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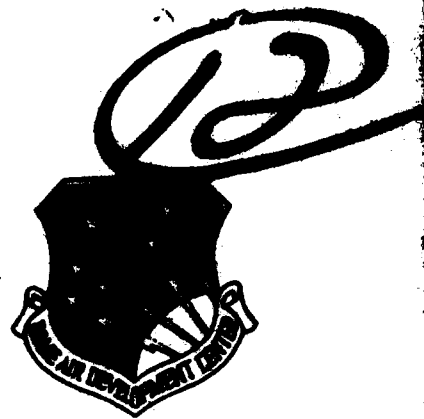
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Interim Report

May 1981

LEVEL II



ANALYSIS AND COMPUTER STUDIES FOR MAGNETOSTATIC SURFACE WAVE TRANSDUCERS

University of Lowell

Jacob I. Weinberg

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RADC-TR-81-96 has been reviewed and is approved for publication.

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19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-81-96 ✓	2. GOVT ACCESSION NO. AD-A102207	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYSIS AND COMPUTER STUDIES FOR MAGNETOSTATIC SURFACE WAVE TRANSDUCERS	5. TYPE OF REPORT & PERIOD COVERED Interim Report.	6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) Jacob I. Weinberg	8. CONTRACT OR GRANT NUMBER(s) F19628-80-C-0029	
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Lowell 450 Aiken Street Lowell MA 01854	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2305J528	
11. CONTROLLING OFFICE NAME AND ADDRESS Deputy for Electronic Technology (RADC/EEAC) Hanscom AFB MA 01731	12. REPORT DATE May 81	13. NUMBER OF PAGES 106
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same	15. SECURITY CLASS. (of this report) Unclassified	15a. DECLASSIFICATION DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) Same		
18. SUPPLEMENTARY NOTES RADC Project Engineer: James C. Sethares (EEAC)		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Magnetostatic Waves Microwave Magnetics Yttrium Iron Garnet Ferrimagnetic Films		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report summarizes results of work on magnetostatic surface wave transducers. Presented are the results for the dispersion relation, radiation resistance and insertion loss for magnetostatic surface wave transducers which may include a gap and apodization. Independent conductors as well as normal modes are considered. Also presented are the results for the dispersion relation for surface waves for a variety of alignments of the externally applied magnetic biasing field. Com-		

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Magnetostatic Surface Wave Transducers

Introduction

The purpose of this report is to summarize the results of the work on magnetostatic surface wave transducers under contract number F 19628-80-C-0029 from the U.S. Air Force ESD RADC EEA at Hanscom AFB, Ma.

To be presented are the results for the dispersion relation, radiation resistance, radiation reactance and insertion loss for magnetostatic surface wave transducers which may include a gap and apodization. Independent conductors as well as normal modes are considered. Also presented are the results for the dispersion relation for surface waves for a variety of alignments of the externally applied magnetic biasing field.

Computerized results including computerized graphs of the results are presented here. Comparisons are made with results obtained from the analysis of the microstrip model which is also here presented.

Basic Theory

The basic theory leading to the dispersion relation, magneto-static wave power, radiation resistance, radiation reactance and insertion loss for surface waves when the applied magnetic biasing field is in the direction of the Z axis (see Figure 1) has been previously detailed [1],[2],[6],[8],[12]. This theory will here be outlined.

We start with Maxwells equations

$$\begin{aligned}\bar{\nabla} \times \bar{H} &= \frac{\partial \bar{D}}{\partial t} & ; & & \bar{\nabla} \cdot \bar{B} &= 0 \\ \bar{\nabla} \times \bar{E} &= - \frac{\partial \bar{B}}{\partial t} & ; & & \bar{\nabla} \cdot \bar{D} &= 0\end{aligned}\tag{1}$$

and the constitutive relations in each of the four regions

$$\begin{aligned}\bar{B} &= \mu_0 (\bar{H} + \bar{M}) \\ \bar{D} &= \epsilon \bar{E}\end{aligned}\tag{2}$$

where $\bar{M}$ is taken as zero in all regions except the YIG region. We utilize the gyromagnetic relation in the YIG region

$$\frac{\partial \bar{M}}{\partial t} = -\gamma \bar{M} \times \bar{H}\tag{3}$$

and retain only first order terms.

We assume the time dependence of all physical quantities to be $e^{j\omega t}$. We also take the magnetostatic approximation

$$\begin{aligned}H_z &= E_x = E_y = 0 \\ \omega \epsilon E_z &= 0\end{aligned}\tag{4}$$

and no variation of any physical quantity in the z direction.

In particular, we obtain

$$\begin{aligned}\frac{\partial E_z}{\partial y} &= -j \omega B_x \\ \frac{\partial E_z}{\partial x} &= +j \omega B_y\end{aligned}\tag{5}$$

and

$$\begin{aligned}B_x &= \mu_0 H_x \\ B_y &= \mu_0 H_y\end{aligned}\tag{6}$$

in all regions except the YIG, while

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \mu_0 \begin{pmatrix} \mu_{11} & -j\mu_{12} \\ j\mu_{21} & \mu_{22} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}\tag{7}$$

in the YIG region where

$$\begin{aligned}\mu_{11} &= \mu_{22} = 1 - \frac{\Omega_H^2}{\Omega^2 - \Omega_H^2} \\ \mu_{21} &= \mu_{12} = \frac{\Omega}{\Omega^2 - \Omega_H^2}\end{aligned}\tag{8}$$

$$\Omega = \frac{f/\gamma}{4\pi M_0}$$

$$\Omega_H = \frac{H_0}{4\pi M_0}$$

$$\gamma = 2.8 \text{ mhz/oe} \quad ; \quad 4\pi M_0 = 1750 \text{ oe}$$

$$f = \omega/2\pi$$

Solutions are sought which satisfy continuity conditions for H_x and B_y at each region junction and satisfy $B_y=0$ at the ground planes. At $y=g$ the condition to be satisfied is that H_x is discontinuous by the surface current density $J(x)$.

We thus assume a solution form of a potential function

$$\psi = F(y) e^{j(\omega t - Kx)} \quad (9)$$

where

$$H_x = \frac{\partial \psi}{\partial x} \quad ; \quad H_y = \frac{\partial \psi}{\partial y} \quad (10)$$

In the non YIG regions we find the form of $F(y)$ to be

$$F(y) = A_i e^{|k|y} + B_i e^{-|k|y} \quad i=1,3,4 \quad (11)$$

while, in the YIG region

$$F(y) = A_2 e^{\beta|k|y} + B_2 e^{-\beta|k|y} \quad (12)$$

where

$$\beta^2 = \mu_{11}/\mu_{22} \quad (13)$$

so that the basic equations (1) - (8) are satisfied. One can see that these solutions consist of waves propagating in the X direction. We carry β along in the analysis even though its value is unity by (13) and (8) because of comparisons to be made later with the analysis for a general direction of the applied biasing field.

The attempt to satisfy the continuity and boundary conditions results first in the requirement to solve^[2]

$$F_T(K) = 0 \quad (14)$$

where

$$F_T(K) = \frac{(\coth|K|t_1 - 1)}{2} [(1 + \alpha_2)e^{-2\beta|K|d} + (1 - \alpha_1)T]e^{-|K|g} - \frac{(\coth|K|t_1 + 1)}{2} [(1 - \alpha_2)e^{-2\beta|K|d} + (1 + \alpha_1)T]e^{|K|g} \quad (15)$$

and

$$\begin{aligned} \alpha_1 &= \mu_{22} \beta + \frac{|K|}{K} \mu_{12} \\ \alpha_2 &= \mu_{22} \beta - \frac{|K|}{K} \mu_{12} \\ T &= \frac{(\alpha_2 + \tanh |K|\ell)}{(\alpha_1 - \tanh |K|\ell)} \end{aligned} \quad (16)$$

Equation (14), a transcendental equation for K as a function of f , is the dispersion relation. Numerical techniques are required for its solution. Two solution curves of K vs. f result; in one solution K is always positive and in the other solution K is always negative. This results in two solution waves which are in opposite directions. Denoting

$$\frac{|K|}{K} = S \quad (17)$$

and the solution values of K by K_S , $S = -1, 1$, we have that the two dispersion relation curves are obtained by solving (14) with (16) for $S = -1$ and $S = 1$.

Equation (14) can also be written as^[8]

$$e^{-2|K|\tau} = \frac{(1-\alpha_2)e^{-2\beta|K|d} + (1+\alpha_1)T}{(1+\alpha_2)e^{-2\beta|K|d} + (1-\alpha_1)T} \quad (18)$$

where

$$\tau = t_1 + g \quad (19)$$

which shows that the effects of material thickness t_1 and g enter the dispersion relation only in combination.

Another useful way of writing the dispersion relation is^[13]

$$e^{-2\beta|K|d} = \frac{(\alpha_1 + \tanh|K|\tau)(\alpha_2 + \tanh|K|l)}{(\alpha_2 - \tanh|K|\tau)(\alpha_1 - \tanh|K|l)} \quad (20)$$

The bandwidth of frequencies for which the solution of (14) can be obtained is given by^[5]

$$\gamma \sqrt{H_0(H_0 + 4\pi M_0)} < f < \gamma (H_0 + 2\pi M_0) \quad (21)$$

Having the dispersion relation curves we can find the group delay

$$V_g = \frac{\partial \omega}{\partial K} \quad (22)$$

for each of the two solution curves.

After equation (14) has been solved we can find all quantities of physical interest for each of the two solutions of (14)^[2]. The magnetostatic wave power is then obtained from^{[2],[3]}

$$P(s) = \frac{1}{2} \int_{-(l+d)}^{\tau} E_z^{(s)} \overline{H_y^{(s)}} dy \quad s=-1,1 \quad (23)$$

where E_z is related to H_y , and H_x in the YIG region, by equations (5) with (6) or (7).

The expression for power is found to be [2]

$$p(s) = \frac{-s \omega \mu_0}{2 K_s^2} A_s G_s^2 \quad s=-1,1 \quad (24)$$

where

$$G_s = \frac{e^{-\beta |K_s| d} |\tilde{J}_1(K_s)|}{\left| \frac{\partial}{\partial K} F_T(K) \right|_{K=K_s}} \quad s=-1,1 \quad (25)$$

$$\begin{aligned} A_s = & \frac{(T_s+1)^2}{\cosh^2 |K_s| \ell} \left(\frac{\sinh 2|K_s| \ell}{4} - \frac{|K_s| \ell}{2} \right) + \frac{(U_s e^{|K_s| g} V_s e^{-|K_s| g})^2}{4 \sinh^2 |K_s| t_1} \\ & \left(\frac{\sinh 2|K_s| t_1}{4} - \frac{|K_s| t_1}{2} \right) \\ & + \frac{1}{4} \left[\frac{U_s^2}{2} (e^{2|K_s| g-1}) - \frac{V_s^2}{2} (e^{-2|K_s| g-1}) - 2 U_s V_s |K_s| g \right] \\ & + \left[\frac{\alpha_1^{(s)}}{2} T_s^2 (e^{2\beta |K_s| d-1}) - \frac{\alpha_2^{(s)}}{2} (e^{-2\beta |K_s| d-1}) - 2 \beta^2 |K_s| T_s d \mu_{22} \right] \quad s=-1,1 \end{aligned} \quad (26)$$

$$\begin{aligned} U_s &= (1-\alpha_2^{(s)}) e^{-\beta |K_s| d} + (1+\alpha_1^{(s)}) T_s e^{\beta |K_s| d} \\ V_s &= (1+\alpha_2^{(s)}) e^{-\beta |K_s| d} + (1-\alpha_1^{(s)}) T_s e^{\beta |K_s| d} \end{aligned} \quad s=-1,1 \quad (27)$$

$$\alpha_1^{(s)} = \alpha_1(K_s), \quad \alpha_2^{(s)} = \alpha_2(K_s), \quad T_s = T(K_s)$$

For independent conductors [4], [12]

$$\tilde{J}_1(K_s) = \sum_{i=1}^N \operatorname{sinc} \frac{a_i K_s}{2\pi} \sqrt{\frac{1}{\epsilon_{1i}}} e^{-j K_s p_i} \quad s=-1,1 \quad (28)$$

For the non-apodized independent conductor case, (28) can be written as [13], with $I_0=1$,

$$\tilde{J}_1(K_S) = I_0 \operatorname{sinc} \frac{a K_S}{2\pi} \frac{1-\eta^N e^{jK_S p N}}{1-\eta e^{jK_S p}} \quad (29)$$

For a truncated array of normal modes we have for the fundamental mode ($n=1$)

$$\tilde{J}_1(K_S) = \sum_{i=1}^N \operatorname{sinc} \frac{2 a_i}{p_i (3-\eta)} \operatorname{sinc} \left[\frac{K_S p_i}{2\pi} - \frac{3+\eta}{4} \right] \eta_i \sqrt{\ell_{1i}} e^{-jK_S p_i i} \quad (30)$$

$s=-1,1$

where ℓ_{1i} , a_i , p_i $i=1,2,\dots,N$ are the conducting strip lengths, conducting strip widths and center to center spacings, respectively, to account for apodization. N is the number of conducting strips and $\eta=-1$ for a meander line and $\eta=1$ for a grating.

The definition

$$\operatorname{sinc} X = \frac{\sin \pi X}{\pi X} \quad (31)$$

is employed in the above.

In the free space case, $t_1=\infty$ and $\ell=\infty$, the dispersion relation is

$$e^{-2\beta|K|d} = \frac{(\alpha_1+1)(\alpha_2+1)}{(\alpha_2-1)(\alpha_1-1)} \quad (32)$$

from (20). The expression for A_s in the power is

$$A_s = \frac{(T_s+1)^2}{2} + \left[\frac{\alpha_1^{(s)}}{2} T_s^2 (e^{2\beta|K_S|d}-1) - \frac{\alpha_2^{(s)}}{2} (e^{-2\beta|K_S|d}-1) - 2\beta^2 T_s \mu_{22} d |K_S| \right] + \frac{V_s^2}{8} \quad (33)$$

The radiation resistance is then obtained from

$$R^{(s)} = \frac{4 |P^{(s)}|}{(1-\eta) + (1+\eta)N^2} \quad s=-1,1 \quad (34)$$

In the above it is typical that the amplitudes for the wave corresponding to $s=-1$ is greater than the amplitudes for the wave corresponding to $s=+1$. Thus the $s=-1$ wave is the stronger of the two and is denoted by the + wave and the $s=1$ wave is denoted by the - wave.

The total radiation resistance is then

$$R_m = R^+ + R^- \quad (35)$$

The radiation reactance contributes meaningfully for surface waves. It is to be obtained from

$$X_m(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{R_m(f')}{f' - f} df' \quad (36)$$

Although this integral contains infinite limits and an apparent singularity it can be computed accurately by numerical techniques [6],[7]. We thus find the radiation reactance from the numerical values of the previously obtained radiation resistance.

The combination of radiation resistance and radiation reactance then results in the complex radiation impedance.

We can now find insertion loss from the radiation resistance, radiation reactance and by including source resistance, conduction loss, matching reactance and propagation loss. We obtain [8]

$$IL(s) = 20 \log_{10} \frac{4R^{(s)} R_g}{(R_g + R_m + R_L)^2 + (X_m + X_L)^2} - \frac{76.4 \Delta H \Delta r}{\partial \omega / \partial K} \quad s=-1,1 \quad (37)$$

where R_g is the source resistance, R_L is the conduction loss and X_L is a series matching reactance. ΔH is a linewidth representing material loss and Δr is a propagation distance.

This completes the basic theory for magnetostatic surface wave transducers.

Microstrip Model

In the microstrip model [9],[11] insertion loss is calculated from input resistance and input reactance of a lossy shorted section of microstrip line and microstrip propagation constants. Apodization is not taken into account in this theory. The conducting strip dimensions ℓ_1 , a and p are thus the same for all strips.

First consider one conducting strip. Here

$$\tilde{J}(K_s) = \text{sinc} \frac{a K_s}{2\pi} \eta \sqrt{\ell_1} e^{-jK_s p} \quad (38)$$

from the independent conductor model, (28).

For N strips we then have, multiplying by an array factor,

$$R(s) = \frac{4|P(s)|}{(1-\eta)+(1+\eta)} \left(\frac{\sin \frac{NK_s p}{2}}{\sin \frac{K_s p}{2}} \right)^2 \quad s=-1,1 \quad (39)$$

for $\eta=1$, and

$$R(s) = \frac{4|P(s)|}{(1-\eta)+(1+\eta)} \left(\frac{\sin \frac{NK_s p}{2}}{\cos \frac{K_s p}{2}} \right)^2 \quad s=-1,1 \quad (40)$$

for $\eta=-1$ and N even where, $P(s)$ is computed for $N=1$ in (39) and (40).

From the above and (35) and (36) we have R^+ , R^- , R_m and X_m .

To obtain insertion loss we first define

$$\begin{aligned}\overline{R^{(s)}} &= \frac{R^{(s)}}{\ell_{1/2}} & s=-1,1 \\ \overline{R_m} &= \frac{R_m}{\ell_{1/2}} \\ \overline{X_m} &= \frac{X_m}{\ell_{1/2}}\end{aligned}\tag{41}$$

Given characteristic impedance Z_{c1} , propagation constant $\overline{\beta}_c$, conduction loss constant α_{cK} and conductivity σ , we have, for $N=1$

$$\begin{aligned}\overline{\beta}_1 &= \overline{\beta}_c f \\ \overline{\alpha}_R &= \frac{\overline{R_m}}{2Z_{c1}} \\ \overline{\alpha}_c &= \alpha_{cK} \sqrt{f/\sigma} / Z_{c1} a\end{aligned}\tag{42}$$

where $\overline{\alpha}_R$ and $\overline{\alpha}_c$ represent radiation attenuation loss and conduction attenuation loss, respectively.

For one or more conducting strips, $N \geq 1$, we now have

$$\begin{aligned}Z_c &= Z_{c1}/N \\ \alpha_R &= \overline{\alpha}_R/N & \eta=1 \\ \overline{\beta} &= \overline{\beta}_1 \\ \alpha_c &= \overline{\alpha}_c\end{aligned}\tag{43}$$

and

$$Z_c = Z_{c1}$$

$$\alpha_R = \bar{\alpha}_R / N \quad \eta = -1 \text{ and } N \text{ even} \quad (44)$$

$$\bar{\beta} = \bar{\beta}_1 N$$

$$\alpha_c = \bar{\alpha}_c N$$

Total attenuation loss is then

$$\alpha = \alpha_R + \alpha_c \quad (45)$$

Then compute

$$\begin{aligned} R_{in} &= \frac{Z_c \tanh 2 \alpha \ell_1}{1 + \cos 2 \bar{\beta} \ell_1 \operatorname{sech} 2 \alpha \ell_1} \\ X_{in} &= \frac{Z_c \sin 2 \bar{\beta} \ell_1 \operatorname{sech} 2 \alpha \ell_1}{1 + \cos 2 \bar{\beta} \ell_1 \operatorname{sech} 2 \alpha \ell_1} \end{aligned} \quad (46)$$

$$Z_{in} = \sqrt{R_{in}^2 + X_{in}^2}$$

where R_{in} , X_{in} and Z_{in} are input resistance, input reactance and the magnitude of the input impedance, respectively. With

$$R_{i,m}^{(s)} = R_{in} \frac{\overline{R^{(s)}} / Z_c}{\alpha_c + \bar{R}_m / Z_c} \quad s = -1, 1 \quad (47)$$

$$R_{i,m} = R_{i,m}^+ + R_{i,m}^-$$

insertion loss can be expressed as

$$IL^{(s)} = 20 \log_{10} \frac{4 R_i R_{i,m}^{(s)}}{(R_i + R_{in})^2 + X_{in}^2} - \frac{76.4}{\partial \omega / \partial K} \frac{\Delta H}{\Delta r} \quad s = -1, 1 \quad (48)$$

where R_i is the source impedance.

Insertion loss can also be written as

$$IL^{(s)} = 20 \log_{10} \frac{4 R_i R_{i,m}^{(s)}}{(R_i + R_c + R_{i,m})^2 + (X_{i,m} + X_\ell)^2} - \frac{76.4 \Delta H \Delta r}{\partial \omega / \partial K} \quad s = -1, 1 \quad (49)$$

where

$$\begin{aligned} R_c &= R_{in} \frac{\alpha_c}{\alpha_c + \bar{R}_m / Z_c} \\ X_\ell &= X_{in} \frac{\bar{\beta}}{\bar{\beta} + \bar{X}_m / Z_c} \\ X_{i,m} &= X_{in} \frac{\bar{X}_m / Z_c}{\bar{\beta} + \bar{X}_m / Z_c} \end{aligned} \quad (50)$$

This completes the theory for the microstrip model.

Complex Impedance-Free Space Case

We here indicate the computation of the magnetic wave power P when all physical quantities are determined by combining the two solutions present. We shall only consider the free space case ($\ell = t_1 = \infty$) and the case of no gap present ($g = 0$). Since $\beta = 1$ in the basic theory, we will eliminate it from the equations.

We write

$$\begin{aligned} \alpha_1^{(s)} &= \mu_{22} + S \mu_{12} \\ \alpha_2^{(s)} &= \mu_{22} - S \mu_{12} \quad s = -1, 1 \\ T_S &= \frac{\alpha_2^{(s)} + 1}{\alpha_1^{(s)} - 1} \end{aligned} \quad (51)$$

and define

$$\begin{aligned} a_s &= \frac{1}{T_s} \\ \bar{G}_s &= \frac{G_s}{a_s} \end{aligned} \quad (52)$$

The magnetic wave power defined as

$$P = \frac{1}{2} \int_{-\infty}^{\infty} E_z \bar{H}_y dy \quad (53)$$

is now

$$P = \frac{1}{2} \left[\int_{-\infty}^{-d} E_{z1} \bar{H}_{y1} dy + \int_{-d}^0 E_{z2} \bar{H}_{y2} dy + \int_0^{\infty} E_{z4} \bar{H}_{y4} dy \right] \quad (54)$$

Considering both solutions we have

$$\begin{aligned} P = \frac{1}{2} & \left[\int_{-\infty}^d (E_{z1}^{(-1)} + E_{z1}^{(1)}) (\bar{H}_{y1}^{(-1)} + \bar{H}_{y1}^{(1)}) dy + \int_{-d}^0 (E_{z2}^{(-1)} + E_{z2}^{(1)}) (\bar{H}_{y2}^{(-1)} + \bar{H}_{y2}^{(1)}) dy + \right. \\ & \left. \int_0^{\infty} (E_{z4}^{(-1)} + E_{z4}^{(1)}) (\bar{H}_{y4}^{(-1)} + \bar{H}_{y4}^{(1)}) dy \right] \end{aligned} \quad (55)$$

Employing (5) with (6) or (7) together with (9)-(12) indicates the form of (55) is

$$\begin{aligned} P = \frac{1}{2} & \left\{ \int_{-\infty}^{-d} \left[E_{-1}(y) e^{-jK_{-1}x} + E_1(y) e^{-jK_1x} \right] \left[H_{y-1}(y) e^{jK_{-1}x} + H_{y1}(y) e^{jK_1x} \right] dy \right. \\ & + \int_{-d}^0 \left[E_{-2}(y) e^{-jK_{-1}x} + E_2(y) e^{-jK_1x} \right] \left[H_{y-2}(y) e^{jK_{-1}x} + H_{y2}(y) e^{jK_1x} \right] dy \\ & \left. + \int_0^{\infty} \left[E_{-4}(y) e^{-jK_{-1}x} + E_4(y) e^{-jK_1x} \right] \left[H_{y-4}(y) e^{jK_{-1}x} + H_{y4}(y) e^{jK_1x} \right] dy \right\} \end{aligned} \quad (56)$$

and, upon insertion of the appropriate solution functions in (55) and the performance of the indicated integrations we obtain the result

$$P = \frac{-\omega\mu_0}{2} \left\{ M_9 + M_8 \cos (K_1 - K_{-1})x + j M_7 \sin (K_1 - K_{-1})x \right\} \quad (57)$$

where

$$M_9 = \sum_{s=-1}^1 \frac{s \bar{G}_s^2}{2 K_s^2} \left[(1+a_s)^2 + \alpha_1^{(s)} (e^{2|K_s|d_{-1}})^{-\alpha_2^{(s)}} (e^{-2|K_s|d_{-1}}) a_s^2 \right. \\ \left. - 2|K_s| a_s (\alpha_1^{(s)} + \alpha_2^{(s)})d + (a_s \alpha_2^{(s)} e^{|K_s|d_{-1}})^{-\alpha_1^{(s)}} e^{|K_s|d} \right] \quad (58)$$

$$M_8 = \frac{\bar{G}_1 \bar{G}_{-1}}{-K_1 K_{-1}} \left\{ \frac{(K_1 + K_{-1})}{(K_1 - K_{-1})} (1+a_{-1})(1+a_1) + \alpha_2^{(1)} a_1 (e^{-(K_1 + K_{-1})d_{-1}})^{-\alpha_1^{(1)}} a_{-1} (e^{(K_1 + K_{-1})d_{-1}}) \right. \\ \left. + \frac{(e^{(K_1 - K_{-1})d_{-1}})^{(\alpha_1^{(-1)})} K_1 + \alpha_1^{(1)} K_{-1} - a_{-1} a_1 (e^{-(K_1 - K_{-1})d_{-1}})^{(\alpha_2^{(-1)})} K_1 + \alpha_2^{(1)} K_{-1}}{(K_1 - K_{-1})} \right. \\ \left. + \frac{(K_1 + K_{-1}) (a_{-1} \alpha_2^{(-1)} e^{K_{-1}d_{-1}})^{(\alpha_1^{(-1)})} e^{-K_{-1}d} (a_1 \alpha_2^{(1)} e^{-K_1d_{-1}})^{(\alpha_1^{(1)})} e^{K_1d}}{(K_1 - K_{-1})} \right\}$$

$$M_7 = \frac{\bar{G}_1 \bar{G}_{-1}}{-K_1 K_{-1}} \left\{ (1+a_{-1})(1+a_1) + (a_{-1} \alpha_2^{(-1)} e^{K_{-1}d_{-1}})^{(\alpha_1^{(-1)})} e^{-K_{-1}d} (a_1 \alpha_2^{(1)} e^{-K_1d_{-1}})^{(\alpha_1^{(1)})} e^{K_1d} \right. \\ \left. + \frac{(K_1 - K_{-1})}{(K_1 + K_{-1})} \left[\alpha_2^{(1)} a_1 (e^{-(K_1 + K_{-1})d_{-1}})^{-\alpha_1^{(1)}} a_{-1} (e^{(K_1 + K_{-1})d_{-1}}) \right] \right. \\ \left. + \frac{(e^{(K_1 - K_{-1})d_{-1}})^{(\alpha_1^{(-1)})} K_1 - \alpha_1^{(1)} K_{-1} - a_{-1} a_1 (e^{-(K_1 - K_{-1})d_{-1}})^{(\alpha_2^{(-1)})} K_1 - \alpha_2^{(1)} K_{-1}}{(K_1 - K_{-1})} \right\} \quad (60)$$

Note that (57) is of the form

$$P = P_R + j P_I \quad (61)$$

where

$$\begin{aligned} P_R &= \frac{-\omega\mu_0}{2} \left[M_9 + M_8 \cos (K_1 - K_{-1})x \right] \\ P_I &= \frac{-\omega\mu_0}{2} M_7 \sin (K_1 - K_{-1})x \end{aligned} \quad (62)$$

The results obtained reduce to that obtained earlier in (24), (25) and (33) when the two solutions are considered separately and (50) and the dispersion relation (31) is utilized. In this case the only terms present in (57) are the two terms in M_9 , one for each value of S , from (58).

In general, the complex impedance is taken as

$$Z = \frac{4 P}{(1-\eta) + (1+\eta)N^2} \quad (63)$$

similar to (34) and has real and imaginary parts. The spatial average of this generalized impedance gives the radiation resistance, while the spatially dependent part gives rise to resistance and reactance terms related to the width of the transducer in the x direction. These terms are assumed to be of second order and have not been incorporated into the present model.

Generalized Dispersion Relation

In this section we obtain the dispersion relation for surface waves when the biasing field is not restricted to be parallel to the Z axis (see Figure 1).

In the YIG region the components of the permeability tensor (7) are now given by [10]

$$\begin{aligned}
\mu_{11} &= 1 + \frac{\gamma^2 H_0 (4\pi M_0) (\sin^2 \theta \sin^2 \phi + \cos^2 \theta)}{\gamma^2 H^2 - f^2} \\
\mu_{22} &= 1 + \frac{\gamma^2 H_0 (4\pi M_0) \sin^2 \theta}{\gamma^2 H^2 - f^2} \\
-j\mu_{12} &= \frac{j \gamma (4\pi M_0) \sin \theta (f \sin \theta + j\gamma H_0 \cos \phi \cos \theta)}{\gamma^2 H^2 - f^2} \\
j\mu_{21} &= \frac{-j \gamma (4\pi M_0) \sin \theta (f \sin \theta - j\gamma H_0 \cos \phi \cos \theta)}{\gamma^2 H^2 - f^2}
\end{aligned} \tag{64}$$

These relations reduce to those given by (8) for the case of the biasing field lying along the z axis; $\theta=90^\circ$ and $\phi=90^\circ$.

The satisfaction of (1)-(3) yields solutions as in (8), the expression in the YIG region being modified to

$$F(y) = e^{-jKby} (A_2 e^{\beta|K|y} + B_2 e^{-\beta|K|y}) \tag{65}$$

instead of (12). Here

$$\beta^2 = \frac{(\mu_{21} - \mu_{12})^2 + 4\mu_{11} \mu_{22}}{4\mu_{22}^2} > 0 \tag{66}$$

instead of (13), and

$$b = \frac{-j(\mu_{21} - \mu_{12})}{2\mu_{22}} \tag{67}$$

We note that b is real and that the term containing b indicates that there is an additional propagation component in the y direction. For the standard surface wave case of $\theta=90^\circ$ and $\phi=90^\circ$ we note that (65) and (66) reduce to (12) and (13) with $b=0$.

The dispersion relation is obtained by satisfying the boundary and continuity conditions as before. With

$$\begin{aligned}\alpha_1 &= \beta \mu_{22} - j \frac{|K|}{K} (j \mu_{21} + b \mu_{22}) \\ \alpha_2 &= \beta \mu_{22} + j \frac{|K|}{K} (j \mu_{21} + b \mu_{22})\end{aligned}\tag{68}$$

the dispersion relation, for the case of $g=0$, is

$$e^{2\beta|K|d} = \frac{(\alpha_2 - \tanh |K|t_1)(\alpha_1 - \tanh |K|\ell)}{(\alpha_2 + \tanh |K|\ell)(\alpha_1 + \tanh |K|t_1)}\tag{69}$$

Again, for the case of $\theta=90^\circ$ and $\phi=90^\circ$ equations (68) coincide with (16) and (69) is then the same as (20).

Thus (69) with (68), (67), (66) and (64) give the dispersion relation for surface waves with the orientation of the biasing field kept arbitrary.

