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Techniques for Measuring Liquid Water Content Along a Trajectory

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Chief Scientist

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The standard procedure for estimating the point value of liquid water content (M) along the reentry path of a ballistic missile is to use a radar derived value of radar reflectivity factor (Z), with the M to Z relation obtained from aircraft measurements of the particle spectrum. This report contains descriptions of the various techniques used to make the radar and aircraft data compatible. The results of various methods to predict the liquid water content values are compared with aircraft measured values. → next page

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The extrapolation of instrumentally truncated aircraft measurements to include the entire range of particle sizes is treated in Section 1. Section 2 contains a discussion of the method of deriving nominal M-Z relations, applies the method to data acquired at Kwajalein Missile Range, and provides M-Z relations for rain, small snow, large snow, and bullet rosettes. The introduction of the factor "k" to convert from aircraft measurements to radar derived values is also discussed in Section 2. The relationship of k with altitude and temperature for data from Kwajalein and Wallops Island is covered in Section 3. Finally, Section 4 contains a determination of the accuracy to which liquid water content can be estimated from climatological storm data for moderate to heavy stratiform winter storms at Wallops Island. The results of Section 4 demonstrate that situations which deviate significantly from climatological averages still require time-specific M-Z relations derived from simultaneous aircraft-radar measurements.

Foreword

The necessity of determining the liquid water content along a trajectory or line-of-sight recurs ever more frequently as more stringent performance specifications are placed on navigation, communication and weapon systems operating at ever smaller wavelengths in the microwave spectral region. Scientists at AFGL (and its predecessor, AFCRL) have long had an interest in the effects of clouds and precipitation on microwave propagation, and have done much theoretical work relating observed physical characteristics of hydrometeors to attenuation and backscatter.[1,2] However, it was not until 1971 that the opportunity arose to combine aircraft measurements of cloud and precipitation with ground-based radar observations on a systematic basis. At the request of SAMSO (now BMO), AFGL scientists provided environmental data for ballistic missile test firings, first at NASA's Wallops Flight Test Center (WFTC) in Wallops Island, Virginia, and later (1973) at the Army's Kwajalein Missile Range (KMR) in the Marshall Islands.[3] The task as originally stated was to determine the liquid water content of water and ice hydrometeors encountered by a vehicle passing through the atmosphere at hypersonic speeds. This was later extended to include the determination of particle spectra of all hydrometeors, both solid and liquid.

The requirements of the weather erosion program were more rigorous than any imposed by the microwave studies. While the latter could be met by an integrated, or average, value of the critical parameter, the application to nosecone erosion required point by point values along the trajectory. This is because the erosion is a function of missile velocity and air density, or altitude. It is not possible to make aircraft measurements along the expected missile trajectory or at selected altitudes close to the time of reentry. Therefore, it was determined that the best way to meet this requirement was by the radar measurement of the radar reflectivity factor along the reentry trajectory. The problem then became the determination of accurate and precise relations between the observed reflectivity and the point values of liquid water content. These relations were established by combined radar and aircraft measurements taken before and after the missile reentry.

Cloud and precipitation particle size spectra measurements taken aboard an aircraft are characterized by small sampling volumes and loss of information at larger sizes because of the reduced probability of detecting these fewer particles and also because of instrument truncation. Radar measurements, on the other hand, give only averages of the summation of the particle backscatter over a large volume, and do not record contributions from the lower end of the size spectrum because of sensitivity limitations. Both sampling volume and sensitivity are functions of radar range. The problem inherent in relating aircraft data and radar measurements consumed much of the effort devoted to this task. Some of the solutions arrived at are described in the first two sections of this report. Section 1 describes this method of extrapolating an instrumentally-truncated, aircraft-obtained distribution. The introduction of the parameter k and its use in converting from aircraft measurements of the hydrometeors to radar derived values of M is contained in Section 2.

The scope of this report is limited to methods of determining M which includes the mass of all cloud and precipitation particles, whether ice or water. Other parameters, such as average particle diameter, while important, do not readily lend themselves to radar observation without previous assumptions concerning the shape of the size spectrum.

No attempt has been made in this report to derive and evaluate water content-radar reflectivity relations within the melting layer. At this time our ignorance of the exact physical processes and their interactions taking place within the melting layer, precludes the development of any standard procedure for remote measurements at these altitudes.

Rosemary M. Dyer

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1. Dyer, R.M. (1973) Radar studies of precipitation and their applications to Air Force problems. Proceedings Air Force Systems Command 1973 Science and Engineering Symposium, Vol. I, AFSC-TR-73-003.
2. Falcone, R.J., Jr., and Dyer, R. (1970) Refraction, Attenuation, and Back-Scattering of Electromagnetic Waves in the Troposphere: A revision of Chapter 9, Handbook of Geophysics and Space Environments. Air Force Surveys in Geophysics No. 214, AFCRL-70-007, AD 703310.
3. Barnes, A.A., Jr., Nelson, L.D., and Metcalf, J.I. (1974) Weather Documentation at Kwajalein Missile Range, Air Force Surveys in Geophysics No. 292, AFCRL-TR-74-0439, AD A000925.

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TECHNIQUES FOR MEASURING LIQUID WATER CONTENT ALONG A TRAJECTORY

1. Estimated Distributions From Instrumentally Truncated Data

Robert O. Berthel

1.1 INTRODUCTION

The specialized instrumentation used aboard the AFGL cloud physics aircraft includes two types of optical array probes manufactured by Particle Measuring Systems, (PMS) Inc., of Boulder, Colorado. Both are specifically designed to characterize and count hydrometeors. The one-dimensional system (PMS 1-D) classifies hydrometeors as to their physical size in one dimension and also measures the number density.[4] The two-dimensional system (PMS 2-D) provides an electronically produced shadowgraph from which a two-dimensional area can be determined and particle shape inferred.[5]

Using the data from the 1-D, the liquid water content (M) of the sampled hydrometeors can be determined by

$$M = C \sum_{i=1}^{i=n} N_i D_i^3 \qquad g\ m^{-3} \tag{1.1}$$

(Received for publication 11 March 1981)

4. Knollenberg, R.G. (1970) The optical array: An alternative to scattering or extinction for airborne particle size determination, J. Appl. Meteor. 9:86-103.

5. Knollenberg, R.G. (1976) Three new instruments for cloud physics measurements: The 2-D spectrometer, the forward scattering spectrometer probe and the active scattering aerosol spectrometer. Preprints International Conference on Cloud Physics, Amer. Meteor. Soc., July 26-30, 1976, Boulder, Colorado, pp 554-561.

where N_i (m^{-3}) is the number concentration in a class having D_i (mm) as the mid-size equivalent melted diameter and

$$C = \frac{\pi}{6} \times 10^{-3} \rho_\omega \tag{1.2}$$

where ρ_ω (g cm^{-3}) is the density of liquid water. The equivalent radar reflectivity factor (Z) (defined as the reflectivity factor that would produce the observed return signal if all particles were spherical and only Rayleigh scattering were present,[6]) can also be calculated from the spectral data by

$$Z = \sum_{i=1}^{i=n} N_i D_i^6 \quad mm^6\, m^{-3}\,. \tag{1.3}$$

In the case of an ice hydrometeor, the D in the equations is taken as the equivalent melted diameter of the ice crystal and is defined by

$$D = \alpha \ell^{\beta} \tag{1.4}$$

where α and β are values that change in accord with the crystal type and ℓ is the physical dimension of the ice particle as measured by the PMS equipment.[7]

If the crystal habit is known, the M and Z of the distribution can be determined from the spectral information supplied by the PMS 1-D. A problem develops, however, when some of the crystals grow to sizes that exceed the measuring capabilities of the instrument. This situation occurs mainly in large snow, but could exist in rain if the instrument's limit is less than the breakup size of a waterdrop,[8] but is not a problem in the small snow or ice crystal regions. When particle sizes exceed the range of the instrument, the spectral distribution is instrumentally truncated and the derived M and Z are valid only for the portion of the spectrum that was actually measured. The PMS 2-D spectrometer instrument, on the other hand, is capable of recording parts of these large particles which gives some knowledge of the physical size, although the number density and size classification are not well defined.

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6. Ryde, J.W. (1946) Echo Intensities and Attenuation Due to Clouds, Rain, Hail, Sand and Dust Storms at Centimetre Wavelengths, GEC Report No. 7831, October 1941, also GEC Report No. 8516, Aug. 3, 1944, by J.W. Ryde and D. Ryde, corrections.

7. Cunningham, R.M. (1978) Analysis of particle spectral data from optical array (PMS) 1-D and 2-D sensors. Preprints Fourth Symposium on Meteorological Observations and Instrumentation, Amer. Meteor. Soc. April 10-14, 1978, Denver, Colorado, pp 345-350.

8. List, R., and Gillespie, J.R. (1976) Evolution of raindrop spectra with collision-induced breakup, J. Atmos. Sci. 33:2007-2013.

An investigation of the instrument truncation problem has resulted in a method of extrapolating the 1-D truncated distributions to give reasonable approximations of the non-truncated spectrum. The details of this study have been reported elsewhere[9] and only a brief description of the technique is described here.

1.2 EXTRAPOLATION OF A TRUNCATED DISTRIBUTION

The problems of extrapolating a distribution are twofold. Which method is to be used to extend the distribution beyond the upper truncation limit and what is the limit of the extension, or the maximum particle size?

When addressing the first problem, it is logical to assume that the form of the number-density distribution will remain relatively constant beyond the physical size limitation. Therefore, the trend of the distribution should be established from the measured portion of the spectra. It was found using the 1-D large-snow data that in the majority of cases the distribution is adequately described by an exponential of the form

$$N_D = N_o e^{-\Lambda D} \quad \text{Number m}^{-3}\ \text{mm}^{-1} \qquad (1.5)$$

where N_o is the intercept of the distribution function, with units of number per cubic meter per millimeter bandwidth. Λ is the slope of the distribution, with number per millimeter bandwidth, and D is the drop diameter (or in the case of ice hydrometeors, the equivalent melted diameter) in millimeters. The maximum particle size (D_m) has previously been found empirically to be related to the slope of the distribution by an expression of the form

$$C = \Lambda D_M \qquad (1.6)$$

where C is a dimensionless constant.[10, 11]

9. Berthel, R.O. (1980) A Method to Predict the Parameters of a Full Spectral Distribution from Instrumentally Truncated Data, Environmental Research Papers No. 689, AFGL-TR-80-001, AD A085950.

