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A TUTORIAL FOR USING THE MONTE CARLO
METHOD IN VEHICLE BALLISTIC
VULNERABILITY CALCULATIONS.

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William Beverly

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
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compared with their known closed form solutions.

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I. INTRODUCTION

An ongoing effort is being conducted at the US Army Armament Research and Development Command, Ballistic Research Laboratory (USA ARRADCOM/BRL) to improve the methodologies that are used to calculate the vulnerability of military vehicles to ballistic threats. The Internal Point Burst model¹ whose distinguishing feature is its ability to separate the debilitating effects of spall from those of the main penetrator constitutes the most advanced methodology being developed at the BRL and elsewhere. In such a methodology, the vulnerability of a target vehicle is calculated as the expected value of vehicle incapacitation due to the direct impacts of primary penetrators and any associated metal debris fragments.

In a simplified version of a more detailed stochastic representation of the point-burst vulnerability process^{2,3}, the probability equations defining the expected value of incapacitation per projectile (either primary penetrator or secondary metal-debris fragment) for a vehicle component exposed to a ballistic threat are formulated as definite integrals whose basic form is given by

$$\lambda = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0) f(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1 | \mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0, \mathbf{g}) Q(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1) d\mathbf{r}_1 d\mathbf{w}_1 d\mathbf{b}_1 d\mathbf{r}_0 d\mathbf{w}_0 d\mathbf{b}_0, \quad (1A)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0) d\mathbf{r}_0 d\mathbf{w}_0 d\mathbf{b}_0 = 1. \quad (1B)$$

In this expression, the boldface letters $(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0)$ and $(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1)$ are the values associated with the random variables (R_0, W_0, B_0) and (R_1, W_1, B_1) , respectively, where R quantifies the location and W quantifies the direction of motion of a projectile. Correspondingly, the set of variables B is used to quantify (characterize) some, but not all, of the remaining significant features of a projectile. The subscript 0 is used to identify the variables associated with a penetrator at its origin and the subscript 1 is used to identify the variables associated with a penetrator at impact with a critical component (Figure 1) in the vehicle. The quantities $d\mathbf{r}$, $d\mathbf{w}$, and $d\mathbf{b}$ are the infinitesimal hypervolumes associated with $\mathbf{r}$, $\mathbf{w}$, and $\mathbf{b}$, respectively.

The quantity $S(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0)$ is a continuous function used to give the probability density that the random variables associated with some projectile at its source will have the values $(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0)$. The function $f(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1 | \mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0, \mathbf{g})$ is the unnormalized conditional probability density that a projectile created as $(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0)$ will perforate a barrier characterized by the set of random variables G having a set of values $\mathbf{g}$ and impact the critical component as $(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1)$. This latter function differs from the conventional definition of a probability density function (PDF) in that it does not normalize to unity, but has a normalization which for each set of values $(\mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0, \mathbf{g})$ lies on the range from 0 to 1, that is

$$0 \leq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\mathbf{r}_1, \mathbf{w}_1, \mathbf{b}_1 | \mathbf{r}_0, \mathbf{w}_0, \mathbf{b}_0, \mathbf{g}) d\mathbf{r}_1 d\mathbf{w}_1 d\mathbf{b}_1 \leq 1. \quad (1C)$$

This quantity, herein identified as the perforation PDF, is continuous and finite in $\mathbf{b}_1$ for those

¹J.R. Rapp, and F.T. Brown, "An Assessment of Existing Point-Burst Models of Armored Vehicle Vulnerability," Ballistic Research Laboratory Memorandum Report No. 02963, October 1979, (UNCLASSIFIED). (AD #B043965L)

²W.B. Beverly, "A Detailed Stochastic Ballistic Vulnerability Model for Armored Military Vehicles," Journal of Ballistics, Vol. III, No. 3, 1979.

³W.B. Beverly, "A Stochastic Representation of the Vulnerability of a Critical Component in a Military Vehicle to Metal Fragments," Submitted to the Journal of Ballistics for Publication.

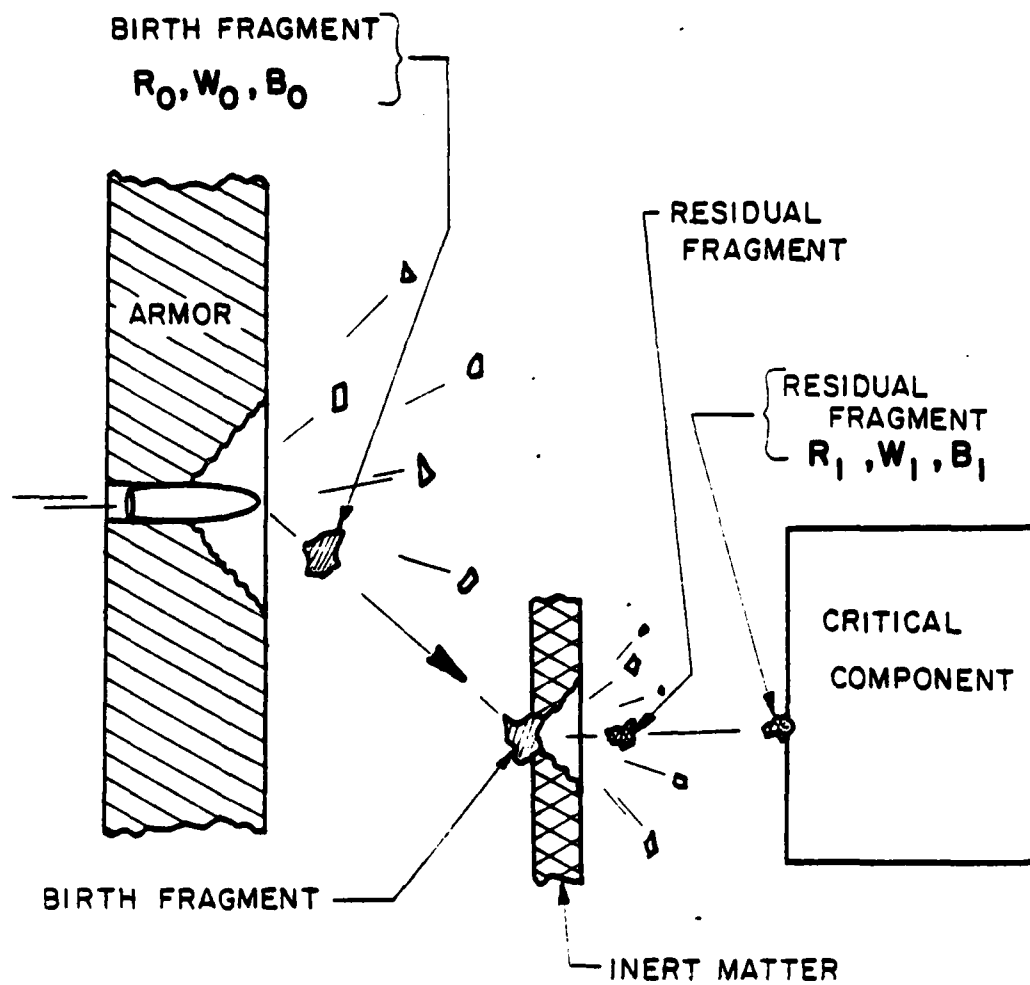


Figure 1. The Mathematical Notation in a Simple Spall Example

cases where a birth projectile must perforate a barrier before impacting a critical component in the vehicle, and is the product of Dirac delta functionals⁴ in b_1 for those cases where the projectile can travel unimpeded to impact the critical component. In either case, the perforation PDF is Dirac delta functionals in r_1 for projectiles impacting the critical component and is identically zero over its total range for projectiles which either fail to perforate the barrier or impact the critical component. The function $Q(r_1, w_1, b_1)$ is any well-behaved function whose values on the impacted surface of a critical component give the average incapacitation caused by all penetrators whose "used" random variables have the values (r_1, w_1, b_1) . In these integrals, $S(r_0, w_0, b_0)$ is assumed to vanish at large distances from its maximum value so that an integration over all values of all variables (identified by placing the infinity symbol at the bottom of the integral sign) will yield a finite expected value λ for the incapacitation.

