in Fig. 4 was formulated and solved. The wave functions in the waveguide were solved using predominantly polarized approximation.* The propagation attenuation constants, e.g. the loss parameters, were calculated. Details of the theoretical formulation and calculation is given in Appendix C.

Citations of research results, which are presented in conferences or submitted for publication, are listed in Appendix D.

*D. Marcuse, Bell Syst. Tech. J., 48, 3187 (1969).

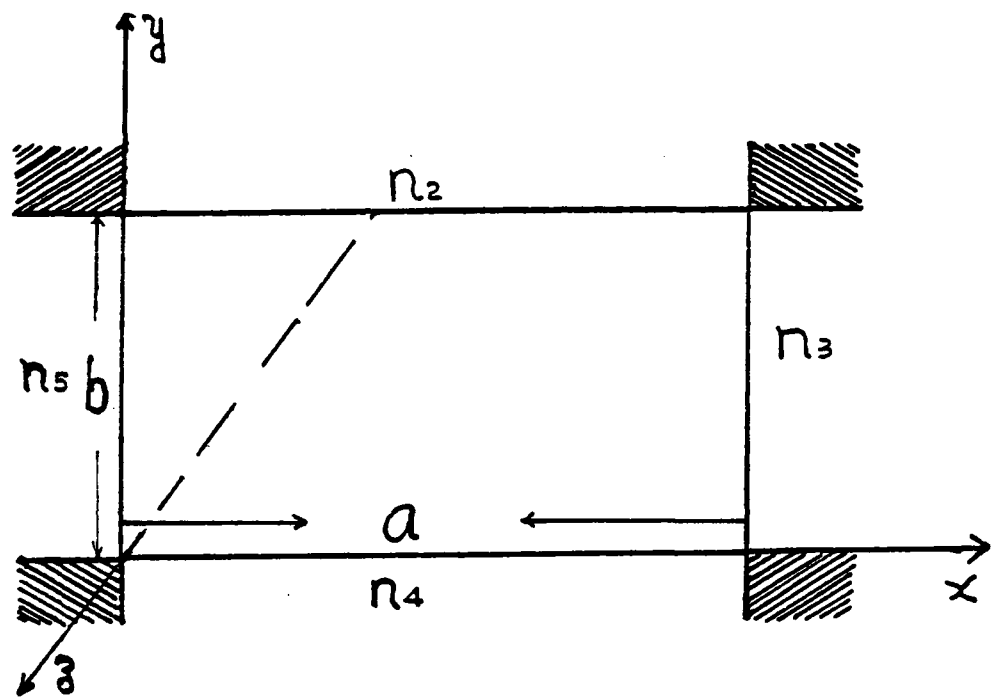


Fig. 4. Cross section of a rectangular dielectric waveguide with dimensions a and b . n_1 , n_2 , n_3 , n_4 and n_5 are the indexes of refraction of regions 1 to 5, respectively.

APPENDIX A

The following paper was presented in the Conference on Determination of Surface Structure by LEED, IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y., June 19-20, 1980

CHAIN METHOD OF LEED/MEED INTENSITY CALCULATION FOR DIATOMIC SURFACES

In low energy electron diffraction (LEED), crystal surface is probed by bombarding electrons of energy $E < 200$ eV at normal incidence to the surface and analyzing the intensity of those elastically scattered electrons which are back reflected from the surface, whereas in medium energy electron diffraction (MEED) due to an increase in the energy of incident electrons ($200 \text{ eV} < E \leq 5 \text{ keV}$) the surface sensitivity of the technique is maintained by taking an oblique angle of incidence. Interpretation of LEED/MEED intensity spectra requires detailed calculations of the diffracted intensities for a series of trial models. For such a technique to work, a rapid and accurate method of calculation is needed.

Layer-KKR method is conventionally employed in the calculation of LEED intensity spectra. In this formalism, the crystal is divided into a number of layers parallel to the crystal surface and scattering calculations are split into two parts, intralayer and interlayer. In the intralayer scattering calculation, due to the assumed spherical symmetry of the atomic potential, an angular momentum representation is used. This involves large matrices and their inverses and becomes cumbersome with increasing energy and more complex surfaces. In such a situation we propose to use the chain method of intralayer multiple scattering calculation.

In the chain method the two dimensional intralayer multiple scattering calculations are further divided into two one dimensional steps, scattering within a chain of atoms and between the chains. At each stage of calculation the electron wave function is represented in terms of an appropriate set of basis functions. The scattering by an atom, a chain of atoms and a layer of chains is expressed in terms of spherical, cylindrical and plane wave representation respectively. As we move from one stage to another, transformation from one basis set to another is carried out. Due to its one dimensional lattice summations the chain scattering formalism has many computational advantages over the layer-KKR method both for normal incidence LEED and off normal incidence MEED.

APPENDIX B

The following paper is submitted for presentation in the Eighth Annual Conference on the Physics of Compound Semiconductor Interfaces, Williamsburg, Virginia, January 27-29, 1981.

ATOMIC STRUCTURE OF GaP (110) FACE

ABSTRACT

Low energy electron diffraction (LEED) intensities have been measured for the (110) face of GaP and analyzed using a dynamical multiple-scattering model of the diffraction process. The intralayer multiple scattering is treated exactly, while for the interlayer multiple scattering, the renormalized-forward-scattering method is used. Comparison of the calculated and observed LEED intensities suggests that both the Ga and the P atoms on the (110) face may exhibit a contracted outermost layer spacing. The surface layer is compressed by about 5% such that the top layer spacing is reduced by $0.1 \pm 0.02\text{\AA}$. The rippled geometry of surface reconstruction is not clearly observed. This indicates that the GaP (110) surface atomic structure is different to that of GaAs (110). By comparing the structures and properties of the GaP (110) face and the GaAs (110) face, it is concluded that the GaP (110) face is relatively unstable and reactive.

APPENDIX C

The problem of a wave propagating along z-direction in a rectangular dielectric waveguide with dimension a and b as shown in Figure 4 is formulated and solved. An exact analytical treatment of rectangular waveguide is practically impossible. Therefore the approximated analytical approach developed by Marcatili (a) is followed in solving the problem.

Assuming a traveling wave propagating in z-direction as shown in Fig. 4, Maxwell equations

$$\nabla \times \vec{H} = \epsilon_0 n^2 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = \mu_0 \frac{\partial \vec{H}}{\partial t}$$

can be solved to give

$$E_x = [-i/(n^2 k^2 - \beta^2)] [\beta \frac{\partial E_z}{\partial x} + \mu_0 \omega \frac{\partial H_z}{\partial y}] \quad \text{--- (1)}$$

$$H_y = [-i/(n^2 k^2 - \beta^2)] [\beta \frac{\partial H_z}{\partial y} + \omega \epsilon_0 n^2 \frac{\partial E_z}{\partial x}] \quad \text{--- (2)}$$

$$E_y = [-i/(n^2 k^2 - \beta^2)] [\beta \frac{\partial E_z}{\partial y} - \omega \mu_0 \frac{\partial H_z}{\partial x}] \quad \text{--- (3)}$$

$$H_x = [-i/(n^2 k^2 - \beta^2)] [\beta \frac{\partial H_z}{\partial x} - \omega \epsilon_0 n^2 \frac{\partial E_z}{\partial y}] \quad \text{--- (4)}$$

$$E_z = \frac{1}{\epsilon_0 n^2 i \omega} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \quad \text{--- (5)}$$

$$H_z = \frac{i}{\mu_0 \omega} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \quad \text{--- (6)}$$

where n is the index of refraction of the dielectric; k is the wavevector in free space; ω is the angular frequency of the wave and β is the propagation constant in z-direction.

Substituting (2) and (4) into (5), and (1) and (3) into (6), we have

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (n^2 k^2 - \beta^2) E_z = 0 \quad \text{--- (7)}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (n^2 k^2 - \beta^2) H_z = 0 \quad \text{----- (8)}$$

Two different types of modes can be supported in the waveguide^(b), e.g.,

E_{pq}^x : polarized predominately in x-direction and

E_{pq}^y : polarized predominately in y-direction,

where p and q are positive integers indicating the modes of the wave propagating in the waveguide.

E_{pq}^x MODE

In region 1 with refraction index n_1 as shown in Fig. 4, the wave equation (7) can be solved to give

$$E_z(x,y) = A \cos k_x (x+\xi) \cos k_y (y+\eta) \quad \text{----- (9a)}$$

with $H_x = 0$, where ξ and η are the phase factors of E_z .

Other components of the wavefunction can be obtained by substituting (9a) into eqs. (1), (3) and (4):

$$H_z = -A \frac{n_1^2}{\beta} \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \left(\frac{k_y}{k_x}\right) \sin k_x (x+\xi) \sin k_y (y+\eta) \quad \text{----- (9b)}$$

$$E_x = iA \frac{(n_1^2 k^2 - k_x^2)}{\beta k_x} \sin k_x (x+\xi) \cos k_y (y+\eta) \quad \text{----- (9c)}$$

$$E_y = \frac{-iA k_y}{\beta} \cos k_x (x+\xi) \sin k_y (y+\eta) \quad \text{----- (9d)}$$

Substituting (9a) into (7) we have

$$n_1^2 k^2 - \beta^2 = k_x^2 + k_y^2 \quad \text{----- (10)}$$

For small incident angle, $\beta \gg (k_x^2 + k_y^2)$, eq. (10) can be approximated to be $\beta \doteq n_1 k \cos \theta \doteq n_1 k$ for $\theta < 5^\circ$.