10. Plank, V.G. (1974) Hydrometeor Parameters Determined From the Radar Data of the SAMS Rain Erosion Program, Environmental Research Papers No. 477, AFCRL/SAMS Report No. 2, AFCRL-TR-74-0249, AD 786454.

11. Plank, V.G. (1977) Hydrometeor Data and Analytical-Theoretical Investigations Pertaining to the SAMS Missile Flights of the 1972-73 Season at Wallops Island, Virginia, Environmental Research Papers, No. 603, AFGL/SAMS Report No. 5, AFGL-TR-77-0149, AD A051192.

These findings are illustrated in Figures 1. 1 and 1. 2, which show the plotted number density vs the equivalent melted diameter for two MC-130E passes taken on 4 July 1978 over the Kwajalein Atoll. The hydrometeors of these distributions are classified as large snow and have maximum physical sizes larger than the 1-D instrument is able to measure. These distributions are typical of the experimental data collected by AFGL in past years, exhibiting a systematic decrease in the number density over the mid- to large-size diameter range. They also demonstrate that a straight line, on a log-linear plot, can adequately describe the population for the major part (approximately 92%) of the total measured size range. Deviation from the straight line occurs at or about the junction of the cloud and precipitation probes with the cloud probe spectra having much steeper slopes. The contributions to M and Z from the precipitation probe data (M_P, Z_P) constitute a high percentage of the measured value and indicate, at least in truncated situations, that the M_C and Z_C derived from the cloud probe spectra are minor contributions. This is shown in Table 1. 1.

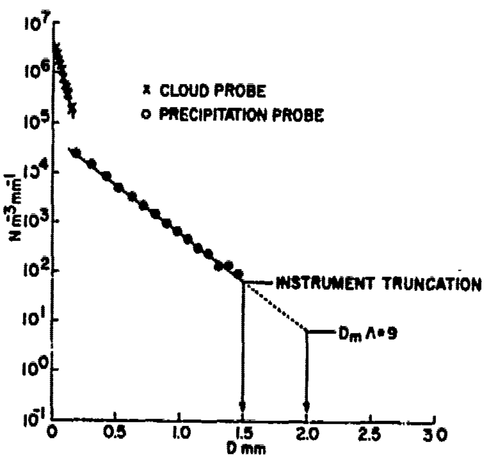

Figure 1. 1. Number-Density Plot of Large Snow Hydrometeors as Recorded by the PMS 1-D on the MC-130E, Pass 3 at an Altitude of 5. 1 km Over the Kwajalein Atoll on 4 July 1978 (142 sec average)

When the data from the precipitation probe of the 1-D instrument are considered by themselves, it is apparent that the size distribution can be reasonably described by a function of exponential type which agrees with the observations made by other investigators for snow.[12, 13] Although this occurrence is evident in the majority of cases, occasional sampling levels may exhibit number density distributions that do not conform to an exponential fit and for which the following relations do not hold.

12. Gunn, K. L. S., and Marshall, J. S. (1958) The distribution with size of aggregate snowflakes, J. Meteorol. 15:452-479.

13. Sekhon, R. S., and Srivastava, R. C. (1970) Snow size spectra and radar reflectivity, J. Atmos. Sci. 27:299-307.

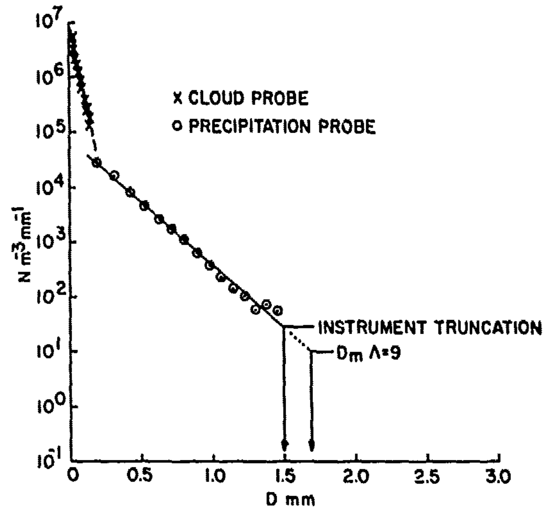

Figure 1.2. Number-Density Plot of Large Snow Hydrometeors as Recorded by the PMS 1-D on the MC-130E, Pass 5 at an Altitude of 6.6 km Over the Kwajalein Atoll on 4 July 1978 (212 sec average)

Table 1.1 Comparison of the M and Z Contributions Derived From the Measurements of the Cloud and Precipitation Probes of the 1-D Instrument

		Cloud Probe	Precip Probe	Total	Percent Cloud	Percent Precip
M gm^{-3}	Pass 3	0.0283	0.3279	0.3504	6.4	93.6
	Pass 5	0.0244	0.3062	0.3306	7.4	92.6
Z mm^6 m^{-3}	Pass 3	0.054	544.1	544.154	0.01	99.99
	Pass 5	0.040	336.4	336.440	0.01	99.99

For a distribution that can be described by Eq. (1.5), the total number of hydrometeors between a minimum diameter (d) and a maximum (D_M) is

$$N_T = \int_d^{D_M} N \, dD \qquad \text{Number } m^{-3} \qquad (1.7)$$

or

$$N_T = \frac{N_o \, r_N}{\Lambda} \qquad \text{Number } m^{-3} \qquad (1.8)$$

where r_N is the truncation ratio of the number of hydrometeors contained within the d and D_M limits to the total N as

$$r_N = \frac{\int_d^{D_M} N \, dD}{\int_o^{\infty} N \, dD} \quad (1.9)$$

or

$$r_N = e^{-d\Lambda} - e^{-D_M \Lambda} . \quad (1.10)$$

The hydrometeor liquid water content is distributed with diameter as

$$M_D = \frac{\pi}{6} \times 10^{-3} \rho_\omega \, N_o \, D^3 \, e^{-\Lambda D} \ g \ m^{-3} \ mm^{-1} , \quad (1.11)$$

and the total M from d to D_M is

$$M = \int_d^{D_M} M_D \, dD \ g \ m^{-3} \quad (1.12)$$

or

$$M = \frac{\pi \times 10^{-3} \rho_\omega N_o r_M}{\Lambda^4} \quad g \ m^{-3} \quad (1.13)$$

where r_M is the truncation ratio of the liquid water content contained within the limits d and D_M to the total M as

$$r_M = \frac{\int_d^{D_M} M_D \, dD}{\int_o^{\infty} MD \, dD} \quad (1.14)$$

or

$$r_M = \frac{1}{6} \left\{ e^{-d\Lambda} [(d\Lambda)^3 + 3(d\Lambda)^2 + 6d\Lambda + 6] - e^{-D_M\Lambda} [(D_M\Lambda)^3 + 3(D_M\Lambda)^2 + 6D_M\Lambda + 6] \right\}. \quad (1.15)$$

The radar-reflectivity distributed values for the same population are

$$Z_D = N_o D^6 e^{-\Lambda D} \quad mm^6\, m^{-3}\, mm^{-1}, \quad (1.16)$$

and the total Z within the limits d to D_M is

$$Z = \int_d^{D_M} Z_D\, dD \quad mm^6\, m^{-3} \quad (1.17)$$

or

$$Z = \frac{720\, N_o\, r_Z}{\Lambda^7} \quad mm^6\, m^{-3} \quad (1.18)$$

where r_Z is the truncation ratio of the radar reflectivity contained within the limits d to D_M to the total Z as

$$r_Z = \frac{\int_d^{D_M} Z_D\, dD}{\int_0^{\infty} Z_D\, dD} \quad (1.19)$$

or

$$r_Z = \frac{1}{720} \left\{ e^{-d\Lambda} [(d\Lambda)^6 + 6(d\Lambda)^5 + 30(d\Lambda)^4 + 120(d\Lambda)^3 + 360(d\Lambda)^2 + 720d\Lambda + 720] - e^{-D_M\Lambda} [(D_M\Lambda)^6 + 6(D_M\Lambda)^5 + 30(D_M\Lambda)^4 + 120(D_M\Lambda)^3 + 360(D_M\Lambda)^2 + 720(D_M\Lambda) + 720] \right\}. \quad (1.20)$$

When Eqs. (1.13) and (1.18) are solved for N_o they can be equated as

$$\frac{M\Lambda^4}{\pi \times 10^{-3} \rho_\omega r_M} = \frac{Z\Lambda^7}{720\, r_Z} \quad (1.21)$$

and solved for Λ as

$$\Lambda = \left(\frac{M\ r_Z\ 720}{Z\ \pi \times 10^{-3}\ \rho_\omega\ r_M} \right)^{1/3} \quad mm^{-1}\ . \tag{1.22}$$

For $\rho_\omega = 1\ g\ cm^{-3}$, this reduces to

$$\Lambda = 61.2 \left(\frac{M\ r_Z}{Z\ r_M} \right)^{1/3} \quad mm^{-1}\ . \tag{1.23}$$

Since both r_M and r_Z incorporate Λ as a term [Eqs. (1.15) and (1.20)], then a trial and error method can be used to solve Eq. (1.23) whereby the value of Λ is adjusted until both sides of the equation are equal. Once Λ has been found, N_o can be determined using either Eq. (1.13) or (1.18) and the total number of particles may be calculated from Eq. (1.8).

If the M and Z derived from the truncated precipitation probe measurements of the 1-D instrument are used in these equations with the appropriate diameter limits, where D_M now becomes the upper truncation diameter D_T, then the distribution properties of the spectra can be described by a function of exponential type. The plotted exponentials of Figures 1.1 and 1.2 (solid lines) were calculated in this manner where d = 0.133 mm and D_T = 1.496 mm. It is apparent from these figures that the solid line representing the exponential number distribution establishes the trend of the spectrum and an estimation of that portion of the population missed because of instrument truncation can be made by extrapolating the line to some larger diameter.

Now the second part of the problem becomes paramount and that is the establishment of a new D_M value which reflects the maximum size of the non-truncated spectra. Past studies of observed versus exponential distributions conducted at AFGL have shown that D_M can be related to Λ through Eq. (1.6). The dimensionless constant C in Eq. (1.6) is dependent upon crystal type and density and remains relatively constant for any specific snow type occurring in a particular storm. The normal range of C is between 9 and 12.