The dimensionality of these integrals and the complexity of the geometrical and compositional features of the target vehicles from which these integrals are derived could make their evaluation by deterministic methods prohibitively expensive. An alternate Monte Carlo method has been outlined by Beverly⁵ which could greatly increase the efficiency of vulnerability calculations. The objective of this study is to analyze the Monte Carlo procedures used in Reference 5 and to illustrate their use by evaluating simple definite integrals.

In the next section, we will initially analyze the Monte Carlo evaluation of simple one-dimensional integrals. We will then extend the procedures developed for the one-dimensional case to multiple integrals. We will also analyze integrals having the form illustrated in equation 1A. Then, in Section III, simple integrals will be constructed which have the form of the integrals discussed in Section II. These integrals will be solved using the outlined Monte Carlo procedures and the results will be compared with the known closed form solutions.

II. THE EVALUATION OF DEFINITE INTEGRALS BY USING THE MONTE CARLO METHOD

A. A Simple One-Dimensional Integral

A simple one-dimensional integral having the form

$$\lambda = \int_{x_1}^{x_2} H(x)G(x) dx, \quad (2)$$

where $H(x)$ and $G(x)$ are well-behaved functions on the interval from x_1 to x_2 , is generally regarded as the area under the curve $H(x)G(x)$ from x_1 to x_2 . However, the integral can be viewed from a different perspective if the integrand is rearranged to obtain

$$\lambda = C \int_{x_1}^{x_2} \frac{H(x)G(x)}{C} dx = C \int_{x_1}^{x_2} H(x)f(x) dx, \quad (3A)$$

where

$$C = \int_{x_1}^{x_2} G(x) dx, \quad (3B)$$

⁴C. Eisenhart, and M. Zelen, "Elements of Probability," Handbook of Physics, E.U. Condon, and H. Odishaw, Editors, McGraw-Hill Book Company, Inc., New York, 1958.

⁵W.B. Beverly, "The Application of the Monte Carlo Method to the Solution of the Internal Point Burst Vehicle Ballistic Vulnerability Model," Ballistic Research Laboratory Technical Report in Preparation.

and

$$f(x) = \frac{G(x)}{C}, \quad (3C)$$

when

$$x_1 \leq x \leq x_2, \quad (3D)$$

and

$$f(x) = 0, \quad (3E)$$

when

$$x < x_1, \quad x > x_2. \quad (3H)$$

The integral can now be regarded as C times the expected value of $H(x)$ where the probability density of values for x is given by the PDF $f(x)$ (Reference 4).

Viewing the integral from the second perspective and applying the strong law of large numbers (Reference 4), the integral can be estimated as

$$\bar{\lambda} = \frac{1}{J} \sum_{j=1}^J C H(x_j), \quad (4A)$$

where the x_j are a series of mutually independent values for x whose common PDF is $f(x)$. This mean value estimate is identified by attaching a bar to λ . A measure of the confidence level of an estimate is taken to be its standard deviation $\delta\bar{\lambda}$, that is⁶

$$\delta\bar{\lambda} = \left[\frac{\sum_{j=1}^J C^2 H^2(x_j) - J\bar{\lambda}^2}{J(J-1)} \right]^{\frac{1}{2}}. \quad (4B)$$

According to the central limit theorem (Reference 4), this measure of confidence can be interpreted as predicting that approximately 68 percent of a large number of similar estimates of λ will fall within $\pm\delta\bar{\lambda}$ of λ .

A step-by-step outline of a Monte Carlo evaluation of λ is given below and illustrated in Figure 2.

1. Pick a value x_j by sampling the PDF $f(x)$. A variety of methods have been developed for conducting such sampling^{7,8}. An efficient method for the case where $f(x)$ is given in histogram form is to pick each x_j by solving the integral equation⁹

$$\int_{x_1}^{x_j} f(x) dx = [RN(0,1)]_j. \quad (5)$$

The quantities $RN[0,1]_j$ are a set of independent random numbers where each random number is picked with equal probability on the range of 0 to 1. The reader should note that the

⁶Y. Beers, "Introduction to the Theory of Error," Addison-Wesley Publishing Company, Inc., Reading, Mass., 1962.

⁷W. Guber, R. Nagel, R. Goldstein, P.S. Mittelman, and M.H. Kalos, "A Geometric Description Technique for Computer Analysis of Both the Nuclear and Conventional Vulnerability of Armored Military Vehicles," Mathematical Applications Group, Inc. Report No. MAGI-6701, August 1967.

⁸E.D. Cashwell, C.J. Everett, and O.W. Rechar, "A Practical Manual on the Monte Carlo Method for Random Walk Problems," Los Alamos Scientific Laboratory Report No. LA-2120, December 1957.

⁹Y.A. Schneider, "The Monte Carlo Method," Pergamon Press, Long Island City, New York, 1966.

