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ON THE APPLICATION OF SYMMETRIZATION TO
THE TRANSMISSION OF ELECTROMAGNETIC
WAVES THROUGH SMALL CONVEX
APERTURES OF ARBITRARY SHAPE

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California Institute of Technology
Pasadena, CA 91125

June 1980

Final Report

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AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base, NM 87117

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This technical report has been reviewed and is approved for publication.

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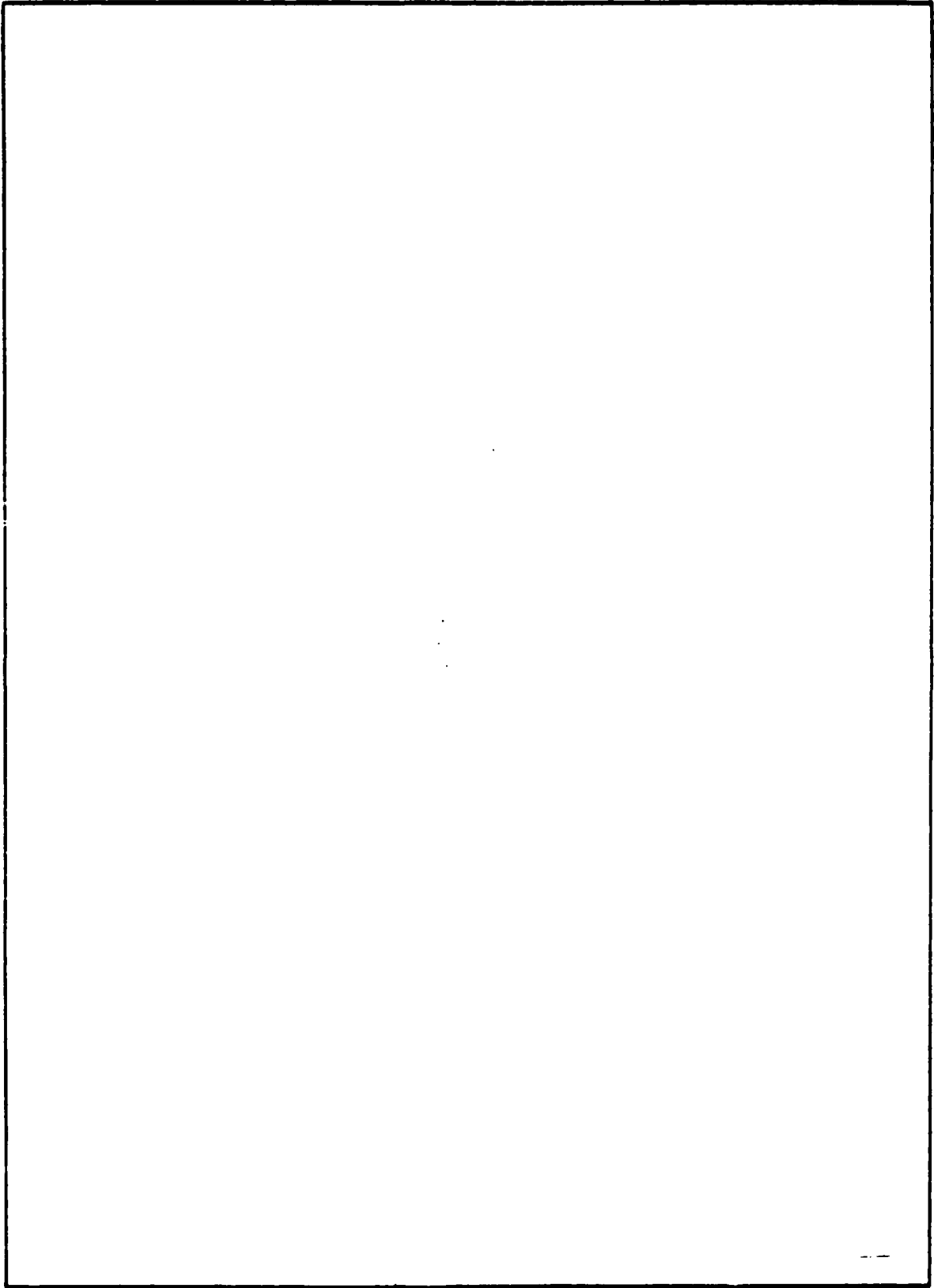
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PREFACE