The wavefunctions in regions 2, 3, 4 and 5 with indices of refraction n_2 , n_3 , n_4 and n_5 , respectively, as shown in Fig. 4, are solved. When boundary conditions are matched between region 1 and the neighboring regions, i.e., regions 2, 3, 4 and 5, we obtain the four transcendental equations:

$$\tan k_x a = n_1^2 k_x (n_3^2 \gamma_5 + n_5^2 \gamma_3) / (n_3^2 n_5^2 k_x^2 - n_1^2 \gamma_3 \gamma_5) \quad (11a)$$

$$\tan k_x \xi = \left(\frac{n_5}{n_1}\right)^2 (k_x / \gamma_5) \quad (11b)$$

$$\tan k_y b = k_y (\gamma_2 + \gamma_4) / (k_y^2 - \gamma_2 \gamma_4) \quad (12a)$$

$$\tan k_y \eta = -\gamma_4 / k_y \quad (12b)$$

where $\gamma_2 = [(n_1^2 - n_2^2)k^2 - k_y^2]^{1/2}$

$$\gamma_3 = [(n_1^2 - n_3^2)k^2 - k_x^2]^{1/2}$$

$$\gamma_4 = [(n_1^2 - n_4^2)k^2 - k_y^2]^{1/2}$$

$$\gamma_5 = [(n_1^2 - n_5^2)k^2 - k_x^2]^{1/2}$$

A computer program is currently being developed using Newton's method to solve equations (11) and (12) to obtain k_x , k_y , β , and the power attenuation constant α .

REFERENCES:

- (a) Marcatili, E. A. J., Bell Syst., Tech. J. 48, 2071-2102 (1969).
- (b) D. Marcuse, "Theory of Dielectric Optical Waveguide", Academic Press, New York, 1974.
- (c) I. P. Kaminow, W. L. Mammel, and H. P. Weber, Applied Optics, Vol 13, No. 2/ Feb. 1974.

APPENDIX D

Publication citations and presentations supported or partially supported by this research grant:

1. "Decomposition of Aluminum oxide by Electron Bombardment." B. W. Lee and J. M. Kuo, B. Am. Phys. S., 25 (3), 238, 1980.
2. "Study of MIS Polycrystalline Silicon Solar Cell Using Auger Electron Spectroscopy," J. M. Kuo, B. W. Lee, B. Lalevic and W. A. Anderson, B. Am. Phys. S., 25 (3), 409, 1980.
3. "Chain Method of LEED/MEED Intensity Calculation for Diatomic Surfaces," N. Masud, C. G. Kinniburgh, D. J. Titterington. Presented in Conference on Determination of Surface Structure by LEED, IBM, T. J. Watson Research Center, Yorktown Heights, NY, June, 1980.
4. "Study of MIS Silicon Cell by ESCA and AES," Y. S. Wang, H. J. Yu, C. C. Hsu, B. W. Lee and W. A. Anderson, presented in the 27th National Symposium of American Vacuum Society, Detroit, October, 1980. Submitted to J. Vac. Sci. Technol for publication.
5. "Stability of MIS Silicon Solar Cell," B. W. Lee, J. M. Kuo, B. Lalevic and W. A. Anderson, submitted to J. Vac. Sci. Technol. for publication.
6. "Atomic Structure of GaP(110) Face," B. W. Lee, R. K. Ni and N. Masud, submitted for presentation in the Eighth Annual Conference on the Physics of Compound Semiconductor Interfaces, Williamsburg, Virginia, Jan. 1981.

A NONLINEAR MAXIMUM ENTROPY METHOD FOR SPECTRAL ESTIMATION

Summary

An intensive research study is made of the nonlinear maximum entropy spectral analysis method proposed by P. F. Fougere of AFGL. The research not only provides a much better understanding of the properties of the method including spectral resolution, convergence, etc. but also shows that the method can be performed effectively with the PDP 11/45 minicomputer. The successful implementation of the method is described in the program listings in Appendix I. Extensive computer results are presented on various data. These results clearly confirm that the nonlinear method is superior to the Burg's maximum entropy spectral analysis.

Preliminary results on the multichannel (multivariate) maximum entropy spectral analysis, the two-dimensional maximum entropy spectral analysis, and computer graphics for the spectral display are also presented. An extensive bibliography of the maximum entropy spectral analysis is given in Appendix III.

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I. Review of Research Progress

This research is concerned with the nonlinear maximum entropy spectral analysis (MESA) method proposed by Dr. Paul Fougere of AFGL. As verified in this research work performed at the PDP 11/45 minicomputer, the method not only provides a much better spectral resolution than the Burg's method but also removes the line-splitting and frequency shifting phenomena for sinusoidal signals, as experienced in the Burg's method. By using the double precision, the minicomputer results are reasonably close to those obtained at the CDC 6600 computer at AFGL.

The final computer program developed for the nonlinear complex signal maximum entropy spectral analysis is shown in Appendix IA. The program follows the mathematical development of Fougere[1] but is quite different from the original computer program developed by Dr. Fougere. Appendix IB is the computer program for Burg's complex signal maximum entropy spectral analysis. By using the computer programs and the two-channel radar data as shown in Fig. 1 (see also[2]), the spectrum of the nonlinear method is shown in Fig. 2a (linear plot) and Fig. 2b (logarithmic plot) for 10 filter weights. The Burg's result is shown in Fig. 3a (linear plot) and Fig. 3b (logarithmic plot), also for 10 filter weights. More detailed tabulation of the major frequency components outside the clutter bandwidth ($-0.167f_s$, $+0.167f_s$) is given in Fig. 4. The nonlinear method which is very close to the true answer clearly is much better than the Burg's method. 30 iterations are used in the nonlinear method which appears to be an optimum

number. The optimum filter weight is around 10 or 11 as determined by Fig. 5 which shows the linear prediction error (FPE) according to the Akaike criterion. The solid curve is the lower bound and the dashed curve is the upper bound as tabulated in Fig. 6. The upper bound was originally proposed in the research proposal[2] and it seems to be better than the lower bound due to Akaike.

Major documentations already made which describe the research progress are as follows:

1. C. H. Chen, J. Chen, and C. Yen, "A minicomputer implementation of Fougere's maximum entropy spectral analysis method," Technical report prepared for the Mini-grant, August 20 1980. This report has detailed results of comparison between Burg's and the nonlinear methods for sinewave and sunspot data.
2. C. H. Chen, "Spectral resolution of Fougere's maximum entropy spectral analysis," to be published in the Proceedings of IEEE, June 1981. This journal paper based on the work performed under the Mini-grant provides a good comparison between the nonlinear and Burg's methods, and the other method for complex sinusoids. The Cramer-Rao bound is used as a reference.
3. C. H. Chen and C. Yen, "Note on computer graphics for maximum entropy spectral analysis," Technical report prepared for the Mini-grant, March 23, 1981. This report provides a three-dimensional spectral display of sinewaves for both nonlinear and Burg's methods.

A number of important results are included in the new research proposal submitted to AFOSR in December 1980. Appendix III provides an extensive list of references on the maximum entropy spectral analysis. The following sections describe some new research areas with preliminary results.

II. Multichannel (multivariate) Maximum Entropy Spectral Analysis

A mathematical presentation of this topic is given in Appendix II. Several computer programs for multichannel maximum entropy spectral analysis were provided by Dr. Fougere. The following results are based on the time series of sunspot numbers, northern light activity, and earthquake activity by using the third multichannel program. The data are tabulated in [3].

Fig. 7a is the first channel (sunspot number) auto-spectrum with linear (left) and logarithmic (right) scales, and 16 lags.

Fig. 7b is the second channel (northern light activity) auto-spectrum with linear (left) and logarithmic (right) scales, and 16 lags.

Fig. 7c is the third channel (earthquake activity) auto-spectrum with linear (left) and logarithmic (right) scales and 16 lags.

Fig. 7d is the cross-spectrum between channels 1 and 2 with real part (left) and imaginary part (right), and 16 lags.

Fig. 7e is the cross-spectrum between channels 2 and 3 with real part (left) and imaginary part (right), and 16 lags.

Fig. 7f is the cross-spectrum between channels 1 and 3 with real part (left) and imaginary part (right), and 16 lags.

By way of verification, it is interesting to note that the spectral peak for the sunspot numbers is determined accurately.

III. Two-Dimensional Maximum Entropy Spectral Analysis

We consider a very simple separable case in the two-dimensional spectral analysis. The signal considered is $\sin(2\pi x) \sin(2\pi y)$.

In this case the power spectrum is the product of the power spectra

of $\sin(2\pi x)$ and $\sin(2\pi y)$. In each spatial dimension, the power spectrum can be determined by using the Fourier, nonlinear and Burg's methods. The two-dimensional spectra are shown in Figs. 8, 9 and 10 respectively, based on the Fourier, nonlinear, and Burg's methods. The nonlinear method clearly is much better. Extension of the above procedure to a more general two-dimensional spectral analysis is not possible. Although some two-dimensional maximum entropy spectral analysis work has been reported (see Appendix III), the success is very limited. Further research is much needed.

IV. Conclusions and Recommendations

The nonlinear maximum entropy spectral estimation method proposed by P. F. Fougere has provided superior spectral estimation over the Burg's method in a number of data considered. The computational requirement of the nonlinear method is, however, significantly higher. Typical number of iterations needed is 20 to 30. The use of minicomputer has not created significant computational problem as predicted. It is our strong belief that the nonlinear method will become very popular in high resolution spectral analysis for a wide range of applications in geophysics, sonar, radar areas, etc.

Recommended further studies include the multichannel (multivariate) maximum entropy spectral analysis, the two-dimensional nonlinear maximum entropy spectral analysis, computer graphics for the spectral display, signal decomposition, and

signal prediction and extrapolation.