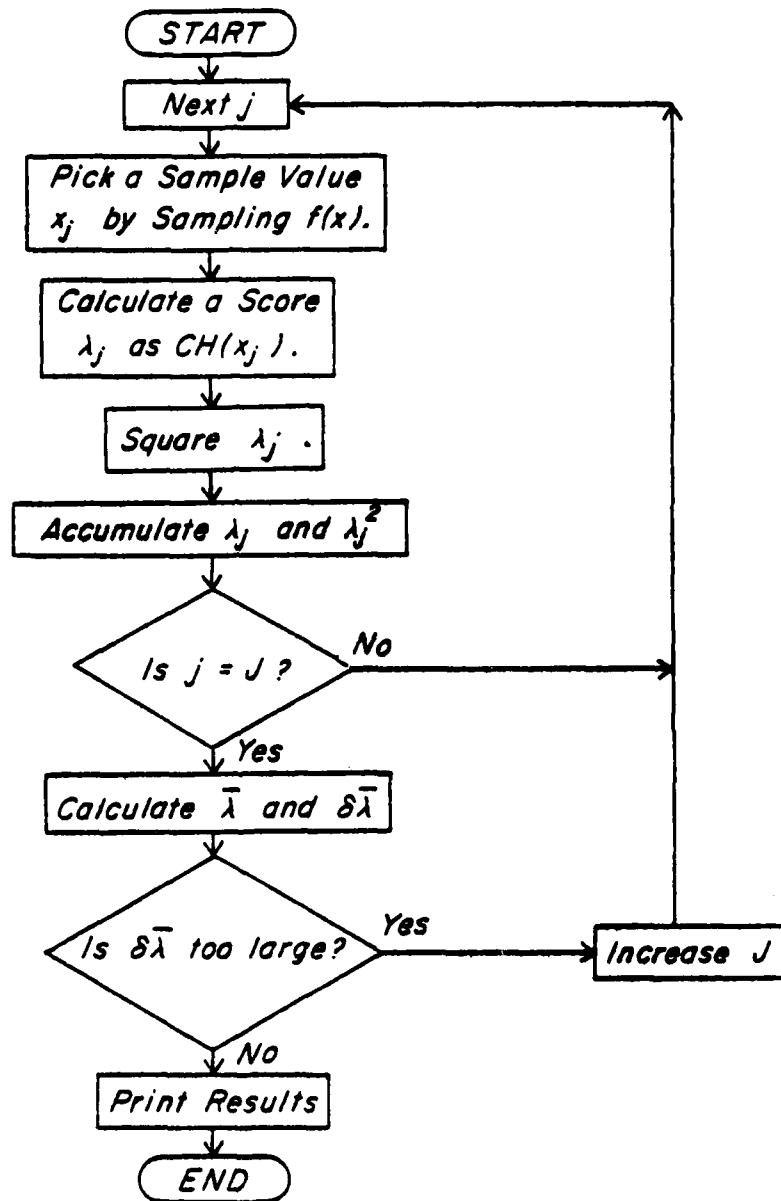


Figure 2. The Monte Carlo Estimation of a Simple One-Dimensional Integral

sampling as described by equation 5 generally requires that the PDF be normalized to unity. We will assume that this method is used in the following outlines of Monte Carlo procedures.

2. Calculate the event score λ_j as $CH(x_j)$.
3. Accumulate the score in a bin reserved for this operation.
4. Calculate the square of the preceding score.
5. Accumulate the squared score in another bin reserved for this operation.
6. Reiterate steps 1-5 for a total of J events.
7. Calculate $\bar{\lambda}$ using equation 4A.
8. Calculate the standard deviation $\delta\bar{\lambda}$ using equation 4B.
9. Determine if the confidence level of $\bar{\lambda}$ as determined by $\delta\bar{\lambda}$ is adequate. Conduct more events and merge their results with those already obtained if $\delta\bar{\lambda}$ is too large.

Step 9 completes an outline of a Monte Carlo evaluation of a one-dimensional integral. The procedures outlined above will be extended in the following Subsection to the evaluation of multiple integrals.

B. Multiple Integrals

A multiple integral of dimensionality I which has the form

$$\lambda = \int_{A_1}^{B_1} \cdots \int_{A_I}^{B_I} H(x_1, \dots, x_I) \cdot G(x_1, \dots, x_I) dx_1 \cdots dx_I, \quad (6A)$$

can be evaluated by reiterating the procedures outlined in Section IIA for evaluating one-dimensional integrals. The integrations are assumed to be started at the innermost integral and conducted toward the outermost integral where I is the running index over the variables. The A_i and B_i define the boundaries of the integration region accordingly, that is

$$\begin{array}{ll} A_1 = \text{constant}, & B_1 = \text{constant}; \\ A_2 = A_2(x_1), & B_2 = B_2(x_1); \\ \vdots & \vdots \\ A_i = A_i(x_1, \dots, x_{i-1}), & B_i = B_i(x_1, \dots, x_{i-1}); \\ \vdots & \vdots \\ A_I = A_I(x_1, \dots, x_i, \dots, x_{I-1}), & B_I = B_I(x_1, \dots, x_i, \dots, x_{I-1}). \end{array} \quad (6B)$$

We will first rearrange the integrand in equation 6A to obtain

$$\lambda = C_1 \int_{A_1} \cdots \int_{A_i} \cdots \int_{A_I} H(x_1, \dots, x_i, \dots, x_I) \cdot f(x_1, \dots, x_i, \dots, x_I) dx_1 \cdots dx_i \cdots dx_I, \quad (7A)$$

where

$$C_1 = \int_{A_1}^{B_1} \cdots \int_{A_i}^{B_i} \cdots \int_{A_I}^{B_I} G(x_1, \dots, x_i, \dots, x_I) dx_1 \cdots dx_i \cdots dx_I, \quad (7B)$$

and

$$f(x_1, \dots, x_i, \dots, x_I) = \frac{G(x_1, \dots, x_i, \dots, x_I)}{C_1}, \quad (7C)$$

when the x_i have values located within the region bounded by the A_i and B_i , and

$$f(x_1, \dots, x_i, \dots, x_I) = 0, \quad (7D)$$

when the x_i have values which lie outside the region bounded by the A_i and B_i . Then, using the same perspective as that used in Subsection IIA, the multiple integral of equation 7B can be interpreted as the expected value of $H(x_1, \dots, x_i, \dots, x_I)$ where the variables x_i have values predicted by the joint PDF $f(x_1, \dots, x_i, \dots, x_I)$.

However, in a Monte Carlo estimation of λ , a sample value $(x_i)_j$ for each x_i during trial j has to be picked individually and in its turn. The probability density of values for x_1 is the marginal PDF $f_1(x_1)$ which is obtained by the integration

$$f_1(x_1) = \int_{A_2}^{B_2} \cdots \int_{A_i}^{B_i} \cdots \int_{A_I}^{B_I} f(x_1, \dots, x_i, \dots, x_I) dx_2 \cdots dx_i \cdots dx_I. \quad (8A)$$

Now, similar to the procedure used in Section IIA, a sample value $(x_1)_j$ can be derived from the integral equation

$$\int_{A_1}^{(x_1)_j} f_1(x_1) dx_1 = RN[0,1]_{1j}. \quad (8B)$$

Continuing, a marginal PDF $f_2[(x_1)_j, x_2]$ is then constructed for the variable x_2 as

$$f_2[(x_1)_j, x_2] = \frac{1}{C_2} \int_{A_3}^{B_3} \cdots \int_{A_i}^{B_i} \cdots \int_{A_I}^{B_I} f[(x_1)_j, x_2, \dots, x_i, \dots, x_I] dx_3 \cdots dx_i \cdots dx_I, \quad (8C)$$

where

$$C_2 = \int_{A_2}^{B_2} \cdots \int_{A_i}^{B_i} \cdots \int_{A_I}^{B_I} f[(x_1)_j, x_2, \dots, x_i, \dots, x_I] dx_2 \cdots dx_i \cdots dx_I. \quad (8D)$$

Similarly, a sample value $(x_2)_j$ is derived for the variable x_i from the integral equation

$$\int_{A_2}^{(x_2)_j} f_2[(x_1)_j, x_2] dx_2 = RN[0,1]_{2j}, \quad (8E)$$

where A_2 now has the constant value

$$A_2 = A_2[(x_1)_j]. \quad (8F)$$

This procedure is continued until a sample value has been picked for each variable x_i . A score λ_j is then calculated for the event as

$$\lambda_j = C_1 H[(x_1)_j, \dots, (x_i)_j, \dots, (x_I)_j]. \quad (9)$$

The remaining procedure is identical to that already outlined in Subsection IIA for evaluating one-dimensional integrals. This procedure is outlined below for multiple integrals and illustrated in Figure 3.

1. Pick a set of sample values $[(x_i)_j]$ for the variables x_i . These operations are performed in the following steps:

A. Construct the marginal PDF $f_1(x_1)$ by using equation 8A.

B. Pick a sample value $(x_1)_j$ by sampling $f_1(x_1)$ (equation 8B).

C. Construct the marginal PDF $f_2(x_2)$ by using equations 8C and 8D.

D. Pick a sample value $(x_2)_j$ by sampling $f_2(x_2)$ (equation 8E).

E. In a similar manner, pick a sample value $(x_i)_j$ for each remaining x_i by using the appropriate marginal PDF $f_i(x_i)$. The reader should note that the marginal PDF in each case is derived from a joint distribution of dimensionality decremented by one from the preceding sampling event.

