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COMMENTS ON

THE MEASUREMENT OF LINEAR DEPENDENCE AND FEEDBACK BETWEEN MULTIPLE TIME SERIES

by John Geweke

by Emanuel Parzen

Institute of Statistics

Texas A&M University

Technical Report No. N-27

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This paper is a discussion presented August 13, 1981 at the American Statistical Association Annual Meeting in Detroit of an invited paper by John Geweke entitled "The Measurement of Linear Dependence and Feedback Between Multiple Time Series." It shows how the results presented by Geweke can be derived and clarified using the approaches to multiple time series modeling published in Parzen (1967), (1969), (1977), and Parzen and Newton (1979). Its aim is to indicate directions for further research. It is emphasized that Geweke's paper is excellent.

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COMMENTS ON

"THE MEASUREMENT OF LINEAR DEPENDENCE AND FEEDBACK BETWEEN MULTIPLE TIME SERIES"

BY JOHN GEWEKE

A

by EMANUEL PARZEN

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I would like to congratulate Professor Geweke on an interesting paper. I believe its most valuable contribution is to stimulate us to develop improved methods for modeling multiple time series with the aim of determining which variables are significantly related.

The problem of modeling multiple time series is one on whose theory I have written extensively in Parzen (1967), (1967*a*), (1969), (1977), and Parzen and Newton (1980). I would like to show how results and notation from these papers help us to derive and clarify the results presented by Geweke.

Let X(t) and Y(t) be multiple time series, with zero means, jointly normal, and jointly covariance stationary. To study the relations between $X(\cdot)$ and $Y(\cdot)$, one models

$$Z(t) = \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} .$$

The covariance matrix $R(v) = E[Z'(t)\, Z(t+v)]$ is assumed to be summable so that the spectral density matrix

$$f(w) = \sum_{v=-\infty}^{\infty} e^{-2\pi i v w} R(v), \quad 0 \le w \le 1,$$

exists. Then $R(v) = \int_0^1 e^{2\pi i v w} f(w)\, dw$.

The joint covariance and spectral density matrices of X and Y are described by the blocks in the partitioned matrices

$$R(v) = \begin{bmatrix} R_{XX}(v) & R_{XY}(v) \\ R_{YX}(v) & R_{YY}(v) \end{bmatrix},$$

$$f(w) = \begin{bmatrix} f_{XX}(w) & f_{XY}(w) \\ f_{YX}(w) & f_{YY}(w) \end{bmatrix}$$

Autoregressive analysis models Z(t) by a joint infinite order autoregressive scheme:

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = A_{X,Y}(1) \begin{bmatrix} X(t-1) \\ Y(t-1) \end{bmatrix} + \ldots + A_{X,Y}(m) \begin{bmatrix} X(t-m) \\ Y(t-m) \end{bmatrix} + \ldots$$

$$+ \eta(t),$$

$$A_{X,Y}(j) = \begin{bmatrix} A_{XX}(j) & A_{XY}(j) \\ A_{YX}(j) & A_{YY}(j) \end{bmatrix},$$

$$\eta(t) = \begin{bmatrix} \eta_X(t) \\ \eta_Y(t) \end{bmatrix} \quad \text{joint innovations}$$

$$\Sigma = \begin{bmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{bmatrix} = E[\eta'(t)\eta(t)]$$

A preferred notation for Σ is $\Sigma(X,Y|X^-,Y^-)$.