COMPUTER PROGRAMS

A. Basic Theory

A computer program which incorporates the results of the basic theory has been made operational on the CDC 6600 at Hanscom AFB, Ma. The program produces plots of the various physical quantities as functions of frequency. There are plots of wave number, group delay, radiation resistance and insertion loss for each of the two solution waves. There are also plots of the normalized dispersion for the + wave, total radiation resistance and the corresponding total radiation reactance. The program also provides for print out of these quantities.

The program is designed for flexibility in that independent conductors as well as a truncated infinite array of normal modes can be accommodated. The case of uniform conducting strips can be handled as well as apodization in strip length, strip width and/or center to center spacing. In addition, the program automatically computes the relevant frequency range by utilizing (21).

There now follows a detailed description of the input cards to the program which shows how to use the features described above.

Card 1 - H_0 , t_1 , d , g , ℓ , N , n

These seven quantities are here supplied, separated by commas. All lengths are in meters. Columns 1-72 may be used.

Card 2 - first ℓ_1 , $\Delta\ell_1$, ℓ_1 option

Card 3 - first a , Δa , a option

Card 4 - first p , Δp , p option

Each of the above three cards, applying to λ_1 , a and p , respectively, contain three items, separated by commas. The first item is the dimension of the first strip. If the third item (option) is 0 then the dimensions of the $(N-1)$ strips following the first are successively incremented by the increment (Δ) of the second item. If the third item is 1 then the dimensions of the first $\frac{(N-1)}{2}$ strips following the first strip are successively incremented by the increment of the second item and the dimensions of the next $\frac{(N-1)}{2}$ strips are successively decremented by the same value. N must be odd when the option is 1. Note that one may use a negative number for the increment of item 2. Also note that the quantities λ_1 , a and p are handled entirely independent of each other. If an increment value is 0 then there is no apodization in the corresponding quantity and the option is immaterial.

Card 5 - $\Delta H, \Delta r$

These two quantities, separated by a comma, are here supplied.

Card 6 - heading

Only columns 1-20 are used for this card. For the normal modes case, the first ten columns on this card should contain NORM_MODE_. For independent conductors, the first ten columns should contain IND__COND_. This card serves as the top heading line on the plots as well as to signify the program whether the case is one of normal modes or uniform conductors.

Card 7 - heading

Card 8 - heading

Card 9 - heading

These three additional heading cards are required and will appear in order on the computer plots under the heading of card 6. Columns 1-70 may be used.

There follows a listing of the entire computer program as it is used on the CDC 6600 at Hanscom AFB, Ma. Omitted are the required control cards consisting of the standard job card, Fortran compile and execute cards and the standard control cards for plots.

```

PROGRAM ROCT (INFUT,CLTPUT)
  DIMENSION PROGID(3)
  DIMENSION F(1200),FM(1200),FP(1200),CAP(1200),CAM(1200),VGM(1200),
  YPP(1200),PM(1200),RP(1200),RM(1200),RT(1200),PX(1200),SERP(1200),
  XSERM(1200),VNM(1200),VGP(1200)
  DIMENSION HEAC(2),HEAC1(7),HEAD2(7),HEAD3(7)
  DIMENSION FN(50),VM(50)
  COMMON FL,AL1,AL2,B,C,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
  X,LMODE
  READ *,H,T1,C,G,EL,FN,ETA
  READ *,ELBEGN,ELDEL,ELOPT
  READ *,AREGIN,ADEL,ACFT
  READ *,PREGIN,PDEL,POPT
  READ *,DELH,DIST
  READ 102,HEAD
  READ 100,HEAD1
  READ 101,HEAD2
  READ 101,HEAD3
  LMODE=1
  IF (HEAD(1) .EQ. "NORM MODE ") LMODE=2
  N=FN
  PL=30.
  PL=75.
  DO 41 I=1,N
    FL1(I)=ELBEGN+(I-1)*ELDEL
    AA(I)=AREGIN+(I-1)*ADEL
41  PE(I)=PREGIN+(I-1)*PDEL
    NEL=(N+1)/2
    IF (ELOPT .EQ. 0.) GO TO 42
    DO 43 I=NEL,N
43  FL1(I)=FL1(NEL)-(I-NEL)*ELDEL
42  IF (AOPT .EQ. 0.) GO TO 44
    DO 45 I=NEL,N
45  AA(I)=AA(NEL)-(I-NEL)*ADEL
44  IF (PCPT .EQ. 0.) GO TO 46
    DO 47 I=NEL,N
47  PE(I)=PE(NEL)-(I-NEL)*PDEL
46  CONTINUE
    FLO=2.8*SQRT(F*(H+1750.))
    FHI=2.8*(H+75.)
    FDEL=1.
    FREG=AINI(FLO)+1.
    NF=INT(FHI)-INT(FLO)-1
    DO 40 I=1,NF
40  F(I)=FREG+(I-1)*FDEL
    IF (F(1) .LT. FLO) PRINT *,"FREQUENCIES TOO LOW"
    IF (F(NF) .GT. FHI) PRINT *,"FREQUENCIES TOO HIGH"
    PRINT 60
    FACT=1.
    IF (ETA .GT. -2.) GO TO 4
    ETA=-1.
    FACT=(2./ENI)**2
    PRINT *," PI GRATING CASE"
4  CONTINUE
    PRINT 61,H,T1,D,G,EL ,EN,ETA,NF
    PRINT *," DELTA H = ",DELH," DISTANCE =",DIST
    PRINT *," LOSS IS ",RL

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PRINT 81, (I, EL1(I), I=1, N)
PRINT 82, (I, AA(I), I=1, N)
PRINT 83, (I, PE(I), I=1, N)
PPRINT 60
IF (ETA .EQ. -1.) ELL = .87*.4E-8*EN
IF (ETA .EQ. 1.) ELL = .23*.4E-8/EN
ELL=0.
DATA PROGID/8HSETHAPES,4H3724,10HJ.WEINBERG/
CALL FLTID3(PROGID,200.,12.,1.)
P=J.
A=J.
AY=0.
PI=3.141592654
PG=50.
AYT=.5*AY*((1.-ETA)+EN*(1.+ETA))
UO=4.*PI*1.E-7
K=1
I=1
J=1
OMH=H/1750.
50 EF=F(K)
OM=EF/(2.*1750.)
U11=1.-OMH/(OM**2-OMH**2)
U22=U11
U12=OM/(OM**2-OMH**2)
R=SQRT(U11/U22)
L=1
30 IF (L .EQ. 2) GO TO 2
1 S=1.
GO TO 3
2 S=-1.
3 CONTINUE
AL1=U22*B+S*U12
AL2=U22*B-S*U12
IF (J .GT. 1) GO TO 53
CAO=.5*SQRT(U22/U11)*ALOG(1.+4.*SQRT(U11*U22)/
*(U12**2-(SQRT(U11*U22)+1.)**2))
CAO=CAO/D
53 CONTINUE
IF (I .EQ. 1) GO TO 51
IF (J .EQ. 1) GO TO 51
IF (L .EQ. 1) CAO=CAP(I-1)
IF (L .EQ. 2) CAO=CAM(J-1)
51 M=1
5 DEL=.02*CAO
CAOP=CAO+DEL
CAQM=CAO-DEL
CAQD=CAO*D
CAOD=ABS(CAOD)
CAOG=ABS(CAOG)
IF (CAOD .GT. 650.) GO TO 35
IF (CAOG .GT. 650.) GO TO 35
FTCO=FT(CAO)
FTCP=FT(CAOP)
FTCM=FT(CAQM)
CA1=CAO-2.*DEL*FTCO/(FTCP-FTCM)
IF (ABS(CA1) .GT. 1.E7) GO TO 35

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CA1D=CA1*D
CA1D=ABS(CA1D)
CA1G=ABS(CA1*G)
IF(CA1D .GT. 650.) GO TO 35
IF(CA1G .GT. 650.) GO TO 35
FTC1=FT(CA1)
IF (ABS((CA1-CA0)/CA0) .LT. .001) GO TO 10
CA0=CA1
M=M+1
IF(M .GT. 10) GO TO 35
GO TO 5
10 IF (L .EQ. 2) GO TO 20
CA=CA1
IF (ABS(FTC1) .GT. 1.) GO TO 35
IF (CA .LT. 0.) GO TO 35
FP(I)=FF
CAP(I)=CA
I=I+1
L=2
GO TO 30
20 CA=CA1
IF (ABS(FTC1) .GT. 1.) GO TO 35
IF (CA .LT. 0.) GO TO 35
FM(J)=EF
CAM(J)=CA
J=J+1
IF (J .EQ. 1) GO TO 15
IF (J .GT. 1) GO TO 31
I=J-1
K=K-1
31 J=J-1
15 K=K+1
IF (K .LE. NF) GO TO 50
PRINT 60
I1=I-1
J1=J-1
GO TO 24
35 PRINT *, "ITERATION DOES NOT CONVERGE. F= ", EF, " S= ", S
IF (L .EQ. 2) GO TO 15
L=2
GO TO 2
24 CONTINUE
PRINT 63, (FP(I), CAP(I), I=1, I1, 10)
PRINT 64, (FM(J), CAM(J), J=1, J1, 10)
PRINT 65
DO 22 J=1, J1
IF (J .EQ. 1) VGM(J)=5./PI*(CAM(2)-CAM(1))/FDEL
IF (J .EQ. J1) VGM(J)=5./PI*(CAM(J1)-CAM(J1-1))/FDEL
IF (J .NE. 1 .AND. J .NE. J1) VGM(J)=
X 5./PI*(CAM(J+1)-CAM(J-1))/FDEL*.5
22 CONTINUE
PRINT 85, (FM(J), VGM(J), J=1, J1, 10)
PRINT 60
DO 21 I=1, I1
IF (I .EQ. 1) VGP(I)=5./PI*(CAP(2)-CAP(1))/FDEL
IF (I .EQ. I1) VGP(I)=5./PI*(CAP(I1)-CAP(I1-1))/FDEL
IF (I .NE. 1 .AND. I .NE. I1) VGP(I)=

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X5./PI*(CAP(I+1)-CAP(I-1))/FDEL*.5
21 CONTINUE
PRINT 87, (FP(I), VGP(I), I=1, I1, 10)
J0=J1/2
CAM0=CAM(J0)
DO 23 J=1, J1
CAMN=CAM0-CAM(J)
CAMN=CIST*CAMN/PI
CAMN=AMOD(CAMN, 2.)
IF (CAMN .GT. 1.) CAMN=CAMN-2.
IF (CAMN .LT. -1.) CAMN=CAMN+2.
VNM(J)=180.*CAMN
23 CONTINUE
PRINT 60
PRINT 86, (FM(J), VNM(J), J=1, J1, 10)
S=1.
DO 25 I=1, T1
FF=FP(I)
CA=CAP(I)
O=2.*PI*FF*1.E6
OM=FF/(2.*1750.)
U11=1.-OMH/(OM**2-OMH**2)
U22=U11
U12=OM/(OM**2-OMH**2)
R=SQRT(U11/U22)
AL1=U22*R+S*U12
AL2=U22*R-S*U12
P1=(T(CA)+1.)**2*(TANH(CA*EL)-CA*EL*(SECH(CA*EL))**2)
P2=(R1(CA)*EXP(CA*G)-R2(CA)*EXP(-CA*G))**2/4.
P3=(COTH(CA*T1)-CA*T1*(CSCH(CA*T1))**2)
P3=.25*(P1(CA)**2)*(EXP(2.*CA*G)-1.)-.25*(R2(CA)**2)
X*(EXP(-2.*CA*G)-1.)-R1(CA)*R2(CA)*CA*G
P4=AL1*(T(CA)**2)*(EXP(2.*R*CA*D)-1.)-AL2*(EXP(-2.*R*CA*D)-1.)
X-4.*P**2+U22*CA*D*T(CA)
GECA=GE(CA)
OP(I)= OM*U0 GECA **2*(P1+P2+P3+P4)/5./CA**2*.5
PP(I)=4.*PP(T)
PP(I)=ARS(FP(I))/((1.-ETA)+(1.+ETA)*EN**2)*4.
RP(I)=FACT*PP(I)
25 CONTINUE
S=-1.
DO 26 J=1, J1
FF=FM(J)
O=2.*PI*FF*1.E6
CA=CAM(J)
OM=FF/(2.*1750.)
U11=1.-OMH/(OM**2-OMH**2)
U22=U11
U12=OM/(OM**2-OMH**2)
R=SQRT(U11/U22)
AL1=U22*R+S*U12
AL2=U22*R-S*U12
P1=(T(CA)+1.)**2*(TANH(CA*EL)-CA*EL*(SECH(CA*EL))**2)
P2=(R1(CA)*EXP(CA*G)-R2(CA)*EXP(-CA*G))**2/4.
Y*(COTH(CA*T1)-CA*T1*(CSCH(CA*T1))**2)
P3=.25*(P1(CA)**2)*(EXP(2.*CA*G)-1.)-.25*(R2(CA)**2)
Y*(EXP(-2.*CA*G)-1.)-R1(CA)*R2(CA)*CA*G

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      P4=AL1*(T(CA)**2)*(EXP(2.*B*CA*D)-1.)-AL2*(EXP(-2.*E*CA*D)-1.)
      X=-4.*B**2*U22*CA*D*T(CA)
      GECA=GE(CA)
      PM(J)= 0*U0* GECA **2*(P1+P2+P3+P4)/./CA**2 .5
      PM(J)=4.*PM(J)
      RM(J)=ABS(PM(J))/((1.-ETA)+(1.+ETA)*EN**2)*4.
      PM(J)=FACT*PM(J)
26  CONTINUE
      PRINT 60
      PRINT 71,(FP(I),RP(I),I=1,I1,10)
      PRINT 72,(FM(J),RM(J),J=1,J1,5)
      IF (I1 .NE. J1) GO TO 90
      DO 84 I=1,I1
84  RT(I)=RP(I)+RM(I)
      PRINT 60
      PRINT 75,(FP(I),RT(I),I=1,I1,10)
      FP1=FP(1)
      FPL=FP(I1)
      CALL FTRAN(PT,PX,I1,FP1,FPL)
      PRINT 60
      M=NM
      M=I1-1
      PRINT 73,(FP(I),PX(I),I=1,M ,20)
      DO 54 I=1,M
      XL=2.*PI*FP(I)*ELL
      XL=XL*1.E6
      L=I
      IF (I .EQ. 1) L=2
      SERP(I)= 20.*ALOG10((4.*RP(I)/RG)/
      X ((1.+(RT(I)+RL)/RG)**2
      X +((PX(I)+XL)/RG)**2))
      SERP(I)=SERP(I)-76.4*DELH*DIST/
      X(2.*PI*1.E6*(FP(I+1)-FP(L-1))/(CAP(I+1)-CAP(L-1)))
      SERM(I)= 20.*ALOG10((4.*RM(I)/RG)/
      X ((1.+(RT(I)+RL)/RG)**2
      X +((PX(I)+XL)/RG)**2))
      SERM(I)=SERM(I)-76.4*DELH*DIST/
      X(2.*PI*1.E6*(FM(I+1)-FM(L-1))/(CAM(I+1)-CAM(L-1)))
54  CONTINUE
      PRINT 60
      PRINT 77,(FP(I),SERP(I),I=1,M ,10)
      PRINT 60
      PRINT 78,(FP(I),SERM(I),I=1,M ,10)
      YMIN=3500.
      DX=50.
      YMIN=2500.
      XMIN=2400.
      FRECO=.01*FREG
      XMIN=AIN(T(FRECO)*100.
      YX=AIN(T(.01*FHI))-AIN(T(.01*FLO))+1.
      CX=100.
      YMIN=0.
      DY=200.
      DY=10000.
      DO 86 J=1,J1
      IF (CAM(J) .GT. 100000.) CAM(J)=100000.
      IF (VGM(J) .GT. 1000.) VGM(J)=1000.

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96  CONTINUE
    DO 97 I=1,I1
      IF(CAF(I) .GT. 100000.) CAF(I)=100000.
      IF (VGP(I) .GT. 1000.) VGP(I)=1000.
97  CONTINUE
    CALL PLOT(15.0.,-3)
    CALL SYMBOL(.5,9.8,.1,HEAD,0,20)
    CALL SYMBOL(.5,9.6,.1,HEAD1,1,70)
    CALL SYMBOL(.5,9.4,.1,HEAD2,0,70)
    CALL SYMBOL(.5,9.2,.1,HEAD3,0,70)
    CALL AXIS(0.,0.,21H WAVE NUMBER (+) (1/M),21,10.,90.,YMIN,DY,10.)
    CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
    CALL LINE(FM,CAM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
    DY=100.
    CALL PLOT(16.0.,-3)
    CALL AXIS(0.,0.,
X      25H GROUP DELAY/CM (+) (NSEC),25,10.,90.,YMIN,DY,10.)
    CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
    CALL LINE(FM,VGM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
    DY=10000.
    CALL PLOT(16.0.,-3)
    CALL AXIS(0.,0.,21H WAVE NUMBER (-) (1/M),21,10.,90.,YMIN,DY,10.)
    CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
    CALL LINE(FP,CAP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
    DY=100.
    CALL PLOT(16.0.,-3)
    CALL AXIS(0.,0.,
X      25H GROUP DELAY/CM (-) (NSEC),25,10.,90.,YMIN,DY,10.)
    CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
    CALL LINE(FP,VGP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
    YMIN=-180.
    DY=36.
    CALL PLOT(16.0.,-3)
    CALL AXIS(0.,0.,22H NORMAL DISPERSION (+),22,10.,90.,YMIN,DY,10.)
    CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
    JJ=0
58  CONTINUE
    DO 59 J=1,J1
      IF ((JJ+J) .GT. J1) GO TO 57
      FN(J)=FM(JJ+J)
      VM(J)=VNM(JJ+J)
      IF (J .EQ. 1) GO TO 59
      IF (VM(J) .LT. VM(J-1)) GO TO 59
      GO TO 57
59  CONTINUE
57  J2=J-1
    CALL LINE(FN,VM,J2,1,0,1,XMIN,DX,YMIN,DY,.08)
    JJ=JJ+J2
    IF ((JJ+1) .LE. J1) GO TO 58
    YMIN=0.
    DY=30.
    DO 27 I=1,I1
      IF (RF(I) .GT. 300.) RP(I)=300.
      IF (RP(I) .GT. 2000.) RP(I)=2000.
27  CONTINUE
    CALL PLOT(16.0.,-3)
    CALL AXIS(0.,0.,27H RAD. PES., MINUS WAVE (OHMS),27,10.,90.,

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X  YMIN,DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)
CALL LINE(FP,RP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
DO 28 J=1,J1
  IF (RM(J) .GT. 300.) RM(J)=300.
  IF (RM(J) .GT. 2000.) RM(J)=2000.
  IF (RT(J) .GT. 300.) RT(J)=300.
  IF (RT(J) .GT. 2000.) RT(J)=2000.
28 CONTINUE
CALL PLOT(17.,0.,-3)
CALL AXYS (0.,0.,27HRAD. RES., PLUS WAVE (OHMS),27,10.,90.,
X  YMIN,DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)
CALL LINE(FM,PM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(17.,0.,-3)
CALL AXYS (0.,0.,22HRAD. RES. TOTAL (OHMS),22,10.,0.,YMIN,DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)
CALL LINE(FP,FT,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
I1=M
J1=M
YMIN=-10000.
YMIN=-250.
DY=2000.
DY=50.
DO 92 I=1,I1
  IF (PX(I) .LT. -10000.) PX(I)=-10000.
  IF (PX(I) .LT. -250.) PX(I)=-250.
  IF (PX(I) .GT. 10000.) PX(I)=10000.
  IF (PX(I) .GT. 250.) PX(I)=250.
92 CONTINUE
CALL PLOT(17.,0.,-3)
CALL AXYS (0.,0.,22HRAD. REAC TOTAL (OHMS),22,10.,90.,YMIN,DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)
CALL LINE(FP,PX,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
DO 93 I=1,I1
  IF (SERP(I) .LT. -80.) SERP(I)=-80.
  IF (SERM(I) .LT. -80.) SERM(I)=-80.
93 CONTINUE
YMIN=-80.
DY=10.
CALL PLOT(16.,0.,-3)
CALL SYMBOL(.5,9.8,.1,HEAD,0,20)
CALL SYMBOL(.5,9.6,.1,HEAD1,0,70)
CALL SYMBOL(.5,9.4,.1,HEAD2,0,70)
CALL SYMBOL(.5,9.2,.1,HEAD3,0,70)
CALL AXYS (0.,0.,26H-INS. LOSS,MINUS WAVE (DB),26,8.,90.,YMIN,
X  DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)
CALL LINE(FP,SERP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(16.,0.,-3)
CALL SYMBOL(.5,9.8,.1,HEAD,0,20)
CALL SYMBOL(.5,9.6,.1,HEAD1,0,70)
CALL SYMBOL(.5,9.4,.1,HEAD2,0,70)
CALL SYMBOL(.5,9.2,.1,HEAD3,0,70)
CALL AXYS (0.,0.,26H-INS. LOSS, PLUS WAVE (DB),26,8.,90.,YMIN,
X  DY,10.)
CALL AXYS (0.,0.,15HFREQUENCY (MHZ),-15,XX ,0.,XMIN,DX,10.)

```

```

      CALL LINE(FM, FERM, I1, 1.0, 1, XMIN, DX, YMIN, DY, .08)
      CALL ENDPLOT
      STOP
90    PRINT 76
      STOP
60    FORMAT(1H1)
61    FORMAT(5X, " H= ", E15.7/5X, " T1= ", E15.7/5X, " D= ", E15.7/
X5X, " G= ", E15.7/5X, " L= ", E15.7/
X5X, " N= ", E15.7/5X, " ETA= ", E15.7///9X, "NO. OF F'S ARE ", I5)
62    FORMAT(///10(E15.5/))
63    FORMAT(///50X, "S=1"///10X, "F= ", E15.7, 10X, "K (-) = ", E15.7//)
64    FORMAT(///50X, "S=-1"///10X, "F= ", E15.7, 10X, "K (+) = ", E15.7//)
66    FORMAT(///10X, "F= ", E15.7, 10X, "P (-) = ", E15.7//)
67    FORMAT(///10X, "F= ", E15.7, 10X, "P (+) = ", E15.7//)
68    FORMAT(///5X, "L1= ", E15.7/5X, "A= ", E15.7/5X, "P= ", E15.7/
X5X, "IC= ", E15.7/5X, "N= ", E15.7/5X, "ETA= ", E15.7)
71    FORMAT(///10X, "F= ", E15.7, 10X, "RAD. RES. (-) = ", E15.7//)
72    FORMAT(///10X, "F= ", E15.7, 10X, "RAD. RES. (+) = ", E15.7//)
73    FORMAT(///10X, "F= ", E15.7, 10X, "RAD. REAC. TOTAL= ", E15.7//)
75    FORMAT(///10X, "F= ", E15.7, 10X, "RAD. RES. TOTAL = ", E15.7//)
76    FORMAT ("1 A K ROOT EXISTS FOR ONE WAVE ONLY")
77    FORMAT(///10X, "F= ", E15.7, 10X, "INS. LCSS (-) = ", E15.7//)
78    FORMAT(///10X, "F= ", E15.7, 10X, "INS. LCSS (+) = ", E15.7//)
81    FORMAT(///10X, "L1(", I4, ") = ", E15.7//)
82    FORMAT(///10X, "A(", I4, ") = ", E15.7//)
83    FORMAT(///10X, "P(", I4, ") = ", E15.7//)
85    FORMAT(///10X, "F= ", E15.7, 10X, "GROUP DELAY (+) = ", E15.7//)
86    FORMAT(" F= ", E15.7, " NORM DISPERSION (+) = ", E15.7)
87    FORMAT(" F= ", E15.7, " GROUP DELAY (-) = ", E15.7)
102   FORMAT(2A10)
100   FORMAT(7A10)
101   FORMAT(7A10)
      END

```

```

FUNCTION SECH(CA)
COMMON EL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
SECH=0.
IF (CA .LE. 740.) SECH=1./COSH(CA)
RETURN
END

```

```

FUNCTION CSCH(CA)
COMMON EL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
CSCH=0.
IF (CA .LE. 740.) CSCH=1./SINH(CA)
RETURN
END

```

```

FUNCTION COTH(CA)
COMMON EL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
COth =1./TANH(CA)
RETURN
END

```

```

FUNCTION T(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
T = (AL2+TANH(CA*EL))/(AL1-TANH(CA*EL))
RETURN
END

```

```

FUNCTION R1(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
R1 = (1.-AL2)*EXP(-B*CA*D) + (1.+AL1)*T(CA)*EXP(B*CA*D)
RETURN
END

```

```

FUNCTION R2(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
R2 = (1.+AL2)*EXP(-B*CA*D) + (1.-AL1)*T(CA)*EXP(B*CA*D)
RETURN
END

```

```

FUNCTION FT(CA)
COMMON EL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
FT = .5*((COth(CA*T1)-1.)*R2(CA)*EXP(-CA*G)*EXP(-B*CA*D)
X-(COth(CA*T1)+1.)*R1(CA)*EXP(CA*G)*EXP(-B*CA*D))
RETURN
END

```

```

FUNCTION FT1(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
FTT =EXP(-B*CA*D)*(R1(CA)*EXP(CA*G)-R2(CA)*EXP(-CA*G))
X*S*T1*(CSCH(CA*T1))**2
FTT2=
X-S*G*EXP(-B*CA*D)*((COTH(CA*T1)+1.)*R1(CA)*EXP(CA*G)
X+(COTH(CA*T1)-1.)*R2(CA)*EXP(-CA*G))
FTT3=
X+2.*B*S*D*EXP(-2.*B*CA*D)*((COTH(CA*T1)+1.)*(1.-AL2)
X*EXP(CA*G)-(COTH(CA*T1)-1.)*(1.+AL2)*EXP(-CA*G))
FTT5=
X+S*EL*(AL1+AL2)*(SECH(CA*EL)**2)/(AL1-TANH(CA*EL))**2
FTT6=
X*((COTH(CA*T1)-1.)*(1.-AL1)*EXP(-CA*G)-(COTH(CA*T1)+1.)*
X*(1.+AL1)*EXP(CA*G))
FTT4=FTT5*FTT6
FT1=.5*(FTT+FTT2+FTT3+FTT4)
RETURN
END

```

```

FUNCTION SINC(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
PI=3.141592654
SINC=(SIN(PI*CA))/(PI*CA)
RETURN
END

```

```

FUNCTION GAY(CA)
COMPLEX C,CS
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
X,LMODE
PI=3.141592654
N=EN
C=CMPLX(0.,0.)
DO 1 I=1,N
CS=CMPLX(COS(CA*I*PE(I)),-SIN(CA*I*PE(I)))
IF (LMODE.EQ. 2) GO TO 2
C=C+SINC(.5*AA(I)*CA/PI)*ETA**I*SQRT(EL1(I))*CS
GO TO 3
2 CONTINUE
C=C+SINC(2.*AA(I)/(PE(I)*(3.-ETA)))*
XSINC((CA*PE(I)*.5/PI) -.25*(3.+ETA))*ETA**I*SQRT(EL1(I))*CS
3 CONTINUE
1 CONTINUE
GAY=CABS(C)
RETURN
END

```

```

FUNCTION GE(CA)
COMMON EL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
GE =ARS(GAY(CA)*EXP(-B*CA*D)/FT1(CA))
RETURN
END

```

```

SUBROUTINE HTRAN(R,X,N,FBEG,FEND)
DIMENSION R(3),X(3)
PI=3.14159265359
FDEL=(FEND-FBEG)/(N-1)
F=FBEG+.5*FDEL
INC=MOD(N,2)
NI=N+INC-1
NM1=N-1
NM2=NI-2
DO 33 I=1,NM1
X(I)=0.
IF (I.EQ. 1) RX=(3.*R(1)+6.*R(2)-R(3))/8.
IF (I.EQ. NM1) RX=(-R(N-2)+6.*R(NM1)+3.*R(N))/8.
IF (I.EQ. 1 .OR. I.EQ. NM1) GO TO 20
RX=(-R(I-1)+9.*R(I)+9.*R(I+1)-R(I+2))/16.
20 CONTINUE
FI=FBEG
DO 28 IP=1,NM2,2
X(I)=X(I)+4.*(R(IP+1)-RX)/(FI+FDEL)**2-F**2)
X      +2.*(R(IP)-RX)/(FI      **2-F**2)
FI=FI+2.*FDEL
28 CONTINUE
FEN=FEND
IF(INC.EQ. 0) FEN=FEND-FDEL
X(I)=X(I)+(R(NI)-RX)/(FEN**2-F**2)
X      -(R(1)-RX)/(FBEG**2-F**2)
X(I)=FDEL/3.*X(I)
IF(INC.EQ.1) GO TO 30
X(I)=X(I)+.5*FDEL*(R(NI)-RX)/(FEN**2-F**2)
X      +(R(N)-RX)/(FEND**2-F**2)
30 X(I)=?./PI*F*X(I)+RX/PI*ALOG
X ((1.-F/FEND)/(1.+F/FEND)*(F+FBEG)/(F-FBEG))
F=F+FDEL
33 CONTINUE
NM2=N-2
X1=(15.*X(1)-10.*X(2)+3.*X(3))/8.
X2=(3.*X(1)+6.*X(2)-X(3))/8.
DO 31 I=3,NM2
XT=(-X(I-2)+9.*X(I-1)+9.*X(I)-X(I+1))/16.
X(I-2)=X1
X1=X2
X2=XT
31 CONTINUE
X(N)=(15.*X(NM1)-10.*X(NM2)+3.*X(N-3))/8.
X(N-1)=(3.*X(NM1)+6.*X(NM2)-X(N-3))/8.
X(N-2)=X2
X(N-3)=X1
RETURN
END

```

B. Microstrip Model

A second computer program incorporating the microstrip model has also been completed for the CDC 6600 at Hanscom AFB, Ma. The physical quantities graphically displayed by this program are the wave number, group delay and insertion loss for both solutions, normalized dispersion for the + wave, input resistance, corresponding reactance and the magnitude of the impedance. Print out is provided by the program as for the basic theory model.

Note that apodization and normal modes are not permitted here and that additional input constants are required. Thus the use of the input cards are modified from the basic theory program by the following:

Cards 2-4 - The increment values are always 0 and the option values should be 0.

Card 5A - Z_{c1} , $\bar{\beta}_c$, σ , α_{cK}

This is a new card to be inserted between card 5 and card 6. The indicated constants, required by the microstrip model, are to be inserted here, separated by commas.

Card 6 - columns 1-70 may be used. The first ten columns should contain IND__COND__.

