

AD-A103 855

WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER

F/G 12/1

A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHING--ETC(U)

JUL 81 D 91

DAAG29-80-C-0041

UNCLASSIFIED

MRC-TSR-2242

NL

1 1
2 2



END
DATE
FORMED
G R
DTIC

AD A103855

MRC Technical Summary Report #2242 ✓

A NEW WAY FOR CONSTRUCTING
HIGHER ORDER ACCURACY SPLINE
SMOOTHING FORMULAS

Dong-Xu Qi

Mathematics Research Center
University of Wisconsin-Madison
610 Walnut Street
Madison, Wisconsin 53706

July 1981

(Received June 2, 1981)

DTIC
ELECTE
SEP 8 1981
A

Approved for public release
Distribution unlimited

FILE COPY

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

81 9 08 043

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

A NEW WAY FOR CONSTRUCTING HIGHER ORDER
ACCURACY SPLINE SMOOTHING FORMULAS.

DONG-XU/QI

Technical Summary Report, #2242

July 1981

ABSTRACT

In this paper the author introduces the operator $\bar{\Delta}^{(n)} := P_n(\mu)\bar{\Delta}$ with higher order accuracy for approximation to the differential operator D , where $\bar{\Delta}$ denotes centered difference operator, μ denotes averaging operator,

$$P_n(\mu) = \sum_{m=0}^n C_m(\mu-I)^m, \quad C_m = -\frac{m}{2m+1} C_{m-1}, \quad C_0 = 1.$$

A class of new many-knot spline basis $\Omega_{k,n} := (P_n(\mu))^k N_k$ was suggested. The smoothing formulas

$$f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n}\left(\frac{\cdot-t}{h}\right) f(t) dt \quad \text{and} \quad S_{k,n} f = \sum f_i \Omega_{k,n}$$

are discussed.

AMS (MOS) Subject Classification: 41A15

Key Words: Spline, smoothing, many-knot, Higher order accuracy

Work Unit Number 3 - Numerical Analysis and Computer Science

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Avail and/or	
Dist	Special
A	

Department of Mathematics, Jilin University, Changchun, China.

Sponsored by the United States Army under Contract No. DAAG29-88-C-0041

SIGNIFICANCE AND EXPLANATION

I. J. Schoenberg studied B-splines and established some smoothing formulas for fitting data. In particular the smoothing approximation

$$S_k f = \sum_i f_i N_{i,k} \quad (\text{where } N_{i,k} \text{ are B-splines and } f \text{ is an arbitrary function})$$

has been successfully used in curve fitting. The paper proposes a new class of spline function denoted $\Omega_{i,k}$ instead of $N_{i,k}$. The new approximation

$S_{k,n} f = \sum_i f_i \Omega_{i,k}$ achieves higher order accuracy. To construct $\Omega_{i,k}$, we first introduce the averaging operator $P_n(\mu)$, $P_n(x) = \sum_{m=0}^n C_m (x-1)^m$, $C_m = -\frac{m}{2m+1} C_{m-1}$, $C_0 = 1$, and then define $\Omega_{i,k} := [P_n(\mu)]^k N_{i,k}$. The smoothing formulas for function f are given by $f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n}(\frac{\cdot-t}{h}) f dt$ and $S_{k,n} f = \sum_i f_i \Omega_{i,k,n}$.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHING FORMULAS

Dong-Xu Qi^{*}

The modern mathematical theory of spline approximation was introduced by I. J. Schoenberg in 1946. In the paper [6] he studied so-called "B-spline basis". A B-spline basis can be normalized in various ways. One of them is the so called normalized B-spline, see [2], denoted by $N_{i,k}$ for the B-spline function of degree $k - 1$ having support (x_i, x_{i+k}) . The spline smoothing formula for degree $k - 1$ to an arbitrary function f can be represented by $S_k f = \sum f_i N_{i,k}$. This approximation has been used in curve fitting successfully [1], [4].

