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DYNAMIC DECISION-MAKING IN MULTI-TASK

ENVIRONMENTS: THEORY AND EXPERIMENTAL RESULTS

Krishna R. Pattipati
David L. Kleinman

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The recent trend towards higher levels of automation in complex systems, such as in nuclear power plants, air-traffic control and flight management, is changing the role of the human operator from one of a controller to one of a supervisory decision-maker. The operator's primary responsibility in this new role is to extract information from his environment, and to integrate it for action selection and its implementation. The present analytic and experimental research has sought to understand human monitoring, information-processing and task selection procedures in dynamic multi-task environments, as a preliminary

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step towards analyzing and evaluating the human component of a supervisory control system.

A simple yet realistic computer representation of the supervisory decision situation is developed. The experimental paradigm retains the essence of the multi-task decision problem by presenting the human with a dynamic situation wherein tasks of different value, time requirement and deadline compete for his attention. Via this framework, the effects of various task related variables on the human decision-processes are studied.

A normative dynamic decision model (DDM) of human task sequencing performance is developed. The analytic framework of the DDM is based on modern control, estimation and semi-Markov decision process theories, which provide a general methodology for analyzing dynamic decision-making under uncertainty. Two novel features of DDM are its explicit incorporation of human limitations such as reaction time delays, randomness, limited resolving power and limited information-processing capacity, and its suitability to assimilate new elements of the decision task as they become considered and understood. Also, the analytic framework of the DDM has been shown to subsume several problems in single-processor sequencing theory, Markov decision theory and priority queueing systems.

In order to validate the model, several time-history and scalar measures of performance are proposed. Excellent model-data agreement is obtained for all the experimental conditions studied. Moreover, the model has been shown to represent human decision behavior significantly better than several heuristic sequencing rules of scheduling theory. The model has the potential for use in computer-aiding, and could form a significant step towards the modeling of multi-human behavior in complex, multi-level, multi-task systems.

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I. INTRODUCTION AND PROBLEM FORMULATION

An emerging trend in man-machine systems appears to be away from manual control to partial, if not full, automation. In this regard, the role of the human operator is shifting from one of a direct system controller to that of a monitor of multiple tasks, or a supervisor of several semi-automated subsystems. The operator's primary task in these systems is to extract information from his environment, and integrate this information for action selection and implementation. In this context, monitoring, information-processing and dynamic (real-time) decision-making skills of the human operator gain prominence over his sensor/motor skills. In order to properly analyze and evaluate the human component of a supervisory control system, an understanding of the human limitations and capabilities as an information-processor and dynamic decision-maker is essential.

There are two feasible paths that one can follow to develop human operator decision models, supported by concomitant experimental results, in complex supervisory control systems. The first approach starts with one task (or subsystem) and several humans to explore information-sharing and inter-human dynamics, and then adds more tasks (or subsystems). The second approach begins by studying single human dynamic decision-making among multiple tasks, and next introduces multiple decision-makers, composed of human and, possibly, non-human decision-makers. The latter route is advocated in this effort.

The present research seeks to understand human information-processing and task selection procedures in dynamic multi-task environments.

The approach is to assimilate the results of a joint experimental and analytic program into a normative dynamic decision model (DDM) of human task sequencing performance. To this end, a general multi-task decision problem is considered wherein tasks of different value, duration and deadline compete for the operator's attention. This situation occurs in targeting selection, air-traffic control, multiple remotely piloted vehicle (m-RPV) control, process control, power system regulation, production scheduling, as well as in many other supervisory control systems. The model that has emerged may be viewed as a basic building block in the comprehensive understanding of decision-making procedures, an understanding that could facilitate the modeling of multi-human behavior in complex, multi-level, multi-task systems.

1.1 Multi-task Decision Problem

We believe that a complete theory of human behavior in multi-task systems, analogous to Edwards' classification of human response theory [1], should consist of three parts: (i) a theory of how potential tasks are identified for consideration; (ii) a theory of the process of consideration by which all tasks but one are eliminated; and (iii) a theory about how the chosen task is executed. The last topic involves the study of human implementation skills, which are of secondary importance in supervisory control situations. The first topic, that of identifying potential tasks for consideration, is the problem of creative thinking, of which little of significance is known at present. However, this may not be restrictive in most multi-task systems of the type discussed above. In these systems, the tasks are immediately identified, e.g., once a target is detected. The topic of selecting a task for action from amongst many candidate tasks involves monitoring, information-

processing and dynamic (real-time) decision-making, and is the problem of interest here.

Fig. 1 shows the fundamental decision-loop that is addressed in this work. The human decision-process involves 1) whether to process a task or gather more information (i.e., monitor); and 2) which of N tasks (N is time-varying) to act upon in order to maximize the system performance (e.g., maximize reward, minimize regret, etc.). The decision-loop is dynamic in nature. As time evolves, tasks of different value, duration (processing time) and opportunity window (deadline) demand human's attention, while others depart. The opportunity windows shrink with time as the tasks approach their deadlines.

In the following, we provide a taxonomy for behavioral decision theory and show that the multi-task decision problem (MTDP) belongs to the most general class of decision-processes studied to date, viz., the semi-Markov decision processes (SMDP). We also summarize the results of a major literature survey on behavioral decision theory [2], and critically evaluate the previous (albiet limited) research on multi-task decision-making, in order to put the nature of the present work in perspective.

1.2 A Taxomony for Behavioral Decision Theory

A decision-maker's (DM's) choice in any decision task is a consequence of what he can do, what he knows and what he wants [3]. "What he can do" represents the alternatives (possible responses) available to the DM. "What he knows" refers to the information that DM has of the decision situation. This can range from the deterministic situations where all the relevant variables of the decision process are known, to the highly probabilistic situations where little information is available

FIG. 1: DYNAMIC MONITORING/DECISION LOOP FOR A SINGLE OPERATOR IN A
MULTI-TASK ENVIRONMENT

about any variable of interest. Finally, "what he wants" pertains to the DM's perception of the task objectives and his preferences for the various outcomes of a decision. These three concepts are fundamental to every decision-making process.

Most theories of individual choice behavior can be conveniently dichotomized into two distinct classes depending on the nature of the decision task, viz., single-stage and multi-stage decision theories. A detailed classification of individual choice theories is shown in Fig. 2 and is clarified below.

```
                    ┌─────────────────────────────┐
                    │  INDIVIDUAL CHOICE THEORY    │
                    └─────────────────────────────┘

┌─────────────────────────────┐   ┌─────────────────────────────┐
│ SINGLE-STAGE DECISION THEORY │   │ MULTI-STAGE DECISION THEORY  │
│          (E,D,r)             │   │        (S,E,D,T,r)           │
└─────────────────────────────┘   └─────────────────────────────┘

┌─────────────────────────────┐   ┌─────────────────────────────┐
│   DYNAMIC DECISION THEORY    │   │  SEQUENTIAL DECISION THEORY  │
│ (controlled decision process)│   │ (uncontrolled decision process)│
└─────────────────────────────┘   └─────────────────────────────┘

┌─────────────────────────────┐   ┌─────────────────────────────┐
│   MARKOV DECISION THEORY     │   │ SEMI-MARKOV OR MARKOV RENEWAL│
│(stage duration is deterministic│  │       DECISION THEORY        │
│ or irrelevant)               │   │ (stage duration is random with│
│                              │   │  known distribution)         │
└─────────────────────────────┘   └─────────────────────────────┘
```

Legend: S = set of states of the system
         E = set of events
         D = set of possible actions
         T = transformation rule
         r = reward function

FIG. 2: A CLASSIFICATION OF INDIVIDUAL CHOICE THEORIES

1.2.1 Single-Stage Decision Theory

A single-stage or static decision process may be represented as in Fig. 3.

(Decisions)D ⬇⬇   ⬇⬇ E(Events)

```
┌─────────────────────────────┐
│                             │
│        DECISION TASK        │
│                             │
└─────────────────────────────┘
```

⬇ r(Reward)

FIG. 3: FLOW DIAGRAM OF A SINGLE-STAGE DECISION PROCESS

We see that a static decision process can be conveniently characterized by the triple (E,D,r) where

$E = \{e\}$ = a finite non-empty set of external events (also known as states of nature, stimuli, hypotheses or diagnoses)

$D = \{d\}$ = a finite, non-empty set of possible decisions representing "What he can do" (also commonly referred to as alternatives, responses, or actions).

$r = r(e,d)$ = a reward (return) uniquely associated with the combined occurance of event, e, and decision, d.

Single-stage decision-making problems can be further classified into two categories depending on the information that the DM possesses (i.e., "What he knows") about E. These are decisions with certainty (riskless) and decisions with uncertainty (under risk). In the former category, each decision guarantees a reward with certainty, i.e., E is completely known. In the latter category, only a probability can be assigned to each $e \epsilon E$ such that $\sum_{e \epsilon E} p(e) = 1$.

The mechanics of a static decision problem are as follows: the DM chooses and executes a decision, d; an event, e, occurs; he receives a

reward, r(e,d), determined by the joint occurance of the event, e, and decision, d; and his decisions are mutually independent, i.e., he <u>never</u> makes another decision based on whatever he may have learned. It is frequently assumed that the DM chooses his decision to maximize the expected reward to minimize regret (i.e., "what he wants"). The widely studied single-choice gambling paradigms are examples of single-stage decision tasks.

1.2.2 Multi-Stage Decision Theory

In single-stage decision-making, the DM must make a <u>single</u> choice from among a number of alternatives. But in most man-machine and organizational systems, the DM seldom makes a single isolated decision. These situations require that the DM evaluate a number of objects or hypotheses simultaneously as the evidence accumulates sequentially and/or that he make several interdependent decisions. Thus, an understanding of human behavior in multi-stage decision-processes is fundamental to modeling human behavior in dynamic and uncertain environments.

In a multi-stage decision process, the DM makes a sequence of decisions. These types of processes consist of a series of stages such that the output of one stage becomes the input to the succeeding stage. Fig. 4 is representative of a multi-stage decision process [4].

FIG. 4: FLOW DIAGRAM OF AN N-STAGE DECISION PROCESS

Referring to Fig. 4, a multi-stage decision process can be characterized by the pentad (S,D,E,T,r) where

$S = \{\underline{s}_i\}$ = set of states of the system

$D = \{\underline{d}_i\}$ = set of possible decisions

$E = \{e_i\}$ = set of events

$T = \{t_i\}$ = set of transformation rules (laws of motion or transition functions) that describe the changes in state at each stage i

$r = \{r_i\}$ = set of rewards associated with each state transition

The stage-to-stage state transition is governed by the transformation rule

$$\underline{s}_{i+1} = \underline{t}_i \ (\underline{s}_i, \ \underline{e}_i, \ \underline{d}_i) \tag{1.1}$$

The reward at stage i is

$$r_i = r_i \ (\underline{s}_i, \ \underline{d}_i, \ \underline{s}_{i+1}) = \bar{r}_i \ (\underline{s}_i, \ \underline{e}_i, \ \underline{d}_i) \tag{1.2}$$

The DM's information can range from complete knowledge of the event set, $\{e_i\}$, and the set of transformation rules, $\{t_i\}$, to little or no knowledge of these variables. Notice that the transformation rule, t_i, and the reward, r_i, can be stage dependent (i.e., non-stationary). It is commonly assumed that the DM chooses his decisions to maximize his expected reward over N stages. The horizon N may or may not be known to the DM.

In studying multi-stage decision processes, a distinction is often maintained between sequential and dynamic decision processes (see Fig. 2). In sequential decision problems, the evolution of the state of the system $\underline{s}_i$ is independent of the DM's decisions. That is, Eq. (1.1) becomes

$$\underline{s}_{i+1} = \underline{t}_i \ (\underline{s}_i, \ \underline{e}_i) \tag{1.3}$$

Thus, a sequential decision task is an __uncontrolled__ decision process. It consists of a sequence of static decision problems repeated periodically and independently. The information gained from earlier decisions is useful in making later decisions, but the earlier decisions do not affect the transformation rule, $\underline{t}_i$. The operation of a sequential decision process is as follows: given that the system is in state $\underline{s}_i$ at the beginning of a stage i, the DM makes a decision, $\underline{d}_i$, the system moves to state $\underline{s}_{i+1}$ (which may or may not be identical to $\underline{s}_i$) according to the transformation rule, and the DM receives a reward $r_i(\underline{s}_i, \underline{d}_i, \underline{s}_{i+1})$ associated with this transition. Examples of sequential decision tasks are system failure detection, revision of opinion, display monitoring, asset selling and optional stopping.

The dynamic decision processes are multi-stage decision tasks in which the stage-to-stage changes in the state of the system are directly affected by the DM's previous decisions, as well as by environmental factors (events) over which the DM exercises no control (see Eq. (1.1)), i.e., it is a __controlled__ decision process. The set of alternatives and the information available at later stages are contingent upon earlier decisions. Thus, the DM has to consider the effect of each of his decisions on the future states of the system and, consequently, on his future decisions. The dynamic decision processes can be further classified into two categories, viz., Markovian and semi-Markovian (see Fig. 2). The Markov decision process (MDP) has the property that the stages are of deterministic duration, or their duration is irrelevant to the decision problem. Multi-stage betting games, inventory control, search theory and resource allocation are examples of MDP.

The semi-Markov decision process (SMDP), or Markov renewal decision process, is characterized by the fact that the time between state tran-

sitions is a random variable. The decision epochs in a stationary SMDP are the times of state transitions. At a decision epoch i, the system is in state $\underline{s}_i$. The DM chooses a feasible decision, $\underline{d}_i$; the system moves to state $\underline{s}_{i+1}$ after a random holding time, τ_i, according to the transformation rule; and the DM receives a reward $r(\underline{s}_i, \underline{d}_i, \tau_i, \underline{s}_{i+1})$, associated with this transition. The process continues for finite or infinite time. A complete characterization of a semi-Markov decision process includes the hexad $(S,\mathcal{D},E,T,H,r)$ where $S,\mathcal{D},E,T$ and r are as defined earlier, and H is a holding time function that determines how long the system stays in a given state before making a transition to another specified state. The process descriptors $(S,\mathcal{D},E,T,H,r)$ can be time dependent. The non-stationarity of the decision process can enter either in the form of time dependent dimension of the spaces $(S,\mathcal{D},E)$, or in the form of time varying nature of the transformation rules, T; the holding time functions H; and the reward structure, r. If a process is a non-stationary SMDP, the notation $(S(t), \mathcal{D}(t), E(t), T(t), H(t), r(t))$ is employed to emphasize its time dependence. Here, the decisions are, in general, continuous functions of time. Some examples of SMDP are targeting selection, air-traffic control, multi-RPV control, industrial process control, power system regulation and many other multi-task systems. The analysis of these systems is arduous, in view of the non-stationarity of the underlying SMDP. Virtually no significant research has been done by behavioral decision theorists using semi-Markov decision paradigms.

1.3 Summary of Research on Behavioral Decision Theory [2]

A brief and selective overview of the theories of individual choice behavior in static and multi-stage decision tasks was provided earlier in [2]. The primary purpose of this review was to investigate the applicability of this body of knowledge to model human information-processing

and decision-making skills in multi-task systems. The main conclusion
was that the multi-task decision problems are more general than any
considered in behavioral decision theory to date. However, there exist
bits and pieces of relevant models and a wide range of experimental liter-
ature that may be useful in modeling human behavior in multi-task systems.
Specifically, the following observations of the review are relevant to
our discussion. The reader is referred to [2] for additional details.

1.3.1 Single-stage Decision-making

Most of the literature on behavioral decision theory is devoted to
single-stage (static) decision-making under risk. The models of risky
decision behavior may be characterized by two alternative descriptions
of the decision task. The first modeling approach, rooted in mathematics
and economics, describes the decision task in terms of probability dis-
tributions over sets of outcomes (events) with little or no attention
paid to the underlying psychological processes of the individual DM.
This approach led to such moment-based models as the Expected Value (EV),
the Expected Utility (EU), the Subjectively Expected Utility (SEU), and
the Risk Preference models. The second modeling approach, rooted mainly
in psychology, characterizes decision tasks in terms of multi-dimensional
stimuli. It assumes that each stimulus forms a basic risk dimension, and
that the DM integrates these dimensions into a judgement or decision.
Thus, this approach led to explanatory models that view decision-making
under risk as a form of information-processing behavior.

The dominant moment based model for single-stage decision-making is
the subjectively expected utility (SEU) model proposed by Edwards [5].
In this model, the DM is assumed to maximize the subjectively expected
utility of an alternative, d, given by

$$SEU(d) = \sum_{e \in E} p_s(e) \ U[r(e,d)] \qquad\qquad (1.4)$$

where $p_s(e)$ is the subjective (perceived) probability of the event, e;
and $U[r(e,d)]$ is the subjective value (utility) function of the event, e.

In assessing the potential application of moment related versus
multi-dimensional stimuli models to static decision-making under risk, the
following observation was made in [2]: for normative (predictive) pur-
poses, models based on moments can serve as a first approximation or as a
formal standard against which to compare actual performance.

1.3.2 Multi-Stage Decision-Making

The existing literature on multi-stage decision-making problems
may be grouped under three headings: sequential statistical inference,
optional stopping and dynamic decision-making. The topic of statistical
inference is concerned with the information-processing (diagnostic)
ability of the humans, i.e., the human's ability to assess and revise
probabilities. The optional stopping problem combines information-pro-
cessing with simple (usually binary) action selection. Finally, the
existing literature on dynamic decision-making is mainly concerned with
action (control) selection with very little or no consideration to the
aspect of information-processing. It should be emphasized that virtually
no significant research has been done by behavioral decision theorists
using real-time decision paradigms.

The literature in the area of sequential probability inference
shows two different approaches to the modeling problem. The first
approach, advanced by statisticians and psychologists, employs Bayes'
rule as a normative representation of how a DM should revise his probabi-
lity estimates in light of new information. This approach led to the
study of "conservatism" - a suboptimal human behavior that produces

posterior probabilities nearer to the prior probabilities than those specified by Bayes' rule. The second approach, proposed mainly by psychologists, argues that the human is a selective, sequential information-processor with limited capacity and that this leads him to apply simple heuristics and cognitive strategies. This approach led to the discovery of such judgemental heuristics as representativeness, availability, and adjustment and anchoring, which were found to determine probabilistic inferences in many tasks. However, these findings can only be described in qualitative terms and, as yet, no quantitative descriptive theory based on heuristics has emerged.

The optional stopping problem is related to information-seeking ability of the human. In this problem, the DM is provided with an option, at each stage of the process, to seek (purchase, sample) one more observation, or to stop and make the terminal decision. Virtually all the models of optional stopping are normative in construct. They were developed within the Bayesian framework using the subjectively expected loss of the sequential decision process as the minimizing criterion of performance. In model-data comparisons, it was found that all the relevant procedural variables (e.g., pay-offs, prior probabilities, etc.) strongly influenced the number of observations, but not as much as the normative model predicted. It was also found that the optimal expected loss was quite insensitive to large deviations in the optimal decision policy ("curse of insensitivity").

The dynamic decision-making problems have not been studied as extensively as the static or sequential decision-making problems. This is due, mainly, to their inherent complexity, analytic sophistication and difficulties in implementing experiments on a computer. Most of the dynamic decision paradigms considered to date are taken from other fields

such as economics and operations research. Typically, the modeling approach begins with a normative construct based on dynamic programming, and then includes human limitations and constraints to produce normative-descriptive models. A common approach to the derivation of a normative-descriptive model is to first compare observed behavior with that prescribed by the normative (truly optimal) model. The discrepancies are then interpreted either in terms of limitations on the information-processing capacity or the human's misperception of the task. The limitation on the information-processing capacity can be linked to the DM's finite memory, his limited ability to project the effects of his present decisions into the future, his limited attention span, loss of decision time, misaggregation of data, etc. The limitation due to misperception of the task can be handled by postulating non-isomorphic internal models and differing subjective and objective cost functionals. The optimal decision policy is obtained under these cognitive and perceptual constraints, and then compared with the actual behavior. However, at present there does not exist a systematic method of identifying the human limitations beyond the current psychological knowledge. Moreover, the dynamic decision-making models, like those of optional stopping, are plagued with the "curse of insensitivity", i.e., optimal expected loss is insensitive to large deviations in the optimal decision strategy.

In assessing the potential application of the existing behavioral decision models to the MTDP, we conclude that none of them address the real-time decision-making issue of the MTDP. However, there exists a rich experimental literature which can provide insights and ideas into the nature of human limitations in information-processing and decision-making contexts. These issues are explored in section 1.6.

1.4 Multi-Task Decision-Making

Sheridan's work on the optimal allocation of personal presence [6] might be thought of as a preliminary step towards human modeling in a multi-task context. In this work, Sheridan was concerned with the dynamic human choice between two alternatives, viz., direct presence by transporting himself from one location to another, or vicarious presence via communication. He employed a dynamic programming formulation to obtain optimal decisions over the planning horizon, with states being the locations to be considered.

Rouse and Greenstein [7] pose the multi-task decision problem in terms of event detection and attention allocations. They considered a multi-task paradigm in which the subjects are presented with the process histories of several dynamic systems, and are instructed to detect process failures and react to them as quickly as possible. Rouse and Greenstein model human event (failure) detection by generating conditional probabilities of event occurrences, given the observation set, via discriminant analysis. The attention allocation problem was formulated in the framework of a single server queueing model with the object of minimizing the weighted expected waiting time, i.e., unlike the multi-task decision paradigm of our work, the tasks, in Rouse and Greenstein's study, stay in the queue until they are acted upon by the DM. They note the application of the model to computer-aiding, but the theoretical as well as experimental results are inconclusive.

Tulga [8] formulated the multi-task decision problem in the framework of a dynamic, deterministic, single machine-sequencing model. In Tulga's paradigm, the tasks are represented by rectangles of varying height (value density) and width (task duration, processing time). Tasks appear randomly in time and position and move at a constant velo-

city towards a dead-line. The subject's task is to attend to one task

at a time and thus cause that tasks' width to collapse uniformly and, one

hopes, to disappear before the task reaches the dead-line. The reward

earned is the aggregate reduction in the areas of all tasks. Assuming

stationary task parameters, open-loop feedback optimal (OLFO) decision

policy was obtained by solving a deterministic optimization problem every

time a new or expected task arrives, and every time a task is completed.

Dynamic programming with branch and bound strategies was employed to

solve the resulting optimization problem.

The studies of Tulga, and of Rouse and Greenstein are particularly

germane to the present research as they exemplify two of the most popular

modeling approaches to the multi-task decision problem (MTDP), viz.,

sequencing (combinatorial) and queueing-theoretic approaches. In section

1.6, we address at some of the limitations of these two approaches to the

MTDP and indicate how we have overcome their shortcomings via a semi-

Markov decision process (SMDP) approach.

1.5 Experimental Paradigm

The primary focus of this research effort is on human information-

processing and dynamic decision-making behavior in multi-task situations.

In order to minimize extraneous complexities, such as intricate task

structure, resource constraints, etc., we have considered a simple, yet

realistic, computer controlled experimental set-up shown in Fig. 5.

This experimental paradigm is a modified version of the one used by Tulga

[8]. In the experiments, the subjects observe a CRT screen on which

multiple, concomitant tasks are represented by moving rectangualr bars.

The bars appear at the left edge of the screen and move at different

velocities to the right, disappearing upon reaching the right edge.

Thus, the screen width represents an "opportunity window". In the pre-

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subjects' response box

Fig. 5: EXPERIMENTAL APPARATUS

sent experimental paradigm, there can be, at most, a total of five tasks
on the CRT screen, with a maximum of one on each line. This number is
commensurate with the results of Miller [9] on the limitations of human
information-processing capacity.

The height (reward, value) of each bar is either one, two or three
units. The number of dots ($1 \leq m \leq 5$) displayed on a bar represents the time
(in seconds) required to process the task. The subject may process a
task by holding down the appropriate push-button as in Fig. 5. By pro-
cessing a task successfully, the subject is credited with the correspond-
ing reward ($r_i \leq 1$, 2 or 3), and the completed task is eliminated from
the screen. However, no partial credit is given.

The above experimental framework retains the essential features of
the multi-task decision problem in a manageable, yet manipulative, con-

text. Using this formulation, the effects of key task variables on human decision-processes are studied via the following five experimental conditions:

(i) *Condition A*: Equal task velocities.

(ii) *Condition B*: Fixed rewards of 3 units for each task.

(iii) *Condition C*: Equal processing times of 3 sec. for each task.

(iv) *Condition D*: Full blown, where none of the variables is fixed.

(v) *Condition B_y*: Similar to condition B, but parallel monitoring is denied.

In condition B_y, the images of all the bars, except the one being processed, are blanked from the CRT screen. This prevents subjects from monitoring other tasks, and, perhaps, deciding on the next task to be acted upon. Thus, the subjects are forced to act in a serial mode under this experimental condition.

Six subjects, all university of Connecticut graduate Engineering students, were well-trained on the experimental paradigm. The relationships among the tasks' velocities and processing times were carefully chosen as to preclude a perfect score, and to motivate the subjects to use a rational sequencing algorithm. In all cases, the subjects were instructed to maximize the accumulated reward, and were scored using the total score, as well as the percentage of a perfect score. They were informed of their score following each 90 sec. run and were encouraged to keep it as high as possible.

In the data-taking runs each subject was presented with eight replications of each experimental condition, in randomized order. This was achieved via a "scrambling technique" that switched tasks among the five parallel lines for different runs [10]. The tasks were unscrambled at

the time of data analysis. This type of experimental design, when aggregated across subjects, yields ensemble statistics that are indicative of the subjects' population. The source of randomness in this design is the inter-subject variability. This type of design has the added advantage of minimizing artifacts such as the effects of learning.

The data collected were time-histories for each line i of the subject's decisions, $d_i(t)$; the task completion status, $c_i(t)$; and the error sequence, $e_i(t)$. The variables $d_i(t)$, $c_i(t)$ and $e_i(t)$ are binary numbers defined by

$$d_i(t) = \begin{cases} 1 \text{ if a subject was processing a task on line i at time t} \\ 0 \text{ otherwise} \end{cases} \qquad (1.5a)$$

$$c_i(t) = \begin{cases} 1 \text{ if a subject had completed a task on line i by time t} \\ 0 \text{ otherwise} \end{cases} \qquad (1.5b)$$

and

$$e_i(t) = \begin{cases} 1 \text{ if a subject was processing a task on line i at time t,} \\ \text{ which can not be successfully completed} \\ 0 \text{ otherwise} \end{cases} \qquad (1.5c)$$

In Eq. (1.5a), i=0 refers to the "do nothing" or monitoring decision. The variable $c_i(t)$ is set to zero at the end of the opportunity window of the present task, before the arrival of the next task in the sequence. At a sampling rate of 20/sec., each run yielded 1800 datum points for each of the variables recorded. For the same experimental condition, the time-histories were ensemble averaged to obtain the decision probabilities, $P_{di}(t)$; completion probabilities, $P_{ci}(t)$; and error probabilities, $P_{ei}(t)$. The averaging process was first done for each subject, and then *across subjects to obtain the* "grand" averages. The details of data analysis are presented in section 3.1.

1.6 SUMMARY

In previous sections, we have examined the relevant literature on

behavioral decision theory and multi-task decision-making. This overview
has suggested several limitations of the previous work and possible means
to overcome them. The following conclusions and comments in this regard
seem appropriate.

(i) *Status of behavioral decision theory*: Most of the litera-
ture on behavioral decision theory is devoted to single-
stage decision-making. The existing literature on multi-stage
decision-making emphasizes either information-processing
(diagnosis) or action selection. However, any realistic
multi-task system involves diagnosis as well as dynamic
(usually real-time) action selection.

(ii) *Normative versus Descriptive models*: Theories of rational
behavior may be normative or descriptive. The normative
theory attempts to <u>prescribe</u> how decisions <u>should</u> be made in
the face of a given situation. The descriptive theory, on
the other hand, purports to <u>explain</u> how decisions <u>are</u> made
in a given situation. A review of behavioral decision
theory [2] shows that normative (prescriptive) models can
serve as a <u>first approximation</u> to assess human decision be-
havior, or they can be used as a <u>formal</u> <u>standard</u> against
which to compare actual performance. The model developed in
this thesis is normative in construct.

(iii) *Need for good Multi-task paradigms*: Experiments in multi-
task decision-making may, by their very nature, become overly
elaborate and cumbersome. This is especially true when the
experimenter yields to the natural temptation to simulate
the "entire scenario", thereby possibly masking trends in the
resulting data. In summarizing the research on behavioral

decision theory, we noted that the discrepancies between a
normative model and observed behavior can be attributed to
cognitive (intellectual or information-processing) limita-
tions, misperception of the task and procedural variables.
Since there exists no systematic method of identifying the
human limitations beyond current psychological knowledge,
the multi-task supervisory control decision paradigms should
be designed to minimize the limitations due to misperception
of the task and procedural variables. Such an experimental
paradigm was developed in section 1.5. This paradigm is
simple, realistic, easy to understand and to administer.
It retains the essence of the multi-task decision problem
by presenting the human with a dynamic situation wherein
tasks of different value, time requirement and deadline
compete for his attention. Due to its simplicity, the
paradigm minimizes the possibility of human misperception
of the tasks. If we can understand and model the behavior
of well-trained subjects in simple laboratory tasks, then
perhaps this knowledge may be extended to more complex
tasks. The ability to repeat laboratory experiments is a
powerful tool, for it allows us to study intersubject
differences, the effects of different information, and
provides us with a measure of variability inherent in
human's decision process.