The listing of the entire program, except for control cards which are the same as for the basic theory program, now follows.

```

PROGRAM FOOT (INPUT, OUTPUT)
DIMENSION PCONF(7)
DIMENSION F(1200), FM(1200), FP(1200), CAP(1200), CAM(1200), VGM(1200),
YPR(1200), PM(1200), PP(1200), RM(1200), RT(1200), PX(1200), SERP(1200),
XPRM(1200), VNM(1200), VGP(1200), RN(1200), XN(1200), ZM(1200)
DIMENSION HEAD(7), HEAD1(7), HEAD2(7), HEAD3(7)
DIMENSION EN(50), VM(50)
COMMON FL, AL1, AL2, B, D, T1, G, S, ETA, EN, P, AY, A, EL1(40), PE(40), AA(40)
Y, LMODE
READ *, H, T1, D, G, EL, EN, ETA
READ *, TL, BCON, ELDEL, ELOPT
READ *, ARECTN, ADEL, AOFT
READ *, PRECTN, PDEL, POFT
READ *, DELH, DIST
READ *, ZC, PTC, SIG, ACC
READ 100, HEAD
READ 100, HEAD1
READ 100, HEAD2
READ 100, HEAD3
LMODE=1
IF (HEAD(1) .EQ. "NORM MODE ") LMODE=2
N=EN
DO 41 I=1, N
AL1(I)=AL1(PCON)+(I-1)*ELDEL
AL2(I)=ARECTN+(I-1)*ADEL
41 P(I)=PRECTN+(I-1)*PDEL
NEL=(N+1)/2
IF (ELOPT .EQ. 0.) GO TO 42
DO 43 I=NEL, N
43 FL(I)=FL1(NEL)-(I-NEL)*ELDEL
42 IF (AOFT .EQ. 0.) GO TO 44
DO 45 I=NEL, N
45 AA(I)=AA(NEL)-(I-NEL)*ADEL
44 IF (POFT .EQ. 0.) GO TO 46
DO 47 I=NEL, N
47 P(I)=P(NEL)-(I-NEL)*PDEL
46 CONTINUE
ELDEL=ACOS(PI*(H+1750.))
PHI2=PI*(H+75.)
PHI=PI.
PCH=ACOS(ELQ)+1.
NE=INT(THI)-INT(ELQ)-1
DO 40 I=1, NE
40 F(I)=FREQ+(I-1)*FDEL
IF (F(I) .LT. FLO) PRINT *, "FREQUENCIES TOO LOW"
IF (F(NF) .GT. FHI) PRINT *, "FREQUENCIES TOO HIGH"
PRINT *,
PCH=1.
IF (ETA .GT. -2.) GO TO 4
ETA=-1.
PCH=(2./EN)*2
PRINT *, "PT GRATING CASE"
CONTINUE
PRINT *, H, T1, D, G, FL, EN, ETA, NF
PRINT *, "DELTA H = ", DELH, " DISTANCE = ", DIST
PRINT *, " SIGMA TO ", SIG
PRINT *, (I, F(I), I=1, N)

```

```

PRINT 82, (I, AA(I), I=1, N)
PRINT 83, (I, PF(I), I=1, N)
PRINT 84
IF (ETA .EQ. -1.) ELL = .87*.4E-8*EN
IF (ETA .EQ. 1.) ELL = .23*.4E-8/EN
ELL = 1.
DATA PROGID/4HSETHAPES, 4H3724, 10HJ.WEINBERG/
CALL ELTID3(PROGID, 200., 12., 1.)
P=0.
A=0.
AY=0.
PT=3.141592754
PG=PG.
AYT=.7*AY*(1.-ETA)+EN*(1.+ETA)
UG=1.-PT*1.-P
PGP=10.
PRQ=1./ (36.PG*PI)
HO=TA
BO=AA(1)
ED=PRQ*PGP
ET=PG.
K=1
T=1
J=1
OMH=H/1751.
50 EF=EF(K)
OM=EF/(2.*41750.)
U11=1.-OMH/(OM**2-OMH**2)
U22=U11
U12=OM/(OM**2-OMH**2)
R=SQRT(U11/U22)
L=1
30 IF (L .EQ. 2) GO TO 2
1 S=1.
GO TO 3
2 S=-1.
3 CONTINUE
AL1=U22**2+S*U12
AL2=U22**2-S*U12
IF (J .GT. 1) GO TO 53
CAQ=.5*SQRT(U22/U11)*ALOG(1.+4.*SQRT(U11*U22)/
Y(U12**2-(SQRT(U11*U22)+1.)**2))
CAQ=CAQ/P
53 CONTINUE
IF (I .EQ. 1) GO TO 51
IF (J .EQ. 1) GO TO 51
IF (L .EQ. 1) CAQ=CAP(I-1)
IF (L .EQ. 2) CAQ=CAM(J-1)
51 M=1
5 DEL=.02*CAQ
CAOP=CAQ+DEL
CAOM=CAQ-DEL
CAQF=CAQ**2
CAOF=ABS(CAOP)
CAOMF=ABS(CAOM)
IF (CAOF .GT. .99.) GO TO 35
IF (CAOMF .GT. .99.) GO TO 35

```

```

      FTOCP=FT(CA0)
      FTOCP=FT(CA0P)
      FTOCM=FT(C10M)
      CA1=CA0-2.*DEL+FTCO/(FTCP-FTCM)
      IF (ABS(CA1) .GT. 1.E7) GO TO 35
      CA1P=CA1+0
      CA1P=ABS(CA1P)
      CA1P=ABS(CA1P)
      IF (CA1P .GT. 150.) GO TO 35
      IF (CA1P .GT. 150.) GO TO 35
      FTO1=FT(CA1)
      IF (ABS((CA1-CA0)/CA0) .LT. .001) GO TO 10
      CA0=CA1
      MEM+1
      IF (M .GT. 10) GO TO 35
      GO TO 5
10  IF (L .TO. 2) GO TO 20
      CA=CA1
      IF (ABS(FTO1) .GT. 1.) GO TO 35
      IF (CA .LT. 0.) GO TO 35
      FM(I)=FF
      CAP(I)=CA
      I=I+1
      I=2
      GO TO 31
20  CA=CA1
      IF (ABS(FTO1) .GT. 1.) GO TO 35
      IF (CA .LT. 0.) GO TO 35
      FM(J)=FF
      CAM(J)=CA
      J=J+1
      IF (J .EQ. 1) GO TO 15
      IF (J .GT. 1) GO TO 31
      I=J-1
      K=K+1
31  J=J-1
15  K=K+1
      IF (K .LE. N) GO TO 50
      PRINT 47
      I=1-1
      J=J-1
      GO TO 20
30  PRINT 48, "ITERATION DOES NOT CONVERGE. F= ", EF, " S= ", S
      IF (L .TO. 2) GO TO 10
      END
      GO TO 2
24  CONTINUE
      PRINT 47, (FM(I), CAP(I), I=1, I1, 10)
      PRINT 48, (FM(J), CAM(J), J=1, J1, 10)
      PRINT 49
      DO 25 J=1, J1
      IF (J .EQ. 1) VGM(J)=5./PI*(CAM(2)-CAM(1))/FDEL
      IF (J .EQ. 11) VGM(J)=5./PI*(CAM(J1)-CAM(J1-1))/FDEL
      IF (J .NE. 1 .AND. J .NE. J1) VGM(J)=
      VGM(J)=5./PI*(CAM(J+1)-CAM(J-1))/FDEL*.5
25  CONTINUE
      PRINT 49, (FM(J), VGM(J), J=1, J1, 10)

```



```

      OMH=FX/(2.*PI*Z(D,))
      U11=1.-OMH/(OMH**2-OMH**2)
      U12=0.
      U13=OMH/(OMH**2-OMH**2)
      U14=OMH/(OMH**2-OMH**2)
      U15=OMH/(OMH**2-OMH**2)
      U16=OMH/(OMH**2-OMH**2)
      U17=OMH/(OMH**2-OMH**2)
      U18=OMH/(OMH**2-OMH**2)
      U19=OMH/(OMH**2-OMH**2)
      U20=OMH/(OMH**2-OMH**2)
      U21=OMH/(OMH**2-OMH**2)
      U22=OMH/(OMH**2-OMH**2)
      U23=OMH/(OMH**2-OMH**2)
      U24=OMH/(OMH**2-OMH**2)
      U25=OMH/(OMH**2-OMH**2)
      U26=OMH/(OMH**2-OMH**2)
      U27=OMH/(OMH**2-OMH**2)
      U28=OMH/(OMH**2-OMH**2)
      U29=OMH/(OMH**2-OMH**2)
      U30=OMH/(OMH**2-OMH**2)
      U31=OMH/(OMH**2-OMH**2)
      U32=OMH/(OMH**2-OMH**2)
      U33=OMH/(OMH**2-OMH**2)
      U34=OMH/(OMH**2-OMH**2)
      U35=OMH/(OMH**2-OMH**2)
      U36=OMH/(OMH**2-OMH**2)
      U37=OMH/(OMH**2-OMH**2)
      U38=OMH/(OMH**2-OMH**2)
      U39=OMH/(OMH**2-OMH**2)
      U40=OMH/(OMH**2-OMH**2)
      U41=OMH/(OMH**2-OMH**2)
      U42=OMH/(OMH**2-OMH**2)
      U43=OMH/(OMH**2-OMH**2)
      U44=OMH/(OMH**2-OMH**2)
      U45=OMH/(OMH**2-OMH**2)
      U46=OMH/(OMH**2-OMH**2)
      U47=OMH/(OMH**2-OMH**2)
      U48=OMH/(OMH**2-OMH**2)
      U49=OMH/(OMH**2-OMH**2)
      U50=OMH/(OMH**2-OMH**2)
      U51=OMH/(OMH**2-OMH**2)
      U52=OMH/(OMH**2-OMH**2)
      U53=OMH/(OMH**2-OMH**2)
      U54=OMH/(OMH**2-OMH**2)
      U55=OMH/(OMH**2-OMH**2)
      U56=OMH/(OMH**2-OMH**2)
      U57=OMH/(OMH**2-OMH**2)
      U58=OMH/(OMH**2-OMH**2)
      U59=OMH/(OMH**2-OMH**2)
      U60=OMH/(OMH**2-OMH**2)
      U61=OMH/(OMH**2-OMH**2)
      U62=OMH/(OMH**2-OMH**2)
      U63=OMH/(OMH**2-OMH**2)
      U64=OMH/(OMH**2-OMH**2)
      U65=OMH/(OMH**2-OMH**2)
      U66=OMH/(OMH**2-OMH**2)
      U67=OMH/(OMH**2-OMH**2)
      U68=OMH/(OMH**2-OMH**2)
      U69=OMH/(OMH**2-OMH**2)
      U70=OMH/(OMH**2-OMH**2)
      U71=OMH/(OMH**2-OMH**2)
      U72=OMH/(OMH**2-OMH**2)
      U73=OMH/(OMH**2-OMH**2)
      U74=OMH/(OMH**2-OMH**2)
      U75=OMH/(OMH**2-OMH**2)
      U76=OMH/(OMH**2-OMH**2)
      U77=OMH/(OMH**2-OMH**2)
      U78=OMH/(OMH**2-OMH**2)
      U79=OMH/(OMH**2-OMH**2)
      U80=OMH/(OMH**2-OMH**2)
      U81=OMH/(OMH**2-OMH**2)
      U82=OMH/(OMH**2-OMH**2)
      U83=OMH/(OMH**2-OMH**2)
      U84=OMH/(OMH**2-OMH**2)
      U85=OMH/(OMH**2-OMH**2)
      U86=OMH/(OMH**2-OMH**2)
      U87=OMH/(OMH**2-OMH**2)
      U88=OMH/(OMH**2-OMH**2)
      U89=OMH/(OMH**2-OMH**2)
      U90=OMH/(OMH**2-OMH**2)
      U91=OMH/(OMH**2-OMH**2)
      U92=OMH/(OMH**2-OMH**2)
      U93=OMH/(OMH**2-OMH**2)
      U94=OMH/(OMH**2-OMH**2)
      U95=OMH/(OMH**2-OMH**2)
      U96=OMH/(OMH**2-OMH**2)
      U97=OMH/(OMH**2-OMH**2)
      U98=OMH/(OMH**2-OMH**2)
      U99=OMH/(OMH**2-OMH**2)
      U100=OMH/(OMH**2-OMH**2)

```

```

      AR=AP/FN
2L2  CONTINUE
      APC=AP+AC
      DIN=1.+COS(2.*RT*FL1(1))*SECH(2.*ARC*FL1(1))
      PIN=70*TANH(2.*ARC*FL1(1))/DIN
      IF (ABS(PIN) .LE. .0001) PIN=ABS(PIN)
      YTN=70*DIN(2.*RT*FL1(1))*SECH(2.*ARC*FL1(1))/DIN
      PM(T)=RTIN
      YN(I)=XIN
      ZN(T)=SQRT(RTN**2+XIN**2)
      PC=RTIN*AC/(AC+RTT /70)
      YL=YIN*RT/(RT+PXX /70)
      RIMP=PIN*PP /70/(AC+RTT /70)
      RIMM=PIN*PM /70/(AC+RTT /70)
      RIM=RIMP+RIMM
      YIM=XTN*PYX /70/(RT+PXX /70)
      DIF=(RT+TIN)**2+XTN**2
      L=T
      IF (T .EQ. 1) L=2
      SERP(T)=20.*ALOG10((4.*RI*RIMP /70SER)
      SERP(I)=SERP(I)-76.4*DELH*DIST/
      Y(2,RI*1.56*(RP(I+1)-RP(L-1))/(CAP(I+1)-CAP(L-1)))
      SERM(T)=20.*ALOG10((4.*RI*RIMM /70SER)
      SERM(I)=SERM(I)-76.4*DELH*DIST/
      Y(2,RI*1.75*(RM(I+1)-RM(L-1))/(CAM(I+1)-CAM(L-1)))
24  CONTINUE
      PRINT 6
      PRINT 77,(RP(I),SERP(I),I=1,M .10)
      PRINT 81
      PRINT 77,(RM(I),SERM(I),I=1,M .10)
      FREQO=.01*FREQ
      YMIN=AIN(T*FREQO)+100.
      XY=AINT(.01*FHI)-AIN(T*.01*FLO)+1.
      DY=100.
      YMIN=0.
      DY=17000.
      DO 96 J=1,J1
      IF (CAM(J) .GT. 100000.) CAM(J)=100000.
      IF (VGM(J) .GT. 1000.) VGM(J)=1000.
26  CONTINUE
      DO 97 I=1,I1
      IF (CAP(I) .GT. 100000.) CAP(I)=100000.
      IF (VGP(I) .GT. 1000.) VGP(I)=1000.
27  CONTINUE
      CALL PLOT(1,5,0.,-7)
      CALL SYMBOL(1,5,5..1,HEAD,0,70)
      CALL SYMBOL(1,5,6..1,HEAD1,0,70)
      CALL SYMBOL(1,5,4..1,HEAD2,0,70)
      CALL SYMBOL(1,5,2..1,HEAD3,0,70)
      CALL AXIS(0..1,214WAVE NUMBER (+) (1/M),21,10.,9.,YMIN,DY,10.)
      CALL AXIS(0..0,15HERFQUENCY (MHZ),-15,XX ,J.,XMIN,DX,10.)
      CALL LINE(FM,CAM,J1,1,0.1,YMIN,DX,YMIN,DY,.08)
      DY=100.
      CALL PLOT(16,0.,-7)
      CALL AXIS(0.,0.,
      Y  BEHGROUP DELAY/CM (+) (NSEC),25,10.,90.,YMIN,DY,10.)
      CALL AXIS(0..0,15HERFQUENCY (MHZ),-15,XX ,J.,XMIN,DX,10.)

```

```

CALL LINE (FM,VGM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
DY=100.
CALL PLOT(10.,0.,-3)
CALL AXIS(0.,0.,21H WAVE NUMBER (-) (1/M),21,10.,90.,YMIN,DY,10.)
CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
CALL LINE(FM,CAP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
DY=100.
CALL PLOT(10.,0.,-3)
CALL AXIS(0.,0.,21H GROUP DELAY/CM (-) (NSEC),29,10.,90.,YMIN,DY,10.)
CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
CALL LINE(FM,VGP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
YMIN=-100.
DY=100.
CALL PLOT(10.,0.,-3)
CALL AXIS(0.,0.,22H NORMAL DISPERSION (+),22,10.,90.,YMIN,DY,10.)
CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
J1=1
54 CONTINUE
DO 56 J=1,J1
IF ((J+J) .GT. J1) GO TO 57
FN(J)=FM(J+J)
VM(J)=VM(J+J)
IF (J .EQ. 1) GO TO 55
IF (VM(J) .LE. VM(J-1)) GO TO 59
GO TO 57
55 CONTINUE
J=J+1
57 J=J-1
CALL LINE(FM,VP,J2,1,0,1,XMIN,DX,YMIN,DY,.08)
J2=J+J2
IF ((J+J) .GT. J1) GO TO 58
J2=M
J2=M
VMTH=0.
VM=0.
DO 62 I=1,J2
IF (FN(I) .GT. 0.) FN(I)=1.
IF (FN(I) .GT. 300.) FN(I)=300.
IF (VM(I) .GT. 300.) VM(I)=300.
62 CONTINUE
CALL PLOT(10.,0.,-3)
CALL AXIS(0.,0.,27H INPUT RESISTANCE (OHMS),27,10.,90.,YMIN,DY,10.)
CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
CALL LINE(FM,FN,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(10.,0.,-3)
CALL AXIS(0.,0.,27H IMPEDANCE MAGNITUDE (OHMS),2,10.,90.,YMIN,DY,10.)
CALL AXIS(0.,0.,15H FREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
CALL LINE(FM,ZM,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
YMIN=-250.
DY=50.
DO 64 I=1,J2
IF (YN(I) .LT. -250.) XN(I)=-250.
IF (YN(I) .GT. 250.) XN(I)=250.
64 CONTINUE
CALL PLOT(10.,0.,-3)

```

```

CALL AXIS (0.,0.,22HINPUT REACTANCE (OHMS),22,10.,0.,YMIN,DY,10.)
CALL AXIS (0.,0.,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
CALL LINE (FM,YN,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
DO 27 I=1,1
  IF (SERP(I).LT.-20.) SERP(I)=-80.
  IF (SERM(I).LT.-20.) SERM(I)=-80.
23 CONTINUE
  YMIN=-20.
  DY=10.
  CALL PLOT(14.,0.,-3)
  CALL SYMBOL(.5,5.5,.1,HEAD,0,70)
  CALL SYMBOL(.5,5.6,.1,HEAD1,0,70)
  CALL SYMBOL(.5,5.4,.1,HEAD2,0,70)
  CALL SYMBOL(.5,5.2,.1,HEAD3,0,70)
  CALL AXIS (0.,0.,26H-INS. LOSS,MINUS WAVE (DB),26,8.,90.,YMIN,
  Y,DY,10.)
  CALL AXIS (0.,0.,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
  CALL LINE (FM,SERP,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
  CALL PLOT(14.,0.,-3)
  CALL SYMBOL(.5,5.8,.1,HEAD,0,70)
  CALL SYMBOL(.5,5.6,.1,HEAD1,0,70)
  CALL SYMBOL(.5,5.4,.1,HEAD2,0,70)
  CALL SYMBOL(.5,5.2,.1,HEAD3,0,70)
  CALL AXIS (0.,0.,26H-INS. LOSS, PLUS WAVE (DB),26,8.,90.,YMIN,
  Y,DY,10.)
  CALL AXIS (0.,0.,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,10.)
  CALL LINE (FM,SERM,I1,1,0,1,XMIN,DX,YMIN,DY,.08)
  CALL PLOT
  STOP
25 PRINT 74
  STOP
26 FORMAT(1H1)
27 FORMAT('Y, " H= ",E15.7/5X," T1= ",E15.7/5X," D= ",E15.7/
  5X," C= ",E15.7/5X," L= ",E15.7/
  5X," N= ",E15.7/5X,"ETA=",E15.7/5X,"NO. OF F'S ARE ",I6)
28 FORMAT(//10(1H10.0/))
29 FORMAT(///5X,"S=1"//10X,"F= ",E15.7,10X,"K (-) = ",E15.7//)
30 FORMAT(///5X,"S=-1"//10X,"F= ",E15.7,10X,"K (+) = ",E15.7//)
31 FORMAT(///10X,"F= ",E15.7,10X,"P (-) = ",E15.7//)
32 FORMAT(///10X,"F= ",E15.7,10X,"P (+) = ",E15.7//)
33 FORMAT(///5X,"L1= ",E15.7/5X,"A= ",E15.7/5X,"Pe= ",E15.7/
  5X,"T= ",E15.7/5X,"N= ",E15.7/5X,"ETA= ",E15.7)
34 FORMAT(///10X,"F= ",E15.7,10X,"RAD. RES. (-) = ",E15.7//)
35 FORMAT(///10X,"F= ",E15.7,10X,"RAD. RES. (+) = ",E15.7//)
36 FORMAT(///10X,"F= ",E15.7,10X,"RAD. REAC. TOTAL= ",E15.7//)
37 FORMAT(///10X,"F= ",E15.7,10X,"RAD. RES. TOTAL = ",E15.7//)
38 FORMAT('1 A K ROOT EXISTS FOR ONE WAVE ONLY')
39 FORMAT(///10X,"F= ",E15.7,10X,"INS. LOSS (-) = ",E15.7//)
40 FORMAT(///10X,"F= ",E15.7,10X,"INS. LOSS (+) = ",E15.7//)
41 FORMAT(///10X,"L1(",I4,")= ",E15.7//)
42 FORMAT(///10X,"A(",I4,")= ",E15.7//)
43 FORMAT(///10X,"P(",I4,")= ",E15.7//)
44 FORMAT(///10X,"F= ",E15.7,10X,"GROUP DELAY (+) = ",E15.7//)
45 FORMAT(" "C= ",E15.7," NORM DISPERSION (+) = ",E15.7)
46 FORMAT(" "F= ",E15.7," GROUP DELAY (-) = ",E15.7)
47 FORMAT(7A1())
  END

```

```

FUNCTION SECH(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
SECH=C.
IF (CA .LE. 740.) SECH=1./COSH(CA)
RETURN
END

```

```

FUNCTION CSCH(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
CSCH=C.
IF (CA .LE. 740.) CSCH=1./SINH(CA)
RETURN
END

```

```

FUNCTION COTH(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
COTH =1./TANH(CA)
RETURN
END

```

```

FUNCTION T(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
T = (AL2+TANH(CA*EL))/(AL1-TANH(CA*EL))
RETURN
END

```

```

FUNCTION R1(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
R1 = (1.-AL2)*EXP(-B*CA*D) + (1.+AL1)*T(CA)*EXP(-CA*D)
RETURN
END

```

```

FUNCTION R2(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
R2 = (1.+AL2)*EXP(-B*CA*D) + (1.-AL1)*T(CA)*EXP(-B*CA*D)
RETURN
END

```

```

FUNCTION FT(CA)
COMMON FL,AL1,AL2,B,D,T1,G,S,ETA,EN,P,AY,A
FT = .5*((COTH(CA*T1)-1.)*R2(CA)*EXP(-CA*G)*EXP(-B*CA*D)
+ (COTH(CA*T1)+1.)*R1(CA)*EXP(CA*G)*EXP(-B*CA*D))
RETURN
END

```

```

      FUNCTION GAM(CA)
      COMPLEX C,CC
      COMMON FL,AL1,AL2,R,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
      Y,LMODET
      DT=3.141592654
      N=EN
      FOR N>1,EN, EN, REC. IS FIRST COMPUTED AS FOR N=1. IT IS THEN
      MULTIPLIED BY 1 FACTOR FOR ETA=1 AND BY ANOTHER FACTOR FOR ETA=-1 AND N
      EVEN. ADDITION IS NOT PERMITTED.
      N=1
      COMPLEX (0.,0.)
      DO 1 I=1,N
      C=COMPLEX(COS(CA*I*PE(I)),-SIN(CA*I*PE(I)))
      IF (LMODET.EQ. 2) GO TO 2
      C=C*SINCE(AA(I)*CA/PI)*ETA**I*SQRT(EL1(I))*CS
      GO TO 3
      2 CONTINUE
      C=C*SQRT(2. AA(I)/(E(I)*(3.-ETA)))
      C=SINC(CA*I*PI*.5/PI) * -.25 * (3.+ETA)**ETA**I*SQRT(EL1(I))*CS
      3 CONTINUE
      1 CONTINUE
      GAM=C/ABS(C)
      RETURN
      END

      FUNCTION SINC(CA)
      COMMON FL,AL1,AL2,R,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),PE(40),AA(40)
      DT=3.141592654
      SINC=(SIN(PI*CA))/(PI*CA)
      RETURN
      END

      FUNCTION ET1(CA)
      COMMON FL,AL1,AL2,R,D,T1,G,S,ETA,EN,P,AY,A
      ET1 = -EXP(-R*CA*D)*(R1(CA)*EXP(CA*G)-R2(CA)*EXP(-CA*G))
      Y=S*T1*(SECH(CA*T1))**2
      ET12=
      Y+1.*R*S*EXP(-R*CA*D)*((COTH(CA*T1)+1.)*R1(CA)*EXP(CA*G)
      Y+(COTH(CA*T1)-1.)*R2(CA)*EXP(-CA*G))
      ET13=
      Y+2.*R*S*D*EXP(-2.*R*CA*D)*((COTH(CA*T1)+1.)*(1.-AL2)
      Y+EXP(CA*G)-(COTH(CA*T1)-1.)*(1.+AL2)*EXP(-CA*G))
      ET15=
      Y+1.*EL*(AL1+AL2)*(SECH(CA*FL)**2)/(AL1-TANH(CA*EL))**2
      ET16=
      Y*(COTH(CA*T1)-1.)*(1.-AL1)*EXP(-CA*G)-(COTH(CA*T1)+1.)
      Y*(1.+AL1)*EXP(CA*G)
      ET14=ET15+ET16
      ET1=.5*(ET1+ET12+ET13+ET14)
      RETURN
      END

```

```

SUBROUTINE HTEAN(F,Y,N,FBEQ,FEND)
DIMENSION P(3),X(3)
PI=3.14159265359
FDEL=(FEND-FBEQ)/(N-1)
F=FBEQ+.5*FDEL
INC=MOD(N,2)
NT=N+INC-1
NM1=N-1
NIM2=NT-2
DO 23 I=1,NM1
X(I)=F.
IF (I.EQ. 1) PX=(3.*F(1)+6.*R(2)-R(3))/6.
IF (I.EQ. NM1) RX=(-9*(N-2)+6.*R(NM1)+3.*R(N))/6.
IF (I.EQ. 1 .OR. I.EQ. NM1) GO TO 20
PX=(-F(I-1)+9.*P(I)+9.*R(I+1)-R(I+2))/16.
20 CONTINUE
FI=FBEQ
DO 28 IP=1,NIM2,2
Y(I)=Y(I)+4.* (R(IP+1)-PX)/((FI+FDEL)**2-F**2)
Y      +2.* (P(IP)-RX)/(FI      **2-F**2)
FI=FI+2.*FDEL
28 CONTINUE
F=FEND
IF (INC.EQ. 1) FEN=FEND-FDEL
X(I)=Y(I)+(F(NI)-PX)/(FEN**2-F**2)
Y      -(F(1)-RX)/(FBEQ**2-F**2)
Y(I)=FDEL/3.*Y(I)
IF (INC.EQ. 1) GO TO 31
Y(I)=Y(I)+F*FDEL*( (R(NI)-RX)/(FEN**2-F**2)
Y      + (F(N)-RX)/(FEND**2-F**2) )
30 Y(I)=0./PI*F*Y(I)+PX/PI*ALOG
  ( (1.-F/FEND)/(1.+F/FEND)*(F+FBEQ)/(F-FDEL) )
F=F+FDEL
32 CONTINUE
NM2=N-2
Y1=(15.*Y(1)-10.*Y(2)+3.*X(3))/8.
Y2=(7.*Y(1)+9.*X(2)-X(3))/6.
DO 34 I=3,NM2
YI=(-Y(I-2)+5.*X(I-1)+9.*X(I)-X(I+1))/16.
X(I-2)=Y1
Y1=YI
YI=YI
34 CONTINUE
Y(N)=(15.*X(NM1)-10.*X(NM2)+3.*X(N-3))/8.
Y(N-1)=(7.*X(NM1)+9.*X(NM2)-X(N-3))/6.
Y(N-2)=Y2
X(N-2)=Y1
RETURN
END

FUNCTION GF(CA)
COMMON FL,AL1,AL2,3,D,T1,G,S,ETA,EN,P,AY,A,EL1(40),FE(40),AA(40)
GF=ABS(GAY(CA)*EXP(-3*CA*D)/FT1(CA))
RETURN
END

```

C. Generalized Dispersion Relation

Another program has been implemented on the CDC 6600 at Hanscom AFB, Ma which solves the dispersion relation problem for surface waves including any arbitrary orientation of the biasing field. Computer print-outs and plots of wave number, wave length and group delay, as functions of frequency, for both + and - solutions, are provided by the program.

The input cards to this program are:

Card 1 - H_0 , t_1 , d , θ , ℓ , ϕ

These six quantities, separated by commas, are supplied here. The lengths are in meters and the angles are in degrees.

Card 2 - first f , Δf , number of frequency values

Here the user is to provide the first frequency value, the frequency increment and the number of frequency values to be considered, all separated by commas. The maximum number of frequency values permitted is currently 500. Although the program could compute the frequency range itself as in the other programs it was left for input to provide flexibility.

Card 3 - heading

Card 4 - heading

Two heading cards for the graphs are here required. Columns 1-70 may be used.