In order to improve accuracy of the smoothing operator S_k , the author in this paper suggests a new spline basis denoted $\Omega_{i,k,n}$ instead of $N_{i,k}$. Thus, a new way for the construction of spline smoothing formulas is introduced. I prefer calling $S_{k,n} f = \sum f_i \Omega_{i,k,n}$ a smoothing operator with grade n and order k . In here when $n = 0$, $\Omega_{i,k,0}$ is just $N_{i,k}$ and $S_{k,0}$ is the same as S_k . Since $S_{k,n} f \in \varphi_k + \varphi_k^*$, this is a class of many-knot splines.

Concerning higher order accuracy spline smoothing formulas, I. J. Schoenberg [1946] has already discussed in [6] and Z. S. Liang studied the many-knot spline smoothing [4]. My main attempt in this paper is to suggest a new way for constructing them.

^{*} Department of Mathematics, Jilin University, Changchun, China.

1. The smoothing operator

Denote the centered difference operator by $\bar{A}_h$, defined by

$$\bar{A}_h f(x) := f(x + \frac{h}{2}) - f(x - \frac{h}{2}) .$$

For simplicity let $h = 1$, and $\bar{A} := \bar{A}_1$.

The B-spline of order k with equally spaced knots are denoted by N_k , and it can be represented by

$$N_k(x) = (\bar{A}D^{-1})^k \delta(x) , \quad (1.1)$$

$$N_{1,k}(\cdot) := N_k(\cdot-1) , \quad (1.2)$$

where D^{-1} is the integral operator, δ is Dirac δ -function.

It is our purpose to find a more exact difference approximation to the operator D . I would like to choose following ready-made identity.

Fact 1.1 ([5] p. 43)

$$\log(y + \sqrt{1 + y^2}) = \sqrt{1 + y^2} \sum_{m=0}^{\infty} (-1)^m \frac{2^{2m} (m!)^2}{(2m+1)!} y^{2m+1} . \quad (1.3)$$

Fact 1.2 The following expansion

$$x = \operatorname{sh} x \sum_{m=0}^{\infty} C_m (\operatorname{ch} x - 1)^m \quad (1.4)$$

holds. Set

$$(2m+1)!! := (2m+1)(2m-1)\dots 3.1 ,$$

then

$$C_m = -\frac{m}{2m+1} C_{m-1} = (-1)^m \frac{m!}{(2m+1)!!}, C_0 = 1 .$$

Proof From (1.3)

$$\begin{aligned}\log(y + \sqrt{1 + y^2}) &= \sqrt{1 + y^2} \sum_{m=0}^{\infty} (-1)^m \frac{m!}{(2m+1)!!} 2^m y^{2m+1} \\ &= \sqrt{1 + y^2} \sum_{m=0}^{\infty} C_m 2^m y^{2m+1}.\end{aligned}$$

Let $y = \operatorname{sh} \frac{x}{2}$, then $\operatorname{ch} \frac{x}{2} = \sqrt{1 + y^2}$, $x = 2 \log(y + \sqrt{1 + y^2})$. Thus

$$\begin{aligned}x &= 2 \operatorname{ch} \frac{x}{2} \sum_{m=0}^{\infty} C_m 2^m (\operatorname{sh} \frac{x}{2})^{2m+1} \\ &= 2 \operatorname{ch} \frac{x}{2} \operatorname{sh} \frac{x}{2} \sum_{m=0}^{\infty} C_m (2 \operatorname{sh}^2 \frac{x}{2})^m \\ &= \operatorname{sh} x \sum_{m=0}^{\infty} C_m (\operatorname{ch} x - 1)^m.\end{aligned}$$

Introduce operators E and μ_α defined by

$$E^\alpha f(x) := f(x + \alpha),$$

$$\mu_\alpha f(x) := \frac{1}{2} (f(x + \frac{\alpha}{2}) + f(x - \frac{\alpha}{2})), \quad \mu := \mu_1,$$

and notice the relationships between those operators (see [3], p. 230)

$$E = e^D, \quad \operatorname{ch} \frac{D}{2} = \mu,$$

$$2 \operatorname{sh} \frac{D}{2} = e^{\frac{D}{2}} - e^{-\frac{D}{2}} = E^{\frac{1}{2}} - D^{-\frac{1}{2}} = \bar{A}.$$