Acknowledgment

We would like to thank Dr. Henry R. Radoski for support and encouragement, and Dr. Paul F. Fougere for guidance and many helpful discussions.

References

1. P. F. Fougere, "A solution to the problem of spontaneous line splitting in maximum entropy power spectrum analysis of complex signals," Proc. of the RADC Spectrum Estimation Workshop, May 1978.
2. C. H. Chen, Mini-grant Proposal on "A Non-Linear Maximum Entropy Method for Spectral Estimation" for AFOSR, Sept. 1979.
3. E. A. Robinson, "Multichannel Time Series Analysis with Digital Computer Programs," Holden-Day, San Francisco, 1967.

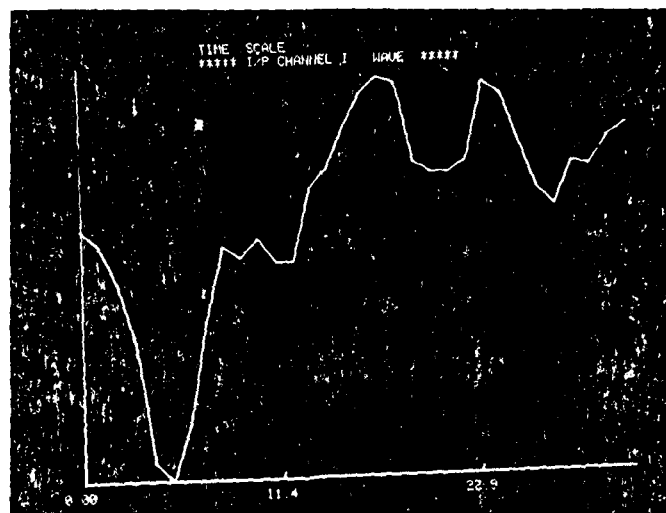


Fig. 1a 32-point I-channel
data, for real part.

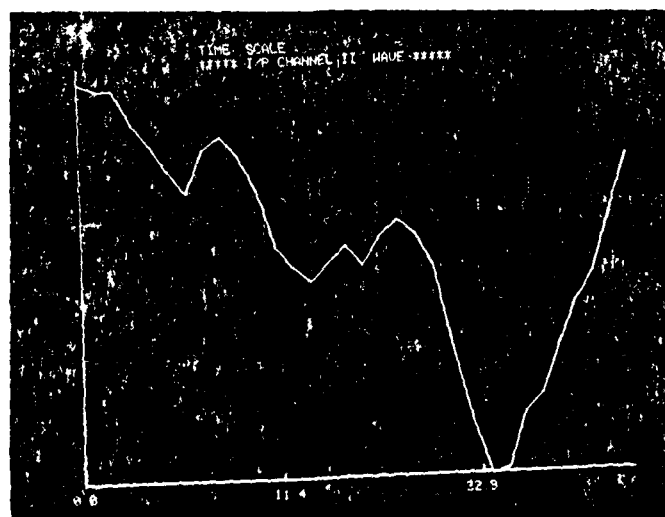


Fig. 1b 32-point Q-channel
data for imaginary
part.

Fig. 2b
Nonlinear
MESA
(log plot)

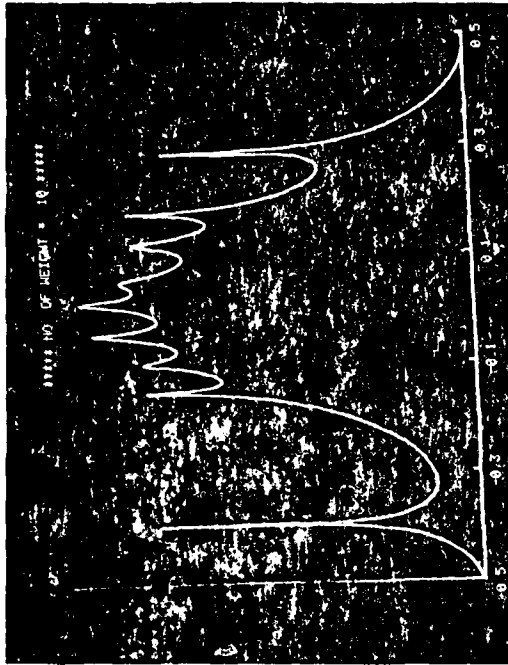


Fig. 2b
Furn's MESA
(log plot)

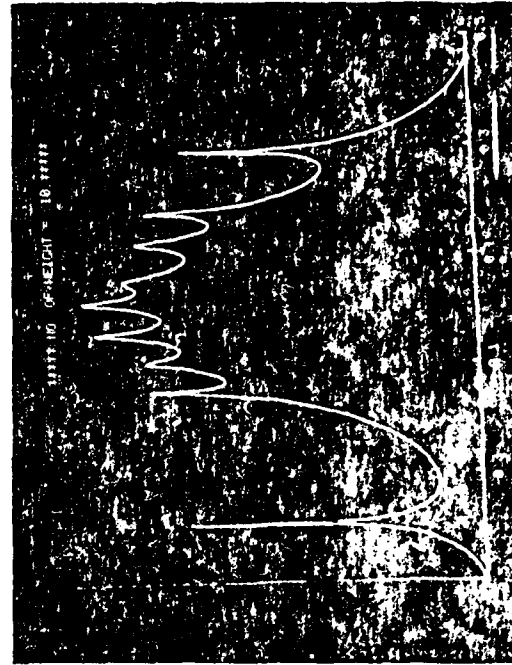


Fig. 2a
Nonlinear
MESA
(linear plot)

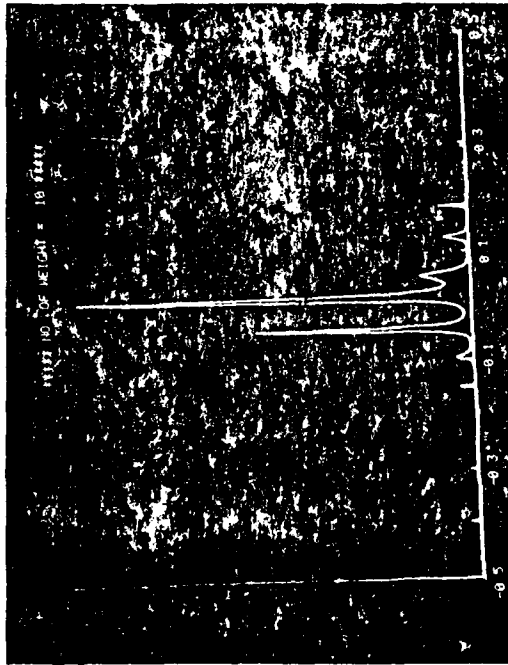
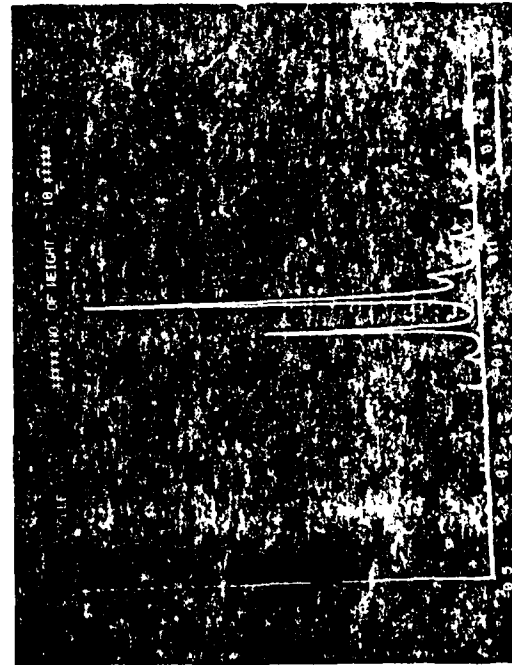


Fig. 2a
Furn's MESA
(linear plot)



Frequency	Amplitude	Relative level	
+0.1836 f_s	0.2184	0	dB
-0.3945 f_s	0.1203	-5.18	dB
+0.2930 f_s	0.0873	-7.96	dB

True Answer

Frequency	Amplitude	Relative level	
+0.18359 f_s	5.48007	0	dB
-0.39453 f_s	3.02523	-5.16	dB
+0.29296 f_s	2.19424	-7.95	dB

Result of Nonlinear Method

Frequency	Amplitude	Relative level	
+0.18164 f_s	3.58956	0	dB
-0.39453 f_s	1.45378	-7.85	dB
+0.29492 f_s	1.40613	-8.14	dB

