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OPTICAL PROCESSING OF ULTRASONIC WAVES

Department Of Mechanical Engineering
Houston, TX

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OPTICAL PROCESSING OF ULTRASONIC WAVES

submitted by

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During the year prior to the contract with ONR, the principal investigator had a similar contract. During that year the principal investigator had developed the theory and algorithms of the prediction of the scattering of light by sound and the inversion procedure of investigating the sound fields from the scattered optical data. For the purpose of this report, these theories will be known as the direct theory and the inversion theory.

During the Spring of 1980, the principal investigator trained two students, Mr. Charles Fray and Mr. John R. Laflin in the aspects of acousto-optic interaction.

The objective of this contract was to allow the principal investigator and his graduate students to work with colleagues in the Physical Acoustics Branch of the Naval Research Laboratory, Washington, DC to implement algorithms on their computer data acquisition system.

During the stay at NRL, Mr. Fray, in conjuction with others, developed a modelling system which would precidt the colored schlieren patterns of ultrasonic fields. The output of this model was by colored television display of computed values. The work of Mr. Fray will be continued by those at NRL.

Mr. Laflin pursued the inversion problem. That is, he developed a computer based experimental system to acquire acousto-optic data and process it to reveal the complicated near field of an ultrasonic transducer.

The principal investigator directed the students, collaborated on a new theory for tomographic processing acousto-optic data, and generally supported the Physical Acoustic Branch with theory and concepts in acousto-optics and scattering of sound.

During the extension period of the contract from September 1980 to May 1981, substantial progress has been made in the graphic routines associated with both the direct theory and inversion theory.

Attachment A is a preprint of a manuscript resulting from the tomographic work which was presented to the Acoustic Imaging conferences, Monterey, CA, Spring 1981. Attachemnt B and C are abstracts of papers presented at the Fall 1980 meeting of the Acoustical Society of America, Los Angeles, CA.

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TOMOGRAPHIC EVALUATION OF SOUND FIELDS FROM ACOUSTO-OPTIC DATA

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ABSTRACT

The principles of computerized transverse tomography can be applied to the acousto-optic reconstruction of the local sound pressure of an ultrasonic field. For sufficiently narrow beams of ultrasound in the low megahertz region, the total optical phase retardation of an interrogating light beam can be considered as a projection of the sound field pressure. (Fourier techniques for the numerical reconstruction of the pressure field yield as intermediate steps a Fourier domain associated with the angular spectrum of plane waves comprising the sound field.) Consequently the sound field can be reconstructed in other regions than the plane of interrogation. In this work we discuss two alternative methods for acquiring data. One method builds the Fourier domain along radial spokes which is inconvenient for numerical processing by DFFT algorithms. The other procedure builds the Fourier domain in a nearly rectangular format compatible with two-dimensional DFFT algorithms. With this latter method, it is possible to evaluate the pressure along a line with limited data and a one-dimensional DFFT.

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1.0 INTRODUCTION [redacted]

A sound field of low ultrasonic power, low ultrasonic frequency, and narrow beam width behaves as an optical phase grating. Collimated light passing through such a sound field experiences an optical phase retardation proportional to the local sound pressure, integrated over the light path. This constitutes a "projection" of the sound field and numerical methods of computerized transverse tomography can be applied to estimate the local sound pressure.

[redacted]

Numerical techniques using Fourier transforms are useful in pressure field evaluation since an intermediate step yields the Fourier domain associated with the angular spectrum of plane waves comprising the sound field. Moreover, by modification of the phase terms of each plane wave of the angular spectrum, it is possible to construct the sound field at different planes. In other words, an estimate of most of the sound field can be computed from a set of acousto-optic data taken over a single transverse plane. The total field, however, cannot be constructed everywhere since evanescent waves near the sound source are not accounted for. This total field concept is valid when the sound field can be described by the Helmholtz equation, thus eliminating application to non-linear or highly attenuated sound fields.

In a series of papers[1-3] directed toward transducer calibration Cook and Berlinghieri have described one method of data collection which we will call Method A. Acousto-optic data is collected at the terminus of the light paths as shown in Figure 1. These light paths are parallel to each other and are in a plane parallel to the surface of the transducer. Sufficient data can be acquired from interrogation of the field in one direction if the field is symmetric. If the field is not symmetric, Method A involves rotation of the transducer about an axis normal to the transducer surface, such as CC'.

Here, we present an alternative method of data collection which we will call Method B. In Method B the transducer is rotated about a line AA' parallel to the transducer surface and located in the plane of the light paths. The line AA' is also perpendicular to the light paths.

