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SYNTHESIS OF TAYLOR AND BAYLISS PATTERNS FOR LINEAR ANTENNA ARR--ETC(U)
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Synthesis of Taylor and Bayliss Patterns for Linear Antenna Arrays

J. P. SHELTON

*Electromagnetics Branch
Radar Division*

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**SYNTHESIS OF TAYLOR AND BAYLISS
PATTERNS FOR LINEAR ANTENNA ARRAYS**

INTRODUCTION

The requirement for low sidelobes from array-type antennas is a long-standing one. The contributions to this theory extend from Dolph's utilization of Chebyshev polynomials, through Taylor's papers on linear and circular apertures, Bayliss's extension to difference-type patterns, and finally to recently developed techniques which provide arbitrary pattern control for linear arrays [1-8].

The purpose of this report is to examine some of the more recent applications of these synthesis techniques in light of their limitations and also the computational capabilities which are now available. For example, at the time Taylor published his synthesis procedure, engineers had only slide rules, mathematical tables, and mechanical desk calculators to generate the distribution functions. The computational capability available to today's engineer is vastly different, and we will show how Taylor's and Bayliss's procedures can be modified to give better results.

A more careful look at the synthesis procedures previously mentioned is presented in Table 1.

Dolph's synthesis is precise and gives minimum beamwidth for given sidelobe levels, but these constant amplitude sidelobes are not desirable for larger arrays because it is possible to radiate most of the energy into the sidelobes. Taylor solved this problem by allowing the far-out sidelobes to fall off as dictated by an amplitude discontinuity at the ends of the aperture. Taylor, and later Bayliss, synthesized continuous distributions and sampled these to obtain array excitations.

Table 1 — Synthesis Procedures for Linear Array Apertures

Procedure/ Date	Continuous or Discrete	Limitations
Dolph/47	Discrete	Poor results for large arrays
Taylor/52	Continuous	Inexact for low sidelobes, small arrays
Bayliss/68	Continuous	Inexact for low sidelobes, small arrays
Hyneman/68	Continuous	Inexact for low sidelobes, small arrays—iterative
Stutzman/72	Continuous	Inexact for low sidelobes, small arrays—iterative
Elliott/76	Continuous	Inexact for low sidelobes, small arrays—iterative
Elliott/77	Discrete	Applies all continuous procedures to discrete arrays

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Some recent applications have called for lower sidelobes and smaller arrays, thereby pressing the limitations of the Taylor and Bayliss synthesis procedures. The problem of discretizing continuous aperture distributions has been treated [9-10]. The technique used in this report is different from those of Winter and of Elliott, but it is mathematically related to Elliot's technique.

REVIEW OF TAYLOR SYNTHESIS PROCEDURE

A brief review of the Taylor synthesis procedure is given here. The key to this procedure is the equal-sidelobe pattern function which is the continuous-aperture analog to the Chebyshev polynomial pattern for arrays:

$$E(u) = \cos \pi \sqrt{u^2 - A^2}, \quad (1)$$

where $u = \pi a \sin \theta / \lambda$, a is the length of the aperture and θ is the angle measured relative to the normal to the array. This function has a maximum value of $\cosh \pi A$ at $u = 0$ and unit sidelobes extending to $u = \pm \infty$. Taylor showed that the pattern of Eq. (1) is not physically realizable from a continuous aperture distribution, just as the Dolph array excitation becomes increasingly impractical in the limit of large arrays. His brilliant solution to this problem was:

1. For all zeros of the synthesized pattern functions, which we will call $E_s(u)$, from the n th from the origin to ∞ , the locations will be the same as those from a uniformly illuminated aperture of the same size. That is,

$$E_s(u) = 0 \text{ for } u = n \text{ for } n \geq \bar{n}.$$

2. For the first $\bar{n} - 1$ zeros, their locations will be determined by the zeros of $E(u)$, scaled so that the n th zero is located at $u = \bar{n}$.