The authors are greatly indebted to Dr. J.P. Castillo of the Air Force Weapons Laboratory and Dr. K.S.H. Lee of the Dikewood Corporation for their valuable assistance. This work was supported by the Dikewood Corporation and the U.S. Air Force Office of Scientific Research.

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SECTION I

INTRODUCTION

The question of how much of the electromagnetic energy that exists on one side of a wall can leak to the other side through a small opening in the wall has become, by virtue of its practical importance, a canonical problem in the theory of EMP (electromagnetic pulse) interactions (ref. 1).

As is well known, the earliest calculation of the transmission of an electromagnetic wave through a small circular aperture in a plane screen of perfect conductivity and zero thickness was performed by Lord Rayleigh. Using potential theory, he calculated the transmitted field of a plane harmonic wave normally incident on an electrically small circular aperture (ref. 2). Years later Bethe derived expressions for the polarizabilities and effective dipole moments of small circular apertures. His results give the transmitted far field for any angle of incidence but not the transmitted near field (ref. 3). Most recently Bouwkamp (ref. 4) and Meixner and Andrejewski (refs. 5 and 6) found an exact solution for both the near and far transmitted fields of a plane wave normally incident on a circular aperture.

Aperture problems can, at least in principle, be solved numerically, but they cannot be solved analytically unless the shape of the aperture happens to be simple enough to permit a separation of the variables and a scalarization of the electromagnetic field. However, from this it should not be inferred that if the aperture problem cannot be solved analytically, a numerical method is the only way to obtain a solution. Actually, as a preferable alternative, one can reformulate the problem so that upper and

lower bounds on the true solution and not the true solution itself would have to be sought. Such a reformulation can be based on Levine and Schwinger's result that when the aperture is electrically small there is a variational principle for the upper bound and another variational principle for the lower bound (refs. 7 and 8). However, this variational approach, which was used by Fikhmanas and Fridberg to find bounds on the electric and magnetic polarizabilities of electrically small apertures (ref. 9), does not lend itself to very easy calculation. Accordingly, it is of some interest to try a simpler method of sandwiching the true solution between upper and lower bounds.

In this report we shall examine how symmetrization, which has yielded interesting results in geometry and mathematical physics (ref. 10), may be used to establish two-sided bounds on the electric and magnetic polarizabilities of differently shaped convex apertures and thereby estimate their transmission properties in a simple economical manner.

SECTION II

SYMMETRIZATION

Of the several kinds of symmetrization that have been invented we shall restrict our attention to the symmetrization of a plane figure with respect to a straight line. To symmetrize a plane figure with respect to a straight line L , we suppose the figure to consist of line segments that are parallel to each other and perpendicular to L (figure 1). We then shift each line segment along its own line until the line segment is bisected by L . The shifted line segments compose the symmetrized figure. For example, a semicircle of radius R , when symmetrized with respect to its bounding diameter, changes into an ellipse with semiaxes R and $R/2$. A further symmetrization can transform the ellipse into a circle of radius $R/2$. Symmetrization leaves the figure's area A unchanged and decreases, or, more accurately, never increases its perimeter P . For the case shown in figure 1, the area is always $\pi R^2/2$ and the perimeter varies from $(2 + \pi)R$ for the semicircle to πR for the circle.

