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FIXED GAIN CONTROLLER DESIGN FOR AIRCRAFT

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30th June 1981

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20. Abstract

A Method for designing fixed gain controllers and filters for systems with large parameter variation is presented.
The approach, based of minmax information criteria, is used to design a non-adaptive back-up control system for a given aircraft and is shown to provide good performance qualities.

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This technical report has been reviewed and is approved for publication.

John W. Bailey

JOHN W. BAILEY
Major, USAF
Chief, Electronics

FOR THE COMMANDER

Gordon L. Hermann

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Lt Colonel, USAF
Deputy Commander

Introduction

A major difficulty associated with aircraft control is that the dynamic characteristics are highly dependent on variable parameters, such as the altitude and the Mach number. Recent years have seen considerable effort towards the design of adaptive control systems, taking the dynamic dependence on varying flight parameters into account. Yet, the design of simple controllers and filters, minimizing the necessary airborn computation and data storage capabilities, on the one hand, and the dependence on accurate air data, on the other, remains of major interest. Fixed gain controllers and filters are commonly used as back-up control systems for advanced high performance aircraft.

In this paper a method for designing fixed-gain controllers and estimators for systems with large parameter variations or uncertainties is described. The approach is based on minmax criteria defined on Kullback's information measure [1] and previously used for solving model simplification problems ([2], [3]). (We note here that other minmax design criteria for deterministic systems have been proposed, e.g. [4]). The design objective is that the maximum possible difference between the optimal system at the actual operating point (i.e. the optimal adaptive system) and the selected fixed-gain system be minimal. Finite time and regulator design criteria are defined. The proposed technique is used to design a longitudinal back-up control system for a given high performance aircraft. Simulated aircraft responses and a pilot rating chart indicate good performance qualities of the resulting fixed gain control system for the entire flight regime.

2. Design Criteria

A. General

In the design problems under consideration we have a stochastic system $M(p,s)$ depending on a parameter p, whose values may vary over a set P and a parameter s, whose value is to be selected from a set S in the design. Employing some optimal design criterion, to each parameter value p there corresponds an optimal value $s(p)$ of s. It is desired, however, to find a single value s^o that will be optimal in some sense for the entire parameter set P. The cost (or damage) of the resulting design at a given parameter value p is defined as the distance, in some measure, between $M(p,s)$ and $M(p,s(p))$. Employing Kullback's information as a distance measure ([1]-[3]) the maximal cost for a given choice of s is

$$\max_{p\in P} E_p^{s(p)} \log \frac{f_p^{s(p)}(Y)}{f_p^{s}(Y)}$$

where Y is a set of observations, $f_p^{s(p)}(Y)$ and $f_p^{s}(Y)$ are the probability densities of Y corresponding to $M(p,s(p))$ and $M(p,s)$, respectively and $E_p^{s(p)}$ denotes expectation with respect to $M(p,s(p))$. The design objective is to minimize the maximal cost, i.e.

$$\min_{s\in S} \max_{p\in P} E_p^{s(p)} \log \frac{f_p^{s(p)}(Y)}{f_p^{s}(Y)}$$

B. Controller Design for a Linear System in Finite Time

Consider a dynamical system in the form

$$x_{n+1} = F_n(p,s)x_n + B_n(p,s)u_n + G_n(p)w_n$$
$$y_n = H_n(p)x_n + v_n \tag{2.1}$$

where x_o is Gaussian with mean $m_o(p)$ and covariance $\psi_o(p)$, u_n is an external (non-feedback) deterministic input, w_n and v_n are zero-mean Gaussian white noise sequences with

$$\text{cov}\ (w_n) = Q_n(p)$$

$$\text{cov}\ (v_n) = R_n(p)$$

p is a variable parameter vector taking values in a set P and s is a vector of controller gains taking values in a set S. Any linear state feedback is accounted for in the matrix $F_n(p,s)$. We denote the above system by M(p,s).