2. Calculate the event score λ_j as $C_1 H[(x_1)_j, \dots, (x_i)_j, \dots, (x_I)_j]$.

3-9. Calculate $\bar{\lambda}$ and $\delta\bar{\lambda}$ as described earlier in this section. These steps are identical to those already described in Subsection IIA for a one-dimensional integral.

Step 9 completes the outline of a Monte Carlo evaluation of multiple integrals. The procedures outlined in Subsections IIA and IIB will be applied in the following subsection to evaluate a multiple integral having the form of the vulnerability integral illustrated in equation 1.

C. The Vulnerability Integral

The vulnerability integral of equation 1A can often be evaluated by more expeditious calculations than those usually needed to evaluate multiple integrals of equivalent dimensionality. The division of the integrand into a source term, a perforation PDF, and a critical component incapacitation function, where the source term is a function only of penetrator birth variables, will simplify the picking of sample states for penetrators. This gain in calculational efficiency is even more pronounced for the general vulnerability equation where the integrals similar to those used in equation 1 are stacked to represent the different stages in a penetrator history (Reference 5).

We will analyze the Monte Carlo evaluation of the vulnerability integral by reformulating equation 1A. In the new form,

$$\begin{aligned} \lambda = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(r_0, w_0, b_0) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(r_1, w_1, b_1 | r_0, w_0, b_0, g) Q(r_1, w_1, b_1) dr_1 dw_1 db_1 dr_0 dw_0 db_0, \\ & - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(r_0, w_0, b_0) D(r_0, w_0, b_0) dr_0 dw_0 db_0, \end{aligned} \quad (10A)$$

where

$$D(r_0, w_0, b_0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(r_1, w_1, b_1 | r_0, w_0, b_0, g) Q(r_1, w_1, b_1) dr_1 dw_1 db_1. \quad (10B)$$

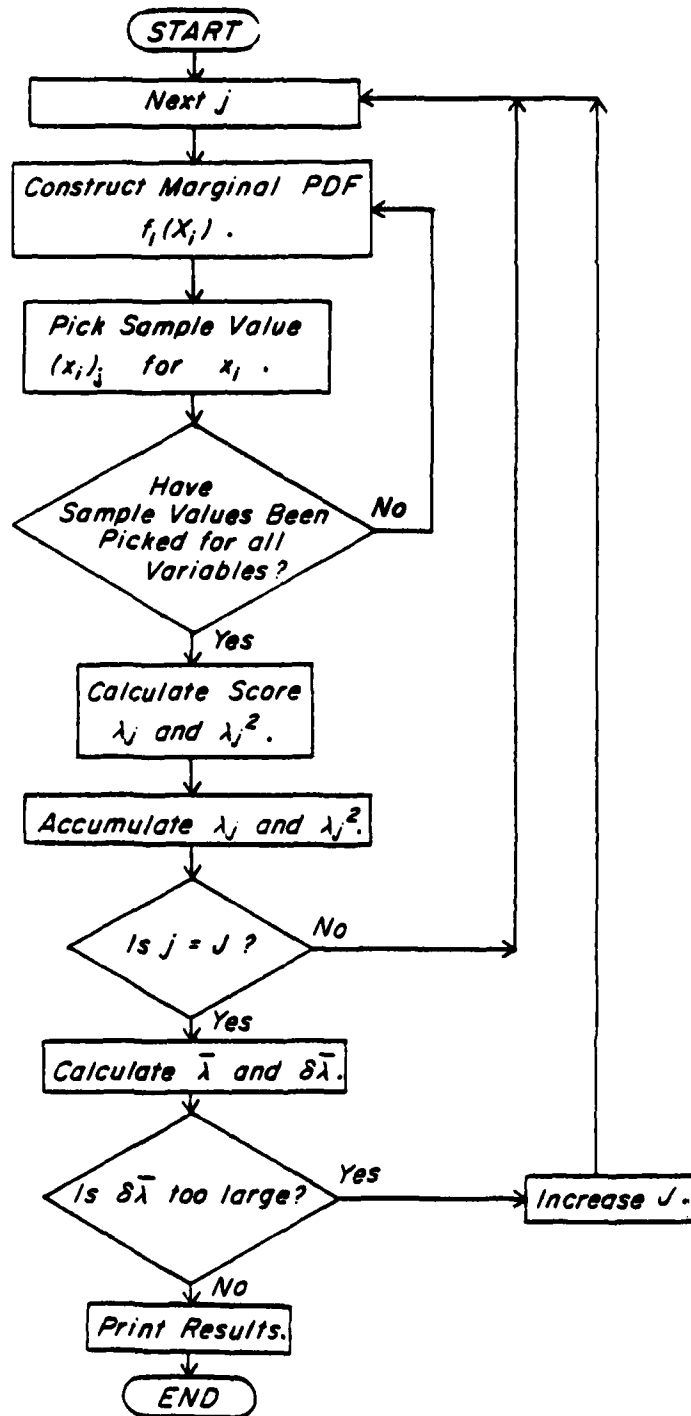


Figure 3. The Monte Carlo Estimation of a Multi-Dimensional Integral

If $D(r_0, w_0, b_0)$ were a known function of the variables (r_0, w_0, b_0) , then λ could be evaluated by using the Monte Carlo method to pick sample values from $S(r_0, w_0, b_0)$ as outlined in Subsection IIB. However, in vulnerability calculations, $D(r_0, w_0, b_0)$ is not generally so tractable and an alternate method of solution must be used.

In a Monte Carlo solution still based on the strong law of large numbers, the multiple integrals in equation 10A could be evaluated by picking sample birth projectiles $[(r_0)_j, (w_0)_j, (b_0)_j]$ from the source term $S(r_0, w_0, b_0)$ by using the procedures outlined in Subsection IIB, and then evaluating each integral $D[(r_0)_j, (w_0)_j, (b_0)_j]$ which is associated with a sample birth projectile. The Monte Carlo evaluation of each $D[(r_0)_j, (w_0)_j, (b_0)_j]$ could also be conducted by using the procedures outlined in Subsection IIB. That is, a sufficient number K of sample residual projectiles is picked from each normalized perforation PDF $f^*[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j]$, defined below, to obtain an accurate estimate $\bar{D}_j$ of the associated integral $D[(r_0)_j, (w_0)_j, (b_0)_j]$ where each estimate is given by

$$\bar{D}_j = (P_C)_j \sum_{k=1}^K Q[(r_1)_k, (w_1)_k, (b_1)_k]. \quad (11A)$$

The quantity λ can then be approximated as the mean of a large number of $\bar{D}_j$, that is

$$\bar{\lambda} \approx \frac{1}{JK} \sum_{j=1}^J (P_C)_j \sum_{k=1}^K Q[(r_1)_k, (w_1)_k, (b_1)_k]. \quad (11B)$$

The normalized perforation PDF is given by

$$f^*[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j] = \frac{f[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j]}{(P_C)_j} \quad (11C)$$

where the normalization factor $(P_C)_j$ is the probability that a projectile birthed as $[(r_0)_j, (w_0)_j, (b_0)_j]$ will perforate the barrier and impact the critical component, that is

$$\begin{aligned} (P_C)_j &= \int \int \int f^*[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j] dr_1 dw_1 db_1. \\ (P_C)_j &= \int \int \int f[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j] dr_1 dw_1 db_1. \end{aligned} \quad (11D)$$

However, the approximation implied in equation 11B isn't necessary and the absolute convergence of $\bar{\lambda}$ to λ can be obtained by applying the Weak Law of Large Numbers for the case where Tchebycheff's Theorem is applicable (Reference 4). In a Monte Carlo estimation based on this law, λ is estimated as

$$\bar{\lambda} = \frac{1}{J} \sum_{j=1}^J (P_C)_j Q[(r_1)_j, (w_1)_j, (b_1)_j]. \quad (12)$$

where only one sample set of values $[(r_1)_j, (w_1)_j, (b_1)_j]$ is picked from each normalized perforation PDF $f^*[(r_1, w_1, b_1) | (r_0)_j, (w_0)_j, (b_0)_j, g_j]$. As noted in Reference 4, the statement concerning the convergence of the sample mean $\bar{\lambda}$ to the universe mean λ is weaker when only the weak law of large numbers is applicable.