The listing of the entire program, excluding the standard control cards, now follows.

```

C   PHT-9 DEG THETA=90 DEG ----- SURFACE WAVES
      PROGRAM VOLM(INPUT,OUTPUT)
      DIMENSION PROGID(3)
      DIMENSION F(501),FP(501),FM(501),CAP(501),CAM(501)
      DIMENSION HEAD1(7),HEAD2(7)
      X,VGP(501),VGM(501),ELAP(501),ELAM(501)
      COMMON D,FL,T1,S,C,AL1,AL2
      READ *,H,T1,D,THET,EL,PHI
      READ *,FR,C,FEEL,NF
      READ 100,HEAD1
      READ 100,HEAD2
      PRINT 60
      P=INT F1,H,T1,D,THET,EL,PHI
      PRINT 60
      DATA PROGID/6HSETHAFES,4H2587,10HJ.WEINBERG/
      CALL FLTP3(PROGID,100.,12.,1.)
      GO 40 I=1,NF
40  F(I)=FRFG+(I-1)*FEEL
      PT=3.141592654
      THET=THET*PT/180.
      PHI=PHI*PI/180.
      FM=1758.
      CAM=2.8
      TU=GAM**2*H*FM
      K=1
      T=1
      J=1
50  FF=F(K)
      TD=(GAM*H)**2-FF**2
      T2=GAM*FM-FF
      UYY=TU/TD*(SIN(THET))**2+1.
      UXX=TU/TD*((SIN(THET)*SIN(PHI))**2+(COS(THET))**2)+1.
      UXXYY=-2.*TU/TD*COS(THET)*COS(PHI)*SIN(THET)
      UXXYYT=T2/TD*SIN(THET)*SIN(PHI)
      TEM=UXXYY**2-4.*UXX*UYY
      TEM=-TEM
      L=2
      IF (TEM .LE. 0.) PRINT *, " NEGATIVE SQUARE ROOT"
      IF (TEM .LE. 0.) GO TO 35
      C=SQRT(TEM) .C/UYY
      L=1
70  IF (L .EQ. 0) GO TO 2
1   S=1.
   GO TO 3
2   S=-1.
3   CONTINUE
      AL2=C*UYY+S*UXXYYI
      AL1=C*UYY-S*UXXYYI
      IF (J .GT. 1) GO TO 53
      TO=(AL2+1.)/(AL1-1.)
      CAO=ALOG((AL2-1.)/(TO*(AL1+1.)))/2./C/D
      IF (CAO .LT. 0.) CAO=100.
53  CONTINUE
      IF (I .TO. 1) GO TO 51
      IF (J .EQ. 1) GO TO 51
      IF (L .EQ. 1) CAO=CAP(I-1)
      IF (L .EQ. 2) CAO=CAM(J-1)

```

```

51 M=1
5 DEL=.02*CA0
  (ADP=CA0+DEL
  CAOM=CA0-DEL
  CAOD=CA0*0.40
  CAOD=ABS(CAOD)
  IF(CAOD .GT. .50.) GO TO 35
  FTOO=FT(CA0)
  FTOP=FT(CADP)
  FTOB=FT(CAOM)
  CA1=CA0-2.*DEL*FTOO/(FTOP-FTOB)
  IF(ABS(CA1) .GT. 1.E7) GO TO 35
  CA1D=CA1*0.40
  CA1D=ABS(CA1D)
  IF(CA1D .GT. .50.) GO TO 35
  FTO1=FT(CA1)
C PRINT *,CA0,FTOO,CA1,FTO1
  IF (ABS((CA1-CA0)/CA0) .LT. .001) GO TO 10
  CA0=CA1
  M=M+1
  IF(M .GT. 10) GO TO 35
  GO TO 5
10 IF (L .EQ. 2) GO TO 20
  CA=CA1
  IF (ABS(FTO1) .GT. 1.) GO TO 35
  IF (CA .LT. 0.) GO TO 35
  FM(I)=EF
  CAP(I)=CA
  I=I+1
C PRINT *, "CA= ",CA,"F= ",EF
  I=2
  GO TO 30
20 CA=CA1
  IF (ABS(FTO1) .GT. 1.) GO TO 35
  IF (CA .LT. 0.) GO TO 35
  FM(J)=EF
  CAM(J)=CA
  J=J+1
C PRINT *, "CA= ",CA,"F= ",EF
  IF (J .EQ. 1) GO TO 15
  IF (J .GT. 1) GO TO 31
  I=J-1
  K=K-1
31 J=J-1
15 K=K+1
  IF (K .LE. N) GO TO 50
  PRINT 60
  I=I-1
  J=J-1
  GO TO 24
35 PRINT *, "ITERATION DOES NOT CONVERGE.F= ",EF," S= ",S
  IF (L .EQ. 2) GO TO 15
  L=2
  GO TO 2
24 CONTINUE
  PRINT 63, (FM(LL),CAP(LL),LL=1,I1,5)
  PRINT 64, (FM(LL),CAM(LL),LL=1,J1,5)

```

```

PRINT 60
DO 11 I=1,I1
11 ELAP(I)=5./PI/CAP(I)*1.E6
DO 12 J=1,J1
12 ELAM(J)=2.*PI/CAM(J)*1.E6
C PRINT 65,(EP(LL),ELAP(LL),LL=1,I1,5)
C PRINT 66,(EM(LL),ELAM(LL),LL=1,J1,5)
DO 21 I=1,I1
IF (I.EQ. 1) VGP(I)=5./PI*(CAP(2)-CAP(1))/FDEL
IF (I.EQ. I1) VGP(I)=5./PI*(CAP(I1)-CAP(I1-1))/FDEL
IF (I.NE. 1 .AND. I.NE. I1) VGP(I)=
X 5./PI*(CAP(I+1)-CAP(I-1))/FDEL*.5
21 CONTINUE
C PRINT 77,(EP(I),VGP(I),I=1,I1,10)
DO 22 J=1,J1
IF (J.EQ. 1) VGM(J)=5./PI*(CAM(2)-CAM(1))/FDEL
IF (J.EQ. J1) VGM(J)=5./PI*(CAM(J1)-CAM(J1-1))/FDEL
IF (J.NE. 1 .AND. J.NE. J1) VGM(J)=
X 5./PI*(CAM(J+1)-CAM(J-1))/FDEL*.5
22 CONTINUE
C PRINT 85,(EM(J),VGM(J),J=1,J1,10)
YMIN=5000.
XY=100.
XMIN=0100.
DY=10.
YMIN=0.
XY=5000.
DO 13 I=1,I1
IF (CAP(I).GT. 500000.) CAP(I)=500000.
IF (ELAP(I).GT. 1000. ) ELAP(I)=1000.
VGP(I)=ABS(VGP(I))
13 IF (VGP(I).GT. 1000.) VGP(I)=1000.
DO 14 J=1,J1
IF (CAM(J).GT. 500000.) CAM(J)=500000.
IF (ELAM(J).GT. 1000. ) ELAM(J)=1000.
VGM(J)=ABS(VGM(J))
14 IF (VGM(J).GT. 1000.) VGM(J)=1000.
X=10.
YY=10.
YY=10.
CALL FLOT(11,5,0.,-3)
CALL SYMBOL(1,5,9.6,.1,HEAD1,0,70)
CALL SYMBOL(1,5,9.4,.1,HEAD2,0,70)
CALL AXIS(0.,.264,WAVE NUMBER K(+) (1/METER),26,X,90.,YMIN,DY,YY)
CALL AXIS(0.,.15,HFREQUENCY (MHZ),-15,X,0.,XMIN,DX,YY)
CALL LINE (EM,CAM,
X J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL FLOT (16,1,0.,-3)
CALL SYMBOL(1,5,9.6,.1,HEAD1,0,70)
CALL SYMBOL(1,5,9.4,.1,HEAD2,0,70)
CALL AXIS(0.,.264,WAVE NUMBER K(-) (1/METER),26,X,90.,YMIN,DY,YY)
CALL AXIS(0.,.15,HFREQUENCY (MHZ),-15,X,0.,XMIN,DX,YY)
CALL LINE (EP,CAP,
Y I1,1,0,1,XMIN,DX,YMIN,DY,.08)
XY=10.
CALL FLOT (17,5,0.,-3)
CALL SYMBOL(1,5,9.6,.1,HEAD1,0,70)

```

```

CALL SYMBOL(.5,.4,.1,HEAD2,1,70)
CALL AXIS(0.,0,24HWAVELENGTH (+) (MICRONS),24,XX,0.,YMIN,DY,YY)
CALL AXIS(0.,0,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,YY)
CALL LINE(FM.FLAM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(13.,0.,-3)
CALL SYMBOL(.5,.4,.1,HEAD1,1,70)
CALL SYMBOL(.5,.4,.1,HEAD2,1,70)
CALL AXIS(0.,0,24HWAVELENGTH (-) (MICRONS),24,XX,90.,YMIN,DY,YY)
CALL AXIS(0.,0,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,YY)
CALL LINE(FP.FLAP,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(13.,0.,-3)
CALL SYMBOL(.5,.4,.1,HEAD1,1,70)
CALL SYMBOL(.5,.4,.1,HEAD2,1,70)
CALL AXIS(0.,0,
X      25HGROUP DELAY/CM (+) (NSEC),25,XX,90.,YMIN,DY,YY)
CALL AXIS(0.,0,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,YY)
CALL LINE(FM.VGM,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL PLOT(13.,0.,-3)
CALL SYMBOL(.5,.4,.1,HEAD1,1,70)
CALL SYMBOL(.5,.4,.1,HEAD2,1,70)
CALL AXIS(0.,0,
Y      25HGROUP DELAY/CM (-) (NSEC),25,XX,0.,YMIN,DY,YY)
CALL AXIS(0.,0,15HFREQUENCY (MHZ),-15,XX,0.,XMIN,DX,YY)
CALL LINE(FP.VGP,J1,1,0,1,XMIN,DX,YMIN,DY,.08)
CALL ENDPLT
STOP
60  FORMAT(1H1)
61  FORMAT(5X," H= ",E15.7/5X," T1= ",E15.7/5X," D= ",E15.7/
Y" THETA= ",E15.7/5X," L= ",E15.7/5X," PHI= ",E15.7)
62  FORMAT(//10(E15.5/))
63  FORMAT(///50X,"S=1"//(10X,"F= ",E15.7,10X,"K (-)= ",E15.7/))
64  FORMAT(///5 X,"S=-1"//(10X,"F= ",E15.7,10X,"K (+) = ",E15.7/))
65  FORMAT(///5 X,"S=1"//(10X,"F= ",E15.7,10X,"LAM(-)= ",E15.7/))
66  FORMAT(///5 X,"S=-1"//(10X,"F= ",E15.7,10X,"LAM(+)= ",E15.7/))
67  FORMAT("      F= ",E15.7,"      GROUP DELAY (-) = ",E15.7)
85  FORMAT(/(10X," F= ",E15.7,10X,"GROUP DELAY (+) = ",E15.7/))
100  FORMAT(7A10)
END

```

```

FUNCTION COTH(CA)
COMMON D,FL,T1,S,C,AL1,AL2
IF (CA .GT. 17.3) GO TO 3
IF (CA .LT. -17.3) GO TO 4
COTH = 1./TANH(CA)
GO TO 2
3   COTH=1.
GO TO 2
4   COTH=-1.
2   CONTINUE
RETURN
END

```

```

FUNCTION T(CA)
COMMON D,FL,T1,S,C,AL1,AL2
CAFL=CA*FL
IF (CAEL .GT. 17.3) GO TO 43
IF (CAEL .LT. -17.3) GO TO 44
T=(AL2+TANH(CAFL))/(AL1-TANH(CAEL))
GO TO 45
43  T=(AL2+1.)/(AL1-1.)
GO TO 45
44  T=(AL2-1.)/(AL1+1.)
45  CONTINUE
RETURN
END

```

```

FUNCTION FT(CA)
COMMON D,FL,T1,S,C,AL1,AL2
FT=EXP(-C*CA*D)*T(CA)*(AL1+COTH(CA*T1)+1.)
Y = (1.-AL2+COTH(CA*T1))*EXP(-C*CA*D)
RETURN
END

```

COMPLETED CASES

In this section there are presented graphical results produced by the computer programs for various cases.

In figures 2-12 are the results of the basic theory for one set of parameters, omitting conduction loss. Non-apodized independent conductors are considered. Graphs are presented for wave number (plus wave), group delay (plus wave), wave number (minus wave), group delay (minus wave), normalized dispersion (plus wave), radiation resistance (plus wave), radiation resistance (minus wave), total radiation resistance, total radiation reactance, negative of insertion loss (plus wave) and negative of insertion loss (minus wave).

In figures 13-16 are the results of the microstrip model for the same set of parameters as above. Presented are graphs of input resistance, input reactance, negative of insertion loss (plus wave) and negative of insertion loss (minus wave).

A second set of parameters is considered in Figures 17-28. The basic theory is employed for non-apodized independent conductors. There are presented graphs for radiation resistance (minus wave), radiation resistance (plus wave), total radiation resistance, total radiation reactance, negative of insertion loss (minus wave) and negative of insertion loss (plus wave). Results are obtained for $N=1$, $N=2$, $N=8$ and $N=100$.

Another set of parameters is considered in all of the figures 29-46.

In figures 29-31 the radiation resistance (minus wave) is presented for the cases of no apodization, apodization in strip width

and apodization in center to center spacing. The basic theory for independent conductors is here considered.

In figures 32-34 there are presented graphs for radiation resistance (minus wave) for the cases of no apodization, apodization in strip width and apodization in center to center spacing. Here the basic theory for normal modes has been considered.

Figure 35 presents the radiation resistance (minus wave) and radiation resistance (plus wave) for the basic theory with independent conductors.

Figure 36 presents the radiation resistance (minus wave) and radiation resistance (plus wave) as above, with no ground planes.

In figures 37-38 are presented the radiation resistance (minus wave) and radiation resistance (plus wave) for the basic theory with normal modes; for the fundamental mode and for $n=3$.

In figure 39 the radiation resistance (minus wave), radiation resistance (plus wave) and total radiation resistance are presented for the basic theory with independent conductors and no ground planes, for $N=1$.

In figure 40 the radiation resistance (minus wave) and radiation resistance (plus wave) for the case as above, with $N=3$, are presented.

In figure 41 the radiation resistances (plus wave) are presented for the basic theory with independent conductors, for $N=1$, $N=2$, $N=3$ and $N=4$.

In figure 42 the radiation resistances (plus wave) for the basic theory with independent conductors, for $N=4$, are presented for three different gap thicknesses.

In figures 43-44 there are presented the radiation resistance (plus wave) and radiation resistance (minus wave) for the basic theory with normal modes, for $N=32$.