(iv)   *Curse of insensitivity*: Most normative decision models of
behavioral research are plagued with the "curse of insensi-
tivity": substantial variations in the optimal decision
policies lead to only a small change in the resulting

cost. This problem could have been minimized, to some extent, by the proper choice of reward and processing time structures, as the discrete format employed in the present experimental paradigm.

(v) *Modeling approaches*: Queueing and sequencing (combinatorial) theoretic approaches [7,8] appear to be the most popular modeling approaches to model human decision strategy in a MTDP. The main shortcoming of classical queueing theory approach is that it is extremely difficult, if not impossible, to determine the structure of an optimal strategy in the MTDP, as it involves a dynamic, endogeneous, preempt-repeat priority discipline with non-conservative[†] customer (task) and server (human) characteristics.[††] The main advantage of this approach is that it can handle stochastic arrivals (which are assumed to occur indefinitely into the future), and stochastic processing times. That is, the approach can incorporate uncertainty in the task characteristics. However, in many practical applications the task characteristics are time dependent and are, to a large extent, predictable. Therefore, it is the randomness associated with the decision-maker that is of primary importance, and the stochastic properties of tasks are a second order effect (but not necessarily negligible). Moreover, the classical queueing theory places great emphasis on finding stationary measures of

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[†] This implies that a customer (task) may leave before being served or the server (human) may refuse to service a low priority customer (task).

[††] With moderate complexity, a stationary, state dependent, non-preemptive priority policy in non-conservative queueing systems can be determined using the tools of dynamic programming. The reader is referred to [11] for details.

system effectiveness, whereas the dominant issue in real
systems is the determination of instant to instant human
decision behavior, while servicing a time dependent demand.
The combinatorial approaches, on the other hand, involve
sequencing a finite number of tasks whose arrival times,
processing times and dead-lines are known deterministically
(if random, mean values are used). This approach can not
handle randomness associated with the decision-maker or the
task parameters easily. Thus, the incorporation of human
randomness into the decision strategy is difficult using a
sequencing theoretic approach. The control and semi-Markov
decision process approach to modeling the human decision
strategy in a MTDP, developed in Chapter II and in [10], sub-
sumes the earlier two approaches and can explicitly incor-
porate human limitations.

(vi) *Drawbacks of Tulga's model*: One major drawback of Tulga's
model is that the fundamental human limitations have not been
identified. First, it is almost impossible for the human to
have perfect estimates of the time available and the time re-
quired to process a task. Second, it is well known [12] that
the humans do not respond to the same stimulus in identical
fashion at different times (due to their limited resolving
power), even when there are no changes in their information
or resources. This makes it difficult to validate/invali-
date the truly normative, sample-path (Monte Carlo) models
of the type espoused by Tulga. Third, it is also well known
that the human is a sequential decision-maker with limited

information-processing capabilities [13]. Thus, it is difficult to justify normative, combinatorial models based on dynamic programming (DP), as they require the specification of complete future courses of action before any task is acted upon. Moreover, the computational load of the DP increases exponentially with the number of tasks to be sequenced. On the other hand, if a finite stage DP is advocated as a compromise, then the nagging question is how to choose the number of stages? The last point is not a peculiarity of Tulga's model alone. It applies to all the behavioral models employing DP formulation. The present dynamic decision model (DDM) overcomes the first two cited limitations of Tulga's model by explicitly including human randomness in the model, and circumvents the combinatorial problem of DP by postulating a myopic (one-stage) decision policy.

(vii) *Modes of Model implementation*: If the subjects do not come from a homogeneous population, in terms of their decision performance, then the sample path (Monte Carlo) models of the type proposed in [8] make little sense. The DDM developed in this thesis can be exercised either in a covariance propagation mode or in a Monte Carlo (sample-path) mode. The first mode gives probabilistic predictions necessary for model-data validation. This is done in chapter III. The second mode is appropriate for using the model as a decision aid. These issues are explored in chapter IV.

II. ANALYTIC MODEL FOR HUMAN TASK SEQUENCING

Our analytic approach to model human decision-making in multi-task
environments is based on, and will extend, the optimal control model (OCM)
of Kleinman et al [14-16]. The optimal control model is a general and
versatile methodology for predicting human response in stochastic, multi-
variable control tasks. The modeling approach, rooted in modern control
and estimation theories, is based on the assumption that a well-trained
and well-motivated human operator behaves in an optimal manner, subject
to his inherent limitations and constraints, and the perceived task
objectives. The OCM has been applied successfully in a variety of manual
control tasks, as well as in tasks that do not involve closed-loop control
[17-18]. However, all these studies emphasize either the continuous con-
trol function of the human, or his ability to test binary hypotheses.
They do not address the decision-making/task-sharing roles of the human
that gain prominence in supervisory control, or in semi-automated sub-
systems of the type discussed in chapter I.

This chapter extends the conceptual framework of the OCM methodology
to multi-task situations in which monitoring, information-processing and
dynamic decision-making (task sequencing) are the operator's main activi-
ties. The basic idea of our modeling approach is to integrate decision-
directed elements within an OCM-like construct. As with the OCM, the
approach is normative, in that we attempt to determine what a well-trained
and well-motivated human operator should do, given the task objectives.

In the sections below, the key elements of OCM are outlined briefly

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and the dynamic decision model (DDM) of human task sequencing performance emerges.

2.1 Optimal Control Model of Human Response - An Overview

2.1.1 Background [14-16]

The basic structure of the OCM is shown in Fig. 6 and consists of the following elements:

(i) *Perceptual model*: The perceptual model translates displayed variables, $\underline{y}(t)$, into noisy, delayed perceived variables $\underline{y}_p(t)$, which is the information upon which the human bases his subsequent estimation, control and/or decision strategies.

(ii) *Human Limitations*: The OCM includes time-delay, human randomness, small signal threshold phenomenon, and scanning effects in its formulation. The time-delay, τ, accounts for the internal human delays associated with visual, central processing and neuromotor pathways. Human randomness is assumed to be manifested as errors in observing/processing displayed quantities and in executing intended control movements. Thus, observation noise, $\underline{v}_y(t)$ and motor noise, $\underline{v}_u(t)$ are lumped representations of controller's central processing and sensory randomness. The non-linear threshold in the OCM captures the "neglect" phenomenon exhibited by humans when observing small stimuli. Finally, the scanning-interference model accounts for the fact that the human must allocate monitoring attention among the various displays [16].

(iii) *Information-Processor*: The information-processor consists of a Kalman filter and predictor that compensate for the psychophysical limitations of the human to generate the "best"
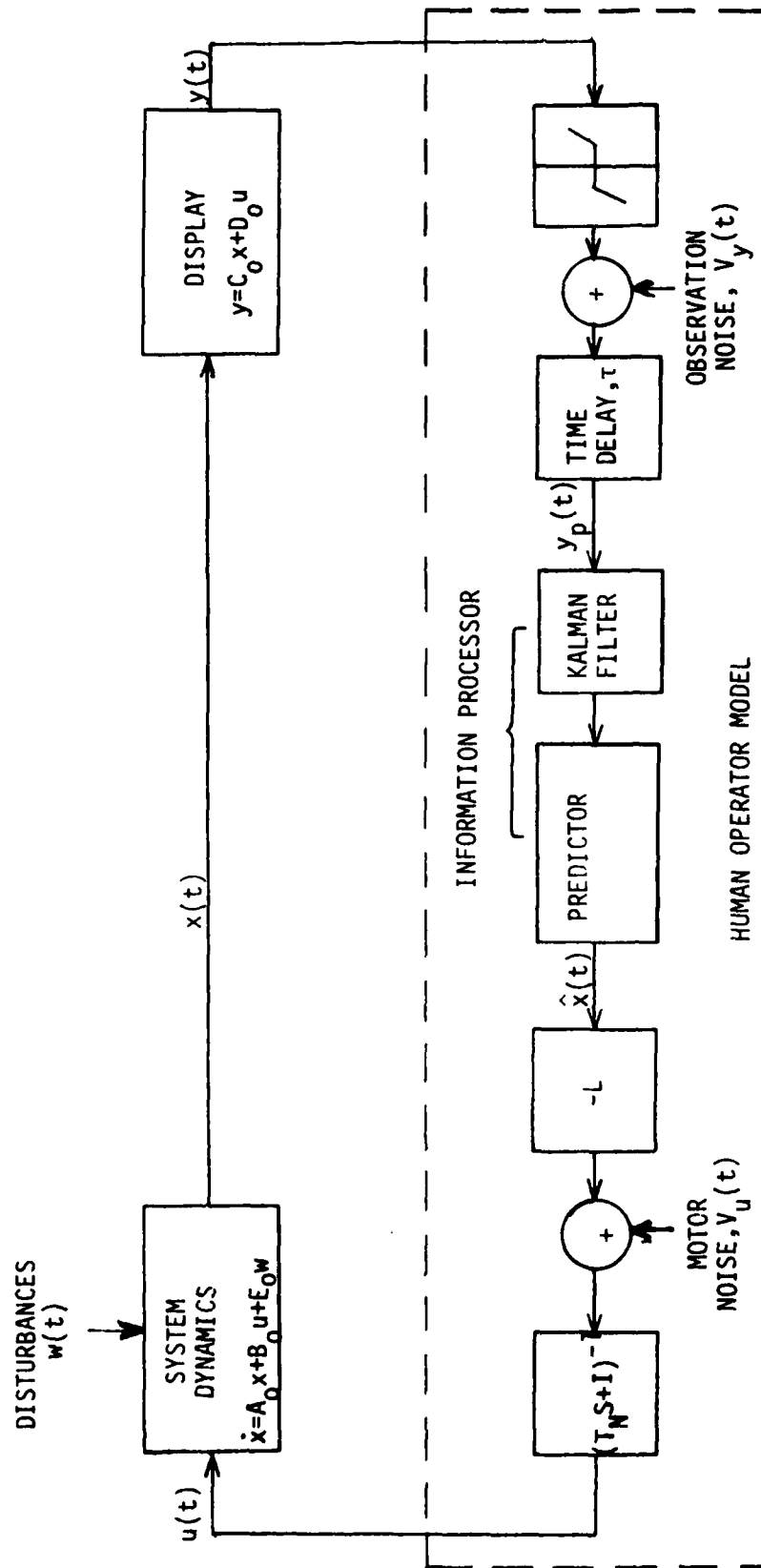
Fig. 6: OPTIMAL CONTROL MODEL OF HUMAN RESPONSE

estimate of the (augmented) system state $\hat{\underline{x}}(t)$ from the perceived information base.

(iv)   *Feedback Gains*: The control task requirements are assumed to be adequately represented by thé minimization of a quadratic cost functional. The operator's commanded control input $\underline{u}_c(t) = -L\hat{\underline{x}}(t)$, where the feedback gains L minimize the cost functional.

(v)   *Motor model*: The motor model accounts for the bandwidth limitations of the human via the neuromotor dynamics, $(T_N s + I)^{-1}$, and his inability to generate noise-free control signals via the motor noise, $\underline{v}_u(t)$.

The Kalman filter-predictor, followed by the feedback gains, represent the adaptations by which the human operator optimizes his performance and compensates for his inherent limitations. In general, these model elements depend on the (human's internal characterization of the) system dynamics, human limitations, and the task requirements. The Kalman filter generates the best estimate of the delayed (augmented) state

$$\underline{p}(t) = E\{\underline{x}(t-\tau)|\underline{y}_p(\sigma), \sigma \leq t\} \tag{2.1}$$

according to an equation of the form

$$\dot{\underline{p}}(t) = A\,\underline{p}(t) + B\,\underline{u}_c(t-\tau) + G_f\,\underline{v}(t) \tag{2.2}$$

where the filter gains, G_f, are determined from a matrix Riccati differential equation. The quantity

$$\underline{v}(t) = \underline{y}_p(t) - C\,\underline{p}(t) \tag{2.3}$$

is the innovation process and represents the difference between the actual and expected observations. Basically, $\underline{v}(t)$ is the new information that

is brought to the filter by $\underline{y}_p(t)$. The predictor generates an estimate of the present state, $\hat{\underline{x}}(t)$, by projecting $\underline{p}(t)$ ahead by τ seconds to compensate for the time-delay.

The state estimate, $\hat{\underline{x}}(t)$, and its associated covariance matrix, $E(t)$, form a sufficient statistic for the closed loop man-machine system. In other words, the pair $\{\hat{\underline{x}}(t), E(t)\}$ can be used as a basis for determining subsequent control/decision strategies. A second quantity of interest in the OCM information-processor is the innovation process, $\underline{v}(t)$, defined in Eq. (2.3). When the internal model of the Kalman filter adequately represents the controlled element dynamics, the process $\underline{v}(t)$ is a zero-mean, white Gaussian noise process with covariance $V_y(t)$ equal to the observation noise covariance. However, when the internal model and system dynamics are not commensurate, the human's estimate of the system behavior deviates from the observed dynamic behavior. These differences produce a non-zero mean, correlated innovation process. This property can be used to develop models of human failure detection [18], and to investigate the effects of training on human performance.

2.1.2 Elements for Decision-making/Detection

A key feature of the OCM's information-processor is that it provides the statistical characteristics of two important variables: the state estimate $\{\hat{\underline{x}}(t), E(t)\}$; and the innovation process $\{\underline{v}(t), V_y(t)\}$. These, in turn, have provided a mechanism for studying selected decision/detection phenomena in man-machine systems. For example, Levison and Tanner [17] studied how well subjects could determine if a signal embedded in noise exceeded a given threshold. Their model assumed that the operator was an optimal decision-maker in the sense of maximizing the subjectively expected utility. For equal penalties on missed detections and false alarms, this rule reduces to a Likelihood ratio test, which was implemen-

ted using the sufficient statistic $\{\hat{\underline{x}}(t), E(t)\}$. In another study, Gai
and Curry [18] used the OCM information-processing submodel to analyze
failure detection in a simple laboratory task and in an experiment simu-
lating pilot monitoring of an automatic landing system. They considered
only instrument failures, and modeled the detection process as a sequen-
tial hypothesis test on the mean of the innovations, $\underline{\nu}(t)$.

These studies demonstrate the potential of modern estimation techni-
ques in decision-making/detection situations. An important feature of
the work in [17-18] is that it provides a validation of the Kalman filter-
predictor submodel in tasks not involving closed-loop control. When these
validation results are combined with the overall verification of the OCM
in manual control tasks, the potential of a control-theoretic construct
for modeling human decision processes emerges.

2.2 Overview of Modeling Approach

Our approach to modeling human decision behavior parallels the opti-
mal control model of human response in spirit, but not in form. In the
OCM, the control and information-processing strategies are separable.
Once an estimate of the system state is available, the linear feedback
control law uses this estimate as if it were the true state. Human limi-
tations affect only the quality of (augmented) state estimates.

This type of separation has been found to be plausible in the present
dynamic decision model (DDM). For any task i in the opportunity window,
it is possible to show that $T_{Ri}(t)$, the time required to complete task i
starting at time t; and $T_{ai}(t)$, the time available/remaining to work on
task i at time t, are valid decision state variables. That is, these two
quantities satisfy the axiomatic definition of a state that it must pro-
vide the complete running summary of past actions (decisions). The joint
density of the decision states of all tasks in the opportunity window is

estimated from the information-processor of the DDM, and provides suffi-
cient information for the decision-process. The statistics of decision
states, along with the task values, $r_i(t)$, and a performance metric, are
used to compute the decision strategy. By analogy to the control theore-
tic OCM, the values $r_i(t)$ play the role of cost functional weights, while
the decision state variables correspond to system state variables.

A block diagram of the DDM is shown in Fig. 7. Each of the N tasks
in the opportunity window is represented by a dynamic subsystem acted on
by disturbances to account for the non-stationarities in task characteris-
tics. The preceived outputs $\{\underline{y}_{pi}\}$ are delayed, noisy versions of the
task states $\{\underline{x}_{Ti}\}$ and are contingent upon the monitoring process. The
preceived outputs are processed to produce the best linear unbiased
estimates of the task states $\{\hat{\underline{x}}_{Ti}\}$, and their associated covariances
$\{E_i\}$ via a Kalman filter-predictor submodel. The statistics of the task
states $\{\hat{\underline{x}}_{Ti}, E_i\}$ are, in turn, used to determine the first and second order
statistics of the decision states $\{\hat{T}_{Ri}, \sigma_{Ri}\}$ and $\{\hat{T}_{ai}, \sigma_{ai}\}$. The statis-
tics of the decision states, along with the task values, $r_i(t)$; are
combined to determine the attractiveness measure, $M_i(t)$, of each task in
the opportunity window. Subsequently, the measures are used to generate
the probability $P_{di}(t)$ of acting on each of the N tasks and the probabil-
ity $P_{d0}(t)$ of not acting on any task (or the monitoring probability,
$P_{dm}(t)$).

The next few sections expand briefly on various features of DDM.

2.3 System Dynamics

In formulating the multi-task decision problem, it is convenient to
differentiate among the process state or Markov state, $\underline{s}$; the set of
task states, $\underline{x}_{Ti}$; and the set of decision states, $\underline{x}_{di}$. In the present

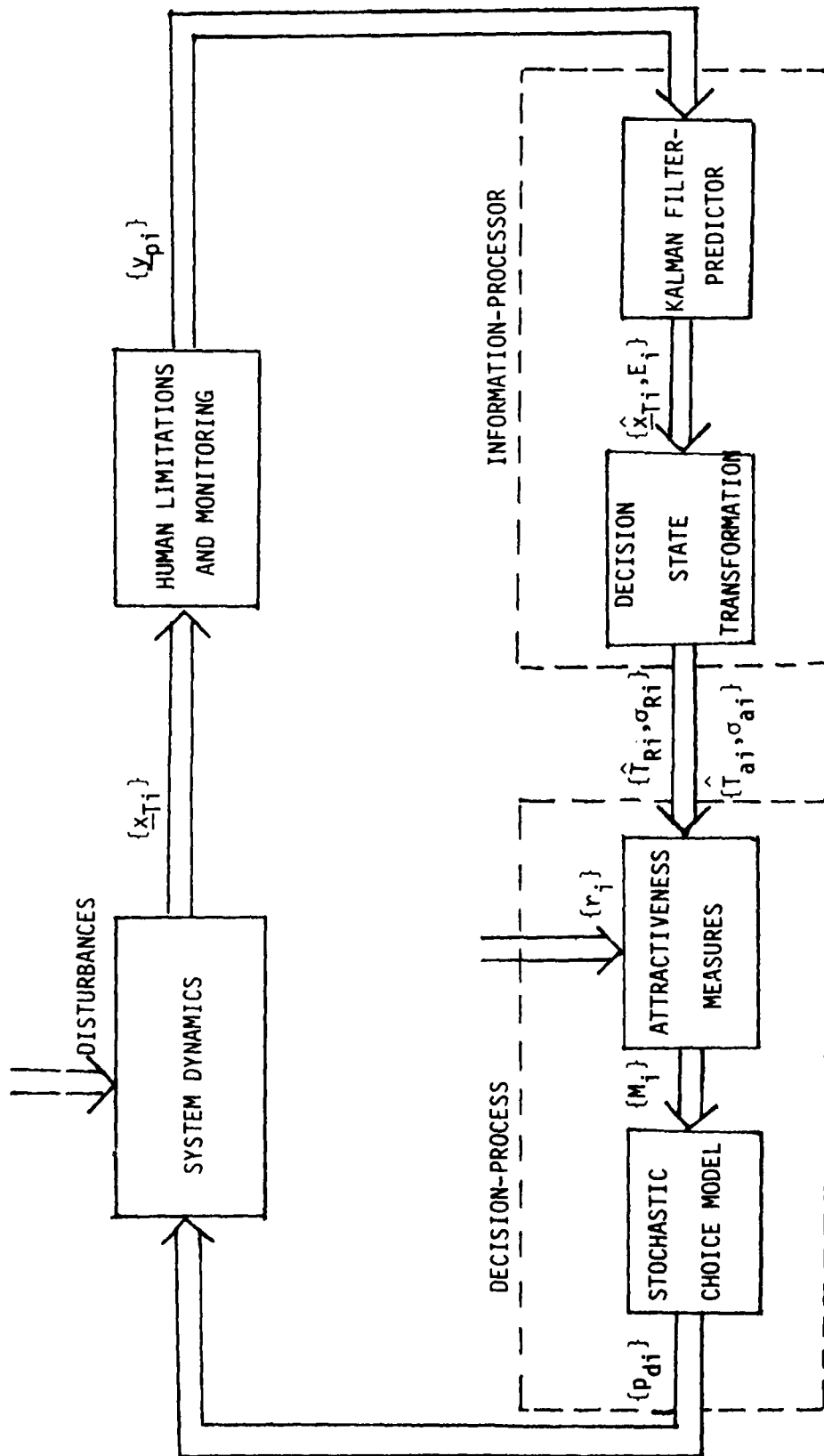Fig. 7: DYNAMIC DECISION MODEL OF HUMAN TASK SEQUENCING PERFORMANCE

experimental context, the process state, $\underline{s}$, is related to the status of the CRT display and indicates whether or not a task is present on each of the K(=5) lines, K being the system capacity. The task state, $\underline{x}_{Ti}$, describes the dynamical variables internal to each task i. In the present experimental paradigm, the task state consists of the instantaneous position and velocity of the bar and the time required to process the task. Finally, the decision state, $\underline{x}_{di}$, consisting of time available and the time required to process task i, is a memoryless functional transformation of the task state, $\underline{x}_{Ti}$. These notions are formalized below.

2.3.1 Process state or Markov state, s

Since the number of tasks in the system is less than or equal to K (system capacity), the process state of the multi-task system at an arbitrary time t can be represented by a K-dimensional row vector as

$$\underline{s} = (I_1 \ I_2 \ldots I_i \ldots I_K) \tag{2.4}$$

where the binary index variable $I_i(t)$ is given by

$$I_i(t) = \begin{cases} 1 & \text{if there exists a task on line i at time t} \\ 0 & \text{otherwise} \end{cases}$$

The total number of possible process states are 2^K. Note that the number of tasks in the opportunity window at any time t is given by

$$N(t) = \sum_{i=1}^{K} I_i(t)$$

We let $A(t)$ denote the set of N available (accessible) tasks in the opportunity window at time t. Formally,

$$A(t) = \left\{ i \middle| I_i(t) = 1; \ i=1,2,\ldots,K \right\} \tag{2.5}$$

Clearly, the decision set $\mathcal{D}(t)$, the set of (N+1) feasible decisions at time t, is given by

$$\mathcal{D}(t) = A(t) + \left\{0\right\}$$

Thus, the (N+1) possible decisions at any time t are to attend to one of the N tasks in the opportunity window, or do nothing. The set of feasible decisions is time-varying as a consequence of estimation and actions by the DM, and as a result of the arrival of new tasks with different attributes.

2.3.2 Task State, $\underline{X}_{Ti}$

For any task on line i, the time required to complete task i, T_{Ri}, the position of the bar from the left edge, ℓ_i, and the velocity, v_i, of the bar constitute the task state variables as shown in Fig. 8.
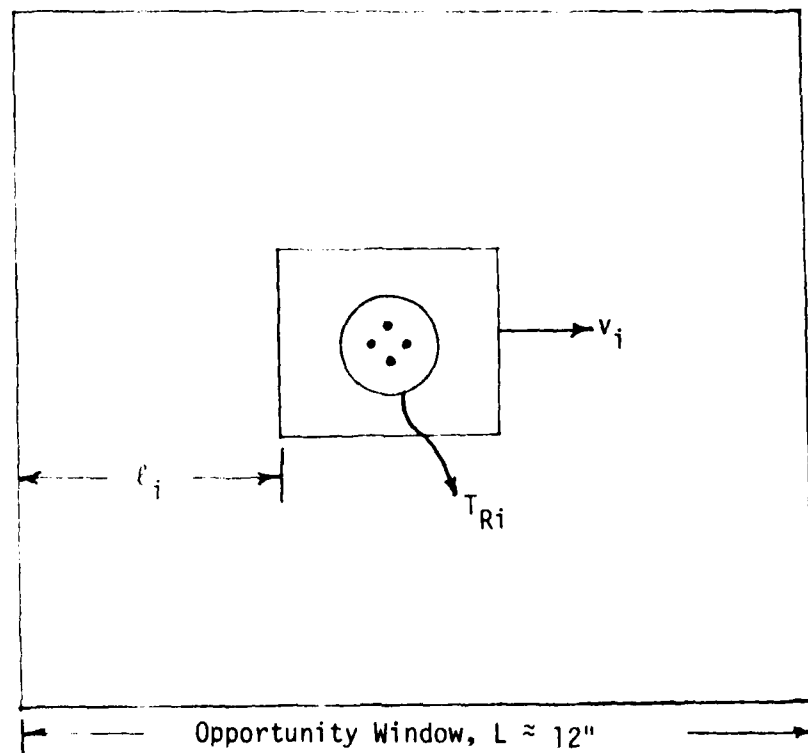

Fig. 8: TASK STATE VARIABLES

The state variable $x_{Ti1}(t)$, denoting the time required to complete task i starting at time t, is action oriented. Its evolution can be characterized by the differential equation

$$\dot{x}_{Ti1}(t) = \dot{T}_{Ri}(t) = -d_i(t) \tag{2.7}$$

where $d_i(t)$ is a binary decision variable given by

$$d_i(t) = \begin{cases} 1 \text{ if the decision is to act on task i at time t} \\ 0 \text{ otherwise} \end{cases}$$

Since the human can act on only one task at any given time, we have the following constraints on the decision variables:

$$d_i(t) = 1 \text{ implies } d_j(t) = 0 \quad i \neq j; \quad i, j \in \mathcal{D}(t)$$

where $d_0(t) = 1$ refers to the "do nothing" or monitoring decision.[†]
The remaining task state variables, representing the position and velocity of the bar, are given by

$$\dot{x}_{Ti2}(t) = \dot{\ell}_i(t) = x_{Ti3}(t)$$

$$\dot{x}_{Ti3}(t) = \dot{v}_i(t) = w_i(t) \tag{2.8}$$

where $w_i(t)$ is a zero-mean, white Gaussian noise with variance $W_i(t)$ that accounts for (perceived) non-stationarities in task velocity.

In vector-matrix form, the dynamics of the task state can be represented

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[†]Note that the defining differential equation for T_{Ri} assumes a preempt-resume processing discipline, while the experimental paradigm was designed to operate in a preempt-repeat mode. The form of Eq (2.7) was chosen after examining the experimental data, which showed that the human seldom preempted a task in all the experimental conditions studied. However, it is straightforward to include the effects of a preempt-repeat mode of processing by reinitializing the dynamical equation for T_{Ri}, everytime $d_i(t)$ switches from 1 to 0 and $T_{Ri}(t)$ is non-zero.

by

$$\dot{\underline{x}}_{Ti}(t) = A \, \underline{x}_{Ti}(t) + \underline{b} \, w_i(t) - \underline{g} \, d_i(t); \quad i\epsilon A(t) \qquad (2.9)$$

where

$$\underline{x}'_{Ti} = [x_{Ti1}, x_{Ti2}, x_{Ti3}] = [T_{Ri}, \ell_i, v_i]$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \, \underline{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad ; \quad \underline{g} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The subsystem state in Eq (2.9) is reinitialized to the new task attributes everytime a new task arrives on line i.

2.3.3 Decision State, $\underline{x}_{di}$

The decision state, $\underline{x}'_{di} = [T_{Ri}, T_{ai}]$, is related to the task state via a functional transformation as

$$\underline{x}_{di}(t) = \underline{F}[\underline{x}_{Ti}(t)]$$

In the present experimental context, the time required to complete task i starting at time t, $T_{Ri}(t)$, is given by

$$T_{Ri}(t) = x_{di1}(t) \equiv x_{Ti1}(t) \qquad (2.10)$$

The other decision state variable $T_{ai}(t)$, the time available to work on task i at time t, is related to the task state $\underline{x}_{Ti}$ via

$$T_{ai}(t) = x_{di2}(t) = \frac{L - x_{Ti2}(t)}{x_{Ti3}(t)} = \frac{L - \ell_i(t)}{v_i(t)} \qquad (2.11)$$

where L is the length of the opportunity window ($\approx 12''$).

In the present experimental paradigm, $T_{ai}(t)$ is assumed to be independent of $T_{Ri}(t)$. This is not a restrictive assumption. If the nature of relationship between $T_{ai}(t)$ and $T_{Ri}(t)$ is known, it can be incorporated into the model formulation in a straightforward manner.