As an instructive example, we apply the principle of symmetrization to the calculation of capacitance C . It is known that the symmetrization of a plane conducting plate decreases (i.e., never increases) the electrostatic capacity of the plate (ref. 10). A plane figure symmetrized infinitely many times becomes a circle and, consequently, of all conducting plates of a given area the circular plate has the minimum capacity. Accordingly,

$$C \geq C_{in} \quad (1)$$

SYMMETRIZATION

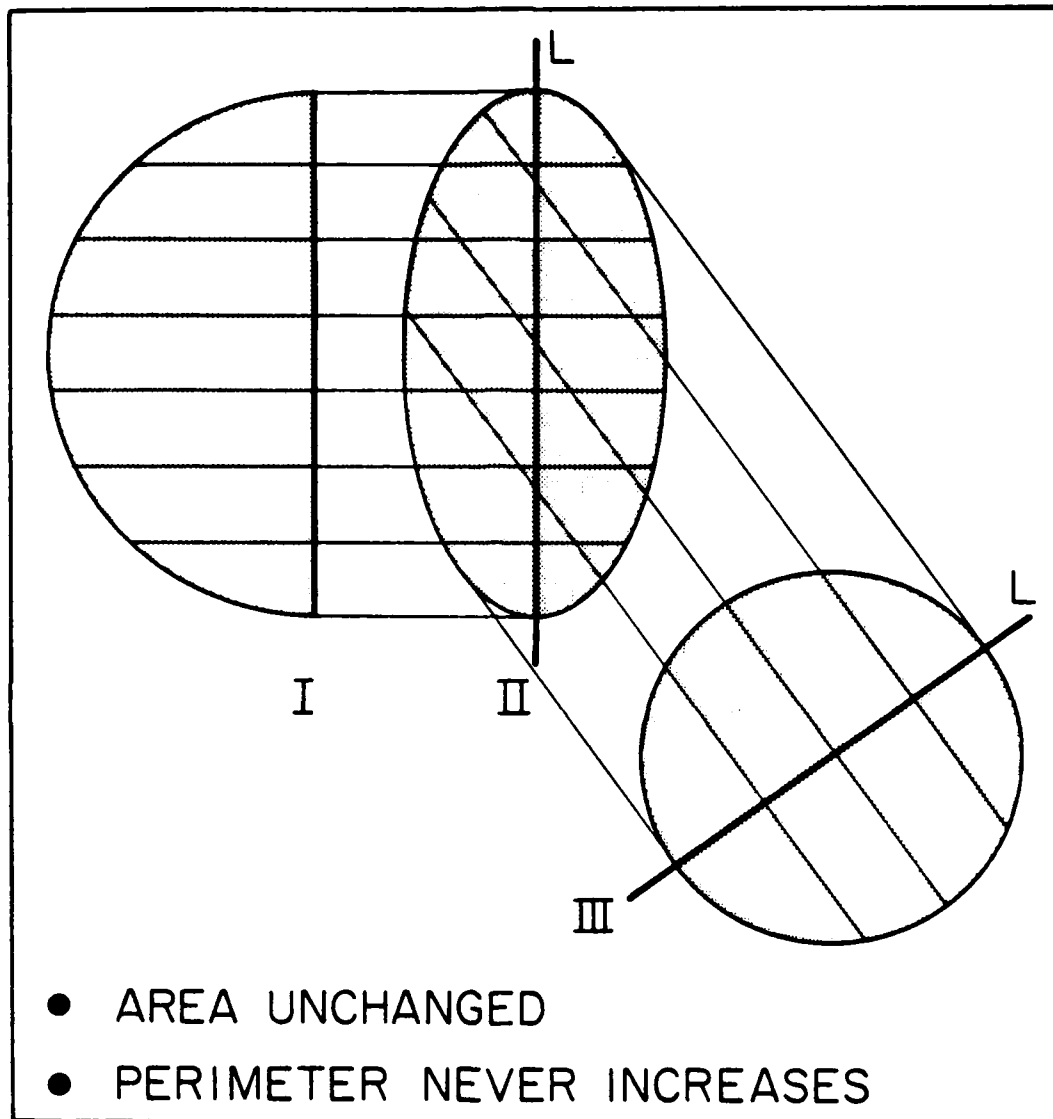


Figure 1. Example of Symmetrization of a Plane Figure with Respect to a Line L . The Semi-Circle of Radius R is Symmetrized with Respect to its Bounding Diameter to Produce an Ellipse with Semi-Axes R and $R/2$. The Ellipse, when Symmetrized, Becomes a Circle of Radius $R/2$. The Area of Each Figure Remains Constant but the Perimeter Decreases with Each Symmetrization.