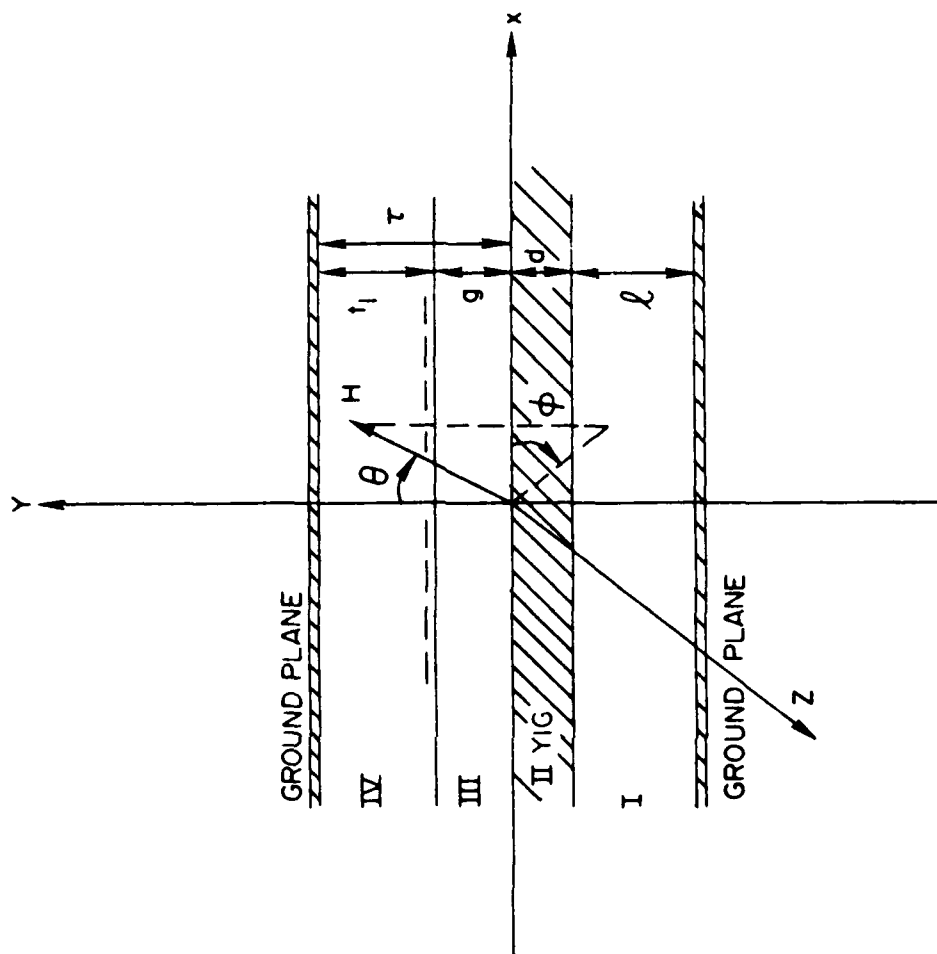


Figure 1

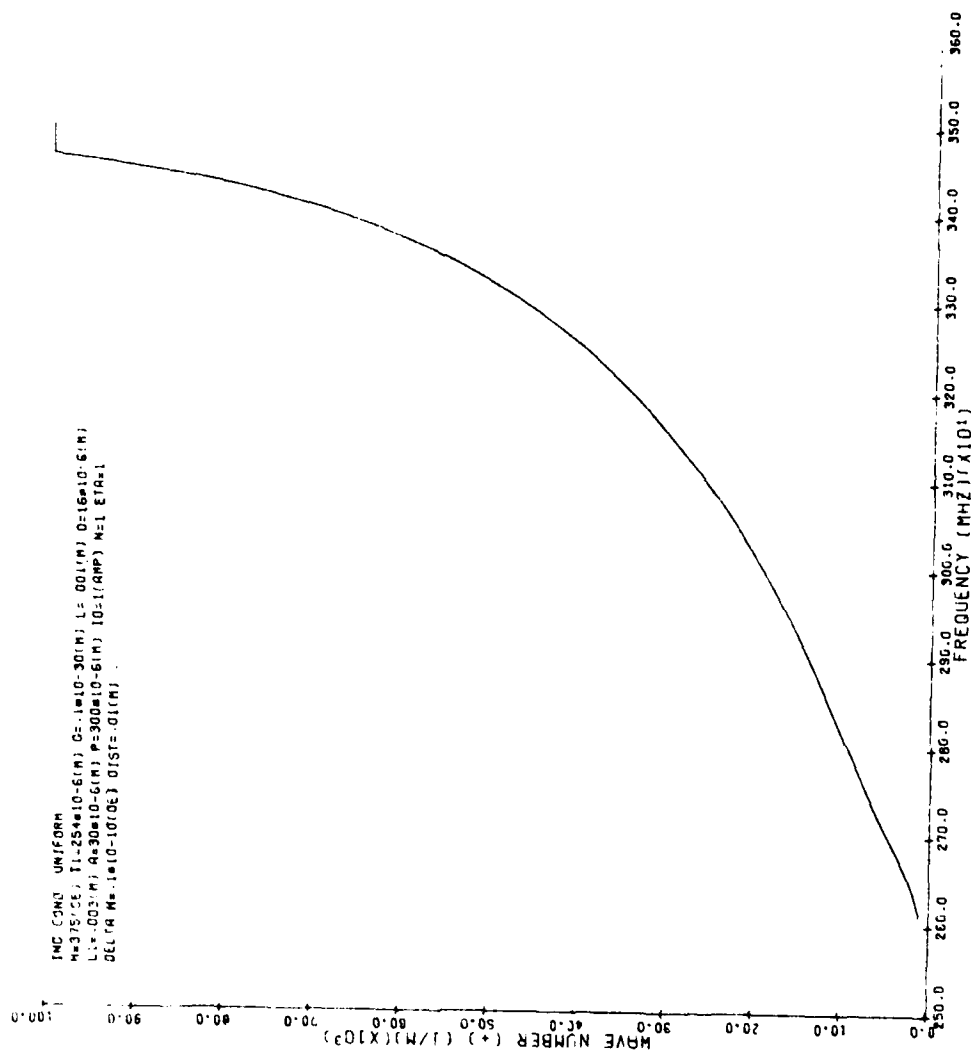


Figure 2

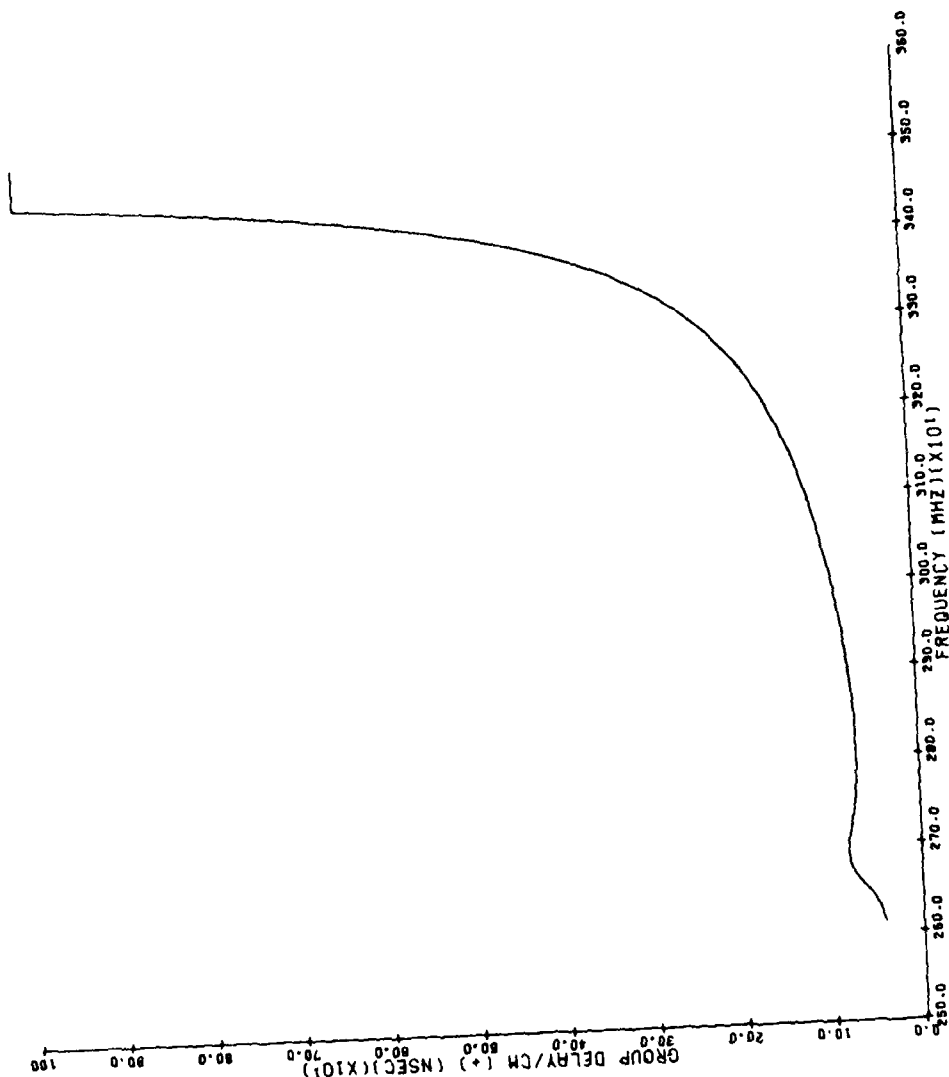


Figure 3

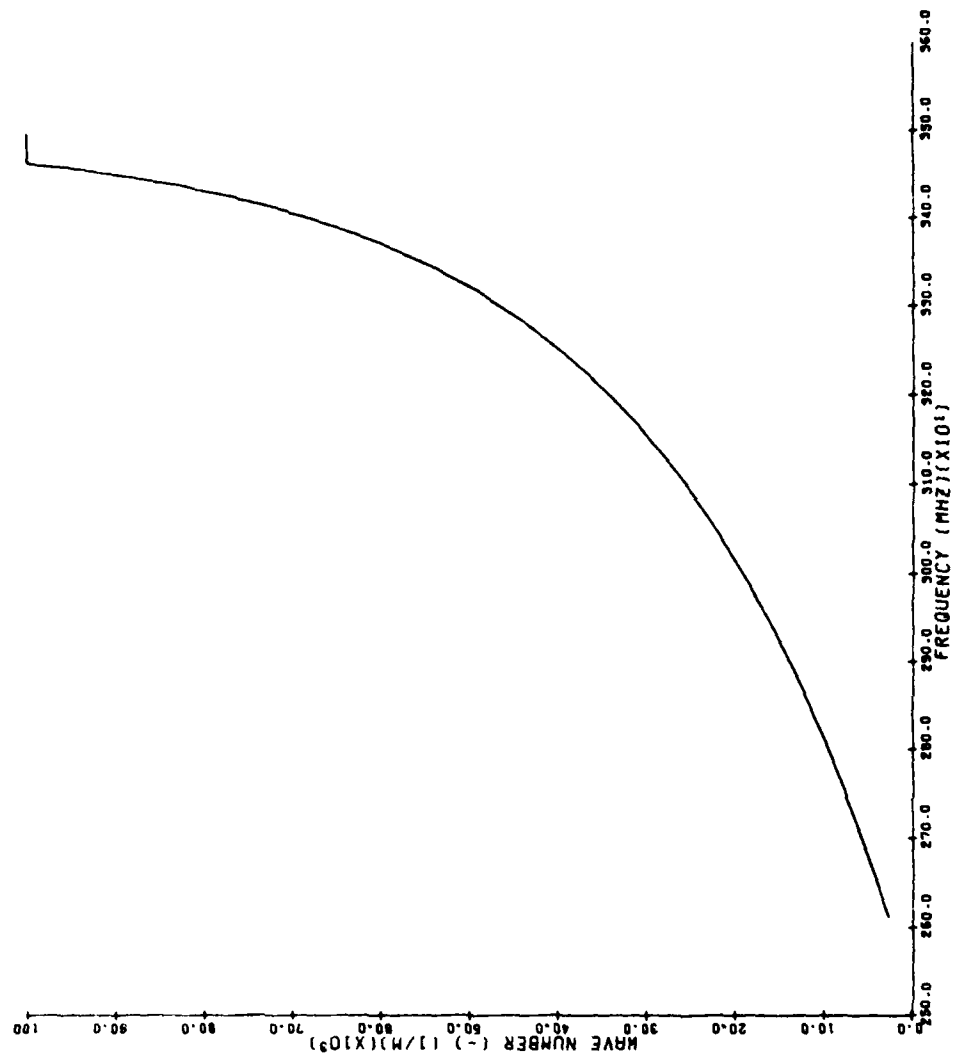


Figure 4

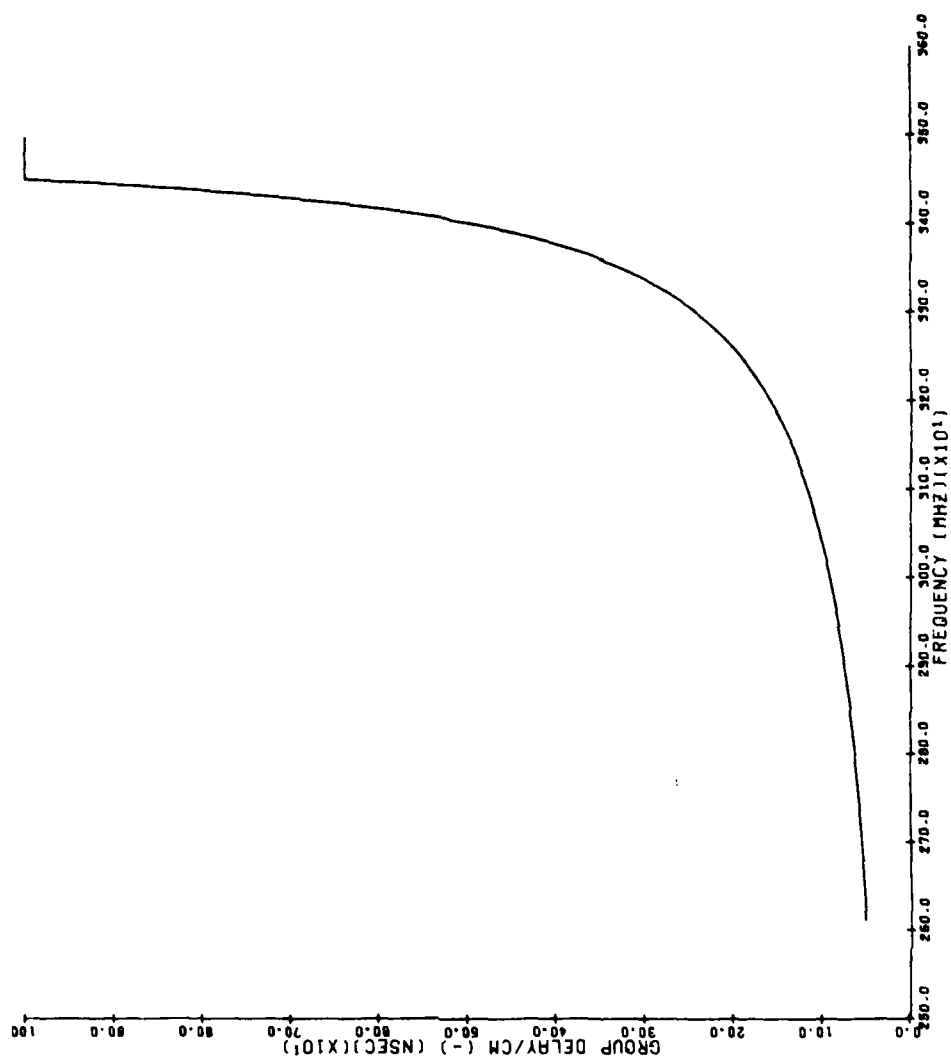


Figure 5

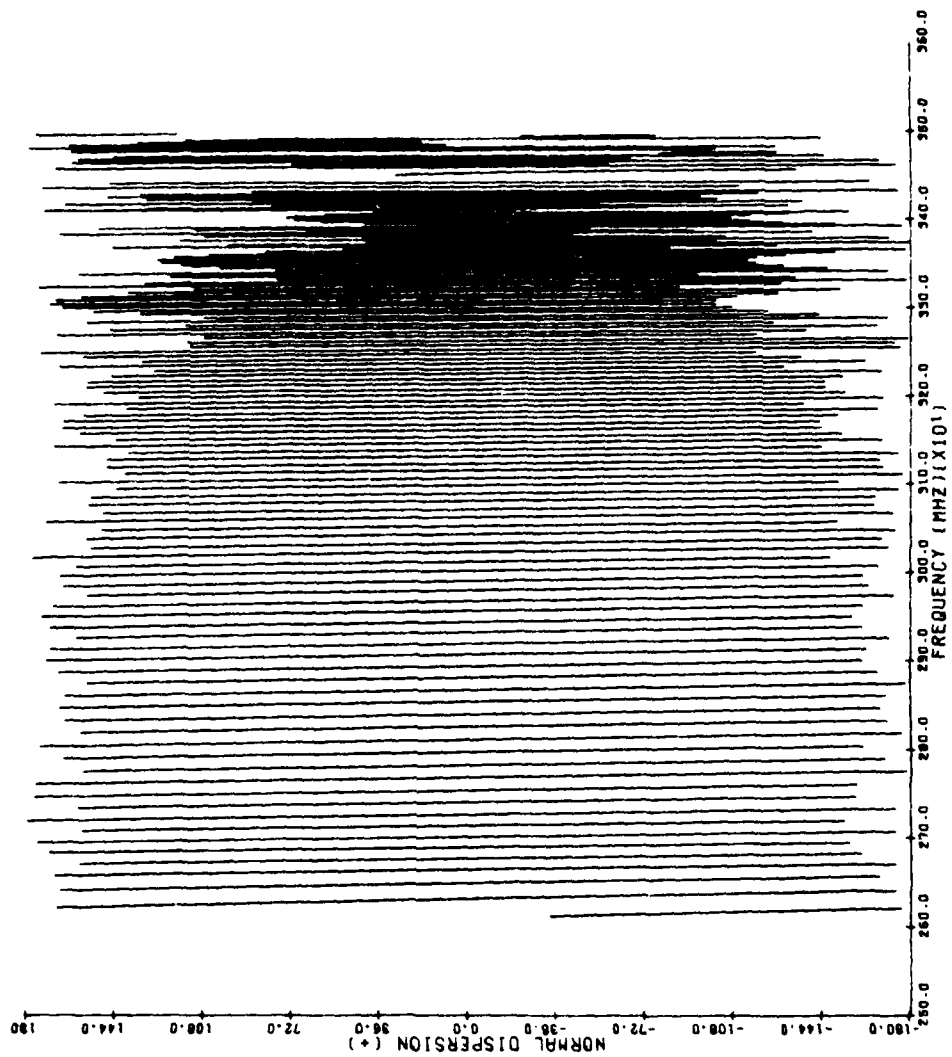


Figure 6

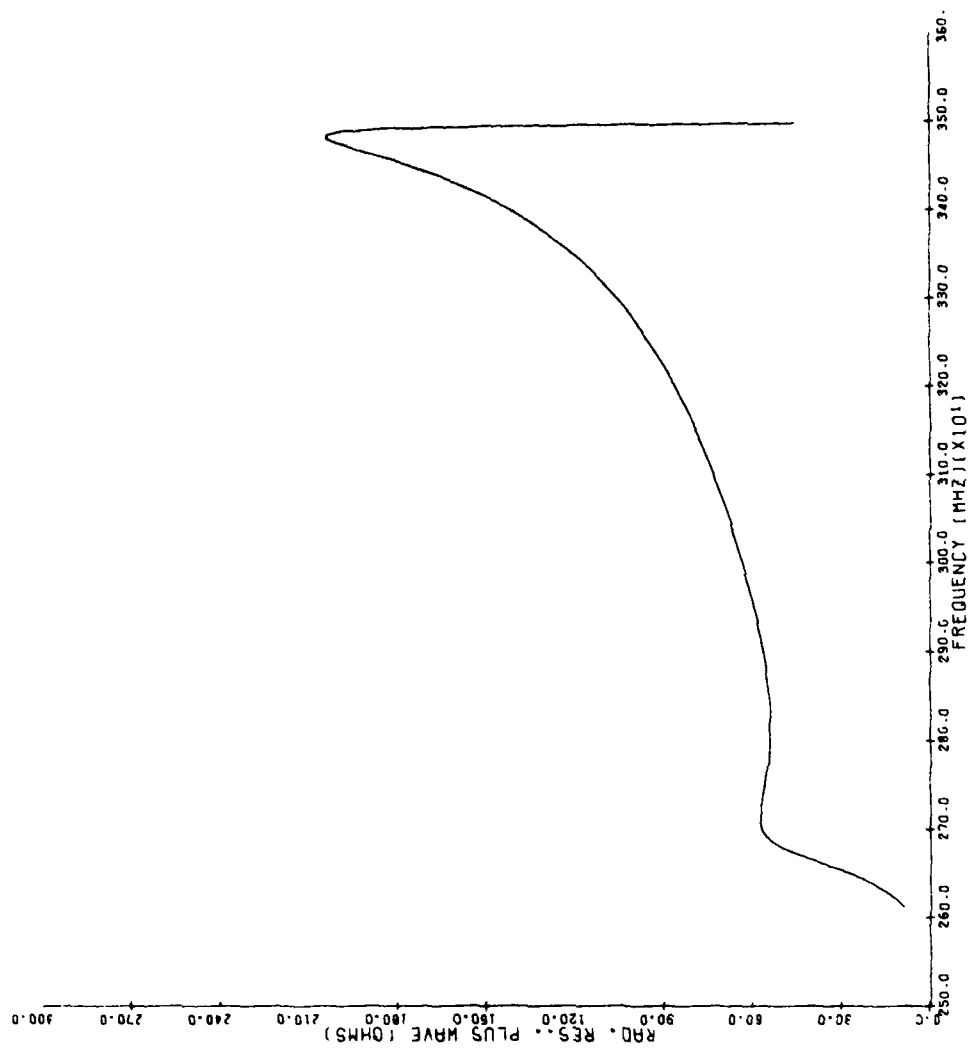


Figure 7

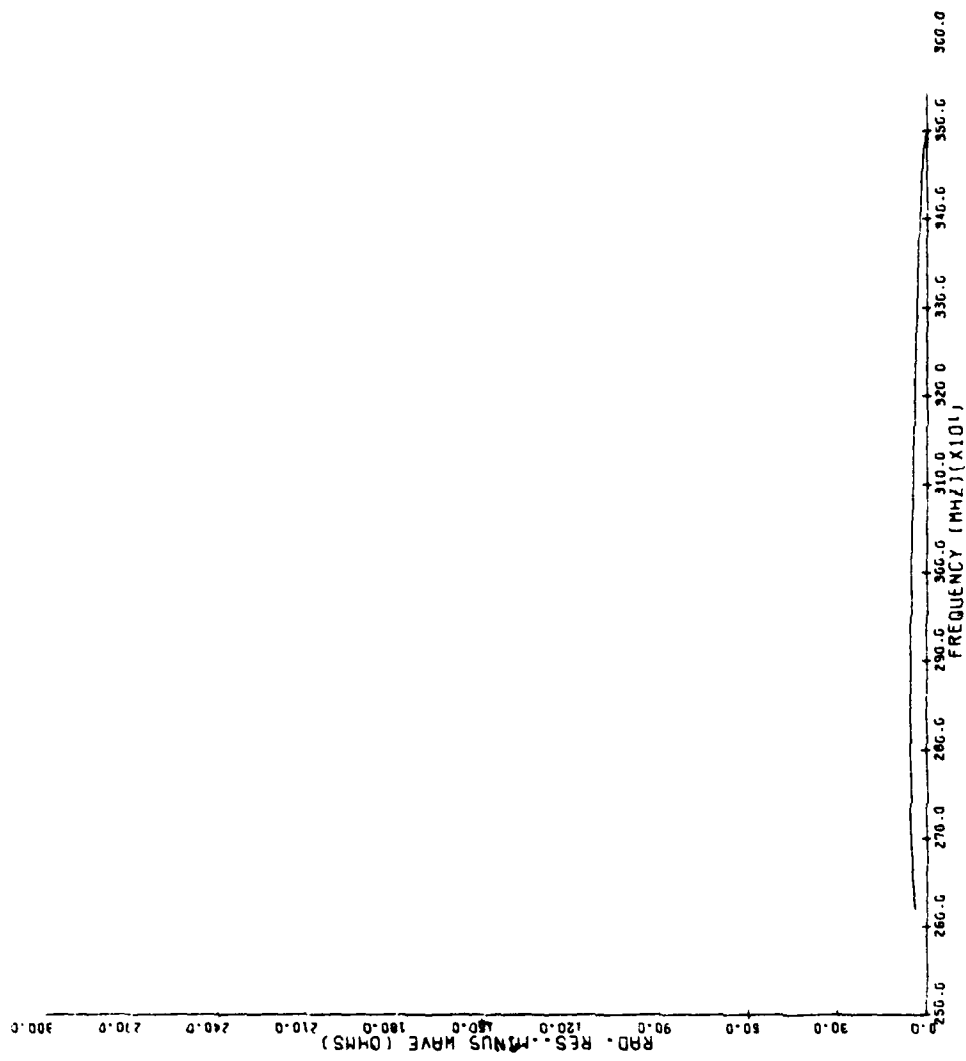


Figure 8

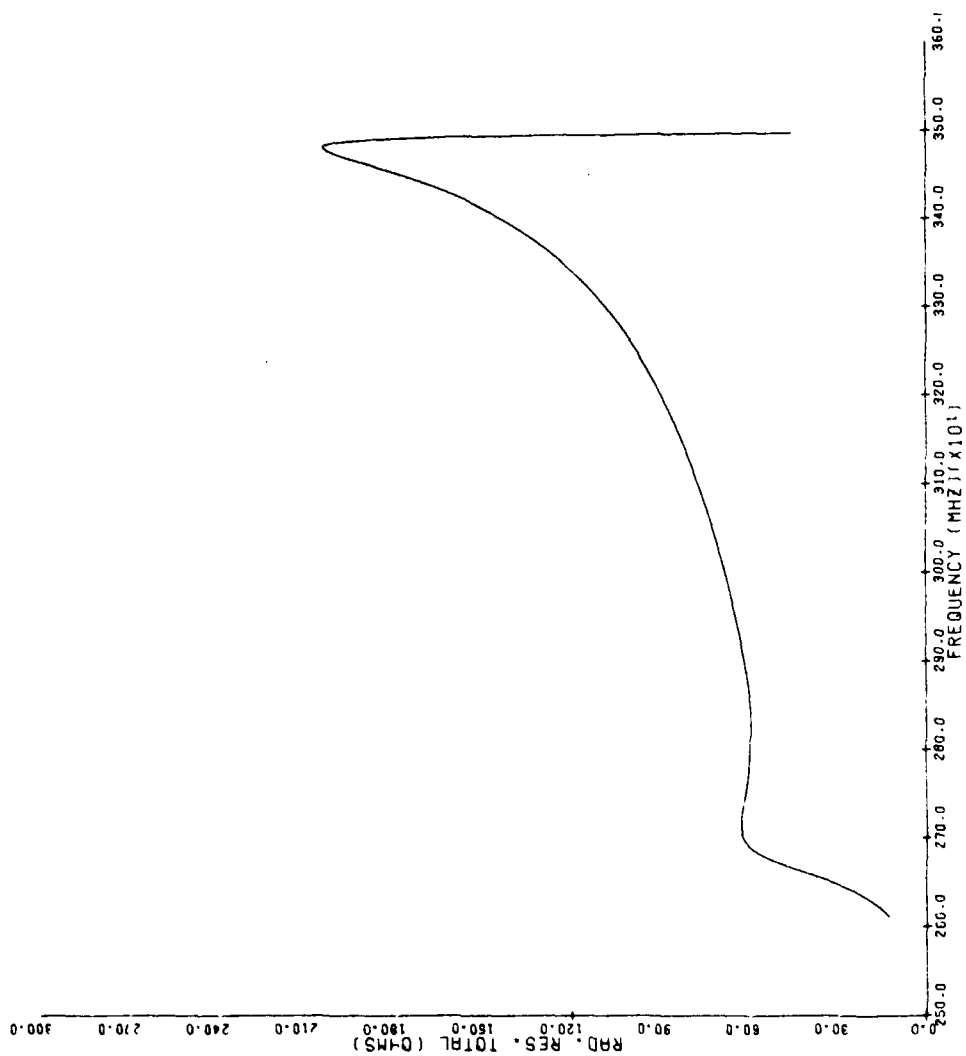


Figure 9

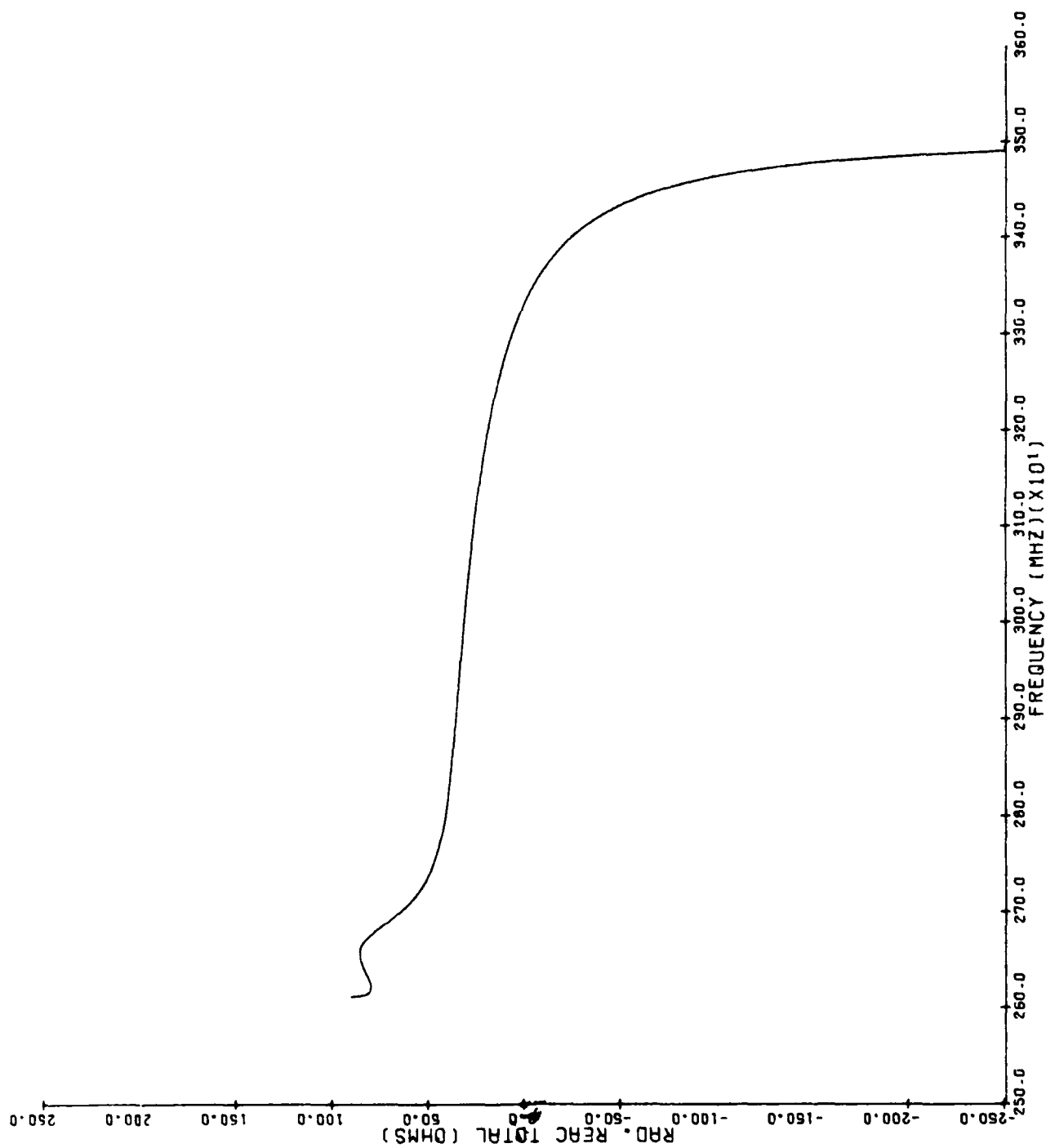


Figure 10

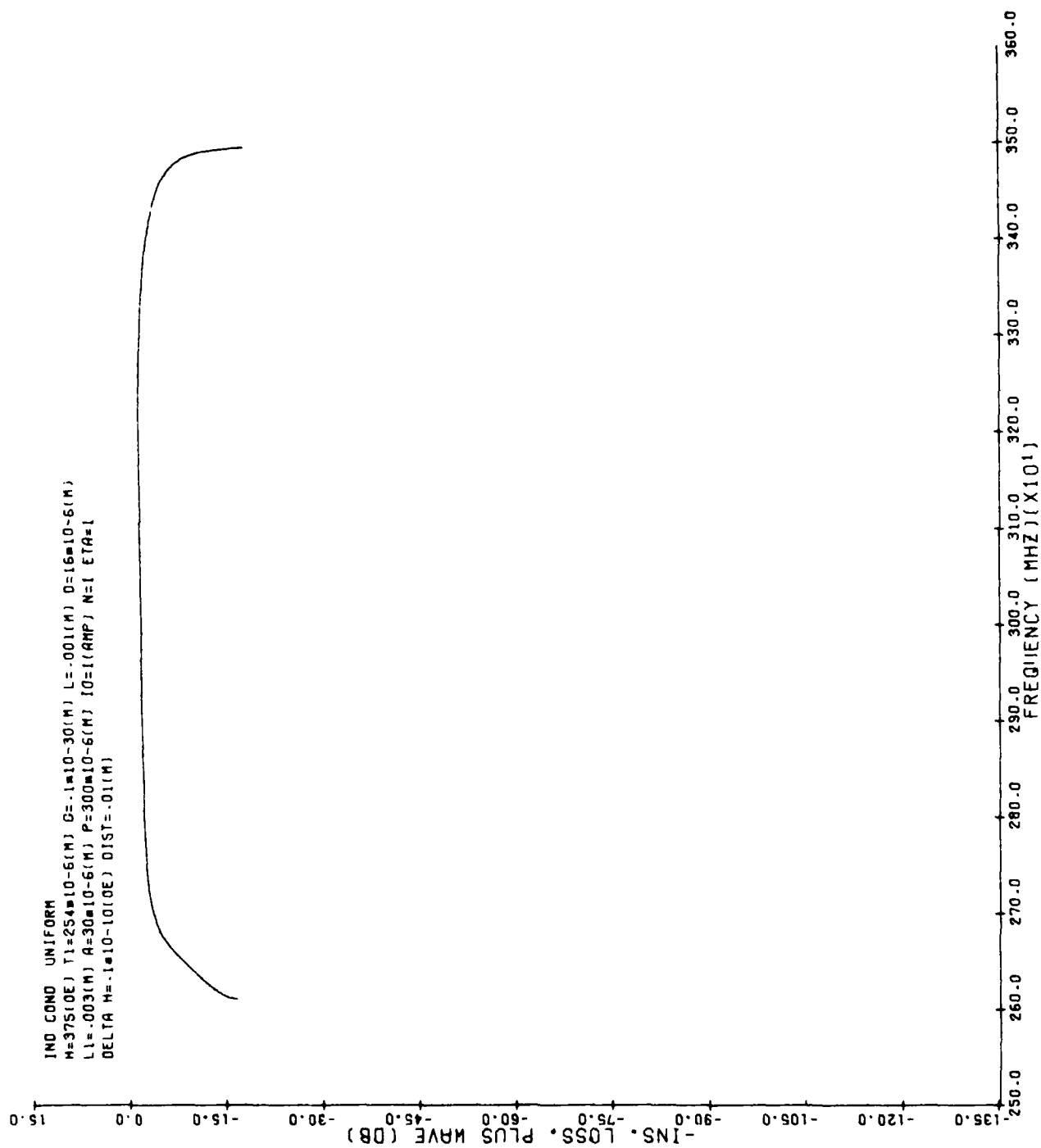


Figure 11

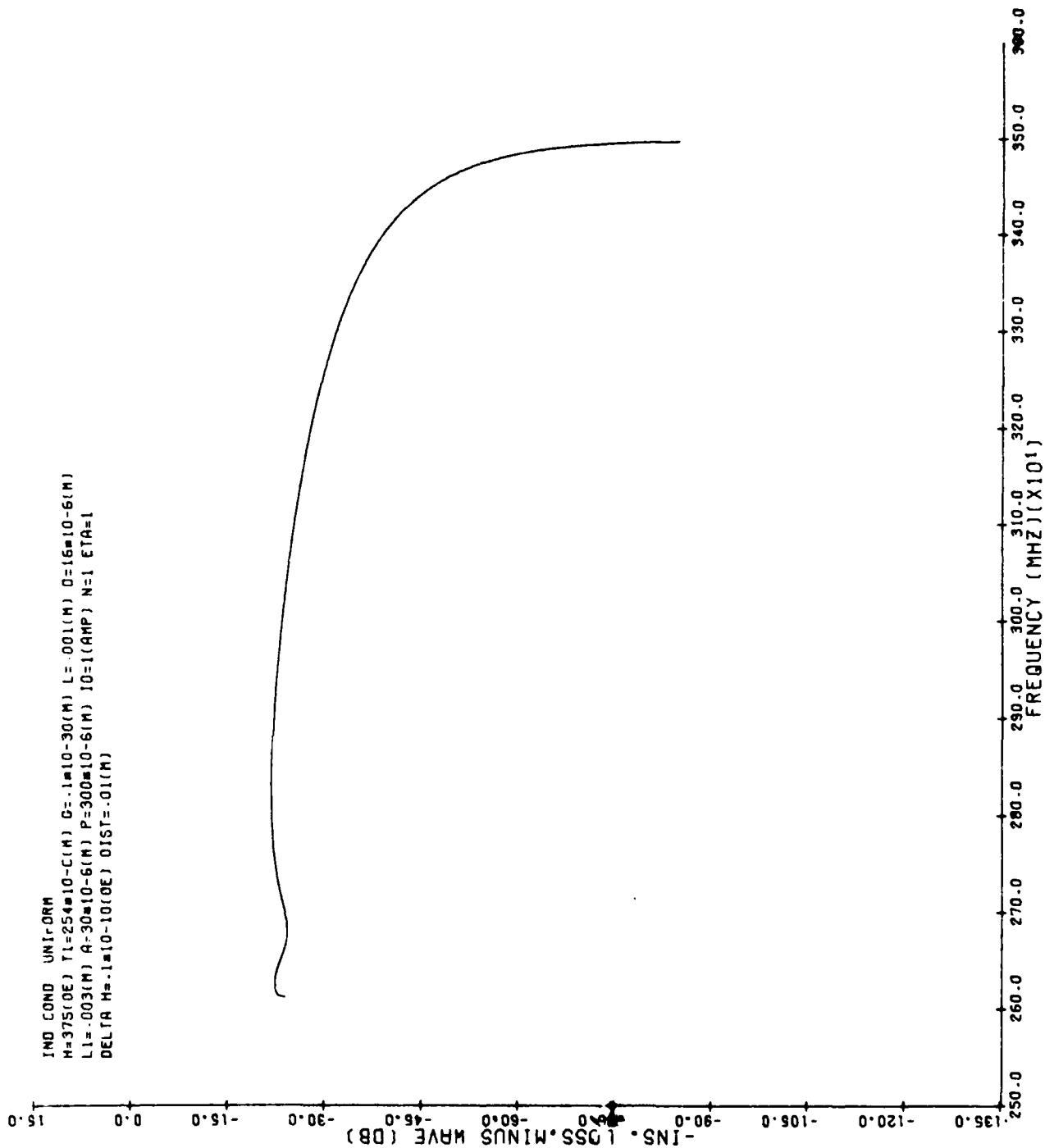


Figure 12

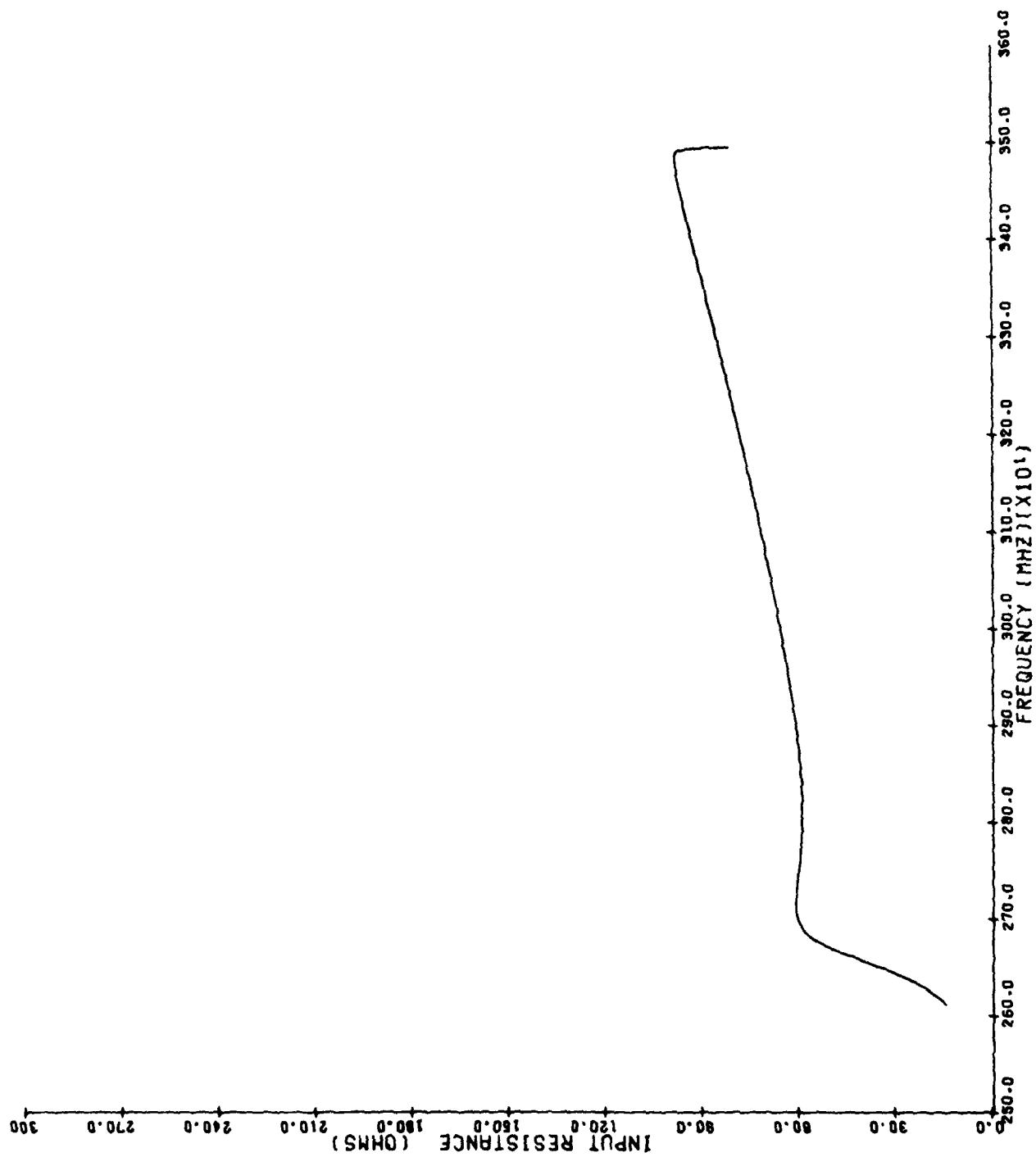
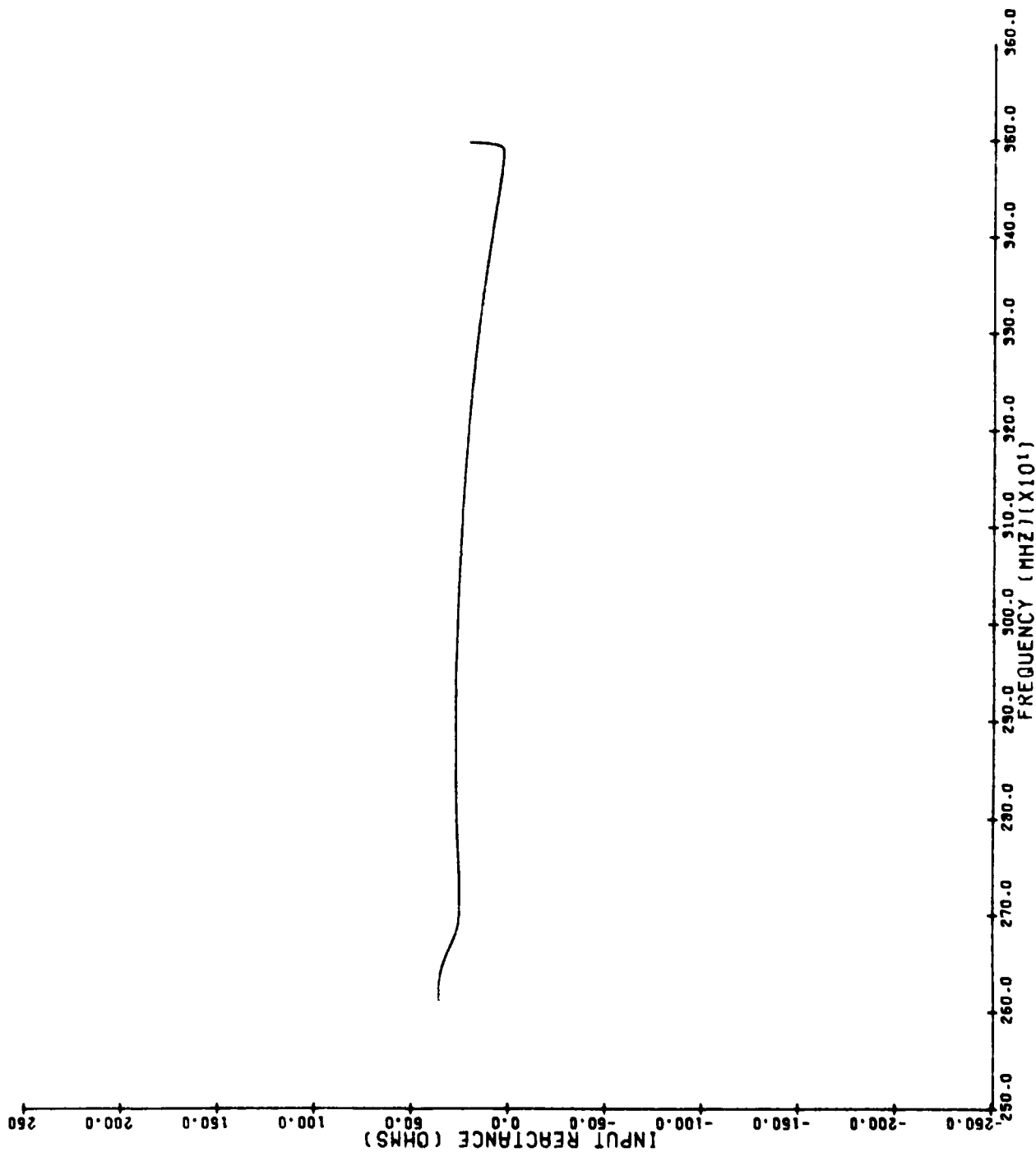