The aperture distribution is determined by performing a Woodward synthesis of $E_s(u)$. That is, we define a set of functions of the form

$$F_n(u) = \sin (u - n)\pi / (u - n)\pi,$$

and then construct $E_s(u)$ from the $F_n(u)$

$$E_s(u) = \sum_{n=-\infty}^{\infty} E_s(n)F_n(u). \quad (2)$$

Since we have defined $E_s(n) = 0$ for $n \geq \bar{n}$, Eq. (3) becomes

$$E_s(u) = \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n)F_n(u). \quad (3)$$

Fourier transformation of Eq. (3) yields the aperture distribution:

$$\begin{aligned}
 A(x) &= \int_{-\infty}^{\infty} E_s(u) e^{j2xu\pi/a} du \\
 &= \int_{-\infty}^{\infty} \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n) F_n(u) e^{j2xu\pi/a} du.
 \end{aligned} \tag{4}$$

That is, $A(x)$ is a weighted sum of integrals of the form,

$$\int_{-\infty}^{\infty} \frac{\sin(u-n)\pi}{(u-n)\pi} e^{j2xu\pi/a} du.$$

Letting $u' = u - n$ results in

$$e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{\sin u'\pi}{u'\pi} e^{j2xu'\pi/a} du'.$$

Since the imaginary part of the integrand is odd, this becomes

$$\begin{aligned}
 &e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{\sin u'\pi \cos 2xu'\pi/a}{u'\pi} du' \\
 &= e^{j2n\pi x/a} \int_{-\infty}^{\infty} \frac{1}{2} \left[\frac{\sin u'\pi(1 - 2x/a) + \sin u'\pi(1 + 2x/a)}{u'\pi} \right] du'.
 \end{aligned} \tag{5}$$

A standard definite integral is

$$\int_{-\infty}^{\infty} \frac{\sin bzdz}{z} = \begin{cases} \pi & \text{for } b > 0 \\ 0 & \text{for } b = 0 \\ -\pi & \text{for } b < 0 \end{cases}$$

Application of this integral to Eq. (5) and thence to Eq. (4) yields

$$\begin{aligned}
 A(x) &= \sum_{n=-\bar{n}+1}^{\bar{n}-1} E_s(n) e^{j2\pi nx/a} \\
 &= E_s(0) + 2 \sum_{n=1}^{\bar{n}-1} E_s(n) \cos 2\pi nx/a \text{ for } |x| \leq a/2 \\
 &= 0 \text{ for } |x| > a/2.
 \end{aligned} \tag{6}$$

The continuous aperture distribution given by Eq. (6) is sampled to give the element excitation values for a discrete array. This last step is approximate, and the pattern function of the array is obviously different from $E_s(u)$. This approximation is acceptable provided that the number of elements in the array is much greater than \bar{n} and the sidelobe level is not extremely low. Figure 1 is an example of a case in which the synthesis procedure gives an unsatisfactory result. For a sidelobe level of 50 dB below mainbeam and $\bar{n} = 8$, a 30-element array has the computed pattern function shown. The near-in sidelobes are unduly low, whereas the first eight sidelobes should be about the same level.

ARRAY PATTERN FUNCTIONS IN TERMS OF ZEROS

Elliott used a synthesis technique which relates the discrete array distribution directly with the array pattern [9]. We also use this relationship, and our procedure achieves identical results with those of Elliott. However, the actual computations are different, and it is desirable to compare the techniques.

Elliott expresses the pattern function as a polynomial in w , where $w = e^{j(2\pi s/\lambda)\sin\theta}$. The zeros of this polynomial are given by w_n , which are normally located on the unit circle. Once he has the w_n properly adjusted, he completes the synthesis by multiplying out the product expression, $\prod(w - w_n)$, into the polynomial. The coefficients of the polynomial are the excitations of the array elements.

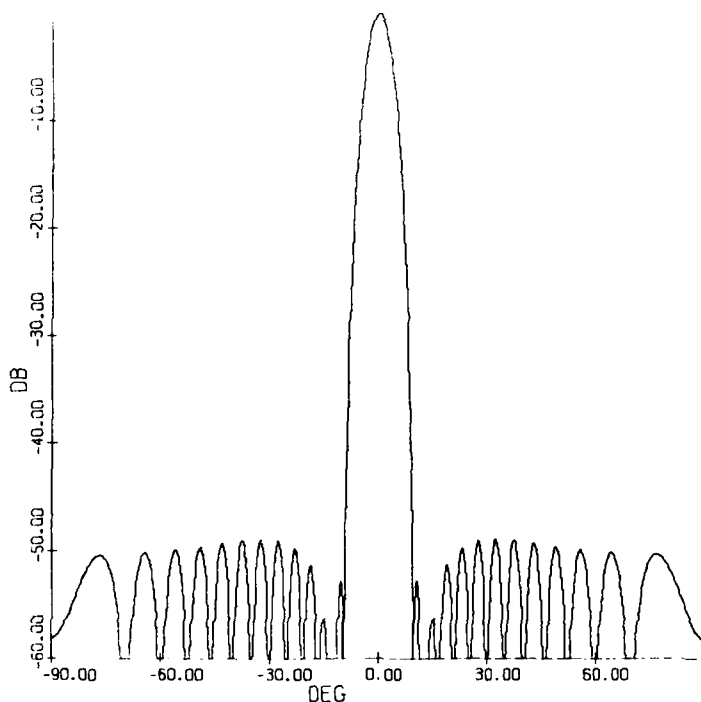


Fig. 1 — Conventional Taylor synthesis, $N = 30$,
 $\bar{n} = 8$, 50-dB sidelobes

