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NONLOCAL EFFECTS IN BRITTLE CRACK PROPAGATION.(U)

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IN BRITTLE CRACK PROPAGATION

M. Ari and A.C. Eringen
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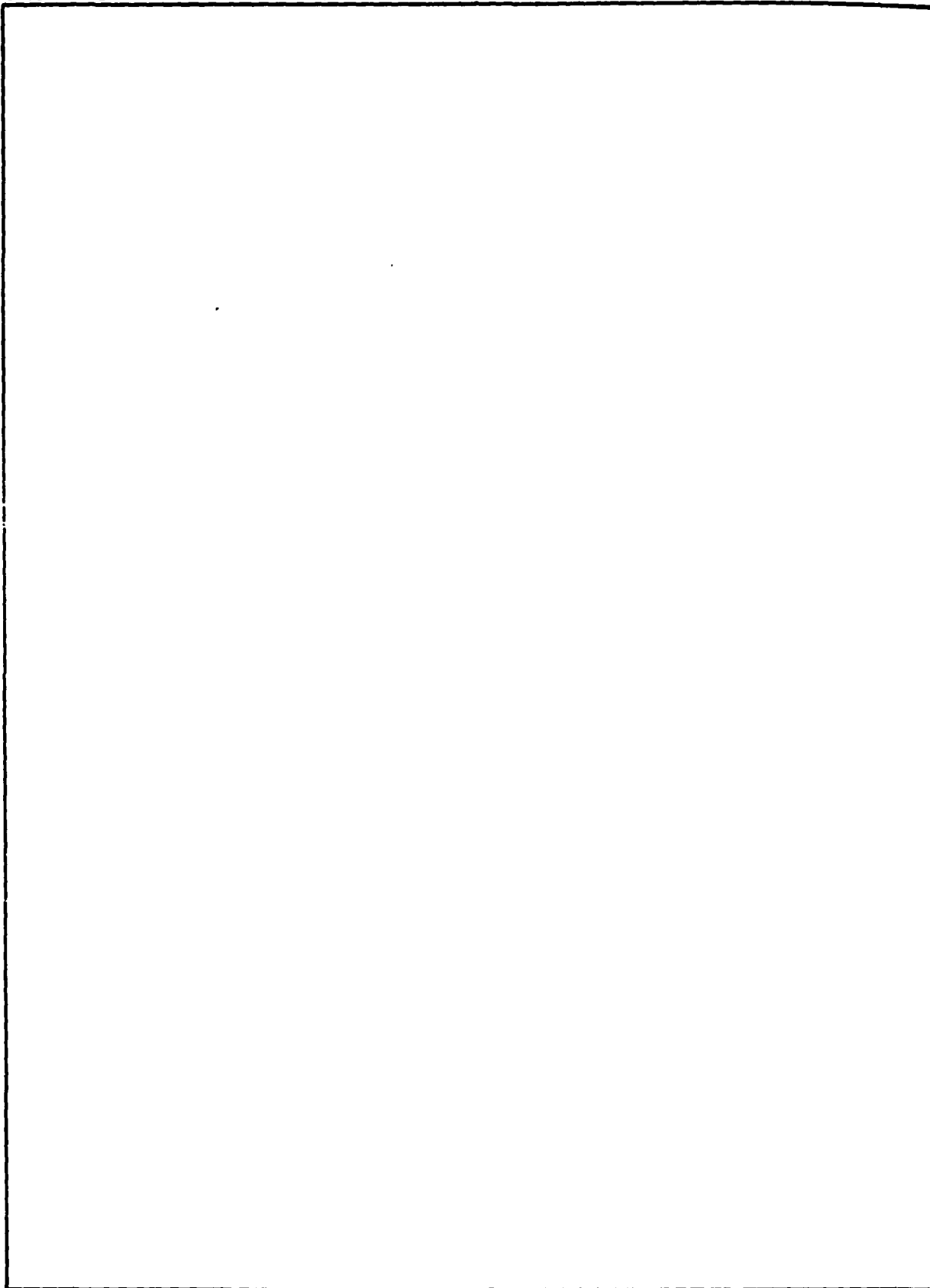
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NONLOCAL EFFECTS IN BRITTLE CRACK PROPAGATION*

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ABSTRACT

Stress distribution near the tip of a constant velocity crack is determined by means of nonlocal theory of elasticity. The maximum stress is finite and it allows one to utilize the maximum stress hypothesis to determine the terminal velocity of running cracks.

1. INTRODUCTION

One of the fundamental mechanisms of dynamic rupture phenomena is that of crack propagation. The angular distribution and the maxima of the crack tip stresses vary strongly with the crack velocity and the dynamic effects become important. One has to solve an elastodynamic problem to obtain these dynamic stresses so that at least a qualitative understanding of the crack motion can be understood.

The present work investigates stress distribution near the tip of a moving crack in a brittle elastic solid by means of the recently developed theory of nonlocal elasticity [1].

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The dynamic crack problems are rather difficult to solve without some simplifying assumptions. Most common among them are the constant crack tip velocity and/or the loading conditions which allow self-similar crack motions. As such, these problems represent a highly contrived picture of the actual rupture phenomenon. Nevertheless, these models prove to be useful in discussing the velocity dependence of the crack tip stresses.

One of the earliest and simplest models of a moving crack is the Yoffe's model [2]. A straight line crack of fixed length $2l$ moves with a constant velocity in an infinite plate subject to uniform tension perpendicular to the crack (Fig. 1). Below, we present a solution of Yoffe's problem within the context of nonlocal elasticity.

The classical elasticity solution of Yoffe's problem as well as other classical treatments of uniformly moving cracks (3-5) have provided certain insights into dynamical aspects of brittle fracture and inspired subsequent research. However, these studies have remained inadequate in explaining a number of important features of dynamic crack propagation. To this end, the following points are worth noting:

- (i) The classical elasticity solution yield the familiar square root stress singularities at crack tips. In order to circumvent the difficulties related to infinite stresses, the dynamic fracture criteria are based on energy considerations (energy release rate). However, the dynamic failure is localized at the crack tip and it depends on the critical stress levels. A maximum stress criterion is therefore more appropriate.