Employing some optimal control criterion, to each parameter value p there corresponds an optimal gain value s(p) and the corresponding system M(p,s(p)). It is desired, however, to find a constant gain s^o that will be optimal in some sense for the entire parameter set P. As in the case of optimal controller design at a given parameter value, the design may be aimed at optimizing the system's response to some input sequence or to an initial condition in finite time. Let us denote

$$I_N(p,s) = \frac{2}{N} E_p^{s(p)} \log \frac{f_p^{s(p)}(Y^N)}{f_p^{s}(Y^N)}$$

where $Y^N = (y_1,\dots,y_N)$. For the Gaussian case under consideration we have

$$I_N(p,s) = \frac{1}{N} \sum_{n=1}^{N} [\log \frac{|\Sigma_n(p,s)|}{|\Sigma_n(p,s(p))|} + \mathrm{tr}\, \Sigma_n^{-1}(p,s)\Gamma_n(p,s) - k] \tag{2.2}$$

where k is the observations' dimension and

$$\Sigma_n(p,s) = E_p^{s} \{[y_n - \hat{y}_{n|n-1}(p,s)][y_n - \hat{y}_{n|n-1}(p,s)]^T\}$$

$$\Sigma_n(p,s(p)) = E_p^{s(p)} \{[y_n - \hat{y}_{n|n-1}(p,s(p))][y_n - \hat{y}_{n|n-1}(p,s(p))]^T\}$$

$$\Gamma_n(p,s) = E_p^{s(p)} \{[y_n - \hat{y}_{n|n-1}(p,s)][y_n - \hat{y}_{n|n-1}(p,s)]^T\}$$

$$\hat{y}_{n|n-1}(p,s) = E_p^{s}\{y_n | Y^{n-1}\}$$

$$\hat{y}_{n|n-1}(p,s(p)) = E_p^{s(p)}\{y_n | Y^{n-1}\}$$

For the given system $\Sigma_n(p,s)$ is obtained from

$$\Sigma_n(p,s) = H_n(p)\psi_n(p,s)H_n^T(p) + R_n(p)$$

where $\psi_n(p,s)$ is obtained from the Ricatti equation

$$\psi_{n+1}(p,s) = F_n(p,s)\psi_n(p,s)F_n^T(p,s) + G_n(p)Q_n(p)G_n^T(p)$$

$$-F_n(p,s)K_n(p,s)\Sigma_n(p,s)K_n^T(p,s)F_n^T(p,s)$$

with

$$K_n(p,s) = \psi_n(p,s)H_n^T(p)\Sigma_n^{-1}(p,s)$$

($\Sigma_n(p,s(p))$ is obtained in a similar manner, replacing s by s(p)). $\Gamma_n(p,s)$ is obtained from

$$\Gamma_n(p,s) = \overline{H}_n(p)[\overline{\psi}_n(p,s) + \overline{X}_n(p,s)\overline{X}_n^T(p,s)]\overline{H}_n(p) + R_n(p) \tag{2.3}$$

where $\overline{\psi}_n(p,s)$ is obtained from

$$\overline{\psi}_n(p,s) = \overline{F}_n(p,s)\overline{\psi}_n(p,s)\overline{F}_n(p,s)^T + \overline{G}_n(p,s)\overline{Q}_n(p)\overline{G}_n^T(p,s) \tag{2.4}$$

initialized at

$$\overline{\Psi}_o(p,s) = \begin{bmatrix} \psi_o(p) & 0 \\ 0 & 0 \end{bmatrix}$$

and $\overline{X}_n(p,s)$ is obtained from

$$\overline{X}_{n+1}(p,s) = \overline{F}_n(p,s)\overline{X}_n(p,s) + \overline{B}_n(p,s)u_n \tag{2.5}$$

initialized at

$$\overline{X}_o = \begin{bmatrix} m_o(p) \\ m_o(p) \end{bmatrix}$$

We have used the notation

$$\overline{F}_n(p,s) = \begin{bmatrix} F_n(p,s(p)) & 0 \\ F_n(p,s)K_n(p,s)H(p) & F_n(p,s)[I-K_n(p,s)H_n(p)] \end{bmatrix}$$

$$\bar{G}_n(p,s) = \begin{bmatrix} G_n(p) & 0 \\ 0 & F_n(p,s)K_n(p,s) \end{bmatrix}$$