where C denotes the electrostatic capacitance of a plane conducting plate and C_{in} denotes the electrostatic capacitance of the circular plate of radius r_{in} , that has been obtained by completely symmetrizing the original plate. This places a lower bound on C . To obtain an upper bound, we invoke the conjecture that of all plates with a given perimeter, the circular plate has the maximum capacitance (ref. 10).

Thus we find

$$C_{out} \geq C \quad (2)$$

where C_{out} is the electrostatic capacitance of a circular plate of radius r_{out} , whose perimeter is equal to that of the perimeter of the original plate. From equations (1) and (2) it follows that

$$C_{in} \leq C \leq C_{out} \quad (3)$$

Since we have

$$r_{in} = (A/\pi)^{1/2} \quad (4)$$

$$r_{out} = P/2\pi \quad (5)$$

and the electrostatic capacitance of a circular plate (disk) in MKS units is given by

$$C = 8\epsilon_0 a \quad (6)$$

where a is the radius of the disk and ϵ_0 is the dielectric constant of free space, upon replacing a by r_{in} and r_{out} we obtain from equations (3) through (6) that the capacitance C of a plate of area A and perimeter P is delimited by

$$(A/\pi)^{1/2} \leq C/8\epsilon_0 \leq P/2\pi \quad (7)$$

Here $\epsilon_0 = (36\pi)^{-1} \times 10^{-9}$ farads per meter.

Both Maxwell (ref. 11) and Rayleigh (ref. 12) made unproven statements concerning bounds on the capacitance of plates, which agree with equation (7). Moreover, the capacitance of an elliptic plate of eccentricity e , as given by

$$C_{\text{ellipse}}/8\epsilon_0 = (A\pi)^{1/2}(1-e^2)^{-1/4}/2K(e^2) \xrightarrow{e \rightarrow 0} (A/\pi)^{1/2}(1+e^2/64) \quad (8)$$

where $K(e^2)$ is the complete elliptic integral of the first kind (ref. 12), clearly satisfies the left side of equation (7). To show that it also satisfies the right side we only need to recall that for an ellipse

$$P_{\text{ellipse}}/2\pi = 2(A/\pi)^{1/2}E(e^2)(1-e^2)^{-1/4}/\pi \xrightarrow{e \rightarrow 0} (A/\pi)^{1/2}(1+3e^2/64) \quad (9)$$

where $E(e^2)$ is the complete elliptic integral of the second kind.

By virtue of the apparent validity of equation (7) for the capacitance of plates of arbitrary size and shape we are led to believe that other quantities of physical interest may be similarly sandwiched between bounds involving only the purely geometric parameters A and P .

SECTION III

POLARIZABILITIES AND TRANSMISSION COEFFICIENTS OF SMALL APERTURES

Let us now consider the transmission of electromagnetic energy through an electrically small aperture which is located in a plane screen of perfect conductivity and zero thickness. Since the aperture is small, the fields on the shadow side of the screen appear to emanate from dipoles located in the aperture. These electric and magnetic dipoles, having moments $\underline{p}$ and $\underline{m}$, radiate in free space and are linearly related to the incident traveling wave through the vector electric polarizability with components α_i and the dyadic magnetic polarizability with components β_{ij} . That is,

$$p_i = \epsilon_0 \alpha_i E_i^{\text{inc}} \quad (i = 1, 2, 3) \quad (10)$$

$$m_i = \mu_0 \sum_{j=1}^2 \beta_{ij} H_j^{\text{inc}} \quad (i = 1, 2) \quad (11)$$

where $\mu_0 = 4\pi \times 10^{-7}$ henries per meter. The incident electric and magnetic fields are plane waves of the form $\underline{E}^{\text{inc}} \exp i(\underline{k} \cdot \underline{r} - \omega t)$ and $\underline{H}^{\text{inc}} \exp i(\underline{k} \cdot \underline{r} - \omega t)$ where $\underline{r}$ is the position vector, $\underline{k}$ is the wave vector and ω is the frequency.