Our procedure also uses the pattern function zeros in a product expression. Since the patterns are symmetric, our expression can be of the form, $\Pi(\cos z - \cos z_n)$, where $z = (2\pi s/\lambda) \sin \theta$. We cannot multiply this product expression out to obtain the coefficients directly since we require terms of the form $\cos nz$ rather than $\cos^n z$. Rather, we carry out a synthesis exactly analogous to that used by Taylor. Uniformly spaced pattern function samples are found by using the product expression. These pattern samples are used in a Fourier series to find the array illumination.

The procedure relies on the equivalent location of pattern function zeros for the line source and for the discrete array. Whereas the zeros for the pattern of a uniform line source distribution are located at $u = n$, the analogous relationship for a discrete array is $z = n\pi/N$, where $z = 2\pi s \sin \theta/\lambda$, where s is element spacing and N is the number of elements in the uniformly excited array.

The transformation of Taylor's procedure is easily seen to consist of locating the zeros in step 1 above at $z = n\pi/N$ for $n \geq \bar{n}$ and then scaling the first \bar{n} zeros of Eq. (1) so that the \bar{n} th zero is located at $z = \bar{n}\pi/N$.

Appendix A lists the resulting equations for Taylor arrays of both even and odd N , and Appendix B lists the equations for Bayliss arrays (yielding monopulse difference patterns) of both even and odd N . Figure 2 is an example of a Taylor array pattern with sidelobe levels of 50 dB with $\bar{n} = 8$ and $N = 30$. These equations can be straightforwardly programmed for automatic processing by a digital computer. Many programmable calculators now have sufficient memory to implement these programs. Appendix C lists programs for carrying out the synthesis and evaluating the pattern functions with an HP-41C programmable calculator.

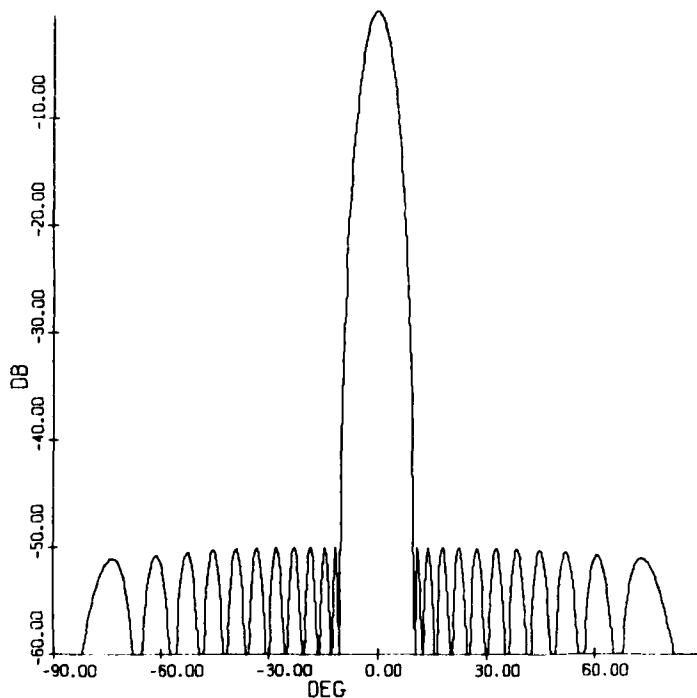


Fig. 2 — Discretized Taylor synthesis,
 $N = 30, \bar{n} = 8, 50\text{-dB}$ sidelobes

ACKNOWLEDGMENT

I thank Dr. Robert J. Adams for his careful and constructive review of the initial draft of this report. His questions led to a clarification of the relationship between this synthesis and that of Elliott [9].

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Appendix A

DESIGN EQUATIONS FOR LINEAR ARRAYS WITH TAYLOR-TYPE PATTERNS

These equations will determine the aperture illumination coefficients for a linear array of N elements to produce a Taylor-type pattern function with \bar{n} sidelobes on each side of the main beam at a level of L dB.