$$\bar{Q}_n(p,s) = \begin{bmatrix} Q_n(p) & 0 \\ 0 & R_n(p) \end{bmatrix}$$

$$\bar{H}_n(p) = [H_n(p) \qquad -H_n(p)]$$

and

$$\bar{B}_n(p,s) = \begin{bmatrix} B_n(p,s(p)) \\ B_n(p,s) \end{bmatrix}$$

The fixed-gain controller is found from

$$s^o = \arg \{\min_{s \in S} \max_{p \in P} I_N(p,s)\}$$

B. Regulator Design.

Suppose that the system is time invariant, i.e.

$$x_{n+1} = F(p,s)x_n + B(p,s)u + G(p)w_n$$

$$y_n = H(p)x_n + v_n$$

with

$$\mathrm{cov}(w_n) = Q(p), \ \mathrm{cov}(v_n) = R(p)$$

Note that the external (non-feedback) input is now assumed constant.

A very attractive alternative, at least from the computation viewpoint, to finite time design in this case is steady state or regulator design. The following result is fundamental to our fixed gain regulator design method.

Theorem 1

Suppose that for each $p \in P$ and each $s \in S$ the system $(F(p,s), G(p), H(p))$ is controllable and observable and the controller $s(p)$ is stabilizing (i.e. $F(p,s(p))$

has all its' eigenvalues inside the unit circle). Then the cost function $I_N(p,s)$ has a finite limit value, given by

$$I(p,s) = \log \frac{|\Sigma(p,s)|}{|\Sigma(p,s(p)|} + \mathrm{tr}\, \Sigma^{-1}(p,s)\Gamma(p,s)-k \tag{2.6}$$

where $\Sigma(p,s)$, $\Sigma(p,s(p))$ and $\Gamma(p,s)$ are the limit values of $\Sigma_n(p,s)$, $\Sigma_n(p,s(p))$ and $\Gamma_n(p,s)$, respectively.

Proof

It is well known that when the system is controllable by the input noise and observable, the state estimation error covariance has a non-singular limit value. It follows that the output estimation error covariance $\Sigma_n(p,s)$ has a non-singular limit value $\Sigma(p,s)$ if $H(p)$ has independent rows, which is the normal case. It is also well known that an equation of the type (2.4) (a Lyapunov equation) has a limit solution for $\psi_n(p,s)$ if $\bar{F}(p,s)$ has all its' eigenvalues inside the unit circle. But due to the structure of $\bar{F}(p,s)$, its' eigenvalues are the eigenvalues of $F(p,s(p))$ together with the eigenvalues of $F(p,s)[I-K(p,s)H(p)]$. Since $s(p)$ is stabilizing, $F(p,s(p))$ has all its' eigenvalues inside the unit circle. Since the system $(F(p,s),G(p),H(p)$ is controllable and observable, the matrix $F(p,s)[I-K(p,s)H(p)]$

has all its eigenvalues inside the unit circle (see, e.g. [5] p. 144). It follows that all the eigenvalues of $\bar{F}(p,s)$ are inside the unit circle and, consequently, (2.4) has a limit solution for ψ_n. In addition since the system (2.5) is stable, $\bar{X}_n(p,s)$ has a finite limit value. It follows that $\Gamma_n(p,s)$ in (2.3) has a finite limit value, $\Gamma(p,s)$. We have thus shown that $\Sigma_n(p,s)$ and $\Gamma_n(p,s)$ have limit values $\Sigma(p,s)$ and $\Gamma(p,s)$. The assertion readily follows from (2.2).