For a circular aperture of radius a the polarizabilities are given by the simple expressions

$$\alpha_i^{\text{circle}} = \frac{8}{3} a^3 \delta_{i3} \quad (i = 1, 2, 3) \quad (12)$$

$$\beta_{ij}^{\text{circle}} = \frac{16}{3} a^3 \delta_{ij} \quad (i, j = 1, 2) \quad (13)$$

$$\text{where } \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

The values 1, 2 and 3 correspond respectively to the directions $\hat{e}_{||}$, $\hat{e}_{\perp}$ and $\hat{e}_n$. The aperture plane is defined by the unit vectors $\hat{e}_{||}$ and $\hat{e}_{\perp}$ and the normal (pointing toward the shadow side) is defined by $\hat{e}_n = \hat{e}_{||} \times \hat{e}_{\perp}$ (figure 2). The polarizabilities are defined here for incident traveling waves and for dipoles radiating in free space. For short circuit incident fields and for dipoles radiating in the presence of a conducting wall, all values of the polarizabilities should be divided by the numeric 4.

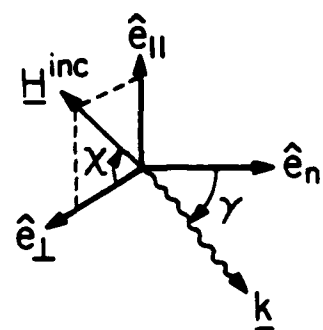
For elliptic apertures with semi-axes a and b along $\hat{e}_{||}$ and $\hat{e}_{\perp}$ respectively we have

$$\alpha_i^{\text{ellipse}} = \frac{4\pi}{3} \frac{ab^2}{E(e^2)} \delta_{i3} \quad (14)$$

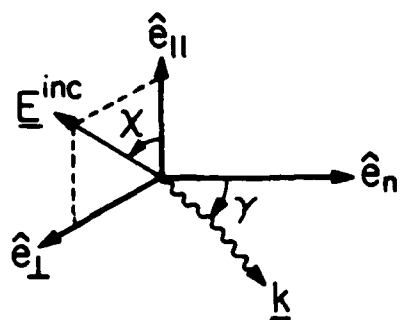
$$\beta_{ij}^{\text{ellipse}} = \begin{cases} \frac{4\pi}{3} \frac{ab^2 e^2}{(1-e^2)[K(e^2)-E(e^2)]} \delta_{i1} & (15) \\ \frac{4\pi}{3} \frac{ab^2 e^2}{E(e^2)-(1-e^2)K(e^2)} \delta_{i2} & (16) \end{cases}$$

where $e = (1-b^2/a^2)^{1/2}$ is the eccentricity of the ellipse and $K(e^2)$ and $E(e^2)$ are elliptic integrals of the first and second kind (ref. 13).

The transmission coefficient τ is defined as the ratio of the total far-field power transmitted through the aperture divided by the total power incident on the aperture. For the case where the principal axes of magnetic polarizability dyadic correspond to $\hat{e}_{||}$ and $\hat{e}_{\perp}$ we find