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- (ii) The dynamic stress intensity factor in Yoffe's solution

$$(1.1) \quad K = \lim_{r \rightarrow 0} [(2\pi r)^{\frac{1}{2}} t_{yy}(r, \theta=0)]$$

turned out to be independent of crack tip velocity and equal to its static value. This result is contrary to experimental observations that K should vary with the crack speed.

- (iii) The analytic expressions of the classical solutions imply that the Rayleigh wave velocity c_R is a natural upper bound for the crack tip velocity. However, for all materials observed, the terminal velocity V_t remained considerably less than c_R [6]. Since Mott [7] first addressed the problem of a modified Griffith criterion for a moving crack, the determination of the limiting velocity V_t has been an open problem. Operating on the premise of a constant surface energy γ_0 , Roberts and Wells arrived at a formula expressing the terminal velocity [8]. However, experimental evidence indicates that the surface energy varies with the crack velocity, i.e. $\gamma = \gamma(V)$. Therefore, Freund [9] considers any agreement of their theoretical terminal velocity with the experimental one rather accidental. An additional difficulty in dealing with surface energy concept is its experimental determination and accuracy.
- (iv) A more involved fracture phenomenon is that of crack bifurcation -- a rapidly moving crack suddenly branches into two new cracks. The precise physical source of this mechanism is not known. Neither is it settled as to which criterion is better suited to determine the

prebranching velocity and the bifurcation angles once the solution for the dynamic problem is given [10].

The motivation for the present work is derived from the foregoing considerations. In order to develop controlled fracture processes and crack arrest mechanisms, the need for addressing the above questions is clear. Encouraged by the results of the static nonlocal crack problems [11-13], we will pursue the solution of the nonlocal Yoffe problem. The nonlocal theories incorporate the long range interactions and the microstructure dependence intrinsically. Hence, they provide a more realistic framework for the treatment of crack problems.

Finally, the formulation of a mixed boundary value problem in nonlocal elastodynamics is in itself important. Difficulties related to mixed boundary conditions have been considered in our earlier work [12,14]. In the present work, in addition, we discuss a modification of the field equations due to the dynamic effects. Throughout the work, emphasis will be on how the nonlocality modifies the dynamic crack tip stresses.

Section 2 formulates the nonlocal boundary value problem. In Section 3, the ensuing dual integral equations are solved. The velocity dependence of the crack tip stresses is given in Section 4. Section 5 compares the nonlocal results with the classical ones and discusses their relevance to dynamic fracture criteria. The quantitative results are gratifying.

2. NONLOCAL FORMULATION

In nonlocal elasticity for isotropic solids, the stress constitutive equations are given by [1].

$$(2.1) \quad t_{k\ell} = \iint_R [\lambda'(|\underline{x}' - \underline{x}|) e_{rr}(\underline{x}') \varepsilon_{k\ell} + 2\mu'(|\underline{x}' - \underline{x}|) e_{k\ell}(\underline{x}')] da(\underline{x}')$$

where the integral is over the two-dimensional plane region $R(x', y')$ and $e_{k\ell}$ is the strain tensor which is related to the displacement vector u_k by

$$(2.2) \quad e_{k\ell} = \frac{1}{2} (u_{k,\ell} + u_{\ell,k})$$

Here, and throughout an index following a comma represent gradient, e.g.

$$u_{k,\ell} = \partial u_k / \partial x_\ell$$

The nonlocal kernels $\lambda'(|\underline{x}' - \underline{x}|)$ and $\mu'(|\underline{x}' - \underline{x}|)$ represent the influence of the neighboring strains at $\underline{x}'$ on the nonlocal stress at a reference point $\underline{x}$. They are usually determined by requiring that the nonlocal field equations yield identical dispersion relations to those derived from atomic lattice dynamics. In this way, in [12], we introduced a two-dimensional kernel

$$(2.3) \quad \lambda'/\lambda = \mu'/\mu = \alpha(|\underline{x}' - \underline{x}|) = \frac{1}{2\pi} \beta^2 K_0 \{ \beta [(x' - x)^2 + (y' - y)^2]^{1/2} \}$$

where K_0 is the zeroth-order modified Bessel function and β is the nonlocality parameter. In the local limit $\beta \rightarrow \infty$, (2.3) reverts into a Dirac-delta functional and (2.1) yields the classical Hooke's law. In addition, (2.3) has the convenient property that

$$(2.4) \quad (1 - \epsilon \nabla^2) \alpha(x,y) = \delta(x,y)$$

where $\epsilon = 1/\beta$ and δ is the Dirac-delta functional.

For nonlocal elasticity (when the effects of nonlocal residuals and the body forces are excluded) the integral balance equations of linear momentum yield the equations of motion

$$(2.5) \quad t_{kl,k} = \rho \ddot{u}_l$$

where a superposed dot represents the material time derivative. Due to the integral form of constitutive equations, (2.5) is of integro-differential character. For such systems, it is very difficult to establish existence and uniqueness theorems and special care is needed in the formulation of mixed boundary value problems. In addition, a closer look at the field equations signals certain difficulties for cases where the displacement fields are discontinuous such as in bodies with cracks. For example, in (2.5), when $\ddot{u}_l$ term is discontinuous, the $t_{kl,k}$ term is very likely not to be discontinuous, since the discontinuities in the displacement fields will be smoothed out by the integral operator in (2.1).

With the appearance of the crack, some interatomic bonds are eliminated and inhomogeneities arise within a narrow boundary layer near the crack surfaces. The nonlocal body forces become important and the use of nonhomogeneous kernels are required. Since, however, the inhomogeneities are confined to a narrow domain, we can approximate the problem with a homogeneous kernel, provided the compatibility of the crack discontinuities with the nonlocal field equations are ensured.