It should be noted that the controllability and observability conditions are structural requirements which are "generically" independent of the specific values of the parameter p and s. The condition that s(p) is stabilizing is normally satisfied in pointwise optimal design. For instance, consider the system

$$\begin{aligned} x_{n+1} &= F(p)x_n + D(p)z_n + B(p)u + G(p)w_n \\ y_n &= H(p)x_n + v_n \end{aligned} \tag{2.7}$$

where z_n is a linear state feedback process, i.e.

$$z_n = -sx_n \tag{2.8}$$

Then it is well known that by an appropriate choice of the gain matrix s, the eigenvalues of the matrix

$$F(s,p) = F(p) - D(p)s \tag{2.9}$$

can be placed inside the unit circle provided that the pair $(F(p), D(p))$ is stabilizable. One such choice of s is obtained by the linear quadratic criterion

$$\min_{z_n} \sum_{n=1}^{\infty} [x_n^T L x_n + x_n^T M z_n + z_n^T N z_n] \tag{2.10}$$

where M and N are positive definite matrices and L is a positive semi definite matrix such that for $C^T C = L$ the pair $(F(p), C)$ is observable. This criterion gives z_n in the form (2.8) with

$$s = s(p) = [D(p)^T \phi(p) D(p) + N]^{-1} [D(p) \phi(p) F(p) + M]$$

where $\phi(p)$ is obtained from the algebraic Ricatti equation

$$\phi(p) = K_1(p) + L - K_2(p) K_3(p) K_2^T(p)$$

where

$$K_1(p) = F^T(p) \phi(p) F(p)$$

$$K_2(p) = F(p) \phi(p) D(p) + M$$

$$K_3(p) = D(p) \phi(p) D(p) + N$$

Example 1

Consider the system (2.7) with

$$F(p) = \begin{bmatrix} 0 & 1 \\ p_1 & p_2 \end{bmatrix}, \quad D(p) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad G(p) = I, \quad H(p) = [1 \quad 0]$$

and state feedback

$$Z_n = - [s_1 \quad s_2] x_n$$

so that

$$F(p,s) = \begin{bmatrix} 0 & 1 \\ p_1 - s_1 & p_2 - s_2 \end{bmatrix}$$

It can be seen that the system $(F(p,s), G(p), H(p))$ is controllable and observable and the pair $(F(p), D(p))$ is stabilizable, independently of the values of p_1, p_2, s_1, s_2. The conditions of theorem 1 are thus satisfied and the cost function $J(p,s)$ has a finite limit value for all p and s. On the other hand, taking $F(p,s)$ as above but with

$$D(p) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad G(p) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad H(p) = [0 \quad 1]$$

it can be seen that the system $(F(p,s),G(p),H(p))$ is controllable and observable everywhere but on the line $p_1 = s_1$ and the pair $(F(p,s),D(p))$ is controllable (hence, stabilizable) everywhere but at $p_1 = 0$. Since these points in the P·S hyperplane produce indefinite values for the cost function $J(p,s)$, they may simply be avoided in the search procedure.

It should be emphasized that while the conditions of theorem 1 ensure that the fixed gain design criterion is well defined, they do not guarantee that the resulting fixed gain controller will be stabilizing at all points in the given parameter set P. In fact, overall stability may not be achievable by a fixed gain controller for given parameter sets. To ensure stability the parameter set must be restricted in such a way that the eigenvalues of $F(p,s)$ are mapped into the unit circle for all values of p and s. This problem has been recently considered in [6].

C. Filter Design.

Once a fixed gain controller has been selected, the system is given by (2.1) with $s=s^o$. To each parameter p there corresponds a linear stochastic model and an optimal estimator (Kalman filter) based on this model. It is now desired to find an estimator, independent of p(i.e., non-adaptive), that will be optimal in some sense for the entire parameter set P. Restricting the search to the set of Kalman filters defined on P, the minmax information criterion defined in Section A implies that the desired estimator is the Kalman filter corresponding to a parameter point $t\epsilon P$, which is found from

$$\min_{t\epsilon P} \max_{p\epsilon P} E_p \log \frac{f_p(Y)}{f_t(Y)}$$

where $f_p(Y)$, $f_t(Y)$ and E_p are the probability densities and the expectation associated with the corresponding parameters. (note that if t is identified as the filter design parameter, then, using the notation of section A, we have

$f_p^{t(p)} = f_p^p = f^p$, $f_p^t = f_t^t = f_t$ and $E_p^{t(p)} = E_p^p = E_p$).