This design procedure involves three steps. The first $\bar{n} - 1$ zeros of the pattern are determined. Then the appropriate pattern function samples are determined. Finally, the array element illumination coefficients are determined by a harmonic analysis of the pattern function samples.

A particular advantage of this synthesis is that the knowledge of all of the pattern function zeros allows the computation of the pattern function as a product rather than as a polynomial. The product computation involves only one trigonometric function evaluation for each pattern function value. All other constants need to be evaluated only once for each array.

The pattern function zeros are given by

$$z_n = \frac{2\pi\bar{n}\sqrt{A^2 + (n - 1/2)^2}}{N\sqrt{A^2 + (\bar{n} - 1/2)^2}} \quad \text{for } n = 1 \text{ to } \bar{n} - 1 \quad (\text{A1a})$$

$$= \frac{2\pi n}{N} \quad \text{for } n = \bar{n} \text{ to } M, \quad (\text{A1b})$$

where

$$M = \text{int} \left(\frac{N - 1}{2} \right)$$

and A is given by

$$A = \frac{1}{\pi} \cosh^{-1} \left[10^{(L/20)} \right] \quad (\text{A2a})$$

$$\approx (L + 6.02)/27.29, \quad (\text{A2b})$$

where L is the sidelobe level (positive) in dB. Equation (A2b) is an excellent approximation, especially for large L .

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The pattern function is given by

$$\begin{aligned}
 E(z) &= \cos \frac{z}{2} \prod_{n=1}^M \left(\frac{\cos z - \cos z_n}{1 - \cos z_n} \right) && N \text{ even} \\
 &= \prod_{n=1}^M \left(\frac{\cos z - \cos z_n}{1 - \cos z_n} \right) && N \text{ odd}
 \end{aligned} \tag{A3}$$

The pattern samples to be used to find the array element illumination coefficients are given by

$$a_m = E \left(\frac{2\pi m}{N} \right) \text{ for } m = 1 \text{ to } \bar{n} - 1. \tag{A4}$$

The element excitation coefficients are given by

$$\begin{aligned}
 e_p &= 1 + 2 \sum_{m=1}^{\bar{n}-1} a_m \cos \frac{m(2p-1)\pi}{N} && N \text{ even, } p = 1 \text{ to } M + 1 \\
 &= 1 + 2 \sum_{m=1}^{\bar{n}-1} a_m \cos \frac{2mp\pi}{N} && N \text{ odd, } p = 0 \text{ to } M,
 \end{aligned} \tag{A5}$$

where p is an index or element number starting at the center and moving to either end of the array.

Appendix B

DESIGN EQUATIONS FOR LINEAR ARRAYS WITH BAYLISS-TYPE DIFFERENCE PATTERNS

Appendix A gave the design equations for linear arrays with Taylor-type patterns, which produce a main beam with slightly larger beamwidth than that of the Dolph synthesis but in general with higher gain. In some applications; such as monopulse, we might require a difference pattern. Bayliss presented a synthesis procedure for difference patterns, analogous to that of Taylor. In this appendix we adapt the Bayliss procedure to discrete arrays.

As in the case of the Taylor synthesis, the application of discrete arrays involves three steps. The first $\bar{n} - 1$ off-axis zeros of the pattern are determined. Then the appropriate pattern function samples are determined. Finally the array element illumination coefficients are determined by a harmonic analysis of the pattern function samples.

The pattern function zeros are given by

$$z_n = \frac{2\pi q_n \left(\bar{n} + \frac{1}{2} \right)}{N\sqrt{A^2 + \bar{n}^2}} \quad \text{for } n = 1, 2, 3, 4 \quad (\text{B1a})$$

$$= \frac{2\pi \left(\bar{n} + \frac{1}{2} \right) \sqrt{A^2 + n^2}}{N\sqrt{A^2 + \bar{n}^2}} \quad \text{for } n = 5 \text{ to } \bar{n} - 1 \quad (\text{B1b})$$

$$= \frac{2\pi \left(n + \frac{1}{2} \right)}{N} \quad \text{for } n = \bar{n} \text{ to } M \quad (\text{B1c})$$

where

$$M = \text{int} \left(\frac{N-2}{2} \right).$$

In this case it is necessary to find both A and q_n from graphs in Bayliss's paper [4]. For 50 dB sidelobes, $A = 2.42$, $q_1 = 2.78$, $q_2 = 3.18$, $q_3 = 3.85$, and $q_4 = 4.65$.