2.3.4 Preceptual Model

Since the processing times are quantized in steps of 1 sec., the displayed information consists of a modified version of the task state, $\underline{x}_{T1}$. Thus,

$$y_1(t) = \begin{bmatrix} Q\ [T_{R1}(t)] \\ \ell_1(t) \\ v_1(t) \end{bmatrix} = \underline{x}_{T1}(t) + \underline{g}\ v_q(t) \qquad (2.12)$$

where $v_q(t)$, the linearized quantization error, is bounded by

$$-0.5 \le v_q(t) \le 0.5 \qquad (2.13)$$

In order to represent the effects of quantization on the estimation process, it is frequently assumed [19] that $v_q(t)$ is uncorrelated with $T_{R1}(t)$, and that it is a stationary, zero-mean, white noise process uniformly distributed over the range of quantization error of Eq (2.13). The autocovariance of the noise process $v_q(t)$ can be shown to be

$$E\ [v_q(t)\ v_q(\sigma)] = V_q\ \delta(t-\sigma) = \frac{1}{12}\ \delta(t-\sigma) \qquad (2.14)$$

Following usual practice, the human is assumed to perceive a noisy, delayed and linearized replica of $\underline{y}_1(t)$ given by

$$\underline{y}_{p1}(t) = \underline{y}_1(t-\tau) + \underline{v}_{y1}(t-\tau) \qquad (2.15)$$

where

τ = the human's time delay ($\approx$ .2 sec)

$\underline{v}_{y1}(t)$ = the observation noise at time t

The observation noise $\underline{v}_{y1}(t)$ is a zero-mean, white Gaussian noise process with diagonal covariance matrix V_{y1}. As with the OCM, the diagonal

elements of the observation noise covariance matrix associated with the task position and velocity are functionally related to the monitoring strategy and the mean-square values of the corresponding output variables according to

$$\left[V_{yi}\right]_{jj} = \frac{\pi \rho_i}{f_i(t)} \quad E\left\{y_{ij}^2(t)\right\} \tag{2.16}$$

where

y_{ij} = j th element of the vector $\underline{y}_i$; $j=1,2,3$

ρ_i = noise to signal ratio (NSR) associated with $\underline{y}_i$ of task
i ($\approx .01$)

$f_i(t)$ = monitoring allocation to task i

There is assumed to be no intratask attention allocation among the individual components of the displayed variables, $\underline{y}_i$, as the task information is presented in an integrated form. Thus, $f_i(t)$ is the monitoring attention to task i. Also since the (linearized) quantization error overshadows the inherent human randomness in perceiving the decision state variable, $T_{Ri}(t)$, the observation noise covariance $\left[V_{yi}\right]_{11}$ can be neglected in comparison to V_q. In summary, the time-histories of $\underline{y}_{pi}(t)$ are the stimuli upon which the human bases his subsequent estimation and decision strategies.

2.4 Monitoring Strategy

The monitoring allocations, $f_i(t)$, affect the subsequent decision strategy. On the other hand, the specifics of the experimental paradigm determine whether or not the monitoring strategy is dependent on the decision strategy. In the present experimental context, if a task i is acted upon at time t (i.e., $d_i(t)=1$), it is also monitored. However, there exist two possibilities for the other tasks $j \neq i$:

(i) *Parallel monitoring:* In this case, all tasks, including the one being acted upon, can be monitored simultaneously. (This corresponds to experimental conditions A-D). Here, an equal (monitoring) attention allocation strategy, i.e., $f_i(t) = \frac{1}{N}$, is found to be adequate for model applications. This result is not surprising, since an overview on the existing monitoring models [10,16] indicates that the overall system performance is not very sensitive to changes in the monitoring process over a reasonable range of variation about the optimal strategy, at least for well-designed displays.

(ii) *No Parallel monitoring:* In this case, tasks, other than that being acted upon, are not available for monitoring (experimental condition B_y), but monitoring of all tasks is an explicit decision alternative. Here, the monitoring process is strongly coupled to the decision strategy. Noting that $f_i(t)$ is the ensemble probability of monitoring task i at time t, we have by the total probability rule

$$f_i(t) = P \{ \text{monitor task i at time t} \}$$

$$= \sum_{j \in \mathcal{D}(t)} P \{ \text{monitor i, act on j} \}$$

$$= \sum_{j \in \mathcal{D}(t)} P \{ \text{monitor i} | \text{act on j} \} \cdot P_{dj}(t)$$

$$= P_{di}(t) + \frac{P_{dm}(t)}{N} \quad ; \quad i \in A(t) \tag{2.17}$$

where it is assumed that the monitoring probability, $P_{dm}(t)$, is equally distributed among N tasks.

2.5 Information - Processor

The information-processor compensates for the human's inherent randomness, time-delay and monitoring allocations to produce the "best" estimate of the decision state from the perceived information base. As with the OCM, the information-processor consists of a Kalman filter and a linear predictor. This choice was motivated by the results of [17-18], which provided an independent verification of the filter-predictor structure for the information-processor in situations not involving closed-loop control. The Kalman filter-predictor submodel generates the best linear unbiased estimates of the task state, $\hat{\underline{x}}_{Ti}(t)$ and its associated covariance matrix, $E_i(t)$. The pairs $\{\hat{\underline{x}}_{Ti}(t), E_i(t)\}$ are subsequently used to compute the first and second order statistics of the decision state, $\underline{x}_{di}(t)$, viz., the pairs $\{\hat{T}_{Ri}(t), \sigma^2_{Ri}(t)\}$ and $\{\hat{T}_{ai}(t), \sigma^2_{ai}(t)\}$ for each task i.

2.5.1 Kalman Filter

The Kalman filter generates the best linear unbiased estimate of the delayed state

$$\underline{p}_i(t) = E\left\{\underline{x}_{Ti}(t-\tau)/\underline{y}_{pi}(\sigma) \; ; \; \sigma \le t\right\}$$

according to an equation of the form

$$\dot{\underline{p}}_i(t) = A\,\underline{p}_i(t) - \underline{g}\,d_i(t-\tau) + G_i(t)\left[\underline{y}_{pi}(t) - \underline{p}_i(t)\right] \tag{2.18}$$

with the initial condition $\underline{p}'_i(t_{0i} + \tau) = [\bar{T}_{Ri}(t_{0i}), 0, v_i(t_{0i})]$. Here t_{0i} is the initial (arrival or ready) time of a task on line i, and $\bar{T}_{Ri}(t_{0i})$ is the a priori mean of the processing time.

The filter gains $G_i(t)$ are given by

$$G_i(t) = \Sigma_i(t)\left[V_{yi}(t-\tau) + \underline{g}\,V_q\,\underline{g}'\right]^{-1} \tag{2.19}$$

where $\Sigma_i(t)$ is generated from the usual Riccati equation

$$\dot{\Sigma}_i = A\, \Sigma_i + \Sigma_i\, A' - \Sigma_i\, [V_{yi}(t-\tau) + \underline{g}\, V_q\, \underline{g}']^{-1}\, \Sigma_i + \underline{b}\, W_i(t-\tau)\, \underline{b}' \qquad (2.20)$$

with the initial condition

$$\Sigma_i\, (t_{0i} + \tau) = \text{diag}\left[\frac{(T_{RH} - T_{RL})^2}{12}\, , \, 0 \, , .01\, \pi\, v_i^2(t_{0i})\right]$$

where T_{RH} and T_{RL} are the a priori maximum and minimum values of the processing time. The initial uncertainty in the velocity estimation is assumed to scale with the square of the velocity in accordance with the Weber's law.

2.5.2 Linear Predictor

Prediction of the present task state, $\hat{\underline{x}}_{Ti}(t)$, is obtained by integrating the vector-matrix linear differential equation

$$\dot{\hat{\underline{x}}}_{Ti}(\sigma) = A\, \hat{\underline{x}}_{Ti}(\sigma) - \underline{g}\, d_i(\sigma) \qquad (2.21)$$

from $\sigma = t-\tau$ to $\sigma = t$ with the initial condition $\hat{\underline{x}}_{Ti}(t-\tau) = \underline{p}_i(t)$.

The error covariance associated with the task state estimate $\hat{\underline{x}}_{Ti}(t)$, denoted by $E_i(t)$, is given by

$$E_i(t) = e^{A\tau}\, \Sigma_i\, e^{A'\tau} + \int_{t-\tau}^{t} e^{A(t-\sigma)}\, \underline{b}\, W_i(\sigma)\, \underline{b}'\, e^{A'(t-\sigma)}\, d\sigma \qquad (2.22)$$

2.5.3 Statistics of the decision state, $\underline{x}_{di}$

The statistics of the decision state variable $T_{Ri}(t)$ are readily computed from those of the task state, $\underline{x}_{Ti}(t)$, as

$$\hat{T}_{Ri}(t) = \text{conditional mean} = \hat{x}_{Ti1}(t)$$

$$(2.23)$$

$$\sigma^2_{Ri}(t) = \text{conditional variance} = E_{i11}(t)$$

The human's perception of the conditional density of the decision state variable $T_{Ri}(t)$ is assumed to be Gaussian with mean $\hat{T}_{Ri}(t)$ and variance $\sigma^2_{Ri}(t)$.

In order to compute the statistics of the remaining decision state variable $T_{ai}(t)$, we note from Eq (2.11) that it involves the ratio of two Gaussian random variables. If the observation signal-to-noise ratio (SNR) is sufficiently high[†], then it can be shown [20] that $T_{ai}(t)$ is approximately a Gaussian random variable. An unbiased estimate of $T_{ai}(t)$ and its variance can be evaluated by linearizing Eq (2.11) about the conditional unbiased estimates $\hat{\ell}_i(t)$ and $\hat{v}_i(t)$ as

$$T_{ai}(t) = \frac{L-\ell_i(t)}{v_i(t)} = \frac{L-\hat{\ell}_i(t)-[\ell_i(t)-\hat{\ell}_i(t)]}{\hat{v}_i(t)+[v_i(t)-\hat{v}_i(t)]}$$

$$\approx \frac{L-\hat{\ell}_i(t)}{\hat{v}_i(t)} - \frac{\ell_i(t)-\hat{\ell}_i(t)}{\hat{v}_i(t)} - \frac{[L-\hat{\ell}_i(t)][v_i(t)-\hat{v}_i(t)]}{\hat{v}_i^2(t)}$$

$$(2.24)$$

Using Eq (2.24), we have

$$\hat{T}_{ai}(t) = \text{Conditional mean} = \frac{L-\hat{\ell}_i(t)}{\hat{v}_i(t)}$$

$$(2.25)$$

$$\sigma^2_{ai}(t) = \text{Conditional variance}$$

$$= \frac{E_{122}(t)+E_{133}(t)\hat{T}^2_{ai}(t)+2E_{123}(t)\hat{T}_{ai}(t)}{\hat{v}_i^2(t)}$$

Due to the scaling nature of the noise processes in the information

[†] $SNR = 10\, \log_{10} \dfrac{v_i^2}{2E_{133}}$ should be (approximately) greater than 12 db. This condition is almost always satisfied in man-machine applications.

processor, one might expect that $E_{122} \approx \epsilon_1 \hat{\ell}_i^2(t)$; $E_{123} \approx \epsilon_2 \hat{\ell}_i(t)\hat{v}_i(t)$; and $E_{133} \approx \epsilon_3 \hat{v}_i^2(t)$ where ϵ_1, ϵ_2, $\epsilon_3 > 0$. Therefore, Eq (2.25) implies, albeit heuristically, that

$$\sigma_{ai}^2(t) \approx \epsilon \hat{T}_{ai}^2(t) \; ; \; \epsilon > 0$$

Thus, the standard deviation of time available, $\sigma_{ai}(t)$, is likely to scale with its conditional mean, $\hat{T}_{ai}(t)$. This is intuitively appealing.

In summary, the decision state variables, $\underline{x}_{di} = [\hat{T}_{Ri}, \hat{T}_{ai}]'$, of the ODM are assumed to be normal with the non-stationary perceived density and distribution functions, $\gamma_i(T_{Ri};t)$, $\Gamma_i(T_{Ri};t)$ and $\phi_i(T_{ai};t)$, $\Phi_i(T_{ai};t)$ respectively. That is,

$$\gamma_i(T_{Ri};t) = N[\hat{T}_{Ri}(t); \sigma_{Ri}^2(t)]$$

$$\phi_i(T_{ai};t) = N[\hat{T}_{ai}(t); \sigma_{ai}^2(t)]$$

(2.26)

The conditional Gaussian statistics of the decision state from an important input to the decision process as shown in Fig. 7.

2.6 Decision Strategy

In this section, the multi-task decision problem is formulated in the framework of a non-stationary, semi-Markov decision process (SMDP). Via this formulation, the combined statistics of the decision states of N tasks are used to compute the transition probabilities among the various process states for each of the decision alternatives. The transition probabilities, along with the task values, are used to determine the attractiveness measures of tasks, employing the subjectively expected value (SEV) as a criterion of performance. These measures form an input to a stochastic choice model that generates the decision probabilities. The decision process is depicted in Fig. 9, and is elaborated next.
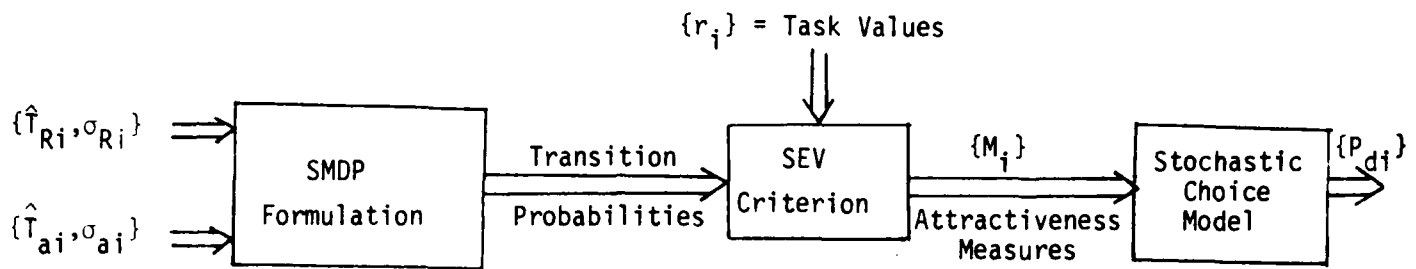
$\{r_i\}$ = Task Values

Fig. 9: HUMAN DECISION PROCESS

2.6.1 Semi-Markov Decision Process Formulation

Recall that a non-stationary SMDP is characterized by the hexad

$(S(t), \mathcal{D}(t), E(t), T(t), H(t), r(t))$, where

$S(t)$ = set of process (Markov) states of the system (state space)

$\mathcal{D}(t)$ = set of possible decisions (action set)

$E(t)$ = set of events (event set)

$T(t)$ = set of transformation rules that describe the changes in the state. This is usually expressed in terms of transition probabilities.

$H(t)$ = Holding time function that determines how long the system stays in a given state before making transition to another specified state. This is expressed in terms of holding time density functions.

$r(t)$ = set of rewards associated with each state transition (reward structure).

Thus, in order to formulate the multi-task decision problem as an SMDP, we need to specify the process descriptors $S(t)$, $E(t)$, $T(t)$, $H(t)$, $r(t)$

and $\mathcal{D}(t)$.

A. State Space, S(t)

The state space S is time invariant and consists of 2^K elements corresponding to 2^K possible realizations of the process state $\underline{s}$, where K is the system capacity. Symbolically,

$$S = \{\text{set of } 2^K \text{ process states, } \underline{s}\}$$

Associated with a process state $\underline{s}$ at time t, there exist N pairs of decision state variables $\{T_{Ri}(t), T_{ai}(t)\}$, $i \epsilon A(t)$. Here, $A(t)$ is the set of N available tasks in the opportunity window at time t.

B. Event set, E(t) and the Transformation Rule, T(t)

The transformation rule (or the law of motion), $T(t)$, is expressed in terms of transition probabilities $p^i_{ss'}(t)$, where $\underline{s}$ is the process state at time t, $\underline{s}'$ is the destination process state after a random holding time τ in process state $\underline{s}$, and $i \epsilon A(t)$ denotes the action on a task i in the opportunity window. The destination state $\underline{s}'$ depends on the values of (N+1) independent random variables, $T_{Ri}(t)$ and $T_{am}(t)$, $m \epsilon A(t)$; associated with the process state $\underline{s}$ and the decision to act on task i at time t.[†] It is clear that a decision to act on task i results in one of the following process state transitions shown in Fig. 10.

   (i)   *Successful Completion or loss of task i*: The task i is said to be successfully completed if the random variable $T_{Ri}(t)$

[†] Note that this formulation assumes complete ignorance of the random variables associated with the future arrivals on the (K-N) empty lines. That is, transitions to process states corresponding to arrivals on empty lines are not included in this formulation. This implies that the decision strategy depends only on the characteristics of tasks in the opportunity window. If the probabilistic information regarding future arrivals is available, it can be incorporated into the decision strategy. The reader is referred to Ref. [10] for details.
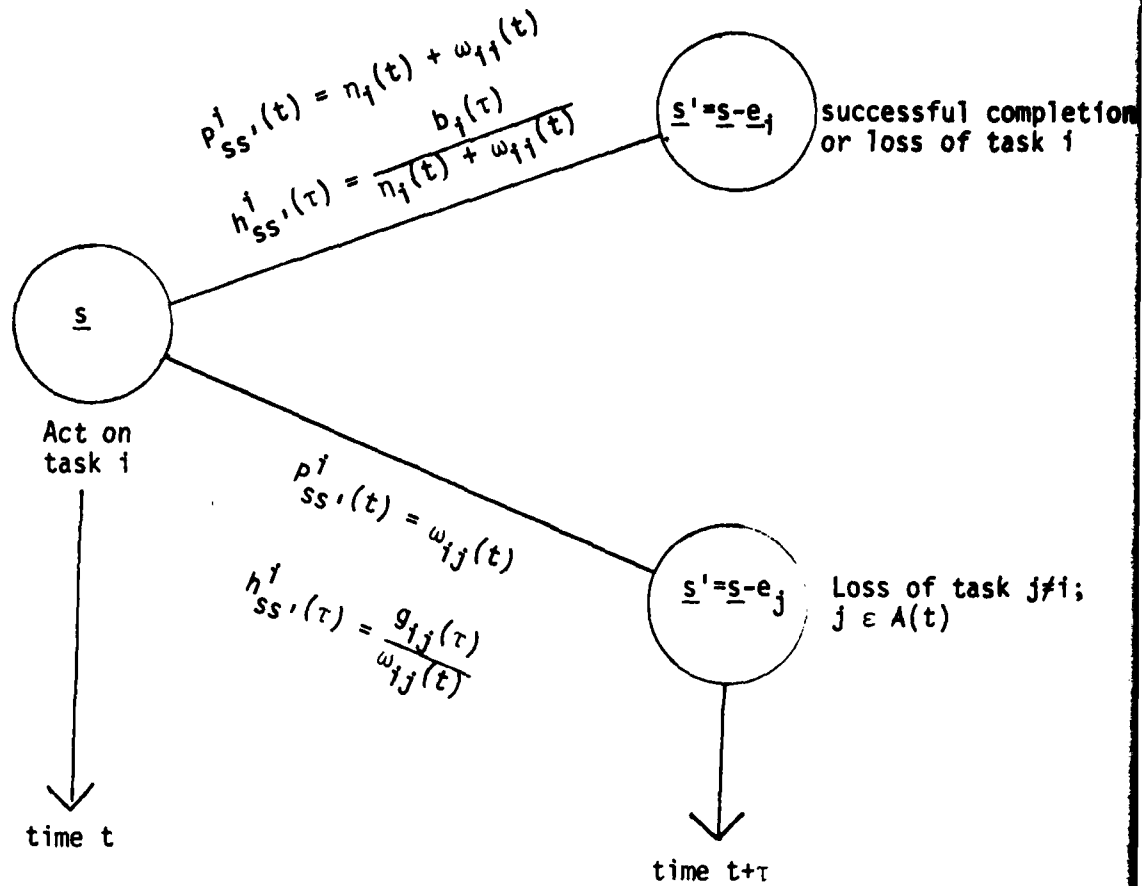
$$p^i_{ss'}(t) = n_i(t) + \omega_{ii}(t)$$

$$h^i_{ss'}(\tau) = \frac{b_i(\tau)}{n_i(t) + \omega_{ii}(t)}$$

$\underline{s}'=\underline{s}-\underline{e}_i$ successful completion or loss of task i

$\underline{s}$

Act on task i

$$p^i_{ss'}(t) = \omega_{ij}(t)$$

$$h^i_{ss'}(\tau) = \frac{g_{ij}(\tau)}{\omega_{ij}(t)}$$

$\underline{s}'=\underline{s}-\underline{e}_j$ Loss of task $j \neq i$; $j \in A(t)$

time t

time $t+\tau$

Fig. 10: PROCESS STATE TRANSITION DIAGRAM OF THE MTDP

of task i is greater than zero, but is less than the available times, $T_{am}(t)$, of all the tasks, including i, in the opportunity window. On the other hand, task i is said to be lost if the random variable $T_{ai}(t)$ is greater than zero, but less than $T_{Ri}(t)$ and $T_{aj}(t)$, $j \neq i$. In any case, the new process state $\underline{s}' = \underline{s} - \underline{e}_i$, where $\underline{e}_i$ is a K-dimensional unit row vector whose i th component is one and whose other components are zero.

(ii) *Loss of a task* $j \neq i$: This event occurs if $T_{aj}(t)$ is greater than zero, but less than $T_{Ri}(t)$ and $T_{am}(t)$, $m \neq j$. When this event occurs, the new process state $\underline{s}' = \underline{s} - e_j$.

Thus, the destination process state, $\underline{s}'$, and the random holding time, τ, depend on the outcome of a race among the $(N+1)$ competing, non-stationary random processes $T_{Ri}(t)$ and $T_{am}(t)$, $m \in A(t)$ associated with the process state $\underline{s}$ and action i. This type of semi-Markov decision process, wherein the state transitions are determined by a race among several random processes is known as a "competing semi-Markov decision process" [21]. It should be emphasized that the analysis of MTDP is complicated by the fact that the transition probabilities and the random holding time functions are non-stationary.

It is clear from the above event description that there are N possible process state transitions of interest from state $\underline{s}$. In general, the destination process state $\underline{s}' = \underline{s} - \underline{e}_m$, $m \in A(t)$. In the following, the transition probabilities for the admissible destination process states are computed. We suppress the time dependence of the density and distribution functions of the decision states for ease of notation.

(a) *Probability of event (i):* This is the probability that the new process state $\underline{s}' = \underline{s} - \underline{e}_i$ given that the present process state is $\underline{s}$ and the decision is to act on task i. Thus,

$$P^i_{ss'}(t) = \eta_i(t) + \omega_{ii}(t) \quad ; \quad \underline{s}' = \underline{s} - \underline{e}_i \qquad (2.27)$$

where

$\eta_i(t) \overset{\Delta}{=} P\{\text{action on task i, task i is successfully completed,}$

$\qquad \text{other channels intact}\}$

$$= \prod_{m \in A(t)} P\{T_{Ri} \leq T_{am}\}$$

$$= \int_0^{\infty} \prod_{m \in A(t)} [1-\Phi_m(\tau)] \, \gamma_i(\tau) \, d\tau$$

$\omega_{ii}(t) \overset{\Delta}{=} P\{\text{action on task i, task i is lost, other channels intact}\}$

$$= P\{T_{ai} \leq T_{Ri}\} \cdot \prod_{\substack{m \in A(t) \\ m \neq i}} P\{T_{ai} \leq T_{am}\}$$

$$= \int_0^\infty [1-\Gamma_i(\tau)] \cdot \prod_{\substack{m \in A(t) \\ m \neq i}} [1-\Phi_m(\tau)] \cdot \phi_i(\tau) \, d\tau$$

(b) Probabilities associated with event (ii): This is the probability that the new state $\underline{s}' = \underline{s} - \underline{e}_j$; $j \neq i$, given that the present state is $\underline{s}$ and decision is to act on task i. Therefore,

$$P_{ss'}^i(t) = \omega_{ij}(t) \; ; \; \underline{s}' = \underline{s} - \underline{e}_j \; ; \; j \neq i \qquad (2.28)$$

where

$\omega_{ij}(t) \overset{\Delta}{=} P\{$action on task i, an accessible task j other than i is

lost, all the other tasks intact$\}$

$$= P\{T_{aj} \leq T_{Ri}\} \cdot \prod_{\substack{m \in A(t) \\ m \neq j}} P\{T_{aj} \leq T_{am}\}$$

$$= \int_0^\infty [1-\Gamma_i(\tau)] \cdot \prod_{\substack{m \in A(r) \\ m \neq j}} [1-\Phi_m(\tau)] \; \phi_j(t) \, d\tau$$

In summary, the N transition probabilities for each $i \in A(t)$ are

$$P_{ss'}^i(t) = \begin{cases} \eta_i(t) + \omega_{ii}(t) \; ; \; \underline{s}' = \underline{s} - \underline{e}_i \\ \omega_{ij}(t) \; ; \; \underline{s}' = \underline{s} - \underline{e}_j \; ; \; j \neq i \end{cases} \qquad (2.29)$$

In Ref [10], numerical quadrature formulae of Hermite [22], and Steen, Byrne and Gelbard [23] were suggested as a means to compute the required transition probabilities. However, the computation of transition probabilities can be greatly simplified using Luce's choice axiom [24-27], which is ideally suited to determine the probability that a certain

random variable is the minimum (or maximum) among a set of random variables. This is precisely the problem of interest in generating the transition probabilities. For example, $\eta_i(t)$, the probability that $T_{Ri}(t)$ is less than $T_{am}(t)$; $m \in A(t)$, can be computed via Luce's choice axiom according to

$$\eta_i(t) \approx \left[1 + \sum_{m \in A(t)} \frac{P\{T_{am}(t) - T_{Ri}(t) < 0\}}{P\{T_{Ri}(t) - T_{am}(t) < 0\}} \right]^{-1} \qquad (2.30a)$$

The main assumption underlying Luce's choice axiom is that the removal of some alternatives (random variable, in our case) does not alter the relative probabilities of choice among the remaining alternatives. In other words, the presence or absence of an alternative is irrelevant to the relative probabilities of choice among the remaining alternatives, although the individual probabilities will generally be affected. The proof of the form of Eq (2.30a) is included in Appendix A.

Since the decision state variables are assumed to be Gaussian, Eq (2.30a) simplifies to

$$\eta_i(t) \approx \left[1 + \sum_{m \in A(t)} \frac{1 + \text{Erf }(\Delta_{im})}{1 - \text{Erf }(\Delta_{im})} \right]^{-1} \qquad (2.30b)$$

where

$$\Delta_{im} = \frac{\hat{T}_{Ri} - \hat{T}_{am}}{\sqrt{\sigma_{Ri}^2 + \sigma_{am}^2}}$$

and $\text{Erf}(a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-u^2} du$; $\text{Erf}(\infty) = 1$

Using a well known result [24] that the logistic function is a good ap-

proximation to the cumulative normal, Eq (2.28b) can be further simplified as

$$\eta_i \approx \left[1 + \sum_{m \in A(t)} \frac{\{1 + \exp~(\Delta_{im})\}}{\{1 + \exp~(-\Delta_{im})\}} \right]^{-1} \qquad (2.30c)$$

The computation of remaining transition probabilites ω_{im}, $m \in A(t)$ proceeds along similar lines to Eq (2.30).

C.   Holding time function, $H(t)$

The holding time function is specified in terms of holding time density functions, $h^i_{ss'}(\tau)$, which determine how long the system stays in process state $\underline{s}$ before making a transition to a specified state $\underline{s}'$. The density function $h^i_{ss'}(\tau)$ can be obtained by first determining the joint probability – probability density function, $f^i_{ss'}(\tau)$, for the event that a system in process state $\underline{s}$ will make its next transition to process state $\underline{s}'$ after a holding time τ, while acting on a task $i \in A(t)$. This event will occur in the competing SMDP only if the random variable representing the destination process state $\underline{s}'$ takes on the value τ and all the other N random variables are greater than τ. Therefore,

$$f^i_{ss'}(\tau) = \begin{cases} b_i(\tau) = \displaystyle\prod_{\substack{m \in A(t) \\ m \neq i}} [1-\Phi_m(\tau)] \cdot \\ \qquad \cdot \{\gamma_i(\tau)[1-\Phi_i(\tau)]+\phi_i(\tau)[1-\Gamma_i(\tau)]\}; \quad \underline{s}' = \underline{s} - \underline{e}_i \\ \\ g_{ij}(\tau) = \phi_j(\tau) \cdot [1-\Gamma_i(\tau)] \cdot \\ \qquad \cdot \displaystyle\prod_{\substack{m \in A(t) \\ m \neq j}} [1-\Phi_m(\tau)]; \quad \underline{s}' = \underline{s} - \underline{e}_j ; \quad j \neq i \end{cases} \qquad (2.31)$$

Note that the transition probabilities, $P_{ss'}^i(\tau)$, of Eq (2.27) are related to $f_{ss'}^i(\tau)$ via

$$P_{ss'}^i(\tau) = \int_0^\infty f_{ss'}^i(\tau) \, d\tau \quad \text{for all } \underline{s}, \, \underline{s}' \in S$$

The holding time density functions, $h_{ss'}^i(\tau)$ are given by

$$h_{ss'}^i(\tau) = \frac{f_{ss'}^i(\tau)}{P_{ss'}^i(\tau)} \quad \text{for all allowable } \underline{s}, \, \underline{s}' \qquad (2.32)$$

It should be emphasized that the functions $f_{ss'}^i(\tau)$ and $h_{ss'}^i(\tau)$ are non-stationary. Closed form expressions for the holding time density functions are not possible and, hence, must be computed numerically [10,22,23].