Result of Burg's Method

Figure 4

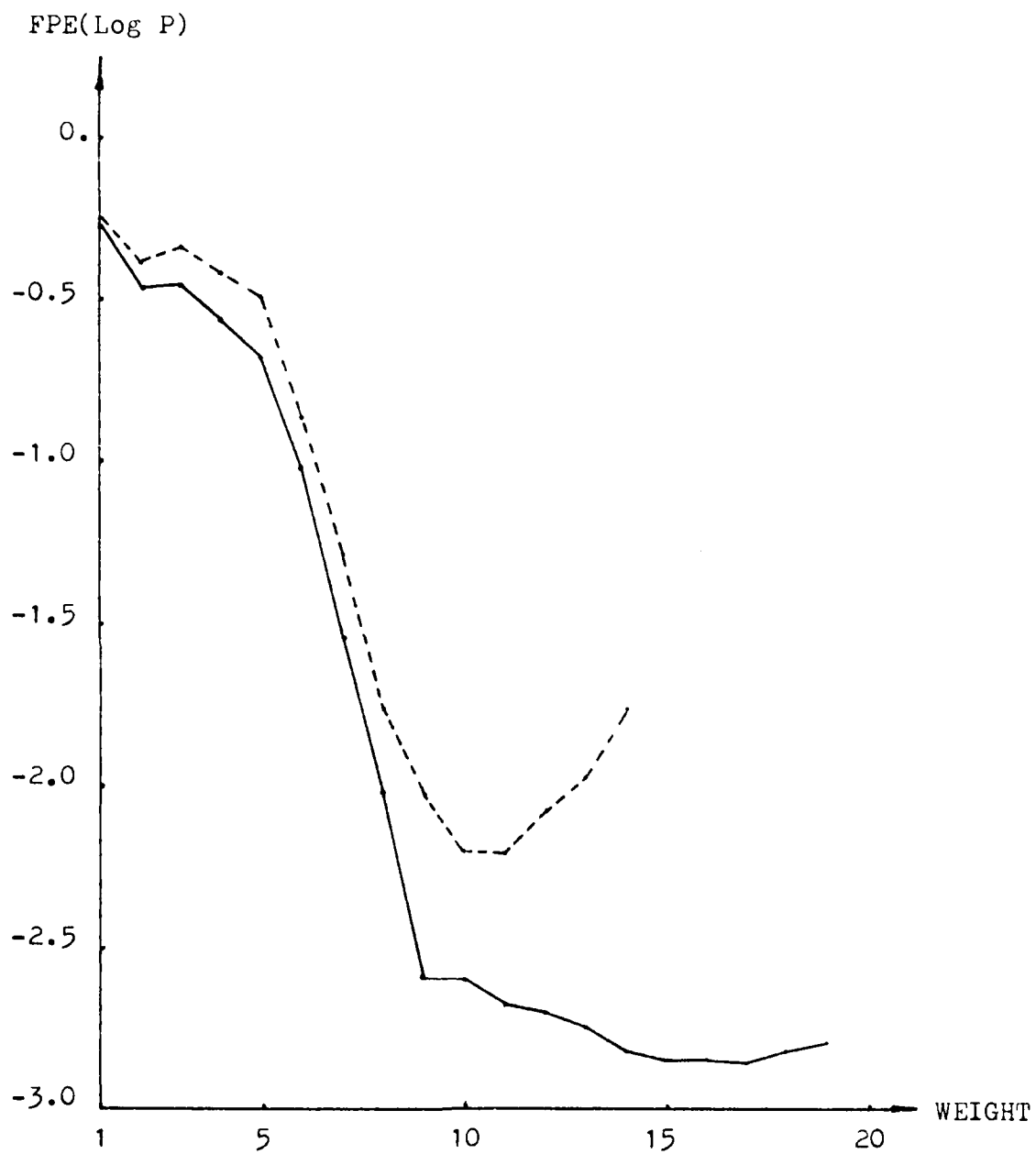


Figure 5

No. of weight	Log. of $\frac{N+(M+1)}{N-(M+1)} \cdot P_m$	Log. of $\frac{N+2(M+1)}{N-2(M+1)} \cdot P_m$
1	-0.279	-0.224
2	-0.463	-0.381
3	-0.445	-0.332
4	-0.562	-0.418
5	-0.674	-0.496
6	-1.080	-0.866
7	-1.539	-1.283
8	-2.063	-1.762
9	-2.587	-2.006
10	-2.597	-2.175
11	-2.678	-2.176
12	-2.686	-2.075
13	-2.738	-1.971
14	-2.815	-1.764
15	-2.839	-
16	-2.824	-
17	-2.833	-
18	-2.810	-
19	-2.799	-

Lower Bound

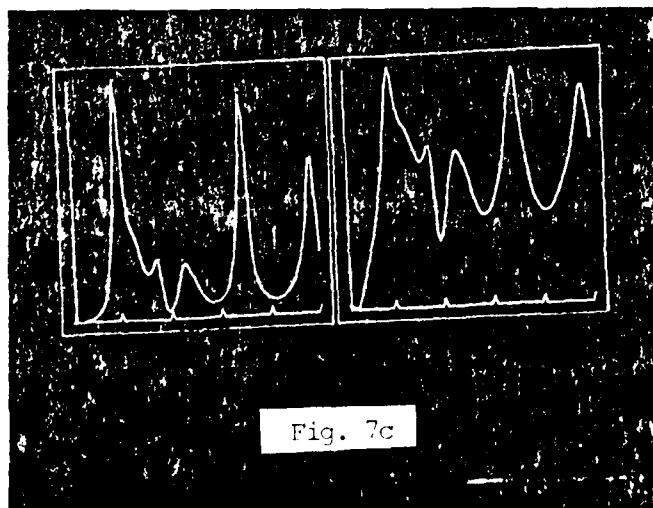
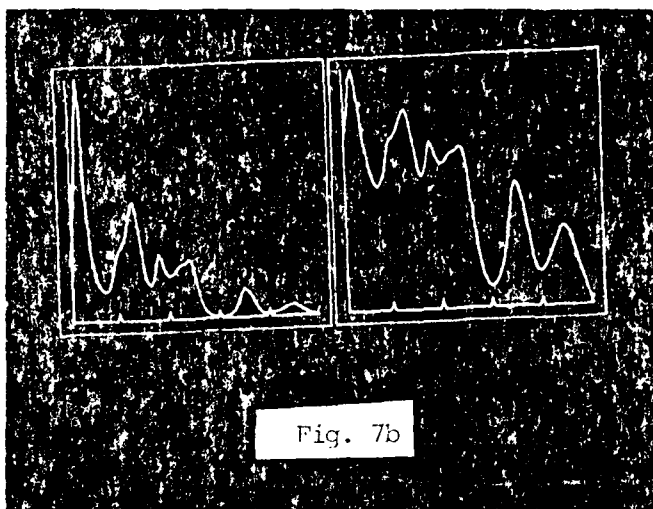
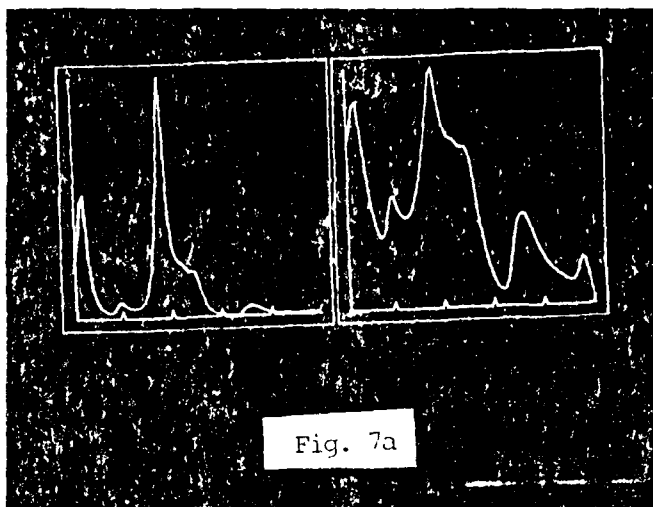
Upper Bound

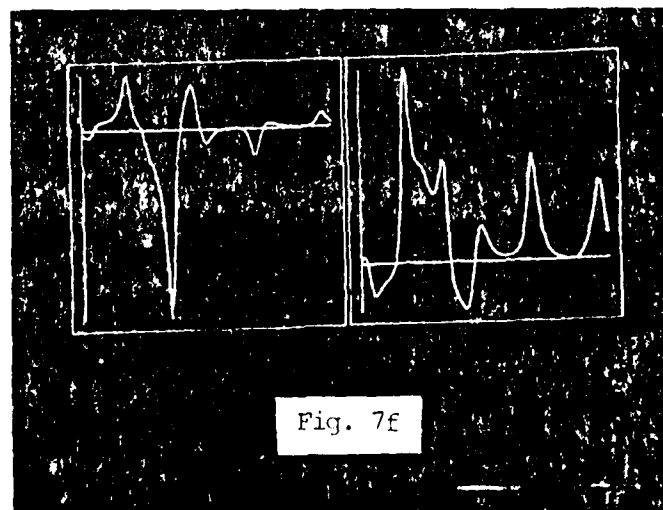
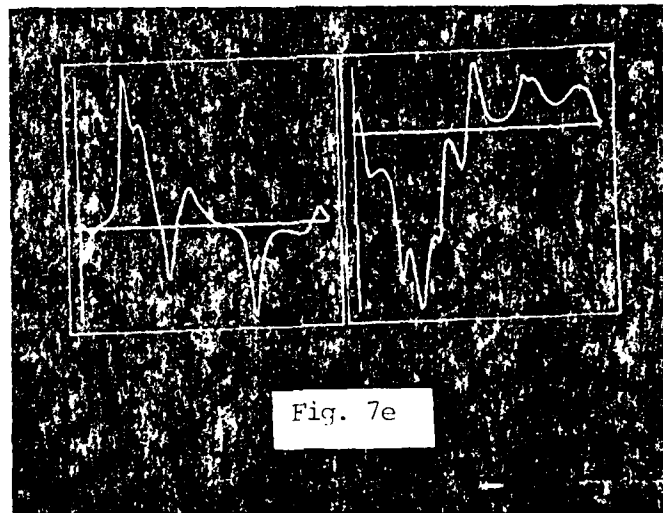
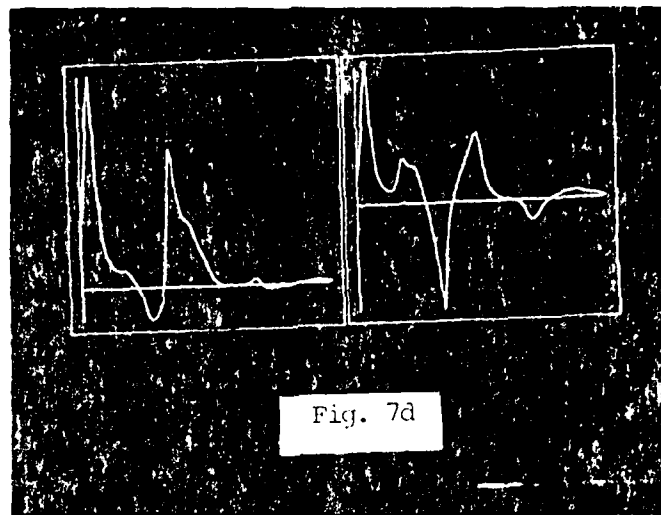
Note: N is the number of data points in each channel (32 in this case)