The pattern function is given by

$$E(z) = \sin \frac{z}{2} \prod_{n=1}^M [\cos z - \cos z_n] / \sin \frac{z_1}{4} \prod_{n=1}^M \left[\cos \frac{z_1}{2} - \cos z_n \right] \quad N \text{ even} \quad (\text{B2})$$

$$= \sin z \prod_{n=1}^M [\cos z - \cos z_n] / \sin \frac{z_1}{2} \prod_{n=1}^M \left[\cos \frac{z_1}{2} - \cos z_n \right] \quad N \text{ odd}$$

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$E(z)$ is normalized to unity at $z = z_1/3$, which is near the pattern maximum. If a more precise pattern maximum is desired, a better multiplying constant can easily be found.

The pattern samples to be used to find the array element illumination coefficients are given by

$$b_m = E\left(\frac{\pi}{N}(2m - 1)\right) \quad \text{for } m = 1 \text{ to } \bar{n}. \quad (\text{B3})$$

The element excitation coefficients are given by

$$\begin{aligned} e_p &= 2 \sum_{m=1}^{\bar{n}} b_m \sin \frac{\pi(2m - 1)(2p - 1)}{2N} \quad \text{for } N \text{ even, } p = 1 \text{ to } M + 1 \\ &= 2 \sum_{m=1}^{\bar{n}} b_m \sin \frac{\pi(2m - 1)p}{N} \quad \text{for } N \text{ odd, } p = 1 \text{ to } M + 1 \end{aligned} \quad (\text{B4})$$

where p is an index of the element number starting with zero at the center of the array. For N odd, the center element of the array always has zero excitation. The excitations on one side of the array are the negative of those on the other side.

Appendix C

PROGRAMS FOR THE HP-41C CALCULATOR

This appendix presents programs for the HP-41C calculator for the design equations of Appendices A and B. The software consists of four programs, SUM, DIF, IN, and SL. "SUM" contains the equations for synthesizing Taylor-type sum patterns; "DIF" contains equations for Bayliss-type difference patterns; "IN" contains subroutines that are used by both programs; and "SL" is a routine for calculating the peaks of the sidelobes of the synthesized array. The number of registers used by the programs and the number of card sides required for storage are:

Program	Registers	Card Sides
SUM	30	2
DIF	42	3
IN	39	3
SL	19	2
	130	10 (5 cards) .

It is possible to synthesize aperture distributions using either SUM and IN or DIF and IN. These programs require at least one additional memory module. Furthermore, the programs use nine registers for variables, indices, and constants. Table C1 correlates the number of registers available for synthesis parameters with the number of additional memory modules in use. The available registers are used for the pattern samples a_m and b_m and for the pattern function zeros (cosines) z_p . The number of these registers is $\bar{n} + M$. Therefore, the size of array that can be synthesized for any given configuration of Table C1 depends on \bar{n} . For a 50-dB sidelobe requirement, \bar{n} will be about 8. Roughly speaking, an array of 55 to 65 elements for difference and 80 to 90 for sum can be synthesized using one memory module by trading programs in and out of the machine, and an array of 90 to 100 elements can be synthesized with all programs loaded using two modules. The maximum array size that can be handled using three modules is 310 to 320 for difference and about 340 for sum. It appears that one or two memory modules should suffice for most requirements.

Table C1 — Registers Available after Loading
Indicated Program Complements

Program Complement	Number of Memory Modules		
	1	2	3
SUM + IN	48	112	176
DIF + IN	35	99	163
SUM + DIF + IN	11	71	135
SUM + DIF + IN + SL	—	54	118

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The procedure for running the programs is:

1. Allocate memory by XEQ SIZE (9 + M + n).
2. Load the appropriate program complement.
3. Enter either XEQ SUM or XEQ DIF.
4. The display will prompt for N, L, and NBAR. N can be even or odd. L is sidelobe level in positive dB. DIF will also prompt for A, Q1, Q2, Q3, Q4.
5. After calculating z_n and loading $\cos z_n$ into registers starting with (9 + \bar{n}), the display will ask whether you want a listing of peak sidelobes (SL) or aperture distribution (EP). After the sidelobes or excitation coefficients are listed, the display will ask whether you want the other set of parameters calculated and listed.