Use $\frac{D}{2}$ and I instead of x and 1 in (1.4)

$$\begin{aligned}
D &= 2 \operatorname{sh} \frac{D}{2} \sum_{m=0}^{\infty} C_m \left(\operatorname{ch} \frac{D}{2} - I \right)^m \\
&= \sum_{m=0}^{\infty} C_m (\mu - I)^m \bar{\Delta} \\
&= \sum_{m=0}^n C_m (\mu - I)^m \bar{\Delta} + R_n, \tag{1.5}
\end{aligned}$$

where

$$R_n := 2 \operatorname{sh} \frac{D}{2} \sum_{m=n+1}^{\infty} C_m \left(\operatorname{ch} \frac{D}{2} - I \right)^m. \tag{1.6}$$

Define $\bar{\Delta}^{(n)}$ as the first part of (1.5), i.e.,

$$\bar{\Delta}^{(n)} := \sum_{m=0}^n C_m (\mu - I)^m \bar{\Delta} = P_n(\mu) \bar{\Delta},$$

where

$$P_n(\mu) = \sum_{m=0}^n C_m (\mu - I)^m = \sum_{j=0}^n 2^{-j} \sum_{m=j}^n (-1)^{m-j} \binom{m}{j} C_m \sum_{i=0}^j \binom{j}{i} \frac{1}{2}^{j-i}. \tag{1.7}$$

In the general case, define

$$\bar{\Delta}_h^{(n)} := P_n(\mu_h) \bar{\Delta}_h. \tag{1.8}$$

This is a collection of operators approximate to D . Beyond doubt $P_n(1) = 1$,

$$P_0(\mu) = I.$$

Fact 1.3 If k is any nonnegative integer, then the sum of all coefficients of items $(\mu_h)^j$ in the expansion $(P_n(\mu_h))^k$ equals to 1.

Notice (1.6), the first term in R_n for any h

$$C_{n+1} D \left[\frac{1}{2!} \left(\frac{hD}{2} \right)^2 \right]^{n+1} = 2^{-3(n+1)} C_{n+1} h^{2(n+1)} D^{2n+3}. \tag{1.9}$$

This implies the following:

Theorem 1.1 Assume that $f \in C^{2n+3}$. Then

$$\bar{\Delta}_h^{(n)} f(x) = Df(x) - 2^{-3(n+1)} C_{n+1} f^{(2n+3)}(\xi) h^{2(n+1)}$$

where $\xi \in [x - \frac{n+1}{2} h, x + \frac{n+1}{2} h]$.

Definition We call the operator $(\bar{\Delta}_h^{(n)} D^{-1})^l$ a smoothing operator with grade n and degree l .

It is to be noted that $(\bar{\Delta}_h^{(0)} D^{-1})^k$ is just as with I. J. Schoenberg's.

Here it is the smoothing operator of grade 0 and degree k .

Fact 1.4 From Theorem 1.1, if $g \in P_{2n+1}$ on $[a, b]$, then

$$\bar{\Delta}_h^{(n)} D^{-1} g = g, \text{ all } x \in [a + \frac{n+1}{2} h, b - \frac{n+1}{2} h].$$