To this end, a modification of the field equations can be obtained by utilizing the relation (2.4), i.e.,

$$(2.6) \quad (1 - \epsilon^2 \nabla^2) t_{kl,k} = \sigma_{kl,k} = (1 - \epsilon^2 \nabla^2) \rho \ddot{u}_l$$

where

$$\sigma_{kl} = \lambda e_{rr} \delta_{kl} + 2\mu e_{kl}$$

is the local (classical) stress tensor. (2.6) together with the appropriate side conditions constitute a sufficient solution for the integral balance equations. The modified equations of motion are singularly perturbed differential equations and they are simpler than the integro-differential system given by (2.5). (2.6) can be derived alternatively from integral balance equations by imposing uniform continuity requirements on nonlocal stresses and displacements [15].

In the two-dimensional formulation, we assume that the displacement fields can be derived from wave potentials $\phi(X,Y,t)$ and $\psi(X,Y,t)$, i.e.

$$(2.7) \quad \begin{aligned} u &= \partial\phi/\partial X + \partial\psi/\partial Y \\ v &= \partial\phi/\partial Y - \partial\psi/\partial X \end{aligned}$$

Substituting (2.7) into (2.6), we obtain two scalar wave equations

$$(2.8) \quad \begin{aligned} (1 - \epsilon^2 \nabla^2) \ddot{\phi} &= c_1^2 \nabla^2 \phi \\ (1 - \epsilon^2 \nabla^2) \ddot{\psi} &= c_2^2 \nabla^2 \psi \end{aligned}$$

where

$$c_1 = [(\lambda + 2\mu)/\rho]^{1/2}, \quad c_2 = (\mu/\rho)^{1/2}$$

which must be solved to determine the displacement field. In the case of the constant velocity crack problem, it is more convenient to introduce a moving coordinate system (Fig. 1),

$$(2.9) \quad x = X - Vt \quad y = Y$$

where V is the crack tip velocity. We assume that in the new coordinate system, the wave potentials become independent of time. Then the field equations (2.8) reduce to

$$(2.10) \quad \left\{ \epsilon^2 V^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + c_1^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - V^2 \frac{\partial^2}{\partial x^2} \right\} \phi(x,y) = 0$$

$$\left\{ \epsilon^2 V^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + c_2^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - V^2 \frac{\partial^2}{\partial x^2} \right\} \psi(x,y) = 0$$

By means of the Fourier transform in the x -direction, we obtain the general solution of (2.10) satisfying the conditions $\phi, \psi \rightarrow 0$ as $(x^2 + y^2)^{1/2} \rightarrow \infty$

$$(2.11) \quad \phi(k,y) = A(k) e^{-\gamma_1(k)y}$$

$$\psi(k,y) = B(k) e^{-\gamma_2(k)y}$$

$y > 0$

where k is the transform variable and

$$(2.12) \quad \gamma_i^2(k) = [(c_i^2 - V^2 - \epsilon^2 k^2 V^2) / (c_i^2 - \epsilon^2 k^2 V^2)] k^2 ; \quad i=1,2$$

The inversion contour of (2.11) requires proper branch cuts so as to ensure that

$$(2.13) \quad \operatorname{Re} \gamma_i(k) \geq 0$$

The boundary conditions for moving crack in the new coordinate system is equivalent to the static case. Therefore, the self-consistent nonlocal boundary conditions are given by, [12] ,

$$(2.14) \quad \begin{aligned} \sigma_{yy}(x,0) &= -t_0 & |x| < \ell \\ v(x,0) &= 0 & |x| > \ell \\ \sigma_{xy}(x,0) &= 0 & \forall x \\ u, v &\rightarrow 0 & \text{as } (x^2+y^2)^{\frac{1}{2}} \rightarrow \infty \end{aligned}$$

(2.14) and (2.11) lead to a set of dual integral equations whose solution will determine $A(k)$ and $B(k)$.

3. SOLUTION OF THE DUAL INTEGRAL EQUATIONS

Substituting (2.11) and (2.7) into (2.14), we obtain

$$(3.1) \quad \sigma_{yy}(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \{[(\lambda+2\mu) \gamma_1^2 - \lambda k^2] A(k) - 2\mu i k \gamma_2 B(k)\} e^{-kx} dk$$

$$= -t_0; \quad |x| < \ell$$

$$(3.2) \quad v(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} [-\gamma_1 A(k) + i k B(k)] e^{-ikx} dk = 0;$$

$$|x| \geq \ell$$

$$(3.3) \quad \sigma_{xy}(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \mu \{2ik \gamma_1 A(k) + (\gamma_2^2 + k^2) B(k)\} e^{-ikx} dk = 0$$

$$; \quad \forall x.$$

An exact solution to the system is yet to be found. The problem is further complicated by the fact that the $\gamma_i(k)$ of (2.12) assumes complex values along the real axis between their respective branch points.

In the static crack problems, it has been shown that the classical solution constitutes a reasonable approximation to the nonlocal one [11,12]. The only difference between the classical and nonlocal versions of the system (3.1)-(3.3) is the dependence of $\gamma_i(k; \epsilon)$ to ϵ , where $\gamma_i(k, \epsilon=0) = s_i k$

$$(3.4) \quad s_i^2 = 1 - (V/C_i)^2.$$

In the limit as $\epsilon \rightarrow 0$, we recover the classical solution. The behaviour of integrands in (3.1)-(3.3) for large k is similar to the classical case,

therefore, it is expected that the limiting process, as $\epsilon \rightarrow 0$, shall be uniform. Hence as an approximation, one might use the classical solution provided that the complex values of $\gamma_i(k)$ are taken into account. To this end, we choose a modified form of the classical solution such that

$\gamma_i(k) A_N(k)$ and $\gamma_i(k) B_N(k)$ remain always real valued and numerically equal to the absolute values of $\gamma_i A_C$ and $\gamma_i B_C$ respectively. These conditions will be satisfied if the nonlocal solutions for A_N and B_N are expressed as

$$(3.5) \quad A_N(k) = P_1(k) + i P_2(k)$$

$$(3.6) \quad B_N(k) = Q_1(k) + i Q_2(k) ,$$

where

$$(3.7) \quad P_1(k) = A_C(k) \quad Q_1(k) = B_C(k) ; \quad \forall k$$

$$(3.8) \quad P_2(k) = -A_C(k) \quad Q_2(k) = -B_C(k); \quad k \in R$$

and R correspond to those regions of the real k -line on which $\gamma_1(k)$ or $\gamma_2(k)$ are not real valued.