|| Polarization



$\perp$ Polarization

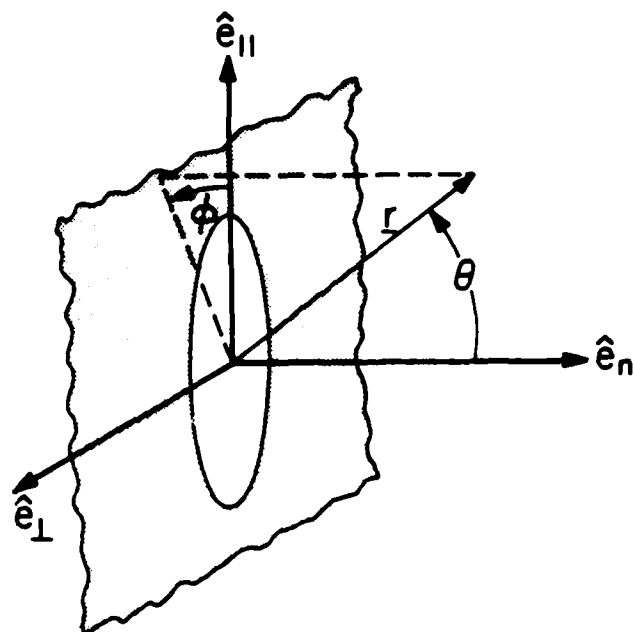


Figure 2. Unit Vectors $\hat{e}_{||}$ and $\hat{e}_\perp$ Lie in the Aperture Plane, and $\hat{e}_n$. For || Polarization $\underline{H}^{inc}$ is Always Parallel to the Aperture Plane and Makes Angle χ with Respect to $\hat{e}_\perp$. For $\perp$ Polarization $\underline{E}^{inc}$ is Always Parallel to the Aperture Plane and Makes Angle χ with Respect to $\hat{e}_{||}$.

$$\tau = \frac{k^4}{12\pi A} \left[\alpha_3^2 \sin^2 \gamma \binom{0}{1} + (\beta_{11}^2 \sin^2 \chi + \beta_{22}^2 \cos^2 \chi) \binom{\cos^2 \gamma}{1} \right] \quad (17)$$

for $\binom{1}{11}$ polarization (ref. 14). Here γ is the angle of incidence, i.e., the angle between $\underline{k}$ and $\hat{e}_n$, and χ is the angle between $\underline{H}^{inc}$ and $\hat{e}_\perp$ for parallel polarization and is the angle between $\underline{E}^{inc}$ and $\hat{e}_\parallel$ for perpendicular polarization (figure 2).

SECTION IV

BOUNDS ON POLARIZABILITIES AND TRANSMISSION COEFFICIENT

Imitating the procedure we followed to establish bounds on the capacitance of plates, we now construct bounds on the mean magnetic polarizability β_m of a convex aperture by replacing the radius a , which appears in expression (13) for the polarizability of a circular aperture, by r_{in} (4) and r_{out} (5) of the aperture. Thus we get

$$\frac{16}{3} \left(\frac{A}{\pi} \right)^{3/2} \leq \beta_m \leq \frac{16}{3} \left(\frac{P}{2\pi} \right)^3 \quad (18)$$

where by definition $\beta_m = (\beta_{11} + \beta_{22})/2$.

To test the plausibility of equation (18) we examine several special cases. For the elliptic aperture of small eccentricity ($e \ll 1$), equation (18) becomes

$$\frac{16}{3} \left(\frac{A}{\pi} \right)^{3/2} \leq \beta_m \leq \frac{16}{3} \left(\frac{A}{\pi} \right)^{3/2} \left(1 + \frac{9}{64} e^4 \right) \quad (19)$$

equations (15) and (16) yield

$$\beta_m = \frac{16}{3} \left(\frac{A}{\pi} \right)^{3/2} \left(1 + \frac{3}{32} e^4 \right) \quad (20)$$

and thus we clearly see that equation (18) is satisfied in the case of mildly eccentric ellipses. It can also be shown that equation (18) holds true for elliptic apertures of arbitrary eccentricity ($0 \leq e \leq 1$) and for other convex apertures such as the rectangular and the rhombical aperture (refs. 14 and 15). The fact that these test cases are in complete agreement

with equation (18) leads us to believe that the assertion (18) is valid for all convex apertures.

Accepting the general validity of equation (18) and recalling that symmetrization reduces P without changing A we conclude that of all convex apertures of fixed area A the circular aperture possesses the smallest mean magnetic polarizability.

