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DETERMINATION OF THE SPECTRA OF MODULATED PULSE TRAINS BY THE SPECTRAL FUNCTION METHOD

(NAKHOZHDENIE SPEKTROV MODULIROVANNYKH POSLEDOVATEL'NOSTI IMPUL'SOV METODOM SPEKTRAL'NYKH FUNKTSII)

by

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AUTHORS' SUMMARY

A means is explained for determining the spectra of modulated pulse trains using the spectral function method. By this method a spectrum of signals is determined with a ChIM-2 frequency pulse modulation for a rectangular train of pulses.

A means is explained for determining the spectra of modulated pulse trains using the spectral function method. By this method a spectrum of signals is determined with a ChIM-2 frequency pulse modulation for a rectangular train of pulses.

By using the spectral function method[1], it is possible to determine the spectrum of a modulated pulse oscillation as the reaction at the output of a linear quadripole during the action on its input of the train of frequency modulated δ pulses, the reaction on each of which is equal to $h(t)$.

Or, using the packet theorem

$$F(t) = h(t)*f\delta(t) . \qquad (1)$$

The function $F(t) = \sum_{k=-\infty}^{\infty} f(t - t_k)$ describes the modulated pulse train of the given form, while

$f\delta(t) = \sum_{k=-\infty}^{\infty} \delta(t - t_k)$ describes the train of modulated δ pulses at the input of the quadripole.

$h(t)$ represents the pulse transient characteristic of the quadripole, the form of whose spectral function depends solely on the form of the pulse.

The direct Fourier transform from (1)

$$F(\omega) = f\delta(\omega) . K(\omega)$$

where $f\delta(\omega)$ is the spectral function of the input effect of the quadripole, while $K(\omega)$ is the transmission coefficient of the quadripole.

Consequently, in order to find the spectrum of the train of modulated pulses it is necessary to find the spectrum of the train of modulated δ pulses at the input of the quadripole; to determine the transmission coefficient of the quadripole; to find the spectrum of the output effect of the quadripole; and by using the packet theorem, to find the pulse train spectrum.

As an example let us find the spectrum of the periodic train of square pulses, frequency modulated as to sequence by a sinusoidal signal

$$a(t) = A_0 . \sin(\Omega t + \varphi_0) ,$$

by the ChIM-2 method (determination of the modulation according to (2).

For the sake of simplicity let us take $\varphi_0 = 0$.

If the periodic pulse train

$$F_0(t) = \sum_{k=-\infty}^{\infty} f_0(t - kT_0) ,$$

where $T_0 = 2\pi/\omega_0$ is frequency modulated as to sequence by the signal $a(t) = A_0 \cdot \sin\Omega t$ by the ChIM-2 method, then it will be also modulated according to time position, at the same time the deflection of the kth pulse is determined by the expression (2)

$$\Delta t_k = -\Delta t_{max} \cdot \cos(kT_0 + \Delta t_k) ,$$

$$\Delta t_{max} = \frac{bA_0}{\Omega} = \frac{\Delta\omega}{\omega_0\Omega} = \frac{1}{\omega_0}\beta ,$$

where b is the coefficient of proportionality; $\Delta\omega$ is the maximum frequency deviation; β is the index of modulation.

The train of the modulated pulses is written as

$$F(t) = \sum_{k=-\infty}^{\infty} f(t - t_k) = \sum_{k=-\infty}^{\infty} f(t - kT_0 - \Delta t_k) \qquad (2)$$

or, passing to δ pulses

$$F\delta(t) = \sum_{k=-\infty}^{\infty} \delta(t - t_k) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0 - \Delta t_k) .$$

It is well known that with increase of the depth of pulse modulation the time interval $t_{k+1} - t_k$ decreases and may even be transformed into zero or become negative. At the same time the order of the sequence of the pulses changes, since the $k + 1$ pulse will appear earlier than the kth pulse. For this not to occur, it is shown in (3) that for ChIM-2 it is necessary to have a value of the modulating function such that there is one solution for each value k .

In the case of a sinusoidal modulating signal this is determined unambiguously, *ie* $t_{k+1} > t_k$, therefore the order of the sequence of the pulses is kept normal.

On the basis of what has been mentioned, we utilise the properties of the generalised functions for the transformation of expression (2).

It is well known from the theory of generalised functions (4), that if $f(t)$ is an infinitely differentiable function having an arbitrary number of simple roots, then $\delta[f(t)]$ is determined by the formula

$$\delta[f(t)] = \sum^{k} \frac{\delta(t - t_k)}{|f'(t_k)|} ,$$

where the summation is performed for all roots of equation $f(t) = 0$.