D.   Reward Structure, r(t)

When the transition from process state $\underline{s}$ to process state $\underline{s}'$ occurs at some time $t + \tau$, the DM earns an expected reward $r_{ss'}^i(\tau)$ in the form of a bonus. That is, the DM earns a lump sum payment at the time of state transition, a payment that depends on process states $\underline{s}$, $\underline{s}'$; the holding time τ, and action i. In the present MTDP, a reward ("bonus") of $r_i(t)$ units is earned while acting on task i if and only if the new process state $\underline{s}' = \underline{s} - \underline{e}_i$ and the task i is successfully completed. The conditional probability that task i is successfully completed, given that the new process state $\underline{s}' = \underline{s} - \underline{e}_i$ and action on task i, is $\eta_i(t)/[\eta_i(t) + \omega_{ii}(t)]$. In addition, if it is assumed that there is a penalty of $q_m(t)$ units for losing a task $m \in A(t)$, then the reward structure can be described by

$$r_{ss'}^i(\tau) = \begin{cases} \dfrac{[r_i(t)\eta_i(t) - q_i(t)\omega_{ii}(t)]}{[\eta_i(t) + \omega_{ii}(t)]} & ; \quad \underline{s}' = \underline{s} - \underline{e}_i \\[2em] - q_m(t) & ; \quad \underline{s}' = \underline{s} - \underline{e}_m \; ; \; m \notin A(t) \end{cases} \qquad (2.33)$$

It should be noted that $r_{ss'}^i(\tau)$ is the conditional expected reward, given the holding time. Even though there are no penalties for missed tasks in the _present_ MTDP, $q_m(t)$ of Eq (2.33) could still represent the subjective losses (utilities) assigned by the DM. A logical choice for the subjective values $q_m(t)$ are the objective rewards $r_m(t)$. The reward structure of Eq (2.33) can be generalized to include decision dependent penalties, as well as a continuous yield rate [10].

E.   Action Set, $\mathcal{D}(t)$

At any time t, the DM is provided with (N+1) choices: act on one of the N tasks in the accessible set $A(t)$ or not act on any task (i.e., do nothing or monitor). Thus, we have

$$\mathcal{D}(t) = A(t) + \{0\}$$

The number of choices may differ from one process state to another. Some process states may have only one alternative and, therefore, choice is constrained whenever such a process state is occupied. The DM's problem is to select the actions (over time) that will make the operation of the system most rewarding.

2.6.2 Attractiveness Measures, $M_1(t)$

The basic assumption underlying the human response modeling is that a well-trained human behaves in a normative, rational manner subject to his inherent limitations. We interpret this, mathematically, in terms of maximizing a specified metric. As with the OCM, the choice of a metric may be either objective (specified by the experimenter), or subjective (adopted by the human in perofrming and relating to the task). In the present experimental context, the objective metric involves the maximization of reward earned. Since the proposed model is normative in construct, we need to specify a subjective metric. If the subjective metric is the

same as the objective metric, then, as shown in [10], a functional equation for the optimal decision strategy can be derived using dynamic programming (DP) and semi-Markov decision process theory. However, the tree-folding back procedure of the DP presents serious computational difficulties ("curse of dimensionality"), and requires the evaluation and specification of all future courses of action before any task is acted upon. The latter point is at variance with the current psychological knowledge of a human's inability to forsee the complete future effects of his present decisions. If a finite stage DP is advocated as a compromise, we are faced with the dilemma of selecting the number of stages. These observations led us to the choice of the subjectively expected value(SEV) of a decision as our metric (or "attractiveness measure") for optimization. It is easy to show [10] that SEV corresponds to a myopic (one-stage) policy, which can be derived from the DP formulation by completely disregarding future rewards. That is, the myopic decision policy acts at every time t, as though the present decision was the final one. Conceptually, this approach is similar to the "open-loop-feedback-optimal" approach of control theory, wherein the present value of future information is neglected.[†]

The attractiveness measure $M_i(t)$ of a decision to act on task i

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[†] The DP formulation of the optimal strategy is of theoretical importance in its own right, as it provides a general and flexible analytic framework for the analysis of dynamic decision-making under uncertainty. This framework covers all cases where the present decisions can affect future information, uncertainties associated with the random processes of the system, future rewards and future actions. More importantly, it was shown in [10] that the optimal decision strategy subsumes Tulga's deterministic, dynamic sequencing formulation of the MTDP [8], as well as the Markov decision problem [21], and several single processor sequencing theoretic rules [28]. The Markov decision formulation was applied and extended in [11] to determine stationary, non-preemptive priority policies in a multi-class queueing system with finite capacity and reneging (i.e., impatient customers).

is simply the subjectively expected discounted value of reward from the first transition out of process state $\underline{s}$ regardless of when it occurs. It is given by

$$M_i(t) = \sum_{\text{all } \underline{s}'} P_{ss'}^i(t) \int_0^\infty e^{-\alpha\tau} r_{ss'}^i(\tau) h_{ss'}^i(\tau) \, d\tau \text{ for all } i \in A(t)$$

The use of discount factor, α in the computation of attractiveness measures, $M_i(t)$, may be interpreted in two ways: First, it can account for the DM's present perception of future rewards. That is, future rewards are worth less at the present time ("reward today is sweeter than reward tomorrow"). Howard [29] calls this explanation for α, the "time preference" or the "greed-impatience trade off". The second interpretation is in terms of the uncertainty associated with the duration of the period during which rewards can be earned.

For the specific MTDP, using Eqs (2.29), (2.32) and (2.33), $M_i(t)$ can be rewritten as

$$M_i(t) = [r_i(t)\eta_i(t) - q_i(t)\omega_{ii}(t)]\tilde{B}_i(\alpha;t) - \sum_{\substack{m \in A(t) \\ m \neq i}} q_m(t)\omega_{im}(t)\tilde{G}_{im}(\alpha;t)$$

$$\text{for all } i \in A(t) \qquad (2.34)$$

where $\tilde{B}_i(\alpha;t)$ and $\tilde{G}_{im}(\alpha;t)$ are the exponential (Laplace) transforms of the holding time density functions $b_i(\tau)/[\eta_i(t) + \omega_{ii}(t)]$ and $g_{im}(\tau)$, respectively. They are given by

$$\tilde{B}_i(\alpha;t) = \frac{1}{\eta_i(t) + \omega_{ii}(t)} \int_0^\infty e^{-\alpha\tau} b_i(\tau) \, d\tau \; ; \; \tilde{B}_i(0;t) = 1$$

and

$$\tilde{G}_{im}(\alpha;t) = \frac{1}{\omega_{im}(t)} \int_0^\infty e^{-\alpha\tau} g_{im}(\tau) \, d\tau; \; \tilde{G}_{im}(0;t) = 1, \; m \neq i$$

The attractiveness measure associated with the "do nothing" decision, $M_0(t)$, or that of monitoring decision, $M_m(t)$, depends on whether or not parallel monitoring is allowed.

(i) *Parallel monitoring*: When parallel monitoring is allowed, $M_0(t)$ can be interpreted as the human's indifference towards, or perception of, small rewards. In the present context, the "do nothing" decision is made only if none of the available tasks can be completed, or if there are no tasks to be processed. We use

$$M_0(t) = - \sum_{m \, \epsilon \, A(t)} q_m(t) \, \omega_{0m}(t) \, \tilde{G}_{0m}(\alpha;t) \qquad (2.35)$$

where $\omega_{0m}(t)$ and $\tilde{G}_{0m}(\alpha;t)$ are computed using a constant "fictitious" processing time for the null task, T_{R0}. Thus, $M_0(t)$ represents the loss due to disappearance of all tasks. The value of T_{R0} is chosen to match the data, but is a constant across experimental conditions (A-D).

(ii) *No Parallel monitoring*: In this case, monitoring of tasks other than the one being acted upon is not allowed (i.e., condition B_y), but monitoring is a separate valid decision. Here, we postulate that the human makes this decision only if the enhanced knowledge of the task characteristics offsets any reward he may have gained by acting on one of the N tasks. That is, $M_m(t)$ is the average value of gathering information for δ sec (inegration time step) starting at time t, and is given by

$$M_m(t) = \frac{1}{N} \sum_{i \, \epsilon \, A(t)} [M_i(t+\delta) - M_i(t)] \qquad (2.36)$$

Thus, the monitoring decision is invoked only if the information value is sufficiently high to preclude action on one of the tasks in the opportunity window. The attractiveness measure $M_m(t)$, in conjunction with the measures $M_i(t)$, is used to compute the monitoring probability, $P_{dm}(t)$.

The form of Eqs. (2.34-2.36) for attractiveness measures is particularly appealing, as it relates to the "net gain" of each of the task alternatives available to the decision-maker at time t. The first term in Eq (2.34) represents the "potential gain" of acting on task i at time t, whereas the summation term represents the "potential loss" due to the disappearance of all the other tasks. The criterion explicitly considers the human's inability to envisage all the future courses of actions, as would be required by DP formulation. Moreover, Eq (2.34) includes human's preference for rewards that are distributed in time via the discount factor, α.

Sensitivity analysis of the DDM (chapter III) has shown that a value of $\alpha = 0$ gives the best possible match to the data. This could imply either of two things: First, humans do not discount rewards distributed over a short-time horizon (one to five seconds in our case). A second and more plausible implication is that the use of discount factor in the analysis of dynamic decision-making may be artificial. That is to say, once the human information-processing limitations are included and a myopic policy is postulated for the human decision strategy, it may not be necessary to employ discount factor, α. In any case, when α is zero, Eq (2.34) simplifies to

$$M_i(t) = r_i(t)\eta_i(t) - \sum_{m \in A(t)} q_m(t)\omega_{im}(t) \quad ; \quad i \in A(t) \qquad (2.37)$$

Thus, there is no need to numerically evaluate the holding time density functions. Note that Eq (2.37) is similar to the SEU model of Eq (1.4) with appropriate interpretation.

In summary, the proposed myopic decision strategy in the general case, but with $\alpha = 0$, involves the computation of only $2N(N+1)$ transition probabilities to evaluate the $(N+1)$ attractiveness measures, $M_m(t)$ and $M_i(t)$, $i \in A(t)$. The required transition probabilities may be computed in a straightforward manner via Luce's choice axiom. Therefore, the computational load of the proposed decision strategy is insignificant compared to that of the truly optimal DP formulation.

2.6.3 Stochastic Choice Model

A decision model that selects the task with maximum attractiveness measure yields a (1-0) response, and suggests that the decision-maker would always make the same sequence of decisions under similar conditions. However, it is well known [12] that people fluctuate in their response to the same stimulus, even when there are no changes in their information or resources. Fluctuations in choice can arise because the subject is unable to discriminate precisely, or because he may make calculating, response or perceptual errors. The stochastic choice models assume that, although the attractiveness measures, $M_i(t)$, could be characterized by a single fixed number, the subjects perceive it as a random variable, $\tilde{M}_i(t)$, with some distribution (usually Gaussian). The randomness may be interpreted in terms of the uncertainties associated with the human perception of task values, $r_i(t)$. Below, we again invoke Luce's choice axiom to compute the decision probabilities, $P_{di}(t)$:

(i)   *Parallel monitoring:*

$$P_{di}(t) = \left[1 + \sum_{\substack{k \in \mathcal{D}(t) \\ k \neq i}} \frac{P\{\tilde{M}_k(t) - \tilde{M}_i(t) > 0\}}{P\{\tilde{M}_i(t) - \tilde{M}_k(t) > 0\}}\right]^{-1} \quad ; \ i \in \mathcal{D}(t)$$

(2.38)

(ii) *No parallel monitoring:*

$$P_{dm}(t) = \left[1 + \sum_{k \in A(t)} \frac{P\{\tilde{M}_k(t) - \tilde{M}_m(t) > 0\}}{P\{\tilde{M}_m(t) - \tilde{M}_k(t) > 0\}}\right]^{-1}$$

(2.39)

The decision probabilities P_{di} are given by a relation similar to Eq (2.38) with $\tilde{M}_m(t)$ replacing $M_o(t)$.

In Eqs (2.38-2.39), we assume that $\tilde{M}_i(t)$ are Gaussian random variables with mean $M_i(t)$ and variance $\sigma_{Mi}^2(t)$ that scales with $M_i^2(t)$. That is,

$$\sigma_{Mi}(t) = c|M_i(t)| \quad (c \approx .2-.4)$$

(2.40)

where c is the co-efficient of variation. Note that the forms of Eqs (2.38-39) can be employed with any decision strategy.

2.7 Model Predictions

The dynamic decision model can be used in a straightforward manner to generate predictions of $P_{di}(t)$, as well as of other response measures that can be computed from the experimental data:

(i) *The completion probability,* $P_{ci}(t)$ is the probability that task i is completed by time t. Thus,

$$P_{ci}(t) = P\{T_{Ri}(t) \leq 0\} = \Gamma_i(0;t) \quad ; \ i \in A(t) \quad (2.41)$$

When $P_{ci}(t) > .99$, the task is assumed to be successfully completed and, therefore, is removed from the model.

(ii) The error probability, $P_e(t)$, is the probability that the human commits an error, i.e., starts acting on a task he can not possibly complete. Thus, $P_e(t)$ is the sum over all tasks of the probability of the joint event: action on task i and the time required to complete task i is greater than the time available to work on it. Therefore,

$$P_e(t) = \sum_{i \in A(t)} P\{T_{Ri}(t) - T_{ai}(t) > 0\} \cdot P_{di}(t) \qquad (2.42a)$$

Since $T_{Ri}(t)$ and $T_{ai}(t)$ are assumed to be independent and conditionally Gaussian random variables, Eq (2.42a) becomes

$$P_e(t) = \sum_{m \in A(t)} \left[\frac{1 - \text{Erf}(\Delta_{ii})}{2}\right] \cdot P_{di}(t) \qquad (2.42b)$$

where Δ_{ii} and $\text{Erf}(\Delta_{ii})$ are defined following Eq (2.30b)

(iii) The average accumulated reward, $\bar{R}(t)$, is the average total reward earned upto the present time t. It is an overall response measure, and is given by

$$\bar{R}(t) = \int_0^t \sum_{i \in A(t)} r_i(t) \max\left(0., \frac{dP_{ci}(\sigma)}{d\sigma}\right) d\sigma \qquad (2.43)$$

(iv) Normalized incremental reward, $W_c(t)$ is the average instantaneous reward-earning rate, and is a measure of instantaneous performance. Thus, $W_c(t)$ is the weighted sum of completion probabilities given by

$$W_c(t) = \frac{1}{K} \sum_{i \in A(t)} r_i(t) P_{ci}(t) \qquad (2.44)$$

where K is the system capacity (= 5).

(v)   *Total expected tasks completed*, $\bar{N}_c$ can be computed by assuming that all tasks $i \in A(t)$ with $P_{ci}(t) > \beta (\approx .99)$ are successfully acted upon. Thus,

$$\bar{N}_c = \int_0^T \left\{ \sum_{i \in A(t)} \delta[P_{ci}(t)-\beta] \right\} dt \qquad (2.45)$$

where $\delta[P_{ci}(t)-\beta]$ is the Dirac delta function and T is the duration of the experiment.

(vi)   *Average time spent on a task on line* i, $\bar{T}_{si}$ is the time the human attends to task i on the average. It is given by

$$\bar{T}_{si} = \int_{t_{0i}}^{t_{fi}} P_{di}(t)\, dt \qquad (2.46)$$

where t_{0i} and t_{fi} are the times between which a task is on line i.

In the next chapter, model predictions of the above response measures are compared with the experimental results for the conditions A, B, C, D and B_y.

2.8   Summary

In this chapter, an analytic model of human task sequencing performance was developed. The modeling approach borrowed from the successful optimal control modeling methodology. The approach taken here and in [10] is quite general, flexible and covers all cases where the present decisions affect future information and future rewards. As with the OCM, the dynamic decision model (DDM) developed in this chapter consists of two separable blocks: information-processor and decision-maker. The information-processor compensates for the human's observation noise,

time-delay and monitoring allocations to produce the best linear unbiased estimates of the "decision state". The conditional Gaussian statistics of the decision state constitute a sufficient statistic of the decision process. The statistics, along with the task values, are used in a myopic decision policy, based on semi-Markov decision process theory, to determine the attractiveness measure of each of the decision alternatives. The measures are subsequently used in a stochastic choice model, that explicitly considers human's inability to discriminate precisely, to generate the decision probabilities.

Some novel features of our modeling approach are in the use of the concept of a decision state; the explicit incorporation of human limitations at the information-processing and decision-making stages; and its suitability to assimilate new elements of the task as they become considered and understood. The last item corresponds to such issues as precedence restrictions, resource constraints, general reward structures, non-stationary task characteristics, and even different experimental paradigms that involve the basic ingredients of monitoring, information-processing and dynamic decision-making. Moreover, the model may be used in a covariance propagation mode or in a sample path mode. The first mode is appropriate for model-data validation efforts presented in chapter III. The second mode is suitable for decision-aiding as discussed in chapter IV.

III. MODEL-DATA VALIDATION STUDIES

In chapter II, the dynamic decision model (DDM) of human task sequencing performance was developed, and the model's ability to generate various response measures of interest, viz., $P_{di}(t), P_{ci}(t), P_e(t)$, etc., was noted. The present chapter proposes several metrics for assessing the "goodness of fit" (or "similarity") between the model predictions and the experimental data, and presents results on the model-data validation efforts.

3.1 Data analysis

As mentioned in section 1.5, the data sampled during each run consisted of the subject's decisions, $d_i(t)$; the task completion status, $c_i(t)$; and the error sequence, $e_i(t)$. These raw data were ensemble averaged to obtain empirical estimates of the following response variables:

(i)   *The decision probability*, $P_{di}^H(t)$, of acting on a task of line i at time t,

$$P_{di}^H(t) = \frac{\displaystyle\sum_{j=1}^{N_s} \sum_{k=1}^{N_{Rj}} d_i^{kj}(t)}{\displaystyle\sum_{j=1}^{N_s} N_{Rj}} \tag{3.1}$$

where

N_s = total number of subjects

N_{Rj} = total number of runs of subject j

and

$$
d_i^{kj}(t) = \begin{cases} 1 \text{ if subject } j \text{ was processing a task on line } i \\ \text{at time } t \text{ during run } k \\ \\ 0 \text{ otherwise} \end{cases}
$$

(ii)   *The completion probability*, $P_{ci}^H(t)$ of having completed a task on line i by time t,

$$
P_{ci}^H(t) = \frac{\displaystyle\sum_{j=1}^{N_s} \sum_{k=1}^{N_{Rj}} c_i^{kj}(t)}{\displaystyle\sum_{j=1}^{N_s} N_{Rj}}
\tag{3.2}
$$

where

$$
c_i^{kj}(t) = \begin{cases} 1 \text{ if subject } j \text{ has completed task } i \text{ by time } t \\ \text{during run } k \\ \\ 0 \text{ otherwise} \end{cases}
$$

Clearly, $P_{ci}^H(t)$ is a monotonically increasing function of time. It is reset to zero at the end of the opportunity window of the present task on line i, i.e., before the arrival of the next task in the sequence.

(iii)   *The error probability*, $P_e^H(t)$, of engaging a task which can not possibly be completed, was calculated from the data via

$$
P_e^H(t) = \frac{\displaystyle\sum_{i=1}^{5} \sum_{j=1}^{N_s} \sum_{k=1}^{N_{Rj}} e_i^{kj}(t)}{\displaystyle\sum_{j=1}^{N_s} N_{Rj}}
\tag{3.3}
$$

where

$$e_i^{kj}(t) = \begin{cases} 1 \text{ if subject } j \text{ was acting on a task of line } i \\ \quad \text{at time } t \text{ during run } k \text{ that can not be suc-} \\ \quad \text{cessfully completed} \\ 0 \text{ otherwise} \end{cases}$$

(iv) *The average accumulated reward,* $\bar{R}^H(t)$ *earned through time t is related to* $P_{ci}^H(t)$ *via an expression similar to Eq (2.43).*

(v) *The normalized incremental reward,* $W_c^H(t)$ *earned by the human is given by an equation similar to Eq (2.44).*

(vi) *The number of expected tasks completed,* $\bar{N}_c^H(t)$, *was computed from*

$$\bar{N}_c^H(t) = \frac{\displaystyle\sum_{m=1}^{N_s} \sum_{k=1}^{N_{Rm}} \sum_{i=1}^{5} \sum_{j=1}^{N_{Ti}} c_i^{km}(t=t_{fij})}{\displaystyle\sum_{m=1}^{N_s} N_{Rm}} \tag{3.4}$$

where c_i^{km} is as defined in Eq (3.2), N_{Ti} is the total number of tasks that appear on line i, and t_{fij} is the time at which a task j of the sequence (i.e., j-th pass) on line i reaches the end of its opportunity window.

(vii) *The average time spent on a task on line* i *that arrived at time* t_{0ij} *and (would have) departed at time* t_{fij} *during the* j-*th pass is given by a relation similar to Eq (2.46). That is,*

$$\bar{T}_{ij}^H(t) = \int_{t_{0ij}}^{t_{fij}} P_{di}^H(t) \, dt \; ; \; i=1,2,\ldots,5 \; ;$$

$$j=1,2,\ldots,N_{Ti} \tag{3.5}$$

where N_{Ti} is as defined in Eq (3.4).

3.2 Measures of Similarity

In order to assess the closeness of model vs. data results and to perform sensitivity studies on the model, it is necessary to define "closeness". In this section, we propose several time-history and scalar measures of similarity, which are subsequently used as a means to validate the model.

3.2.1 Time-history Metrics

These measures compare the ensemble-averaged time-history of a response variable obtained empirically with that predicted by the DDM. Here, we formulate five time-history metrics that appear to be suitable in the present multi-task decision paradigm.

(i) *The decision probability comparisons* $P_{di}^H(t)$ *versus* $P_{di}^M(t); i=0,1,2,\ldots,5.$

(ii) *The completion probability comparisons* $P_{ci}^H(t)$ *versus* $P_{ci}^M(t); i=1,2,\ldots,5.$

(iii) *The normalized incremental reward comparisons*, $W_c^H(t)$ *versus* $W_c^M(t)$. Equivalently, the difference $(W_c^H(t) - W_c^M(t))$, or the rms difference $W_{cr}(t)$ given by

$$W_{cr}(t) = \left\{ \frac{1}{5} \sum_{i=1}^{5} \left[r_i(t) \left(P_{ci}^H(t) - P_{ci}^M(t) \right) \right]^2 \right\}^{1/2} \tag{3.6}$$

may be used as a measure of similarity.

(iv) *The accumulated reward comparisons*, $\bar{R}^H(t)$ *versus* $\bar{R}^M(t)$.

(v) *The error probability comparisons*, $P_e^H(t)$ *versus* $P_e^M(t)$.

3.2.2 Scalar Metrics

Below, we propose six scalar metrics that appear to be pertinent in the multi-task paradigm. The suggested scalar measures are useful in

the model-data validation studies, as well as in understanding the impact
of changes in various model parameters on the DDM predictions.

(i)   *Action metric*, AM computes the normalized time integral of the
squared error differences between the decision probabilities
$P_{di}^{H}(t)$ and $P_{di}^{M}(t)$: That is,

$$AM = \frac{1}{6T}\left\{ \sum_{i=0}^{5} \int_{0}^{T} \left[P_{di}^{H}(t) - P_{di}^{M}(t)\right]^2 dt \right\} \qquad (3.7)$$

where T is the duration of the experiment. The square root
of AM is a measure of the average discrepancy between $P_{di}^{H}(t)$
and $P_{di}^{M}(t)$.

(ii) *Incremental Reward Metric*, IRM is the normalized time integral
of the squared, weighted difference of the completion proba-
bilities $P_{ci}^{H}(t)$ and $P_{ci}^{M}(t)$ given by

$$IRM = \frac{\sum\limits_{i=1}^{5} \int_{0}^{T} \left[r_i(t)\left(P_{ci}^{H}(t) - P_{ci}^{M}(t)\right)\right]^2 dt}{\sum\limits_{i=1}^{5} \int_{0}^{T} r_i^2(t)\, dt} \qquad (3.8)$$

The square root of IRM is a measure of the difference between
the average reward-earning rates of the human and the model.

(iii) *Accumulated Reward Metric*, ARM is the normalized time integral
of the squared difference between the average reward earned
upto that instant of time by the human and the model. There-
fore,

$$ARM = \frac{\frac{1}{T}\left\{ \int_{0}^{T} \left[\bar{R}^{H}(t) - \bar{R}^{M}(t)\right]^2 dt \right\}}{R_{av\ell}^{2}} \qquad (3.9)$$

where $R_{av\ell}$ is the maximum available reward during the run. The square root of ARM is a measure of discrepancy between the average overall performance of the model and the human.

(iv)   *Task Completion Metric*, TCM computes the normalized squared differences between the average number of tasks completed by the human and the model as

$$TCM = \left\{ \frac{\bar{N}_c^M - \bar{N}_c^H}{N_{av\ell}} \right\}^2 \tag{3.10}$$

where $N_{av\ell}$ is the total number of available tasks during the experimental run.

(v)   *Average time on each task metric*, ATTM calculates the normalized root-mean-squared sum of the difference between the times spent on each task by the human and the model according to

$$ATTM = \frac{\left\{ \sum_{i=1}^{5} \sum_{j=1}^{N_{Ti}} \left(\frac{\bar{T}_{ij}^H - \bar{T}_{ij}^M}{T_{Rij}(t_{0ij})} \right)^2 \right\}}{N_{av\ell}} \tag{3.11}$$

where N_{Ti} is defined in Eq (3.4) and $T_{Rij}(t_{0ij})$ is the initial (actual) processing time of a task on line i during the j-th pass.

(vi)   *Error probability metric*, EPM is the normalized time integral of the squared differences between the error probabilities $P_e^H(t)$ and $P_e^M(t)$ and is given by

$$EPM = \frac{1}{T} \int_0^T \left[P_e^H(t) - P_e^M(t) \right]^2 dt \tag{3.12}$$

Note that the normalized scalar measures can range from a value of 0, corresponding to a perfect fit between the model and data, to a maximum value of 1.

3.3 Model vs. Data Comparisons

The application of the DDM to generate predictions of various response measures is straightforward, once we specify the parameter set $\Omega = \{\tau, \rho_i, c, T_{RO}\}$. From experience with the OCM, we choose

τ = human's time-delay = 0.2 sec

ρ_i = observation noise-to-signal ratio = 0.01 (i.e., -20db)

After a sensitivity study was made on the DDM, we selected the remaining parameters as

c = co-efficient of variation = 0.3 (see Eq. (2.40))

T_{RO} = "fictitious" processing time = 3 sec (see Eq. (2.35))

The parameter set was held constant across experimental conditions. In all cases, the subjective values q_i are chosen to be the objective rewards r_i. Pertinent data on task attributes, viz., arrival times, processing times, values and velocities, for the experimental conditions A, B, C, D and B_y may be found in Ref. [10].