2. A class of many-knot splines

As has been already pointed out, the B-spline N_k with equally spaced knots ($h = 1$) is the result of the 0-th grade smoothing operator applied to the Dirac δ -function

$$N_k = (\bar{A}D^{-1})N_{k-1} = (\bar{A}D^{-1})^k \delta. \quad (2.1)$$

Now we use the smoothing operator $\bar{A}^{(n)}D^{-1}$ of grade n for the δ -function repeatedly. We can define a class of spline functions which as more knots than N_k :

$$\Omega_{k,n} := (P_n(u))^{\ell} N_k \quad (2.2)$$

and

$$\Omega_{i,k,n}(\cdot) := \Omega_{k,n}(\cdot - i).$$

If $\ell = k$, then $\Omega_{k,n}(x) = (\bar{A}^{(n)})^k \left\{ \frac{x_+^{k-1}}{(k-1)!} \right\}$ which has knots

$$\xi_j^{(n,k)} = -\frac{(n+1)k-j}{2}, \quad j = 0, 1, \dots, 2(n+1)k, \quad n > 0.$$

We often take $\ell = k$ if without note.

The following facts can be proved easily in the same way as the corresponding facts for N_k .

Fact 2.1:

- (1) $\Omega_{k,n}(x) = \Omega_{k,n}(-x)$;
- (2) $\Omega_{k,n}(x) = 0$ for all $|x| > \frac{(n+1)k}{2}$;
- (3) $D^m \Omega_{k,n}(x) = (\bar{A}^{(n)})^m \Omega_{k-m,n}(x)$, $0 < m < k$;
- (4) $D^{-m} \Omega_{k,n}(x) = (\bar{A}^{(n)})^{\ell} \left\{ \frac{x_+^{k+m-1}}{(k+m-1)!} \right\}$, $m > 0$;
- (5) $\sum_{j=-\infty}^{\infty} \Omega_{k,n}(x+j) = 1$, $\int_{-\infty}^{\infty} \Omega_{k,n}(x) dx = 1$;

(6) $\Omega_{k,n}$ can be represented by the convolution integral

$$\Omega_{k,n}(\cdot) = \int_{-\infty}^{\infty} \Omega_{k-1,n}(\cdot-t) \Omega_{0,n}(t) dt ;$$

$$(7) \quad \Omega_{k,n}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega_{k,n}(\xi) e^{i\xi x} d\xi$$

$$\begin{aligned} \Omega_{k,n}(\xi) &= \int_{-\infty}^{\infty} e^{-i\xi x} \Omega_{k,n}(x) dx \\ &= \left[\frac{\sin(\xi/2)}{\xi/2} P_n(\cos \frac{\xi}{2}) \right]^k \end{aligned}$$

(8) Integration by parts:

$$\int_{-\infty}^{\infty} \Omega_{k,n}(x) f(x) dx = (\bar{\Delta}^{(n)} D^{-1})^k f(0).$$

From the above mentioned facts we have the following theorems:

Theorem 2.1 Assume f is a continuous function or with discontinuity of the first kind on $[a,b]$, and is extended with period $b-a$ to $(-\infty, \infty)$, then

$$\lim_{h \rightarrow 0} \int_{-\infty}^{\infty} \delta_h(x-t) f(t) dt = \frac{1}{2} (f(x+0) + f(x-0)) ;$$

If f is a function whose derivatives of order l is continuous or is a discontinuity of the first kind on $[a,b]$, then

$$\lim_{h \rightarrow 0} \int_{-\infty}^{\infty} \frac{d^l}{dx^l} \delta_h(x-t) f(x) dt = \frac{1}{2} (f^{(l)}(x+0) + f^{(l)}(x-0)) ,$$

where

$$\delta_h(x) := \frac{1}{h} \Omega_{k,n}\left(\frac{x}{h}\right) .$$

This Theorem shows that the many-knot spline function δ_h converges weakly to the Dirac δ -function.