Figure 13



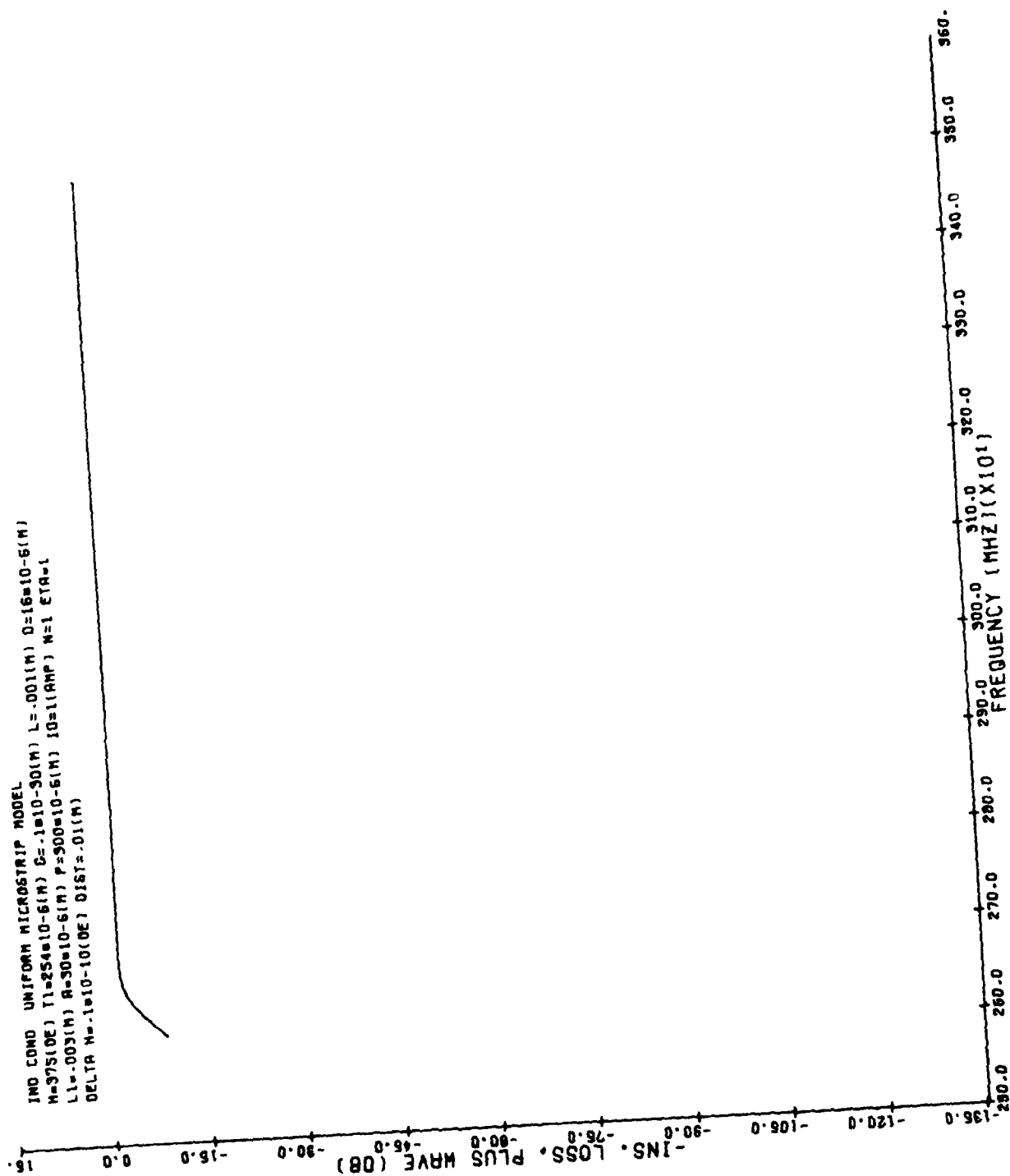


Figure 15

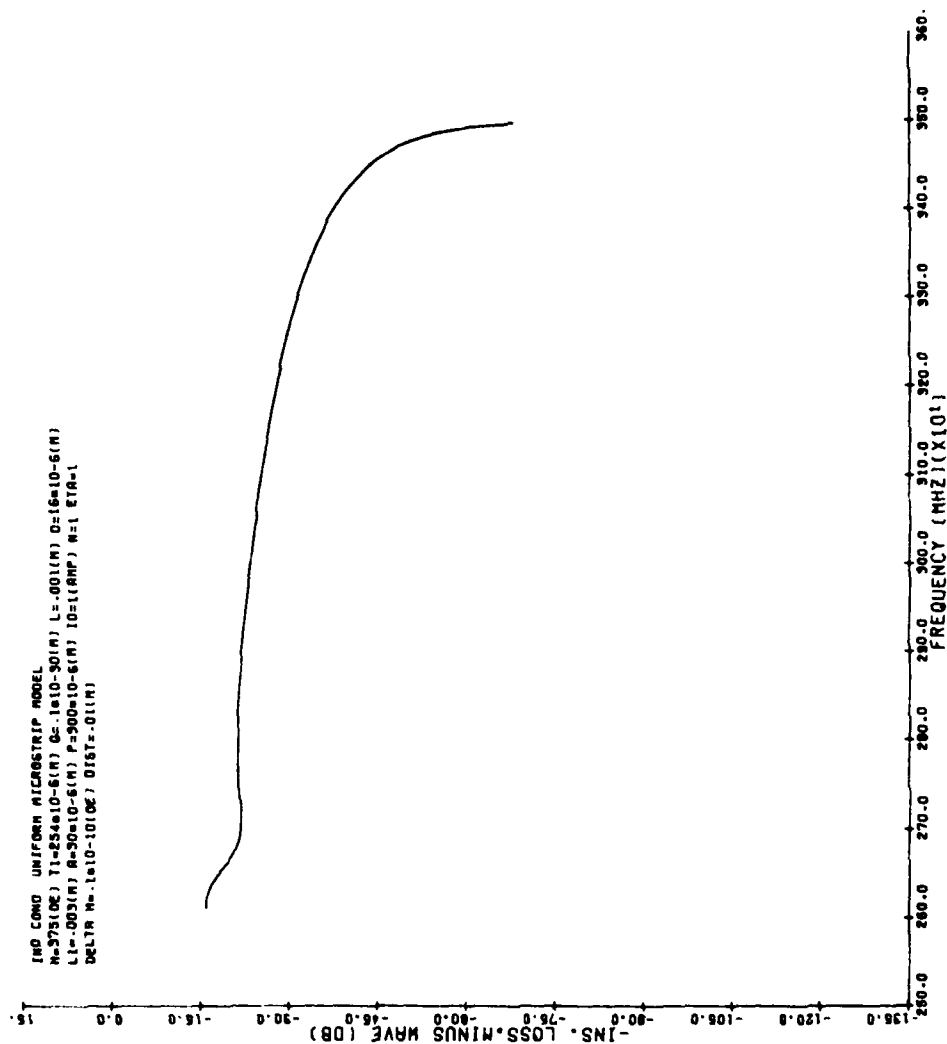


Figure 16

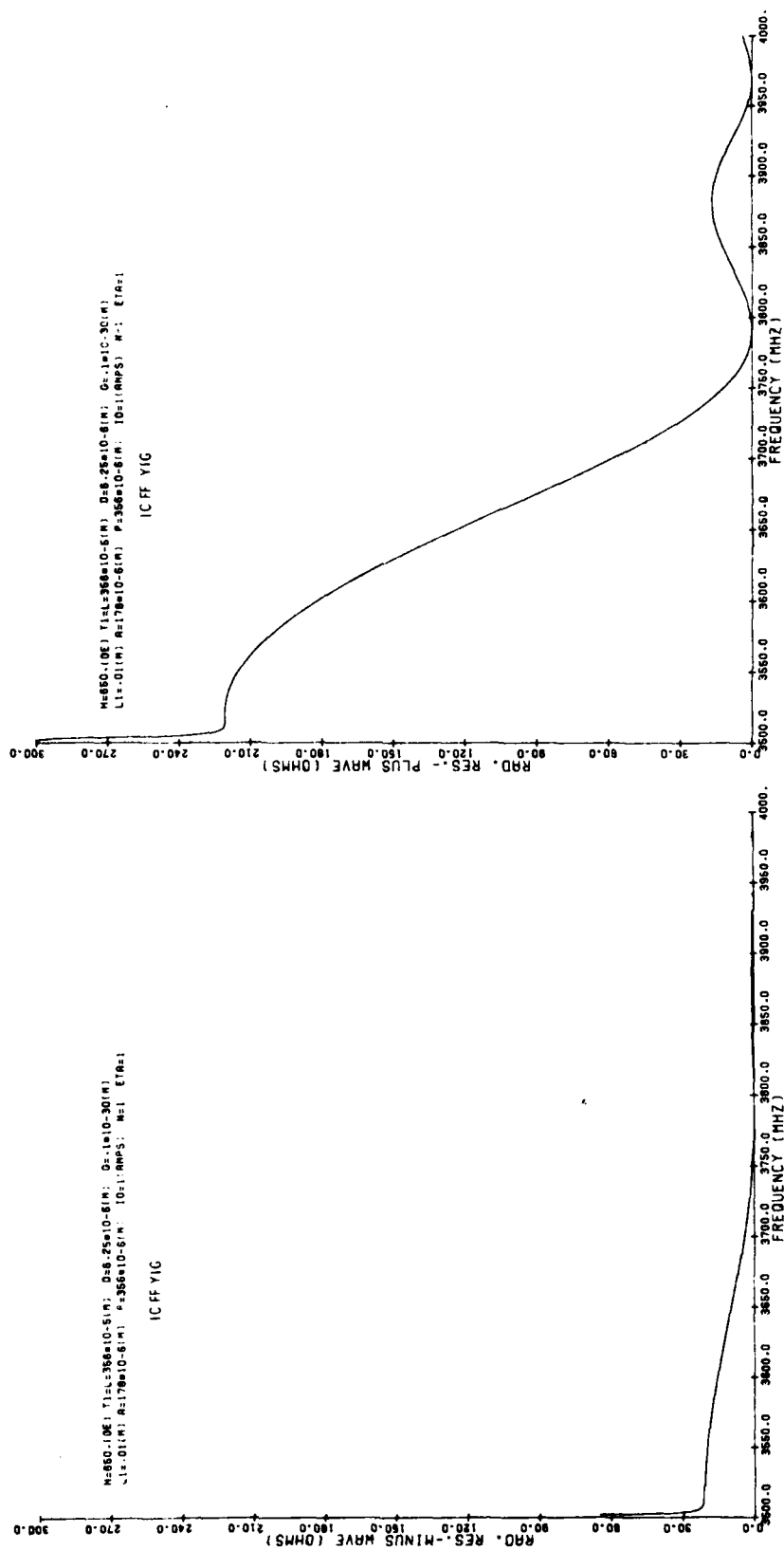


Figure 17

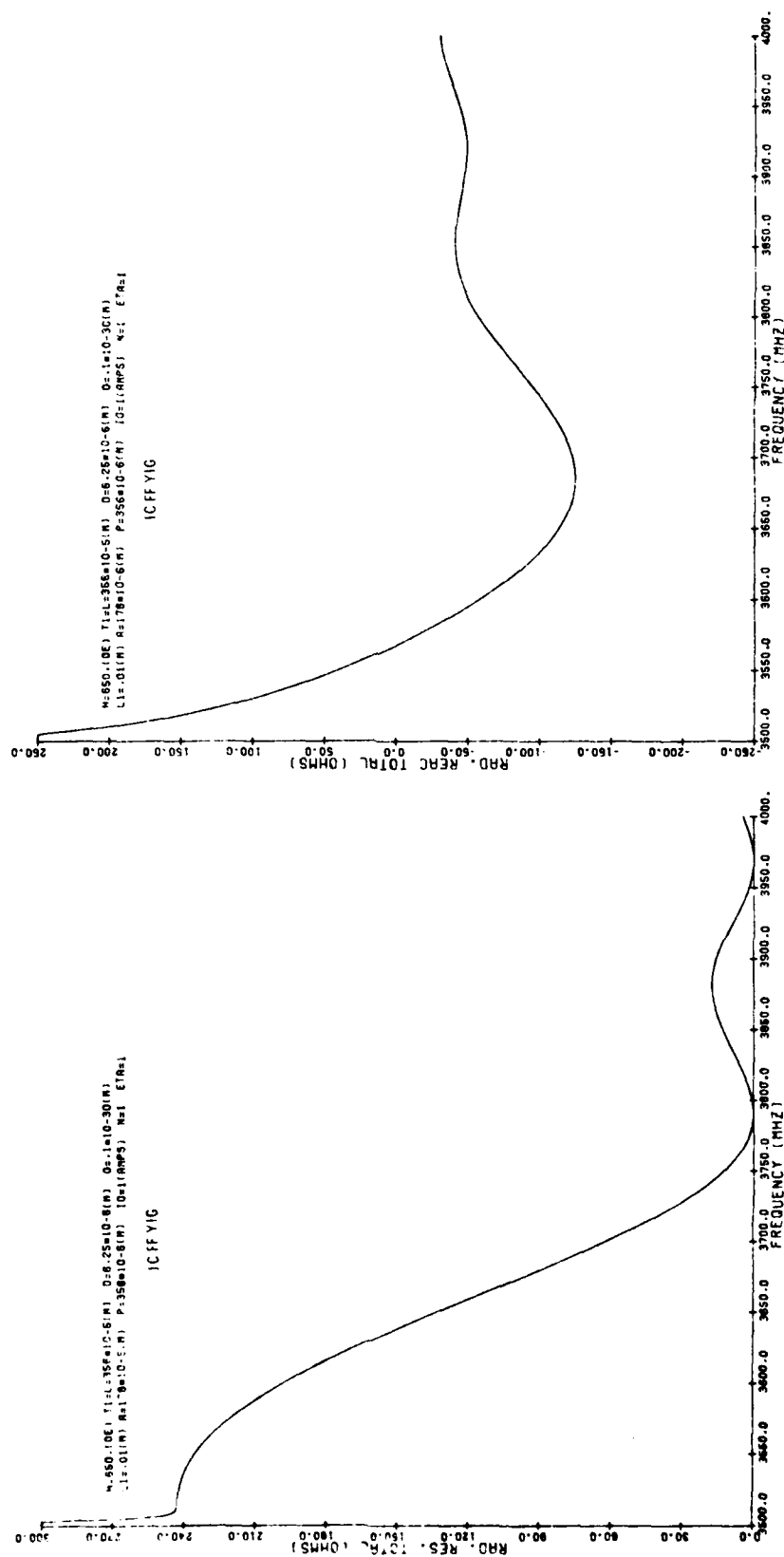


Figure 18

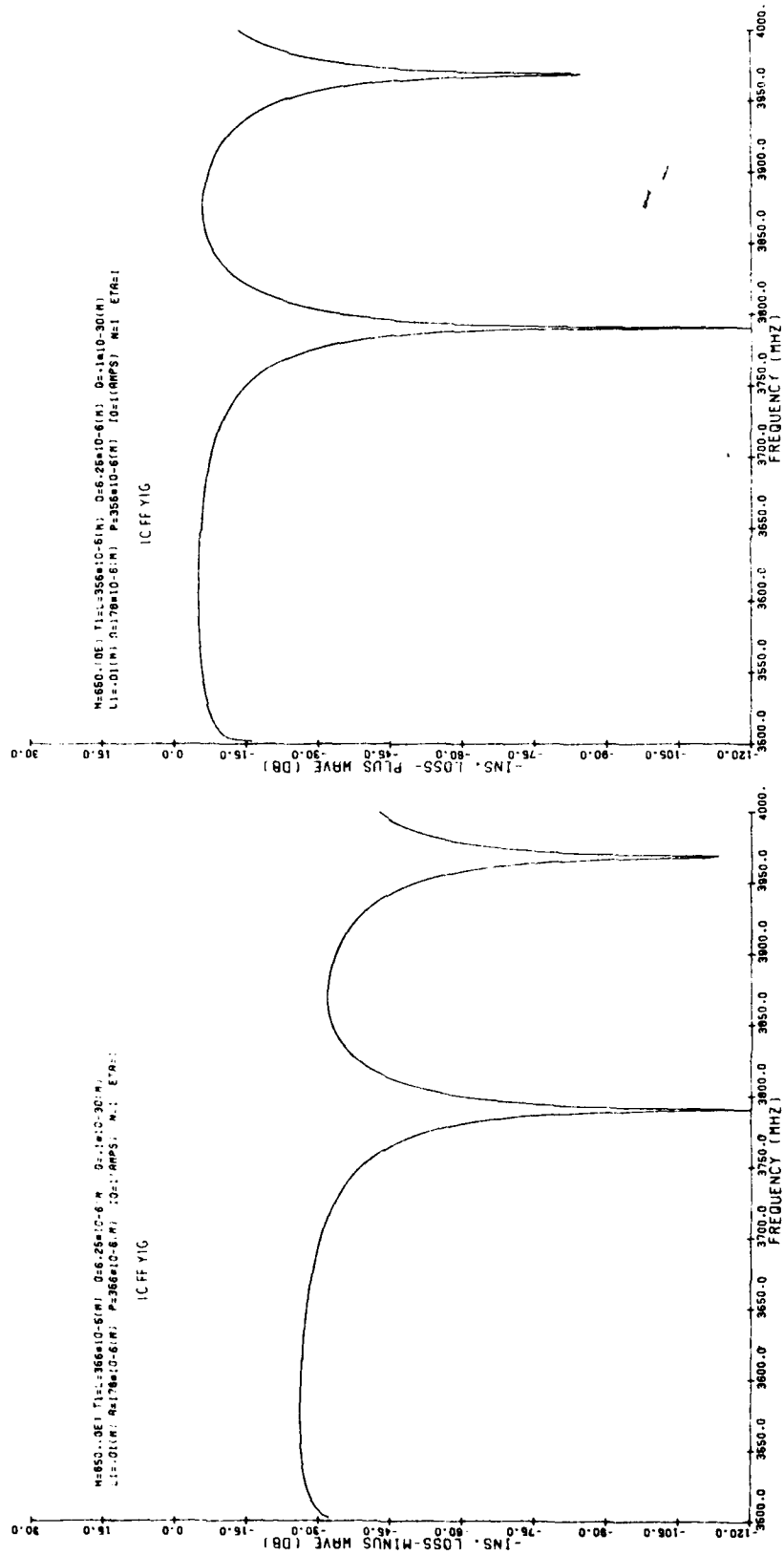


Figure 19

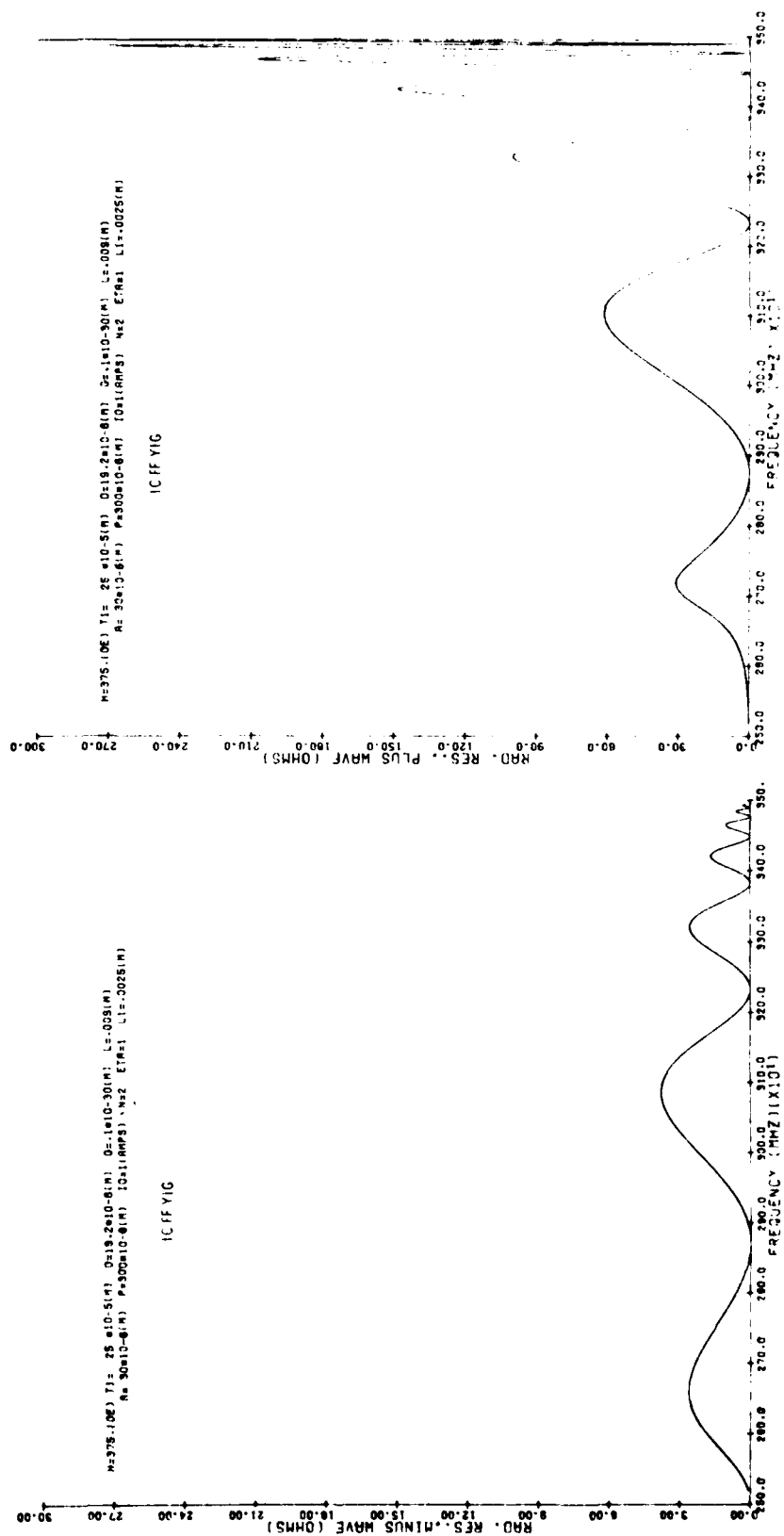


Figure 20

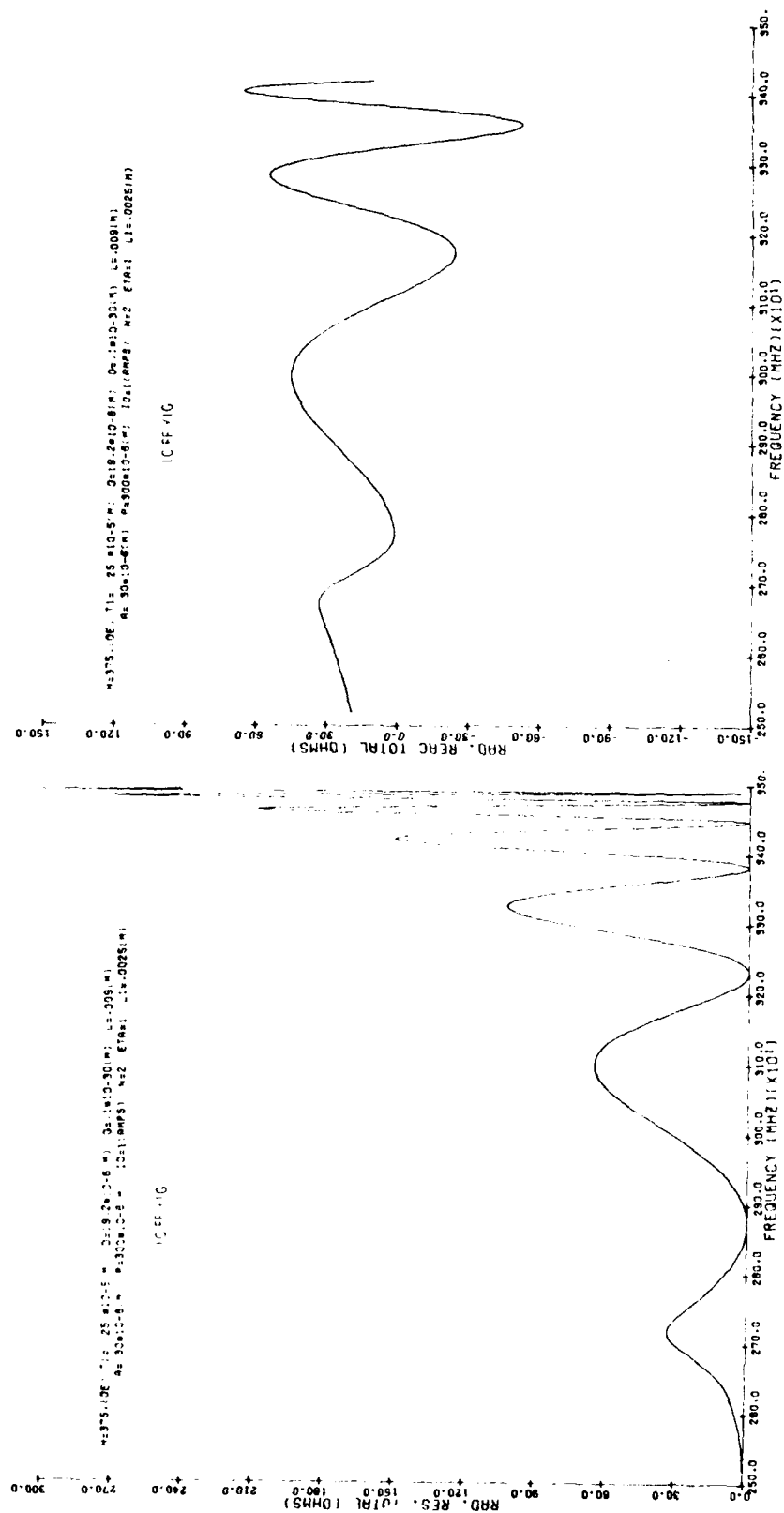


Figure 21

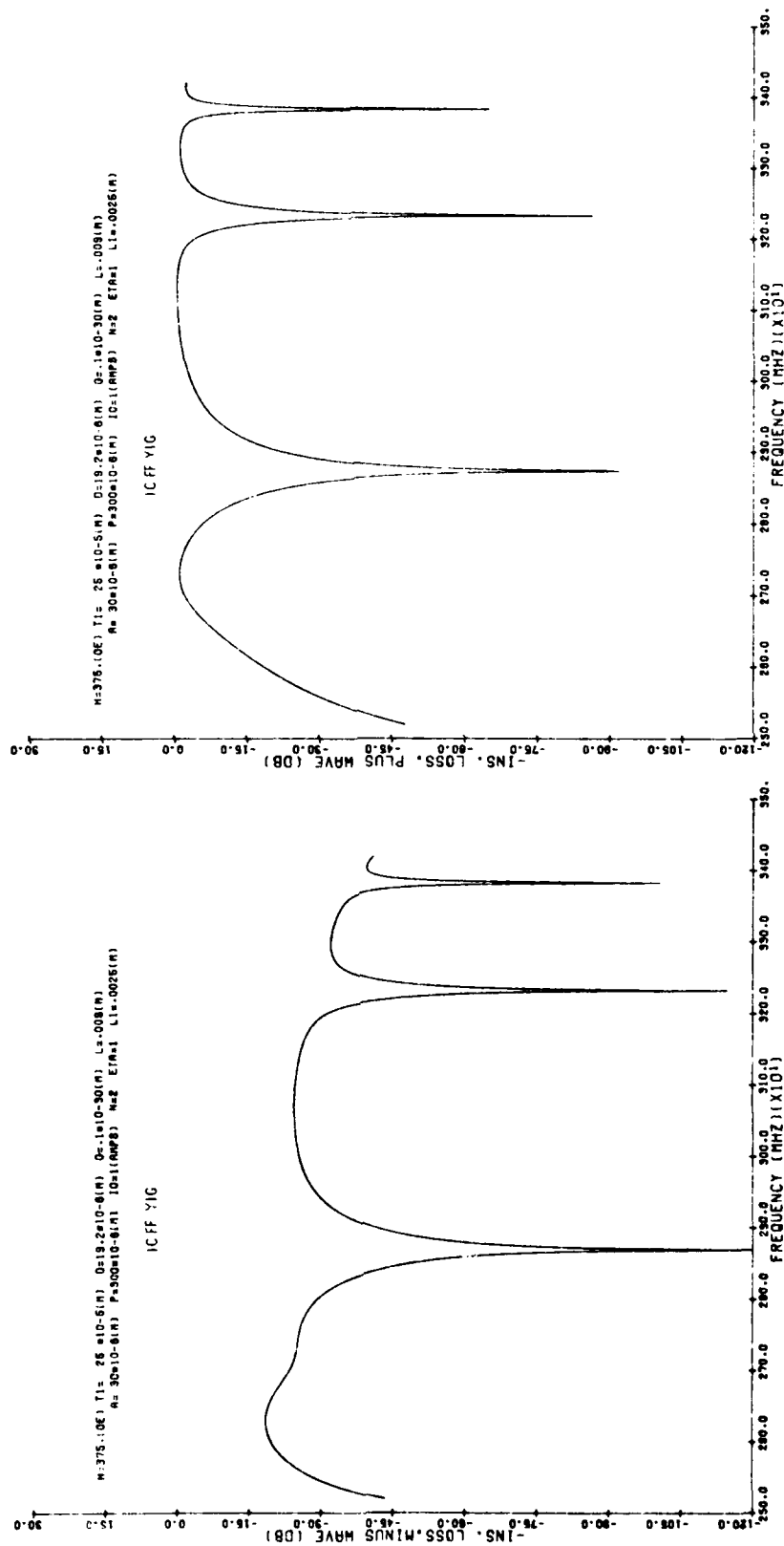


Figure 22

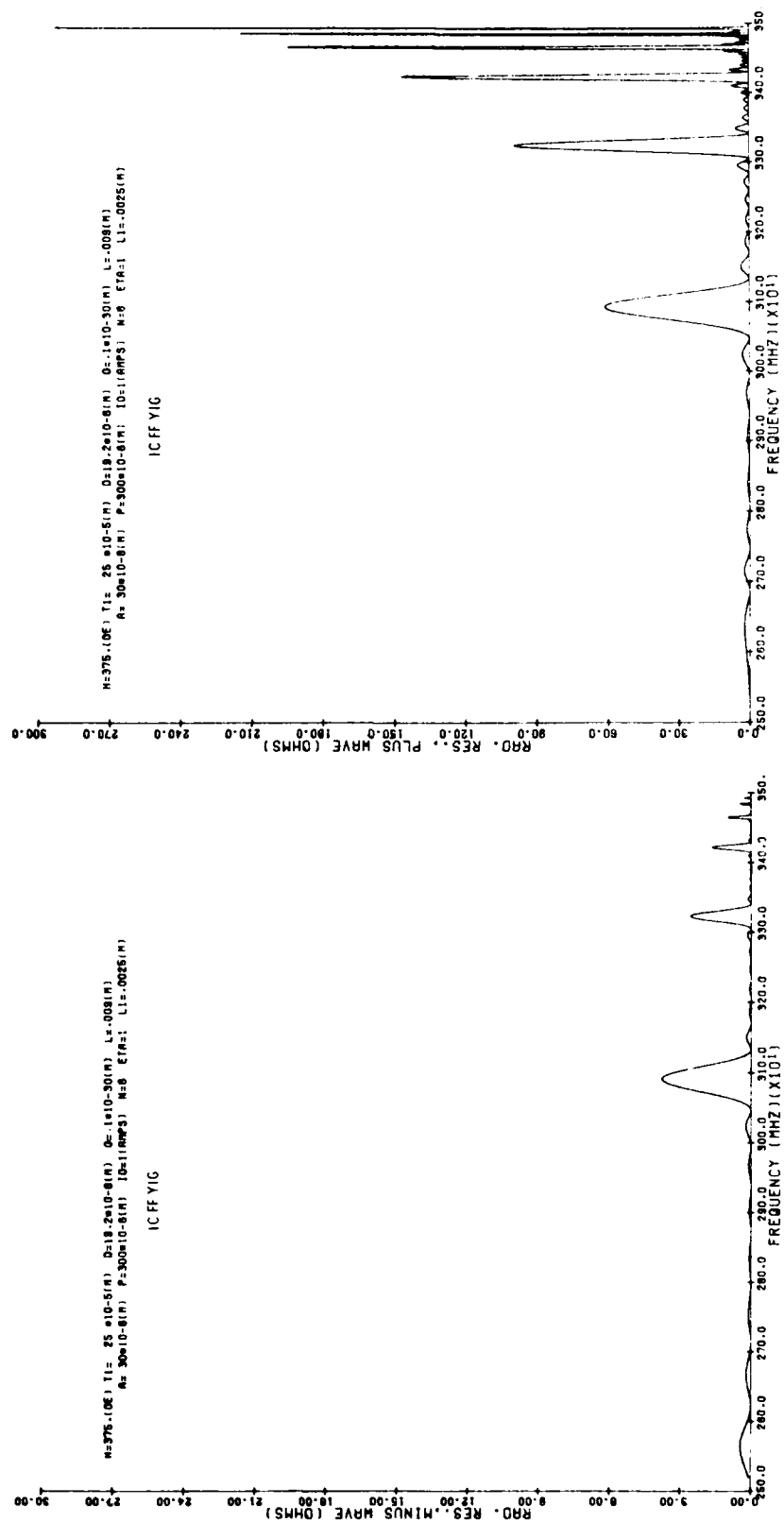


Figure 23

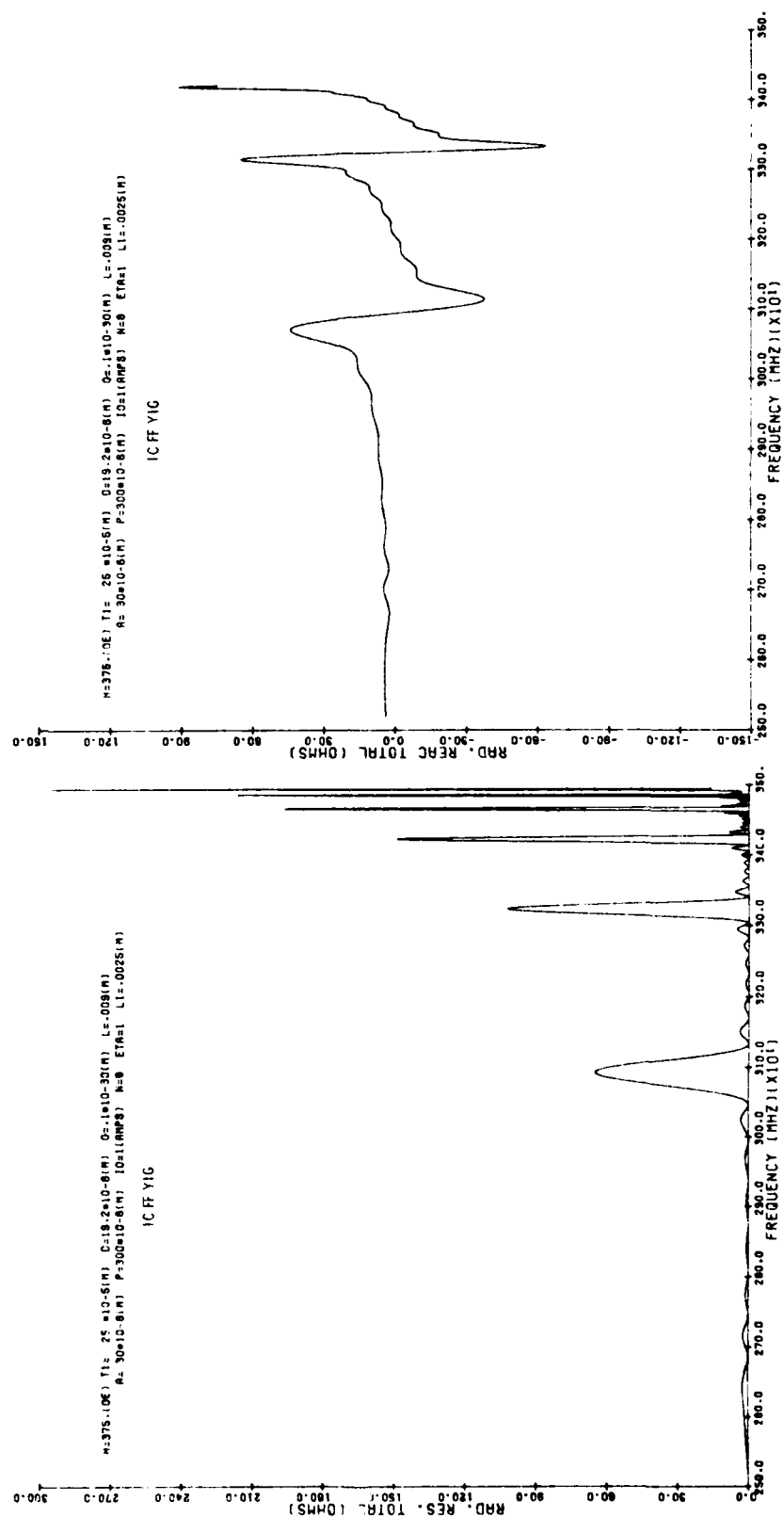
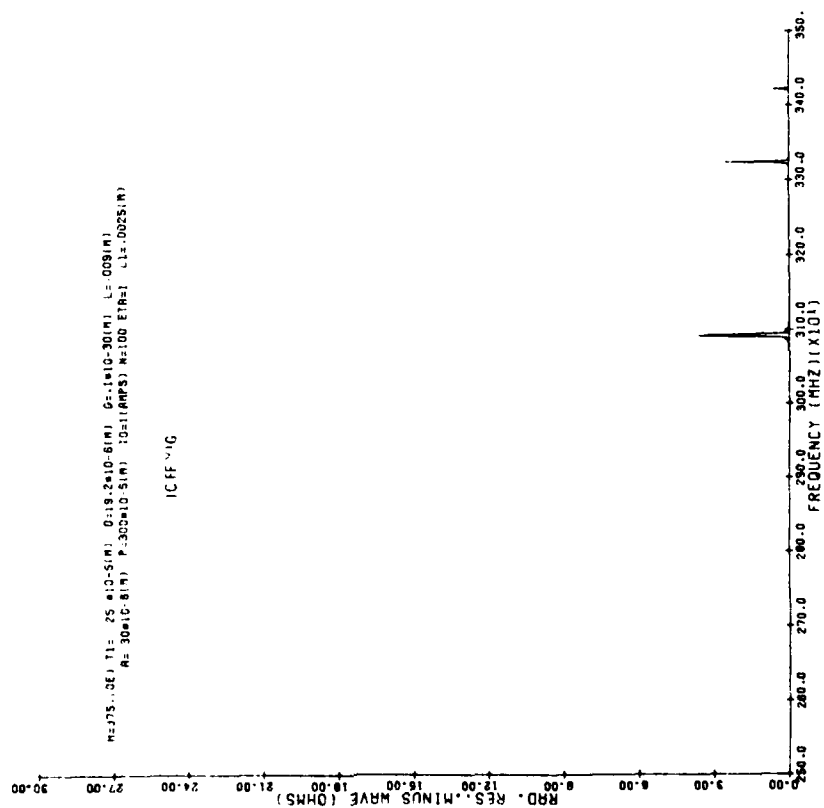
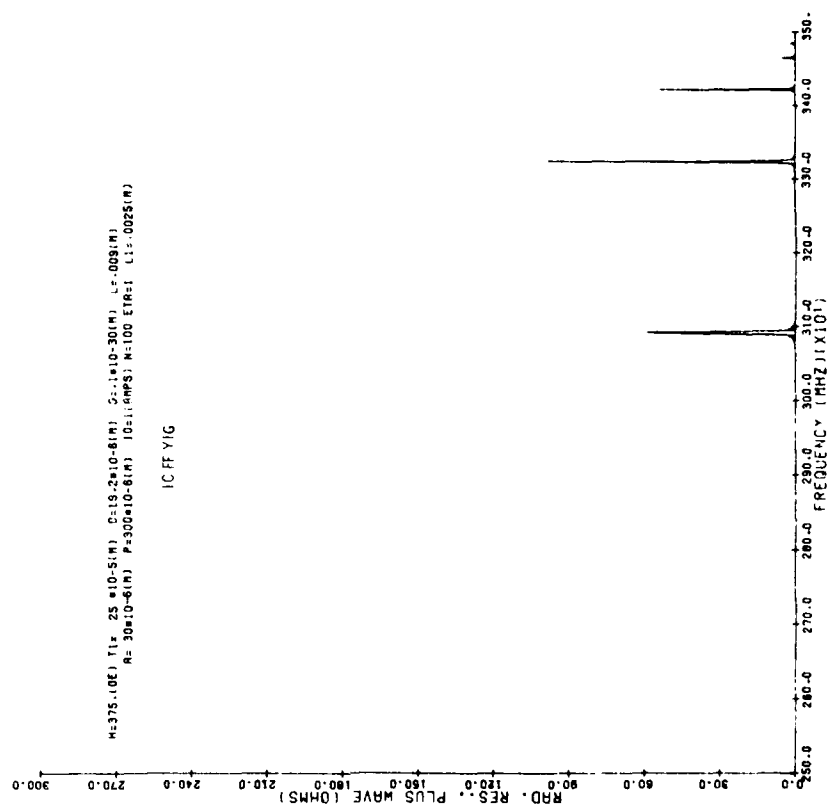