The five time-history metrics generated from the data and the model are compared in Figs. (11-35) for the five experimental conditions A, B, C, D and B_y. The ensemble data were obtained by averaging over NR runs (e.g., NR = 48 for condition A). The results show striking similarity between the data and model predictions. The model-data match is uniformly good to excellent for all the five experimental conditions studied. This is most noteworthy considering that a nominal set of parameters were used throughout, and that the decision problem involved is complex. To be sure, there are some discrepancies, as in decision probability (P_{di}) comparisons: they show that the model predictions exhibit rapid variations when compared to the data. This discrepancy is likely a result of
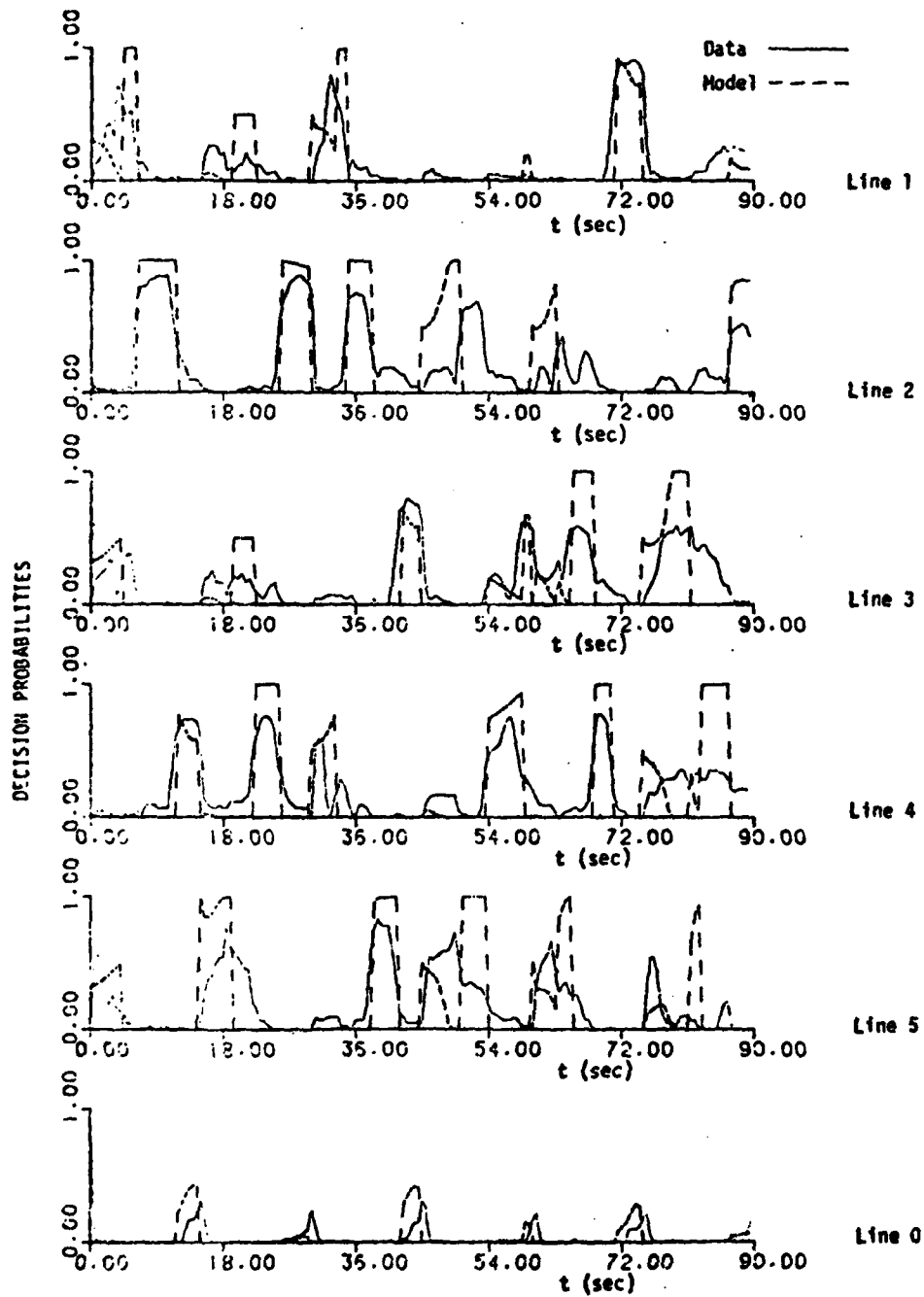
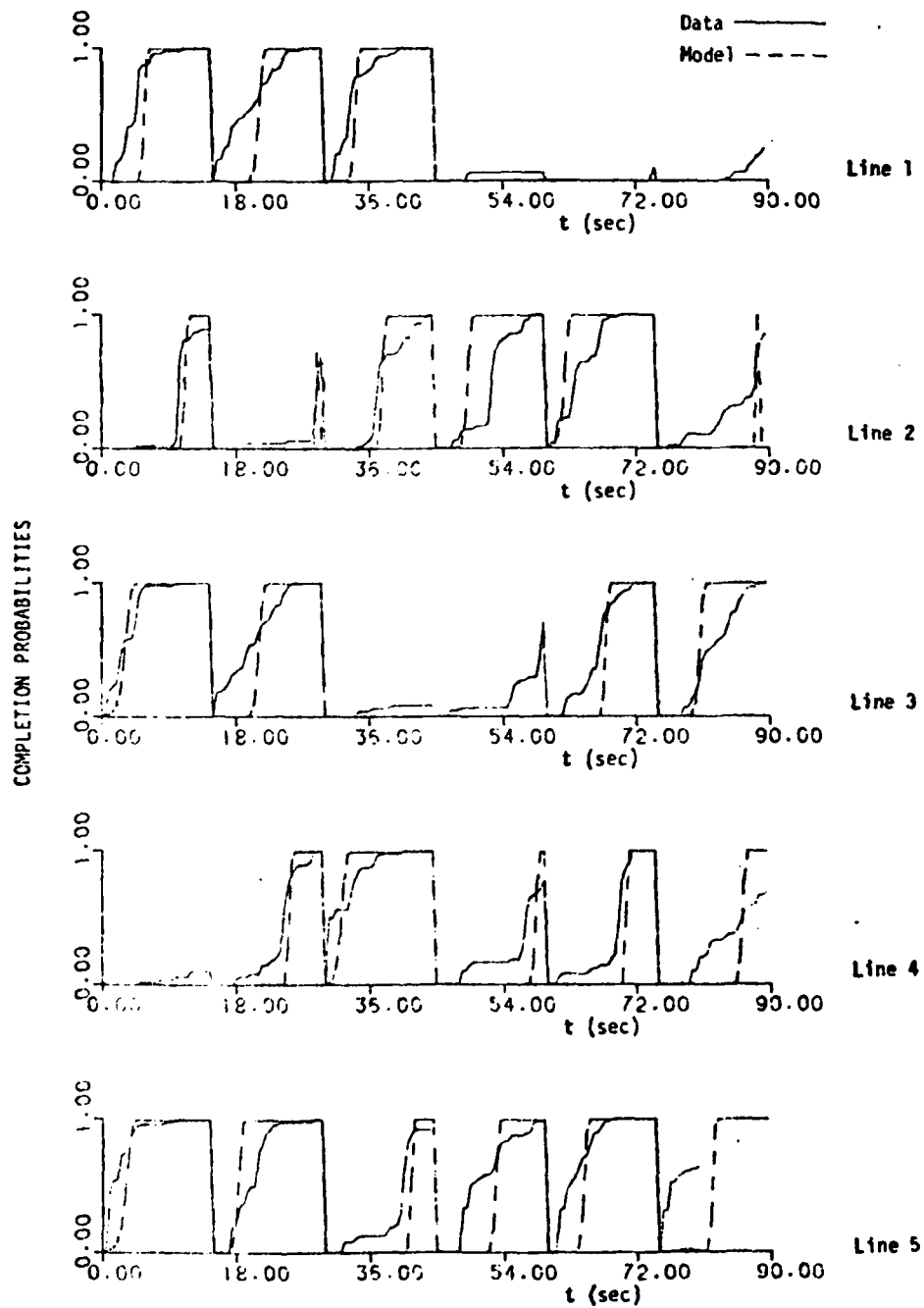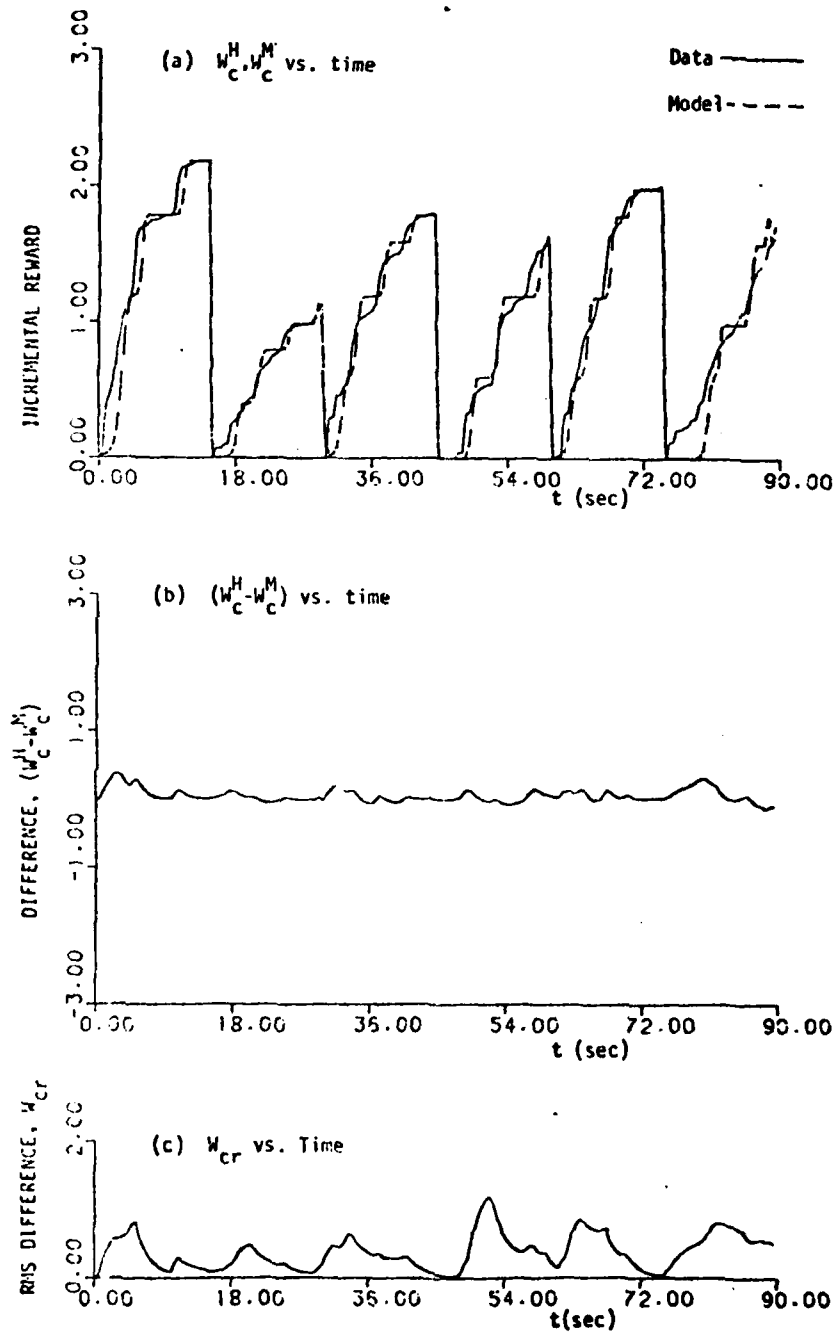Fig. 11: DECISION PROBABILITIES, CONDITION A, NR=48

Fig. 12: COMPLETION PROBABILITIES, CONDITION A, NR=48
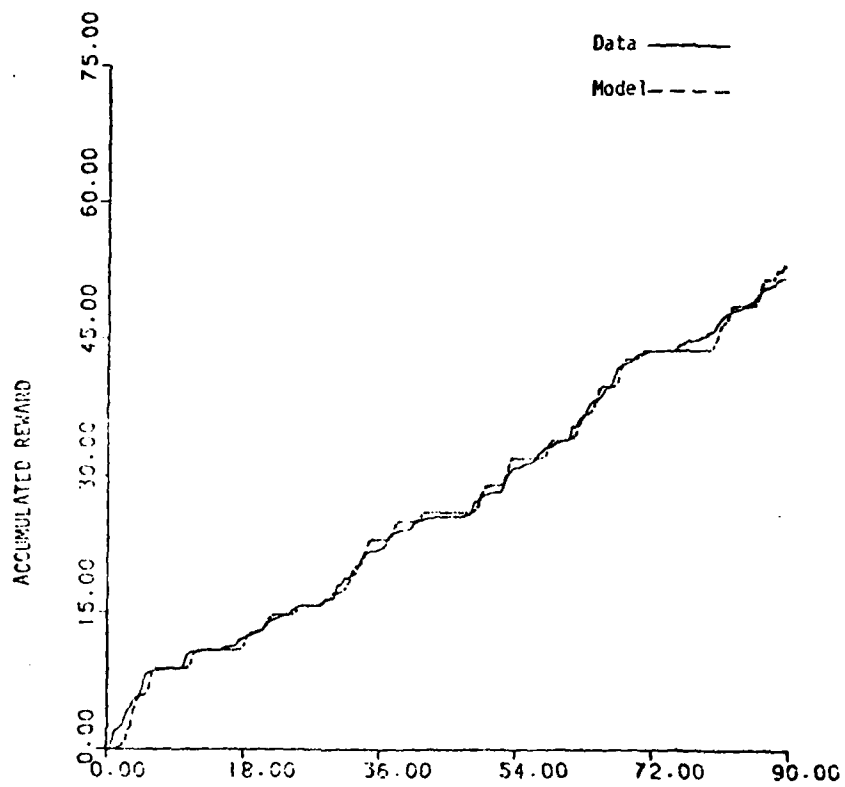
Fig. 13: INCREMENTAL REWARD, CONDITION A, NR=48

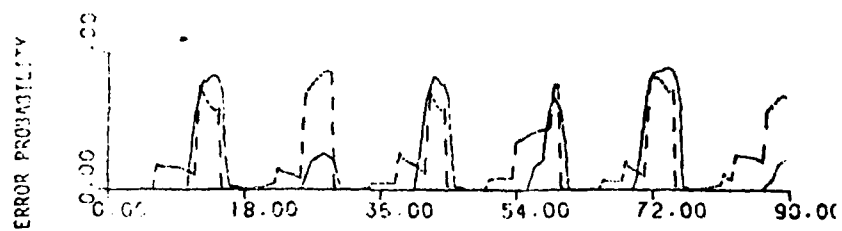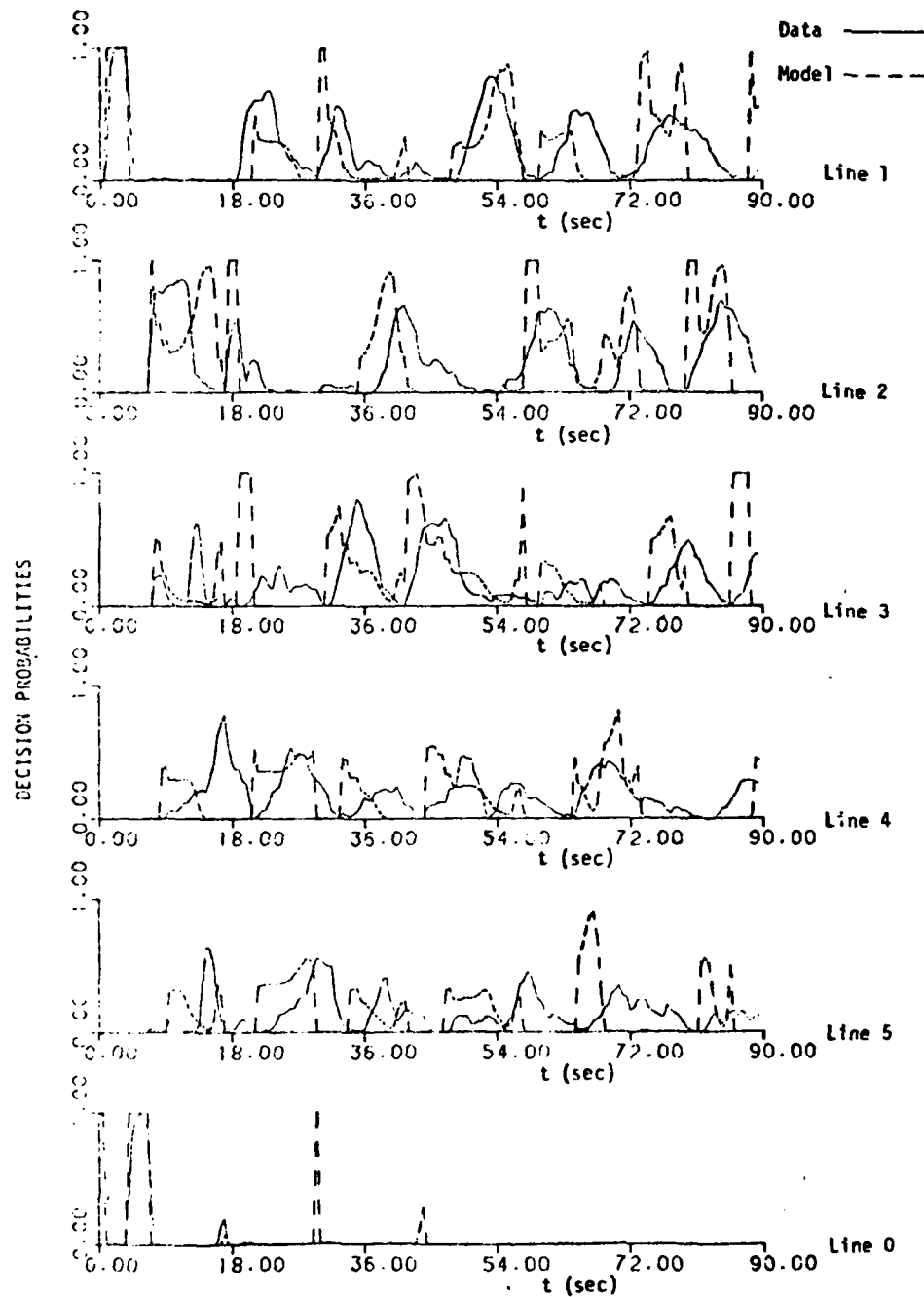**Fig. 14:** ACCUMULATED REWARD, CONDITION A, NR=48

Fig. 15: ERROR PROBABILITY, CONDITION A, NR=48

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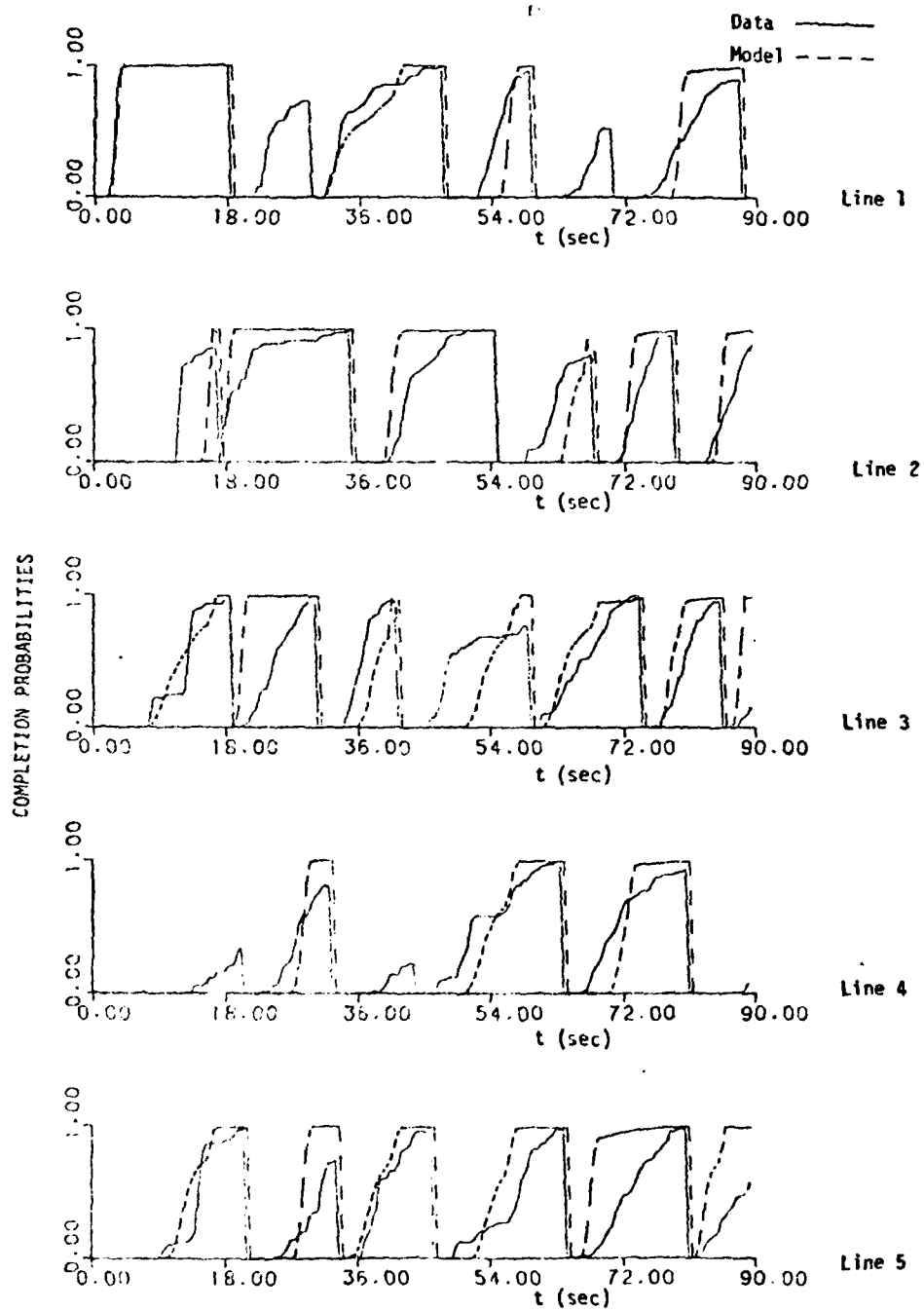Fig. 16: DECISION PROBABILITIES, CONDITION B, NR=48
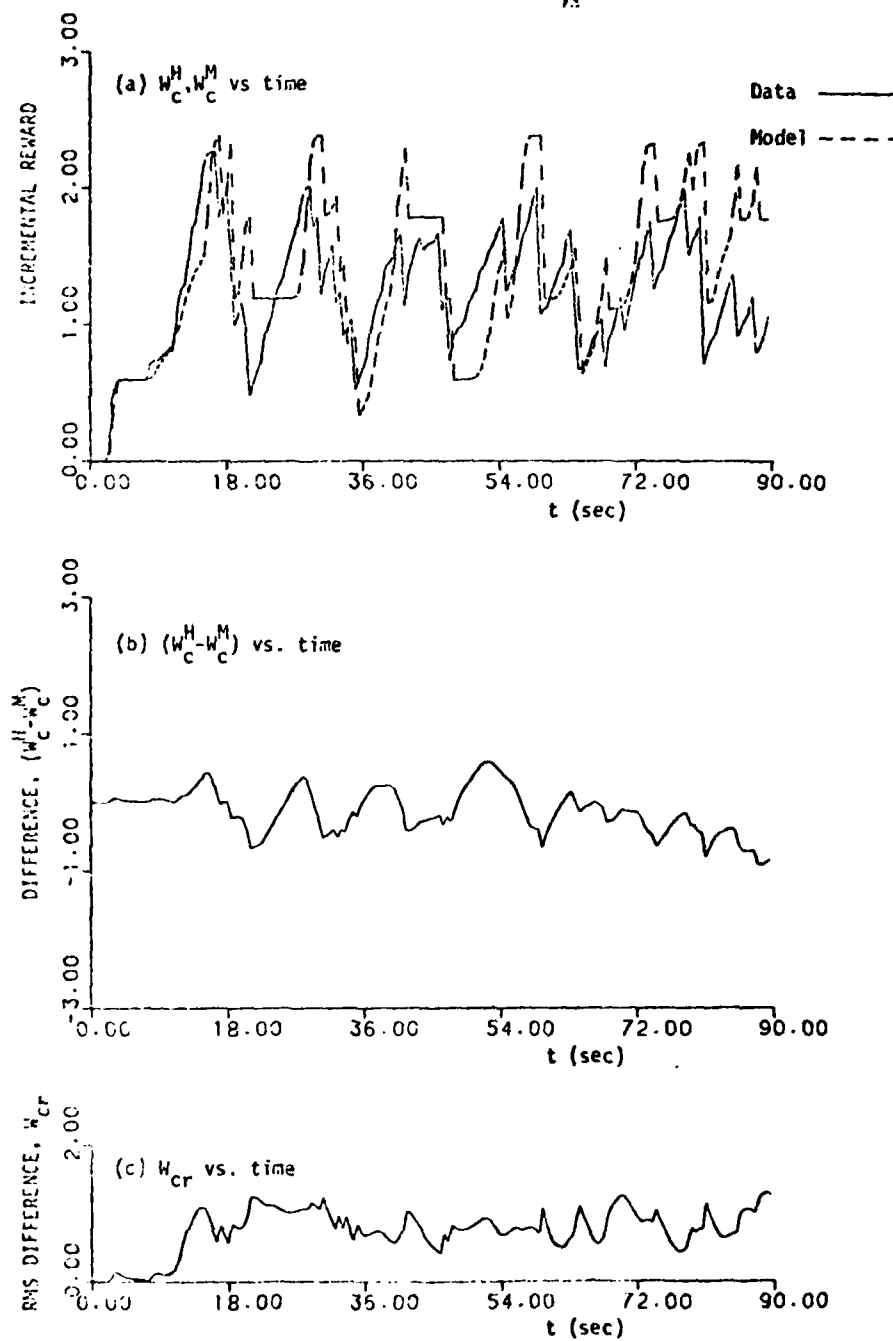
Fig. 17: COMPLETION PROBABILITIES, CONDITION B, NR=48

Fig. 18: INCREMENTAL REWARD, CONDITION B, NR=48
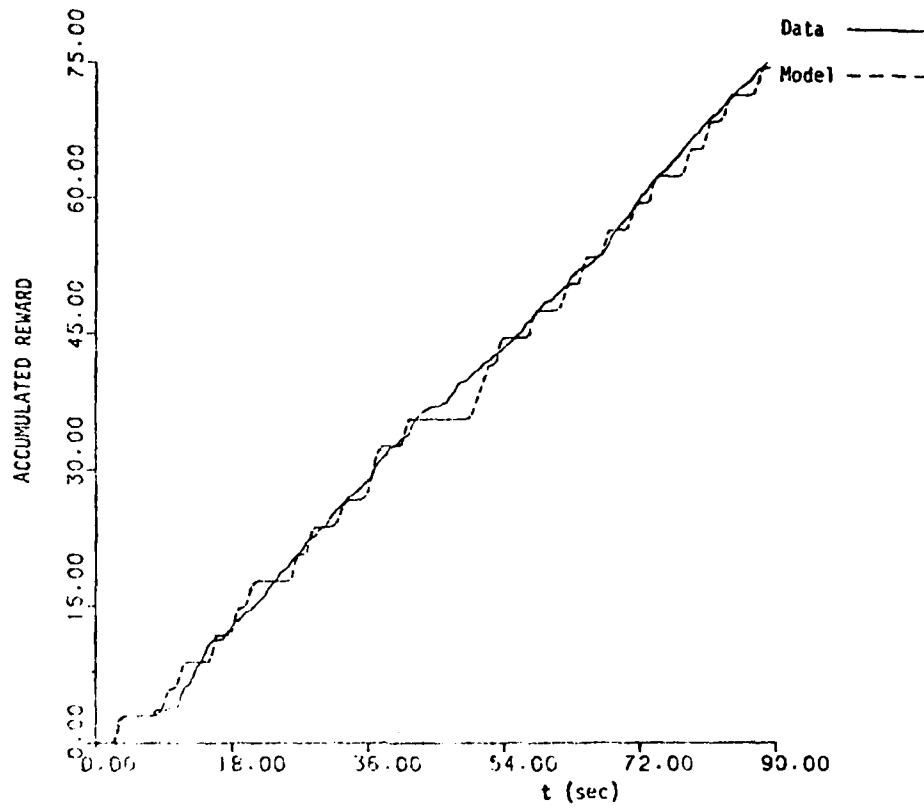
 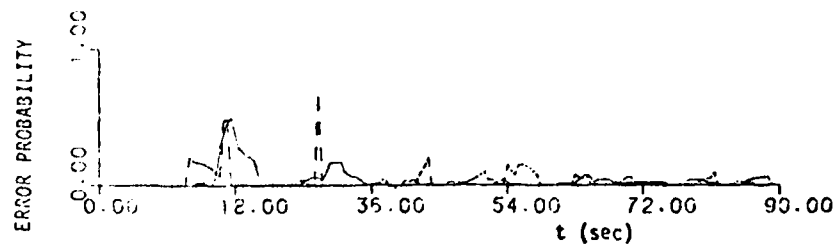
Data
Model
ACCUMULATED REWARD
75.00
60.00
45.00
30.00
15.00
0.00
0.00
18.00
36.00
54.00
72.00
90.00
t (sec)

Fig. 19: ACCUMULATED REWARD, CONDITION B, NR=48

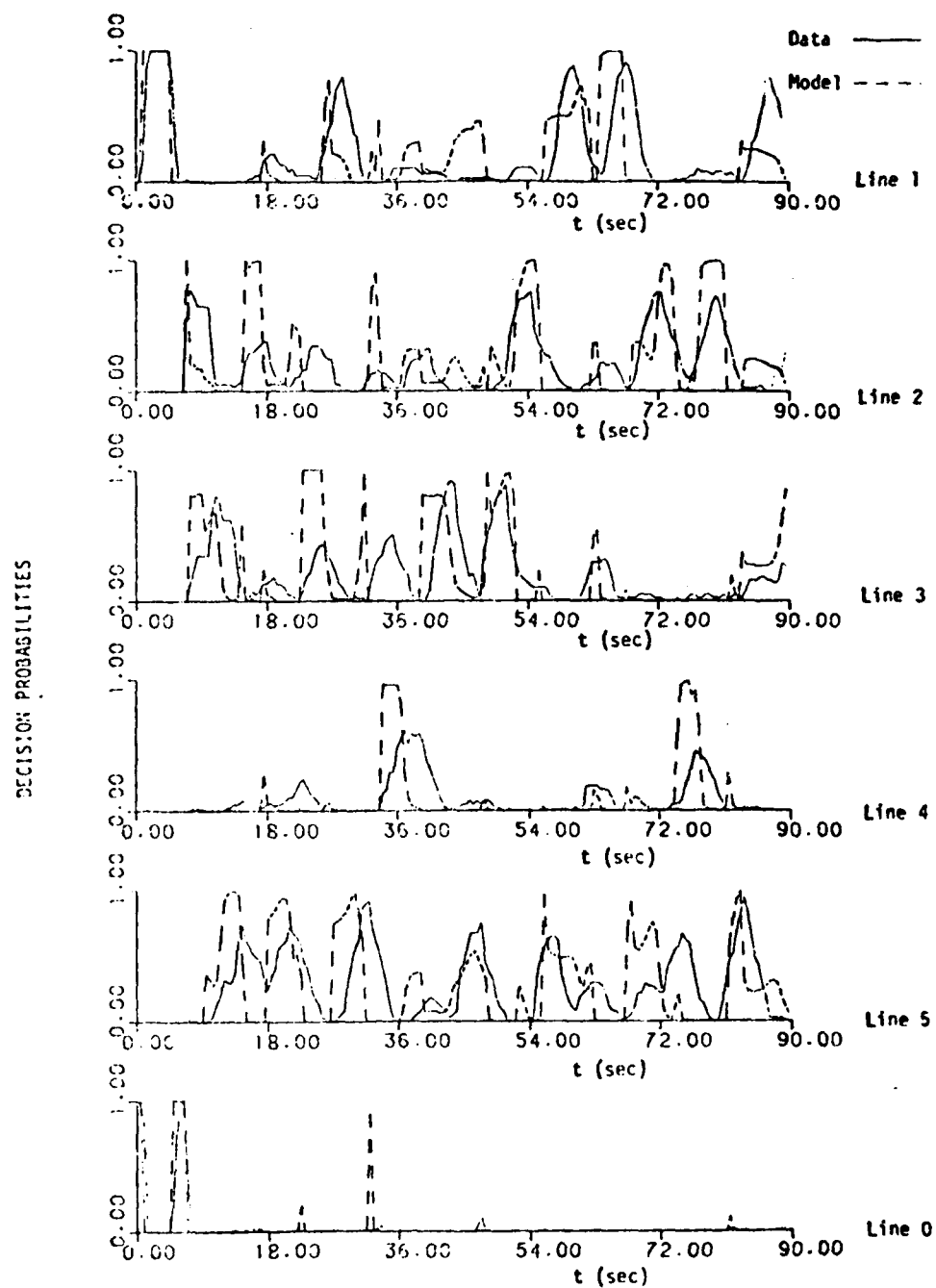

ERROR PROBABILITY
1.00
0.00
0.00
18.00
36.00
54.00
72.00
90.00
t (sec)

