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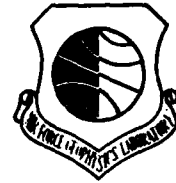
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**An Objective Method  
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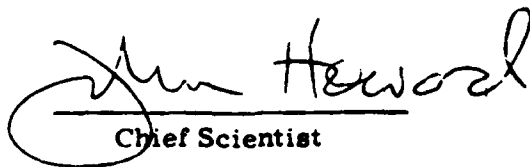
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Solar parameters derived from the region analysis program at the NOAA<br>Space Environment Services Center (SESC) are submitted to a multivariate<br>discriminant analysis (MVDA) in which the parameters relevant to flare<br>prediction are identified and incorporated in a classification procedure to<br>produce a flare forecast. The analysis uses two years of data (6095 solar<br>active region-days). The MVDA forecast is compared with a subjective<br>forecast derived from the SESC forecast during the same period, and is |                       |                                                                                   |

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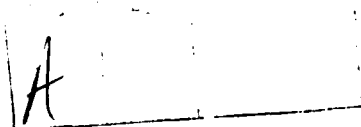
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found to have greater accuracy overall. Specific recommendations are made concerning the application of the technique in a forecasting operation, and in the types of data required for future improvement.

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# An Objective Method for Forecasting Solar Flares

## 1. INTRODUCTION

This report is a continuation of an earlier study (Hirman et al, 1980) in which multivariate discriminant analysis (MVDA) is used in a computer program to produce an objective daily solar flare forecast. The essential feature of the statistics package is the comparison between a number of input parameters and a number of output classes, in which the discrimination between the classes in terms of the input parameters is maximized by constructing appropriate classification functions. In the application to flare prediction, the input parameters are daily solar parameters for each active region on the solar disk, and the output classes are the levels of flare activity occurring the following day within the same active regions. We have used more than two years of data, of which approximately 25 percent has been used to derive the classification functions. The latter are then extrapolated forward in time to produce a true forecast.

The computer program, known as BMD07M, was originally written at UCLA,<sup>1</sup> although the particular version used here was developed further by Seagraves<sup>2</sup> to

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1. Dixon, W. J. (ed.) (1968) Biomedical Computer Programs, University of California Publications in Automatic Computations, No. 2, University of California Press, p. 214a.
2. Seagraves, P. H. (1972) UBC BMD07M Stepwise Discriminant Analysis, University of British Columbia Computing Centre Documentation.

include the Cooley and Lohnes classification procedure,<sup>3</sup> and the Lachenbruch "N-1" technique.<sup>4</sup> The Cooley and Lohnes procedure does not assume uniformity of variance, and this sometimes results in better classification scores. The computational burden, however, is increased because linear classification functions are not possible; instead, canonical variables, constructed from the original input parameters, are used as a transformation to reduce the matrix dimension in the classification formulas. The Lachenbruch technique removes bias when the program classifies its own data base.

A complete description of the mathematics is beyond the scope of this report. The reader may consult Anderson<sup>5</sup> and Rao<sup>6</sup> for references on discriminant analysis. A discussion of the suitability of applying various statistical methods to discrete input variables is contained in Vecchia et al.<sup>7</sup> The latter point is of particular interest because the work of Vecchia et al uses the same discrete data base as used herein, to produce solar flare probability forecasts using discriminant analysis (without the Cooley and Lohnes procedure) and logistic regression analysis.

An important feature of the present study is the comparison of the objective, computer forecast with a subjective, conventional forecast prepared during the same test period for the same active regions on the sun. Without such a benchmark for relative evaluation, the presentation of any forecast method has considerably reduced merit.

## 2. DATA

The data used herein were obtained from the region analysis program at the NOAA Space Environment Services Center (SESC) in Boulder, Colorado. The region analysis program collects daily a variety of solar parameters for each active region on the solar disk. It is important to note that there is no attempt

3. Cooley, W. W., and Lohnes, P. R. (1962) *Multivariate Procedures for the Behavioral Sciences*, Wiley, New York.
4. Lachenbruch, P. A., and Mickey, M. R. (1968) *Technometrics*, 10:1.
5. Anderson, T. W. (1958) *An Introduction to Multivariate Statistical Analysis*, Wiley, New York.
6. Rao, C. R. (1974) *Advanced Statistical Methods in Biometric Research*, Hafner.
7. Vecchia, D. F., Caldwell, G. A., Tryon, P. V., and Jones, R. H. (1980) in *Sol.-Terres. Pred. Proc.*, Vol. 3, R. F. Donnelly (ed.), C-76.

in this program to select the more flare-productive regions. The parameters include radio and X-ray data, but most are derived from optical data supplied by the USAF/AWS SOON system. The parameters contain information which the SESC forecasters consider vital to the preparation of a 24-hour flare forecast. The present study uses data for the period 1 January 1977 to 31 January 1979, containing 6095 active-region days (records) that have been checked for errors and internal consistency. Random scrutiny, however, has shown that errors still remain. After several reassignments of parameter values and definitions, we arrived at the form of the data base shown in Table 1.