When the maximum particle sizes of known crystal habit that are present in any particular pass are measured with the Aluminum Foil Hydrometeor Sampler[14] and/or the PMS 2-D, they can be converted to D_M values through Eq. (1.4) which, in turn, can be used in Eq. (1.6) with the Λ of that pass to define C. Thus, the relationship

14. Church, J.F., Pocs, K.K., and Spatola, A.A. (1975) The Continuous Aluminum Foil Sampler; Design Operation, Data Analysis Procedures, and Operating Instructions, Instrumentation Papers No. 235, AFCRL-TR-75-0370, AD A019630.

of D_m and Λ for the complete pass can then be used to define the D_m of the truncated spectra for any portion of the pass.

The maximum physical sizes recorded by the 2-D for the flight passes plotted in Figures 1.1 and 1.2 were approximately 8 and 6.5 mm. When these crystal sizes were substituted in Eq. (1.4) and the parameters of ℓ and β were given the values of 0.4 and 0.782 which are designated for large snow,[7] the maximum equivalent melted diameters of 2.03 and 1.73 mm resulted. These maximum diameters were then used in Eq. (1.6) with the Λ values from the passes of Figures 1.1 and 1.2 to determine the $D_m \Lambda$ constants. The C's for both passes were approximately 9.0.

Since N_o and Λ are known and a D_M has been established, the estimated values of M and Z can now be determined by finding the M and Z contribution from the extrapolated part of the distribution (dotted line in Figures 1.1 and 1.2) and adding them to the measured values as

$$M_E = M_P + \Delta M \qquad \text{g m}^{-3} \tag{1.24}$$

and

$$Z_E = Z_P + \Delta Z \qquad \text{mm}^6\ \text{m}^{-3} \tag{1.25}$$

where M_E and Z_E are the estimated liquid water content and radar reflectivity, M_P and Z_p are the values derived from the precipitation probe measurements and ΔM and ΔZ are the contributions from the extrapolated D_T to D_M portion.

The equations for M_E and Z_E thus become

$$M_E = \int_d^{D_T} M_D \, dD + \int_{D_T}^{D_M} M_D \, dD \qquad \text{g m}^{-3} \tag{1.26}$$

and

$$Z_E = \int_d^{D_T} Z_D \, dD + \int_{D_T}^{D_M} Z_D \, dD \qquad \text{mm}^6\ \text{m}^{-3} \tag{1.27}$$

Since both first terms are known quantities, the solving of these equations becomes a two-part process where M_P and Z_P are used in Eq. (1.23) to determine the Λ of the exponential function which, in turn, is used in either Eq. (1.13) or (1.18) to find N_o. These parameters are then incorporated into the second term of Eq.'s (1.26) and (1.27) to find ΔM and ΔZ.

Table 1.2 lists the M_P and Z_P that were derived from the measurements from the precipitation probe for the two sample passes compared with the estimated M_E and Z_E from the extrapolated spectra.

Table 1.2. Comparison of Precipitation Probe Data M_P, Z_P With the Estimated Values (M_E, Z_E) Determined from the Extrapolated Distributions

	M_P g m^{-3}	M_E g m^{-3}	FACTOR M_E/M_P	Z_P mm^6 m^{-3}	Z_E mm^6 m^{-3}	FACTOR Z_E/Z_P
Pass 3	0.3824	0.4144	1.084	544.1	845.5	1.554
Pass 5	0.3062	0.3137	1.024	336.4	393.6	1.170

1.3 CONCLUSIONS

Although the amount of M and Z missed because of instrument truncation could vary considerably in different situations, it is clear from these results that it is the Z parameter which shows the largest degradation. In the case of a slightly truncated distribution, the measured M may be acceptable but if the sampled hydrometeors are to be defined in a relationship of aircraft measured M and Z, then new estimated values of Z have to be incorporated.

The reader is referred to Berthel[5] for a more detailed discussion on how instrumentally truncated aircraft data may be utilized in regression analysis.

2. Nominal Equations to Determine Liquid Water Content (Mass) From Radar Reflectivity at Kwajalein

Robert O. Berthel

2.1 INTRODUCTION

Knowledge of the mass of liquid water (M) encountered by a missile is crucial in the evaluation of weather-related missile erosion. In tests conducted at the Kwajalein Missile Range (KMR), the M profile is established from the reflectivity measurements (Z) of the weather radar taken along the trajectory path of the missile which are then associated with the aircraft-radar relationships of M vs Z as determined from that specific time period. These relationships are developed from simultaneous radar and aircraft measurements of cloud and precipitation particles at various altitude levels made before and after missile flight. The number densities measured with the aircraft instrumentation, modified by other meteorological information, are converted into "k" factors (defined by the relation $k = M/\sqrt{Z}$) and are then correlated with the radar returns to give power function relationships which are readily converted into mass.[15]

Until specific aircraft-radar equations are established, any determination of M from radar Z is necessarily based on standard relationships. Up to the time of this study, nominal equations had not been derived from the KMR area and the equations being used as stand ds were those that were used in the weather erosion tests conducted at Wallops Island, Virginia.[10]

15. Plank, V.G., Berthel, R.O., and Barnes, A.A. (1980) An improved method for obtaining the water content values of ice hydrometeors from aircraft and radar data, J. Appl. Meteor. 19:1293-1299.

The purpose of this investigation is to establish standard equations peculiar to the climatological conditions of the Kwajalein Atoll.

2.2 DATA

All aircraft-radar correlated passes that were taken at KMR from January 1977 through January 1979 were considered in this study. The original data tapes from the Learjet (Aeromet Inc., Tulsa, Oklahoma), the MC-130E (AFGL) and the ALCOR and TRADEX radars (from MIT Lincoln Laboratory) were reprocessed through the AFGL computers. Each separate pass was then reanalyzed in accordance with the following criteria:

Aircraft

(a) Instruments operating properly
(b) Particle typing (verification of original type or change)
(c) Level altitude (aircraft not ascending or descending)
(d) Pass time limits (to conform to periods of level flight)

Radar

(a) Instrument operating properly
(b) Correct altitude, range and elevation
(c) Reflectivity did not include returns from aircraft
(d) Sufficient variability in Z to insure a correct regression analysis.

The few passes that did not meet these criteria were discarded.

Forty-three passes were deemed to be of sufficient quality to use in the analysis:

14 June 1977	6 passes	7 January 1978	4 passes
6 August 1977	1 pass	23 June 1978	3 passes
16 September 1977	4 passes	4 July 1978	17 passes
4 November 1977	6 passes	26 June 1979	2 passes

There were 11 cases of Bullet-Rosettes, 21 cases of Small Snow, 8 cases of Large Snow and 3 cases of Rain. Out of the 43 cases used in this analysis, 17 or 39.5% were from the "heavy" weather test of 4 July 1978, the remainder being composed of "light" weather or cloud situations.

2.3 DATA ANALYSIS

In the normal analytical procedure used at AFGL, the multi-sample spectral data, acquired by the Lear and MC-130E along particular flight paths through cloud and storm situations, are used to obtain equations that mathematically define hydrometeor environments. This is done by the conventional method of cross-plotting the logarithmic values of M and Z from the multiple, individual samples. The least-square analyses of this field of data points determines the line of best fit and provides the coefficient and exponent of the power function regression equation. The ultimate goal is to relate M and Z in a regression relationship as

$$M = a\,Z^{b} \qquad (2.1)$$

so that the liquid-water-content of a specific hydrometeor region can be calculated from a given radar reflectivity.

Two methods have been employed in the past to acquire M-Z information. One is the regression of the dependent aircraft derived variables of M_A versus Z_A, and the other the regression of the independent variables M_A versus the radar measured Z_R.[3]

Obtaining the M and Z from aircraft-measured ice hydrometeors requires the knowledge of the sizes of water drops that would be formed if the ice particles were melted.[7] These equivalent melted diameters are determined by

$$D = \alpha\,\ell^{\beta} \qquad (2.2)$$

where ℓ is the measured physical size of an ice particle and α and β are values assigned to a particular type of crystal.

The M of a measured distribution is found by

$$M_A = \frac{\pi}{6} \times 10^{-3}\,\rho_\omega\,N_T\,\alpha^3 \sum_{i=1}^{i=n} \ell_i^{3\beta}\,\gamma_i \qquad g\,m^{-3} \qquad (2.3)$$

where ρ_ω is the density of water (1 g cm^{-3}) and γ_i is the ratio of the number of particles in a class (i) to the total number (N_T) in the distribution.

The Z, which is equivalent to the reflectivity measured by a radar, from a distribution is determined by

$$Z_A = N_T\,\alpha^6 \sum_{i=1}^{i=n} \ell_i^{6\beta}\,\gamma_i \qquad mm^6\,m^{-3}\,. \qquad (2.4)$$

These equations show that the aircraft spectral M_A and Z_A are highly dependent upon the α and β used to convert the measured physical sizes of the ice particles into equivalent melted diameters. When these values are used in the regression analysis Eq. (2.1), any uncertainty in the knowledge of crystal type is reflected in both Z and M. Using Z_R in place of the Z_A in Eq. (2.1) removes these particular uncertainties from one parameter of the analysis since Z_R does not require a prior knowledge of crystal type but only if the particles are water or ice.

Past studies at AFGL[11, 15] have shown that a spectral parameter k, which is defined as

$$k = \frac{M_A}{\sqrt{Z_A}} \quad g\ mm^{-3}\ m^{-1.5} \tag{2.5}$$

when used in the regression analysis as

$$k = a_1 Z_R^{b_1} \tag{2.6}$$

will minimize the uncertainty effect of particle typing. The related M of the sample is then found by

$$M = a_1 Z_R^{b_1 + 0.5} . \tag{2.7}$$

Each correlated pass was computer processed using 3-sec running mean averages to produce individual k-Z_R relationships [Eq. (2.6)]. The cases in each particle type category were then combined with each case weighted equally to form a composite equation to relate M and Z_R [Eq. (2.7)].