A step-by-step outline of a Monte Carlo estimate of the vulnerability integral (equation 1A) is outlined below and illustrated in Figure 4. Stacked vulnerability integrals such as those found in vulnerability methodologies (Reference 5) can be estimated by reiterating the following procedure.

1. Pick a birth projectile by sampling $S(r_0, w_0, b_0)$. This sample projectile is identified as $[(r_0)_j, (w_0)_j, (b_0)_j]$. Since $[r_0, w_0, b_0]$ are each assumed to be composed of several variables, the procedures outlined in Subsection IIB will have to be used.

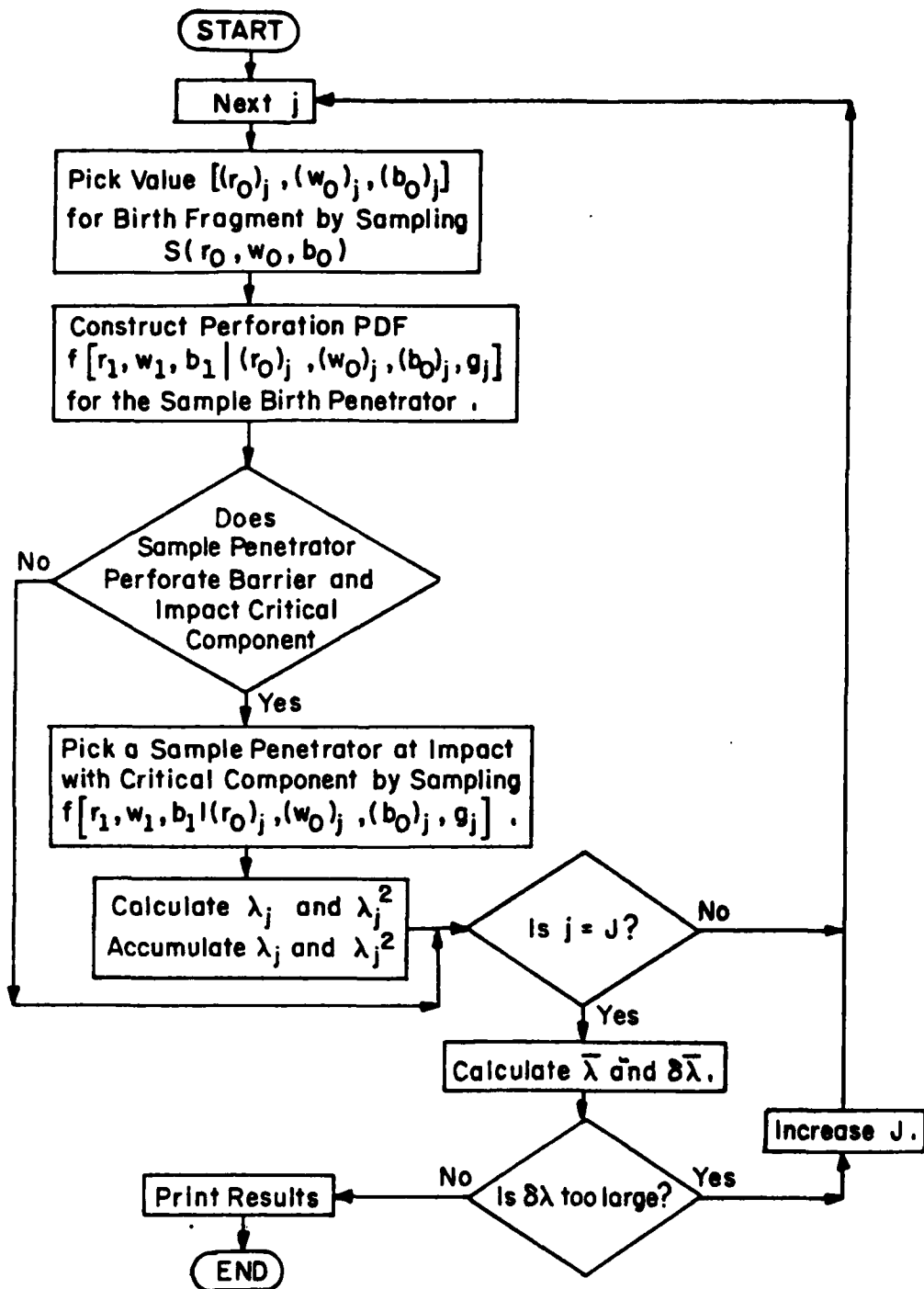


Figure 4. The Monte Carlo Estimation of the Expected-Value-of-Kill Vulnerability Integral

2. Determine the values g_j for the variables associated with the barrier which is impacted by the projectile.

3. Derive the perforation PDF $f[r_1, w_1, b_1 | (r_0)_j, (w_0)_j, (b_0)_j, g_j]$ which predicts the description (states) of the residual projectile at impact with the critical component.

4. Determine if a residual penetrator impacts the critical component by comparing a random number $RN(0,1)_j$ with $P_C[(r_0)_j, (w_0)_j, (b_0)_j]$. Set the event score to zero and go to step 8 if a residual penetrator does not impact the critical component. This "Russian Roulette" played with the residual fragment differs from the procedures implied by equation 12, but the two methods will both converge toward the true value of λ .

5. Pick a residual penetrator at impact with the critical component by sampling the normalized perforation PDF $f^*[r_1, w_1, b_1 | (r_0)_j, (w_0)_j, (b_0)_j, g_j]$. This sample residual penetrator is identified as $[(r_1)_j, (w_1)_j, (b_1)_j]$.

6. Calculate the incapacitation of the critical component expected from the impact. This incapacitation score is identified as λ_j and is given by

$$\lambda_j = Q[(r_1)_j, (w_1)_j, (b_1)_j]. \quad (13B)$$

7. Accumulate λ_j and λ_j^2 in bins reserved for these operations.

8. Conduct a total of J similar events by reiterating Steps 1-7.

9. Calculate an estimate of λ as

$$\bar{\lambda} = \frac{1}{J} \sum_{j=1}^J \lambda_j. \quad (13C)$$

10. Calculate an estimate of $\delta\bar{\lambda}$ as

$$\delta\bar{\lambda} = \left[\frac{\sum_{j=1}^J \lambda_j^2 - J\bar{\lambda}^2}{J(J-1)} \right]^{1/2}. \quad (13D)$$

11. Assess $\delta\bar{\lambda}$ to determine if it is too large. If necessary, calculate more histories and merge their results with those already calculated in order to reduce $\delta\bar{\lambda}$.

Step 11 completes the outline of a Monte Carlo evaluation of a multiple integral having the form illustrated by the vulnerability integral used in equation 1A. The procedures outlined above will be applied in the next section to evaluate a definite integral constructed by using simple analytic functions.