The classical solutions are given by [16]

$$(3.9) \quad A_C(k) = (1 + s_2^2) D_0 J_1(k\ell)/k^2$$

$$(3.10) \quad B_C(k) = -2 s_1 i D_0 J_1(k\ell)/k^2$$

where

$$D_0 = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} t_0 \ell (V/C_2)^2 [4s_1s_2 - (1-s_2^2)^2]^{-1}$$

It can be shown by substitution that the nonlocal boundary conditions are uniformly approximated by (3.5) - (3.6) to the order of $O(\epsilon^{\frac{1}{2}})$ [15].

In the present work, we are primarily interested in the normal stress along the crack line, thus in the next section, we utilize this modified classical solution to express the normal stress along the crack line near the tip.

4. NONLOCAL STRESSES

The constitutive equations for the stress field are given by (2.1). Using the standard integration formulas and substituting (3.5) - (3.10) into (2.1), we obtain for the normal stress along the crack line

$$(4.1) \quad t_{yy} = k_1 [4 s_1 t_B - 2 (1+s_2^2) t_A]$$

where

$$(4.2) \quad \{t_A, t_B\} = \int_0^\infty J_1(k) \cos k\xi \{\alpha_A(\epsilon k), \alpha_B(\epsilon k)\} dk$$

$$(4.3) \quad \alpha_A(\epsilon k) = \frac{(C_1^2 - \eta^2 V^2 - \epsilon^2 k^2 V^2)}{C_1^2} \left[1 - \frac{\epsilon k}{(1+\epsilon^2 k^2)^{\frac{1}{2}}} \left(\left| \frac{(C_1^2 - V^2 - \epsilon^2 k^2 V^2)}{(C_1^2 - \epsilon^2 k^2 V^2)} \right| \right) \right]$$

$$(4.4) \quad \alpha_B(\epsilon k) = \left\{ \left(\left| \frac{C_2^2 - V^2 - \epsilon^2 k^2 V^2}{C_2^2 - \epsilon^2 k^2 V^2} \right| \right) - \frac{\epsilon k}{(1+\epsilon^2 k^2)^{\frac{1}{2}}} \frac{C_2^2 - V^2 - \epsilon^2 k^2 V^2}{C_2^2 - \epsilon^2 k^2 V^2} \right\} \frac{C_2^2 - \epsilon^2 k^2 V^2}{C_2^2}$$

and

$$k_1 = [4 s_1 s_2 - (1 + s_2^2)^2]^{-1} t_0 \ell^{\frac{1}{2}} / 2\pi^{\frac{1}{2}}$$

$$(4.5) \quad \eta^2 = (\lambda + 2\mu)/2\mu$$

$$\xi = x/\ell ; \quad \epsilon = 1/\beta\ell$$

Near the crack tip, stresses are given in terms of generalized hypergeometric functions which are difficult to visualize in graphic terms (see Appendix 1). In the present work, we are most interested in the velocity dependence of the crack tip stresses. Therefore we present the numerical values of $t_{yy}(1,0; V)$ in Table 1.

An alternative method of solution to the system (3.1) - 3.3) is the reduction of the dual integral equations into an equivalent Fredholm integral equation of the second kind. The Fredholm integral was then solved numerically. The crack tip stresses so obtained are comparable to the analytic results as shown in Table 2.

5. DISCUSSION

The following results are worth noting:

- (i) The nonlocal stresses at the crack tip are finite. These results will enable us to extend Eringen's fracture criterion [17] to dynamic rupture phenomena. In the limit as the crack velocity $V \rightarrow 0$, we recover the static nonlocal crack results (see Table 1).
- (ii) The local analysis of the steady-state crack problems yielded stress intensity factors which are independent of the crack velocity and equal to their static values (Yoffe [2] Craggs [3]). These results are clearly undesirable in that they contradict the experimental results which show that the crack tip velocity depends on applied loads.

The nonlocal analysis on the other hand, provided a crack tip stress behaviour which is compatible with experiments. The substantial difference between the local and nonlocal results can be observed in Figure 2.

- (iii) The nonlocal results can be employed further to determine the terminal velocity of propagating cracks. The uniformly moving crack is in a state of dynamic equilibrium. As the crack propagates first, the interatomic bonds at the tip break. Then as the stress around the crack tip relaxes, the next bond is overloaded by the sweeping stress wave. There will be a sudden increase in the stress level above the ultimate stress one atomic distance away from the tip.

At the same time, from Fig. 2, we observe that the crack tip stress decreases with increasing velocity. In the dynamic equilibrium, two changes in the stress level compensate for each other and the crack reaches a uniform propagation velocity.

The stress relaxation at the crack tip is a complex phenomenon. Therefore, it is difficult to calculate the stress overshoot precisely. However, using a harmonic oscillator model, we may assume that it is roughly equal to the difference between the crack tip stress and the stress level one atomic distance away from the tip. Consequently, we propose that the terminal velocity be determined by

$$(5.1) \quad \frac{t_{yy}(1+a; 0)}{t_{yy}(1; 0)} \sim \frac{t_{yy}(1; v_t)}{t_{yy}(1; 0)}$$

$t_{yy}(1+a)/t_{yy}(1)$ is given in [12, Table 1] for the static crack problem. $t_{yy}(v)/t_{yy}(0)$ is plotted on Fig. 2. On Fig. 3, both curves are drawn together -- their crossing point yields the value of the terminal velocity as

$$(5.2) \quad v_t = \rho c_2 \quad \rho \sim (0.5 - 0.6)$$

Note that, in the case one is able to solve the stress relaxation problem at the crack tip, $[t_{yy}(1+a)/t_{yy}(1)]$ in Fig. 3, may not be a straight line any more (i.e. it will be velocity dependent). In that case, the terminal velocity estimates will be slightly modified.