For an observation sequence $Y^N = (y_1,\ldots,y_N)$ let us denote

$$I_N(p,t) = \frac{2}{N} E_p \log \frac{f_p(Y^N)}{f_t(Y^N)}$$

then the filter design criterion in finite time may be written as

$$\min_{t\epsilon P} \max_{p\epsilon P} I_N(p,t)$$

The calculation of $I_N(p,t)$ is similar to that of $I_N(p,s)$ in the controller design problem, replacing in the corresponding equations $F(p,s(p))$, $F(p,s)$, $K(p,s)$, $B(p,s(p))$

and $B(p,s(p))$ by $F(t,s^{o})$, $F(p,s^{o})$, $K(p,s^{o})$, $B(t,s^{o})$ and $B(p,s^{o})$ respectively, where s^{o} is the selected controller gain vector, and replacing $\overline{H}(p)$ by $[H(t) - H(p)]$.

As in the case of controller design, steady-state estimator design offers a computational advantage over finite time design. We have the following result.

Theorem 2

Suppose that for each $p \varepsilon P$ the system $(F(p,s^{o}),G(p), H(p))$ is stable, controllable and observable. Then the cost function $I_N(p,t)$ has a finite limit value

Proof: Substituting the corresponding matrices, the proof is identical to that of Theorem 1.

It should be noted that an intermediate result of the proof is that under the condition of the theorem the state estimation error

$$\psi_n(t,p) = E_t\{[x_n-\hat{x}_{n|n-1}(p)][x_n-\hat{x}_{n|n-1}(p)]^T\}$$

has a finite limit value for any t and p. This means that although the non-adaptive filter is not optimal in the least-squares sense at any point but t, its' mean-square error has a finite limit value. The fixed-gain steady state estimator is obtained from

$$\min_{t \epsilon P} \max_{p \epsilon P} I(p,t)$$

where $I(p,t)$ is the limit value of $I_n(p,t)$.

3. Fixed Gain Controller and Filter Design for Aircraft

The proposed design procedure has been applied to the problem of finding fixed gain values for the longitudinal control system of a given fighter aircraft (the YF-16 CCV data has been used in the design). The linearized short period equations are given by (e.g. [7]).

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} a_{11}(p) & a_{12}(p) & a_{12}(p) \\ 1 & a_{22}(p) & a_{22}(p) \\ 0 & 0 & a_{33}(p) \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ w \end{bmatrix} + \begin{bmatrix} d_1(p) \\ d_2(p) \\ 0 \end{bmatrix} \delta e + \begin{bmatrix} 0 \\ 0 \\ g_3(p) \end{bmatrix} \xi$$

where q is the pitch rate, α is the angle of attack, w is a wind gust disturbance, δ_e *is the elevator angle* (no actuator dynamics are introduced here, for simplicity) and ξ is a zero mean unity covariance white noise component of the wind disturbance. The coefficients $a_{ii}(p)$, $d_i(p)$ and $g_3(p)$ are functions of the flight condition parameters, $p = (h,M)$, where h is the altitude and M is the Mach number. Pitch rate and normal acceleration measurements are used for feedback purposes. The measurement equation is

$$y = \begin{bmatrix} q \\ a_{nz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{22}(p)U_o & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ d_2(p)U_o \end{bmatrix} \delta_e + v$$

where a_{nz} is the normal acceleration, U_o is the forward velocity and v is a zero mean white noise process. Typical sensor accuracy and wind disturbance data can be found in the literature (e.g. [8]). The open loop dynamical system was found to be unstable for most of the given flight conditions.