Table 1. SESC Region Analysis Parameters (Modified)

| PARAMETER                              | ASSIGNED VALUE |
|----------------------------------------|----------------|
| 1. DATE                                |                |
| 2. REGION NUMBER                       |                |
| 3. REGION'S FIRST APPEARANCE LONGITUDE |                |
| 4. CURRENT LONGITUDE                   |                |
| 5. N/S LATITUDE                        |                |
| 6. CURRENT LATITUDE                    |                |
| 7. CARRINGTON LONGITUDE                |                |
| 8. REGION AGE                          |                |
| 9. SPOT CLASS 1                        |                |
| A .....                                | 1              |
| B .....                                | 2              |
| C .....                                | 4              |
| D .....                                | 5              |
| E .....                                | 6              |
| F .....                                | 7              |
| H .....                                | 3              |

Table 1. SESC Region Analysis Parameters (Modified) (continued)

|                                                                                 |            |
|---------------------------------------------------------------------------------|------------|
| 10. SPOT CLASS 2                                                                |            |
| r/x .....                                                                       | 1          |
| s .....                                                                         | 2          |
| a .....                                                                         | 3          |
| h .....                                                                         | 4          |
| k .....                                                                         | 5          |
| 11. SPOT CLASS 3                                                                |            |
| x .....                                                                         | 1          |
| o .....                                                                         | 2          |
| i .....                                                                         | 3          |
| c .....                                                                         | 4          |
| 12. MAGNETIC CLASS                                                              |            |
| No spots .....                                                                  | 0          |
| Alpha .....                                                                     | 1          |
| Beta .....                                                                      | 2          |
| Beta-Gamma .....                                                                | 3          |
| Gamma .....                                                                     | 4          |
| Beta-Delta .....                                                                | 5          |
| Beta-Gamma-Delta .....                                                          | 6          |
| Gamma-Delta .....                                                               | 7          |
| 13. MAGNETIC POLARITY OF STRONGEST FIELD .....                                  | (+/-)      |
| 14. MAGNETIC FIELD STRENGTH .....                                               | (Gauss)    |
| 15. MAGNETIC GRADIENTS .....                                                    | (Gamma/km) |
| 16. INTERACTION WITH ANOTHER REGION                                             |            |
| None .....                                                                      | 0          |
| Spots of opposite polarity converge (from less<br>than two degrees apart) ..... | 1          |
| 17. SUNSPOT DYNAMICS                                                            |            |
| No spots or no motion .....                                                     | 0          |
| Coalescing of spots .....                                                       | 1          |

Table 1. SESC Region Analysis Parameters (Modified) (continued)

|                                                               |   |
|---------------------------------------------------------------|---|
| Spot rotation .....                                           | 2 |
| Relative motion between oppositely poled spots .....          | 3 |
| 18. STAGE OF DEVELOPMENT                                      |   |
| No spots .....                                                | 0 |
| Mature group (stable) .....                                   | 1 |
| Decaying .....                                                | 2 |
| Growing .....                                                 | 3 |
| Rapid decay (spot numbers/areas decrease by $> 50\%$ ) .....  | 4 |
| Rapid growth (spot numbers/areas increase by $> 50\%$ ) ..... | 5 |
| Rapid growth ( $> 100\%$ ) .....                              | 6 |
| 19. LEADER/TRAILER FIELDS                                     |   |
| Structure not definite .....                                  | 0 |
| Returning Region .....                                        | 1 |
| $< 5$ deg of neutral line and out of phase .....              | 2 |
| $> 5$ deg of neutral line and in leader fields .....          | 3 |
| $> 5$ deg of neutral line and in trailer fields .....         | 4 |
| $< 5$ deg of neutral line and in-phase .....                  | 5 |
| 20. RETURNING REGION                                          |   |
| 21. SECTOR BOUNDARY RELATIONSHIP                              |   |
| 22. ASSOCIATED FILAMENT                                       |   |
| None .....                                                    | 0 |
| Filament unchanged .....                                      | 1 |
| Filament growing .....                                        | 2 |
| Filament disappeared within past 24 hrs .....                 | 3 |
| Filament darkens or is active .....                           | 4 |
| 23. EMBEDDED FILAMENT                                         |   |
| None .....                                                    | 0 |
| Filament present .....                                        | 1 |
| Active filament .....                                         | 2 |

Table 1. SESC Region Analysis Parameters (Modified) (continued)

|                                            |   |
|--------------------------------------------|---|
| 24. PLAGE COMPACTNESS                      |   |
| Non-compact .....                          | 0 |
| Compact .....                              | 1 |
| 25. NEUTRAL LINE ORIENTATION               |   |
| Weak structure .....                       | 0 |
| North-south ( $\pm 45$ deg) .....          | 1 |
| East-west ( $\pm 45$ deg) .....            | 2 |
| Hairpin .....                              | 3 |
| Circular .....                             | 4 |
| 26. REVERSE POLARITY                       |   |
| Normal polarity .....                      | 0 |
| Reverse polarity .....                     | 1 |
| 27. NEUTRAL LINE COMPLEXITY                |   |
| Straight line or weak structure .....      | 0 |
| 1-3 Kinks .....                            | 1 |
| 4-6 .....                                  | 2 |
| 7-12 .....                                 | 3 |
| >12 .....                                  | 4 |
| 28. NEUTRAL LINE CHANGES                   |   |
| No trend .....                             | 0 |
| Becoming simple .....                      | 1 |
| Becoming complex .....                     | 2 |
| 29. BRIGHT POINTS                          |   |
| None .....                                 | 0 |
| Occurred, but not along neutral line ..... | 1 |
| Occurred along neutral line .....          | 2 |
| 30. PLAGE FLUCTUATIONS                     |   |
| None .....                                 | 0 |
| Occurred .....                             | 1 |