The number of points used in each category were:

Bullet-Rosettes	2229	Large Snow	1976
Small Snow	4166	Rain	581

2.4 RESULTS

The composite equation for Bullet-Rosettes is

$$M = 0.04483\ Z_R^{0.4602} \tag{2.8}$$

with a standard deviation of 3.13 dBM (a factor of 2.06.).

for Small Snow:

$$M = 0.04300\ Z_R^{0.5231} \tag{2.9}$$

with a standard deviation of 2.07 dBM (a factor of 2.03.),

for Large Snow:

$$M = 0.02024\, Z_R^{0.4406} \qquad (2.10)$$

with a standard deviation of 1.92 dBM (a factor of 1.55.),

for Rain:

$$M = 0.005476\, Z_R^{0.5649} \qquad (2.11)$$

with a standard deviation of 2.79 dBM (a factor of 1.90.).

As previously mentioned, there are only three cases of rain considered in this analysis. The procurement of more correlated data would be expected to change this equation although no significant differences are anticipated.

All eight cases of large snow were obtained on the same date, 4 July 1978, and seven of these cases exhibit some degree of instrument truncation. When the extrapolation procedure outlined in Section 1 is used on these data, the composite equation for large snow [Eq. (2.10)] becomes

$$M = 0.02463\, Z_R^{0.3899}. \qquad (2.12)$$

Comparison of M calculated from the equation derived from truncated data [Eq. (2.10)] and from the extrapolated Spectra [Eq. (2.12)] show that the noncorrected equation gives higher values. For a Z = 300, the increase is 9.7%, and when Z = 700, the increase is 14.6%.

It must be emphasized that these nominal equations are averages that were obtained using all available data. The standard deviations (and factors) include 67% of these data thus, 33% of the data lie outside of the limits and therefore, in some cases, the data points deviate considerably from the average.

The computer processing of radar reflectivity using these nominal equations will give quick and convenient estimations of liquid-water-contents for use in the predictions of specific weather criteria when the sampling aircraft is unable to be on station or when only crystal habit information is available from aircraft. Use of these equations are not intended to be replacements for those individual M-Z relationships that are derived from simultaneous aircraft and radar measurement for specific time periods.

3. Variation of Mk With Altitude and With Temperature

Y. Izumi

3.1 INTRODUCTION

The problem of combining aircraft and radar data led to the introduction of the factor k, where $k = M/\sqrt{Z}$. In the course of data collection at Wallops Island and at Kwajalein, it was noted that k varies with particle type; average values increase as we progress from rain, to large snow, to small snow, and finally to bullet rosettes. Because particle type is to a large extent a function of temperature and therefore altitude, it was decided to search for some relation between k and altitude, or between k and temperature.

This section presents the results of analyzing data from Wallops Island and Kwajalein, and expressing k as a function, first of altitude, then of temperature. In order to simplify the presentation, the parameter $Mk = 1000\,k$ is used. Mk ranges from approximately 1 to 150.

3.2 DATA

The Mk values presented here are pass averages of 3-sec running means obtained during aircraft and radar correlation runs. The Wallops correlation runs were made on eight different days during the period from January through April 1977, and the Kwajalein runs were made on seven different days during the 13-month period starting June 1977.

A number of correlation runs were deleted from this study to avoid misinterpretation. Those deleted were as follows:

(1) Runs with missing or extremely small values of M. The total aircraft M_A, which is the sum of the cloud M_C and the precipitation M_P, and which is directly related to Mk, is not reliable in such cases, particularly in the regime where the contribution of cloud M_C to the total M is expected to be large.

(2) Runs in which the comparison of the arithmetic mean with the geometric mean of the aircraft data showed large discrepancies. In such cases the variability of the data was too great to obtain meaningful averages.

A total of 110 Mk values were thus available for this study, with 65 from Wallops and 45 from Kwajalein.

3.3 RESULTS

The variation of Mk with aircraft altitude for Wallops is shown in Figure 3.1 and for Kwajalein in Figure 3.2. The variation of Mk with aircraft measured temperature for Wallops is shown in Figure 3.3 and for Kwajalein in Figure 3.4. The particle types are indicated in all four figures. The figures show a much greater range of altitude and temperature at Kwajalein than at Wallops Island. The data points scatter widely about the regression lines, but the trend is for Mk to increase with increasing altitude and to decrease with increasing temperature. The regression equations obtained are listed in Table 3.1. In the equations H is the height in kilometers and T is the temperature in degrees Celsius. Along with the equations are tabulated the correlation coefficients and the rms values.

Because of the different temperature and altitude relationships found in the two markedly different climatic regions of Wallops Island (75°W, 38°N) and Kwajalein (168°E, 9°N), significant differences were expected, particularly between the height equations. However, the results showed surprising similarity.

Data were obtained up to greater heights and there was a larger range of heights sampled at Kwajalein. Nevertheless, the differences in the Mk - Height relationships at Wallops and Kwajalein are meteorologically significant. As can be seen by comparing Figures 3.1 and 3.2, these differences were larger at the higher heights and undoubtedly were related to the more extensive depths of the storms at Kwajalein. Wallops Island storms topped out between 8 and 10 km since the storms sampled were limited to winter and early spring, when the tropopause was generally below 10 kilometers. With Kwajalein so close to the equator there was less variation of tropopause height, 16 to 17 kilometers.

The spread of Mk values versus temperature for the various crystal habits was less at Wallops Island than it was at Kwajalein. In addition, it should also be noted that the correlations were smaller and the rms values were larger at Kwajalein as seen in Table 3. 1. This can be attributed to the greater extent and more convective nature of the storms at Kwajalein.

Figures 3. 3 and 3. 4 clearly show different relations between Mk and temperature at the two locations thus indicating that different relationships should be used for tropical and extratropical storm systems.

The above results show that reasonable estimates of Mk as a function of height or temperature can be made for either Wallops Island or Kwajalein. Whether the results are accurate enough depends on the accuracy requirements of the particular experiment.

Table 3. 1. Mk as a Function of Height or Temperature at Wallops Island and Kwajalein

Location	Regression Equations	Correlations	rms
Wallops	$Mk = 4.699\, e^{0.284\, H}$	0.833	0.433
Kwajalein	$Mk = 4.919\, e^{0.205\, H}$	0.772	0.487
Wallops	$Mk = 12.217\, e^{-0.0474\, T}$	-0.868	0.388
Kwajalein	$Mk = 14.264\, e^{-0.0291\, T}$	-0.739	0.516