For our case we have: $f(t) = 0$ when $t - t_k - kT_0 + \Delta t_k$.

Assuming that there are no other zeros, we shall have:

$$\delta(t - kT_0 - \Delta t) = \frac{\delta(t - t_k)}{|f'(t)|} \quad .$$

Since $f'(t) = 1 - \Delta t'$ and, dropping the absolute value on condition that $\Delta t' < 1$, we obtain

$$\delta(t - t_k) = (1 - \Delta t') \,.\, \delta(t - kT_0 - \Delta t) \,. \tag{3}$$

Substituting (3) in (2) we obtain an expression for the modulated periodic train of the δ pulses:

$$F_0(t) = (1 - \Delta t') \sum_{k=-\infty}^{\infty} \delta(t - kT_0 - \Delta t) \quad . \tag{4}$$

Denoting the non-modulated periodic train of the δ pulses by

$$F_0\delta(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0) \ ,$$

we write (4) in the form

$$F\delta(t) = (1 - \Delta t') \,.\, F\delta(t - \Delta t) \quad . \tag{5}$$

The function $F\delta(t - \Delta t)$ is periodic, since its value does not change when replacing t by $t + 2\pi/\omega_0$, therefore it can be expanded into a Fourier series:

$$F\delta(t - \Delta t) = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} e^{im\omega_0(t-\Delta t)} \quad . \tag{6}$$

Substituting (6) in (5) we shall finally obtain

$$F\delta(t) = \frac{1}{T_0} (1 - \Delta t') \sum_{m=-\infty}^{\infty} e^{im\omega_0(t-\Delta t)} \quad . \tag{7}$$

For a sinusoidal modulated signal $\Delta t = - \Delta t_{max} \,.\, \cos \Omega t$ and the expression (7), is written in the following form:

$$F\delta(t) = \frac{1}{T_0}(1 + \Omega\Delta t_{max} \,.\, \sin \Omega t) \sum_{m=-\infty}^{\infty} e^{im\omega_0 t} e^{tm\omega_0 \Delta t_{max} \cos \Omega t}$$

$$= \frac{1}{T_0}\left[1 + \frac{\Omega\Delta t_{max}}{2}\left(\frac{e^{i\Omega t} - e^{-i\Omega t}}{i}\right)\right] \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} i^n I_n(m\omega_0\Delta t_{max}) e^{i(m\omega_0 + n\Omega)t}$$

$$= \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[i^n I_n(m\omega_0\Delta t_{max}) + i^{n-1} I_{n-1}(m\omega_0\Delta t_{max}) + i^{n+1} \times\right.$$

$$\left. \times I_{n+1}(m\omega_0\Delta t_{max})\right] \rightarrow e^{i(m\omega_0 + n\Omega)t} = \frac{1}{2\pi} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{n=-\infty}^{\infty} i^n I_n(m\beta) \frac{m\omega_0 + n\Omega}{m} e^{i(m\omega_0 + n\Omega)t} .$$

The Fourier transform from $F\delta(t)$, which is the spectral function of the input action, is defined as

$$F\delta(\omega) = \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{n=-\infty}^{\infty} i^n I_n(m\beta) \frac{m\omega_0 + n\Omega}{m} \delta(\omega - m\omega_0 - n\Omega) .$$

Taking the transmission coefficient of the quadripole

$$K(\omega) = \frac{2E}{\omega} \,.\, \sin \omega \frac{\tau_0}{2} ,$$

where E is the amplitude, and τ_0 the duration of the pulse and using the theorem of transformation of the packet, we find the train spectrum for the rectangular pulses modulated by the ChIM-2 method

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F\delta(\omega) \,.\, K(\omega) \,.\, e^{i\omega t} d\omega = \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{n=-\infty}^{\infty} i^n I_n \frac{E}{m\pi} I_n(m\beta) \rightarrow$$

$$\rightarrow \sin(m\omega_0 + n\Omega) \frac{\tau_0}{2} e^{i(m\omega_0 + n\Omega)t} . \qquad (8)$$

The 'filtering action' of the δ function was used in determining (8).

In order to demonstrate the characteristic components of the spectrum, it is required to expand expression (8), assuming the value $m = 0$ and taking the real part.

If the form of the pulse is selected arbitrarily with a spectral density $K_1(\omega)$ and amplitude E_1, then clearly the expression for the frequency spectrum having ChIM-2, will assume the form:

$$F_1(t) = \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{n=-\infty}^{\infty} i^n \frac{E_1}{m\pi} (m\omega_0 + n\Omega) I_n(m\beta) \cdot K_1(m\omega_0 + n\Omega) \cdot e^{i(m\omega_0 + n\Omega)t} .$$

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