M is the number of filter weights

P_m is the error power

Figure 6





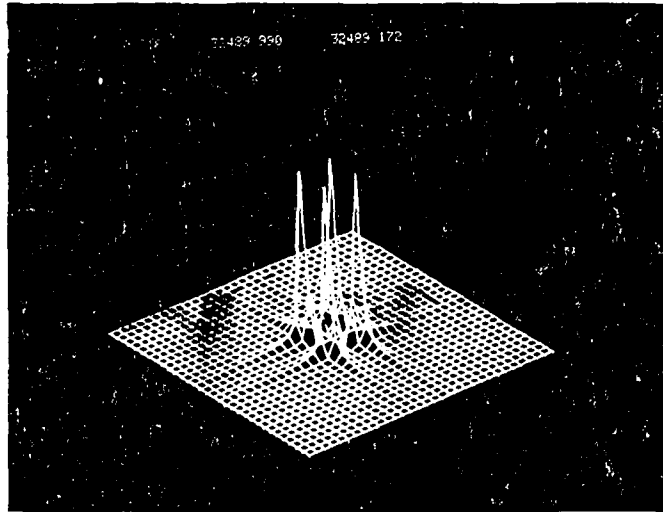


Fig. 8a (linear plot)

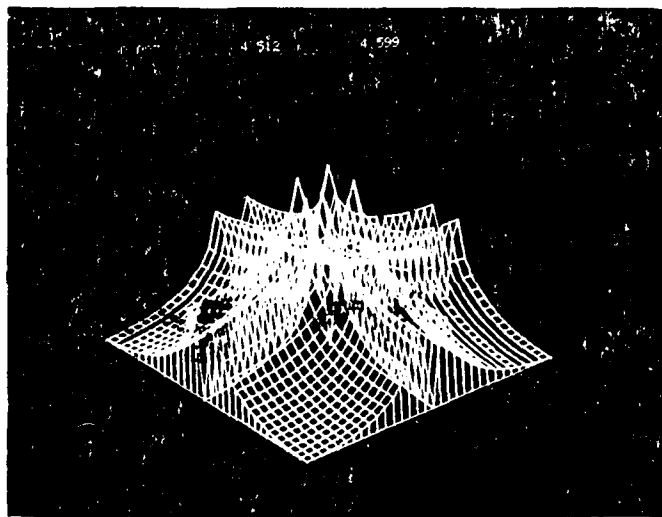


Fig. 8b (logarithmic plot)

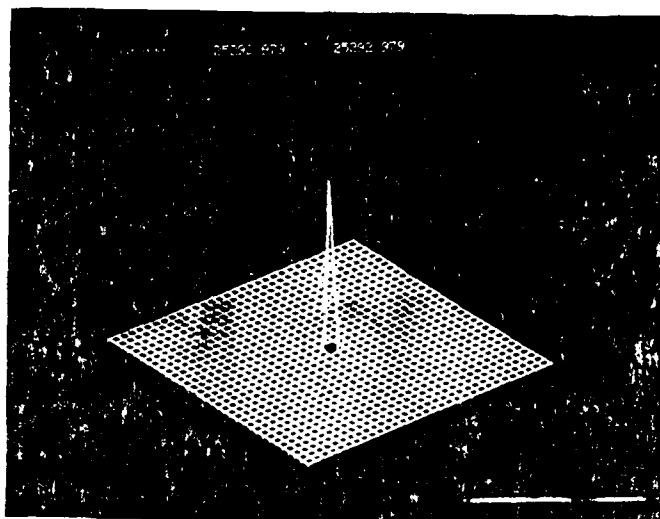


Fig. 9a (linear plot)

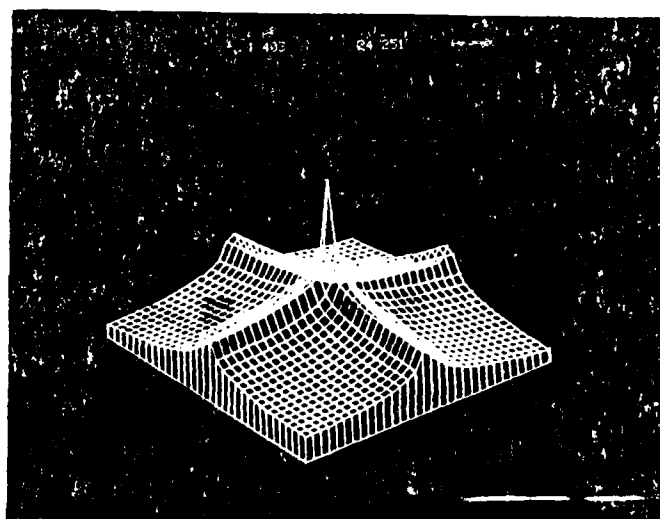


Fig. 9b (logarithmic plot)

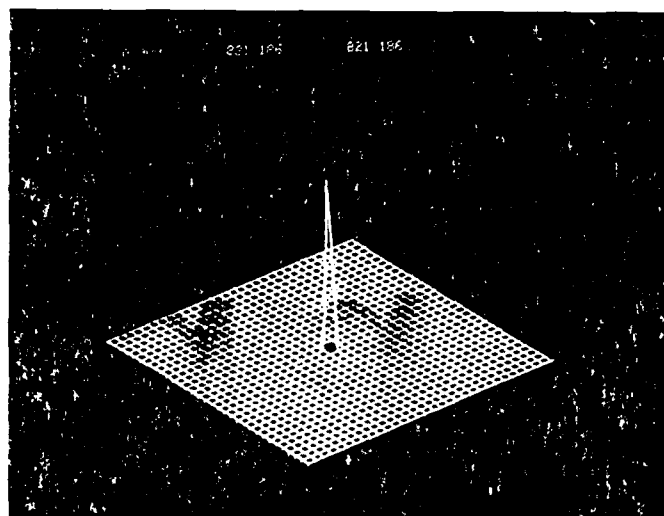


Fig. 10a (linear plot)

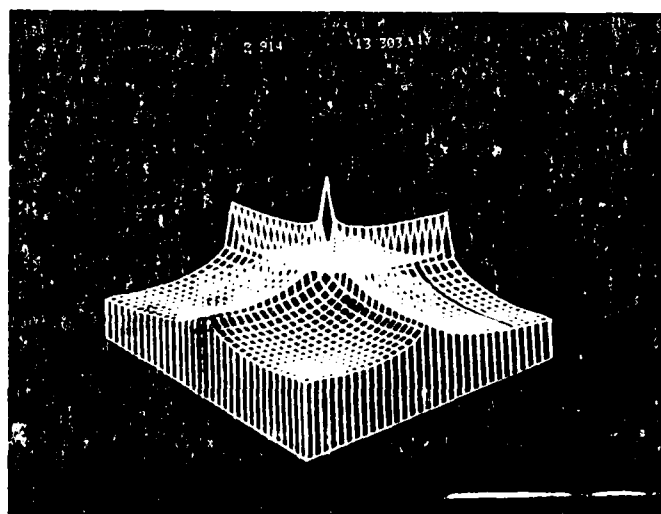


Fig. 10b (logarithmic plot)

Appendix IA Nonlinear Complex Signal MESA Program Listing

[illegible]

```

SUBROUTINE BLRN(10A)
DIMENSION H(10), PER(20), PER(32), PER(4), G(10), GGG(10)
COMPLEX G, PER, PER, H, GGG, PER, D, SIN, TH
DOUBLE PRECISION YR(30), X(30), GR(10), AI(10), BR(10), IC(
1), CI(10,10), U, PR, PWR, THETA(20), PERG, PER, EXTA(80)
COMMON ZEP(20),NR(10),U,PR,PWR,ABLE(10),ZEP(10),ZEP(10)
EQUIVALENCE (EXTA(1),GR(1,1)), (EXTA(10),AI(1,1)),
1 (EXTA(20),AR(1,1)), (EXTA(30),AI(1,1)), (EXTA(20),THETA(1,
2)), (EXTA(30),PER(1)), (EXTA(30),PER(1)), (EXTA(34),PER(1)),
3 (EXTA(34),H(1)), (EXTA(34),G(1)), (EXTA(34),G(1))
PER=0.0
DO 10 J=1,NR
10  P(1)=NR*(X(30)-GR(1)),SIN(21(1))
    DO 10 NR=1,1A
        X=0.0
        NR=
        IF (P(1)) GO TO 30
        DO 10 J=1,NR
        PER(J)=X
        20  PER(J)=X
        30  SIN=X
        DO 10 J=1,NR
        DO 10 J=1,NR
        C= (J+1)+PER(J)
        PER(J)=PER(J)
        SIN=LOG(SIN)+C
        DO 10 J=1,NR
        DO 10 J=1,NR
        GR(NR,NR)=PER(1)
        AI(NR,NR)=AI(1)
        THETA(NR,NR)=THETA(1)
        IF (PER(NR,NR)) PER(NR,NR)=
        IF (PER(NR,NR)) PER(NR,NR)=
        THETA(NR,NR)=DATA(THETA(NR,NR))
        THETA(NR,NR)=DATA(THETA(NR,NR))
        PER(NR,NR)=PER(NR,NR)
        DO 10 J=1,NR
        DO 10 J=1,NR
        K=J+1
        H(J)=GGG(J)-GGG(NR)*CONJ(GGG(K))
        60  CONTINUE
        DO 10 J=1,NR
        GGG(J)=H(J)
        JJ=J+1
        80  DO 10 J=1,NR
        PER(J)=PER(J)+LOG(GGG(NR))*(PER(J)+PER(J+1))
        PER(J)=PER(J+1)+GGG(NR)*(PER(J+1)+PER(J+1))
        90  CONTINUE
        DO 10 J=1,NR
        G(1)=GGG(1)
        110  CONTINUE
        DO 10 J=1,NR
        AR(1)=REAL(G(1))
        120  AI(1)=AIMAG(G(1))
        PER=
        END