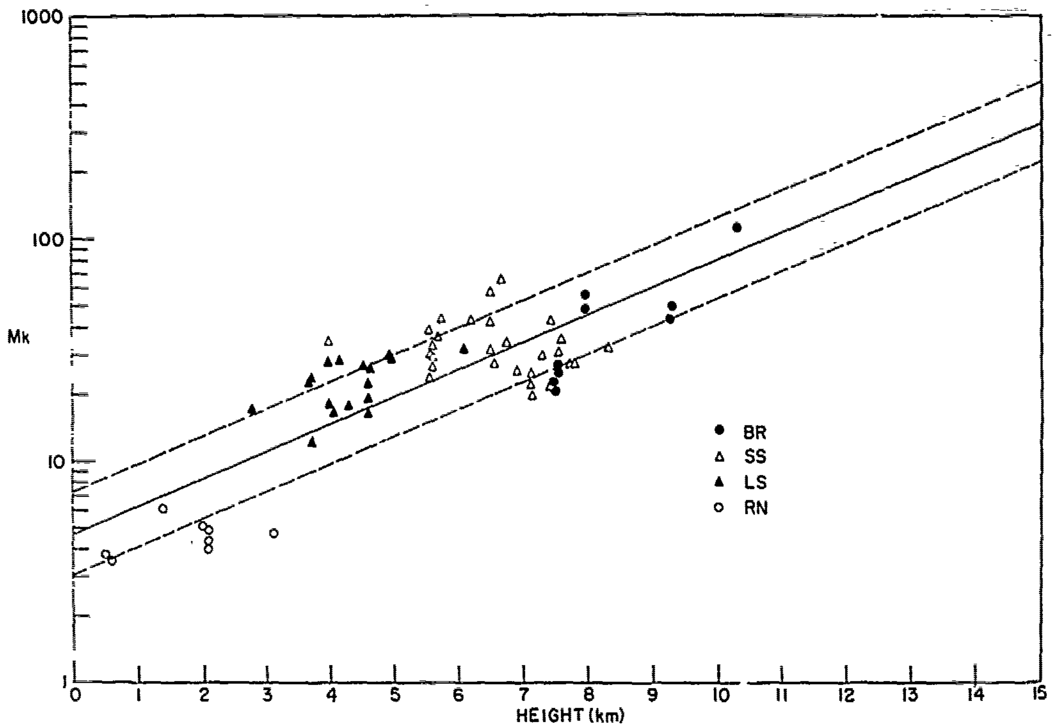

Figure 3. 1 Variation of Mk With Altitude for Wallops Island

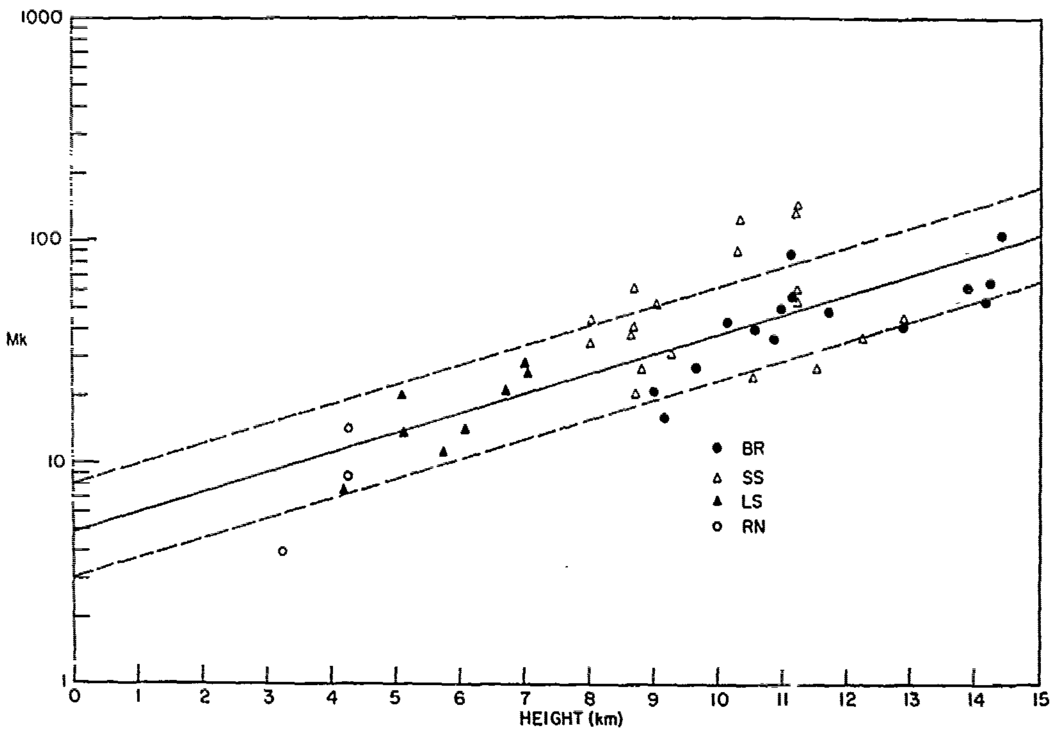

Figure 3.2 Variation of Mk With Altitude for Kwajalein

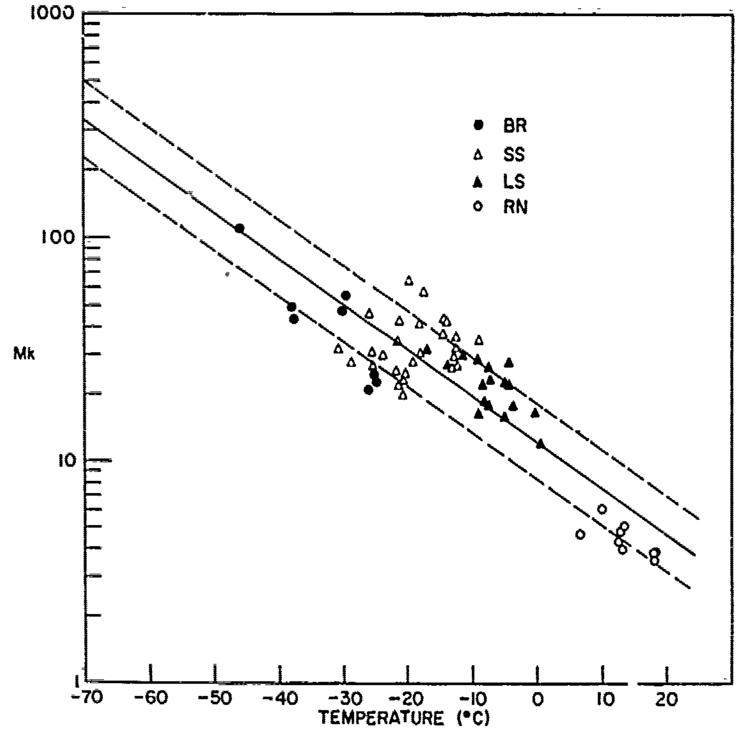

Figure 3.3 Variation of Mk With Temperature for Wallops Island

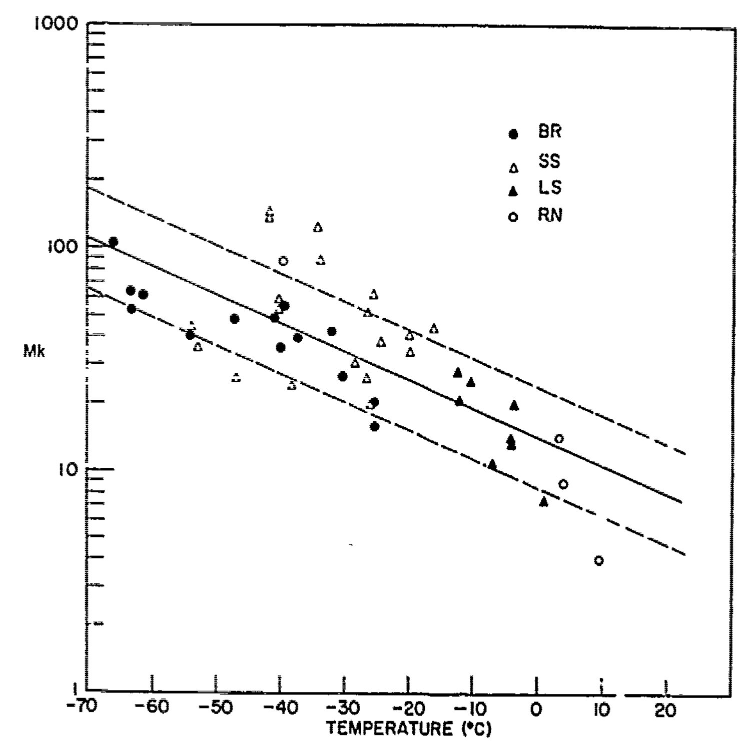

Figure 3.4 Variation of Mk With Temperature for Kwajalein

4. Predicting Trajectory M Values at Wallops Island

R. M. Dyer

4.1 INTRODUCTION

Because it is not possible to make aircraft measurements along the trajectory at the time of missile reentry, a relation between radar Z and trajectory M has to be assumed. At first, this was a power relation of the form $M = aZ^b$, with a and b obtained from results published by observers at locations other than either Wallops Island or Kwajalein. As data were acquired at Wallops Island, especially tailored M-Z relations were derived for each crystal type shortly before and shortly after each reentry. Assuming no temporal change, these relations were then applied to the reentry time.

From the initial Wallops Z_R data using the literature M-Z's, it was possible to derive a climatology of Wallops Island storms.[16] Climatological values of k ($\overline{k}_c$) can be obtained using these data and, once $\overline{k}_c$ has been derived for each particle regime, a climatological M_c - Z_R relation of the form $M_c = \overline{k}_c Z_R^{0.5}$ can then be applied to future situations when radar data are available.

Several questions then arose. Can measured aircraft (M_A) and radar (Z_R) relations be applied to a meteorological situation several hours before or after the measurements are made? How does this differ from a M-Z relation obtained from

16. Berthel, R.O. (1976) A Climatology of Selected Storms for Wallops Island, Virginia, 1971-1975, Environmental Research Papers, No. 563, AFGL/SAMS Report No. 4, AFGL-TR-76-0118, AD A029354.

previous measurements reported in the literature? Does the accuracy improve if climatological values of $\overline{K}_c$ are used?

Finally, are the ranges of M encountered at a given altitude or temperature, or associated with a given particle type, sufficient'y narrow, and is there an adequate amount of data, that an appropriate climatological value of $\overline{M}_c$ could be substituted for radar or aircraft measurements? The latter question applies to missile reentry over areas for which it is not possible to have either radar or aircraft measurements on the spot.

The analysis reported in the following pages is an attempt to answer these questions for Wallops Island. The methodology can be applied to other locations as well.

4.2 THE DATA

During the first four months of 1977, special efforts were made at Wallops Island, Virginia, to make correlated aircraft runs; that is, to measure the cloud physics parameters along a constant altitude path at the same time that a ground-based radar was measuring the radar reflectivity along the path. Each run produced an aircraft-radar relationship of M_A - Z_R.

Several successive aircraft-radar correlation runs were made at virtually the same altitude, at intervals ranging between a few minutes and three hours. Each data point consisted of a 4-sec sample of the hydrometeor spectrum obtained from the PMS 1-D probes aboard the aircraft, and the measured ground-based radar return from the volume of space one range gate short of the gate containing the aircraft echo. A correlation run was useable for the study reported here if and only if more than one pass were made on the same day within the same altitude interval, and with the same predominant particle type.

Useable correlation runs were made on five separate days between 9 January and 4 April, 1977. Altogether, there were 38 constant altitude passes for which both aircraft and radar data were available. The passes, divided into four broad categories of particle type, are tabulated in Table 4. 1. For the purpose of this analysis, they were grouped into pairs. A pair consists of two correlation passes made on the same day, at the same altitude and with the same observed particle type. The M_A - Z_R relationship derived from each run was then applied to the pass-average Z_R of its pair partner, which gave a derived M and then was compared with the partner's observed $\overline{M}$. Using only the criteria of same day, same altitude, and same particle type, it was possible to obtain 42 pairs of correlated pass data that could be used to test the predictive power of the M_A - Z_R relations derived from the combination of aircraft and radar data. However, only 18 of these were completely independent, in the sense that each pass was used only once. As a first exercise, only independent pairs were used. Each pass was paired with the pass closest to it in time, adhering to the same altitude, same particle type requirement.

Table 4. 1. Summary of Correlated Aircraft Radar Data

Particle Type	Altitude	Passes	Pairs	Independent Pairs
Rain	0.5 km	4	6	2
	2.1 km	4	6	2
	3.1 km	2	1	1
		10	13	5
Large Snow	3.8 km	3	3	1
	3.9 km	2	1	1
	4.8 km	4	6	2
		9	10	4
Small Snow	5.8 km	4	6	2
	6.0 km	2	1	1
	6.3 km	3	3	1
	7.0 km	4	6	2
		13	16	6
Bullet Rosettes	7.9 km	2	1	1
	8.5 km	2	1	1
	9.3 km	2	1	1
		6	3	3
Totals		38	42	18

4.3 ANALYSIS PROCEDURE

Five methods of estimating liquid water content at a point were evaluated in this study.

(1) In the absence of any measurements, whether radar or aircraft, it is necessary to rely on climatological data. Because Z_R measurements were made between 1971 and 1975 at Wallops Island, there is an adequate data base for deriving an average $\overline{M}_c$ as a function of hydrometeor type and synoptic situation, using literature M-Z relations. These averages, published by Berthel,[16] were compared with the pass average M obtained from the 1977 correlated aircraft-radar data.

Since all the correlation runs were made in stratiform storm situations, the climatological values were taken from Table A1 for stratiform storms from Berthel's report.[16] They are listed in Table 4.2.

Table 4.2 Climatological Average Liquid Water Content Values at Wallops Island Based on Z_R and Literature M-Z Relations for Stratiform Situations. Taken from Table A1 of Berthel (1976)

Hydrometeor Type	Mean (gm m^{-3})	Standard Deviation (gm m^{-3})	SD/Mean (%)
Bullet Rosettes	0.087	0.073	83.9
Small Snow	0.116	0.052	44.8
Large Snow	0.257	0.124	48.2
Rain	0.161	0.095	59.0

The large standard deviations indicate that the climatological means for stratiform systems will not be accurate indications of liquid-water-content, and certainly are not desirable if either radar or aircraft data are available. However, if Wallops Island were in a location without satellite or conventional weather data, climatological data would be the only available estimator. It is useful, therefore, to determine what errors can arise under such circumstances.