The electric polarizability contributes to transmission through small apertures only when the incident wave is obliquely incident and polarized parallel to the plane of incidence. To construct bounds for the electric polarizability we note that, for a circular aperture of radius a and area A , equation (12) can be written as

$$\alpha_i^{\text{circle}} = \frac{8}{3\pi^2} \frac{A^2}{a} \delta_{13} \quad (21)$$

Then by replacing the radius a of this expression by $r_{\text{in}}(4)$ and $r_{\text{out}}(5)$ we arrive at

$$\frac{16}{3\pi} \frac{A^2}{P} \leq \alpha_3 \leq \frac{8}{3} \left(\frac{P}{\pi} \right)^{3/2} \quad (22)$$

To test the plausibility of equation (22) we again consider the case of a mildly eccentric ellipse ($e \ll 1$). In this case equation (22) becomes

$$\frac{8}{3} \left(\frac{A}{\pi} \right)^{3/2} \left(1 - \frac{3}{64} e^4 \right) \leq \alpha_3^{\text{ellipse}} \leq \frac{8}{3} \left(\frac{A}{\pi} \right)^{3/2} \quad (23)$$

and from equation (14) we have

$$\alpha_3^{\text{ellipse}} = \frac{8}{3} \left(\frac{A}{\pi} \right)^{3/2} \left(1 - \frac{3}{64} e^4 \right) \quad (24)$$

Obviously, expression (24) is equal to the lower bound in equation (23). Furthermore, with the aid of equation (14) it can be verified that the lower bound in equation (22) is precisely the value of the electric polarizability of ellipses of arbitrary eccentricity (refs. 9 and 15). Also, we note that the electrical polarizabilities of rectangular and rhombical apertures satisfy equation (22) (refs. 14 and 15).

Assuming the validity of equation (22) and invoking symmetrization, we find that of all convex apertures of fixed area the circular aperture possesses the largest electric polarizability.

The bounds that have been proposed for the electric (22) and mean magnetic (18) polarizabilities can be used to obtain bounds on the transmission coefficient (17). In some modern applications the quantity of interest is the upper bound for the case where the incident wave is directed and polarized to maximize the transmission through the given aperture. Clearly, maximum possible transmission through a given aperture occurs when the incident wave is parallel polarized and is made to fall on the aperture at grazing incidence. To find the upper bound for maximum possible transmission we use equation (22) and note that $r_{out} \geq r_{in}$. Thus

$$\alpha_3^2 \sin^2 \gamma \leq \frac{64}{9} \left(\frac{A}{\pi} \right)^3 \leq \frac{64}{9} \left(\frac{P}{2\pi} \right)^6 \quad (25)$$

Moreover, in view of equation (18) we can write

$$\beta_{11}^2 \sin^2 \chi + \beta_{22}^2 \cos^2 \chi \leq \frac{1024}{9} \left(\frac{P}{2\pi} \right)^6 \quad (26)$$

Substituting equation (25) into equation (26) into equation (17) we thus obtain the following expression for the maximum possible transmission through a small aperture of area A and perimeter P

$$\tau \leq \frac{68(P/\lambda)^6}{27\pi^3(A/\lambda^2)} \quad (27)$$

where $\lambda = 2\pi/k$ is the wavelength of the incident radiation.

Since symmetrization reduces P and keeps A unchanged we see from expression (27) that the maximum possible transmission decreases as the aperture is symmetrized. This is, the maximum possible transmission decreases as the shape of the aperture approaches that of a circle (ref. 16).

SECTION V

CONCLUSIONS

By delimiting the polarizabilities of a small convex aperture of arbitrary shape and given area we have found upper and lower bounds on its transmission coefficient. Symmetrizing the aperture we see that the maximum possible transmission decreases as the shape of the aperture approaches that of a circle. For example, the maximum possible transmission decreases as the shape of the aperture is changed from that of an equilateral triangle to that of a square and finally to that of a circle.

The bounds are simple to evaluate from a knowledge of the aperture's area and perimeter and therein lies the desirability and economy of this method.

It appears that this method of estimation can be generalized to handle other boundary-value problems and thus provide information as to how their solutions are modified when there is a change of shape.