Theorem 2.2 Given the function f , define its many-knot spline smoothing function by

$$f_{k,n} := \bar{\Delta}_h^{(n)} D^{-1} f_{k-1,n} = (\bar{\Delta}_h^{(n)} D^{-1})^k f . \quad (2.3)$$

Then

$$f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n}\left(\frac{\cdot-t}{h}\right) f(t) dt . \quad (2.4)$$

Theorem 2.3 If $f \in C^l(-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} |f_{k,n}^{(l)}(x)|^2 dx \leq \int_{-\infty}^{\infty} |f^{(l)}(x)|^2 dx . \quad (2.5)$$

Proof Take the derivative of order l for (2.4), and the integration by parts, and notice that

$$\left| \frac{\sin x}{x} P_n(\cos x) \right| \leq 1 .$$

If f is a discrete valued function $y_1 = f(x_1)$, $x_1 = x_0 + ih$, then a numerical smoothing formula is as follows:

$$S_{k,n}^f := \sum_j y_j \Omega_{k,n}\left(\frac{\cdot - x_j}{h}\right) . \quad (2.6)$$

Formula (2.6) can be efficiently applied to the problems of curve fitting for discrete data.

3. Examples

In this section some discussions which are helpful for applications in practice will be given.

From (2.2), with $l = k$, $n = 1$, $k = 1, 2, 3, 4$, we show the particular representations as follows:

$$\Omega_{1,1}(x) = \begin{cases} \frac{7}{6}, & |x| < \frac{1}{2}, \\ \frac{1}{2}, & |x| = \frac{1}{2}, \\ -\frac{1}{6}, & \frac{1}{2} < |x| < 1, \\ -\frac{1}{12}, & |x| = 1, \\ 0, & |x| > 1; \end{cases}$$

$$\Omega_{2,1}(x) = \begin{cases} \frac{50}{36} - \frac{65}{36} |x|, & |x| < \frac{1}{2}, \\ \frac{42}{36} - \frac{49}{36} |x|, & \frac{1}{2} < |x| < 1, \\ -\frac{22}{36} + \frac{15}{36} |x|, & 1 < |x| < \frac{3}{2}, \\ \frac{2}{36} - \frac{1}{36} |x|, & \frac{3}{2} < |x| < 2, \\ 0, & |x| > 2; \end{cases}$$

$$\Omega_{3,1}(x) = \begin{cases} \frac{462}{432} - \frac{878}{432} x^2, & |x| < \frac{1}{2}, \\ \frac{858}{432} - \frac{1584}{432} |x| + \frac{706}{432} x^2, & \frac{1}{2} < |x| < 1, \\ \frac{471}{432} - \frac{810}{432} |x| + \frac{319}{432} x^2, & 1 < |x| < \frac{3}{2}, \\ -\frac{627}{432} + \frac{654}{432} |x| - \frac{169}{432} x^2, & \frac{3}{2} < |x| < 2, \\ \frac{141}{432} - \frac{114}{432} |x| + \frac{23}{432} x^2, & 2 < |x| < \frac{5}{2}, \\ -\frac{9}{432} + \frac{6}{432} |x| - \frac{1}{432} x^2, & \frac{5}{2} < |x| < 3, \\ 0, & |x| > 3, \end{cases}$$

$$\Omega_{4,1}(x) = \begin{cases} \frac{7920}{7776} - \frac{20556}{7776} x^2 + \frac{13059}{7776} |x|^3, & |x| < \frac{1}{2}, \\ \frac{8444}{7776} - \frac{3144}{7776} |x| - \frac{14268}{7776} x^2 + \frac{8867}{7776} |x|^3, & \frac{1}{2} < |x| < 1, \\ \frac{25212}{7776} - \frac{53448}{7776} |x| + \frac{36036}{7776} x^2 - \frac{7901}{7776} |x|^3, & 1 < |x| < \frac{3}{2}, \\ \frac{4152}{7776} - \frac{11328}{7776} |x| + \frac{7956}{7776} x^2 - \frac{1661}{7776} |x|^3, & \frac{3}{2} < |x| < 2, \\ -\frac{22440}{7776} + \frac{28560}{7776} |x| - \frac{11988}{7776} x^2 + \frac{1663}{7776} |x|^3, & 2 < |x| < \frac{5}{2}, \\ \frac{9060}{7776} - \frac{9240}{7776} |x| + \frac{3132}{7776} x^2 - \frac{353}{7776} |x|^3, & \frac{5}{2} < |x| < 3, \\ -\frac{1308}{7776} + \frac{1128}{7776} |x| - \frac{324}{7776} x^2 + \frac{31}{7776} |x|^3, & 3 < |x| < \frac{7}{2}, \\ \frac{64}{7776} - \frac{48}{7776} |x| + \frac{12}{7776} x^2 - \frac{1}{7776} |x|^3, & \frac{7}{2} < |x| < 4, \\ 0, & |x| > 4. \end{cases}$$