III. SAMPLE PROBLEMS

A. A Monte Carlo Solution Which Uses a General Multivariate PDF

We have devised a sample two-dimensional integral which can be used to illustrate the procedures outlined in both Sections IIIB and IIC. In the selected definite integral,

$$\lambda = \int_1^2 \int_1^2 x^2 y^2 dx dy = 225/16 = 14.0625, \quad (14A)$$

the procedures of Subsection ITR are illustrated by first rearranging the integrand to the form

$$\lambda = \frac{9}{4} \int_1^2 \int_1^2 (x^2 y^2) \left(\frac{4xy}{9} \right) dx dy. \quad (14B)$$

Equation 14B can then be further changed to the form used by equation 7A, that is

$$\lambda = C_1 \int_1^2 \int_1^2 H(x,y) f(x,y) dx dy, \quad (15A)$$

by taking:

$$C_1 = \frac{9}{4}, \quad (15B)$$

$$H(x,y) = x^2 y^2, \quad (15C)$$

and

$$f(x,y) = \frac{4xy}{9}, \quad (15D)$$

when

$$(1,1) \leq (x,y) < (2,2), \quad (15E)$$

and

$$f(x,y) = 0, \quad (15F)$$

when

$$(x,y) < (1,1), \quad (x,y) \geq (2,2). \quad (15G)$$

The constant $C_1 = 9/4$ is introduced to normalize $f(x,y)$ to unity over the integration range of the integral, that is,

$$\int_1^2 \int_1^2 f(x,y) dx dy = \int_1^2 \int_1^2 \frac{4xy}{9} dx dy = 1. \quad (16)$$

A Monte Carlo estimate of λ can now be calculated by picking sample values of (x,y) from $f(x,y)$ and then calculating the associated scores using $H(x,y)$. The calculational procedure is given below and is illustrated in Figure 5.

1. Pick a set of values (x_j, y_j) by sampling $f(x,y)$ over the integration range of the problem. This procedure is accomplished in the following steps:

A. Construct the marginal PDF $f_1(x)$ as

$$f_1(x) = \int_1^2 \frac{4xy}{9} dy = \frac{2x}{3}, \quad (17A)$$

where $f_1(x)$ is taken to be zero when x lies outside the integration range $(1,2)$.

B. Pick a sample value x_j by solving the integral equation

$$\int_1^{x_j} \frac{2x}{3} dx = RN(0,1)_{1j}. \quad (17B)$$

C. Construct a marginal PDF $f_2(x_j, y)$ for y as

$$f_2(x_j, y) = \frac{f(x_j, y)}{C_2}, \quad (17C)$$

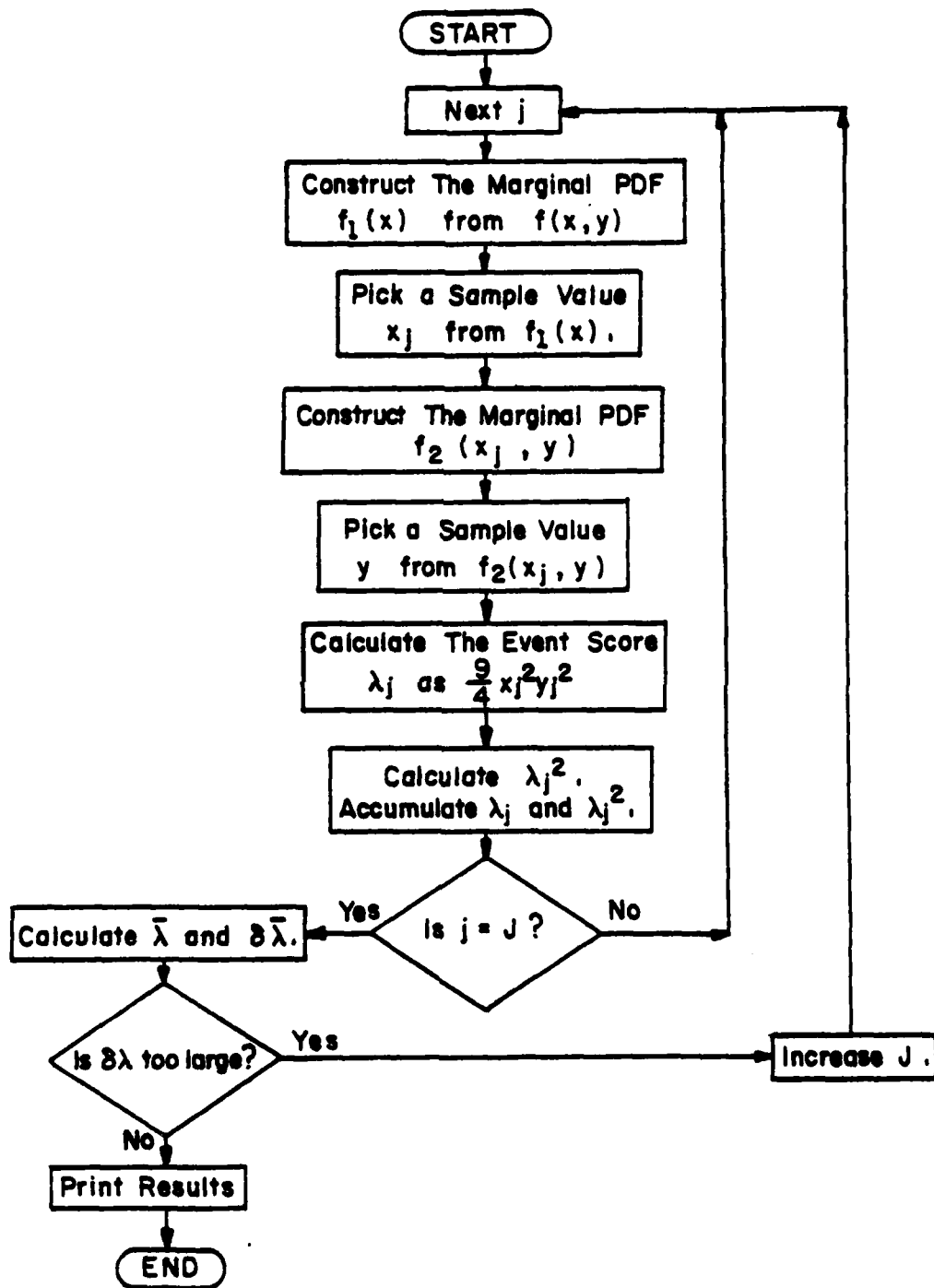


Figure 5. The Monte Carlo Estimation of the Example Problem

where

$$C_2 = \int_1^2 f(x_j, y) dy. \quad (17D)$$

D. Pick a sample value y_j for y by solving the integral equation

$$\int_1^{y_j} f_2(x_j, y) dy = RN(0,1)_{2j}. \quad (17E)$$

The quantity $(C_2)_j$ can be easily shown to be

$$(C_2)_j = \frac{2x_j}{3}, \quad (17F)$$

so that equation 17E simplifies to

$$\int_1^{y_j} \frac{2y}{3} dy = RN(0,1)_{2j}. \quad (17G)$$

2. Calculate the score λ_j for the sample event as

$$\lambda_j = \frac{9x_j^2 y_j^2}{4}. \quad (17H)$$

3. Calculate λ_j^2 . Accumulate λ_j and λ_j^2 in the bins reserved for this operation.

4. Calculate a total of J similar sample events by reiterating steps 1-3.

5. An estimate $\bar{\lambda}$ of λ is calculated as

$$\bar{\lambda} = \frac{1}{J} \sum_{j=1}^J \lambda_j. \quad (17I)$$

6. An estimate of the standard deviation $\delta\bar{\lambda}$ of $\bar{\lambda}$ is calculated as

$$\delta\bar{\lambda} = \left[\frac{\sum_{j=1}^J \lambda_j^2 - J\bar{\lambda}^2}{J(J-1)} \right]^{\frac{1}{2}}. \quad (17J)$$

7. Assess $\delta\bar{\lambda}$ to determine if it is too large. If necessary, calculate more histories and merge their results with those already calculated in order to reduce $\delta\bar{\lambda}$.