(iv) By formal extension of Eringen's fracture criterion to dynamic phenomena, we can predict qualitatively the variation of the surface energy with the propagation velocity. The Griffith definition of surface energy can be formally extended to the velocity dependent case by

$$(5.3) \quad t_c^2 a = \frac{8\mu}{(1-\nu)} C(V)^2 \gamma(V) ,$$

where $\gamma(V)$ is the surface energy, μ, ν are the standard material moduli, t_c is the cohesive stress, a is the inter-atomic distance and $C(V)$ is defined by

$$(5.4) \quad t_{yy}(1,0)/t_0 = C(V) (2\ell/a)^{1/2} .$$

By assuming the cohesive strength, a material property and therefore constant, from (5.3) we can deduce that

$$(5.5) \quad \frac{\gamma(V)}{\gamma(0)} = \left[\frac{C(0)}{C(V)} \right]^2$$

From (5.3) and Table 1, we can deduce that the surface energy increases with increasing crack velocity. Although there is considerable scatter in the experimental values of dynamic surface energies, the nonlocal results are in agreement with their general tendency to increase with crack speed [9].

(v) If one wishes to establish a link with classical fracture theories, the following is interesting to mention. A typical fracture toughness versus crack velocity relationship is shown in Figure 4 [18]. The general character of this relationship is supported by physical considerations (Erdogan [19], Freund [20]), as well as by experiments on structural mild steel [21, 22] and on glassy polymers [23]. On Figure 4, we also plotted the ratio $t_{yy_{max}}(V)/t_{yy_{max}}(0)$.

Figure 4 indicates that in the lower velocity ranges, the fracture toughness decreases faster than the maximum stress with increasing velocity. Thus when

$$(5.6) \quad t_{yy_{max}}(V) > t_{yy_{critical}}(V) ,$$

the crack will accelerate. According to Eringen's fracture criterion, a dynamic equilibrium will be established when

$$(5.6) \quad t_{yy_{max}}(V_t) = t_{yy_{critical}}(V_t) .$$

We can therefore determine the terminal velocity, assuming that it is defined by this dynamic equilibrium. From Figure 4, we observe that the terminal velocity will be on the order of

$$(5.7) \quad V_t \sim 0.55 C_2 .$$

The important feature of this result is in that it predicts a terminal velocity well below the Rayleigh wave speed,^{1/} which is consistent with experimental observations and some numerical simulation studies [24]. Furthermore, it is obtained by a simple, natural fracture criterion, without extraneous arguments. The present approach, however, should be considered with some caution, since the fracture toughness measurements include some global effects such as viscoelasticity and plasticity. Therefore, we prefer the microstructural considerations discussed in (iii), to predict the terminal velocity.

- (vi) The explanation of crack bifurcation phenomena is much more difficult. Experimental observations suggest that [25]

$$(1) \quad \sigma_f C_b^{1/2} = \text{constant}$$

where σ_f is the fracture stress and C_b is the length of the crack at the moment of branching.

- (2) "Crack branching is not a spontaneous event"... but is the eventual outcome of cumulative process of advance cracking.

These characteristics of crack branching suggest that it is not likely to explain crack bifurcation solely in terms of a critical branching velocity. One has to consider the fracture energies of the involved materials (i.e. glasses do have low but tool steels do have high fracture energies). In addition, the accelerating phase of the crack propagation has to be taken into account.

¹For example, Broberg's local analysis [4] predicts $V_t = V_R$.

Our present analysis does not include considerations about advance cracking and acceleration effects. Hence we do not attempt to explain crack branching on the basis of the present result. Nevertheless, from the foregoing conclusions, it is clear that the nonlocal dynamic results are in the right direction.

TABLE IV.1

VARIATION OF CRACK TIP STRESS
WITH THE CRACK TIP VELOCITY(from 4.3, $\lambda = \mu$)

<u>V/C_2</u>	<u>$t_{yy}(1,0;V)/t_{yy}(1,0; V=0)$</u>
0.1	0.96
0.2	0.93
0.3	0.89
0.4	0.84
0.5	0.77
0.6	0.68
0.7	0.55
0.8	0.27