For digital control purposes the equations have been disretized at 0.02 seconds sampling intervals. The resulting system has the form

$$x_{n+1} = F(p)x_n + D(p)\delta_{e_n} + G(p)\xi_n$$

$$y_n = H(p)x_n + v_n$$

As a first step in the design procedure an optimal controller is designed for each of the flight conditions. While any desired criterion could be used at this stage, we have chosen for illustration purposes the linear quadratic criterion [8]

$$\min_{z_n} \sum_{n=0}^{\infty} \left[\frac{q_n^2}{q_{max}^2} + \frac{a_{nz_n}^2}{a_{nz_{max}}^2} + \frac{\delta e_n^2}{\delta e_{max}^2} \right]$$

It has been found that for all given p values the pair (F(p),D(p) is stabilizable and the pair (F(p),C) is observable where

$$C = \begin{bmatrix} \frac{U_o}{100g} & 0 & 0 \\ 0 & \frac{U_o^2 a_{22}(p)}{36g^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Flight Cond. No.	alt. (ft)	Mach No.	Dyn. Press. (lb/ft^2)	s_α	s_q	s_w
1	S.L.	0.6	363.55	-1.377	-2.048	-1.5242
2	15,000	0.6	205.07	-1.121	-1.101	-0.0413
3	30,000	0.6	107.85	-0.9603	-0.608	-0.0177
4	S.L.	0.8	646.32	-1.3916	-3.395	-0.0077
5	15,000	0.8	364.57	-1.197	-1.943	-0.0072
6	30,000	0.8	191.73	-1.079	-1.115	-0.0027
7	S.L.	0.9	818.00	-1.3175	-4.307	-0.0030
8	15,000	0.9	461.41	-1.1903	-2.575	0.0051
9	30,000	0.9	242.66	-1.1057	-1.525	0.0028
10	S.L.	0.95	911.41	-1.26[illegible]	-5.127	0.0261
11	15,000	0.95	514.10	-1.1086	-2.935	0.0456
12	30,000	0.95	270.37	-1.0575	-1.687	0.0225
13	S.L.	1.2	1,454.22	-1.6637	-6.782	0.1931
14	15,000	1.2	820.28	-1.4532	-4.044	0.2061
15	30,000	1.2	431.39	-1.3345	-2.344	0.1092
16	30,000	1.7	865.77	-1.7801	-2.698	0.1281
17	50,000	1.7	335.37	-1.6837	-1.383	0.0554

Table 1. Optimal (Linear Quadratic) Feedback Gains for Given Flight Conditions.

so that the resulting controller (regulator)

$$z_n = -s(p)x_n$$

is stabilizing. In addition, the system $(F(p,s), G(p), H(p))$ is controllable and observable for any s and p so that the conditions of Theorems 1 and 2 are satisfied and the fixed-gain design criteria are well defined.

In searching for a fixed-gain controller we have restricted ourselves to the finite set of optimal gain values s(p) for the 17 given flight conditions. These are given in Table 1. The fixed gain controller design criterion now becomes

$$\min_i \max_j \{I(j,i) \ ; \ i,j = 1,\ldots,17\}$$

The resulting fixed gain controller gains are

$$s_q = -1.4532$$

$$s_\chi = -4.0437$$

$$s_w = 0.2061$$

(these are the optimal controller gains corresponding to flight condition 14). It remains to find a flight condition-independent state estimator. Substituting the fixed controller gains into the dynamical models for each of the flight conditions, we get 17 "closed-loop" models. Employing the filter design criterion presented in section 2.2, and restricting the search to the set of 17 optimal Kalman filters, the filter corresponding to flight condition 3 is found to be the overall (non-adaptive) optimal filter. The resulting control system is given schematically in figure 3.1

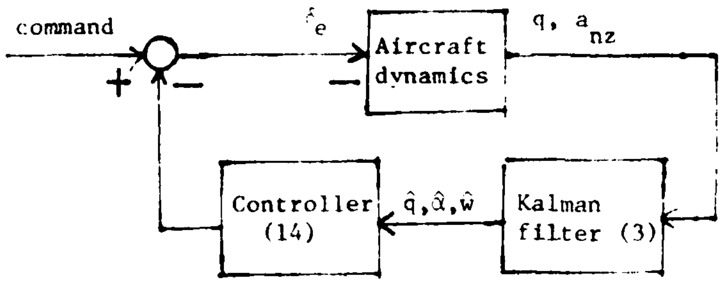


Figure 3.1 Resulting Fixed-Gain Control System for Aircraft

Although we did not specifically introduce performance requirements into the design procedure, it is interesting to examine the performance of the resulting control system at different flight conditions. Loosely speaking, it might be expected that with a fixed gain controller the aircraft responses would be faster at high dynamic pressures than at low pressures. (The dynamic pressure is $p_d = \frac{1}{2}\rho U_o^2$ where ρ is the atmospheric air density and U_o is the forward velocity.) For high performance military aircraft it is necessary that even at low dynamic pressures the control system maintains sufficient maneouvring power (i.e. fast response) which is acceptable to the pilot.