Table 1. SESC Region Analysis Parameters (Modified) (continued)

|                                                                                                                           |   |
|---------------------------------------------------------------------------------------------------------------------------|---|
| 31. ISOLATED POLE                                                                                                         |   |
| 32. EMERGING FLUX                                                                                                         |   |
| None, or region is new .....                                                                                              | 0 |
| New flux emerges within spot group .....                                                                                  | 1 |
| New flux emerges near region (within 5 deg) .....                                                                         | 2 |
| 33. ARCH FILAMENT SYSTEM                                                                                                  |   |
| 34. RADIO BURST/SWEEP                                                                                                     |   |
| None occurred .....                                                                                                       | 0 |
| >250 flux units at 10 cm .....                                                                                            | 1 |
| >1000 flux units at 10 cm .....                                                                                           | 2 |
| Type III .....                                                                                                            | 3 |
| Type IV .....                                                                                                             | 4 |
| Type II and IV .....                                                                                                      | 5 |
| U Burst .....                                                                                                             | 6 |
| Major/complex 10 cm burst .....                                                                                           | 7 |
| >1000 flux units at 10 cm plus a U burst, or<br>Type III and IV, or<br>250 flux units at 10 cm plus Type III and IV ..... | 8 |
| 35. REGION'S FIRST APPEARANCE (TRANSIT HISTORY)                                                                           |   |
| 36. FLARE HISTORY                                                                                                         |   |
| No flares have occurred .....                                                                                             | 0 |
| C class flares have occurred .....                                                                                        | 1 |
| M class flares have occurred .....                                                                                        | 2 |
| X class flares have occurred .....                                                                                        | 3 |
| 37. FLARES TODAY                                                                                                          |   |
| None .....                                                                                                                | 0 |
| C class .....                                                                                                             | 1 |
| M class .....                                                                                                             | 2 |
| X class .....                                                                                                             | 3 |

Table 1. SESC Region Analysis Parameters (Assigned) (continued)

|                                                                                                                                                                                  |   |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
| 38. PROTON HISTORY                                                                                                                                                               |   |
| None occurred .....                                                                                                                                                              | 0 |
| Proton event occurred .....                                                                                                                                                      | 1 |
| Ground level event .....                                                                                                                                                         | 2 |
| 39. PROTONS TODAY                                                                                                                                                                |   |
| None .....                                                                                                                                                                       | 0 |
| Occurred .....                                                                                                                                                                   | 1 |
| 40. REGION FORECASTS (SESC)                                                                                                                                                      |   |
| Probabilities for each class of flare (none, C, M, or X) for each region, for the 24-hour period beginning at 0 hr UT next day. Proton event probabilities are similarly stated. |   |

Most of the parameters in Table 1 have been assigned discrete values according to categories which are subjectively related to increasing flare activity. This subjectivity is the weakest link in any scheme utilizing objective procedures for producing a forecast solely from data. In essence, the situation merely allows the element of subjectivity to reside entirely in the data acquisition process. Probably, this situation is preferable to having subjectivity introduced also in the forecast preparation. There are several parameters (e.g. spot class, flare history, magnetic class) for which assigned values are based upon quantitative studies. Fortunately, (or perhaps therefore!) these parameters are among those from which the objective forecast derives most of its skill.

Perhaps the most unfortunate circumstance is that for a large number of records one or more parameters is missing. In the computer program, missing data codes are replaced by averages for the particular parameter in the set of records used in deriving the classification functions. Missing data, in addition to errors, makes the testing of objective techniques difficult, especially for determining the relative significance of various parameters. In order to portray some feeling for the degree of representation in the data base we note the following: for three commonly observed parameters, "Spot Class 2," "Magnetic Class," and "Flares Today," only 5893 of the total 6095 records contain all three; if "Bright Points," "Spot Class 3," "Spot Class 1," "Magnetic Gradients," and "Sunspot Dynamics" are added to the first three, only 3732 records remain; and for a total of 15 of the 31 usable parameters, only 510 records contain all 15. This is, indeed, a hardship for statistical analysis.

Nevertheless, we are able to show later that at least some of these frequently missing parameters contain valuable predictive information.

The data base contains daily, region-by-region entries for the actual flare activity, in addition to the official SESC subjectively derived flare forecast. Thus, the information required for objective forecast testing, as well as for comparison with the SESC forecast, is contained in the same base. Flares are listed according to their peak soft (1-8 Å) X-ray flux at 1 AU:

Class C:  $10^{-6} < E < 10^{-5}$  watt m<sup>-2</sup>  
 Class M:  $10^{-5} \leq E < 10^{-4}$  watt m<sup>-2</sup>  
 Class X:  $10^{-4} \leq E$  watt m<sup>-2</sup>

From the standpoint of geophysical environment studies, the classes M and X are of greatest importance.

In addition to Table 1, six combination parameters (Table 2), derived from certain original parameters, were included as input parameters. These six were

Table 2. Combination Parameters (Numbers in right-hand column refer to original parameter number in Table 1)

| New Parameter No. | Parameter Formula |
|-------------------|-------------------|
| 1                 | 9•10•11           |
| 2                 | 9•10•11•12        |
| 3                 | 9•10•11•32        |
| 4                 | 14•15•(17+25)     |
| 5                 | 12•17•27          |
| 6                 | 17•(25+27+28)     |

found to have possible predictive significance in the earlier study where twenty such combination parameters were tested.<sup>8</sup> The derivation of combination parameters is based on intuitions about the form in which predictive information might be contained in the data, and about physical quantities (e.g., energy stored in sheared magnetic fields) presumed relatable to flares. The subject of these and other combination parameters will be discussed in a later section.