```


2

```

CALL CH(XR(K+M-1), -XI(K+M-1),
+  PGR(1, 1), -PGR(1, 1), TPER, TMPI)
CALL CAD(XR(K+1), TPER, TMPI)
CALL CH(XR(K+1), -XI(K+1),
+  PGR(1, 1), -PGR(1, 1), TPER, TMPI)
CALL CAD(XR(K+1), TPER, TMPI)
CALL CH(XR(K+M-1), -XI(K+M-1),
+  PGR(1, 1), -PGR(1, 1), TPER, TMPI)
CALL CAD(XR(K+1), TPER, TMPI)
CALL CH(XR(K+1), -XI(K+1), PGR(1, 1), PGR(1, 1), TPER, TMPI)
30 CALL CAD(PXS, PZI, TPER, TMPI)
CALL CH(PXS, PZL, TPER, TMPI)
CALL CH(PXS, PZL, TPER, TMPI)
CALL CH(PXS, PZL, TPER, TMPI)
PZ(J) = PZ(J) + TPER + TMPI
CALL CH(PXS, PZL, PZL, PZL, TPER, TMPI)
CALL CH(PXS, PZL, PZL, PZL, TPER, TMPI)
20 PZ(J+1) = PZ(J+1) + TPER + TMPI
PZ(J) = PZ(J) / DELT(Delta)
10 PZ(J+1) = PZ(J+1) / DELT(Delta)
RETURN
END

```

```

SUBROUTINE CH(XR, XI, YR, YI, ZR, ZI)
DOUBLE PRECISION XR, XI, YR, YI, ZR, ZI
ZR = XR*YR - XI*YI
ZI = XR*YI + XI*YR
RETURN
END

```

```

SUBROUTINE CAD(XR, XI, YR, YI, ZR, ZI)
DOUBLE PRECISION XR, XI, YR, YI, ZR, ZI
ZR = XR + YR
ZI = XI + YI
RETURN
END

```

```

SUBROUTINE CAD(XR, XI, YR, YI)
DOUBLE PRECISION XR, XI, YR, YI
XR = XR + YR
XI = XI + YI
RETURN
END

```



```

      GNRN=GNRN+0.05*GRAT(I+1)
100  DY=DY+0.05*(V(I+1,Q))
      IF (DY) 110, 140, 140
110  IF (GNRN/5NRN+FR) 140, 140, 140
120  F=F
      ALFA=(2.0-(V(I+1)+F)/D)
      AMRDA=1.00
      IF (ALFA) 150, 150, 150
130  IF (ALFA-AMRDA) 140, 150, 150
140  AMRDA=ALFA
150  ALFA=0.00
140  FX=F
      DX=DY
      DO 170 I=1,N
170  X(I)=X(I)+AMRDA*H(I)
      CALL FGR(O)
      FY=F
      DY=0.00
      DO 180 I=1,N
180  DY=DY+GRAT(I+1)*H(I)
      IF (DY) 190, 190, 190
190  IF (FY-FY) 200, 200, 200
200  AMRDA=AMRDA+ALFA
      ALFA=AMRDA
      IF (GNRN*AMRDA-1.0) 210, 210, 210
210  I=I+1
      RE=RE
      I=0.00
220  IF (AMRDA) 230, 230, 240
230  Z=1.0+(FX-FY)/AMRDA-UX-DY
      ALFA=0.05*(GRAT(2), DABS(DX), DABS(DY))
      DIALFA=Z/ALFA
      DMALFA=DIALFA-DIALFA-DY/ALFA*DY/ALFA
      IF (DMALFA) 240, 240, 240
240  U=ALFA-DISORT(DIALFA)
      ALFA=DY-UX+U+W
      IF (ALFA) 260, 270, 260
260  ALFA=(DY-Z+W)/ALFA
      GO TO 280
270  ALFA=(Z+DY-U)/(Z+UX+W+DY)
280  ALFA=ALFA*AMRDA
      DO 290 I=1,N
290  X(I)=X(I)+(1-ALFA)*H(I)
      CALL FGR(O)
      IF (F-FX) 300, 300, 310
300  IF (F-FY) 300, 300, 310
310  DIALFA=0.00
      DO 320 I=1,N
320  DIALFA=DIALFA+GRAT(I+1)*H(I)
      IF (DIALFA) 330, 330, 330
330  IF (F-FX) 340, 340, 340
340  IF (F-FY) 340, 340, 340
350  FX=F
      DX=DY
      F=F
      DMALFA=DMALFA
      AMRDA=AMRDA

```

```

360 IF (P) = F 370, 370, 380
370 IF (QY) = 0 380, 380, 390
380 FY = F
    DY = D/AI F
    AMRDA = AMSDA + RLE A
    GO TO 270
390 IF ((CIDE) = F) 400, DE = F 400, 400, 400
400 DU = 410 J = 1, N
    K = N + J
    H(K) = DBA(H + J) + H(I)
    K = N + K
410 H(K) = X(CJ) + H(I)
    IF K = 0
        IF (EDUN) = H 420, 420, 420
420 T = C J
    Z = 0 DO
    DC 430 J = 1, N
    K = N + J
    U = C K
    E = U + P
    T = T + D + C(H + K)
430 Z = Z + P + H(T)
    IF (PR) = F 440, 440, 440
440 T = T + P 450, 450, 450
450 IF (G) = F 460, 460, 460
460 GE = G DO
    DU = 500 J = 1, N
    K = 2 + N
    W = 0 DO
    DC 470 L = 1, N
    EL = W
    G = G + C(H + L) + H(K)
    IF L = J 470, 480, 480
470 F = S + W
    GO TO 490
480 K = F + I
490 CDB = C DB
    K = N + J
    AI = AI + CLE + W + F + I
500 H(J) = F
    IF (Z * AI) = 0 510, 510, 510
510 K = R 31
    DO 520 L = 1, N
    K = N + J
    DO 530 J = 1, N
    M = N + J
    H(K) = H(K) + H(K) * H(M) / Z - H(L) + H(J) 70, F A
520 F = F + I
    GO TO 540
530 IF F = 0
    RE = F + I
540 DO 550 J = 1, N
    F = F + G
550 X(J) = H(F)
    C(AI) = F + K 0
    IF (DB) = F 560, 560, 560
560 IF (LE) = F 570, 570, 570
    IF R = 1 F = 1
    GO TO 17

```


Appendix IB Burg's Complex Signal MESA Computer Program

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PAGE

1

```

C      NAME: CMPBUG [11,11]      DATE: 14-AUG-80
0001      COMMON INDEX
0002      REAL F(32), PEF(32), PER(32), FNEW(32), GNEW(32)
0003      REAL G(20), GGG(20), GGGM(20), HALF(20), H(20)
0004      REAL SF(513), S(513), YX(1000), CAL(1000)
0005      COMPLEX F, PEF, PER, GGG, GGGM, G, HALF, H
0006      WRITE(6,5)
0007      5      FORMAT(1X, '***** I/P NUM, LG, SN, EXPAND, DT *****')
0008      READ(6,10) NUM, LG, SN, EXPAND, DT
0009      10      FORMAT(2I5, SF12.5)
0010      WRITE(6,15)
0011      15      FORMAT(1X, '***** I/P ISTART, ISTOP, INC, NOUT, IZ, LE *****')
0012      READ(6,20) ISTART, ISTOP, INC, NOUT, IZ, LE
0013      20      FORMAT(6I5)
0014      WRITE(6,25) NUM, LG, ISTART, ISTOP, INC, NOUT, IZ, SN, EXPAND, DT
0015      25      FORMAT(1H1, 5X, /I7, SF12.5)
0016      CALL BELL
0017      NI=NUM*2
0018      DEFINE FILE 2(?, NI, U INDEX)
0019      CALL INPUT(F, NUM, SN, FNEW, GNEW)
0020      WRITE(6,30) NUM
0021      30      FORMAT(30X, ' F DATA :      NO. OF PTS = ', I5)
0022      CALL WR(NUM, F, 5)
0023      TR1=S*ATAN(1.)
0024      CALL BELL
0025      50      CONTINUE
0026      DEN=FLOAT(NOUT-1)*EXPAND
0027      KFUL=INT(DEN/EXPAND-1.)
0028      DO 60 KF=1, KFUL
0029      SF(KF)=FLOAT(KF-1)/DEN+.5
0030      60      SF(KF)=TR1*SF(KF)
0031      DO 65 I=1, NUM
0032      FNEW(I)=REAL(F(I))
0033      GNEW(I)=AIMAG(F(I))
0034      65      CONTINUE
0035      DO 70 I=1, NUM
0036      YX(I)=FLOAT(I)
0037      CALL NEWPAG
0038      CALL PLOT(YX, FNEW, NUM, 50, 1010, 50, 700)
0039      CALL FIG1(YX, CAL, 348, 3)
0040      CALL BELL
0041      WRITE(6,998)
0042      998      FORMAT(20X, '***** I/P CHANNEL I WAVE *****')
0043      READ(6,999) NX
0044      999      FORMAT(I5)
0045      CALL NEWPAG
0046      CALL PLOT(YX, GNEW, NUM, 50, 1010, 50, 700)
0047      CALL FIG1(YX, CAL, 348, 3)
0048      CALL BELL
0049      WRITE(6,997)
0050      997      FORMAT(20X, '***** I/P CHANNEL II WAVE *****')
0051      READ(6,999) NX
0052      R=0.
0053      DO 80 I=1, NUM
0054      80      R=R+F(I)*CONJG(F(I))