(2) In the absence of climatological data, and with only a ground-based radar available, recourse must be made to M - Z relations of the type presented by Plank.[10] These were obtained from the published literature, without reference to synoptic situation or geographic location. The derivation of the literature M - Z relations is shown at the extreme right of Figure 4.1. The absence in the diagram of intermediate calculations and derived quantities between the selection of crystal type and the ultimate statistic (the M_R - Z_R relation) does not indicate that these computations, with their attendant assumptions and sources of error, were not required, but only that we have not tabulated the different assumptions of the various authors who derived the equations originally. The relations used are listed in Table 4.3.

(3) If a ground-based radar is available, and a sufficient body of data has been collected at the same site in the past, it is possible to derive an M-Z relation for the storm type and geographic location of interest. The steps used in this derivation are shown in Figure 4.1, under the heading "Previous Measurements (Climatology)". Using an appropriate M-Z relation from the literature, each radar return Z_R is converted to a measure of M. The climatological $\overline{k}_c$ should be determined from the individual ks, that is, $\overline{k}_c \equiv M/Z^{0.5}$. The original data from Wallops Island

were no longer available so the climatological $\bar{k}_c$ values were obtained from the average M ($\bar{M}_c$) and average Z ($\bar{Z}_c$) values[16] using $\bar{k}_c = M_c/Z_c^{0.5}$. Using these climatological k_c values an estimate of M can then be obtained from the observed Z_R values and the relation $M = \bar{k}_c Z_R^{0.5}$.

The climatological k_c values used here were derived from the Berthel data.[16] For bullet rosettes, $\overline{Mk}_c$ ($k_c \times 1000$) is 65; in small snow $\overline{Mk}_c$ is 31; in large snow, 19; and in rain, 4.

Table 4.3. M - Z Relations Published in the Literature and Applied to Wallops Measurements

Bullet Rosettes	$M = 0.38\ Z^{0.529}$
Small Snow	$M = 0.0145\ Z^{0.538}$
Large Snow	$M = 0.00495\ Z^{0.596}$
Rain	$M = 0.00314\ Z^{0.576}$

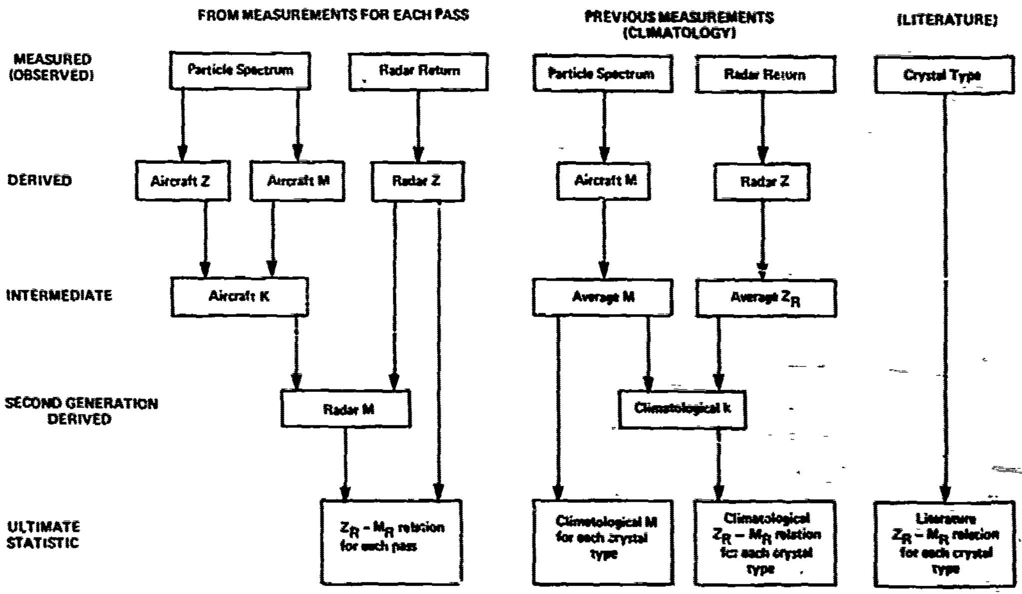

Figure 4.1. Flow Diagram Showing the Methods of Deriving M

(4) The ideal situation is to have both aircraft and radar data available at the time and place of interest. In practice, it is not possible to make aircraft

measurements along the reentry trajectory at the time of reentry. The next best thing is to make measurements as close to the reentry time as possible, then to apply the M - Z relations derived during the calibration pass to radar measurements taken at the reentry time. This was done for each of the passes listed in Table 4.1. The power relation $M_A = aZ_R^b$ was determined for each pass, and this relation was then applied to the average Z_R of the pass which was paired with the calibration pass.

(5) The fifth estimate of trajectory M is the pass average M_R obtained from the regression formula derived from the aircraft and radar data measured during the pass itself and reapplied to Z_R values. It was assumed for the purpose of this study that these data represent the conditions prevailing during the missile reentry. In practice, it is not feasible to take aircraft measurements during reentry, and the closest measurements would have to be those of case 4. However, it was necessary to establish the most accurate, or reference, value of M, and that is why this fifth method is included. Even this reference value has an error associated with it, namely, that resulting from applying an average M-Z relation obtained along a constant altitude pass to any point on that pass. This is the minimum error of any point estimate, and results from the variability in M along the path. It is expressed in Table 4.5 as the standard error of estimate of M from the pass derived M - Z relation.

4.4 RESULTS

Details of each pass and the results of the computations are shown in Tables 4.4a through 4.4d. The errors (or more accurately, the uncertainties) when using each method of prediction are expressed as percent deviations from the mean M_R. The standard error of estimate is assumed to be an expression of the minimum possible uncertainty. Whenever a predictive method produces an apparently lower error (those values in parenthesis), it is adjudged to be as accurate as meteorological variability will allow. Only the results of using the pass average next closest in time are listed in the table.

Table 4.4a. Summary of Analysis of Correlated Aircraft-Radar Data for Rain

Parameter	13 March 1977, Mean Altitude 0.53 km				13 March 1977, Mean Altitude 2.1 km				13 March 1977, Mean Altitude 3.1 km	
Pass Number	1	2	3	4	5	6	7	8	9	10
Start Time	1545	1550	1606	1609	1442	1445	1450	1453	1430	1435
Duration (sec)	280	186	189	121	89	63	169	189	223	35
Altitude (km)	0.59	0.51	0.53	0.49	2.1	2.0	2.1	2.1	3.1	3.1
Z_R Average Range	73.3 7-303	167.6 2-977	991.8 81-2399	890.0 309-1905	792.0 195-1549	15.2 12.0-19.5	563.0 93-2951	198.0 120-1230	231.8 11.2-1549	78.3 57.5-112.2
M_R Average Range	0.028 0.008-0.092	0.039 0.003-0.152	0.105 0.027-0.225	0.112 0.069-0.238	0.110 0.047-0.182	0.020 0.012-0.035	0.104 0.050-0.366	0.053 0.009-0.149	0.182 0.011-0.650	0.059 0.033-0.084
kx1000 Average Range	3.6 2.1-7.2	3.4 1.6-5.9	3.7 1.8-5.3	3.8 2.5-6.3	4.0 2.5-5.8	5.1 3.3-9.0	4.8 3.0-8.6	4.4 2.8-8.9	4.7 2.8-7.1	6.7 4.0-8.9
$M = aZ^b$ a b	0.0025 0.587	0.0020 0.621	0.0028 0.541	0.0034 0.516	0.0029 0.549	0.0066 0.386	0.0095 0.364	0.0041 0.510	0.0039 0.531	0.008 0.465
Percent Errors										
Standard Error of Estimate	20.1	15.6	16.1	19.3	17.5	22.5	25.1	17.9	18.7	14.9
M = 0.161 (Climatology)	47.7	310	56.3	43.8	46.9	712	54.9	201	3.5	173
k_cx1000 = 4.1	(10.5)	(5.3)	(9.9)	(7.3)	(1.6)	(19.4)	(12.3)	(5.4)	(17.0)	38.8
Literature M-Z	33.1	53.8	59.1	40.1	33.4	24.7	(15.9)	24.6	68.2	34.4
M-Z From Nearest Pass	(8.7)	(1.8)	(1.8)	(3.2)	18.3	34.8	(6.6)	19.6	22.7	33.4
k From Nearest Pass	(8.4)	(7.6)	(1.8)	(3.1)	26.3	(21.3)	(5.9)	(10.8)	35.7	29.9
Average Standard Error = 18.8%										

Table 4.4b. Summary of Analysis of Correlated Aircraft-Radar Data for Large Snow

Parameter		10 January 1977, Mean Altitude 3.8 km			24 February 1977, Mean Altitude 3.9 km		10 January 1977, Mean Altitude 4.8 km			
Pass Number		1	2	3	4	5	6	7	8	9
Start Time		0154	0253	0327	2112	2155	0038	0131	0240	0341
Duration (sec)		205	280	148	248	223	287	287	228	230
Altitude (km)		3.8	3.0	3.8	3. -	3.9	4.8	4.8	4.8	4.8
Z_R	Average	62.3	94.0	40.6	149	252	121	117	155	30.1
	Range	35.5-95.5	40.0-151	32.4-57.5	43.7-389	42.7-977	55-302	35-331	41-203	15.0-65.1
M_R	Average	0.175	0.224	0.140	0.179	0.438	0.279	0.201	0.1'M	0.099
	Range	0.120-0.289	0.134-0.329	0.106-0.176	0.081-0.359	0.116-1.144	0.139-0.573	0.073-0.599	0.105-0.292	0.058-0.168
k×1000	Average	22.1	23.2	22.0	15.9	27.3	20.2	21.9	16.0	18.2
	Range	16.2-29.9	10.0-28.4	17.0-27.3	11.2-24.8	16.3-43.7	18.9-33.0	12.3-32.9	13.5-10.3	13.1-23.4
$M = aZ^b$	Average	0.0086	0.0087	0.0284	0.0484	0.0054	0.0032	0.0136	0.026	0.012
	Range	0.730	0.717	0.4297	0.286	0.602	0.455	0.635	0.403	0.633
Percent Errors										
Standard Error of Estimate		10.8	0.5	8.9	14.7	16.0	7.5	9.8	7.1	10.6
M = 0.257 (Climatology)		46.9	14.5	83.6	43.4	41.4	7.8	27.9	32.1	161.4
k×1000 = 19		14.3	19.0	13.6	24.5	35.5	25.0	(2.3)	19.5	(3.4)
Literature M-Z		68.0	66.9	67.8	45.4	69.5	69.0	55.4	48.5	82.0
M-Z From Nearest Pass		(4.0)	(4.5)	11.5	63.0	54.3	92.6	86.1	47.8	(1.9)
k From Nearest Pass		(4.2)	(5.8)	(5.6)	78.9	46.0	16.1	32.9	14.4	12.9