8. Hirman, J. W., Neidig, D. F., Seagraves, P. H., Flowers, W. E., and Wiborg, P. H. (1980) in Sol.-Terrest. Pred. Proc., Vol. 3, R.F. Donnelly (ed.), C-64.

### 3. PROCEDURE

The region analysis parameters for today are independent of any information on flare activity occurring tomorrow; therefore, they can be used in practice, today, to produce a flare forecast for tomorrow, assuming that predictive information is present in the parameters. We have used the first  $N$  records (with  $N = 1500$ , as described below) as a "training set" in order to derive the classification functions for three possible outcomes: "No Flare," "C Flare," and "M or X Flare." M and X flares were grouped together as a single class in order to reduce statistical noise caused by the relatively few cases of larger flares. The classification functions were then applied to new records, using only the input parameters, in order to produce a true forecast. The latter procedure was accomplished in steps of 250 records each, with the training set sliding forward in time, 250 records (approximately one month) after each step. Thus, for a 1500-record training set, the remaining 4595-record test set requires 19 individual subtests of 250 records each (except for the nineteenth). This sliding base technique maintains a constant  $N$  records in the training set, thereby assuring that the program is trained on recent data relative to the test subset. This, combined with the relatively small size of the test subset, minimizes the effects of secular trends, either of observational or solar origin, which might be present in the data.

The computer program was trained on the X-ray class of the largest event (No Flare, C Flare, or M & X Flare) occurring in the region in the 24-hour period following the acquisition date of the input parameters. Thus, the computer forecast is expressed in terms of probabilities for the largest event to be in one of these classes. The outcomes are mutually exclusive, with the sum of probabilities over all classes equal to unity. The SESC forecast, however, is a probability forecast for the occurrence of each class of event; i.e., a non-exclusive format. In order to assess the quality of the computer forecast, we derived a comparison forecast in the "exclusive" format by selecting the largest event class in the SESC forecast that was assigned a probability greater than or equal to 0.5. Although this is not an SESC forecast, it is probably representative of what would be extant if the SESC chose to cast their predictions in this mode.

In the following test results we present the forecasts according to both the standard multivariate discriminant analysis (MVDA) and the Cooley and Lohnes procedure (MVDA/CL). There are important differences in the character of these two forecasts, which, as will be shown later, may be used to advantage.

## 1. TEST RESULTS

### 1.1 Preliminary Discussion

We are concerned mainly with the behavior of the computer forecasts relative to the comparison forecast when, for example, changes are made in the size of the training set, choice of input parameters, solar activity levels, and percent of missing data. In all cases we present the computer forecast along with the comparison forecast for the same set of test records. Also included is a list of input parameters submitted to analysis, along with their frequency of selection in classifying the three outcomes. Note, however, that due to the 250-record increment the training sets are independent of each other only when separated by six or more subsets.

As a first step, we eliminated 11 parameters which were not selected in any of the 19 subsets. Following this, the program was run again using the remaining 20 input parameters. The results are given in Tables 3, 4, and 5. This test (A) will serve as an example for the displays used elsewhere in this report.

Table 3 shows the actual matrix of region-day forecasts vs region-day largest events, for the three forecasts. Table 5, derived from the data in Table 3, summarizes the following:

- F Percent of forecasts correct in the given event class
- E Percent of region-day largest events which were forecasted
- A  $(F + E)/2$
- C Climatology (percent of the total number of events in the class)
- U Unweighted mean of the A's for all three-event classes
- W Weighted mean forecast accuracy (the sum of the matrix diagonal elements divided by the total number of forecasts, or events, in all classes)
- Off 1 Percent of forecasts that are one matrix element away from the diagonal
- Off 2 Percent of forecasts that are two matrix elements away from the diagonal

These various scores are of interest because of the several ways in which forecasts can be used. For example, the F score, or percentage of forecasts that are correct, is the quantity of interest to a customer who cannot tolerate false alarms. A quite different requirement applies, however, in a situation where surprise flares are unwelcome. In the latter case, the E score is the important measurement. Of course, knowing the customer's need in advance allows the forecast to be biased either toward underprediction, which tends to improve the F score, or toward overprediction, which improves the E score.

Table 3. Comparison of Forecasts--Test A  
(1500-record training set)

| Largest Event<br>Forecasted | Largest Event Observed |     |     | Total<br>Forecasts |
|-----------------------------|------------------------|-----|-----|--------------------|
|                             | No Flare               | C   | M&N |                    |
| COMPARISON                  |                        |     |     |                    |
| No Flare                    | 3376                   | 213 | 25  | 3614               |
| C                           | 501                    | 199 | 59  | 759                |
| M&N                         | 77                     | 82  | 63  | 222                |
| Total Events                | 3954                   | 494 | 147 | 4595               |
| MVDA                        |                        |     |     |                    |
| No Flare                    | 3349                   | 190 | 26  | 3565               |
| C                           | 513                    | 206 | 51  | 770                |
| M&N                         | 92                     | 98  | 70  | 260                |
| Total Events                | 3954                   | 494 | 147 | 4595               |
| MVDA/CL                     |                        |     |     |                    |
| No Flare                    | 3739                   | 316 | 50  | 4105               |
| C                           | 185                    | 142 | 50  | 377                |
| M&N                         | 30                     | 36  | 47  | 113                |
| Total Events                | 3954                   | 494 | 147 | 4595               |

As a measure of the "balanced" accuracy of a forecast in a given event class we, therefore, introduce the average of F and E, given by A.