We call $\eta(t)$ the joint innovations, and Σ the joint innovation covariance matrix. We can define $\eta(t)$ as infinite

memory <u>prediction errors</u>:

$$\eta(t) = \begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} - E\left[\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} \middle| X(t-1),\ Y(t-1), \ldots X(t-m), Y(t-m), \cdots\right].$$

The joint innovations should be contrasted with the <u>individual innovations</u>

$$\tilde{X}(t) = X(t) - E[X(t)|X(t-1),\ldots,X(t-m),\ldots]$$

$$\tilde{Y}(t) = Y(t) - E[Y(t)|Y(t-1),\ldots,Y(t-m),\ldots]$$

which provide individual infinite order autoregressive models

$$X(t) = A_{X|X}(1)\ X(t-1)+\ldots+A_{X|X}\ X(t-m)+\ldots+\tilde{X}(t)$$

$$Y(t) = A_{Y|Y}(1)\ Y(t-1)+\ldots+A_{Y|Y}\ Y(t-m)+\ldots + \tilde{Y}(t).$$

The individual innovation covariance matrices are denoted

$$\Sigma_{\tilde{X}} = \Sigma(X|X^-) = E[\{X(t)\}'X(t)],$$

$$\Sigma_{\tilde{Y}} = \Sigma(Y|Y^-) = E[\{Y(t)\}'\ Y(t)].$$

The <u>innovation innovations</u> are defined to be the joint innovations of the joint time series $\begin{bmatrix} \tilde{X}(t) \\ \tilde{Y}(t) \end{bmatrix}$ of individual innovations. A <u>remarkable theorem</u> is that the innovation innovations are identical with the joint innovations. Thus in practice, one can determine the joint innovations of X(t) and Y(t) by first "prewhitening" them to form $\tilde{X}(t)$ and $\tilde{Y}(t)$, whose joint innovations are then determined.

The general theory of multiple time series discussed below is phrased in terms of general stationary time series X(t) and Y(t). However it works best in practice when applied to time series which have been somewhat pre-whitened.

In several papers on model identification, Parzen shows that the individual innovations of a time series are essentially unique, while the whitening filter which generates them may be expressed in diverse ways as a series of filters representing detrending, deseasonalizing, and innovations operations.

To model Y(t) we compare the properties of the prediction errors, and prediction error covariance matrices, corresponding to five sets of explanatory variables:

Prediction error	Covariance Matrix
$(Y^{\nu}\|Y^{-})(t) = Y(t) - E[Y(t)\|Y(t-1),\ldots,Y(t-m)]$	$\Sigma(Y\|Y^{-})$
$(Y^{\nu}\|X^{-},Y^{-})(t) = Y(t) - E[Y(t)\|X(t-1),\ldots,X(t-m), Y(t-1),\ldots,Y(t-m)]$	$\Sigma(Y\|X^{-},Y^{-})$
$(Y^{\nu}\|X^{+},Y^{-})(t) = Y(t) - E[Y(t)\|X(t),X(t-1),\ldots,X(t-m), Y(t-1),\ldots,Y(t-m)]$	$\Sigma(Y\|X^{+},Y^{-})$
$(Y^{\nu}\|X)(t) = Y(t) - E[Y(t)\|X(s), -\infty<s<\infty]$	$\Sigma(Y\|X)$
$(Y^{\nu}\|X,Y^{-})(t) = Y(t) - E[Y(t)\|X(s), -\infty<s\infty, Y(t-1),Y(t-2),\ldots,Y(t-m)]$	$\Sigma(Y\|X,Y^{-})$

It should be noted that after a conditioning sign |, X^{-} represents the past of X, X^{+} the past and present of X, and X the past, present, and future of X.

The spectral densities of the various error series are denoted $f_{Y^\nu|Y^-}(w)$, $f_{Y^\nu|X^-,Y^-}(w)$, $f_{Y^\nu|X^+,Y^-}(w)$, $f_{Y^\nu|X}(w)$, $f_{Y^\nu|X,Y^-}(w)$.

(1) $\underline{Y|Y^-}$: $\Sigma(Y|Y^-)$ is the individual innovation covariance matrix $\Sigma_{\tilde{Y}}$; $(Y^\nu|Y^-)(t)$ is white noise.

(2) $\underline{Y|X^-,\ Y^-}$: $\Sigma(Y|X^-,\ Y^-)$ is the block Σ_{YY} in the joint innovation covariance matrix; $(Y^\nu|X^-,\ Y^-)(t)$ is white noise.