Figure 24



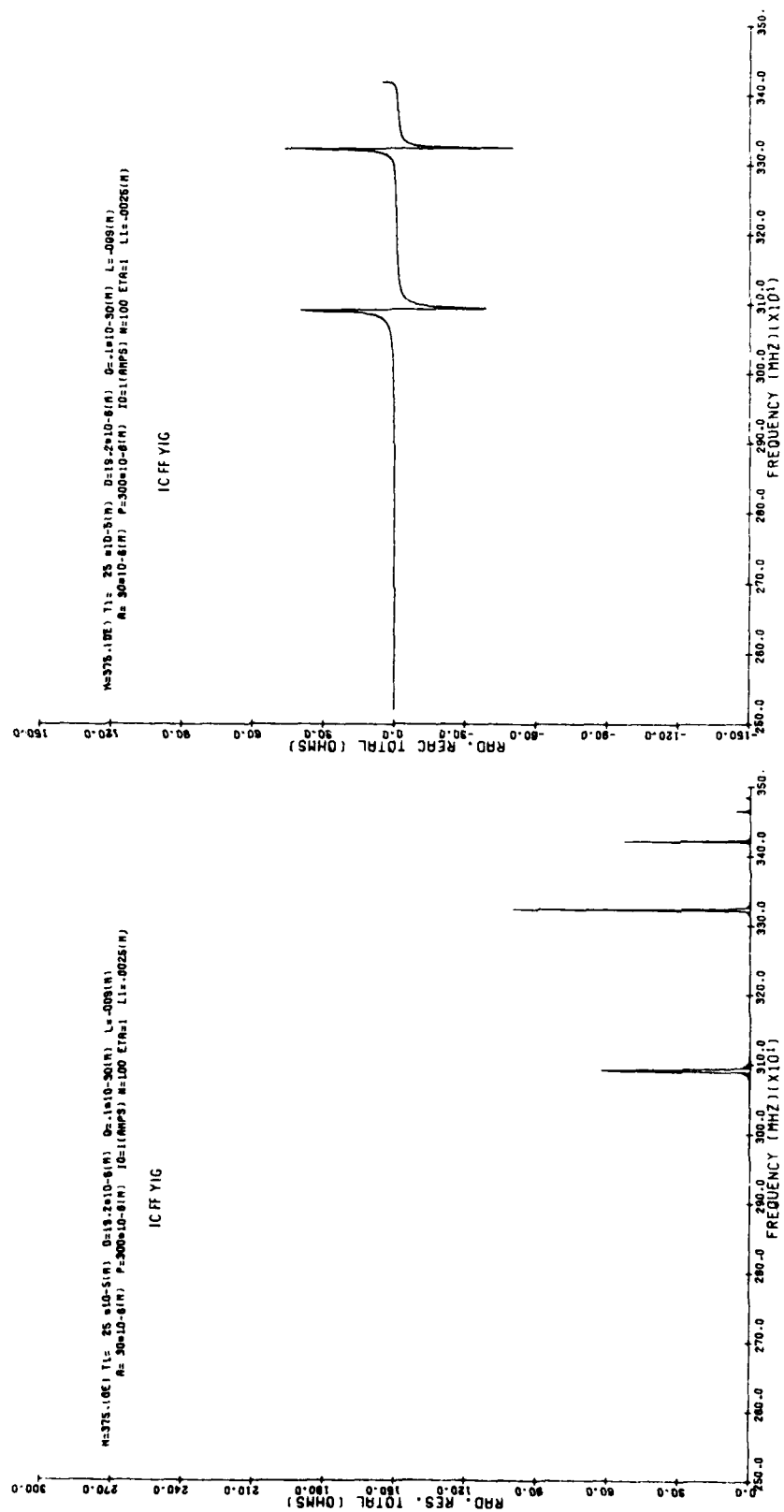


Figure 27

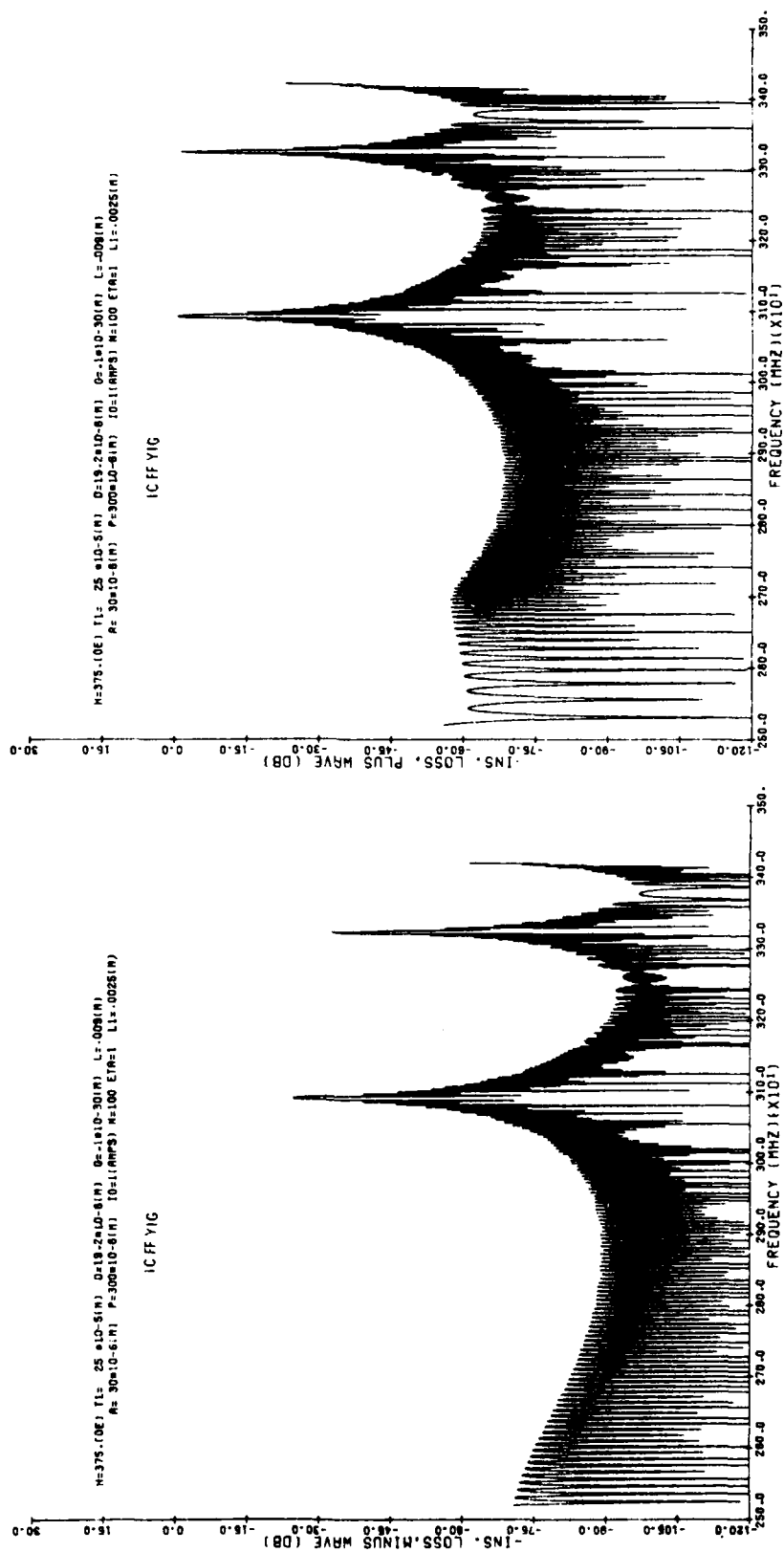


Figure 28

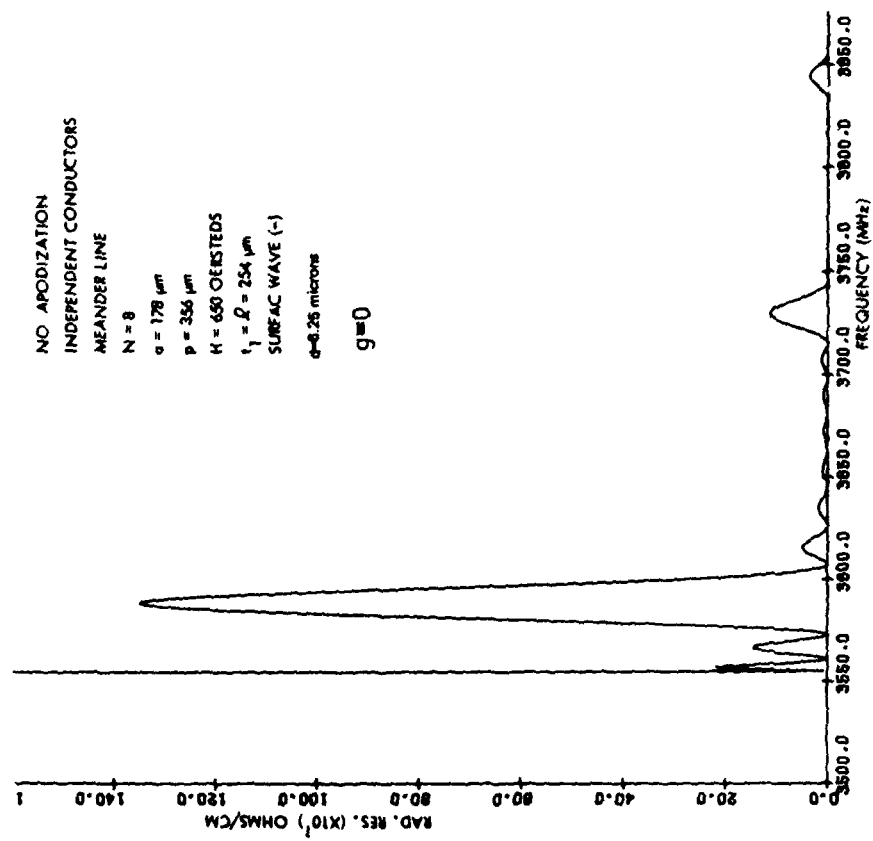


Figure 29

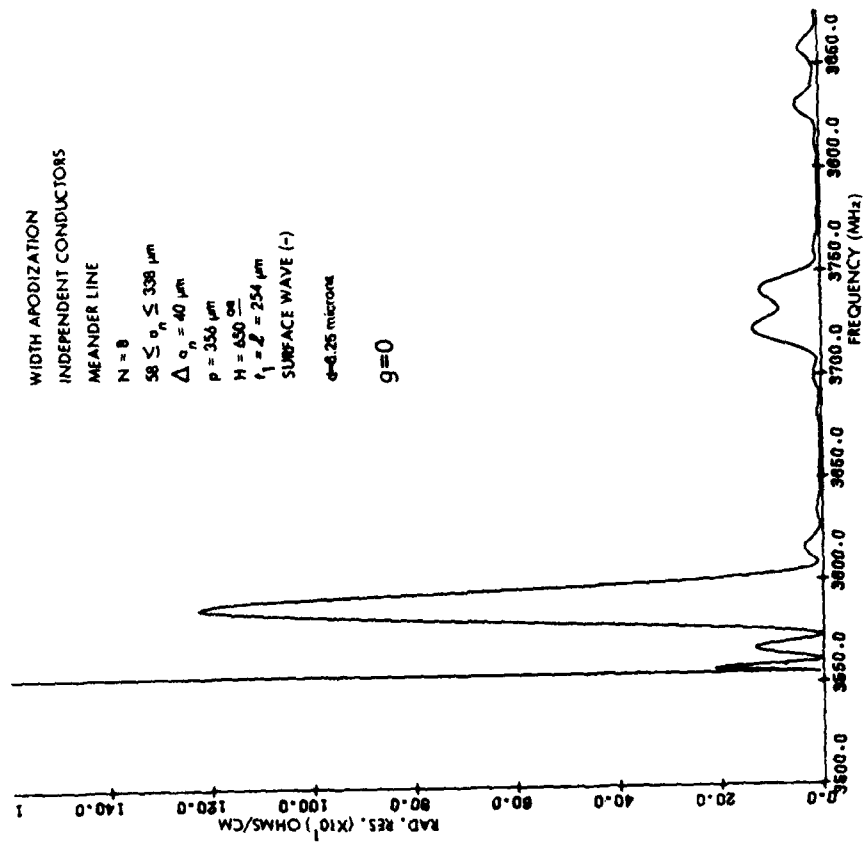


Figure 30

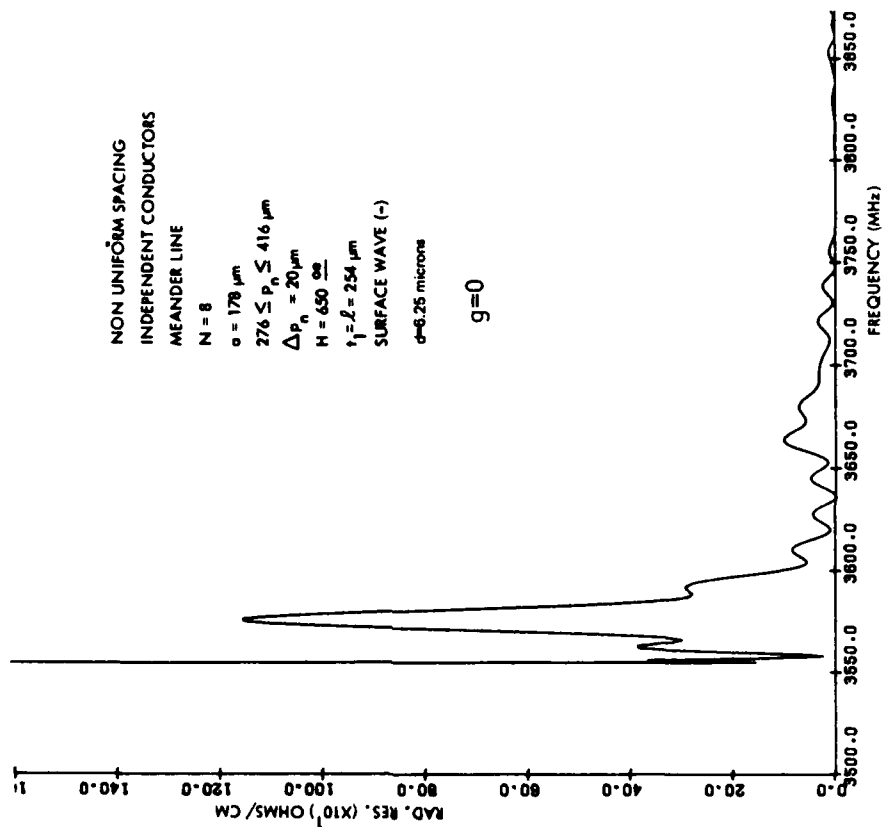


Figure 31

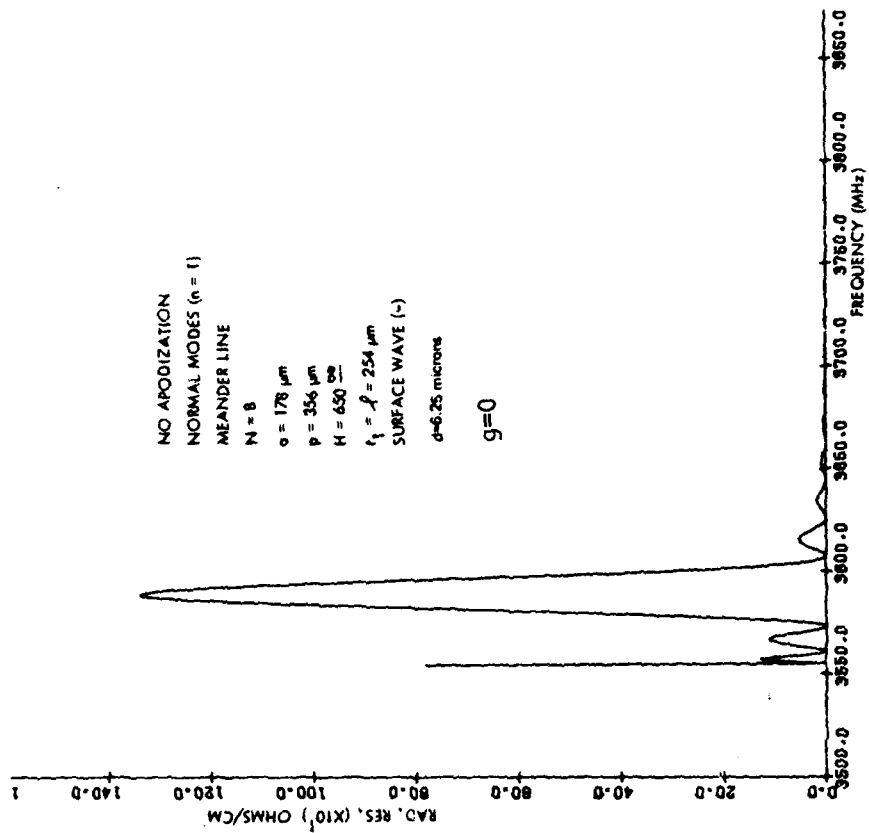


Figure 32

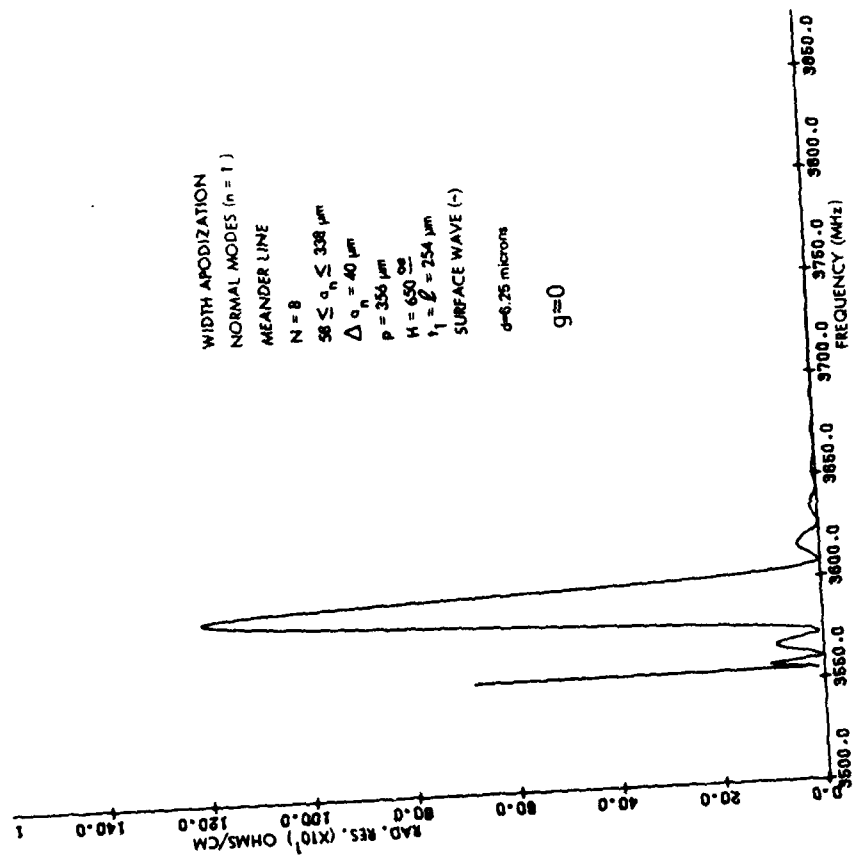


Figure 33

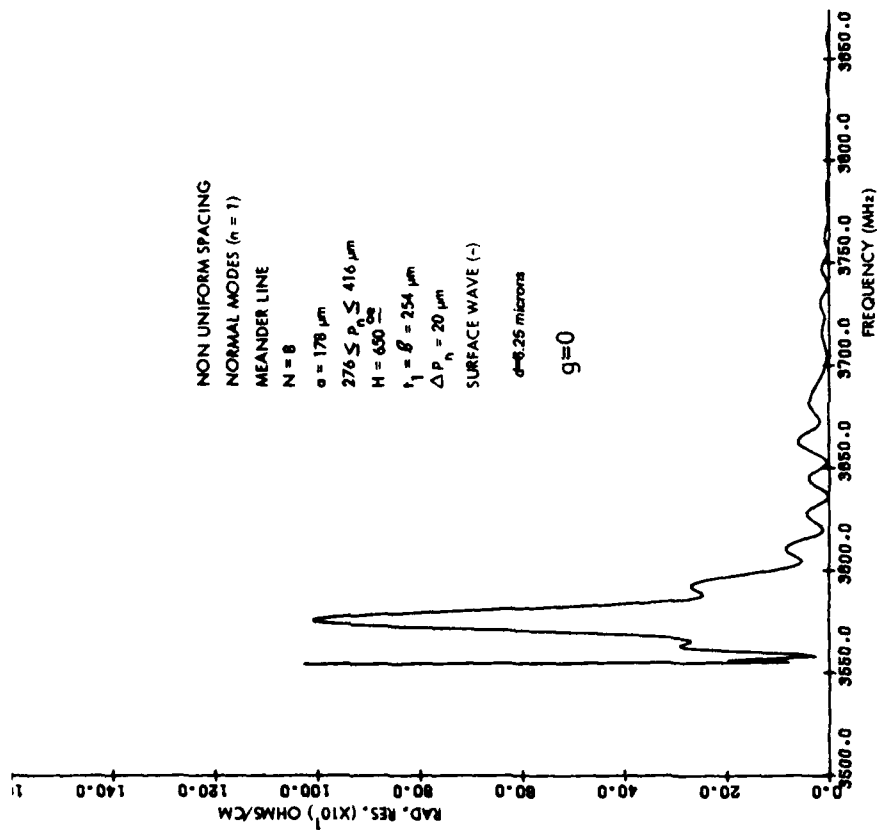


Figure 34

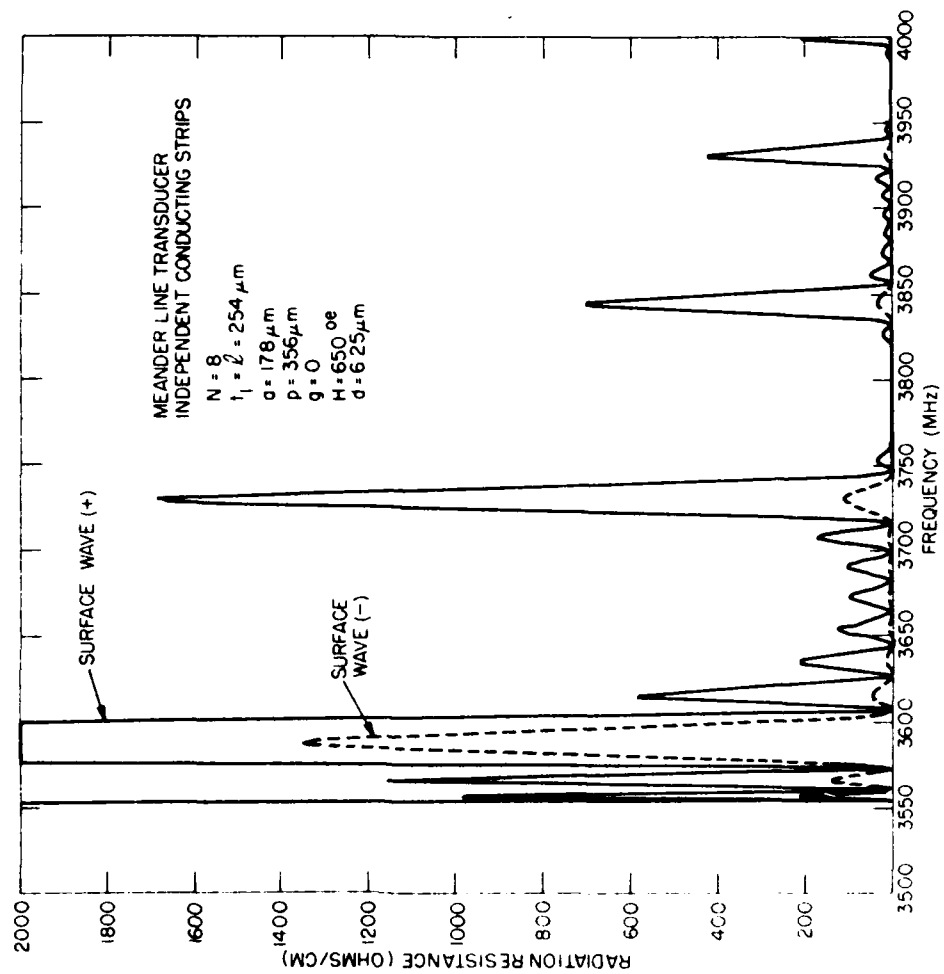


Figure 35

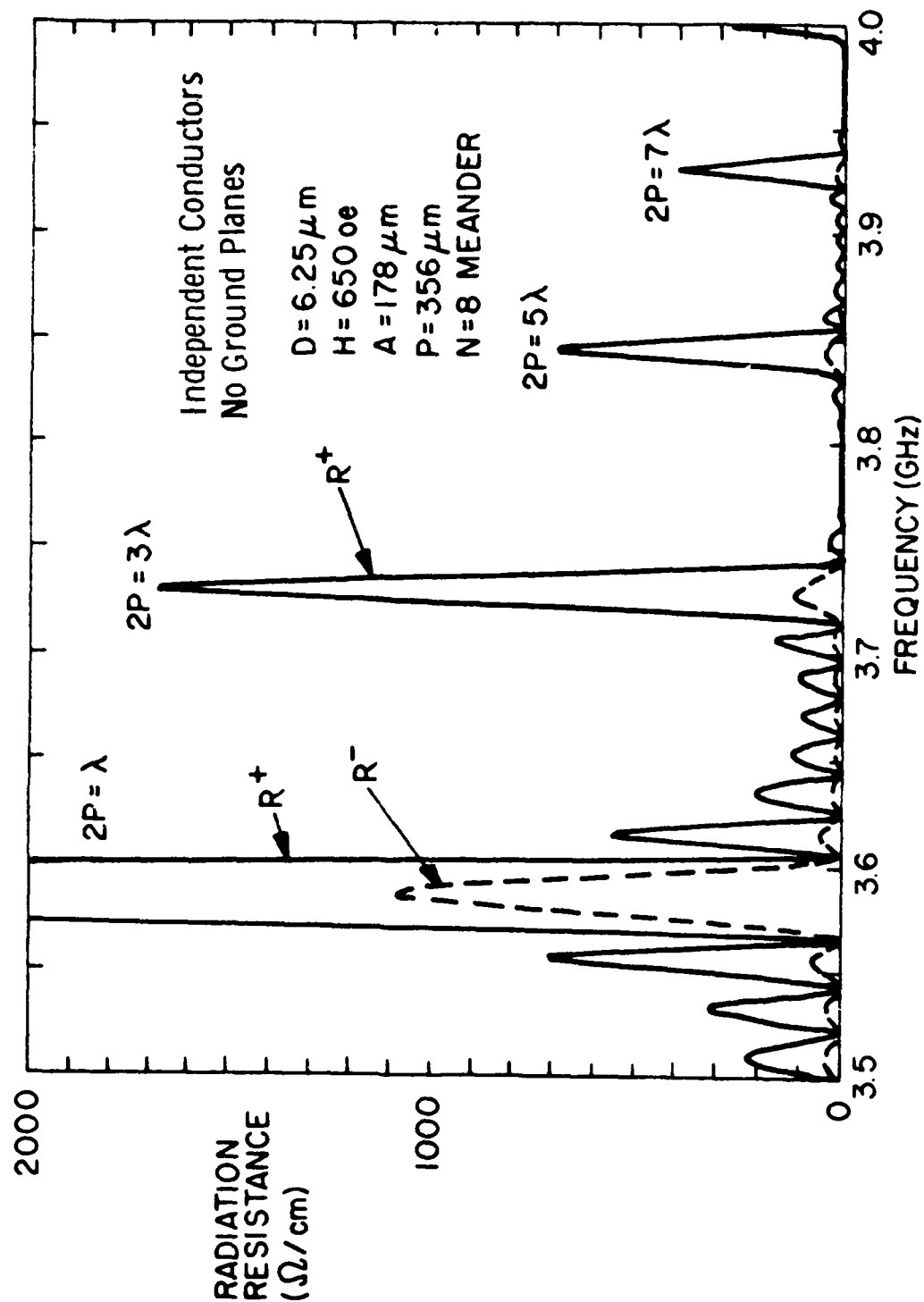