The accuracy of a forecast is always dependent upon the climatology for the event being forecasted. Higher climatological probabilities tend to improve the chances for predictions to be correct. For example, it is easy to predict "No Flare" with 90 percent accuracy, simply because no flare occurs in almost 90 percent of all active-region days. In comparing cumulative scores between forecasts it is imperative to note the climatology which prevailed during the test period. Climatology is affected by a number of factors, including event classification criteria, duration of forecast interval, and level of solar activity.

Table 4. Parameters Submitted to Analysis and Their Frequency of Selection in 19 Subsets--Test A

|               |    |             |   |               |   |
|---------------|----|-------------|---|---------------|---|
| Flares Today  | 19 | New No. 1   | 9 | Mag. Pol.     | 5 |
| Bright Points | 19 | Mag. Grad.  | 9 | Neut. I. Chg. | 5 |
| New No. 2     | 17 | Mag. Class  | 6 | Spot Class 3  | 4 |
| Spot Dynam.   | 12 | Radio B/S   | 6 | Spot Class 2  | 3 |
| New No. 5     | 12 | Flare Hist. | 6 | Spot Inter.   | 3 |
| Proton Hist.  | 11 | New No. 3   | 6 | Emerg. Flux   | 1 |
| Spot Class 1  | 9  | New No. 4   | 6 |               |   |

Table 5. Comparison of Forecast Scores--Test A

| Forecaster | Event    | F    | E    | A    | C    | U    | W    | Off 1 | Off 2 |
|------------|----------|------|------|------|------|------|------|-------|-------|
| COMPARISON | No Flare | 93.4 | 85.4 | 89.4 | 86.1 |      |      |       |       |
|            | C        | 26.2 | 40.3 | 33.3 | 10.8 | 52.8 | 79.2 | 18.6  | 2.2   |
|            | M&N      | 28.4 | 42.9 | 35.6 | 3.2  |      |      |       |       |
| MVDA       | No Flare | 93.9 | 84.7 | 89.3 | 86.1 |      |      |       |       |
|            | C        | 26.8 | 41.7 | 34.3 | 10.8 | 53.6 | 78.9 | 18.4  | 2.6   |
|            | M&N      | 26.9 | 47.6 | 37.3 | 3.2  |      |      |       |       |
| MVDA/CL    | No Flare | 91.1 | 94.6 | 92.8 | 86.1 |      |      |       |       |
|            | C        | 37.7 | 28.7 | 33.2 | 10.8 | 54.3 | 85.5 | 12.8  | 1.7   |
|            | M&N      | 41.6 | 32.0 | 36.8 | 3.2  |      |      |       |       |

In essence, climatology is directly dependent upon "bin size." Failure to state climatological conditions clearly (an unfortunately common practice) makes intercomparison of forecasts almost impossible.<sup>9</sup> It seems that this point cannot be emphasized enough.

Because "No Flare" constitutes the majority of situations on the sun, it comes as no surprise that solar flare forecasts are usually quite accurate overall; i.e., their weighted means (W) are high. It is of greater interest, however, to predict flares than quiet conditions and, for this reason, the unweighted score U, given simply by the mean of the A scores over all classes, has been included in Table 5.

Finally, we note that if a forecast is in error, it is better to be wrong by one event class than by two. Thus, the tendency for the off-diagonal entries in the matrix to cluster near the diagonal is an important measure when comparing forecast scores which are similar otherwise. Table 5 includes a measure of this error distribution in the form of the Off 1 and Off 2 scores.

The scores (F, E, and A) have uncertainties of approximately  $\pm 1$ ,  $\pm 3$ , and  $\pm 5$  for No Flare, C Flare, and M & X Flare, respectively. The U and W scores have uncertainties of about  $\pm 1$ . Thus, in terms of A and U, the three forecasts in Table 5 are essentially identical. The MVDA/CL forecast definitely excels in the W score, although this is mainly due to its tendency for underprediction, which places a large number of forecasts in the No Flare column. The tendency for underprediction in the MVDA/CL forecast is evident also in the F scores for C, and M & X flares, being significantly higher than the corresponding E scores. On the other hand, both the comparison and the MVDA forecast are biased toward overprediction. Their overall similarity is quite striking.

#### 4.2 Effect of Training Set Size

The number of records to be used in the training set should be large enough to provide sufficient statistics to train the computer program, yet small enough to avoid the effects of trends in the data. The optimum number, while not known from theory, may be determined empirically by varying the training set size and comparing the scores of the resulting forecasts. Table 6 shows the results for training sets of 750 and 2095 records. Together with Table 5 (1500-record training set) we find differences of only small significance. A close examination of

9. Simon, P., Smith, J. B., Ding, Y., Flowers, W., Guo, Q., Harvey, K. L., Hedeman, R., Martin, S. F., McKenna Lawlor, S., Lin, V., Neidig, D., Obridko, V. N., Dodson Prince, H., Rust, D., Speich, D., Starr, A., and Stepanyan, N. N. (1980) in Sol.-Terres. Pred. Proc., Vol. 2, R. F. Donnelly (ed.), p. 287.