The aircraft response in the pitch rate and in the normal acceleration to a 4^{o} disturbance ("initial condition") in the angle of attack is shown in Figure 3.2. A linearized aircraft model has been used in the simulation. The flight conditions considered, ordered by increasing dynamic pressure are 3, 1, 5 and 11. It is seen that, as expected, the response speed generally increases with the dynamic pressure (Note, however that this dependence is not simple. For instance, the response is significantly faster at flight condition 5 than at flight condition 1, although the corresponding dynamic pressures are nearly the same.) Note that an acceptable settling time of 2-3 sec. is obtained for the lower dynamic pressure range (e.g. flight conditions 1 and 3) while for the higher pressure range it is much shorter (1 sec).

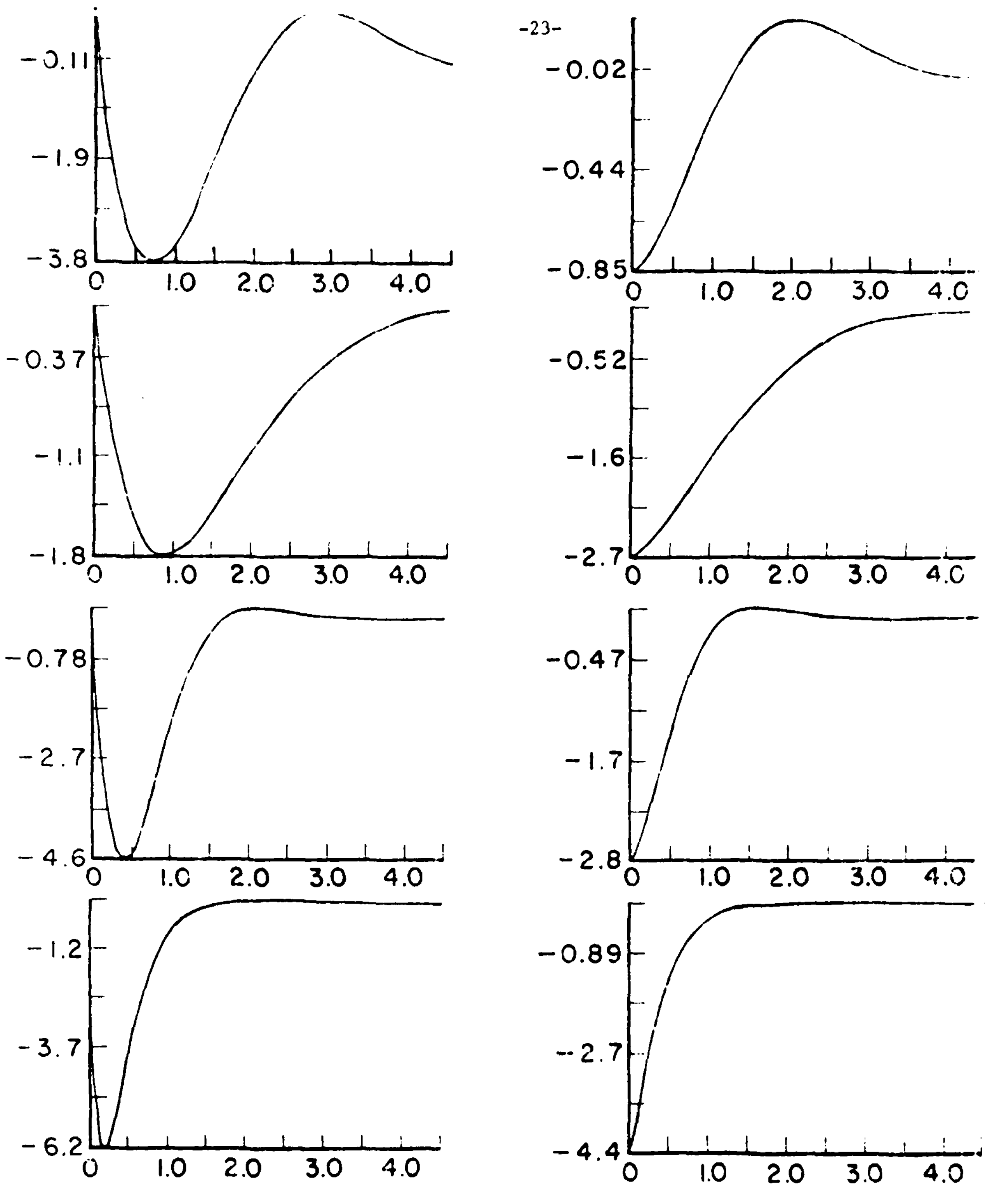
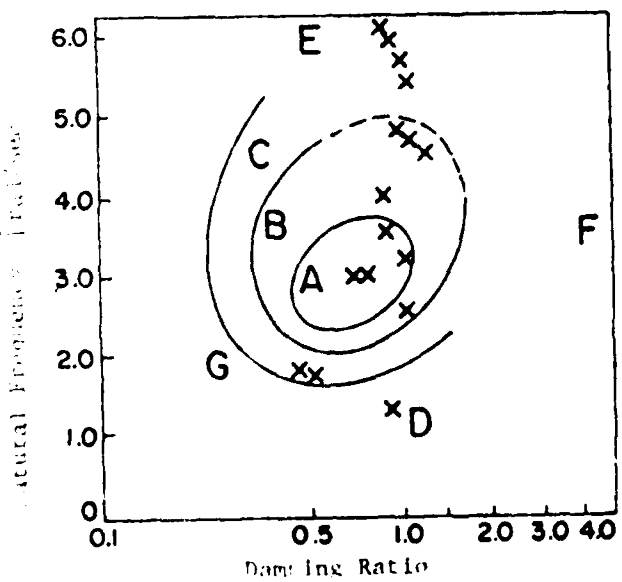