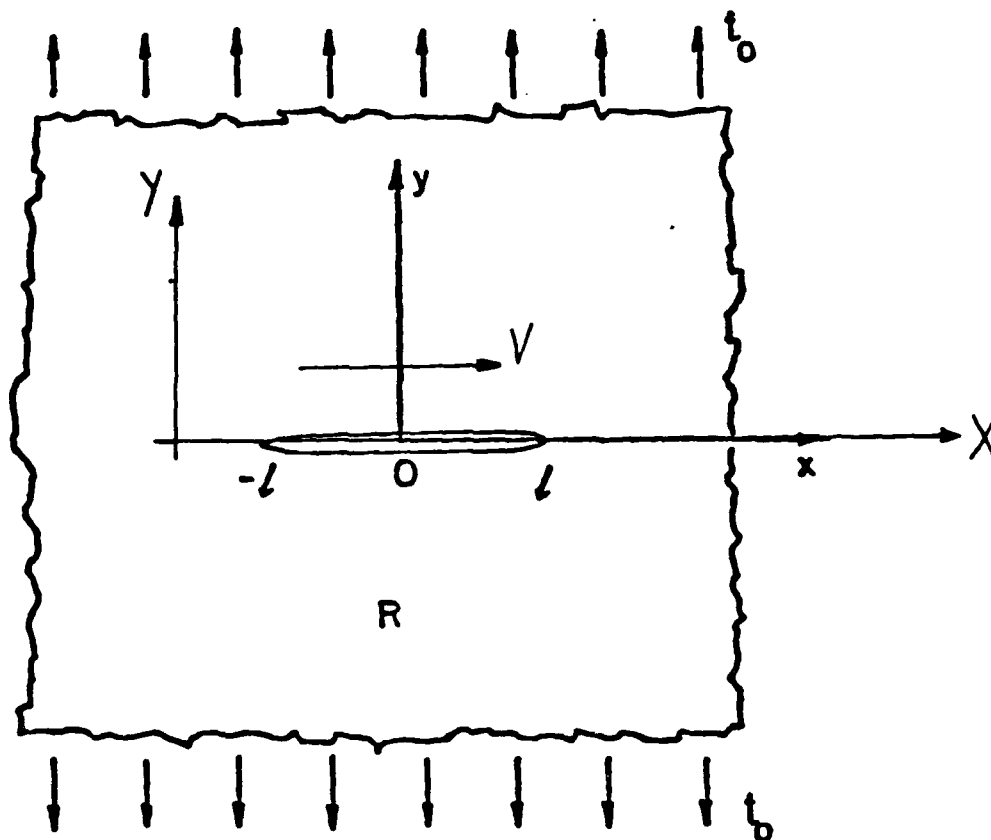


FIGURE 1

PLATE WITH MOVING LINE CRACK

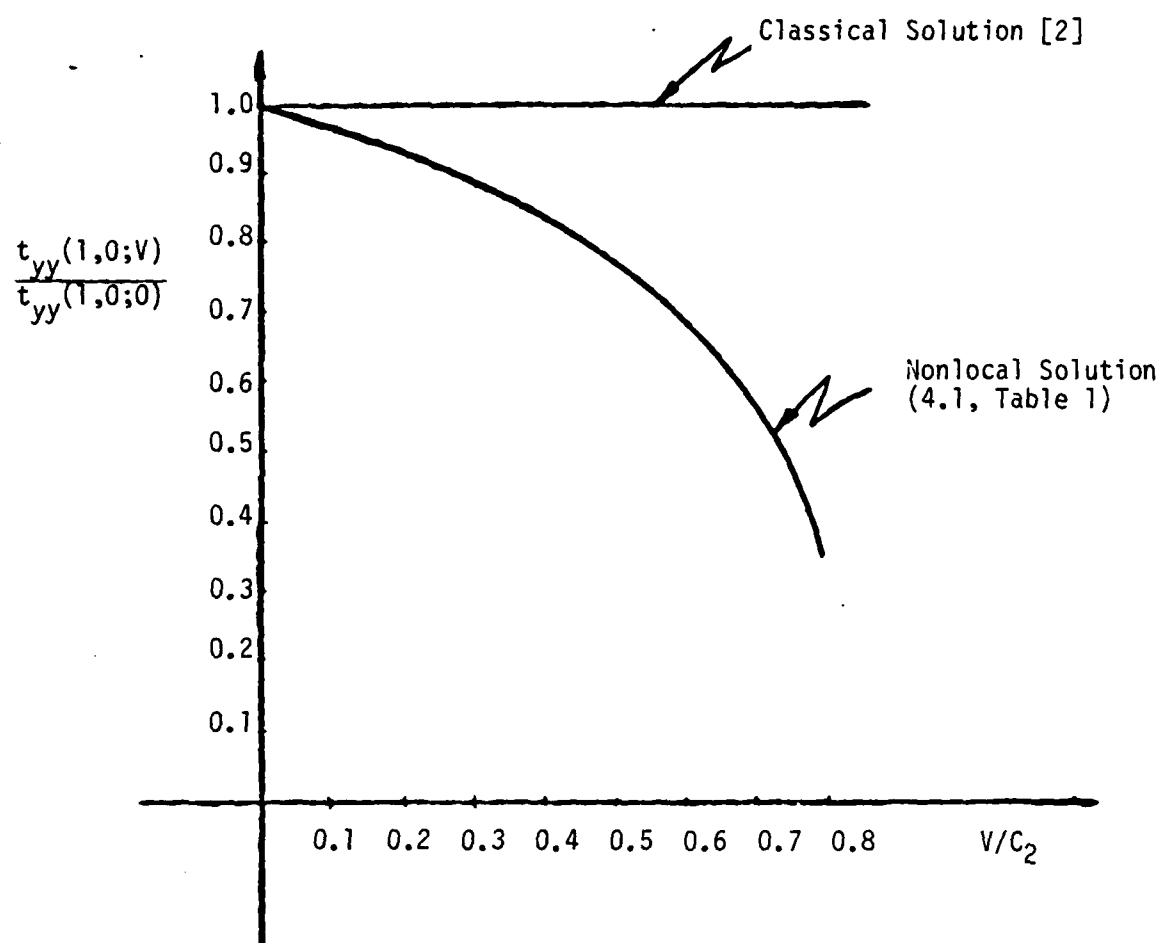


FIGURE 2
COMPARISON OF LOCAL AND NONLOCAL
CRACK TIP STRESSES

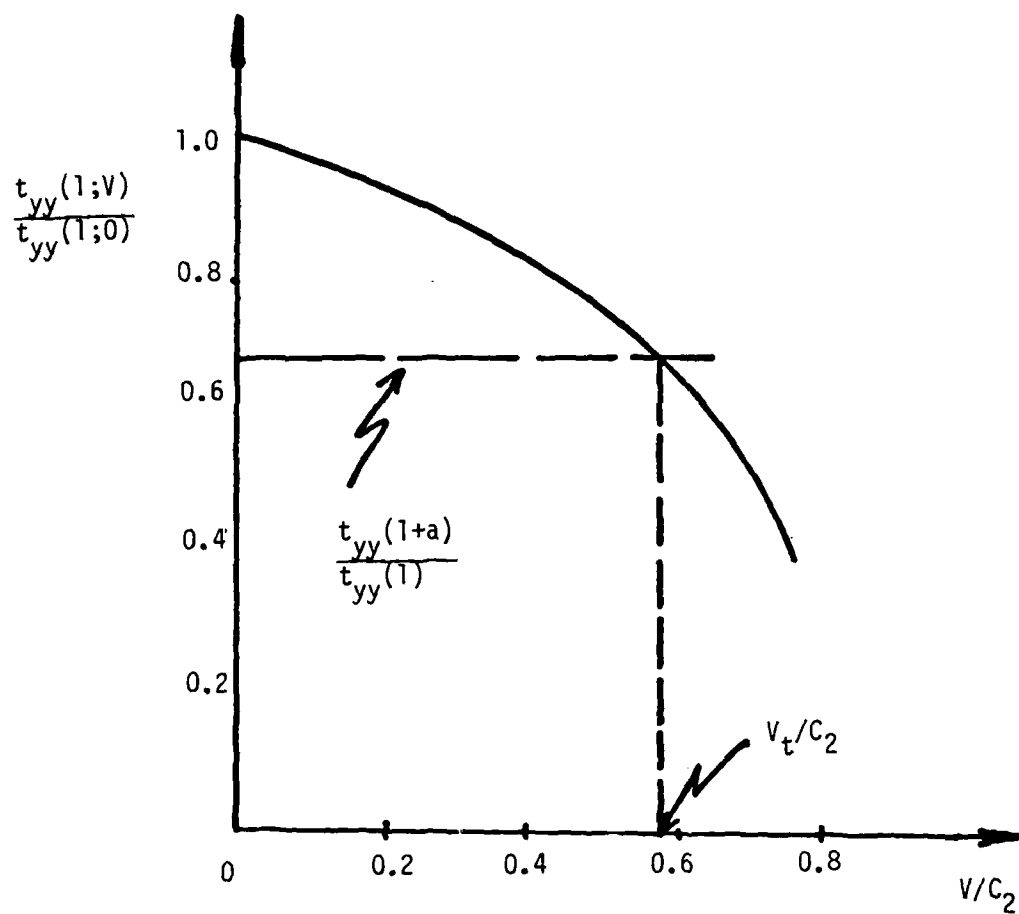


FIGURE 3

TERMINAL VELOCITY