Table 6. Comparison of Scores Using 750-Record and 2095-Record Training Sets--Test B and C

| Forecaster              | Event    | F    | E    | A    | U    | W    | Off 1 | Off 2 |
|-------------------------|----------|------|------|------|------|------|-------|-------|
| MVDA<br>750 Records     | No Flare | 93.9 | 86.0 | 89.9 |      |      |       |       |
|                         | C        | 28.4 | 41.3 | 34.8 | 53.3 | 80.0 | 17.0  | 3.0   |
|                         | M & N    | 25.2 | 45.1 | 35.1 |      |      |       |       |
| MVDA/CL<br>750 Records  | No Flare | 91.7 | 93.7 | 92.7 |      |      |       |       |
|                         | C        | 36.2 | 32.7 | 34.4 | 52.8 | 85.1 | 13.1  | 1.8   |
|                         | M & N    | 36.3 | 26.0 | 31.2 |      |      |       |       |
| MVDA<br>2095 Records    | No Flare | 93.9 | 84.0 | 89.0 |      |      |       |       |
|                         | C        | 25.9 | 42.8 | 34.4 | 53.6 | 78.4 | 19.4  | 2.2   |
|                         | M & N    | 28.6 | 46.4 | 37.5 |      |      |       |       |
| MVDA/CL<br>2095 Records | No Flare | 91.0 | 95.0 | 93.0 |      |      |       |       |
|                         | C        | 38.0 | 29.4 | 33.7 | 54.3 | 85.8 | 12.8  | 1.4   |
|                         | M & N    | 45.8 | 26.4 | 36.1 |      |      |       |       |

the trend in the various scores, however, suggests that there may be some improvement, especially in the MVDA/CL forecast, as the size of the training set is increased from 750 to 1500 records. The improvement is less certain in increasing the set from 1500 to 2095. According to motivations which will be described later, the E score is of interest in the case of the MVDA forecast, while the F score is of prime importance for MVDA/CL. Noting these, the U scores, and the fact that we do not wish to make the training set unnecessarily large, we have decided to use 1500 records in all training sets.

#### 4.3 Inclusion of Additional Combination Parameters

Table 4 indicates that five of the six combination parameters from Table 2 were retained for analysis after the initial parameter selection. Because several of these ranked highly in frequency of selection in Test A, we decided to test additional combination parameters. As in the case of the original six, the additional parameters were derived on the basis of intuition. Their formulas are given in Table 7.

The 20 new combination parameters, in addition to the 20 parameters used in Test A, were submitted to analysis in Test D (Tables 8 and 9). It is convenient to defer the discussion of the latter to the following section.

Table 7. Additional Combination Parameters (Numbers in right-hand column refer to original parameter numbers in Table 1)

| New Parameter No.  | Parameter Formula           |
|--------------------|-----------------------------|
| Rates of Change    |                             |
| 7                  | 29 (today) - 29 (yesterday) |
| 8                  | 37 - 37                     |
| 9                  | (9•10•11•12) - (9•10•11•12) |
| 10                 | 17 - 17                     |
| 11                 | (12•17•27) - (12•17•27)     |
| 12                 | 38 - 38                     |
| 13                 | 9 - 9                       |
| 14                 | (9•10•11) - (9•10•11)       |
| 15                 | 15 - 15                     |
| 16                 | 12 - 12                     |
| Parameters Squared |                             |
| 17                 | 29 <sup>2</sup>             |
| 18                 | 37 <sup>2</sup>             |
| 19                 | (9•10•11•12) <sup>2</sup>   |
| 20                 | 17 <sup>2</sup>             |
| 21                 | 9 <sup>2</sup>              |
| 22                 | (New 7) <sup>2</sup>        |
| 23                 | (New 8) <sup>2</sup>        |
| 24                 | (New 9) <sup>2</sup>        |
| 25                 | (New 10) <sup>2</sup>       |
| 26                 | (New 13) <sup>2</sup>       |

Table 8. Parameters Submitted to Analysis and Their Frequency of Selection in 19 Subsets--Tests D, E, F, G, and H

| No. of<br>Test Parameters |    |              |    |               |    |              |    |
|---------------------------|----|--------------|----|---------------|----|--------------|----|
| D                         | 40 | Flares Today | 19 | Radio B/S     | 6  | Flare Hist.  | 2  |
|                           |    | New 18       | 17 | New 9         | 6  | New 4        | 2  |
|                           |    | New 2        | 16 | New 12        | 6  | New 23       | 2  |
|                           |    | Bright Pts.  | 14 | New 1         | 5  | Spot Class 2 | 1  |
|                           |    | New 19       | 12 | Neut. L. Chg. | 5  | Emerg. Flux  | 1  |
|                           |    | New 15       | 10 | New 5         | 5  | New 17       | 1  |
|                           |    | Mag. Grad.   | 9  | New 14        | 5  | New 19       | 1  |
|                           |    | Proton Hist. | 9  | New 21        | 5  | Spot Class 1 | 0  |
|                           |    | New 3        | 9  | New 22        | 5  | New 11       | 0  |
|                           |    | New 8        | 9  | Mag. Pol.     | 4  | New 13       | 0  |
|                           |    | New 20       | 8  | New 7         | 4  | New 16       | 0  |
|                           |    | Mag. Class   | 7  | Spot Inter.   | 3  | New 26       | 0  |
|                           |    | New 10       | 7  | New 25        | 3  |              |    |
|                           |    | Spot Class 3 | 6  | New 20        | 2  |              |    |
| A                         | 20 | See Table 4  |    |               |    |              |    |
| E                         | 15 | Flares Today | 19 | Spot Class 3  | 14 | Radio B/S    | 6  |
|                           |    | Bright Pts.  | 19 | Spot Dynam.   | 13 | Spot Class 1 | 5  |
|                           |    | Mag. Class   | 16 | Proton Hist.  | 11 | Mag. Pol.    | 5  |
|                           |    | Mag. Grad.   | 16 | Flare Hist.   | 10 | Emerg. Flux  | 4  |
|                           |    | Spot Class 2 | 14 | Neut. L. Chg. | 6  | Spot Inter.  | 3  |
| F                         | 8  | Flares Today | 19 | Spot Class 2  | 16 | Spot Dynam.  | 11 |
|                           |    | Bright Pts.  | 19 | Spot Class 3  | 14 | Spot Class 1 | 5  |
|                           |    | Mag. Class   | 17 | Mag. Grad.    | 13 |              |    |
| G                         | 5  | Flares Today | 19 | Mag. Class    | 18 | Spot Class 3 | 15 |
|                           |    | Bright Pts.  | 19 | Spot Class 2  | 16 |              |    |
| H                         | 3  | Flares Today | 19 | Mag. Class    | 19 | Spot Class 2 | 19 |