Step 7 completes an outline of the calculation of an estimate of λ by using the Monte Carlo procedures given in Section IIB. A computer program MCTP1 for performing this calculation was written in BASIC and is given in the appendix at the end of this report. The results of calculations using MCTP1 are given in TABLE 1 at the end of this section.

B. A Monte Carlo Solution Which Uses the Vulnerability Integral Approach

The procedures of Subsection IIC are illustrated by first rearranging the integrand of the example integral (equation 14A) to the form

$$\lambda = \int_1^2 \int_1^2 (x^2)(xy^2)(y) dy dx \quad (18A)$$

where the integration is assumed to proceed from the inner integral to the outer integral. Equation 18A can be compared with equation 1A as

$$\lambda = \int_1^2 \int_1^2 S(x) f(y|x) Q(y) dy dx = \int_1^2 \int_1^2 S^*(x) f^*(y|x) W(x) Q(y) dy dx \quad (18B)$$

by taking

$$S(x) = x^2, \quad (19A)$$

$$Q(y) = y, \quad (19B)$$

and

$$f(y|x) = xy^2. \quad (19C)$$

The quantity $W(x)$, identified here as a weighting function, is introduced to compensate for the normalization of the preceding source term and PDF to unity.

We define a normalized source term $S^*(x)$ as

$$S^*(x) = \frac{3S(x)}{7} \quad (19D)$$

where

$$\int_1^2 \frac{3S(x)}{7} dx = \int_1^2 S^*(x) dx = \int_1^2 \frac{3x^2}{7} dx = 1. \quad (19E)$$

Applying the Theorem of Bayes¹⁰, a normalized PDF $f^*(y|x)$ is associated with xy^2 by

$$xy^2 = \frac{7x f^*(y|x)}{3}, \quad (19F)$$

or

$$f^*(y|x) = \frac{3y^2}{7}. \quad (19G)$$

Equation 18A can now be rewritten as

$$\lambda = \int_1^2 \int_1^2 \frac{7}{3} S^*(x) \frac{7x f^*(y|x)}{3} Q(y) dy dx, \quad (20A)$$

and rearranged to

$$\lambda = \int_1^2 \int_1^2 S^*(x) f^*(y|x) \left[\frac{7}{3} \cdot \frac{7x}{3} \right] Q(y) dy dx. \quad (20B)$$

The weighting function is given by

$$W(x) = \frac{7}{3} \cdot \frac{7x}{3} \quad (20C)$$

where $\frac{7x}{3}$ corresponds to the quantity P_C used earlier in outlining a solution of the vulnerability integral (Subsection IIC).

¹⁰C. Kim, "Statistical Analysis for Induction and Decision," The Dryden Press, Inc., Hinsdale, Illinois, 1973.

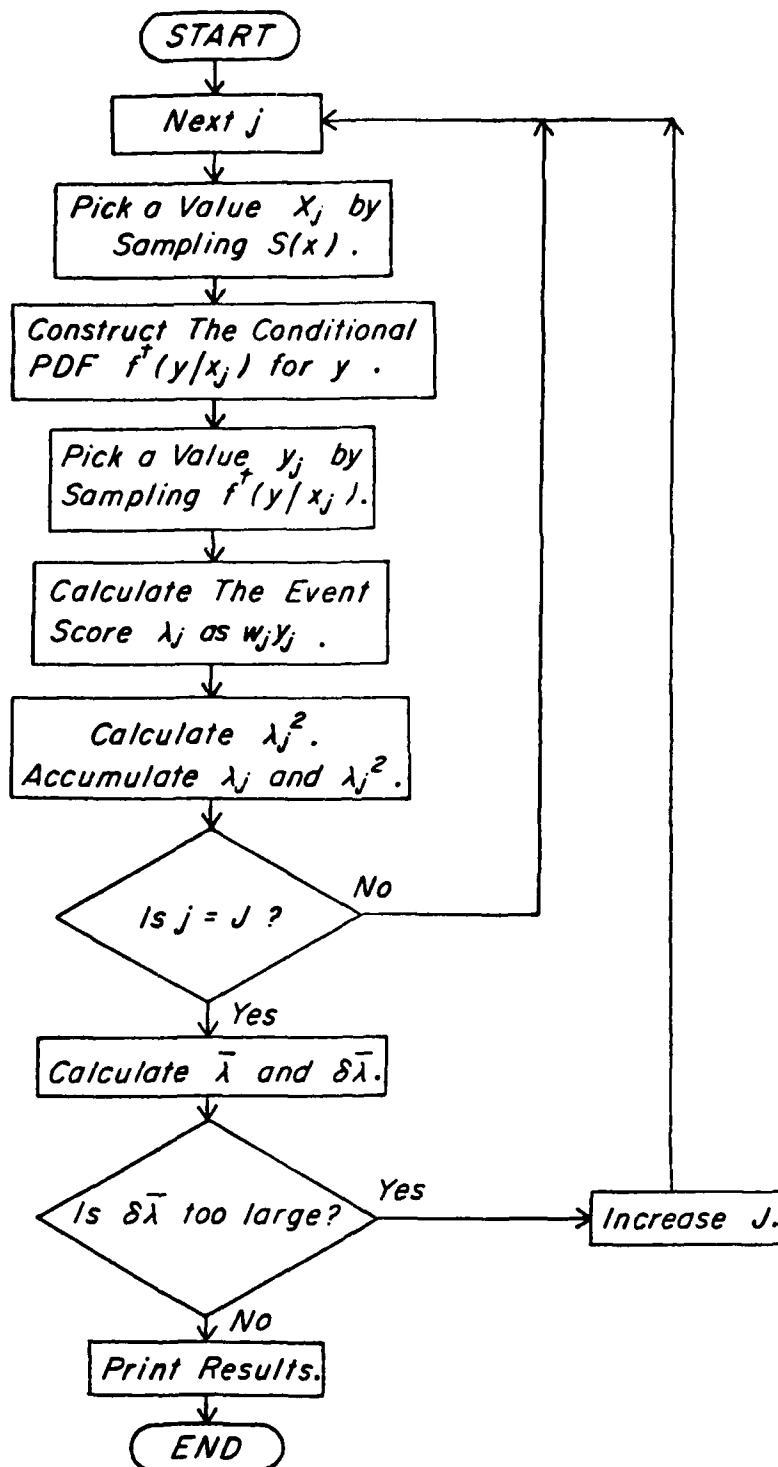


Figure 6. The Monte Carlo Estimation of the Example Problem where the Integrand Is Structured in a Form Similar to That of the Vulnerability Integral

Equation 20B is in a form which is tractable to Monte Carlo evaluation. A general solution is given below and outlined in Figure 6. The steps are:

1. Pick a sample value x_j for x by sampling $S^+(x)$. The sample value is derived from the integral equation

$$\int_1^{x_j} \frac{3x^2 dx}{7} = [RN(0,1)]_{1,j}. \quad (21A)$$

The quantity $\left[\frac{7}{3}\right]$ in equation 20A is saved and used later as a factor during the calculation of the event score.