Figure 36

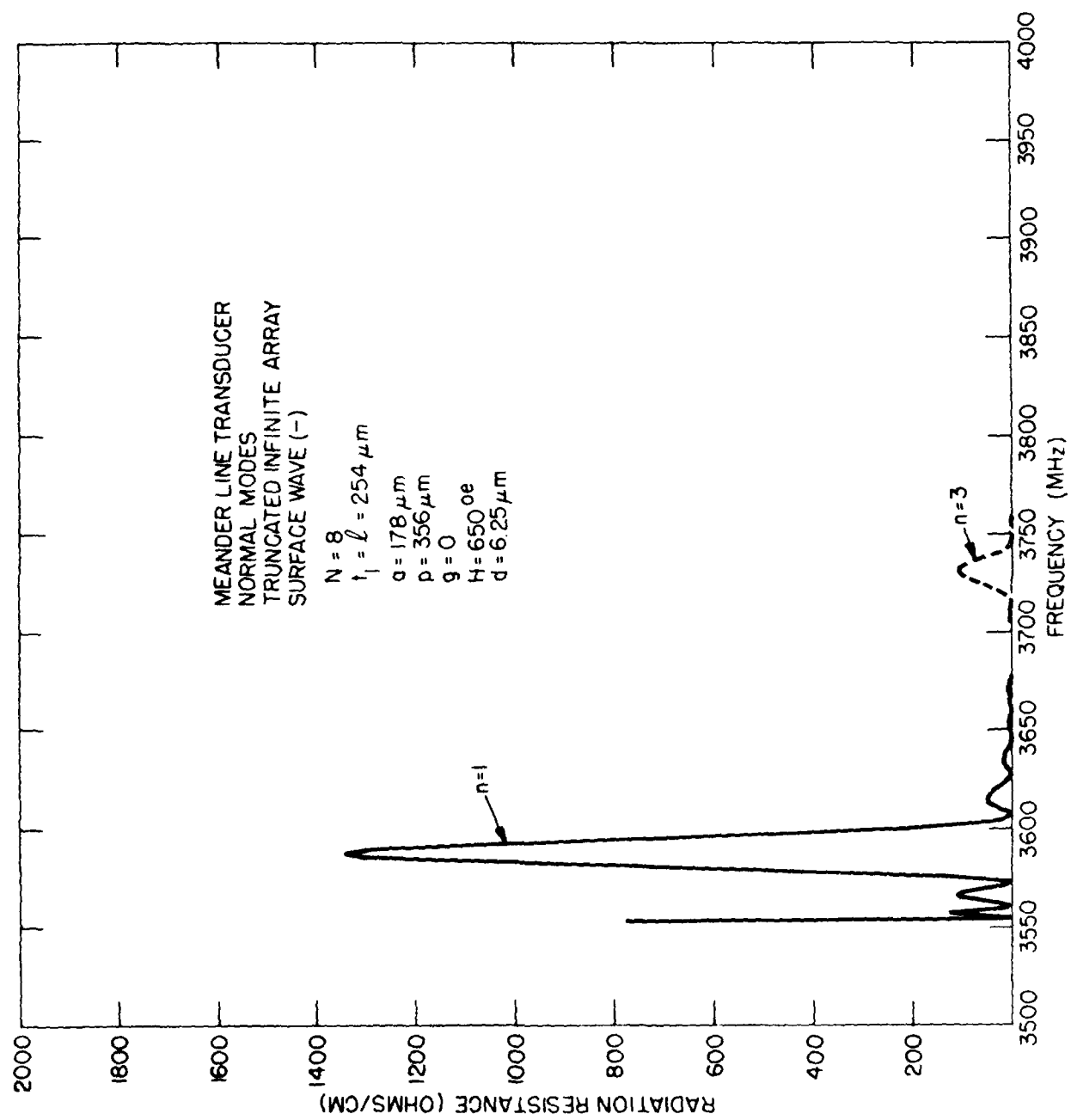


Figure 37

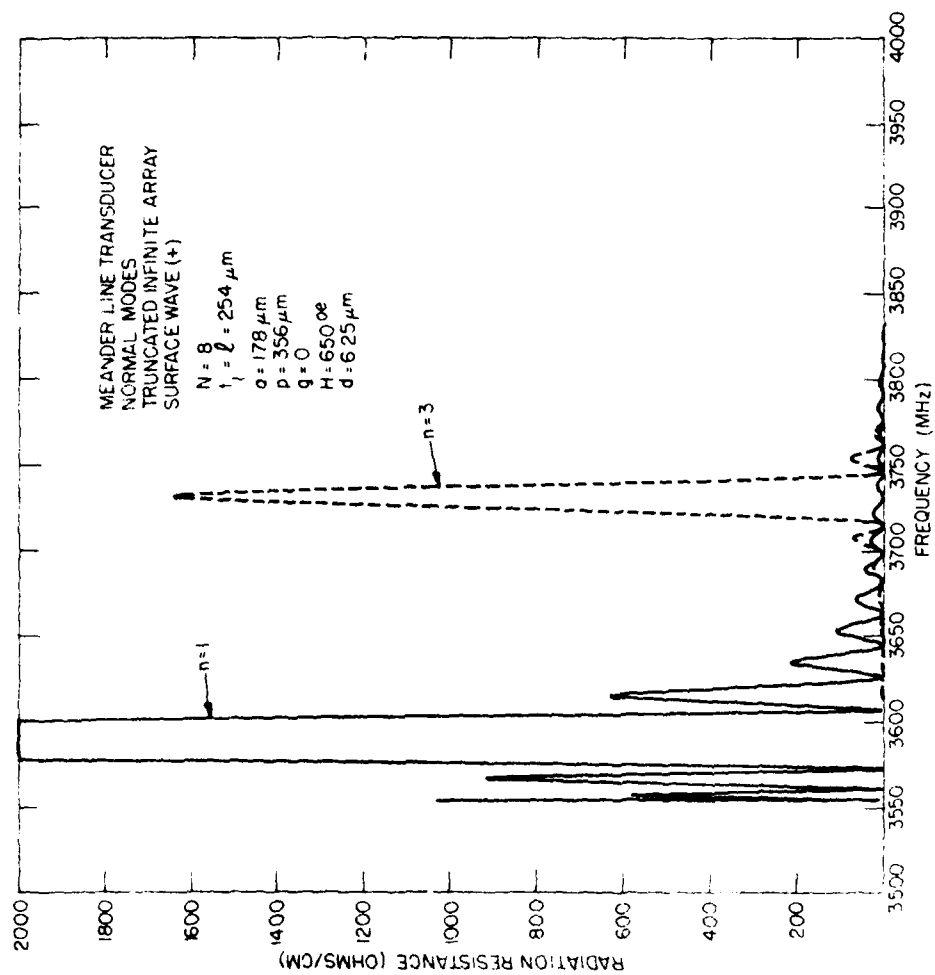


Figure 38

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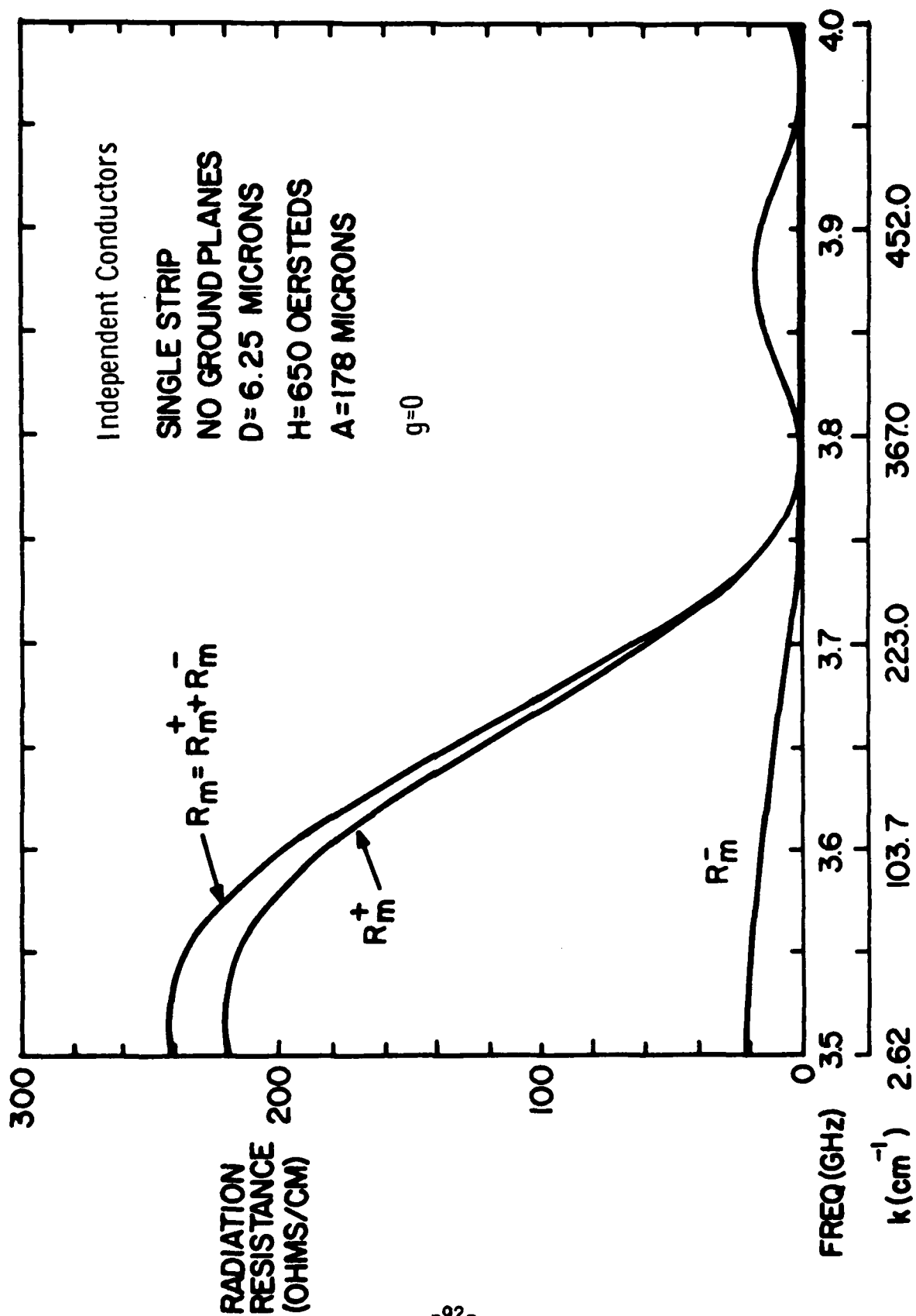


Figure 39

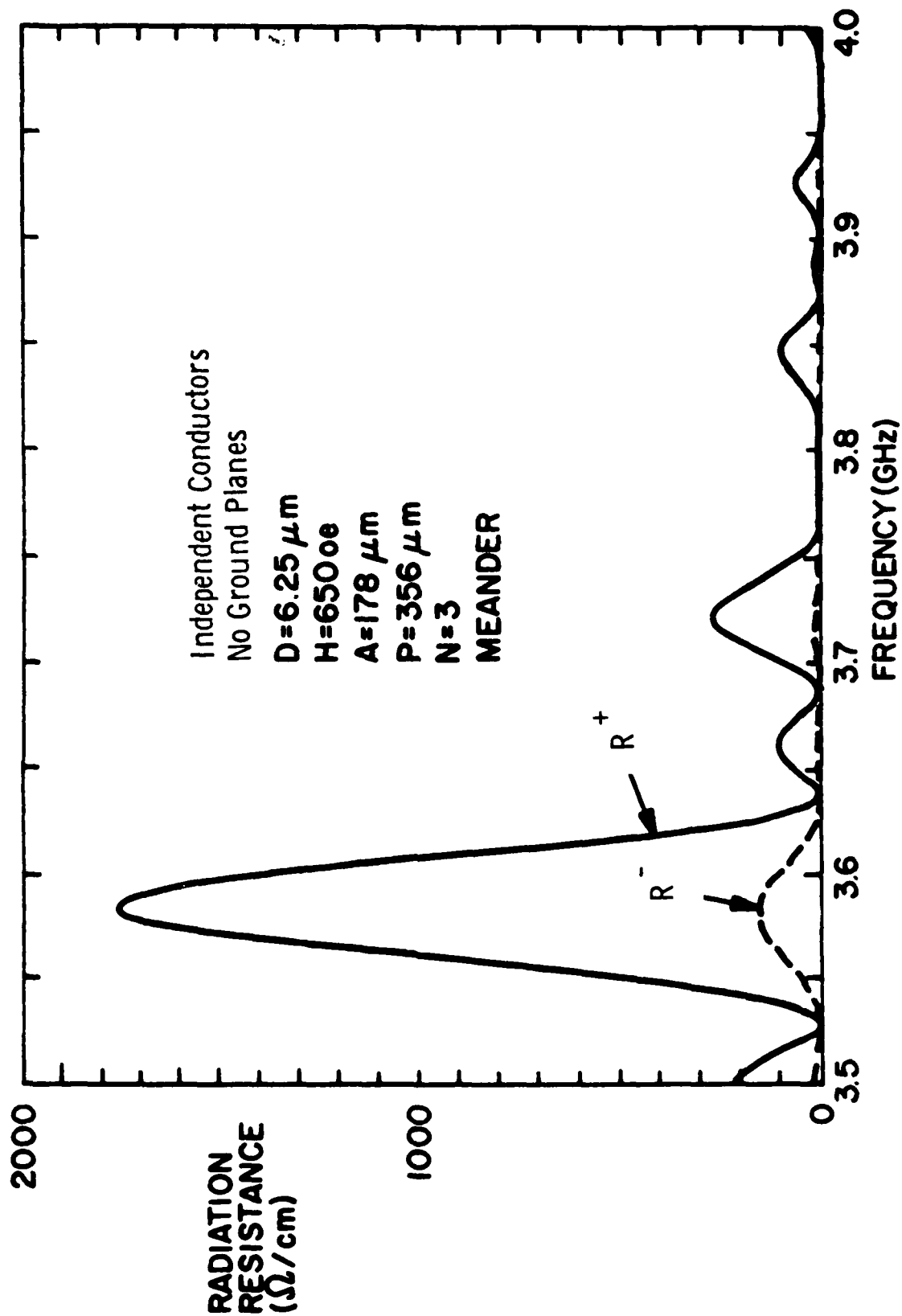


Figure 40

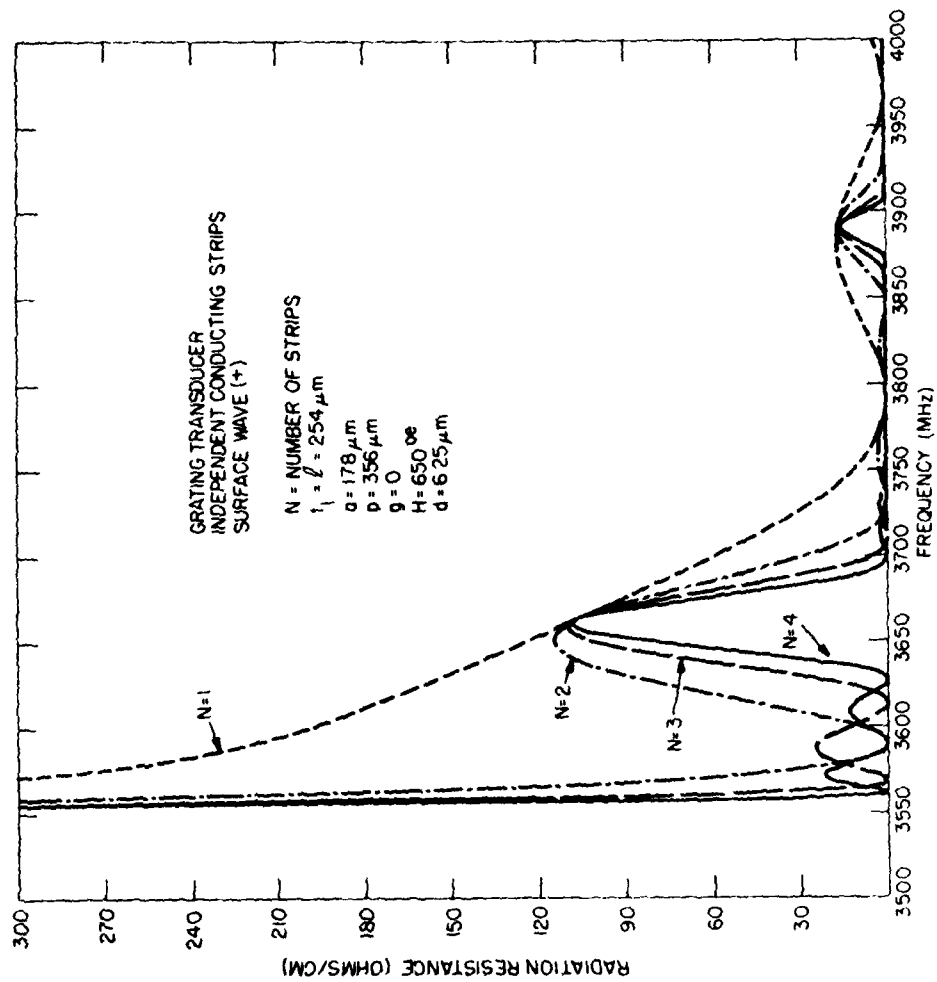


Figure 41

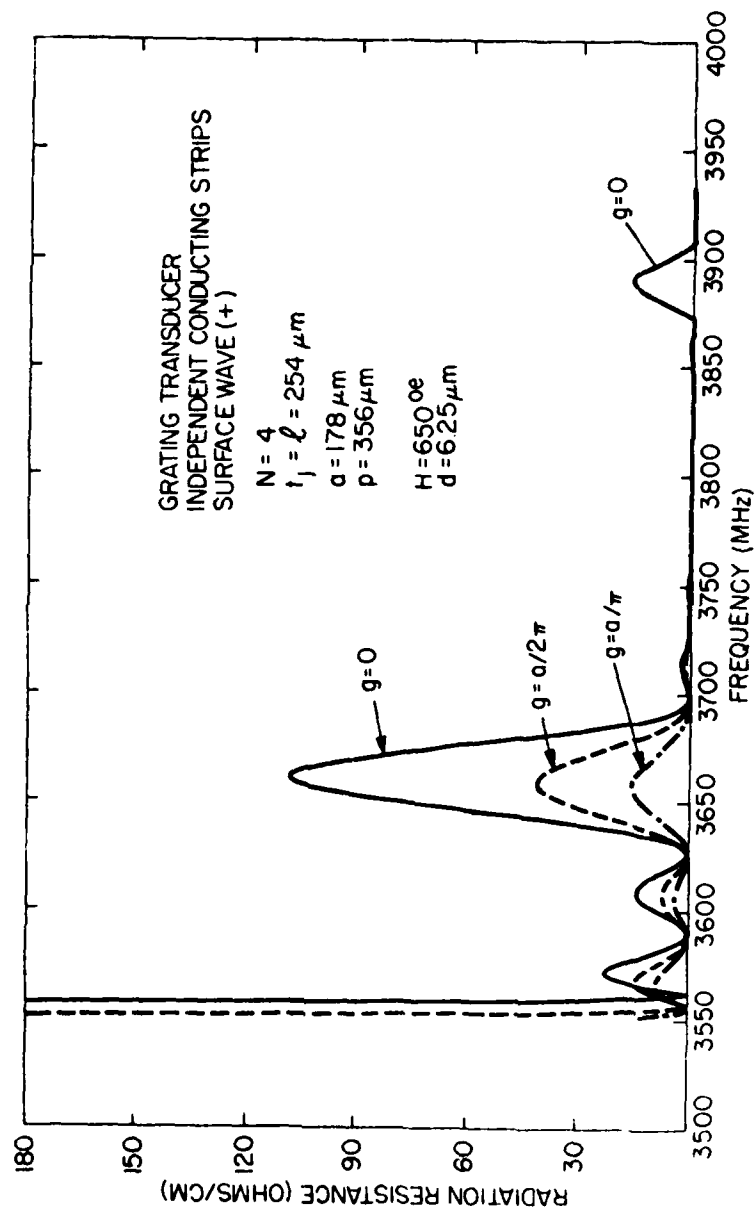


Figure 42

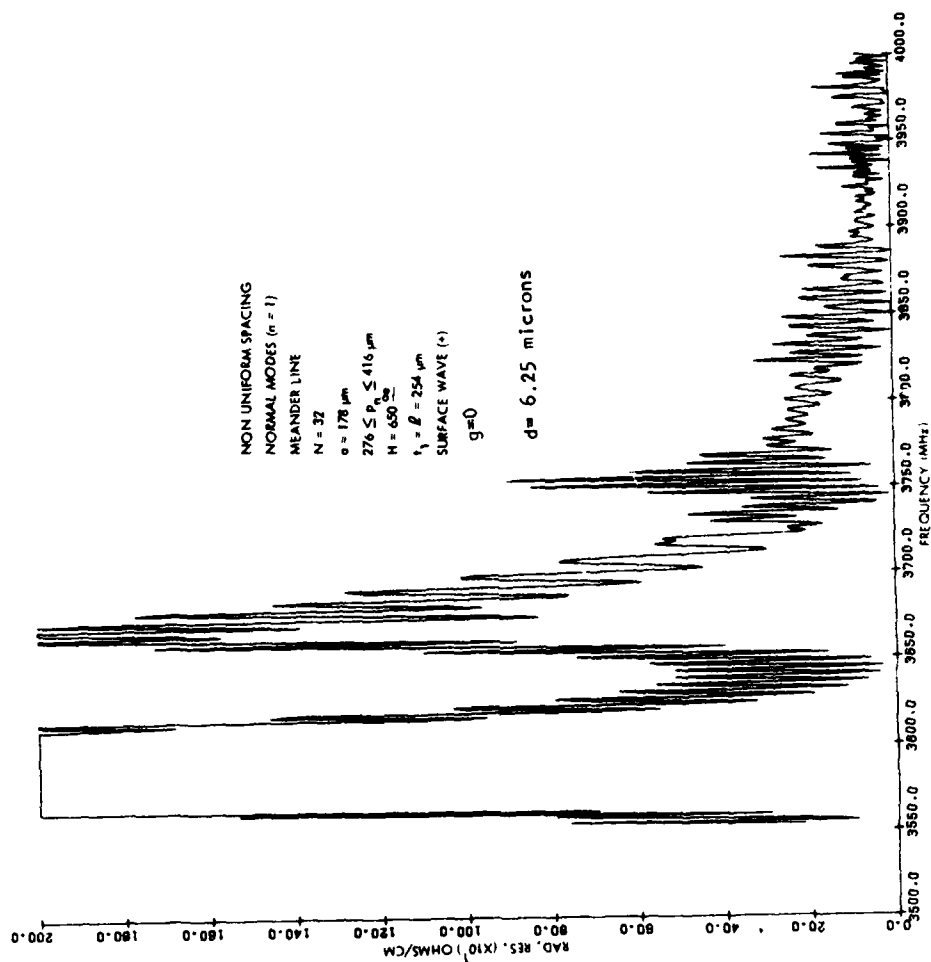


Figure 43

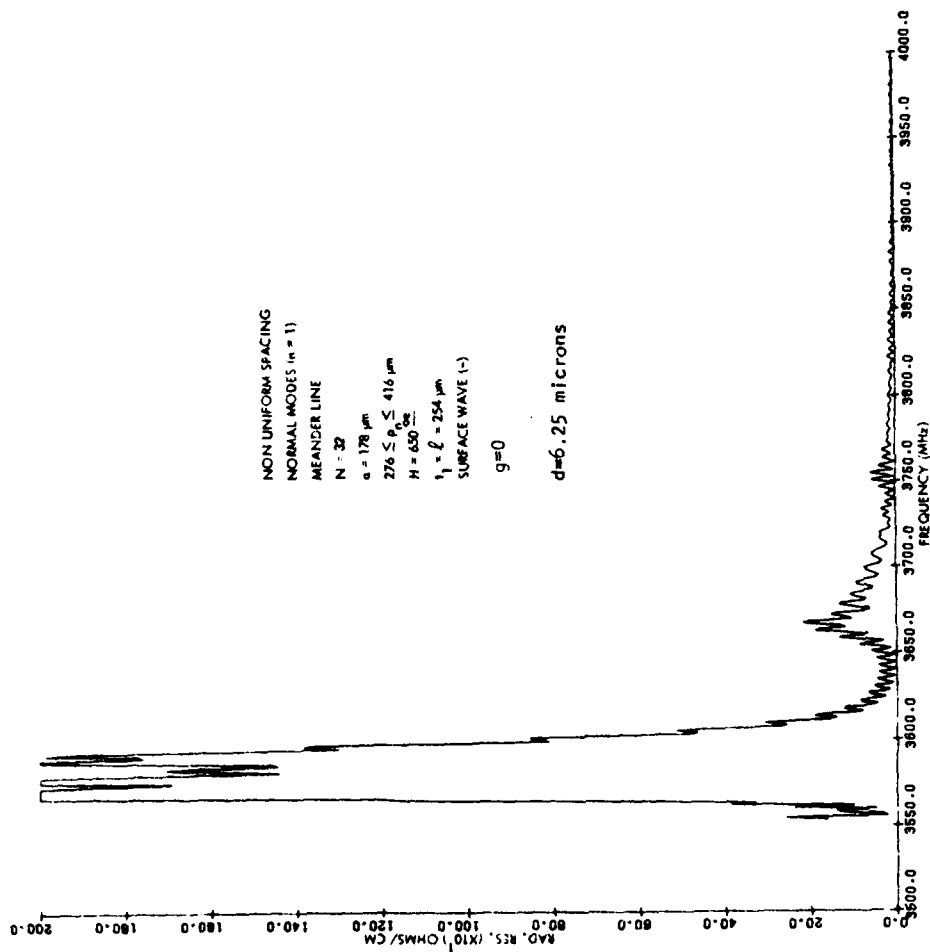


Figure 44

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