Figure 3.2. Pitch rate (left) and normal acceleration (right) responses of resulting fixed-gain control system at flight conditions 3 (top), 1, 5, 11 (bottom). The dynamic pressure is lowest at f.c.3 and highest at f.c. 11.

Next, the fixed gain design is evaluated using a pilots' opinion chart proposed in [7]. The damping ratio and the natural frequency of the closed-loop system at each of the flight conditions have been calculated and placed on the chart as shown in Figure 3. It is seen that the system's performance at most of the flight conditions is rated as satisfactory or acceptable. (The tendency towards the fast response range and away from the slow response range is also desirable for high performance fighter aircraft). This is particularly remarkable in view of the large variability of the flight conditions and the corresponding aircraft dynamics, and in view of the fact that performance or handling qualities have not been specifically considered in the design.



A - Satisfactory

B - Acceptable

C - Poor

D - Very slow response, large control motion to manoeuvre, difficult to trim

E - Initial response fast, oversensitive, light stick forces

F - Sluggish, large stick motion and forces

G - Slow initially, then oscillatory, tendency to overcontrol

Figure 3.3 Performance evaluation of proposed fixed-gain design at given flight conditions by pilot rating contours.

4. Concluding Remarks

A method for designing fixed-gain controllers and filters for stochastic systems with large parameter variation has been presented with particular attention to the control problem of high performance aircraft. The resulting design has shown good performance qualities, although performance specifications have not been introduced directly into the design. This may be done by first designing each of the pointwise optimal controllers (at each of the flight conditions) so as to meet the specifications, and then imposing the specifications as constraints on the minmax search procedure. In this paper we have found a fixed-gain control system for the entire flight regime. However, since the fixed-gain controller would normally be used as a back-up system in emergency situations, a small subset of the flight conditions corresponding to those situations may be considered in the design, so as to meet tighter performance requirements.

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