(Nonlocal Determination)

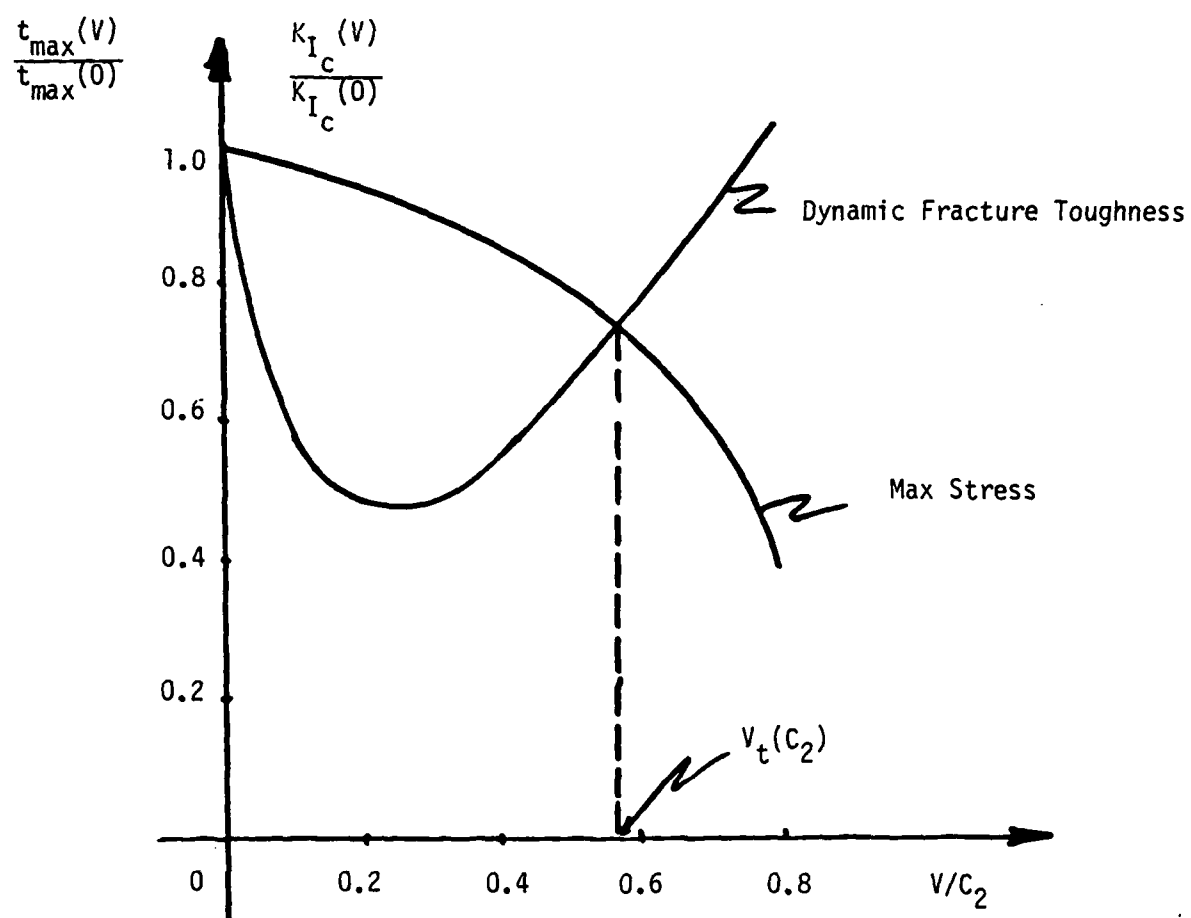


FIGURE 4

TERMINAL VELOCITY DETERMINATION
BY FRACTURE TOUGHNESS DATA

APPENDIX 1

The evaluation of the stress tip integrals can be facilitated by the use of the asymptotic expansion of Bessel functions, i.e.