Fig. 20: ERROR PROBABILITY, CONDITION B, NR=48
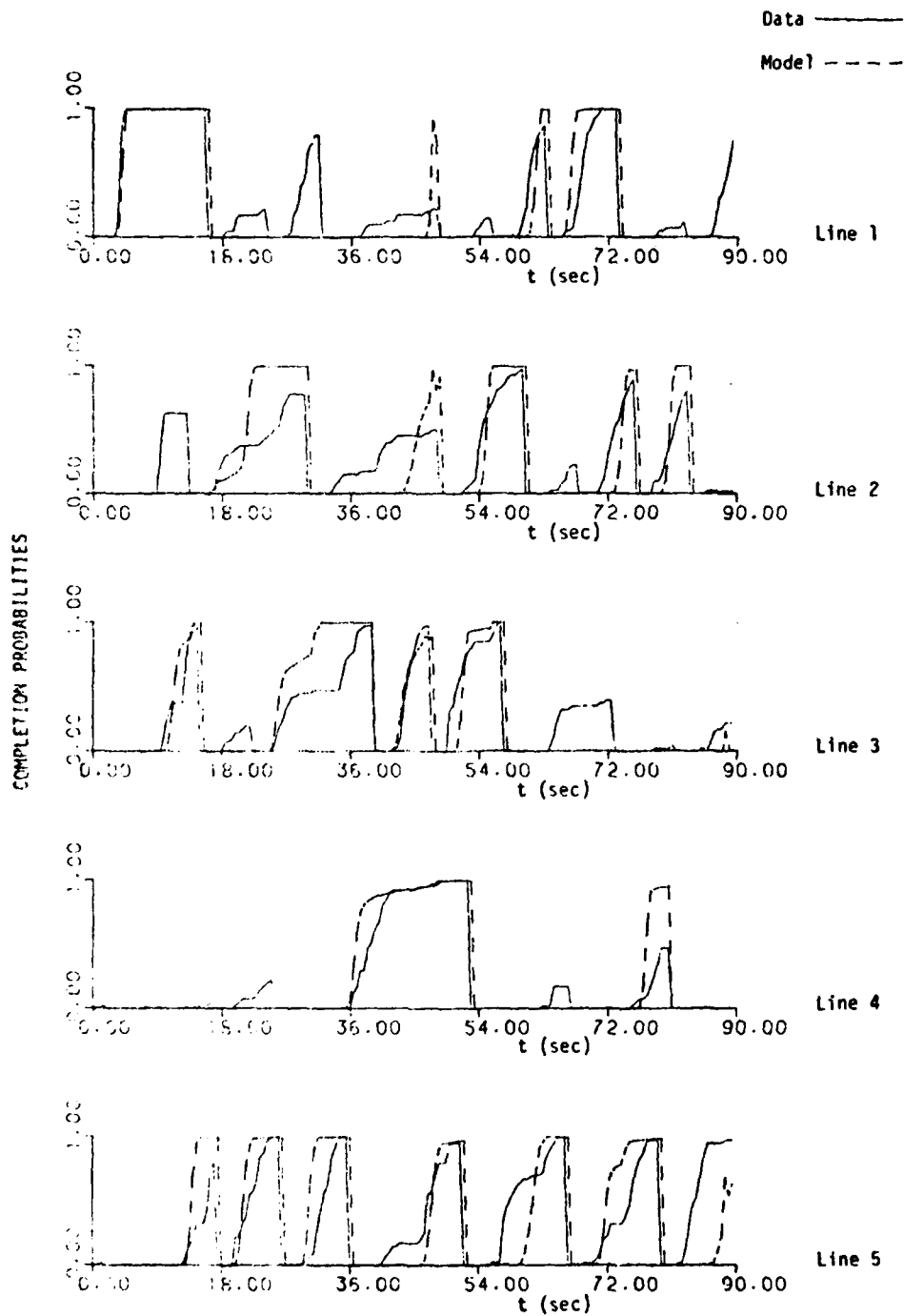
Fig. 21: DECISION PROBABILITIES, CONDITION C, NR=40

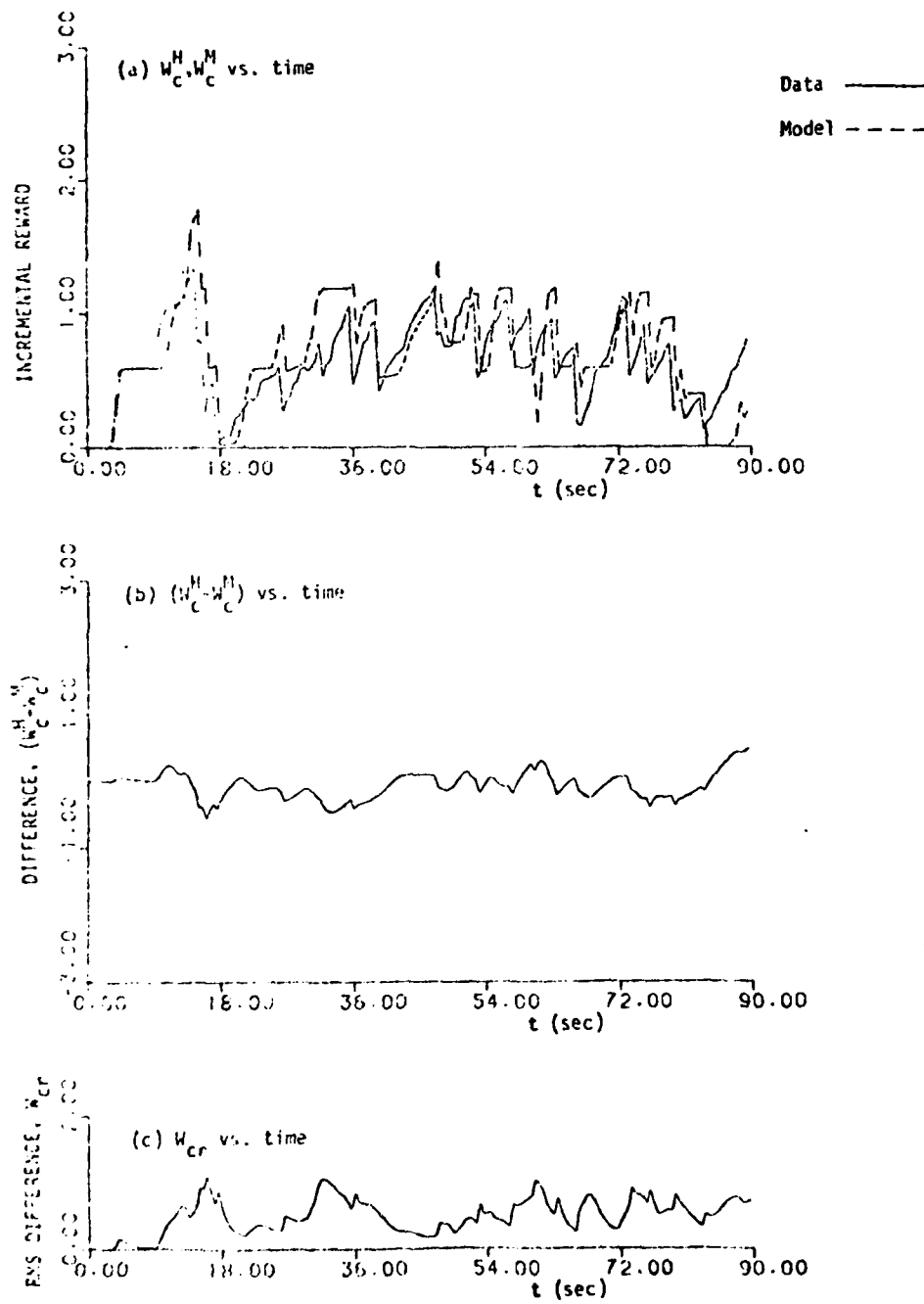Fig. 22: COMPLETION PROBABILITIES, CONDITION C, NR=40

Fig. 23: INCREMENTAL REWARD, CONDITION C, NR=40

Fig. 24: ACCUMULATED REWARD, CONDITION C, NR=40

Fig. 25: ERROR PROBABILITY, CONDITION C, NR=40

Fig. 26: ACTION PROBABILITIES, CONDITION D, NR=32

Fig. 27: COMPLETION PROBABILITIES, CONDITION D, NR 32

Fig. 28: INCREMENTAL REWARD, CONDITION D, NR=32

Data ———

Model — — —

ACCUMULATED REWARD

Fig. 29: ACCUMULATED REWARD, CONDITION D, NR=32

ERROR PROBABILITY

Fig. 30: ERROR PROBABILITY, CONDITION D, NR=32

Fig. 31: DECISION PROBABILITIES, CONDITION B_y, NR=32

Fig 32: COMPLETION PROBABILITIES, CONDITION B_y, NR=32
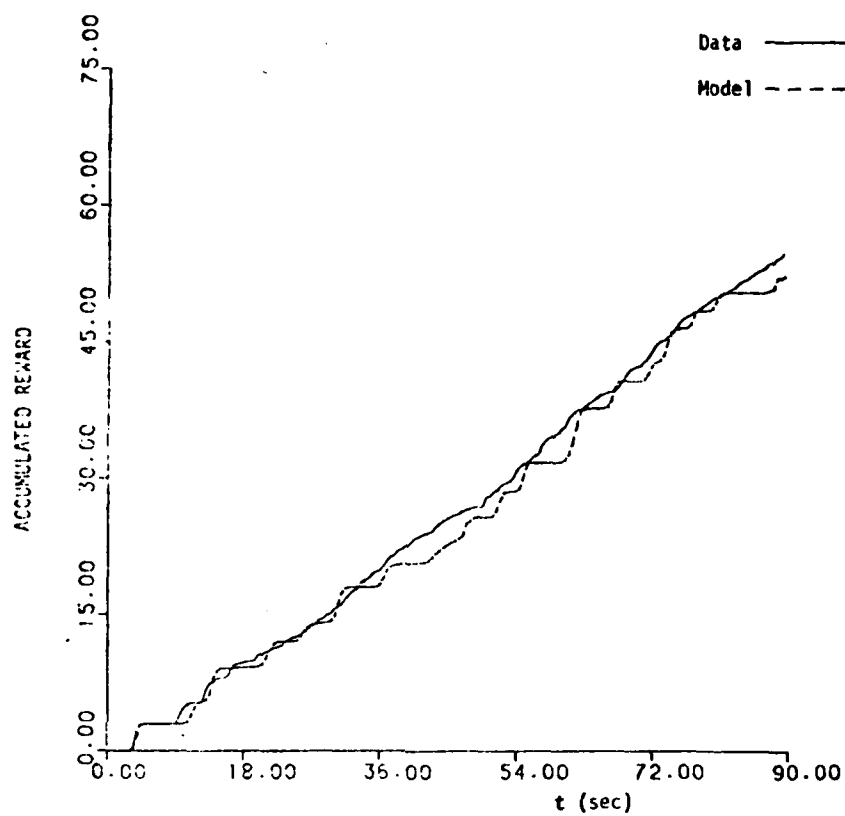
Fig. 33: INCREMENTAL REWARD, CONDITION B_y, NR=32

Fig. 34: ACCUMULATED REWARD, CONDITION B_y, NR=32
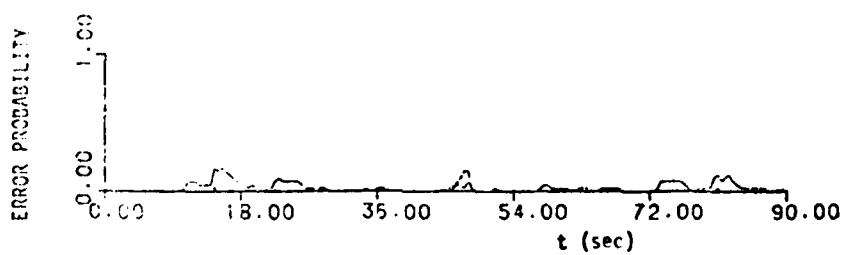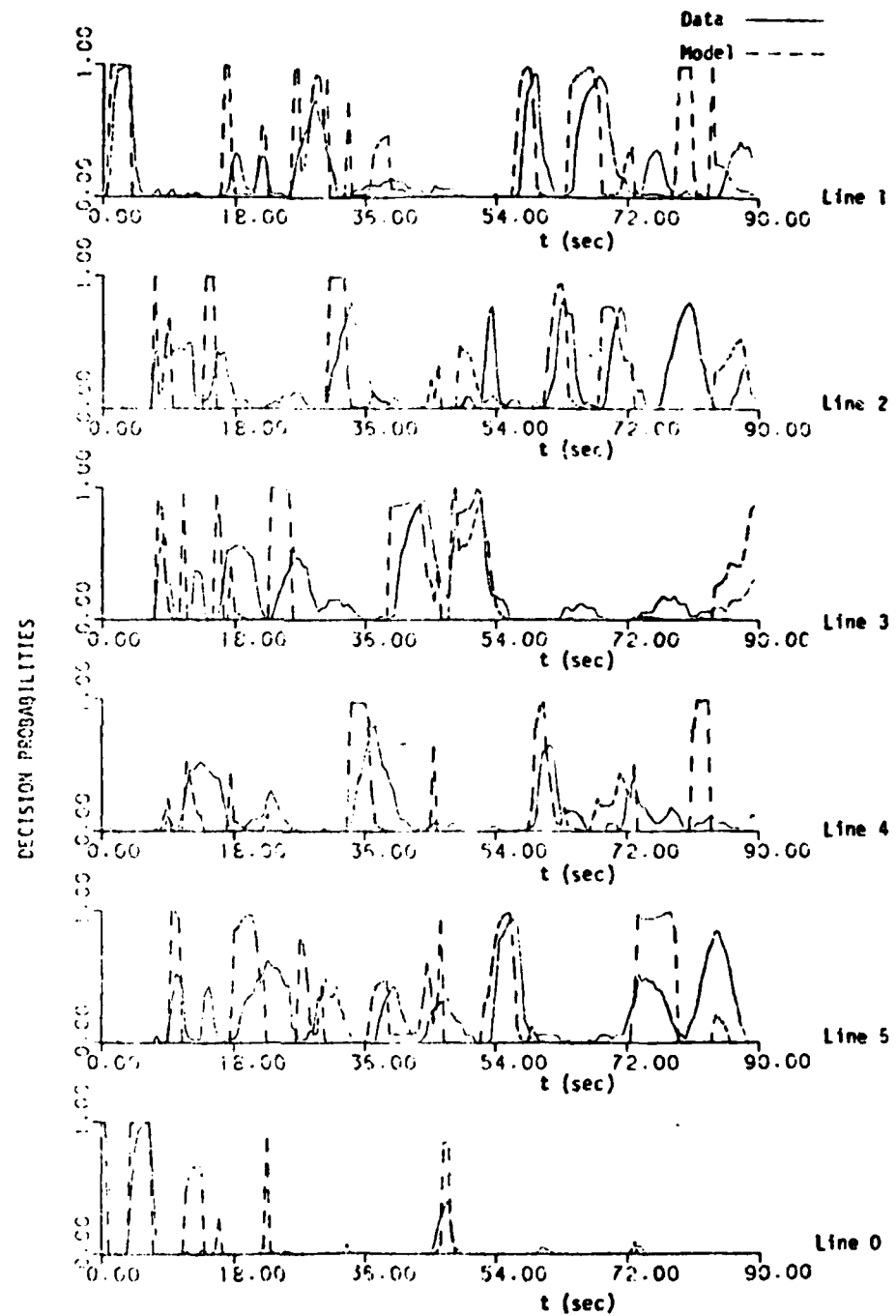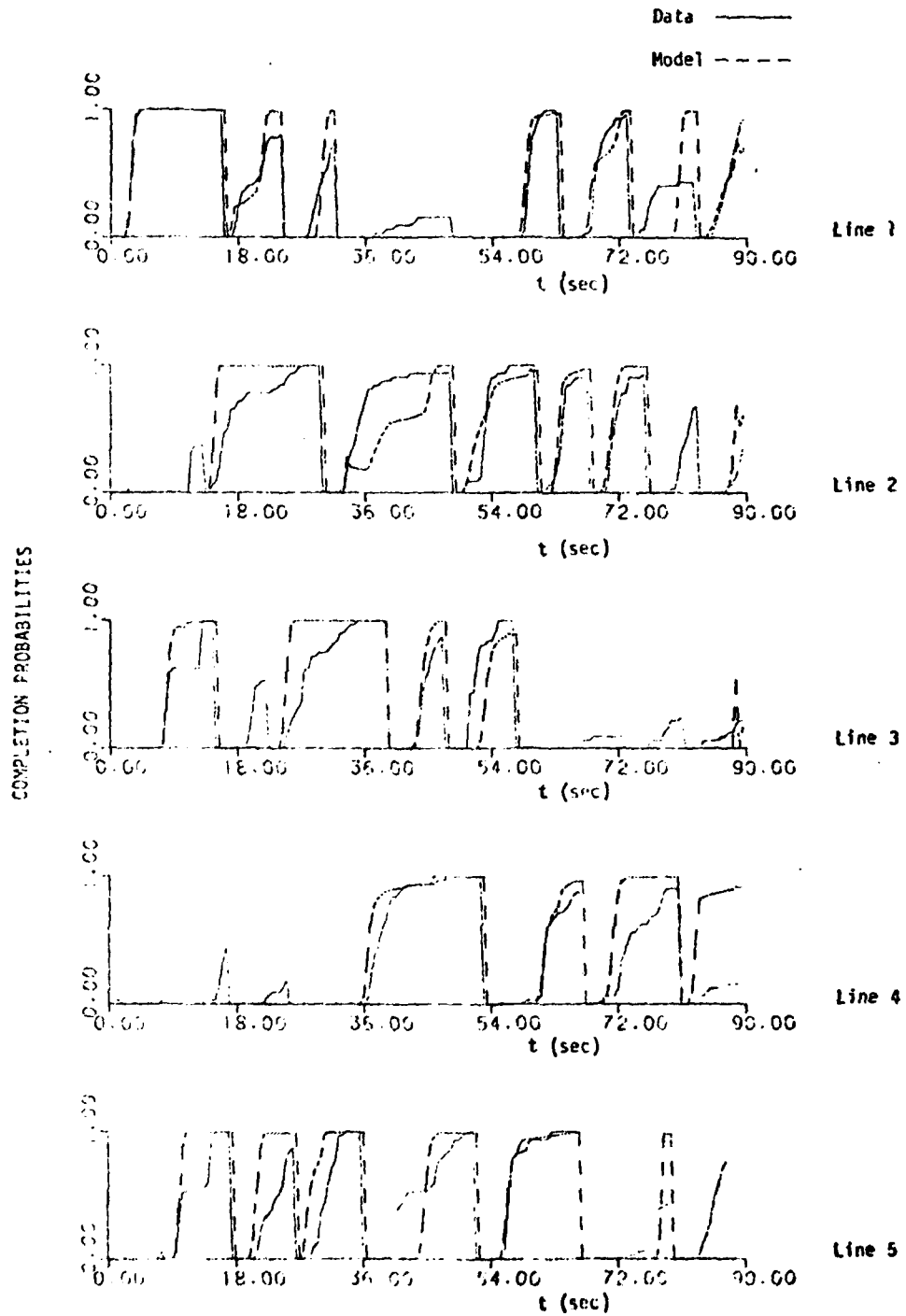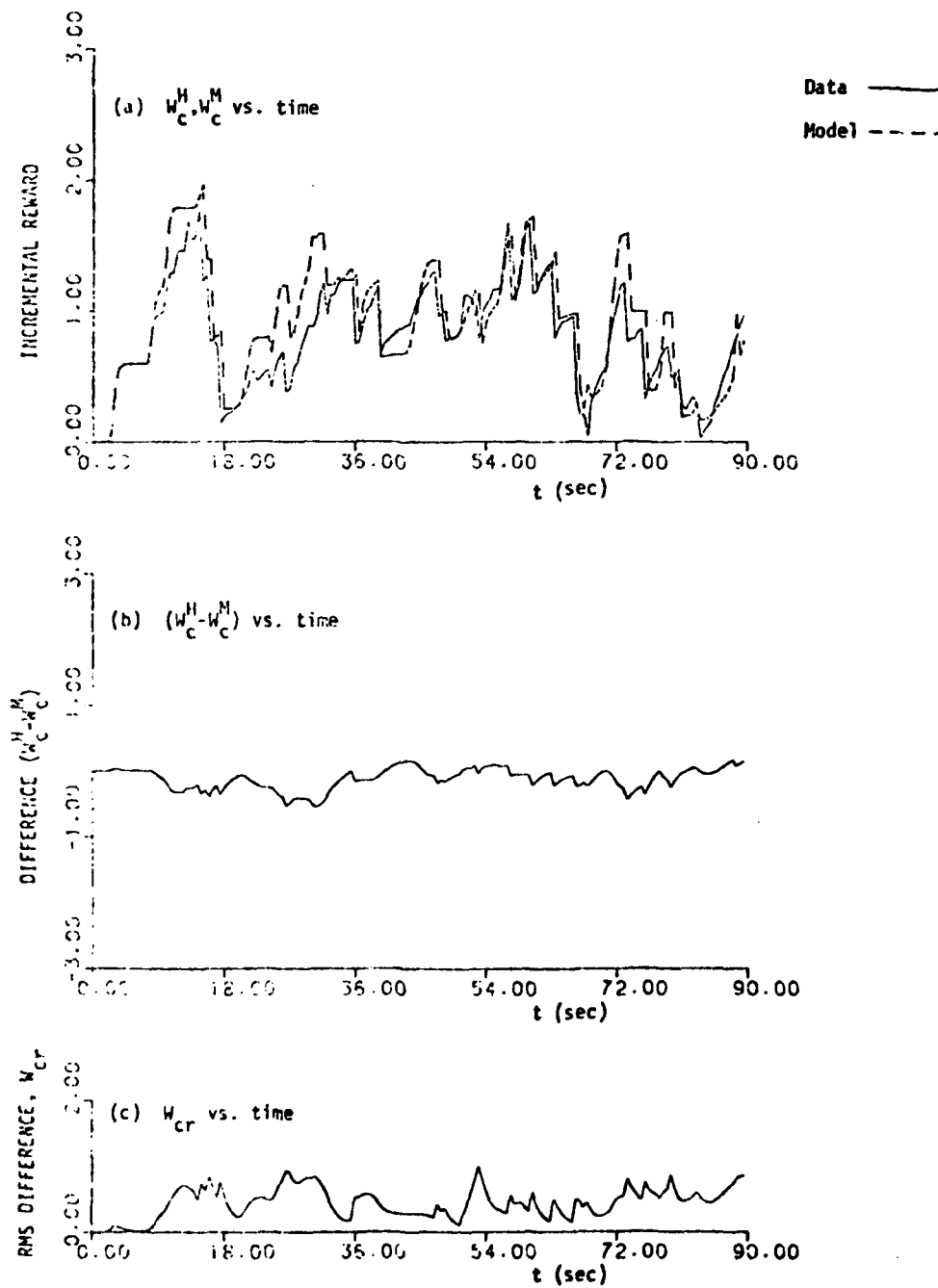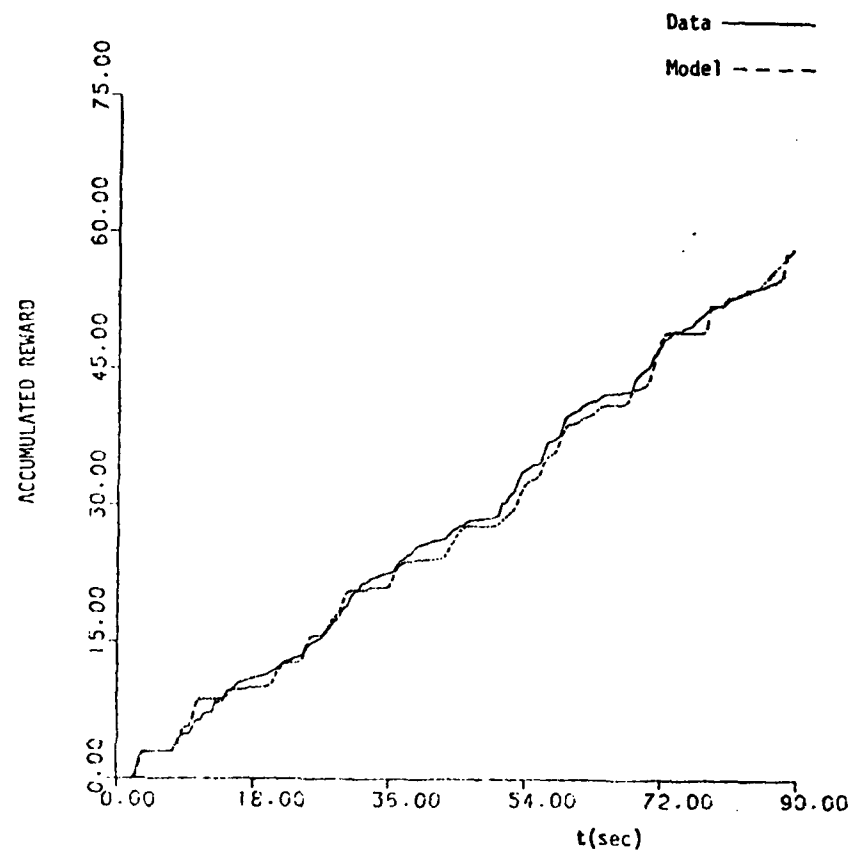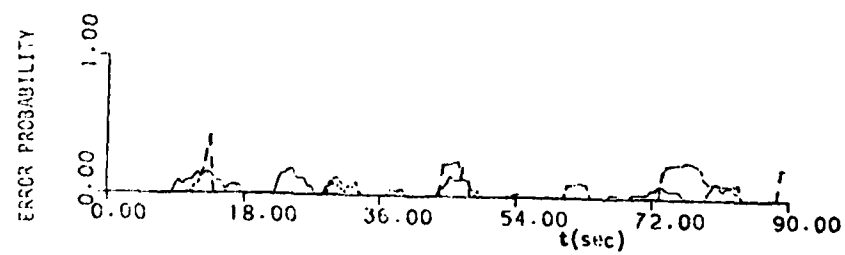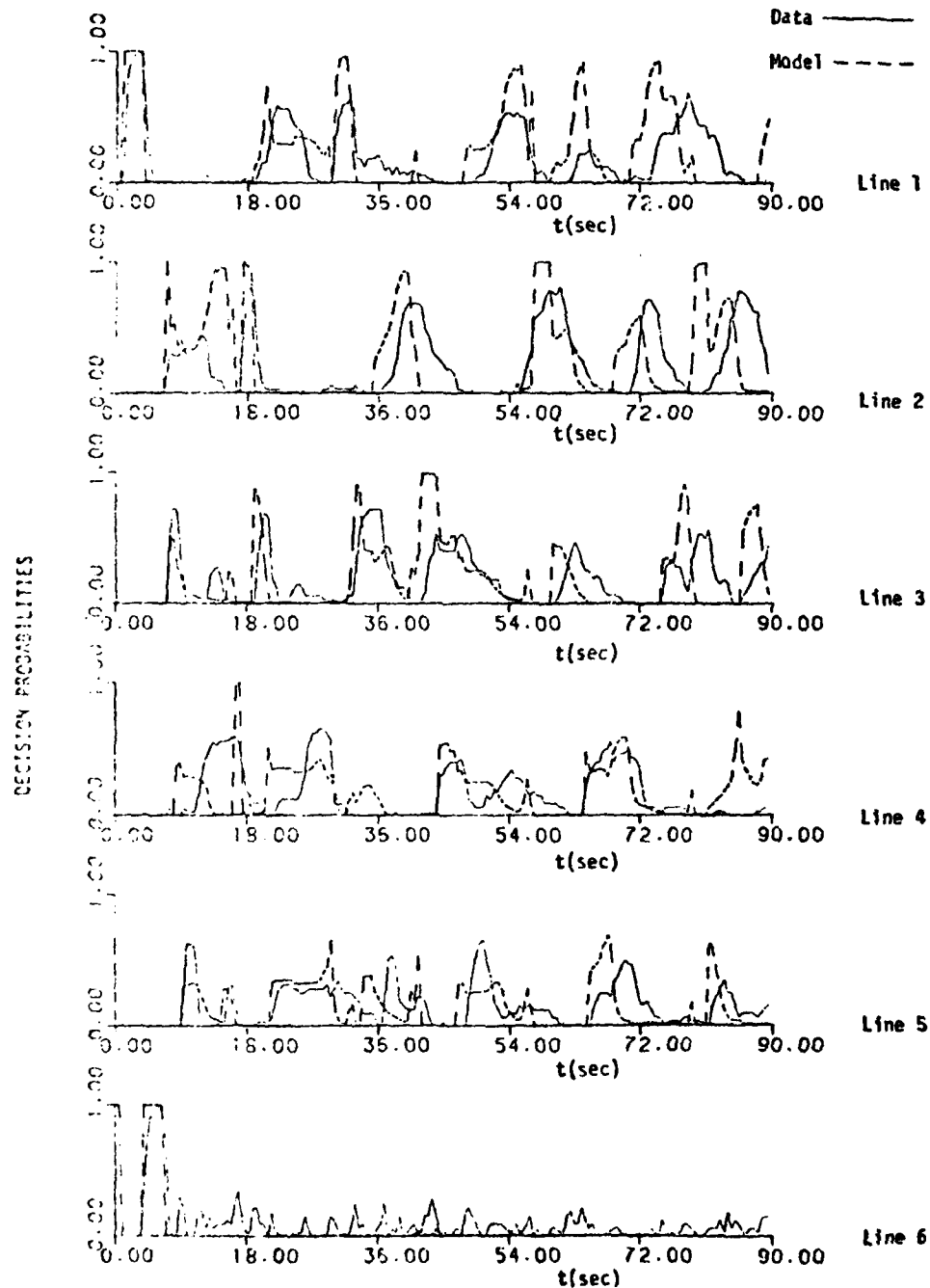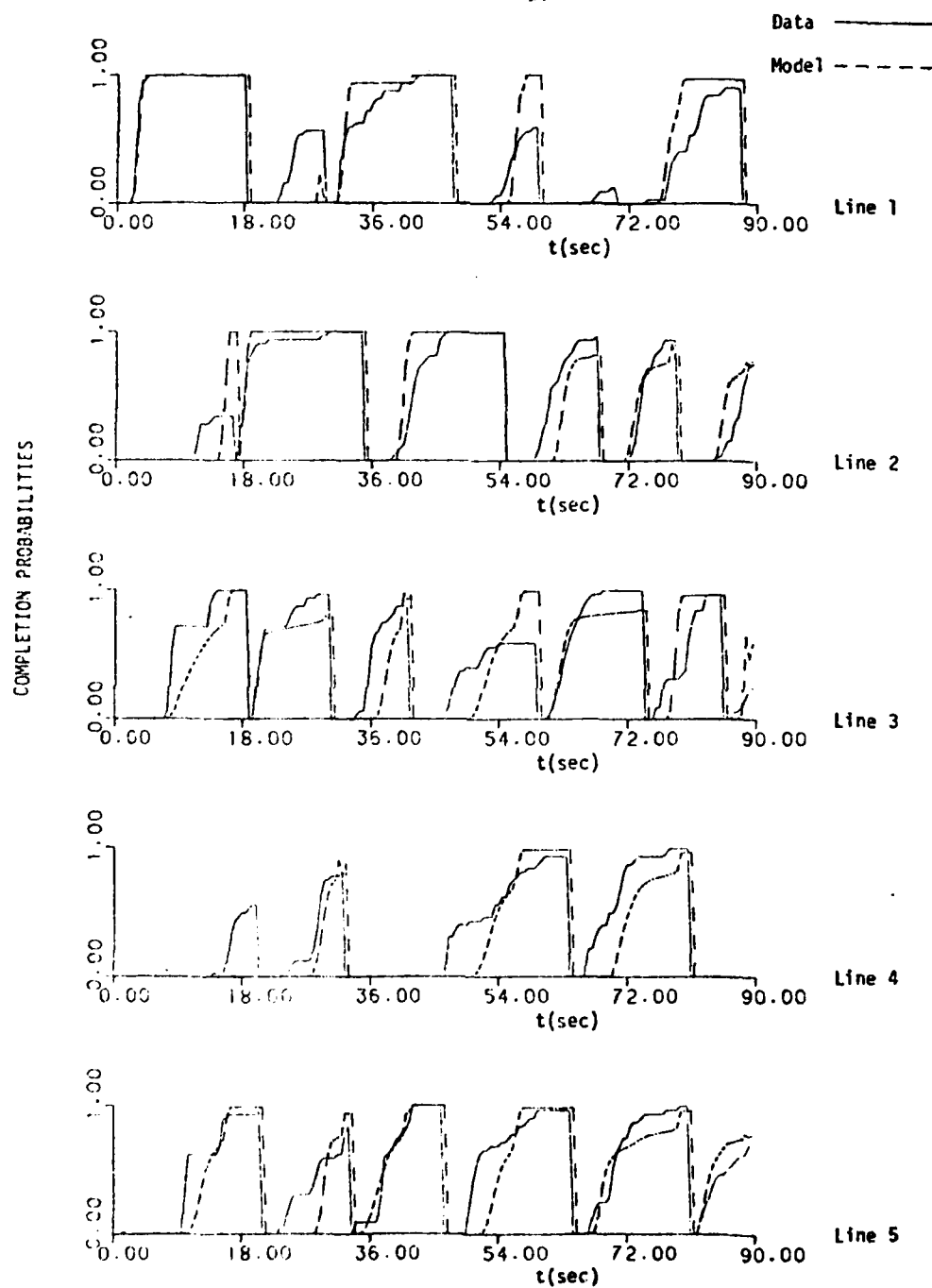