Table 9. Effects of Reduction in the Number of Input Parameters

| Forecaster |         | Number of<br>Parameters | U    | W    | Off 1 | Off 2 | R    |
|------------|---------|-------------------------|------|------|-------|-------|------|
| COMPARISON |         |                         | 52.8 | 79.2 | 18.6  | 2.2   | 2.22 |
| TEST D     | MVDA    | 40                      | 53.8 | 79.5 | 18.4  | 2.1   | 2.38 |
|            | MVDA/CL |                         | 54.6 | 85.1 | 13.4  | 1.5   | 0.73 |
| TEST A     | MVDA    | 20                      | 53.6 | 78.9 | 18.4  | 2.6   | 2.63 |
|            | MVDA/CL |                         | 54.3 | 85.5 | 12.8  | 1.7   | 0.60 |
| TEST E     | MVDA    | 15                      | 52.4 | 78.1 | 18.7  | 3.2   | 2.80 |
|            | MVDA/CL |                         | 53.9 | 85.4 | 12.8  | 1.8   | 0.65 |
| TEST F     | MVDA    | 8                       | 52.2 | 78.0 | 18.6  | 3.4   | 2.83 |
|            | MVDA/CL |                         | 53.3 | 85.0 | 13.4  | 1.6   | 0.71 |
| TEST G     | MVDA    | 5                       | 53.0 | 77.6 | 18.5  | 3.9   | 3.02 |
|            | MVDA/CL |                         | 53.7 | 84.4 | 14.0  | 1.7   | 0.83 |
| TEST H     | MVDA    | 3                       | 51.5 | 78.2 | 17.5  | 4.3   | 2.72 |
|            | MVDA/CL |                         | 53.5 | 85.2 | 13.3  | 1.5   | 0.61 |

#### 4.4 Reduction in the Number of Parameters

The computer forecast was subjected to a series of reductions (Tests E, F, G, and H) in the number of input parameters, according to Table 8, with the corresponding forecast results summarized in Table 9. Table 9 displays the effects of parameter reduction beginning with 40 parameters and ending with only three. In addition to the previously used scores we introduce R, the ratio of the number of matrix entries below the diagonal to the number above the diagonal. This ratio provides a measure of the asymmetry of the forecast, with values greater than unity indicating overprediction, and values less than unity indicating underprediction.

Table 9 clearly illustrates that the reduction in the number of parameters has a small but unfavorable effect on the computer forecasts. We may regard the tendencies for R to depart further from unity, for Off 2 to increase, and for U to decline, as evidence for progressively worsening forecasts. These three effects are most noticeable in the MVDA forecast, while the latter effect alone is marginally evident in MVDA/CL.

The effects of the parameter reduction are offset by the increase in the number of records containing all or most of the parameters submitted for analysis in the reduced sets. This improvement in representation occurs because in the reduction steps we usually eliminated those parameters that were least significant; i.e., those chosen least often in the subsets of the previous test; and, generally, the lower the significance of a parameter, the more often it is missing from the data base. It is concluded, therefore, that the decline in forecast quality in Table 9 would have been more pronounced had all parameters been present in all records. This proves that there is valuable predictive information contained in at least some of the "less significant" parameters. It is emphasized that, perhaps to a large degree, the lower significance of these parameters is due only to their frequent absence from the data base.

A final word must be noted regarding the combination parameters. Table 8 indicates that a number of these new parameters have been selected by the computer program as significant in classifying the outcomes. Due to the complex intercorrelations among various parameters, however, in addition to possible variance stabilization effects and other statistical phenomena, we do not fully understand the true significance of these combination parameters. Questions such as this probably must await further testing on data bases containing fewer missing parameters.

#### 4.5 Tests on a Fully Represented Data Base

The most important test of the computer forecast is achieved in the case where all the parameters submitted to analysis are present in all records of the data base. Such a test, using the full set of parameters, is impossible with the presently available data. A test can be made on a fully represented base, however, if, for example, only eight parameters are used, and we are willing to accept a reduced base of 3732 records, of which only 2232 remain in the test set. Such a test (I) was performed, and the results are shown in Tables 10, 11, and 12.