```

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```

0055      P=P/FLOAT(NUM)
0056      SAVE=P
0057      WRITE(5,55)P
0058  55      FORMAT(21X,'P =',E15.7)
0059      LA1=NUM-1
0060      DO 150 NN=1,LA1
0061          NNN=NN+1
0062          CALL BPEC(NUM,NN,PFF,PER,F,G,GGG,GGGM,H)
0063          P=(1.-G(NN)*CONJG(G(NN)))*P
0064          HELP=P
0065  100      IF(NNN.GT.ISTOP)GO TO 150
0066          IF(NNN.LT.ISTART)GO TO 150
0067          IF(MOD((NNN-ISTART),INC).NE.0)GO TO 150
0068          CALL SPEC(NN,G,S,SE,KFUL,P,DT)
0069          CALL BELI
0070          CALL BELI
0071          READ(6,999)NX
0072          CALL NEWPAG
0073          CALL KOBLOT(YX,S,NOUT,50,1010,50,700)
0074          WRITE(6,120)NNN
0075          WRITE(5,120)NN
0076  120      FORMAT(30X,'***** NO. OF WEIGHT = ',I3,'*****')
0077          CALL UR(NN,G,5)
0078          CALL FIG1(YX,CAL,12,2)
0079          IF(I.P.F0.1)CALL PRIT(NOUT,DT,S,YX)
0080          IF(LG.LE.0)GO TO 140
0081          READ(6,999)NX
0082          CALL ALG(S,NOUT)
0083          CALL NEWPAG
0084          CALL WR(NN,G,5)
0085          CALL KOBLOT(YX,S,NOUT,50,1010,50,700)
0086          WRITE(6,120)NNN
0087          CALL FIG1(YX,CAL,12,1)
0088  140      P=HELP
0089  150      CONTINUE
0090          CALL BELI
0091          CALL EXIT
0092          END

```

ROUTINES CALLED:

BELI , INPUT , UR , ATAN , FLOAT , INT , REAL
 AIMAG , NEWPAG, KOBLOT, FIG1 , CONJG , BPEC , MOD
 SPEC , PRIT , ALG , EXIT

OPTIONS =/OP:2

BLOCK	LENGTH
MAIN	8185 (037762)*
****	2 (000004)

COMPILER ----- CORF
 PHASE USED FREE
 DECLARATIVES 00866 14517
 EXECUTABLES 01263 14115
 ASSEMBLY 01774 18744

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```

001      SUBROUTINE INPUT(X,N1,SN,Y,Z)
002      COMMON INDEX
003      COMPLEX X(1)
004      REAL Y(N1),Z(N1)
005      INDEX=1
006      READ(2,INDEX)Y
007      INDEX=2
008      READ(2,INDEX)Z
009      DO 5 J=1,N1
010      Z(J)=Z(J)
011      5 CONTINUE
012      DO 10 I=1,N1
013      X(I)=CMPLX(Y(I),Z(I))+SN*CMPLX(GAUSS(1,0),GAUSS(1,0))
014      10 CONTINUE
015      WRITE(5,20)
016      20 FORMAT(/'RX, 'CHANNEL I',5X,'CHANNEL II')
017      DO 40 J=1,N1
018      WRITE(5,30)Y(J),Z(J)
019      30 FORMAT('EX F9.3,5X,F9.3')
020      40 CONTINUE
021      RETURN
022      END

```

ROUTINES CALLED:
CMPLX , GAUSS

OPTIONS =/UP:2

BLOCK	LENGTH
INPUT	273 (001042)*
####	2 (000004)

##COMPILER ----- CORE##

PHASE	USED	FREE
DECLARATIVES	00627	14756
EXECUTABLES	00786	14592
ASSEMBLY	01218	18800

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```

001      SUBROUTINE WR(NN, G, IC)
002      REAL G(1)
003      COMPLEX G
004      DO 10 I=1, NN
005      A=REAL(G(I))
006      B=AIMAG(G(I))
007      IF (IC.EQ.5) WRITE(5,20) A, B
008      IF (IC.EQ.6) WRITE(6,20) A, B
009 20     FORMAT(28X, '(1,F15.7, /, /, F15.7, /)')
010 10     CONTINUE
011      RETURN
012      END

```

ROUTINES CALLED:
REAL , AIMAG

OPTIONS =/OP:2

BLOCK LENGTH
WR 150 (000434)*

COMPILER ----- CCRP
 PHASE USED FREE
DECLARATIVES 00702 14676
EXECUTABLES 00783 14595
ASSEMBLY 01097 18921

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```

0001      FUNCTION GAUSS(XBAR, SIGMA)
0002      ROOT=1./SQRT(8.*ATAN(1.))
0003 10     CONTINUE
0004      X=10.*(RAN(0,1)-.5)
0005      Y=ROOT*EXP(-X*X/2.)
0006      YTRY=ROOT*RAN(0,1)
0007      IF(YTRY.GT.Y) GO TO 10
0008      GAUSS=X*SIGMA+XBAR
0009      RETURN
0010      END

```

ROUTINES CALLED:
SQRT , ATAN , RAN , EXP

OPTIONS =/OP:2

BLOCK LENGTH
GAUSS 171 (000526)*

COMPILER ----- CCRP
 PHASE USED FREE
DECLARATIVES 00672 14776
EXECUTABLES 00783 14595
ASSEMBLY 01033 18985

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```

0001      SUBROUTINE BFEC(NUM, NN, PFF, PFR, F, G, GGG, GGGH, H)
0002      DIMENSION H(1), PFF(1), PFR(1), F(1), G(1), GGG(1), GGGH(1)
0003      COMPLEX G, PFF, PFR, H, GGG, GGGH, F, P, D, SN, SD
0004      XERO=(0., 0.)
0005      N=NN-1
0006      IF (N.NE. 0) GO TO 20
0007      DO 10 J=1, NUM
0008      PFF(J)=XERO
0009      PFR(J)=XERO
0010      SN=XERO
0011      SD=XERO
0012      JJ=NUM-N-1
0013      DO 30 J=1, JJ
0014      Q=F(J+N-1)-PFF(J)
0015      P=F(J)+PFR(J)
0016      SN=SN+CONJG(P)*Q
0017      SD=SD+P*CONJG(P)+Q*CONJG(Q)
0018      30 CONTINUE
0019      GGG(NN)=-2.*SN/SD
0020      IF (N.EQ. 0) GGGH(1)=GGG(1)
0021      IF (N.EQ. 0) GO TO 60
0022      DO 40 J=1, N
0023      K=H-J-1
0024      H(J)=GGG(J)+GGG(NN)*CONJG(GGG(K))
0025      40 CONTINUE
0026      DO 50 J=1, N
0027      GGG(J)=H(J)
0028      GGGH(NN)=GGG(NN)
0029      JJ=JJ-1
0030      60 DO 70 J=1, JJ
0031      PFR(J)=PFR(J)+CONJG(GGG(NN))*(PFF(J)+F(J+NN))
0032      PFF(J)=PFF(J+1)+GGG(NN)*(PFR(J+1)+F(J+1))
0033      70 CONTINUE
0034      DO 80 I=1, NN
0035      80 G(I)=GGG(I)
0036      RETURN
0037      END

```

ROUTINES CALLED:
CONJG

OPTIONS =/DP: 2

BLOCK	LENGTH
BFEC	523 (002026)*

COMPILER ----- CORE

PHASE	USED	FREE
DECLARATIVES	00422	14756
EXECUTABLES	00863	14515
ASSEMBLY	01205	18813

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```

001      SUBROUTINE SPEC(NN,G,S,SF,KFUL,P,DT)
002      DIMENSION S(1),SF(1),G(1)
003      COMPLEX G,SCS,ONE,GK
004      ONE=(1.,0.)
005      WRITE(5,5)P
006      5    FORMAT(15X,'***** M.F. P= ',E15.7,' *****')
007      PQ=7.*P*DT
008      DO 20 KF=1,KFUL
009      CA=1.
010      SA=0.
011      SCS=ONE
012      ADD=SF(KF)
013      CT=COS(ADD)
014      ST=SIN(ADD)
015      DO 10 K=1,NN
016      GK=G(K)
017      TEMP=CT*CA-ST*SA
018      SA=CT*SA+ST*CA
019      CA=TEMP
020      SCS=SCS+GK*CMPLX(CA,SA)
021      10    CONTINUE
022      S(KF)=PQ/(SCS*CONJG(SCS))
023      20    CONTINUE
024      RETURN
025      END

```

ROUTINES CALLED:

COS , SIN , CMPLX , CONJG

OPTIONS =/OP:2

BLOCK	LENGTH
SPEC	355 (00:166)*

```

**COMPILER ---- CORE**
  PHASE      USED  FREE
DECLARATIVES 00622 14756
EXECUTABLES  00943 14435
ASSEMBLY      01273 18745