Average Standard Error = 10.4%

Table 4.4c. Summary of Analysis of Correlated Aircraft-Radar Data for Small Snow

Parameter		10 January 1972, Mean Altitude 5.8 km				4 April 1972, Mean Altitude 6.0 km		13 March 1972, Mean Altitude 6.3 km			4 March 1972, Mean Altitude 7.0 km			
Pass Number		1	2	3	4	5	6	7	8	9	10	11	12	13
Start Time		0008	0112	0233	0350	1632	1638	1642	1647	1650	2132	2136	2138	2153
Duration (sec)		347	210	225	318	118	228	30	101	139	136	32	78	271
Altitude (km)		5.7	5.7	5.8	5.9	6.1	6.0	6.3	6.3	6.4	7.0	7.1	7.0	7.0
Z_R	Average	27.4	14.0	46.5	4.2	67.4	29.3	48.3	78.7	25.6	2.0	3.8	8.6	4.2
	Range	16.2-43.7	10.5-19.5	32.4-58.0	3.0-6.3	14.0-70.0	11.0-60.0	1.0-470	1.7-204	0.7-44.8	1.2-3.2	2.0-4.8	3.7-11.6	2.6-5.9
M_R	Average	0.152	0.109	0.161	0.054	0.252	0.193	0.365	0.360	0.033	0.034	0.048	0.059	0.046
	Range	0.096-0.218	0.088-0.142	0.129-0.210	0.034-0.075	0.111-0.518	0.132-0.301	0.001-7.14	0.040-0.700	0.005-0.095	0.013-0.065	0.034-0.059	0.020-0.104	0.026-0.064
λx1000	Average	29.1	29.3	23.6	28.4	32.5	35.9	56.4	42.8	7.8	24.5	24.9	19.8	22.4
	Range	22.2-38.5	25.1-33.0	19.2-29.6	17.4-74.0	26.2-42.1	30.8-45.5	13.0-272.3	31.4-58.0	2.1-13.4	11.0-49.8	22.0-28.7	9.5-94.1	12.9-29.7
$M=aZ^b$	a	0.0148	0.0233	0.0413	0.0154	0.0266	0.0260	0.0120	0.0240	0.0019	0.0244	0.0208	0.0020	0.0135
	b	0.102	0.585	0.353	0.680	0.550	0.596	0.874	0.632	0.462	0.777	0.637	1.335	0/458
Percent Errors														
Standard Error of Estimate		13.1	6.1	7.2	11.1	11.6	7.6	95.5	11.6	52.6	49.1	5.9	23.3	15.1
M = 0.116 (Climatology)		24.2	6.4	20	113.4	53.9	40.1	(12.5)	86.6	247	244	140	95.0	153
λx1000=31		(5.9)	6.2	31.2	16.1	(6.0)	14.5	(53.9)	29.2	287	(27.7)	24.0	51.4	-38.5
Literature M-Z		77.1	45.0	40.7	42.1	44.6	53.6	230	143	154	(38.1)	38.0	(21.8)	31.8
M-Z From Nearest Pass		(6.1)	13.4	181	25.5	185	13.1	(37.3)	43.7	382	(8.0)	17.1	(4.4)	58.9
λ From Nearest Pass		(0.3)	(0.6)	12.9	11.8	(8.8)	10.3	(36.4)	25.9	449	(2.6)	(2.0)	(9.4)	(11.6)

Average Standard Error of Estimate = 23.0%

Table 4.4d. Summary of Analysis of Correlated Aircraft-Radar Data for Bullet Rosettes

Parameter		4 April 1977, Mean Altitude 7.9 km		4 April 1977 Mean Altitude 8.5 km		10 January 1977, Mean Altitude 9.3 km	
Pass Number		1	2	3	4	5	6
Start Time		1535	1542	1606	1615	0.0045	0.0126
Duration (sec)		222	119	205	216	188	230
Altitude (km)		7.9	8.0	8.6	8.5	9.3	9.3
Z_R	Average	3.3	5.9	1.6	0.93	0.40	0.30
	Range	1.4-8.3	2.7-9.3	0.62-41.7	0.49-3.6	0.24-0.66	0.10-0.52
M_R	Average	0.047	0.059	0.057	0.052	0.025	0.025
	Range	0.024-0.074	0.030-0.088	0.036-0.095	0.030-0.082	0.018-0.036	0.012-0.043
kx1000	Average	26.6	24.3	47.3	55.5	4[illegible].8	49.4
	Range	18.6-41.6	15.7-30.1	34.5-71.8	2[illegible].9-84.0	34.8-55.8	26.4-72.9
$M=aZ^b$	a	0.0290	0.0177	0.050	0.054	0.047	0.093
	b	0.420	0.679	0.296	0.434	0.578	1.047
Percent Errors							
Standard Error of Estimate		18.0	9.1	11.4	17.8	7.7	20.7
M = 0.087 (Climatology)		83.6	44.4	57.4	76.0	230	247
kx1000 = 65		143	161	46.9	24.7	58.6	40.6
Literature M-Z		51.8	64.6	13.8	20.6	13.0	26.0
M-Z From Nearest Pass		(17.8)	(1.1)	17.4	(2.3)	38.6	(7.5)
k From Nearest Pass		(9.1)	(6.8)	24.7	(9.2)	20.8	(5.2)

Average Standard Error = 14.1%

It is immediately obvious that for stratiform storms at Wallops Island the competition for most accurate method of prediction lies between using correlated aircraft and radar data from a same-day constant altitude pass (case 4), and using only radar data and a climatological k_c value determined by the predominant crystal type. In Table 4.5, the average improvement in accuracy using correlated aircraft-radar data rather than climatological $\overline{k}_c$ values is expressed as the average change in percent error. Negative values indicate that for Wallops Island the climatological k_c method yielded better correspondence than either the pass average M-Z or k-Z relations. Only oullet rosette situations (6 passes) resulted in any improvement when aircraft data were used. These results were not changed when all aircraft data for a given day were combined to make the prediction. Nor was there any improvement when the appropriate k was selected from Figures 3.1 and 3.3 according to the altitude or temperature at which the pass was made. However, using the average k measured during the closest correlated pass did improve the prediction in several instances.

Table 4.5. Changes in Percent Error of M: Climatological k_c Method vs Pass Regression Method

	All Passes	Independent Pairs Only	Correlation With Time From Prediction
Rain	-4.6	-6.5	0.06
Large Snow	-21.3	[illegible]	-0.17
Small Snow	-2.4	-13.0	-0.50
Bullet Rosettes	+197.	+197.	-0.15

Except for some insignificant changes in the magnitude of the errors, the results were the same, whether the "average error" was defined as the average of the errors in the individual point estimates as shown in Table 4.5, or the error in the estimate of the average M along the pass. In either case, the ranking of the estimates remained unchanged. These rankings are shown in Table 4.6. A ranking of 1 was given to the method of estimate with the smallest average error. Again, parenthesis enclose those methods of estimate which yielded an average error lower than the standard error.

Table 4. 6a. Relative Rankings of Methods of Estimating M for Rain

Methods of Estimate	Pass Number									
	1	2	3	4	5	6	7	8	9	10
Climatology	5	5	4	5	5	5	5	5	(1)	5
Climatological k_C	(3)	(2)	(3)	(3)	(1)	(1)	(3)	(1)	(2)	4
Literature $M = aZ^b$	4	4	5	4	4	3	(4)	4	5	3
Nearest Pass $M = aZ^b$	(2)	(1)	1	(2)	2	4	(2)	3	3	2
Nearest Pass k	(1)	(3)	(1)	(1)	3	(2)	(1)	(2)	4	1

Table 4. 6b. Relative Rankings of Methods of Estimating M for Large Snow

Methods of Estimate	Pass Number								
	1	2	3	4	5	6	7	8	9
Climatology	4	3	5	2	2	1	2	3	5
Climatological k_C	3	4	3	1	1	3	(1)	2	(2)
Literature $M = aZ^b$	5	5	4	3	5	4	4	5	4
Nearest Pass $M = aZ^b$	(1)	(1)	2	4	4	5	5	4	(1)
Nearest Pass k	(2)	(2)	(1)	5	3	2	3	1	3

Table 4. 6c. Relative Rankings of Methods of Estimating M for Small Snow

Methods of Estimate	1	2	3	4	5	6	7	8	9	10	11	12	13
Climatology	5	3	2	5	5	4	(4)	4	2	(5)	5	5	5
Climatological k_C	(2)	2	3	2	(1)	3	(3)	2	3	(3)	3	4	3
Literature $M = aZ^b$	4	5	4	4	4	5	(5)	5	1	(4)	4	(3)	2
Nearest Pass $M = aZ^b$	(3)	4	5	3	3	2	(2)	3	4	(2)	(2)	(1)	4
Nearest Pass k	(1)	(1)	1	1	(2)	1	(1)	1	5	(1)	(1)	(2)	(1)

Table 4.6d. Relative Rankings of Methods of Estimating M for Bullet Rosettes

	Pass Number					
Methods of Estimate	1	2	3	4	5	6
Climatology	4	3	5	5	3	3
Climatological k_C	5	5	4	3	5	5
Literature $M = aZ^b$	3	4	1	4	1	4
Nearest Pass $M = aZ^b$	(2)	(1)	2	(1)	4	(2)
Nearest Pass K	(1)	(2)	3	(2)	2	(1)

The elapsed time between the pass during which the M - Z relation was derived, and the pass for which the relation was applied to estimate M, varied between three minutes and three hours, two minutes. Before this study was undertaken, it was expected that the accuracy of the estimate would be highest for the shortest elapsed time between passes, and that there would be a systematic decrease in accuracy as the time between passes increased. This was not the case. As the figures in the last column of Table 4.5 show, the correlation coefficient between accuracy and time between the derivation of the M - Z relation and the prediction is not significant. Indeed, except for the rain cases, this coefficient was negative, indicating that if the correlation were significant the M - Z relations derived from measurements taken only a few minutes before the prediction would be less accurate than those taken hours before! This conclusion runs counter to our intuition. Additional analyses using independent data are required to further investigate this point and, at present, these data are not available. It should be restated here that all of these correlation runs were made in stratiform, moderate to heavy intensity storm situations during winter months at Wallops Island, Virginia. Attempts to characterize predictability based on synoptic type must await further data.