(3) $\underline{Y|X^+,\ Y^-}$: Parzen (1967) shows (p. 401)

$$(Y^\nu|X^+,Y^-)(t) = (Y^\nu|X^-,Y^-)(t) - \Sigma_{YX}\,\Sigma_{XX}^{-1}\,(X^\nu|X^-,Y^-)(t);$$

$$\Sigma(Y|X^+,Y^-) = \Sigma_{YY} - \Sigma_{YX}\,\Sigma_{XX}^{-1}\,\Sigma_{XY}$$

(4) $\underline{Y|X}$: Parzen (1967) shows that a joint autoregressive model for X(t) and Y(t) provides f(w) from which one can compute the statistical parameters of the representation

$$Y(t) = (Y^\mu|X)(t) + (Y^\nu|X)(t),$$

by

$$(Y^\mu|X)(t) = B(L)\ X\ (t),\ B(z) = \sum_{k=-\infty}^{\infty} B(k)z^k$$

$$B(e^{2\pi iw}) = f_{YX}(w)\ f_{XX}^{-1}(w)$$

$$f_{Y^\nu|X}(w) = f_{YY}(w) - f_{YX}(w)\ f_{XX}^{-1}(w)\ f_{XY}(w)$$

$$\Sigma(Y|X) = \int_0^1 f_{Y^\nu|X}(w)\ dw.$$

In practice, to estimate the time domain coefficients B(k) from an estimator $\hat{B}\ (e^{2\pi iw})$ one uses regression methods.

(5) $\underline{Y|X,\ Y^-}$: As Geweke shows, find an autoregressive model for $(Y^\nu|X)(t)$;

$$g_{Y^\nu|X}(L)\ (Y^\nu|X)(t) = \varepsilon(t).$$

It generates a white noise sequence $\varepsilon(t)$ which can be identified with $(Y^\nu|X,Y^-)(t)$. Further a model for $Y(t)$ as a function of $X(s)$, $-\infty<s<\infty$ and $Y(s)$, $s\leq t-1$ is given by

$$g_{Y^\nu|X}(L)\ Y(t) = g_{Y^\nu|X}(L)B(L)\ X(t) + \varepsilon(t);$$

$$\Sigma(Y|X,\ Y^-) = \Sigma_\varepsilon = \text{covariance matrix of } \varepsilon(t).$$

The time domain coefficients of the filter with input $X(t)$ have Fourier transform

$$g_{Y^\nu|X}\ (e^{-2\pi iw})\ B(e^{-2\pi iw}).$$

If one needs only the log determinant of Σ_ε, it can be calculated without fitting an autoregressive scheme:

$$\log|\Sigma_\varepsilon\ | = \int_0^1 \log|f_{Y^\nu|X}(w)|\,dw.$$

One need not actually calculate the spectral density since Geweke's Theorem 1 shows that

$$\ell n\ |\ \Sigma(Y|X,Y^-)| = \ell n|\Sigma(X,Y|X^-,Y^-)| - \ell n\ |\Sigma(X|X^-)|$$

The meaning of the various definitions of feedback, and the formulas for them given by Geweke's Theorem 1, is easily understood if one employs the notation we have introduced.