Fig. 35: ERROR PROBABILITY, CONDITION B_y, NR=32

human inertias, e.g. neuro-muscular lags, decision time losses, etc. It can be corrected by employing subjective values that depend on previous actions, or by incorporating a switching cost in the attractiveness measure of Eq (2.37). Since the discrepancies were not major in terms of the overall performance comparisons $\bar{R}^M(t)$ vs. $\bar{R}^H(t)$, and since our focus was on developing the structure of human decision model rather than the fine-tuning of it, these modifications were not explored in detail.

The average times spent on each task by the model and the human, along with the six scalar measures of similarity for experimental conditions A, B, C, D and B_y are displayed in Tables 1 through 5. They also indicate a reasonably close agreement between the model and data results.

Line \ Pass	1	2	3	4	5	6
1	2.659 (2.619)	1.603 (1.464)	2.767 (2.607)	0.361 (0.321)	3.233 (4.155)	0.313 (2.155)
2	5.510 (5.333)	4.132 (4.167)	3.619 (3.976)	3.763 (3.500)	1.922 (1.690)	2.454 (2.417)
3	1.763 (1.583)	1.603 (1.631)	1.909 (3.250)	1.319 (2.643)	3.614 (3.833)	4.693 (4.548)
4	2.022 (3.417)	3.588 (4.071)	1.921 (1.607)	4.532 (4.167)	2.549 (2.726)	5.551 (3.929)
5	1.763 (1.536)	3.812 (3.607)	3.531 (3.500)	4.645 (4.440)	2.769 (2.583)	1.582 (1.631)

TABLE 1a: AVERAGE TIME SPENT ON EACH TASK IN EACH PASS FOR CONDITION A (Brackets: Data)

SCALAR MEASURE	VALUE
AM	0.05569
IRM	0.05421
ARM	0.00018
TCM	0.00019
ATTM	0.0431
EPM	0.05867

TABLE 1b: SCALAR MEASURES OF SIMILARITY FOR CONDITION A (A Value of 0 Corresponds to a Perfect Fit)

Pass Line	1	2	3	4	5	6	7
1	2.792 (3.010)	2.121 (3.615)	2.494 (2.625)	4.491 (4.281)	1.828 (3.042)	4.346 (4.062)	1.020 (0.062)
2	5.582 (4.771)	1.514 (1.823)	3.547 (3.708)	4.213 (3.781)	2.428 (2.375)	4.467 (4.115)	—— (——)
3	1.762 (1.615)	1.659 (1.573)	3.379 (3.521)	5.557 (4.385)	1.267 (1.656)	2.176 (2.458)	2.539 (0.802)
4	1.585 (3.385)	3.473 (3.104)	1.427 (1.427)	3.357 (3.573)	3.379 (3.323)	0.560 (1.229	—— (——)
5	1.364 (1.740)	3.542 (2.906)	1.376 (1.677)	2.409 (2.573)	2.376 (2.510)	1.376 (0.917)	—— (——)

TABLE 2a: AVERAGE TIME SPENT ON EACH TASK IN EACH PASS FOR CONDITION B
(Brackets: Data)

SCALAR MEASURE	VALUE
AM	0.06656
IRM	0.09250
ARM	0.00019
TCM	0.028×10^{-6}
ATTM	0.06706
EPM	0.00933

TABLE 2b: SCALAR MEASURES OF SIMILARITY FOR CONDITION B

Pass Line	1	2	3	4	5	6	7	8	9
1	3.744 (4.095)	0.202 (0.905)	1.239 (2.838)	3.251 (0.811)	0.019 (0.405)	3.665 (3.270)	3.613 (3.608)	0.040 (0.514)	1.306 (2.932)
2	0.810 (2.622)	3.651 (2.905)	3.245 (1.770)	3.541 (3.486)	0.300 (0.851)	3.387 (3.419)	3.766 (2.892)	1.235 (0.486)	—— (——)
3	3.520 (3.392	0.166 (0.716)	3.738 (3.608)	3.288 (3.527)	3.672 (3.541)	0.491 (1.338)	0.030 (0.230)	2.640 (1.149)	—— (——)
4	0.023 (0.284)	0.173 (0.892)	0.077 (0.027)	3.402 (3.676)	0.152 (0.703)	3.292 (1.716)	0.244 (0.027	—— (——)	—— (——)
5	3.566 (2.865)	3.755 (3.622)	3.699 (3.635)	2.288 (3.514)	3.513 (3.635)	3.377 (3.541)	3.094 (3.459)	—— (——)	—— (——)

TABLE 3a: AVERAGE TIME SPENT ON EACH TASK IN EACH PASS FOR CONDITION C
(Brackets: Data)

SCALAR MEASURE	VALUE
AM	0.06676
IRM	0.09269
ARM	0.00045
TCM	0.00370
ATTM	0.07856
EPM	0.00200

TABLE 3b: SCALAR MEASURES OF SIMILARITY FOR CONDITION C

Pass Line	1	2	3	4	5	6	7	8	9
1	2.792 (3.031)	1.624 (1.172)	3.613 (2.734)	1.438 (1.219)	0.012 (0.016)	2.436 (2.531)	4.636 (4.344)	2.494 (1.156)	1.119 (1.531)
2	0.915 (2.547)	1.725 (1.672)	3.428 (3.313)	1.239 (1.422)	2.398 (2.344)	2.729 (2.328)	0.010 (3.359)	2.123 (0.875)	—— (——)
·	1.761 (1.641)	1.044 (2.516)	2.688 (2.672)	2.677 (4.391)	4.227 (4.594)	0.019 (0.531)	0.076 (0.859)	3.029 (1.109)	—— (——)
4	0.962 (2.656)	0.374 (0.953)	0.026 (0.063)	3.502 (3.563)	2.296 (2.188)	1.587 (1.500)	2.268 (0.531)	—— (——)	—— (——)
5	1.494 (1.656)	3.713 (3.406)	1.474 (1.688)	2.783 (2.688)	3.407 (3.516)	5.351 (2.750)	0.392 (3.891)	—— (——)	—— (——)

TABLE 4a: AVERAGE TIME SPENT ON EACH TASK IN EACH PASS FOR CONDITION D
(Brackets: Data)

SCALAR MEASURE	VALUE
AM	0.08489
IRM	0.08151
ARM	0.00020
TCM	0.00122
ATTM	0.12574
EPM	0.00345

TABLE 4b: SCALAR MEASURES OF SIMILARITY FOR CONDITION D

Line \ Pass	1	2	3	4	5	6	7
1	2.791 (2.906)	3.368 (2.750)	2.576 (2.859)	4.437 (2.875)	2.059 (1.000)	4.326 (4.328)	0.825 (0.031)
2	5.543 (2.250)	1.495 (1.859)	3.513 (3.859)	4.428 (4.656)	2.292 (2.656)	4.220 (3.766)	———— (————)
3	1.371 (1.797)	1.205 (1.672)	3.439 (3.313)	5.272 (3.656)	1.398 (1.828)	2.380 (2.859)	2.074 (0.922)
4	1.889 (3.563)	3.273 (3.016)	0.802 (0.016)	3.344 (3.719)	3.328 (3.828)	1.919 (0.125)	———— (————)
5	1.546 (1.766)	3.275 (3.281	1.541 (1.828)	2.377 (2.703)	2.352 (2.688)	1.227 (1.266)	———— (————)

TABLE 5a: AVERAGE TIME SPENT ON EACH TASK IN EACH PASS FOR CONDITION B_y
(Brackets: Data)

SCALAR MEASURE	VALUE
AM	0.06105
IRM	0.06944
ARM	0.00066
TCM	0.00094
ATTM	0.07897
EPM	0.00132

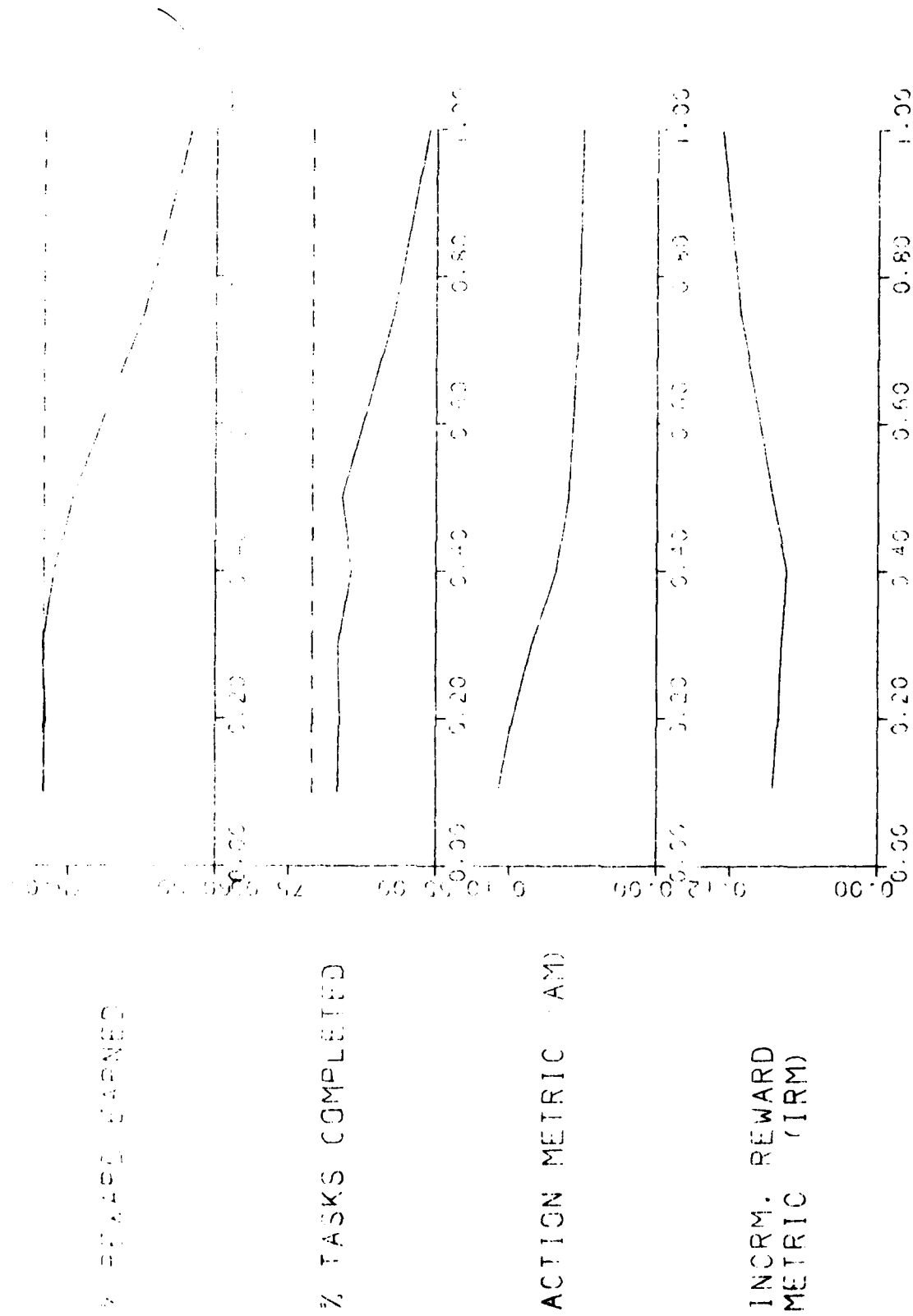TABLE 5b: SCALAR MEASURES OF SIMILARITY FOR CONDITION B_y

3.4   Sensitivity Analysis of the DDM

Sensitivity studies were made on the DDM with respect to the parameter set Ω. The study showed that the model predictions exhibit greater sensitivity to the parameter c, the co-efficient of variation, than to the remaining parameters τ, ρ_i, T_{RO}. Therefore, only the results of varying the parameter c are presented in detail for experimental conditions B and D, and results for the other parameters and the discount factor, α are briefly summarized.

   (i)   *Variations of co-efficient of variation,* c: The parameter

c was varied in the range 0.1 - 1.0 and the model predictions of percent reward earned, percent tasks completed and the scalar measures of similarity are plotted in Figs. 36 and 37 for the experimental conditions D and B, respectively.[†]

As the value of c increases, the percent reward earned and the percent tasks completed by the model decreases. This is because the model allocates attention equally among tasks at high values of c. This results in a reduction in the number of tasks being completed and, hence, the reward, since the value is credited only at the end of a successful task completion. The tendency of the model to uniformly allocate attention among tasks at large values of c, causes a decrease in the measure AM. However, all the other measures of similarity, IRM, ARM, ATTM and EPM, generally increase with increasing c. Note, in particular, that ARM, which is a

[†]The measure TCM is not shown, as it is similar to comparing percent tasks completed by the human and the model.

COEFFICIENT OF VARIATION

Fig. 36: SCALAR MEASURES VERSUS CO-EFFICIENT OF VARIATION FOR EXPERIMENTAL CONDITION D

(cont'd)

COEFFICIENT OF VARIATION

ACCUM. REWARD
METRIC (ARM)

AVERAGE TIME ON EACH
TASK METRIC (ATTM)

ERROR PROB.
METRIC (EPM)

Fig. 36: SCALAR MEASURES VERSUS CO-EFFICIENT OF VARIATION FOR EXPERIMENTAL CONDITION D

% REWARD EARNED

% TASKS COMPLETED

ACTION METRIC (AM)

INCRM. REWARD
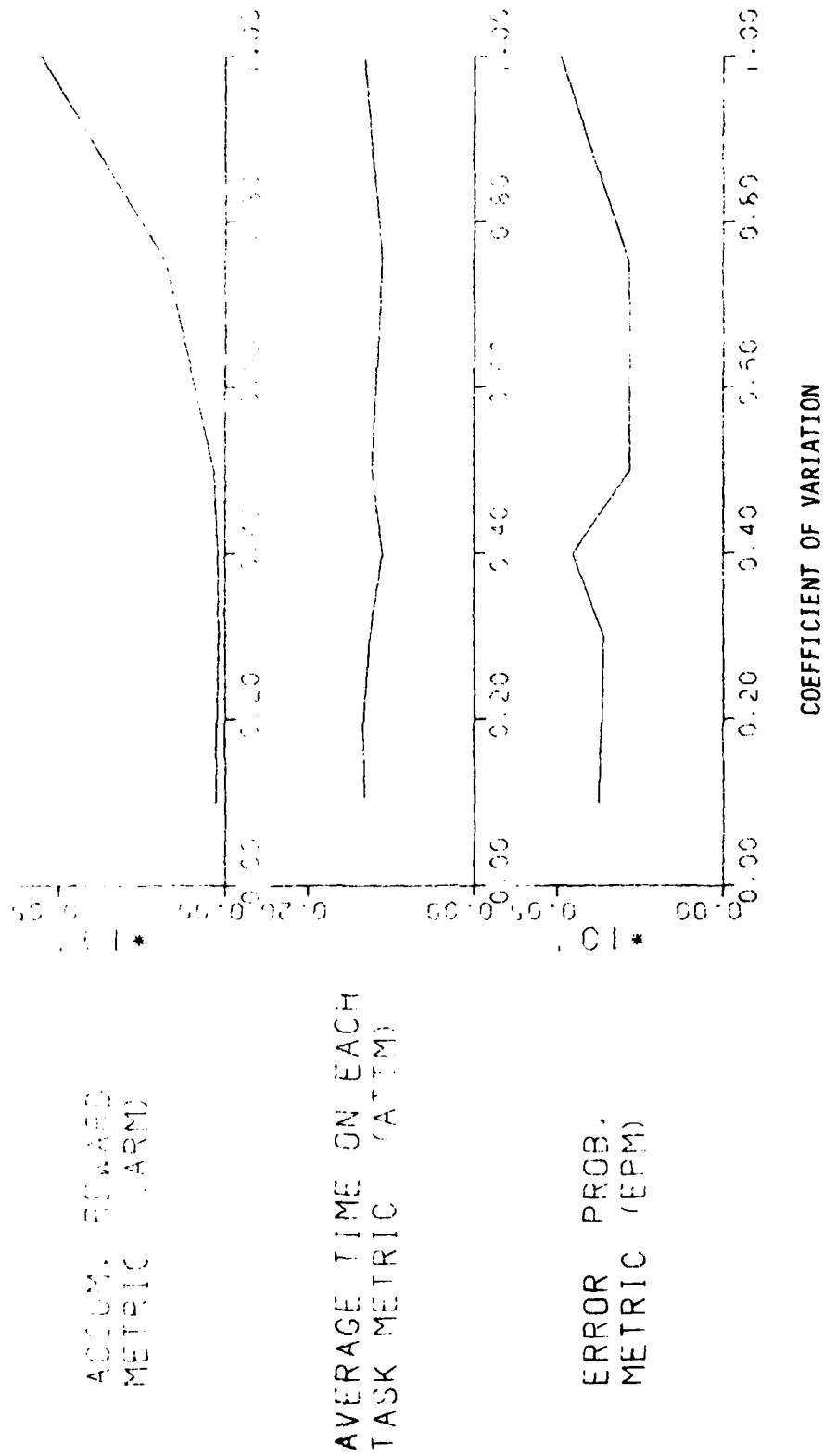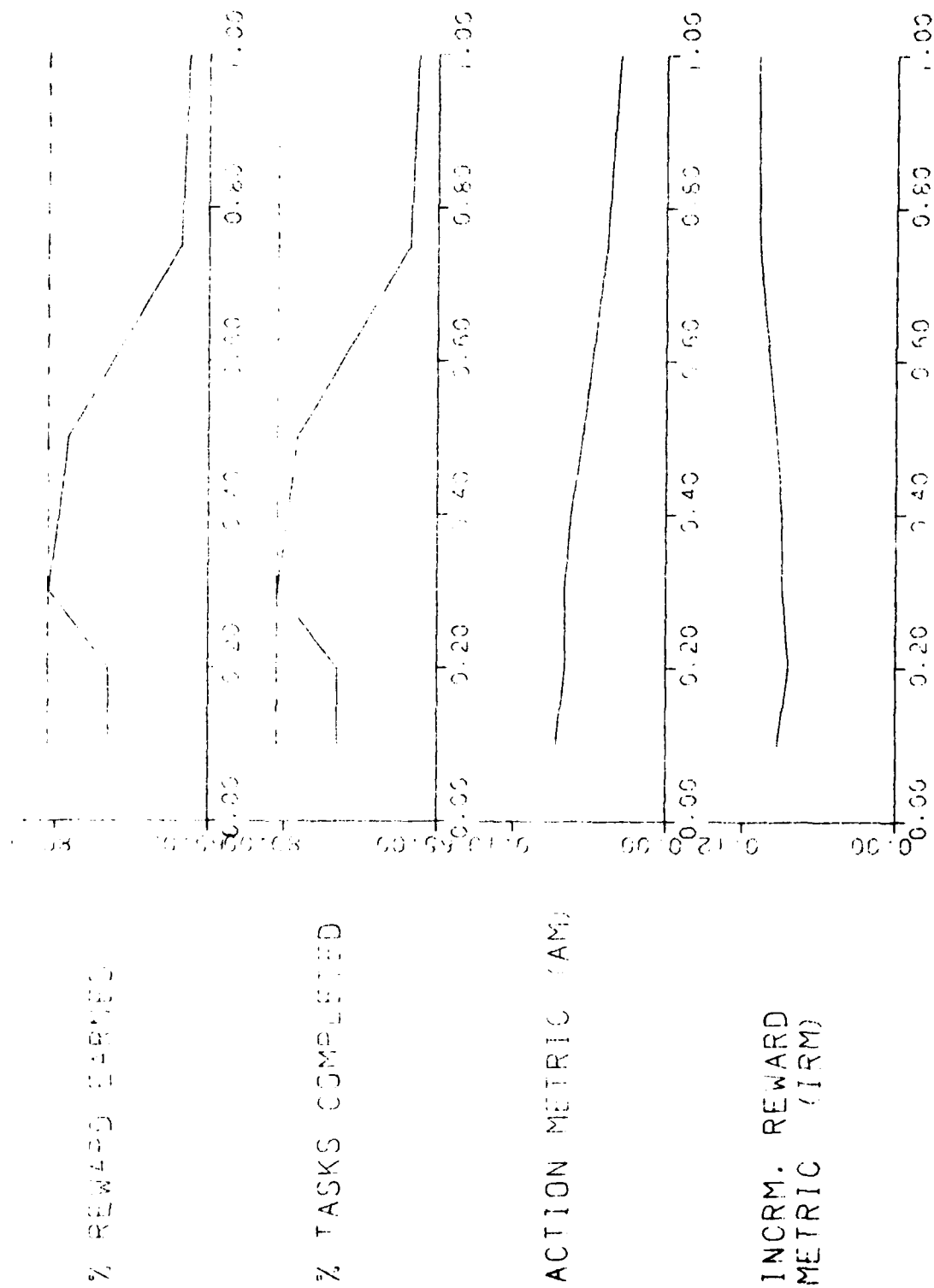METRIC (IRM)

COEFFICIENT OF VARIATION

Fig. 37: SCALAR MEASURES VERSUS CO-EFFICIENT OF VARIATION FOR EXPERIMENTAL CONDITION B
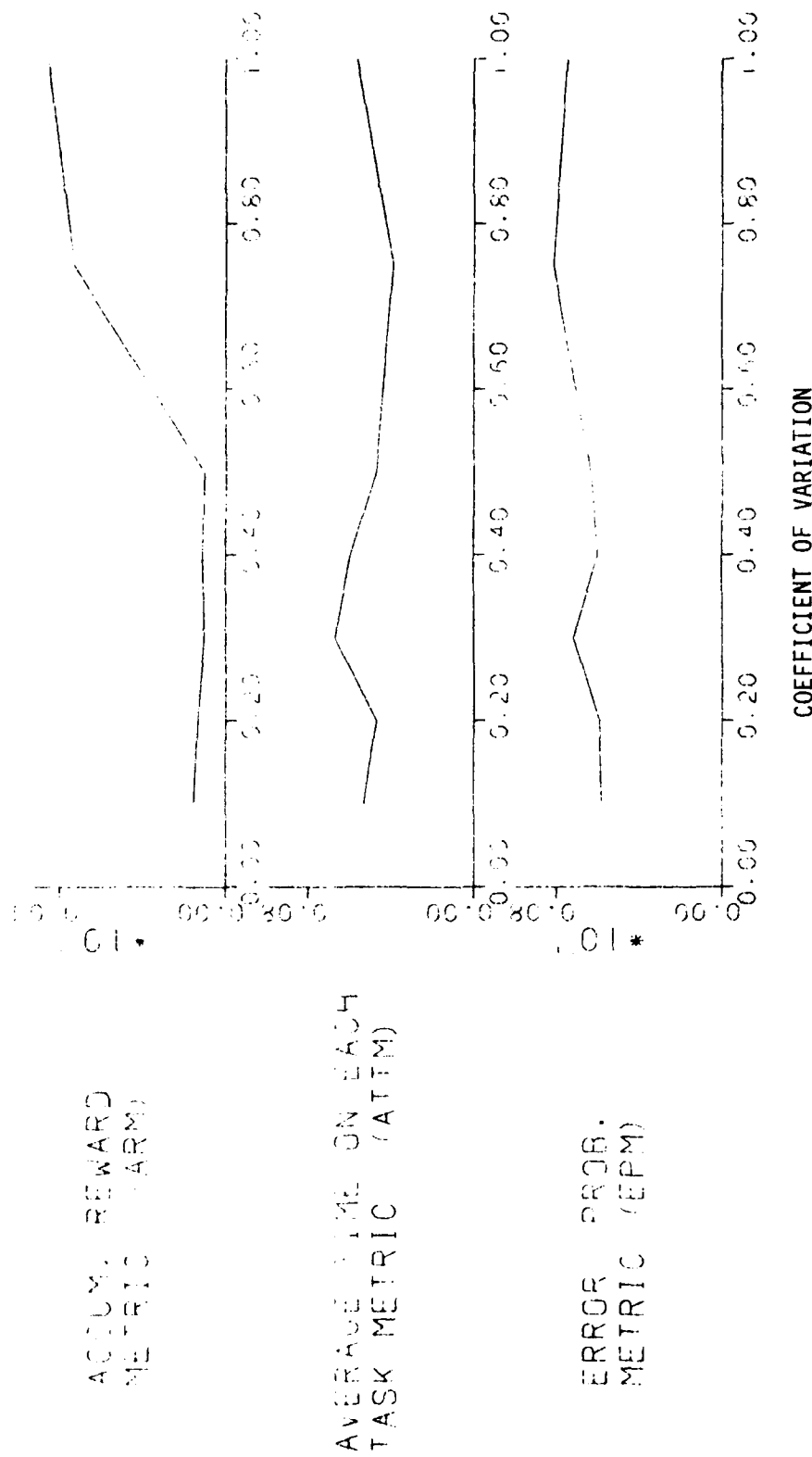
(cont'd)

COEFFICIENT OF VARIATION

Fig. 37: SCALAR MEASURES VERSUS CO-EFFICIENT OF VARIATION FOR EXPERIMENTAL CONDITION B

measure of overall performance of the model, *exhibits good sensitivity* to c when compared to IRM, which is a measure of incremental performance. Overall, the results indicate that a value of c in the range 0.3 ± 0.1 gives a good fit to the experimental data.