REFERENCES

- [1] Ricketts, L.W., J.E. Bridges, and J. Miletta, EMP Radiation and Protective Techniques, John Wiley and Sons, New York, 1976.
- [2] Rayleigh, Lord, "On the Passage of Waves through Apertures in Plane Screens, and Allied Problems," Phil. Mag. XLIII, pp. 259 - 272, 1897; also "On the Incidence of Aerial and Electric Waves upon Small Obstacles in the Form of Ellipsoids or Elliptic Cylinders, and on the Passage of Electric Waves through a Circular Aperture in a Conducting Screen," Phil. Mag. XLIV, pp. 28 - 52, 1897.
- [3] Bethe, H.A., "Theory of Diffraction by Small Holes," Phys. Rev. 60, pp. 163 - 182, 1942.
- [4] Bouwkamp, C.J., "On Bethe's Theory of Diffraction by Small Holes," Philips Res. Rep. 5, pp. 321 - 332, 1950; also "On the Diffraction of Electromagnetic Waves by Small Circular Disks and Holes," Philips Res. Rep. 5, pp. 401 - 422, 1950.
- [5] Meixner, J. and W. Andrejewski, "Strenge Theorie der Beugung ebener Electromagnetischer Wellen der Volkommen Leitenden ebener Schirm," Ann. Phys. 7, pp. 157 - 168, 1950.
- [6] Andrejewski, W., "Die Beugung Electromagnetischer Wellen an der Leitenden Kreisscheibe und der Kreisförmigen Öffnung in Leitenden ebener Schirm," Z. Angew. Phys. 5, pp. 178 - 186, 1953.
- [7] Levine, H. and J. Schwinger, "On the Theory of Electromagnetic Wave Diffraction by an Aperture in an Infinite Plane Conducting Screen," Commun. Pure Appl. Math. 3, pp. 355 - 391, 1950.
- [8] Borgnis, F.E. and C.H. Papas, Randwertprobleme der Mikrowellenphysik, Springer-Verlag, Berlin, 1955.
- [9] Fikhmanas, R. and P. Fridberg, "Variational Estimate of Upper and Lower Bounds on the Coefficient of Polarizability in the Theory of Pinhole Diffraction," Sov. Phys.-Doklady 14, pp. 1155 - 1157, 1970;

REFERENCES (Continued)

- also "Theory of Diffraction at Small Apertures. Computation of Upper and Lower Boundaries of the Polarizability Coefficients," Radio Eng. Electron. Phys. 18, pp. 824 - 829, 1973.
- [10] Pólya, G. and G. Szegő, Isoperimetric Inequalities in Mathematical Physics, Princeton Univ. Press, Princeton, 1951; also "Inequalities for the Capacity of a Condenser," Amer. J. Math. 67, pp. 1 - 32, 1945. Payne, L.E., "Isoperimetric Inequalities and their Application," SIAM Review 9, pp. 453 - 487, 1967.
- [11] Maxwell, J., ed., The Scientific Papers of the Honorable Henry Cavendish, vol. 1, 1897, reprinted by Cambridge University Press, Cambridge, 1921.
- [12] Rayleigh, Lord, The Theory of Sound, vol. 2, 1896, reprinted by Dover, New York, 1945.
- [13] See for example Montgomery, C., R. Dicke and E. Purcell, Principles of Microwave Circuits, McGraw Hill, New York, 1948 or Collin, R., Field Theory of Guided Waves, McGraw Hill, New York, 1960.
- [14] Jaggard, D.L., "Transmission through One or More Small Apertures of Arbitrary Shape," California Institute of Technology Antenna Lab. Tech. Report No. 83, California Institute of Technology, May 1977.
- [15] Latham, R., "Small Holes in Cable Shields," EMP Interaction Notes, Note 118, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, September 1972.
- [16] Papas, C.H., "An Application of Symmetrization to an EMP Shielding Problem," California Institute of Technology Antenna Lab. Tech. Report No. 80, California Institute of Technology, December 1976.

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