Test I shows a dramatic improvement in the MVDA/CL computer forecast in all scores, while the MVDA and comparison forecasts show smaller improvements. These improvements occur despite the somewhat lower flare climatology that applies to this particular test set. The fact that the comparison (subjective) forecast scores are higher indicates that the more complete observational coverage during this sample of records somehow benefits the subjective methods also.

Due to the reduced number of records, the errors associated with the Test I scores are about 50 percent higher than those stated earlier. Nevertheless, there now seems no question that the MVDA/CL forecast is superior to the others.

Table 10. Comparison of Forecasts Using a Fully  
Represented Data Base--Test I (1500-record training set)

| Largest Event<br>Forecasted | Largest Event Observed |     |       | Total<br>Forecasts |
|-----------------------------|------------------------|-----|-------|--------------------|
|                             | No Flare               | C   | M & X |                    |
| COMPARISON                  |                        |     |       |                    |
| No Flare                    | 1754                   | 90  | 8     | 1852               |
| C                           | 193                    | 70  | 24    | 287                |
| M & X                       | 28                     | 31  | 34    | 93                 |
| Total Events                | 1975                   | 191 | 66    | 2232               |
| MVDA                        |                        |     |       |                    |
| No Flare                    | 1707                   | 67  | 10    | 1784               |
| C                           | 232                    | 92  | 24    | 348                |
| M & X                       | 36                     | 32  | 32    | 100                |
| Total Events                | 1975                   | 191 | 66    | 2232               |
| MVDA/CL                     |                        |     |       |                    |
| No Flare                    | 1829                   | 97  | 14    | 1940               |
| C                           | 145                    | 82  | 33    | 260                |
| M & X                       | 1                      | 12  | 19    | 32                 |
| Total Events                | 1975                   | 191 | 66    | 2232               |

Table 11. Parameters Submitted to Analysis and  
Their Frequency of Selection in 9 Subsets--Test I

|              |   |             |   |              |   |
|--------------|---|-------------|---|--------------|---|
| Flares Today | 9 | Mag. Class  | 8 | Spot Class 2 | 5 |
| Bright Pts.  | 9 | Mag. Grad.  | 8 | Spot Class 1 | 1 |
| Spot Class 3 | 8 | Spot Dynam. | 8 |              |   |

Table 12. Comparison of Forecast Scores--Test 1

| Forecaster | Event    | F    | E    | A    | C    | U    | W    | Off 1 | Off 2 |
|------------|----------|------|------|------|------|------|------|-------|-------|
| COMPARISON | No Flare | 94.7 | 88.8 | 91.8 | 88.5 |      |      |       |       |
|            | C        | 24.4 | 36.6 | 30.5 | 8.6  | 55.5 | 83.2 | 15.1  | 1.6   |
|            | M & X    | 36.6 | 51.5 | 44.1 | 3.0  |      |      |       |       |
| MVDA       | No Flare | 95.7 | 86.4 | 91.1 | 88.5 |      |      |       |       |
|            | C        | 26.4 | 48.2 | 37.3 | 8.6  | 56.2 | 82.0 | 15.9  | 2.1   |
|            | M & X    | 42.0 | 48.5 | 40.2 | 3.0  |      |      |       |       |
| MVDA/CL    | No Flare | 94.7 | 82.6 | 93.5 | 88.5 |      |      |       |       |
|            | C        | 31.5 | 42.9 | 37.2 | 8.6  | 58.3 | 86.5 | 12.9  | 0.7   |
|            | M & X    | 59.1 | 28.8 | 44.1 | 3.0  |      |      |       |       |

### 5. CONCLUSIONS AND RECOMMENDATIONS

The conclusions of this study may be summarized as follows:

1. The standard MVDA forecast is very similar to the comparison forecast used in this study in terms of overall accuracy and bias toward overprediction.
2. The MVDA/CL forecast is superior overall to either the MVDA or the comparison forecast, and is biased toward underprediction.
3. The optimum size for the training set is probably about 1500 records for the climatologies that prevailed during 1977 and 1978.
4. "Flares Today" is the most valuable prediction parameter in the data base used here, with the "Bright Points" parameter a very close second. Other important parameters are "Magnetic Class," "Magnetic Gradient," "Spot Class," and "Sunspot Dynamics."
5. Combination parameters, although their role is not fully understood, seem to improve forecast scores.
6. Some of the often missing parameters (which probably, therefore, only appear to be less significant as predictors) contain valuable predictive information. Probable candidates include "Radio Burst/Sweep," "Neutral Line Changes," "Neutral Line Complexity," and "Emerging Flux."

The MVDA/CL procedure may be capable of producing forecasts superior to any presently available using conventional, subjective techniques. It has been shown that its skill becomes markedly evident when complete parameter representation is achieved in the data base. On the basis of this, we predict that with

improvements in data consistency, as well as the inclusion of new, objective parameters in the future, the computer forecast scores will continue to improve.

This study has led us to make the following recommendations concerning the use of the two computer forecasts:

1. Provide a flare forecast derived from MVDA/CL for those customers who cannot tolerate false flare alarms (note the comparison of F scores in Table 12).
2. Provide a flare forecast derived from standard MVDA for those customers who need to be forewarned of flares as often as possible (compare E scores in Table 12).
3. Improve the coverage for the parameters in Table 1 that are deemed "less significant" by virtue of their frequent absence in the data base.
4. Improve the objectivity and consistency of all parameters.

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