From (2.2) the following tables are given:

Table 1:

x	$N_1(x)$	$\Omega_{1,1}(x)$
0	1	$\frac{7}{6}$
$\pm \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
± 1		$-\frac{1}{12}$

Table 2:

0	$N_2(x)$	$\Omega_{2,1}(x)$	
		$\ell = 1$	$\ell = 2$
0	1	$\frac{7}{6}$	$\frac{100}{72}$
$\pm \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{35}{72}$
± 1		$-\frac{1}{12}$	$-\frac{14}{72}$
$\pm \frac{3}{2}$			$\frac{1}{72}$

Table 3

x	$N_3(x)$	$\Omega_{3,1}(x)$			$\Omega'_{3,1}(x)$
		$\ell = 1$	$\ell = 2$	$\ell = 3$	
0	$\frac{3}{4}$	$\frac{40}{48}$	$\frac{270}{288}$	$\frac{1848}{1728}$	0
$\pm \frac{1}{2}$	$\frac{1}{2}$	$\frac{25}{48}$	$\frac{156}{288}$	$\frac{970}{1728}$	$-\frac{878}{432}$
± 1	$\frac{1}{8}$	$\frac{4}{48}$	$\frac{8}{288}$	$-\frac{80}{1728}$	$+\frac{172}{432}$
$\pm \frac{3}{2}$		$-\frac{1}{48}$	$-\frac{12}{288}$	$-\frac{105}{1728}$	$\pm \frac{147}{432}$
± 2			$\frac{1}{288}$	$\frac{20}{1728}$	$-\frac{22}{432}$
$\pm \frac{5}{2}$				$-\frac{1}{1728}$	$\pm \frac{1}{432}$

Table 4

x	$N_4(x)$	$\Omega_{4,1}(x)$				$\Omega'_{4,1}(x)$	$\Omega''_{4,1}(x)$
		$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 4$	
0	$\frac{2}{3}$	$\frac{210}{288}$	$\frac{1392}{1728}$	$\frac{9332}{10368}$	$\frac{63360}{62208}$	0	$-\frac{13704}{2592}$
$\pm \frac{1}{2}$	$\frac{23}{48}$	$\frac{144}{288}$	$\frac{902}{1728}$	$\frac{5647}{10368}$	$\frac{35307}{62208}$	$-\frac{14349}{10368}$	$-\frac{645}{2592}$
± 1	$\frac{1}{6}$	$\frac{40}{288}$	$\frac{176}{1728}$	$\frac{545}{10368}$	$-\frac{808}{62208}$	$+\frac{6772}{10368}$	$\frac{8222}{2592}$
$\pm \frac{3}{2}$	$\frac{1}{48}$	0	$-\frac{39}{1728}$	$-\frac{480}{10368}$	$-\frac{4359}{62208}$	$+\frac{1771}{10368}$	$\frac{321}{2592}$
± 2		$-\frac{1}{288}$	$-\frac{8}{1728}$	$-\frac{26}{10368}$	$\frac{256}{62208}$	$\pm \frac{752}{10368}$	$-\frac{1340}{2592}$
± 5			$\frac{1}{1728}$	$\frac{16}{10368}$	$\frac{155}{62208}$	$+\frac{265}{10368}$	$\frac{323}{2592}$
± 3				$\frac{1}{10368}$	$-\frac{24}{62208}$	$\pm \frac{28}{10368}$	$-\frac{30}{2592}$
$\pm \frac{7}{2}$					$\frac{1}{62208}$	$-\frac{1}{10368}$	$\frac{1}{2592}$