The routine SL computes the sidelobe level relative to the main beam level by evaluating the pattern value at a point midway between pattern zeros. This computation is admittedly approximate because the pattern maximum is in general not exactly midway between zeros. The main beam pattern value is computed for $z = 0$. The difference pattern maximum is computed for $z = z_1/3$. This factor was found to be accurate for 50 dB sidelobes. The exact multiplying factor will be somewhat larger for higher sidelobes ($L < 50$), and it can be found quickly by obtaining z_1 and executing PA:

```
RCL (9 +  $\bar{n}$ )    gives  $\cos z_1$ 
ACOS           gives  $z_1$ 
k             new multiplying factor, such as .4
*
COS
STO 02
XEQ PA .
```

Alternatively, k can be found from Fig. 4 of Bayliss^{C1}, which defines the beam maximum by p_0 , where $k = p_0/\$1$. ($\1 corresponds to our z_1)

Once the desired value of k has been found, go to lines 110, 111 in DIF, and exchange k , * for 3,/. It is now necessary to reload the reference main beam pattern value into R08. This calculation starts at line 61 of SUM and 105 of DIF. Alternatively, you can simply rerun the program.

The pattern value, in voltage and normalized to mainbeam level, is found by keying in the value of z in degrees, then keying COS, STO 02, XEQ PA.

The registers used are:

00	N
01	M
02	A^2 and $\cos z_m$ for PA
03	n
04, 05	loop indices
06	multiplying constants

^{C1}E. T. Bayliss, BSTJ, May-Jun 1968, pp. 623-650.

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07 accumulator for $E(z)$, e_p
08 main beam reference value
09 to $(8 + \bar{n})$ computed values of a_m , b_m
 $(9 + \bar{n})$ to $(8 + \bar{n} + M)$ computed values of $\cos z_n$.

Program IN contains the following subroutines:

IN Asks for input data N , L , $NBAR$
ZN Completes calculation and storage of $\cos z_n$
BR Asks for choice of sidelobes or aperture distribution and branches to EP or SL
PR Prints element excitations e_p
PA Computes pattern value for a_m , b_m , or SL routines
EP Completes calculation of e_p .

The programs use flags 00 and 01 to indicate the following conditions:

Flag 00 is set for N even
clear for N odd

Flag 01 is set for DIF execution
clear for SUM execution.

The use of registers by program PA precludes the use of the plot subroutines resident in the printer.

Note that the sidelobes and pattern values obtained with these programs are all relative to the main beam level. No information concerning gain or aperture illumination efficiency is computed. The aperture distribution can be used to compute aperture efficiency or gain.

The programs and sample printouts are listed on the following pages.

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00 LBL 01	55 LBL 02	108 +	23 XTC
01 SF 01	56 RCL 04	109 +	24 +
02 XEQ "1A"	57 INT	110 STO 04	25 SQRT
03 RCL 00	58 XEQ "2N"		26 /
04 /	59 ISG 04	111 LBL 04	27 RCL 03
05 /	60 STO 00	112 1	28 .5
06 /	61 1	113 RCL 03	29 +
07 INT	62 STO 02	114 1	30 *
08 STO 01	63 STO 08	115 -	31 STO 06
09 360	64 XEQ "FA"	116 1 E-3	32 5
10 RCL 03	65 STO 08	117 +	33 RCL 02
11 +	66 XEQ "BR"	118 +	34 XCVT
12 RCL 00		119 STO 05	35 GTD 07
13 /	67 LBL "3"	120 0	36 1.000
14 RCL 00	68 RCL 05	121 STO 07	37 STO 04
15 /	69 1		38 GTD 06
16 RCL 03	70 -	122 LBL 05	
17 .5	71 1 E-3	123 XEQ "EP"	39 LBL 07
18 -	72 +	124 ISG 05	40 1
19 X+2	73 1	125 GTD 05	41 -
20 RCL 07	74 +	126 2	42 1 E-3
21 +	75 STO 05	127 RCL 07	43 +
22 SQRT	76 360	128 +	44 +
23 /	77 RCL 00	129 1	45 STO 04
24 STO 06	78 /	130 +	
25 RCL 03	79 STO 06	131 XEQ "PR"	46 LBL 06
26 1		132 GTD 04	47 RCL 04
27 -	80 LBL 03	133 END	48 INT
28 1 E-3	81 RCL 06		49 "ENT 0
29 +	82 RCL 05		50 FIX 6
30 1	83 INT		51 ARCL X
31 +	84 +	PRP "DIF	52 PROMPT
32 STO 04	85 COS		53 XCVT
	86 STO 02	01 LBL "DIF"	54 "0"
33 LBL 01	87 XEQ "PR"	02 SF 01	55 ARCL X
34 RCL 04	88 RCL 05	03 XEQ "IN	56 RCL
35 INT	89 8	04 RCL 06	57 "0"
36 .5	90 +	05 2	58 RCL
37 -	91 XCVT	06 -	59 RDN
38 X+2	92 STO IND Y	07 2	60 FIX 2
39 RCL 02	93 ISG 05	08 /	61 ACX
40 +	94 GTD 03	09 INT	62 PRBUF
41 SQRT	95 96	10 STO 01	63 XEQ "2N"
42 XEQ "7N"	96 RCL 00	11 "R="	64 ISG 04
43 ISG 04	97 /	12 PROMPT	65 GTD 06
44 GTD 01	98 STO 06	13 FIX 2	66 5
45 RCL 01	99 1	14 ARCL X	67 RCL 03
46 1 E-3	100 1	15 PRA	68 1
47 *	101 FCP 00	16 X+2	69 -
48 RCL 05	102 0	17 STO 02	70 XCVT
49 +	103 FCP 00	18 360	71 GTD 10
50 STO 04	104 0	19 RCL 06	72 1 E-3
51 360	105 RCL 01	20 /	73 +
52 RCL 00	106 +	21 RCL 00	74 +
53 /	107 1 E-3	22 RCL 03	75 STO 04