2. Construct a conditional PDF $f^+(y|x_j)$ to be used to pick a sample value y_j for y . The conditional PDF used here (equation 19G) is independent of x_j , but this independence may not exist for cases where functions other than xy^2 are used in the construction.

3. Pick a sample value y_j by sampling $f^+(y|x_j)$. This sample value is derived from the integral equation

$$\int_1^{y_j} f^+(y|x_j) dy = \int_1^{y_j} \frac{3y^2}{7} dy = [RN(0,1)]_{2,j}. \quad (21B)$$

The quantity $\left[\frac{7x}{3}\right]$ in equation 20A is saved and used later as a factor during the calculation of the event score.

4. Calculate the score for the event as

$$\lambda_j = \left(\frac{7}{3}\right) \left[\frac{7x_j}{3}\right] Q(y_j) = \frac{49}{9} x_j y_j. \quad (21C)$$

5. Calculate λ_j^2 . Accumulate λ_j and λ_j^2 in the bins reserved for this operation.

6. An estimate $\bar{\lambda}$ of λ is calculated as

$$\bar{\lambda} = \frac{1}{J} \sum_{j=1}^J \lambda_j. \quad (21D)$$

7. An estimate $\delta\bar{\lambda}$ of the standard deviation of $\bar{\lambda}$ is calculated as

$$\delta\bar{\lambda} = \left[\frac{\sum_{j=1}^J \lambda_j^2 - J\bar{\lambda}^2}{J(J-1)} \right]^{\frac{1}{2}}. \quad (21E)$$

8. Assess $\delta\bar{\lambda}$ to determine if it is too large. If necessary, conduct more histories by reiterating Steps 1-5 and merge these results with those obtained earlier.

Step 8 completes the outline of the Monte Carlo evaluation of the example problem by using the procedures which would be used to evaluate a vulnerability integral of the form shown in equation 1. The results of test calculations of this example problem are given below in Subsection IIIC.

C. Example Problem Results

A computer program was written in BASIC (APPENDIX) to solve the sample integral by using both Monte Carlo procedures described in the preceding sections. The results of test calculations where 5000 histories were conducted for each estimate are given below in Table I. The reader should note that standard deviations of magnitude ten times that of the sample calculations could be obtained by using approximately fifty histories. Confidence levels of this order of magnitude would usually be acceptable in vulnerability calculations.

IV. CONCLUSION

We have outlined the use of the Monte Carlo method for solving expected value integrals of the type encountered in ballistic vulnerability calculations. As an aid to the comprehension of these methods, the outlined procedures (Subsection IIB and IIC) were applied to a sample problem for which a closed form solution existed and could be readily calculated. The insignificant differences between the computer-generated Monte Carlo solutions and the exact solution (Table I) serve to indicate the viability of the Monte Carlo techniques.

The reasons for using the Monte Carlo method in preference to deterministic methods to solve vulnerability problems can be summarized as:

1. Regardless of the dimensionality of the integration, a Monte Carlo estimate of multiple integrals converges toward the true value as $\frac{1}{\sqrt{N}}$ where N is the number of trials used in calculating the estimate. Therefore, the Monte Carlo method may be the most efficient procedure for solving expected-value problems when the dimensionality of the associated multiple integrals is large and confidence levels for the estimate on the order of five percent are acceptable.
2. The probability of introducing systematic errors such as those which might be introduced in deterministic calculations using a computerized phantom of the target vehicle is greatly reduced.
3. Computational simplicity can often be obtained by the division of the integrand into one part used in calculating scores and another part used to derive the sampling PDF's. In particular, the PDF's used in describing the ballistic phenomena are often directly derivable in terms of the "used" random variables B and G.
4. The calculations can be organized so that the sample events correspond in a one-to-one manner with actual ballistic events. This organization would aid vulnerability analysts in conducting calculations and interpreting results.
5. In many vulnerability problems, sample events can be obtained more easily than the associated PDF. Such problems must generally be solved by using the Monte Carlo method.

<i>METHOD OF SOLUTION</i>	<i>RUN NO.</i>	<i>ESTIMATE</i>	<i>STANDARD DEVIATION</i>
<i>VULNERABILITY INTEGRAL</i>	<i>1</i>	<i>14.058</i>	<i>0.048</i>
	<i>2</i>	<i>14.158</i>	<i>0.048</i>
	<i>3</i>	<i>14.043</i>	<i>0.048</i>
<i>MULTI-VARIATE SAMPLING</i>	<i>1</i>	<i>14.153</i>	<i>0.102</i>
	<i>2</i>	<i>14.082</i>	<i>0.101</i>
	<i>3</i>	<i>14.044</i>	<i>0.100</i>
<i>CLOSED FORM</i>		<i>14.0625</i>	

Table 1. The Monte Carlo Estimates of the Example-Problem Integral

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APPENDIX. THE COMPUTER PROGRAM USED TO SOLVE THE EXAMPLE PROBLEMS

```

1 REM MC TEST PROG 1 MCTP1
10 INPUT "R1,N",R1,N
20 FOR I=1TO N
30 GOSUB 170
40 X=SQR(1+3*R)
50 GOSUB 170
60 Y=SQR(1+3*R)
70 Z=X*Y*X*Y
80 S1=S1+Z
90 S2=S2+2*Z
100 SELECT PRINT 005: PRINT I,Z
110 NEXT I
120 S1=S1/N
130 S2=S2-N*S1*S1
140 S2=S2/(N*(N-1))
150 S2=SQR(S2)
160 SELECT PRINT 215
170 PRINT 9*S1/4, 9*S2/4
180 END
4000 DEFFN 170
4001 REM RN SR
4005 R1=25*R1
4010 GOSUB 171
4015 R1=5*R1
4020 GOSUB 171
4025 R=R1/67108864
4030 RETURN
4050 DEFFN 171
4051 REM RN SR2
4055 R2=R1/67108864
4060 R3=67108864*INT(R2)
4065 R1=R1-R3
4070 RETURN

```

```

1 REM MC TEST PROG 2 MCTP2
10 INPUT "R1,N",R1,N
20 FOR I=1TO N
30 GOSUB 170
40 X=(1+7*R)/0.333333
50 GOSUB 170
60 Y=(1+7*R)/0.333333
70 Z=X*Y
80 S1=S1+Z
90 S2=S2+2*Z
100 SELECT PRINT 005: PRINT I,Z
110 NEXT I
120 S1=S1/N
130 S2=S2-N*S1*S1
140 S2=S2/(N*(N-1))
150 S2=SQR(S2)
160 SELECT PRINT 215
170 PRINT 49*S1/9, 49*S2/9
180 END
4000 DEFFN 170
4001 REM RN SR
4005 R1=25*R1
4010 GOSUB 171
4015 R1=5*R1
4020 GOSUB 171
4025 R=R1/67108864
4030 RETURN
4050 DEFFN 171
4051 REM RN SR2
4055 R2=R1/67108864
4060 R3=67108864*INT(R2)
4065 R1=R1-R3
4070 RETURN

```

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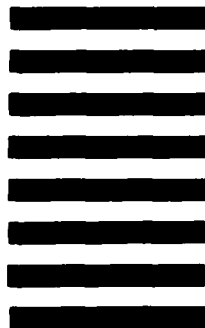


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