<u>Measure of linear dependence</u> (or <u>information</u>) $F_{X,Y} =$

$= \ln \det \Sigma(X|X^-) - \ln \det \Sigma(X|X^-,Y)$

$= \ln \det \Sigma(Y|Y^-) - \ln \det \Sigma(Y|Y^-,X)$

<u>Measure of instantaneous linear feedback</u>: $F_{X\cdot Y} =$

$= \ln \det \Sigma(X|X^-,Y^-) - \ln \det \Sigma(X|X^-,Y^+)$

$= \ln \det \Sigma(Y|X^-,Y^-) - \ln \det \Sigma(Y|X^+,Y^-)$

<u>Measure of linear feedback from Y to X</u>: $F_{Y\to X} =$

$= \ln \det \Sigma(X|X^-) - \ln \det \Sigma(X|X^-,Y^-)$

$= \ln \det \Sigma(Y|X^+,Y^-) - \ln \det \Sigma(Y|X,Y^-)$

<u>Measure of linear feedback from X to Y</u>: $F_{X\to Y} =$

$= \ln \det \Sigma(Y|Y^-) - \ln \det \Sigma(Y|Y^-,X^-)$

$= \ln \det \Sigma(X|X^-,Y^+) - \ln \det \Sigma(X|X^-,Y)$

Theorem 1 in Geweke's paper shows that there is a crucial identity from which the equivalence of the foregoing <u>definitions</u> follows immediately:

$\ln \det \Sigma(X|X^-,Y^+) + \ln \det \Sigma(Y|X^-,Y^-)$

$= \ln \det \Sigma(X|X^-,Y) + \ln \det \Sigma(Y|Y^-)$

$= \ln \det \Sigma(X,Y|X^-,Y^-)$

These feedback measures seem to me to be most clearly interpreted as measuring the significance of various variables as independent variables in a model for Y(t).

Measure	Variables in Model	In Words	Variables tested for inclusion	Add
$F_{X\to Y}$	Y^-	Past Y	Y^-, X^-	Past X
$F_{X\cdot Y}$	Y^-, X^-	Past Y, Past X,	Y^-, X^+	Present X
$F_{Y\to X}$	Y^-, X^+	Past Y, Past X, Present X	Y^-, X	Future X
$F_{X,Y}$	Y^-	Past Y	Y^-, X	All X

This table also exhibits the information decomposition of the identity:

$$F_{X,Y} = F_{X\to Y} + F_{X\cdot Y} + F_{Y\to X}$$

One uses $F_{X,Y}$ to compare the hypotheses H_o: model Y by Y^-; H_1: model Y by Y^-, X. In addition one should compute $\Sigma(Y|X)$ in order to compare the hypotheses H_o: model Y by X; H_1: model Y by X, Y^-.

In any empirical <u>multiple time series</u> analysis, one should compute, and report, $\Sigma_{\tilde{X}}$, $\Sigma_{\tilde{Y}}$, Σ (the individual and joint innovation covariances). Then one should compute (and test for significant difference from zero)

$$F_{X\to Y} = \ln \det \Sigma_{\tilde{Y}} - \ln \det \Sigma_{YY}$$

$$F_{X\cdot Y} = -\ln \det (I - \Sigma_{YY}^{-1} \Sigma_{YX} \Sigma_{XX}^{-1} \Sigma_{XY})$$

$$= \ln \det \Sigma_{\tilde{X}} + \ln \det \Sigma_{\tilde{Y}} - \ln \det \Sigma$$

$$F_{Y\to X} = \ln \det \Sigma_{\tilde{X}} - \ln \det \Sigma_{XX}$$

The computation of these determinants, and additional insight into the relations between variables, could be attained by computing the eigenvalues and eigenvectors of the matrices,

such as $\Sigma^{-1}(Y|Y^-,X^-)\Sigma(Y|Y^-,X^+)$, whose log determinants are being calculated. The eigenvalues can be interpreted in terms of various canonical correlations (see Parzen and Newton (1979)).

Finally, to estimate (from data) parameters such as Σ, Σ_X, and Σ_Y, it is strongly recommended that one use approximating autoregressive schemes, and order-determining criteria such as CAT (see Parzen (1974), (1977)).

As with any excellent piece of research, Geweke's paper raises many open questions, some of which have been alluded to in my discussion.

The definition of the feedback spectral measure $f_{Y\to X}(w)$ given by Geweke is impressive. Only experience can show us whether it should be routinely computed in empirical research; it may suffice to use as the feedback spectral measure

$$f_{X,Y}(w) = \ln \det I - f_{YY}^{-1}(w)\, f_{YX}(w)\, f_{XX}^{-1}(w)\, f_{XY}(w)$$

computed using canonical spectral analysis (see Brillinger (1981), chapter 10 for references to this literature).

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