Another approach would be to use M - Z relations derived from passes which showed the same characteristics as the pass which is to be predicted. Because the radar-measured Z_R is the one parameter obtainable in real-time during reentry, it was decided to test the hypothesis that the M - Z relation obtained during an aircraft pass could be applied to any other pass with the same hydrometeor type and approximately the same average Z_R. Data to test this hypothesis proved to be very sparse. There were no correlation passes in bullet rosettes for which the average Z_R's were comparable. Three passes in small snow at altitudes between

4.7 and 6.4 km had average Z_R's between 25.6 and 29.2, and two passes in large snow had altitudes and Z_R's of 3.9 km and 149 $mm^6\ m^{-3}$ and 4.8 km and 155 mm^6 m^{-3} respectively. The power relations obtained during these passes were used to estimate the liquid water content for all other passes having approximately the same average Z_R. In the case of rain, there were two passes taken, 1 hr 10 min apart, for which the altitudes differed widely, but the Z_R's were approximately the same. A comparison was made between these two passes to determine whether, in this case, more accurate predictions could be made using calibrations obtained at similar Z_R's rather than for passes close to the time and altitude for which the prediction is to be made.

The results are shown in Table 4.7. For the rain case the nearest pass was by far the better predictor. In the case of large snow, using the same Z_R pass data with the closest Z_R (obtained more than a month previously) was preferable to either the nearest pass or the climatological $\bar{k}_c$ calibration. There was little difference between the two results for two of the three small snow passes. For the third one, a highly variable case, the nearest pass calibrator was far superior. Though far from conclusive, the results indicate that further comparison of correlation runs at similar altitudes and with comparable Z_R's should be undertaken to confirm these conclusions and to obtain more definitive results.

The average absolute error in the estimate of M was the same for both the climatological $\bar{k}_c$ and the nearest pass estimate, 0.05 g m^{-3}. The standard deviation was higher for the climatological $\bar{k}_c$ predictions, 0.10 g m^{-3} vs 0.08 g m^{-3} for the nearest pass predictions. In general, there was a tendency for the percent variability along a pass to be larger for the smaller average M's, thus pushing the absolute errors toward the central value of M. However, the correlation between percent error and magnitude of M (-0.31 for the combined data) was not significant.

4.5 CONCLUSIONS

With the exception of the bullet rosette case, there now seems to be a large enough data base for moderate to heavy stratiform storms at Wallops Island to permit prediction of pass average liquid-water-content using the radar alone and a climatological relation between Z and M. For the stratiform storms the most stable method appears to be to use a climatological value $\bar{k}_c$, based on crystal type, and the relation $M = \bar{k}_c Z^{0.5}$.

Table 4.7 Accuracy of M - Z Relations Obtained From Passes With Same Average Z_R

Hydrometeor Type	Date	Alt (km)	$\overline{Z}_R$ (Pass Average)	M_R (4-sec Average)	Percent Errors			
					Standard Error	Nearest Pass in Time	Pass With Similar Value Z_R	Climatological $\overline{k}_c$
Rain								
(1 hr 10 min apart)	13 Mar 1977	0.59	73.3	0.028	20.1	(2.8)	110.4	25.3
	13 Mar 1977	3.1	78.3	0.059	14.9	33.0	45.2	38.3
Large Snow	24 Feb 1977	3.9	148	0.179	14.7	67.3	(9.1)	29.7
	10 Jan 1977	4.8	155	0.194	7.1	45.1	(4.8)	21.8
Small Snow 1.	10 Jan 1977	5.7	27.6	0.153	11.1	(6.1)	(7.2)(2) 75.8(3)	(5.9)
2.	4 Apr 1977	6.0	29.2	0.193	7.6	11.8	18.1(1) 80.3(3)	13.2
3.	13 Mar 1977	6.4	25.6	0.033	52.6	(4.8)	336(1) 37.8(2)	79.4

It cannot be emphasized too strongly that the analysis reported here does not address the question of accuracy in measuring M, Z or k. That subject has been treated elsewhere.[17] This analysis was confined to a comparison of different methods of estimating M along a trajectory, assuming that all input parameters were completely error-fee.

Under ideal conditions (errorless aircraft data, perfect radar measurements and a Z-M relation which remains constant over time and space), there would still be an error in the estimate of liquid water content at a point along the path, due to scatter about the Z-M regression line. The deviations of a and b result in errors in the estimate of M. For the Wallops Island data used here, the average standard error in M was ± 18.8% in rain, ± 10.4% in large snow, ± 23.0% in small snow, and ± 14.1% in bullet rosettes. This scatter is due to the meteorological variability inherent in even stratiform situations, and the figures cited here may be taken as the normal atmosphere variability in any estimate of point values of M.

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5. Conclusions and Recommendations

The first three sections of this report, though approaching the problem from different angles, have a common objective—the analysis of aircraft measured particle data to yield relations between liquid-water-content and some other parameter (radar reflectivity, temperature, altitude) which may then be applied to determine the liquid water water content along a trajectory. There are three possible ways of determining a trajectory M in the absence of storm specific M-Z correlations. In the first, described in Section 2, nominal M-Z_R equations were derived from one-second aircraft spectra and simultaneous radar reflectivity measurements for each hydrometeor type. The second and third methods, described in Section 3, consisted of finding relations between k and temperature and k and altitude, and using the relation of $M = k Z^{0.5}$.

A comparison was made between these approaches using the mean values of Z_R, temperature and altitude of the hydrometeor types from the Kwajalein data in Section 2. The mean Z_R was substituted in the nominal equation to give one estimate of M for each type. The mean temperature was used with the Kwajalein specific k vs temperature relationship to give a second. A third M was determined from the mean altitude and the k vs altitude equation. Table 5.1 lists the percentage differences that result in the derived M values when the methods from Section 3 are compared with that calculated from the nominal equations.

Table 5.1 A Comparison of M's Derived From the k Relationships of Temperature and Altitude at Kwajalein With Those Determined From the Nominal Equations

Hydrometeor Type	Percentage $\Delta M/M$ (Nominal Eq) From k vs T	k vs Km
Bullet Rosettes	6.6	7.7
Small Snow	-25.3	-23.6
Large Snow	21.0	30.9
Rain	51.0	30.9

These comparisons highlight the variability between the methods using pass averages and climatological parameters with that derived from the one-second aircraft and radar measurements. This is also true in the analysis reported in Section 4. Although the data used in Section 4 were from Wallops Island, and less variable than those from the tropical convective storms of Kwajalein, the limiting factor in defining the liquid water content at a point along the trajectory was still the scatter of the individual estimates around the pass average. The results of Section 4, particularly the large uncertainties when average values are applied in the bullet rosette case, demonstrate that situations which deviate significantly from climatological averages still require time-specific M-Z relations derived from simultaneous aircraft-radar measurements.

Much work remains to be done in this field. In addition to an adequate theory of how the melting process affects radar reflectivity, mathematical models explaining the evolution of M-Z relations at a given altitude with time are essential before remote measurements of M will be possible. As data become available, further studies of the merits of using M-Z relations tailored for particular Z profiles will be investigated.

LIST OF SYMBOLS AND ABBREVIATIONS

a	power-function coefficient
b	power-function exponent
c	constant
°C	degrees Celsius
d	minimum drop or equivalent melted diameter
D	drop or equivalent melted diameter
D_i	drop or equivalent melted diameter of classified data
D_M	maximum drop or equivalent melted diameter
D_T	maximum drop or equivalent melted diameter that is able to be measured with a particular instrument
H	altitude in kilometers
i	index of summation
k	aircraft spectral parameter
k_c	climatological average value of k
ℓ	measured physical size of ice hydrometeors
M	liquid or ice water content
M_A	liquid or ice water content from aircraft data
M_C	liquid or ice water content derived from cloud probe
M_c	climatological liquid of ice water content
M_D	distributed liquid or ice water content
M_E	estimated liquid or ice water content from an extrapolated distribution
M_P	liquid or ice water content derived from precipitation probe

N number concentration of the hydrometeor drops or particles per m^3 per mm bandwidth

N_D number concentration at diameter D

N_i number concentration for classified data

N_o the "zero intercept" of a distribution function of exponential type

N_T total number of the hydrometeor particles of a given population per m^3

n number of terms in summation

r_M truncation ratio of liquid water content

r_N truncation ratio of number concentration

r_Z truncation ratio of radar-reflectivity factor

T temperature in ° Celsius

Z radar-reflectivity factor for Rayleigh scattering

Z_A radar-reflectivity factor computed from aircraft data

Z_C radar-reflectivity factor from cloud probe

Z_c climatological radar-reflectivity factor

Z_E estimated radar-reflectivity factor from an extrapolated distribution

Z_p radar-reflectivity factor from precipitation probe

Z_R radar-reflectivity factor as measured by radar

ΔZ radar-reflectivity factor from the extrapolated part of the distribution

α coefficient of the ℓ to D equation

β exponent of the ℓ to D equation

Λ exponential "slope factor" in the distribution function for the number concentration of the drops or particles

ρ_ω density of liquid water (g/cm^3)

γ_i ratio of the number of particles in class i to the total number, N_T

AFCRL Air Force Cambridge Research Laboratory, Hanscom AFB, MA

AFGL Air Force Geophysics Laboratory, Hanscom AFB, MA

BMO Ballistic Missile Organization, Norton AFB, CA

KMR Kwajalein Missile Range, Kwajalein Atoll, Marshall Islands

NASA National Aeronautics and Space Administration

SAMSO Space and Missile Systems Organization, Los Angeles AFS, CA

WFTC Wallops Flight Test Center, VA

PMS Particle Measuring Systems, Boulder, CO

1-D One-dimensional PMS System

2-D Two-dimensional PMS System

MIT Massachusetts Institute of Technology

TRADEX L & S Band Radar used at KMR

ALCOR C-Band Radar used at KMR

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