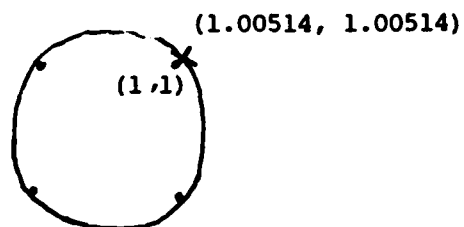
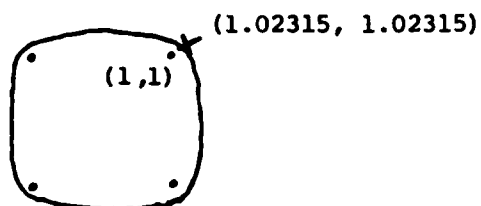
From (2.6), set $n = 1$, $\ell = k$, $k = 3, 4$. We obtain

$$s_{3,1}f(x_1) = y_1 + \frac{5}{432} \bar{\Delta}^4 y_1,$$

$$s_{4,1}f(x_1) = y_1 + \frac{4}{7776} \bar{\Delta}^4 y_1 - \frac{3}{7776} \bar{\Delta}^6 y_1.$$

Assume four points in the plane are given:

$(0,0), (1,0), (1,1), (0,1)$.



t	$s_{3,1}^f$		$s_{4,1}^f$	
	x(t)	y(t)	x(t)	y(t)
1.6	0.65648	1.11870	0.61804	1.13213
1.8	0.89167	1.08685	0.83711	1.09813
2.0	1.02315	1.02315	1.00514	1.00514
2.2	1.08685	0.89167	1.09813	0.83711
2.4	1.11870	0.65648	1.13213	0.61804

Acknowledgement

I would like to thank Professor Carl de Boor for reading this paper and for his very valuable suggestions.

REFERENCES

- [1] R. E. Barnhill and R. F. Riesenfeld, Computer Aided Geometric Design, Academic Press, New York-San Francisco-London, 1974.
- [2] C. de Boor, On calculating with B-splines, J. Approximation Theory, 6 (1972), 50-62.
- [3] F. B. Hildebrand, Methods of Applied Mathematics, Prentice Hall, Inc., New York, 1952.
- [4] Z. S. Liang and D. X. Qi, On the smoothing method by many-knot spline function, J. of Num. Math. of Universities of China, 2 (1979), 196-209.
- [5] I. M. Ryshik and I. S. Gradstein, Tables of series, products, and integrals, (Translation from the Russian), 1963.
- [6] I. J. Schoenberg, Contributions to the problem of approximation of equidistant data by analytic functions, Quart. Appl. Math. 4 (1946), 45-99 and 112-141.

DXQ/jvs

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #2242	2. GOVT ACCESSION NO. AD-A103	3. RECIPIENT'S CATALOG NUMBER 855
4. TITLE (and Subtitle) A New Way for Constructing Higher Order Accuracy Spline Smoothing Formulas		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Dong-Xu Qi		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of 610 Walnut Street Wisconsin Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 3 - Numerical Analysis and Computer Science
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE July 1981
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 13
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Spline, smoothing, many-knot, Higher order accuracy		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this paper the author introduces the operator $\bar{\Delta}^{(n)} := P_n(\mu)\bar{\Delta}$ with higher order accuracy for approximation to the differential operator D , where $\bar{\Delta}$ de- notes centered difference operator, μ denotes averaging operator, $P_n(\mu) = \sum_{m=0}^n C_m(\mu-I)^m, C_m = -\frac{m}{2m+1} C_{m-1}, C_0 = 1.$ A class of new many-knot spline basis $\Omega_{k,n} := (P_n(\mu))^k N_k$ was suggested. The		

ABSTRACT (continued)

smoothing formulas

$$f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n} \left(\frac{\cdot - t}{h} \right) f(t) dt \quad \text{and} \quad s_{k,n} f = \sum f_i \Omega_{k,n}$$

are discussed.