(ii) *Variations of time-delay*, τ: As time-delay increases, the uncertainty associated with the estimation of the decision state increases. This, in turn, leads to a smaller number of tasks being completed, and smaller reward being earned. The measures AM and IRM were found to be relatively insensitive (within 10 percent) to time-delay variations in the range 0.15 - 0.50 sec, wheras ARM was quite sensitive to τ. Also, the measures ATTM and EPM exhibited modest increases with time-delay. The overall results indicated that a value of τ in the range 0.20 ± 0.05 sec is the best choice, a range consistent with that employed in the OCM.

(iii) *Variations of discount factor*, α: As α increases, the model allocates attention to tasks with small processing times. This results in a decrease of total reward earned, although the number of tasks (of less value) completed may increase. The measures AM, IRM and ATTM generally increase with α, whereas the overall measure ARM is insensitive (within 10 percent) to variations in the discount factor. Overall, a value of $\alpha \approx 0$ was found to give the best possible match to the data. Therefore, the parameter α was discarded from the model.

(iv) *Variations of observation noise ratio, ρ_i:* The model response was relatively insensitive (within 10%) to observation noise ratio in the range -15 db to -25 db. However, the results showed some perplexing trends. At high values of ρ_i (i.e., less negative), the model earned more reward and completed more number of tasks than at low values of ρ_i. Therefore, the measures IRM and ARM decrease with increases in ρ_i, but the measure AM appears to increase slightly. This apparent anomaly may be due to complex interaction between ρ_i and the co-efficient of variation, c.

(v) *Variations of "fictitious" processing time, T_{RO}:* As T_{RO} increases, the attractiveness measure, $M_0(t)$ becomes more negative. This reduces the "do nothing" probability, $P_{do}(t)$ and results in a non-decreasing total reward. A value of $T_{RO} \approx 3$ to 5 sec was found to be a reasonable choice in the present experimental context.

The above sensitivity results show that the choice of the parameter set Ω is not critical, at least within a reasonable range of variation. However, future research could determine whether or not the parameter set remains constant with modified decision paradigms, such as those suggested in section 4.1.

3.5 Comparison with other Decision Models

Since the decision situation basically involves dynamic sequencing of tasks under uncertainty, a logical question is: "Couldn't we have used one of the many sequencing rules that appear in the scheduling literature [28] to model human decision strategy as effectively as the DDM?" In this section, we answer this question in the negative by comparing the DDM with four heuristic sequencing rules of scheduling

theory. We also contrast DDM with two other decision rules, which may be interpreted as special cases. The results illustrated here are for condition D only, but they are representative of the other conditions as well.

3.5.1 Comparison with Heuristic Sequencing Rules

The following four decision rules were selected for comparison with the DDM:

(i) *Weighted shortest remaining processing time (WSRPT) rule*: At any time t, this rule chooses a task with maximum $[r_i(t)|T_{Ri}(t)]$. Some advantages of WSRPT rule are: (a) it minimizes the weighted completion times as well as the weighted waiting times of tasks being sequenced, and (b) it does not require any look-ahead features, even though tasks become available intermittently, i.e., it is a dispatching decision rule. The major drawbacks of this rule are: (a) it stipulates a (1,0) type of decisions and does not consider randomness in human response; (b) it does not take into account the time available to work on a task, although it does minimize average lateness of tasks (if allowed to work even after deadline has exceeded); (c) It assumes that $T_{Ri}(t)$ is deterministic; and (d) it discriminates among tasks to the greatest possible extent, resulting in increasingly excessive waiting time for low priority tasks. The first two cited limitations of WSRPT are removed in the decision rules (ii) and (iii), respectively.

(ii) *WSRPT with stochastic choice*: This rule is similar to

(i), except that it employs Luce's choice *axiom to render*
the decision rule random as in the DDM.

(iii) *Modified WSRPT*: At any time t, this rule selects a task
with maximum $[r_i(t)|T_{Ri}(t)] \cdot u[T_{ai}(t) - T_{Ri}(t)]$, where $u(\cdot)$
is a unit step function. This rule is similar to (i), but
takes into consideration the time available to work on a
task via a unit step function involving slack time,
$(T_{ai}(t) - T_{Ri}(t))$.

(iv) *Weighted Slack time (WST) rule*: At any time t, this rule
selects a task with maximum $[r_i(t)|(T_{ai}(t) - T_{Ri}(t))]$. This
scheme is often used with WSRPT sequencing to overcome the
limitation (d) of WSRPT rule.

Table 6 compares DDM performance with those of the heuristic se-
quencing rules (i) - (iv) via the scalar measures of similarity for the
experimental condition D. The figures in brackets display the percent
decrement in performance of a heuristic sequencing rule, using measures
for DDM as a base. The results clearly indicate that the performance of
DDM is significantly better than the sequencing rules (i) - (iv). It
should also be noted that WSRPT rule with stochastic choice does better
than a pure WSRPT, thereby confirming randomness in human decision be-
havior, as well as the inadequacy of Monte Carlo models of the type
espoused by Tulga [8]. These results also cast doubt on models that
assume perfect human perception of the task attributes.

3.5.2 Comparison with Related Decision Models

Several decision models that are derivatives of the DDM were
studied; two particularly interesting ones are discussed here.

MODEL / SCALAR MEASURE	ODM	WSRPT $\frac{r_i}{T_{Ri}(t)}$	WSRPT with Stochastic Choice	Modified WSRPT $\frac{r_i}{T_{Ri}(t)}$ u[$T_{ai}(t)-T_{Ri}(t)$]	WST $\frac{r_i}{(T_{ai}(t)-T_{Ri}(t))}$
Action Metric (AM)	.0849	.1431 (68.6%)	.1272 (49.8%)	.1385 (63.1%)	.1419 (67.1%)
Incremental Reward Metric (IRM)	.0815	.1501 (84.2%)	.1381 (69.45%)	.1492 (83.1%)	.2720 (233.7%)
Accumulated Reward	$.0199 \times 10^{-2}$	$.0786 \times 10^{-2}$ (294.9%)	$.0822 \times 10^{-2}$ (3.3%)	$.0221 \times 10^{-1}$ (1010%)	$.0648 \times 10^{-1}$ (3155%)
Averate Time on each Task Metric (ATTM)	.1257	.1940 (54.3%)	.1680 (33.7%)	.2062 (64.0%)	.2101 (67.1%)
Error Probability Metric (EPR)	.0036	.0037 (8.9%)	.0037 (8.9%)	.00544 (60.0%)	.0037 (8.9%)

TABLE 6: COMPARISON OF DDM WITH THE HEURISTIC SEQUENCING RULES

Legend

WSRPT: Weighted shortest remaining processing time

WST: Weighted slack time

(i) *Related model* 1: This model assumes that the subjective looses, $q_i(t)$, are zero. Thus, the attractiveness measures of Eqs. (2.35) and (2.37) become, respectively

$$M_0(t) = 0$$

$$M_i(t) = r_i(t) \, \eta_i(t) \; ; \; i \in A(t)$$

(3.13)

(ii) *Related model* 2: In this model, the attractiveness measures are given by

$$M_0(t) = - \sum_{j \in A(t)} r_j(t) \, P\left\{T_{aj}(t) \leq T_{R0}\right\}$$

$$M_i(t) = r_i(t) \, P\left\{T_{Ri}(t) \leq T_{ai}(t)\right\} - \sum_{j \in A(t)} r_j \, P\left\{T_{aj}(t) \leq T_{Ri}(t)\right\}$$

(3.14)

This model may be derived from Eqs (2.35) and (2.37) by letting all the available times $T_{am}(t) = \infty$, $m \neq j$ while computing ω_{ij}, and setting $T_{aj} = \infty$, $i \neq j$ in evaluating $\eta_i(t)$. The form of the attractiveness measures in Eq (3.14) is similar to those of "information-integration rules" of behavioral decision theory [2]. A notable feature of this model is that it affords simple computation, and does not require any (numerical) approximations in its evaluation.

The results of Table 7 show that the simplified models perform almost as well as the DDM. Model 1 matches the data well with respect to measures AM and IRM, but performs poorly with respect to error probability measure, EPM. In fact, this is what motivated us to include the subjective losses in the attractiveness measures.

MODEL / SCALAR MEASURE	DDM	Related Model 1	Related Model 2
Action metric (AM)	.0849	.0788 (-7.2%)	.0849 (0%)
Incremental Reward Metric (IRM)	.0815	.0881 (8.1%)	.1035 (27%)
Accumulated Reward Metric (ARM)	$.0199 \times 10^{-2}$	$.0331 \times 10^{-2}$ (66.3%)	$.0497 \times 10^{-2}$ (150.8%)
Average time on Each Task	.1257	.1413 (12.4%)	.1301 (3.5%)
Error Probability metric (EPM)	.0036	.0389 (980.5%)	.0051 (41.7%)

Table 7: COMPARISON OF DDM WITH RELATED DECISION RULES

3.6 Summary

This chapter described the results on model-data validation efforts. In order to validate the model, several time-history and scalar measures of similarity between the model predictions and the experimental data were proposed. The model-data validation effort consisted of comparing the time-history metrics, such as the decision probabilities, completion probabilities, incremental reward, accumulated reward and error probability. Validation on the basis of scalar measures consisted of comparing the average time spent on each task, the difference between the incremental and accumulated rewards of the model and data, etc.

When viewed in total, the model-data comparisons for all the cases studied are excellent. This is achieved with a simple, intuitively appealing decision model, using a nominal set of parameters throughout. To be sure, there are some discrepancies, as in decision probability comparisons. However, these mismatches are not major, and can be corrected by minor model modifications. The model predicted trends generally agree with the data.

Sensitivity analysis of the DDM has shown that the choice of the parameter set is not critical, at least within a reasonable range of variation. The performance of DDM was contrasted with those of several heuristic sequencing rules of scheduling theory, as well as some related decision models. The results point to the clear superiority of the DDM in representing human task sequencing performance.

IV. DISCUSSION AND EXTENSIONS

The primary purpose of this research has been to gain an understanding of human information-processing and task selection procedures in dynamic multi-task environments. The approach has been to combine the results of a joint analytic and experimental program into a normative dynamic decision model (DDM) of human task sequencing performance. To this end, a general multi-task paradigm was developed that retains the essential features of human task selection in a manageable, yet manipulative, context. Via this framework, we have studied the effects of various task related variables on the human decision processes. The model that has emerged from this effort could form a small, but significant, step towards human modeling in complex supervisory control systems. In the following, several suggestions for further research are given. These include model refinements, model application to decision-aiding and the modeling of multi-human decision-making in multi-task systems.

4.1 Modifications of the Decision Paradigm

The concept of a decision state is fundamental to our analytic modeling approach. The human's decision strategy depends directly on his estimates of the decision state, once the task values and a performance metric are given. In the present experimental context, the decision state is related to the task state via a simple functional transformation. Also, the tasks are assumed to be independent and the task values are constant as the bar moves across the CRT screen. This simplicity in the experimental paradigm enabled us to develop the DDM by focusing on the underlying

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structural aspects of the human decision-processes, without the attendant task complexities. However, the future tests of DDM should consider more intricate task structures, such as those involving non-stationary task attributes, task dependency and resource constraints. These and other extensions are described below:

(i) *Non-stationary task attributes*: In many realistic situations, the task attributes (e.g., value and velocity) may evolve in time, or they may vary as a function of human's decisions. For example, a AAA gunner who fires at an enemy target may find that the target has changed course and is diving towards the gunner's position. This results in a change of target's value and the time available to engage the target. The present experimental paradigm can be modified to include time-varying task characteristics in a straightforward manner. However, the analytic framework of the model is valid almost in toto for this case.

(ii) *Task dependency*: Since the subsystems are interconnected physically in a complex system, the tasks are, in general, correlated. This correlation may assume the form of precedence relations and/or dependency among the attributes of different tasks. Precedence relations pertain to the existence of technological restrictions on the task sequence, or the partial ordering among the tasks. The precedence relations generally take the form of an assembly tree or a branching tree. A relevant example of such a situation is the problem of multi-RPV control, where some RPVs (e.g., ECM) must be brought over the target area before the others.

This situation can be incorporated into the experimental paradigm by not allowing the subject to engage certain tasks until he has successfully completed their prerequisite tasks. In this case, the analytic modeling of the decision process involves a two step procedure in which sequencing phase is preceeded by a labeling phase that identifies feasible action subsets. Thus, only the set of feasible decisions, $\mathcal{D}(t)$, along with any human limitations (e.g., loss of decision time in the labeling phase), need to be identified. On the other hand, the task correlations due to dependency among the attributes of different tasks can be incorporated in the form of coupled subsystems in a state space formulation. This will undoubtedly increase the computational complexity of the model. Hopefully, only a small number of tasks have such interactions.

(iii) *Resource constraints*: In practice, resources, such as fuel and ammunition, are finite. Since the availability of resources has implications in the human decision-making processes, future research should investigate human decision behavior with resource constraints. In the present experimental paradigm, a displayed resource may be the total time a subject can expend in processing tasks. This research could delineate the nature of differences in human behavior under constrained and unconstrained situations.

(iv) *A Related Paradigm*: Although the present experimental paradigm is well-suited to understand the basic issues of a multitask decision problem, it is far too abstract to be of use in

a specific application. A means to overcome this limitation and, at the same time, be close to the manual control paradigms is to design an experimental situation wherein human interacts simultaneously with several dissimilar dynamic processes. The task characteristics can be manipulated by varying the nature and occurrence of disturbances acting on the processes. This experimental paradigm is ideally suited to study all the issues of a multi-task decision problem, viz., task detection, task sequencing and task implementation. The conceptual framework of the DDM is still valid. The modeling process poses interesting, albeit solvable, control theoretic problems.

4.2 Computer-aided Decision-making

With rapid advances in technology and higher levels of automation, computers are increasingly being used in decision-making situations. If the computer is to be accepted by human as a decision-aid, or if decision-making responsibility is to be allocated between human and computer, then there must exist a symbiotic relationship between the two. Computer-made decisions and/or information displays should be compatible with human processing goals, implying that the computer would require a model of the human! Successful interaction between human and computer could reduce human work load, increase probability of correct decisions and reduce system risk.

The DDM developed in chapter II is used in a covariance propagation mode to predict ensemble or averaged statistics of human response. However, for decision-aiding applications, one needs a Monte Carlo (or sample-path) simulation of the model. The implementation of the sample-path version of the DDM is similar to that of OCM [31]. That is, the

model mimics the human actions, complete with random number generators that reproduce inherent human randomness. The simulation generates time-histories of human decisions in response to any given task arrival pattern. Using a Monte Carlo model, one can study the potential application of computer-aiding at various levels as outlined below:

(i)   *Information-processing mode*: In this mode, the computer, using a model of the human or its own internal model, displays information relevant to decision-making. The displayed infor-mation can be of various types: an assessment of the present and, possibly, future task states ("raw data") or of the decision states ("reduced data"); or the detection of new tasks while human is attending to a task. Note that, in this mode, the computer provides information at the pre-decision level. If this type of aiding is to be effective, the infor-mation must be accessible in real time, and it must reduce memory load of the human.

(ii)   *Decision-prompting mode*: In this mode, the model provides the human with guidelines for making a decision so that he can concentrate on few vital decision alternatives. The comput-erized model may be exercised to rank-order the importance of various decisions via the attractiveness measures. These metrics are used only as prompting information with the DM free to select any of the alternatives. If the model is truly representative of human decision-processes, there should be high correlation between human and computer decisions. Moreover, this mode of aiding may be used to investigate the human's ability to detect decision blunders by the computer, and it may answer the important question: Should a machine, in

order to help or replace us, act like us?

(iii) Decision-sharing mode: In situations where the human poten-
tially encounters more tasks than he can satisfactorily per-
form, allocation of decision-making responsibility between
human and computer may be the best mode of human-computer
interaction. In order that the human-computer interaction be
efficient, the actions of the computer must be transparent to
the human, and the computer should be able to infer what the
human is doing. Thus, a model of the human allows for covert
communication between the human and computer, and reduces the
need for overt communication. Moreover, a model of the human
can be used to predict future courses of action by the human
so that the computer can strive to avoid them. This results
in a reduction of conflicts, a particularly desirable feature
under high work load situations.

4.3 Multiple Human Decision-makers

The study of a multi-task system with multiple decision-makers can
be approached at various levels of complexity. The analytic framework of
the ODM can be extended, at least conceptually, to a centralized decision-
making system in which tasks arrive at a central supervisor who, in turn,
routes them to various subordinate decision-makers. The individual
decision-makers have the responsibility of sequencing tasks in their
respective queues. The overall decision-process involves finding a global
routing strategy for the supervisor and local sequencing strategies for
the individual subordinates, taking into account inherent and interhuman
randomness.

A more realistic and challenging problem is the modeling of multiple
DMs in distributed multi-task systems. Here tasks arrive at each individ-

ual DM. An individual DM has to determine whether to keep an arriving task for himself or send it to someone else; and which task, if any, he should process. Thus, the decision-process requires the specification of a local routing strategy and a local sequencing strategy for each DM. The decision process is affected by the communication, information-pattern at each DM, hierarchical structures, inter-human randomness and variability, to name but a few.

4.4 Summary

In this chapter, we have delineated three logical extensions of the present research. The first relates to exercising the model in more complex multi-task situations such as those involving non-stationary task attributes, task dependency or resource allocation constraints. This research serves to refine and validate the DDM. The second extension seeks to use the model for studying computer-aided technology. In this context, three modes of interaction between the human and computer are identified, viz., the information-processing mode, the decision-prompting mode and the decision-sharing mode. It was concluded that in all modes of operation, computer must have, as a reference, an internal model of the human for effective man-computer interaction. Finally, the third extension relates to developing models suitable for multi-task systems with multiple decision-makers. This research poses problems of immense analytic difficulty, but, if solved, will be extremely useful in understanding the human component of a complex supervisory control system. It is hoped that future contributions will be along these lines.

APPENDIX A

LUCE'S CHOICE AXIOM

The observed inconsistency and uncertainty associated with human decision behavior have led to two classes of probabilititistic choice models. These are the random utility models and the constant utility models. The random utility models (called the "discriminable dispersion models" by psychologists and "probit analysis" by statisticians) assume that the utility, or the value, of each alternative is intrinsically variable at the subjective level, and that the alternative with the highest momentary value is chosen. Thus, in these models, the uncertainty in choice is attributed to the randomness in utility. The constant utility models, on the other hand, assume that the value assigned to each of the alternatives is fixed, but that the choice is a probabilistic function of these values. Here, the randomness in choice is attributed to uncertainty in the decision rule. Although these two types of choice representation are very different in psychological terms, they are somewhat compatible in mathematical terms. This is because some forms of probabilistic choice models can be interpreted as either random or constant utility models [24,25].The random utility models have their origins in the works of Thurstone on psychophysical scaling [31] and later Block and Marschak on probabilistic theories of response [32], whereas the constant utility models have largely been influenced by Luce's choice axiom [24-27].

Luce's choice axiom is a probabilistic formulation of Arrow's [33]

famed "law of irrelevant alternatives". The axiom, in essence, says that our preferences between two alternatives (stimuli) do not change when other alternatives are added to, or discarded from, the overall set of alternatives. The axiom has been invoked implicitly or explicitly, in psychophysical scaling, utility theory, decision theory, stochastic learning theory and in many psychometric models. This is because of its simplicity and the resulting computational attractiveness. In the following, the axiom is formally stated and its implications for developing a stochastic choice model are discussed.

A.1 Notation and Preliminaries

Let $T = \{x, y, z \ldots\}$ denote a finite set of independent alternatives (e.g., x is the minimum value of some random variables associated with a process state transition, x is the maximum attractiveness measure, etc.). We use $A, B, C, \ldots$ to denote the non-empty subsets of T. We let $P_A(x)$ represent the probability of choosing an alternative x when only the subset A of alternatives is offered to the DM. The usual probability axioms are assumed to hold for all A. Clearly, $P_T(x)$ is the probability of selecting x when the entire set T is presented to the DM and

$$P_T(A) = \sum_{x \in A} P_T(x)$$

For brevity, we let $P(x:y)$ denote $P_{\{x,y\}}(x)$, the probability that the DM selects x when asked to choose between x and y. Also, we assume that $P(x:x) = \frac{1}{2}$.

A.2 Choice Axiom

The choice axiom, in essence, states that the removal of some alternatives does not alter the relative probabilities of choice among the remaining alternatives. That is, the presence or absence of an alter-

native is irrelevant to the relative probabilities of choice between *two* other alternatives, although the absolute values of these probabilities will generally be affected. Formally, for all $x \in A \subseteq T$

$$P_A(x) = P_T(x/A) \tag{A.1}$$

whenever the conditional probability exists.

The choice axiom says that the choice from the subset A is independent of what else may have been available. In other words, even when the entire set T is offered to the DM, if we only look at those occasions when the choices are made from the subset A, then the probability of selecting x from A, $P_T(x/A)$, is identical to the probability of selecting x from A, $P_A(x)$, when only the subset A was presented to the DM in the first place.

By the definition of conditional probability,

$$P_T(x/A) = \frac{P_T(x,A)}{P_T(A)} = \frac{P_T(x)}{P_T(A)}$$

Eq. (A.1) can be rewritten as

$$P_T(x) = P_T(A) \cdot P_A(x) \tag{A.2a}$$

or

$$P_A(x) = \frac{P_T(x)}{P_T(A)} \tag{A.2b}$$

Eq. (A.2) provides an alternate interpretation of the choice axiom. It says that the overall probability of choosing an element x from the set T, $P_T(x)$, may be viewed as a multi-stage process. First, the probability of choosing A from T, $P_T(A)$, is estimated, and then the probability of choosing x from A, $P_A(x)$, is computed. Note that the subset A may be subdivided a number of times until a single element x remains. Moreover, the axiom implies that the product $P_T(A) \cdot P_A(x)$ is independent

of the way in which T is partitioned into subsets! Clearly, intuition suggests that the axiom can not be expected to hold in complex interdependent situations. We will discuss the limitations of the axiom later.

Below, we prove some trivial consequences of the choice axiom as a prelude to deriving a stochastic choice model.

Lemma 1

Suppose that the choice axiom holds for all A, $x \in A \subseteq T$.

(i)   If $P_T(x) \neq 0$, then $P_A(x) \neq 0$

(ii)   If $P_T(x) = 0$ and $P_T(A) \neq 0$, then $P_A(x) = 0$

(iii)   If $P_T(y) = 0$ and $y \neq x$, then $P_T(x) = P_{T-\{y\}}(x)$

Proof

(i)   Since $x \in A$, $P_T(x) \neq 0$ implies $P_T(A) \neq 0$.

Thus, $P_A(x) = P_T(x/A) = \dfrac{P_T(x)}{P_T(A)} \neq 0$.

(ii)   Since $P_T(x) = 0$ and $P_T(A) \neq 0$, $P_A(x) = \dfrac{P_T(x)}{P_T(A)} = 0$.

(iii) $P_T(y) = 0$ implies $P_T(T-\{y\}) = 1$

Using this and the fact that $x \neq y$, we have

$$P_T(x/T-\{y\}) = P_T(x)$$

By the choice axiom

$$P_T(x/T-\{y\}) = P_{T-\{y\}}(x) = P_T(x)$$

The result (iii) shows that an alternative that is never chosen may be removed from the set without affecting the choice probabilities. The fact that this process may be repeated in any order, until all the choice probabilities are positive, is guaranteed by (i) and (ii).

A.3 Stochastic Choice Model

Here, we prove that if the choice axiom holds, all the choice pro-

babilities are determined by the pairwise probabilities. In the following, we assume, without loss of generality, that the choice probabilities are positive.

Theorem 1

If for all $x \in T$, $P_T(x) \neq 0$ and if the choice axiom holds for all x and A such that $x \in A \subseteq T$, then

(i) $$\frac{P(x:y)}{p(y:x)} = \frac{P_T(x)}{P_T(y)} = \frac{P_A(x)}{P_A(y)} \qquad (A.3)$$

and

(ii) $$P_T(x) = \left[1 + \sum_{y \in T - \{x\}} \frac{P(y:x)}{P(x:y)} \right]^{-1} \qquad (A.4)$$

Proof

(i)   By the choice axiom

$$P_T(x) = P_{\{x,y\}}(x) \cdot P_T(\{x,y\})$$

$$= P(x:y) \, [P_T(x) + P_T(y)]$$

so

$$P_T(x) \, [1 - P(x:y)] = P(x:y) P_T(y)$$

Noting that $P(y:x) = 1 - P(x:y)$, we have

$$\frac{P(x:y)}{P(y:x)} = \frac{P_T(x)}{P_T(y)}$$

The result can be extended to include any subset $A \subseteq T$ that contains the alternatives x and y. The condition in Eq (A.3) states that the odds of x being chosen over y from any set containing them equals the odds of a binary choice of x over y. This, rather important, consequence of choice axiom is variously referred to as the independence from irrelevant alternatives in economics and Clarke's constant Ratio Rule in psychology, since it was independently proposed by Clarke [34].

(ii)   To prove part (ii), consider the term

$$1 + \sum_{y \in T-\{x\}} \frac{P(y:x)}{P(x:y)} = \frac{P_T(x)}{P_T(x)} + \sum_{y \in T-\{x\}} \frac{P_T(y)}{P_T(x)}$$

$$= \frac{1}{P_T(x)} \sum_{y \in T} P_T(y)$$

$$= \frac{1}{P_T(x)}$$

The required result immediately follows. Eq (A.4) is similar to Eqs.
(2.30), (2.38) and (2.39). Other consequences of the choice axiom, such
as stochastic transitivity and the existence of a ratio scale, may be
found in [24-27].

A.4 Discussion

Luce's choice axiom provides a powerful means to construct a ration-
al, probabilistic theory of individual choice behavior. The empirical
evidence [27] suggests that it works very well in some situations, and
not so well in others. Here, we summarize the advantages and limita-
tions of the axiom, and indicate decision situations where it can pro-
fitably be applied.

The primary advantages of the choice axiom are that it allows for
easy computation of choice probabilities via pairwise comparisons, and
that it provides a simple means to add new alternatives or subtract from
existing ones. The latter also points to a weakness in the axiom in
that the independence of irrelevant alternatives is implausible in
situations where some of the alternatives are similar. This is exem-
plified by the often cited objection of Debreu [35] to the choice
axiom. Suppose, we are choosing among a pony (x), a bicycle (y) and
another bicycle (z). That is, T = {x,y,z}. Assume that all pairwise

choice probabilities equal $\frac{1}{2}$. Since y and z are duplicates of each other, one expects that $P_T(x) = \frac{1}{2}$, while $P_T(y) = P_T(z) = \frac{1}{4}$. Data supports this intuition. However, if choice axiom is assumed to hold, then all trinary choice probabilities equal $\frac{1}{3}$. This example shows that two alternatives (x and y), which are equivalent in one context (i.e., $P(x:y) = \frac{1}{2}$) are not equivalent in another context (i.e., $P_T(x) \neq P_T(y)$), contrary to independence of irrelevant alternatives. Thus, the application of choice axiom should be restricted to situations where the alternatives can be assumed to be distinct and independent, such as those in the present work.

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