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76* LBL 01	128	81* LBL 11*	51 PROMPT
77 RCL 02	129 STO 0A	82 F1 0	52 X=00
78 RCL 04		83 OF 00	53 RTN
79 INT	130* LBL 14	84 -	54 GTO 10
80 +	131 RCL 01	85 PROMPT	
81 -	132 INT	86 ARCL X	56* LBL 18
82 000	133 2	87 RCL	57 E1
83 XEQ "D"	134 *	88 STO 0A	58 RCL 04
84 100 04	135 -	89 2	59 INT
85 GTO 00	136 -	90 -	60 F1 0
	137 RCL 06	91 ENTER	61 ARCL Y
86* LBL 10	138 +	92 FRC	62 RCR
87 360	139 COS	93 X=00	63 F1 4
88 RCL 00	140 STO 02	94 OF 00	64 -
89 /	141 XEQ "PAR"	95 L=1	65 RCR
90 STO 0A	142 RCL 05	96 PROMPT	66 RCR
91 RCL 03	143 0	97 ARCL X	67 RCR
92 RCL 01	144 +	98 PPR	68 PREDF
93 1 E-3	145 XCRN	99 20	69 ISG 04
94 +	146 STO IND Y	100 -	70 RTN
95 +	147 ISG 05	101 2	71 RCR
96 STO 04	148 GTO 14	102 LOG	72 "SLT 0"
	149 RCL 01	103 +	73 PROMPT
97* LBL 11	150 1	104 1	74 X=00
98 RCL 04	151 +	105 E1X	75 GTO "SL
99 INT	152 1 E-3	106 LOG	76 STOP
100 JC	153 *	107 F1	
101 +	154 1	108 *	77* LBL "PR
102 XEQ "2M"	155 +	109 /	78 RCL 01
103 ISG 04	156 STO 04	110 X+2	79 1 E-3
104 GTO 11	157 90	111 STO 02	80 +
105 RCL 03	158 RCL 00	112 "NBAR="	81 1
106 0	159 /	113 PROMPT	82 +
107 +	160 STO 06	114 ARCL X	83 STO 04
108 RCL IND 1		115 PPR	84 1
109 ACOS	161* LBL 15	116 STO 03	85 STO 07
110 3	162 1	117 FTK	
111	163 RCL 03	118* LBL 12*	86* LBL 00
112 COS	164 1 E-3	119 RCL 04	87 RCL 04
113 STO 02	165 *	120 RCL 04	88 RCL 03
114 1	166 +	121 *	89 +
115 STO 00	167 STO 05	122 COS	90 5
116 XEQ "PAR"	168 0	123 RCL 03	91 +
117 STO 08	169 STO 07	124 0	92 RCL 02
118 XEQ "BR"		125 +	93 RCL IND 1
	170* LBL 16	126 RCL 04	94 -
119* LBL "PR"	171 XEQ "EF"	127 ISG 04	95 ST* 07
120 RCL 03	172 ISG 05	128 STO 00	96 ISG 04
121 1 E-3	173 GTO 16	129 1	97 STO 00
122 *	174 2	130 FTK	98 1
123 1	175 RCL 07	131* LBL "BR"	99 FSK 06
124 +	176 *	132* LBL "MERN"	100 GTO 01
125 STO 05	177 XEQ "PR"		101 FCL 01
126 100	178 GTO 15		102 GTO 07
127 RCL 04	179 STOP		103 RCL 03
	180 END		