```

FORTRAN V06.13

00:34:02

25-MAR-81

PAGE

1

```
0001      SUBROUTINE PRIT(NOUT,DT,S,YX)
0002      REAL S(1),YX(1)
0003      FREQ1=1./FLOAT(NOUT-1)/DT
0004      YX(1)=-0.50
0005      DO 10 I=2,NOUT
0006 10      YX(I)=YX(I-1)+FREQ1
0007      DO 30 J=1,NOUT
0008      WRITE(5,20)J,YX(J),S(J)
0009 20      FORMAT(10X,I3,5X,E15.7,5X,E15.7)
0010 30      CONTINUE
0011      RETURN
0012      END
```

ROUTINES CALLED:
FLOAT

OPTIONS =/OP.2

BLOCK	LENGTH
PRIT	173 (000532)*

```
**COMPILER ----- CORP**
  PHASE      USED  FREE
DECLARATIVES 00672 14756
EXECUTABLES  00783 14595
ASSEMBLY      01141 18877
```

ORTRAN V06 18

00:34:23

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PAGE

1

```

001      SUBROUTINE FIG1(YX,CAL,I2,KG)
002      REAL CAL(1),YX(1)
003      CAL(1)=330.
004      DO 100 I=1,960
005      IF(I.EQ.1)GO TO 100
006      I2=I-I/I2*I2
007      IF(I1.EQ.0)GO TO 50
008      CAL(I)=0.
009      GO TO 100
010      50      CAL(I)=5.
011      100     CONTINUE
012      DO 200 J=1,960
013      200     YX(J)=FLOAT(J)
014      CALL KBPLOT(YX,CAL,960,50,1010,50,725)
015      IF(KG.EQ.2)WRITE(6,205)
016      205     FORMAT(5X,'LINEAR SCALE')
017      IF(KG.EQ.1)WRITE(6,210)
018      210     FORMAT(5X,'LOG SCALE')
019      IF(KG.EQ.3)WRITE(6,215)
020      215     FORMAT(20X,'TIME SCALE')
021      CALL BELL
022      RETURN
023      END

```

ROUTINES CALLED:
 FLOAT , KBPLOT, BELL

OPTIONS =/OP:2

BLOCK	LENGTH
FIG1	285 (001072)*

```

**COMPILER ----- CORF**
  PHASE      USED  FREE
DECLARATIVES 00622 14756
EXECUTABLES  00863 14515
ASSEMBLY      01177 18541

```

ORTRAN V06.13

00.34:53

25-MAR-81

PAGE

1

```
001      SUBROUTINE ALG(S,KFUL)
002      REAL S(1)
003      DO 10 I=1,KFUL
004      IF(S(I).GE.0.)S(I)=ALOG10(S(I))
005 10     CONTINUE
006      RETURN
007      END
```

ROUTINES CALLED:
ALOG10

OPTIONS =/DP:2

BLOCK	LENGTH
ALG	74 (000224)*

```
**COMPILER ----- CORP**
  PHASE      USED  FREE
DECLARATIVES 00702 14676
EXECUTABLES  00702 14676
ASSEMBLY     00943 19077
```

Appendix II

On Multichannel (Multivariate) Maximum Entropy Spectral Analysis

1. Introduction

The univariate maximum entropy spectral analysis has now been well developed and applied to many defense research areas such as radar, sonar and geophysics. There has been some work done to extend the maximum entropy spectral analysis to multivariate case. Whittle [1] and Robinson [2][3] generalized the Levinson-Durbin recursion to the multivariate case by fitting both forward and backward autoregressions in a stepwise fashion. In this thesis, Burg [4] has mentioned about the multichannel case. However the computer programs for both multichannel and multivariate maximum entropy spectral analysis were only recently developed successfully. Morf et.al [5] developed an algorithm for direct estimation of the normalized reflection coefficients from the observed data for maximum entropy spectral analysis. They also compared the spectral estimation with the methods of Jones [6], Nuttall [7] and Strand [8], which are more of a direct extension of Burg's work to the multichannel (multivariate) case. Burg's algorithm does not generalize directly since the forward and backward autoregression matrices are not the same in the multivariate case, and the forward and backward one-step prediction error covariance matrices are different, although they have the same determinant. In this report, the programs developed by Strand and Jones are applied to real multichannel data and imagery data in addition to a set of test signals. The merits of these methods are closely examined. In spite of programming complexity the multichannel and multivariate maximum entropy spectral analysis will have increased application as the real data are almost always gathered in several channels. Data from several channels form a vector for multivariate study.

2. Brief Mathematical Analysis

Let $x_1, x_2, \dots, x_n$ denote n zero mean vectors of dimension d each. The sample estimate of covariance sequence for lag j is

$$\hat{R}_j = \frac{1}{n} \sum_{t=1}^{n-j} x_{t+j} x_t' \quad (1)$$

where the prime denotes the transposed vector. The forward and backward predicting autoregressions of order p are given, respectively, as

$$\begin{aligned} \hat{x}_t^{(f)} &= \sum_{k=1}^p A_k^{(p)} x_{t-k} \\ \hat{x}_t^{(b)} &= \sum_{k=1}^p B_k^{(p)} x_{t+k} \end{aligned} \quad (2)$$

where $A_k^{(p)}$ and $B_k^{(p)}$ are $d \times d$ matrices, and can be determined recursively [6] by making use of the estimated covariance matrix in Eq. (1). The recursion starts with

$$S_0^{(f)} = S_0^{(b)} = R_0 \quad (3)$$

The one-step forward and backward prediction error covariance matrices are

$$\begin{aligned} S_p^{(f)} &= (I - A_p^{(p)} B_p^{(p)}) S_{p-1}^{(f)} \\ S_p^{(b)} &= (I - B_p^{(p)} A_p^{(p)}) S_{p-1}^{(b)} \end{aligned} \quad (4)$$

The forward and backward residuals are, respectively,

$$e_t^{(p)} = x_t - \sum_{k=1}^p A_k^{(p)} x_{t-k}, \quad t = p+1, \dots, n \quad (5)$$

$$\beta_t^{(p)} = x_t - \sum_{k=1}^p B_k^{(p)} x_{t+k}, \quad t = 1, \dots, n-p$$

The recursive equations are then given by

$$\begin{aligned} e_t^{(p)} &= e_t^{(p-1)} - A_p^{(p)} \beta_{t-p}^{(p-1)}, \quad t = p+1, \dots, n \\ \beta_t^{(p)} &= \beta_t^{(p-1)} - B_p^{(p)} e_{t+p}^{(p-1)}, \quad t = 1, \dots, n-p \end{aligned} \quad (6)$$

The least squares estimates for the forward and backward autoregression matrices are

$$\begin{aligned} A_p^{(p)} &= UV^{-1} \\ B_p^{(p)} &= U'W^{-1} \end{aligned} \quad (7)$$

where U is the sum of cross products of forward and backward residuals at lag p ,

$$U = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (\beta_t^{(p-1)})' \quad (8)$$

and V and W are estimates of $(n - p)S_{p-1}^{(b)}$, $(n - p)S_{p-1}^{(f)}$ respectively,

$$V = \sum_{t=1}^{n-p} \beta_t^{(p-1)} (\beta_t^{(p-1)})', \quad (9)$$

$$W = \sum_{t=1}^{n-p} e_{t+p}^{(p-1)} (e_{t+p}^{(p-1)})', \quad (10)$$

Although the forward and backward autoregression matrices and the prediction error covariance matrices are different, the multivariate spectra should be identical when calculated from the forward and backward fits by

$$S(f) = h[A(f)]^{-1} S_p^{(f)} [A^*(f)]^{-1}$$

or by

$$S(f) = h[B(f)]^{-1} S_p^{(b)} [B^*(f)]^{-1}$$

where

$$A(f) = I - \sum_{k=1}^p A_k^{(p)} e^{2\pi i k h f}$$

$$B(f) = I - \sum_{k=1}^p B_k^{(p)} e^{-2\pi i k h f}$$

h is the sampling period and $*$ denotes complex conjugate transpose.

The above approach based on the work of Jones [6] does not guarantee stability and does not generally produce a non-negative definite spectrum as has been pointed out by Nuttall [7]. Subsequently, Nuttall [7] and Strand [8] applied a weighted arithmetic mean error criterion in order to provide model stability and to ensure positive definite stationary spectra. Another procedure suggested by Morf, et.al. [5] that also meets the spectral requirements is to compute the spectrum from the normalized reflection coefficient matrix ρ . To obtain this matrix, W and V are factored by using Cholesky decomposition into the product of lower triangular matrices times their transposes. A new recursive procedure for $S_p^{(f)}$ and $S_p^{(b)}$ by using ρ in place of Eqs. (4) can then be obtained. Other recursive algorithm has been proposed [9] for the solution of the normal equations for both single and multichannel data.

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Appendix III

A BIBLIOGRAPHY ON MAXIMUM ENTROPY SPECTRAL
ANALYSIS AND RELATED TECHNIQUES

I. Introduction

Recently there has been strong research interest on high resolution spectral analysis techniques. This is an important area of defense research because of the numerous applications to radar, sonar, and geophysical areas of defense interest. An excellent publication is the Proceedings of the 1978 and 1979 RADC Spectrum Estimation Workshop. Maximum entropy spectral analysis is one of a number of high resolution spectral analysis techniques. The impact of the Burg's maximum entropy spectral analysis method is far more significant than the technique itself. Thus in this report we present not only the bibliography of the maximum entropy methods in one and two spatial dimensions but also a number of related methods of high resolution spectral analysis. One common assumption with all these methods is that the data record is short and thus the conventional fast Fourier transform method of spectral analysis is not suitable. Probably because of the short length record, the maximum entropy spectral computation is fairly sensitive to the presence of noise. In the following sections, references are arranged in the first author's alphabetical order. Each reference is listed only once in the report. Effort has been made to provide as complete list of references as possible.

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