$$(A.1) \quad J_1(k) \sim k^{-1/2} \cos(k - 3\pi/4) + O(k^{-3/2}); \quad |k| \rightarrow \infty$$

Substituting (A.1) into (4.2), we obtain the following results as $\xi \rightarrow 1^+$

$$(A.2) \quad t_A = [(C_1^2 - n^2 v^2)/C_1^2] (t_a + t_c + t_e) - (t_b + t_d + t_c)$$

$$(A.3) \quad t_B = t_g + t_i + t_k$$

where

$$(A.4) \quad t_a = 2 s_1^{1/2} M_1^{1/2} - 0.5 s_1^{5/2} M_1^{3/2} \frac{\pi^{1/2}}{2^{5/4}} \frac{\Gamma(3/2) \Gamma(3/4)}{\Gamma(9/8) \Gamma(13/8)} {}_3F_2(1/2, 3/2, 3/4; 13/8, 9/8; q_a)$$

$$(A.5) \quad t_b = 0.4 s_1^{5/2} M_1^{1/2} - 0.5 s_1^{9/2} M_1^{3/2} \frac{\pi^{1/2}}{2^{9/4}} \frac{\Gamma(3/2) \Gamma(7/4)}{\Gamma(13/8) \Gamma(17/8)} {}_3F_2(1/2, 3/2, 7/4; 17/8, 13/8; q_a)$$

$$q_a = -s_1^4 M_1^2/4$$

$$(2.6) \quad t_c = 2(1-s_1^{1/2}) M_1^{1/2} - 0.5 M_1^{-3/2} s_1^{-1/2} B(1/2, 3/2)$$

$$\sum_{n=0} \left[\frac{(1/2)_n (3/2)_n}{(2)_n} \frac{(-1)^n}{n!} M_1^{-2n} {}_2F_1(1/4, n+3/2; n+2; q_c) \right]$$

$$(A.7) \quad t_d = 0.4 (1-s_1^{5/2}) M_1^{1/2} - 0.5 M_1^{-3/2} s_1^{3/2} B(1/2, 3/2)$$

$$\sum_{n=0}^{\infty} \left[\frac{(1/2)_n (3/2)_n}{(2)_n} \frac{(-1)^n}{n!} M_1^{-2n} {}_2F_1(-3/4, n+3/2; n+2; q_c) \right]$$

$$q_c = -M_1^{-2} s_1^{-1}$$

$$(A.8) \quad t_e = 0.5 M_1^{1/2} \left[4 + \frac{\Gamma(1/2) \Gamma(-1/4)}{\Gamma(1/4)} \left[\sum_{n=0}^{\infty} \frac{(-M_1^{-2})^n}{n!} \frac{(1/2)_n (-1/4)_n}{(1/4)_n} {}_2F_1(-1/2, n-1/4; n+1/4; s_1^2) \right] \right]$$

$$(A.9) \quad t_f = 0.5 M_1^{1/2} \left[0.4 + \frac{\Gamma(1/2) \Gamma(-5/4)}{(-3/4)_n} \left[\sum_{n=0}^{\infty} \frac{(1/2)_n (-5/4)_n}{(-3/4)_n} \frac{(-M_1^{-2})^n}{n!} {}_2F_1(-1/2, n-5/4; n-3/4; s_1^2) \right] \right]$$

$$(A.10) \quad t_g = 0.5 M_2^{1/2} [s_2^{3/2} B(3/2, 1/4) {}_2F_1(-1/2, 1/4; 7/4; s_2^2) - s_2^{7/2} M_2 B(2, 3/4) {}_2F_1(1/2, 3/4; 11/4; -s_2^2/M_2^2)]$$

$$(A.11) \quad t_i = 0.5 [M_2^{-7/2} s_2^{-3/2} B(3/2, 3/2) {}_2F_1(3/4, 3/2; 3; - (M_2 s_2)^{-2}) - [(4/7) M_2^{-2} q_i^{7/4} F(7/4, 9/4; 11/4; q_i) - s_2^{7/2} {}_2F_1(7/4, 9/4; 11/4; s_2^2) - (4/3) s_2^2 q_i^{3/4} F(3/4, 5/4; 7/4; q_i) - s_2^{3/2} {}_2F_1(3/4, 5/4; 9/4; s_2^2)]]$$

$$q_i = c_2^2 / (c_2^2 + v^2)$$

$$\begin{aligned}
 (A.12) \quad t_k = & (1/3) M_2^{-3/2} s_2^2 - 0.5 M_2^{-1/2} \left[\sum_{n=2} \frac{(-1/2)_n}{n!} \frac{M_2^{-2}}{(n-5/4)} {}_2F_1(3/4, 1; n-1/4; s_2^2) \right. \\
 & \left. - \sum_{n=2} \frac{(1/2)_n}{n!} (-M_2^{-2})^n \left[\frac{1}{n-5/4} - \frac{s_2^2}{n-1/4} \right] \right]
 \end{aligned}$$

where

$$\begin{aligned}
 M_i &= c_i/V \\
 s_i^2 &= 1 - (V/c_i)^2
 \end{aligned}
 \quad i=1,2.$$

The above results are obtained either by standard integration formulae or when necessary, by expanding the integrands and integrating term by term. The integration intervals are naturally divided by the presence of the branch points [15].

Depending on the values of (V/c_i) , all series can be cast into rapidly converging forms by proper analytic continuation. The numerical value of the hypergeometric functions can also be found easily by evaluating the already existing sharp inequalities for lower and upper bounds. The relevant formulae for these inequalities and the necessary analytic continuation identities can be found in standard references.

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