SHELTON

104 ACOS
105 SIN
106 GTO 03

107*LBL 01
108 RCL 02
109 ACOS
110 2
111 /
112 FSP 01
113 GTO 02
114 COS
115 GTO 03

116*LBL 02
117 SIN

118*LBL 03
119 RCL 07
120 *
121 RCL 06
122 /
123 RTN

124*LBL EP
125 -1
126 FCP 00
127 0
128 RCL 04
129 INT
130 2
131 *
132 +
133 RCL 05
134 INT
135 2
136 *
137 FCP 01
138 GTO 03
139 1
140 -

141*LBL 03
142 *
143 RCL 06
144 *
145 FSP 01
146 GTO 04
147 COS
148 GTO 05

149*LBL 04
150 SIN

151*LBL 05
152 RCL 05
153 3
154 +
155 XYY
156 RCL IND
157 *
158 ST+ 07
159 RTN
160 END

PRP *SL*

01*LBL *SL*
02 *SL PEAKS, DE*

03 PRP
04 FIX 2
05 1
06 RCL 03
07 1
08 -
09 1 E-3
10 *
11 +
12 STO 05

13*LBL 00
14 RCL 03
15 8
16 +
17 RCL 05
18 +
19 RCL IND X
20 ACOS
21 XXY
22 1
23 +
24 XXY
25 RCL IND Y

26 ACOS
27 +
28 2
29 /
30 COS
31 XEQ 03
32 ISG 05
33 GTO 00
34 RCL 03
35 RCL 01
36 1 E-3
37 *
38 +
39 STO 05

40 120
41 RCL 00
42
43 STO 02

44*LBL 0
45 1
46 FSP 01
47 0
48 RCL 05
49 INT

50 2
51 *
52 +
53 RCL 04
54 +
55 COS
56 STO 02
57 XEQ 03
58 ISG 05
59 GTO 01
60 ADV
61 STOP

62*LBL 03
63 STO 02
64 XEQ *PR*
65 ABS
66 LOG
67 20
68 *
69 CHS
70 PRX
71 RTN
72 *EP? 0
73 PROMPT
74 *00
75 STOP
76 FSP 01
77 GTO *0
78 GTO *S*
79 END

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N=20
L=50
NBAP=8
Z1= 1.9452
Z2= 1.8435
Z3= 1.6549
Z4= 1.4671
Z5= 1.2166
Z6= 0.8294
Z7= 0.5884
Z8= 0.3527
Z9= 0.1903
Z10= 0.0965

N=21
L=50
NBAP=8
Z0= 1.9579
Z1= 1.9111
Z2= 1.7763
Z3= 1.5703
Z4= 1.3156
Z5= 1.0483
Z6= 0.7766
Z7= 0.5291
Z8= 0.3306
Z9= 0.1812
Z10= 0.0957

SL PEAKS. 06

50.33	***
49.93	***
49.82	***
49.72	***
49.60	***
49.45	***
49.29	***
49.09	***
49.09	***

SL PEAKS. 06

50.35	***
49.95	***
49.86	***
49.77	***
49.67	***
49.56	***
49.43	***
49.26	***
49.29	***
49.31	***

N=20
L=50
NBAP=8
A=2.42
Q1= 2.78
Q2= 3.18
Q3= 3.85
Q4= 4.65
E1= 0.4866
E2= 1.3673
E3= 1.9531
E4= 2.2448
E5= 2.1558
E6= 1.9061
E7= 1.3879
E8= 0.8180
E9= 0.4184
E10= 0.1751

N=21
L=50
NBAP=8
A=2.42
Q1= 2.78
Q2= 3.18
Q3= 3.85
Q4= 4.65
E1= 0.9093
E2= 1.6548
E3= 2.1187
E4= 2.2515
E5= 2.0835
E6= 1.7613
E7= 1.2231
E8= 0.7650
E9= 0.3353
E10= 0.1726

SL PEAKS. 06

49.54	***
48.41	***
47.91	***
46.89	***
47.09	***
46.90	***
46.59	***
46.56	***
45.64	***

SL PEAKS. 06

49.56	***
48.44	***
47.94	***
46.89	***
47.10	***
46.90	***
46.64	***
46.59	***
45.59	***