We will demonstrate how both methods allow the generation of data in the angular spectrum (plane wave decomposition) domain with both phase and amplitude information to allow evaluation of the pressure field at the plane of measurement using Fourier transforms.

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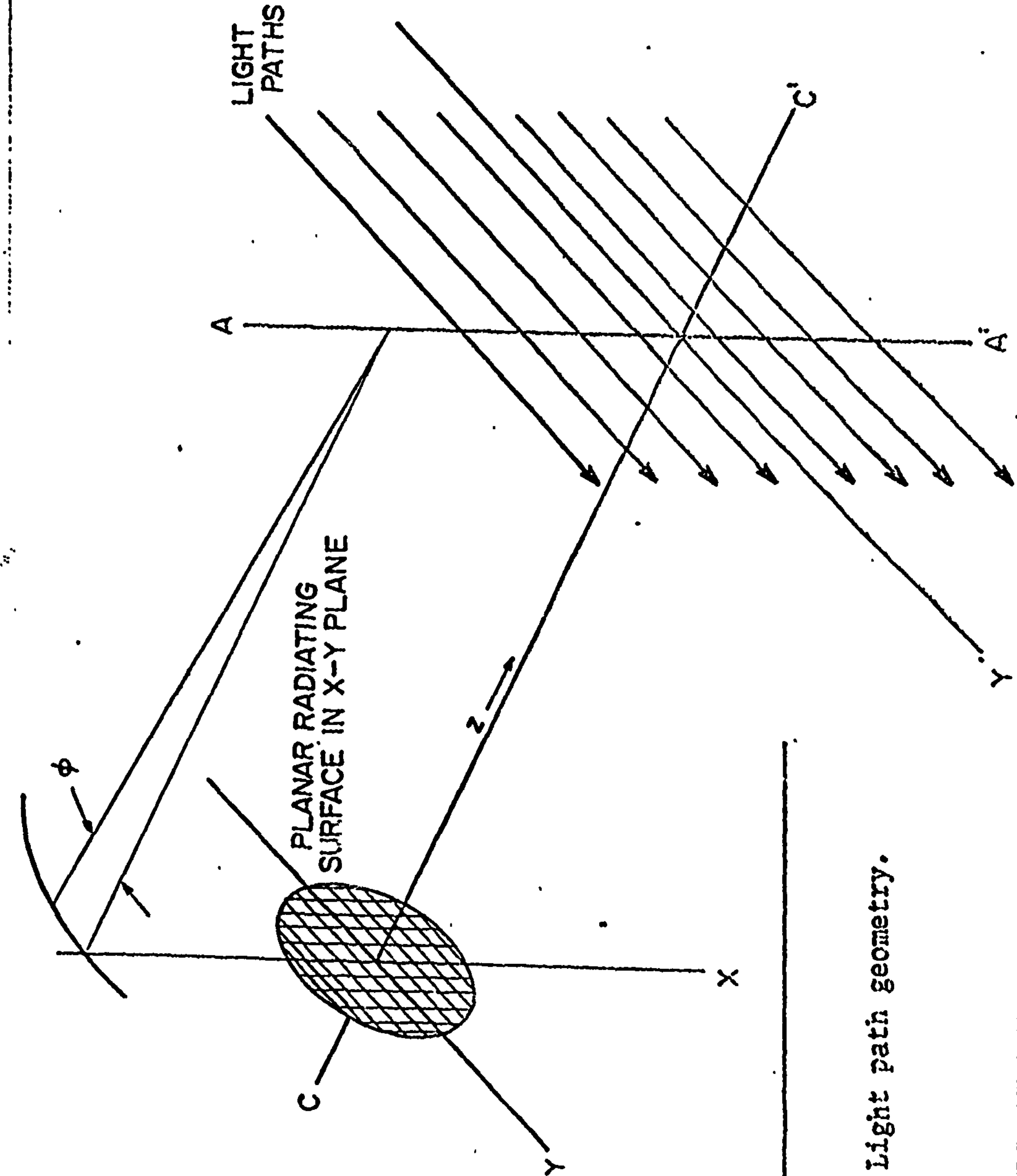

Fig. 1. Light path geometry.

[redacted]

[redacted]

Data acquired using Method A builds the angular spectrum domain in a polar format through a series of one-dimensional Fourier transforms. The pressure field at the plane of measurement can then be obtained by a two- dimensional inverse Fourier transform. DFFT algorithms, however, require data to be in a rectangular format. Two alternatives are to interpolate the rectangular data from the polar data or to perform a Hankel (Fourier-Bessel) transform. The polar data becomes less dense away from the origin so interpolation becomes questionalble there. On the other hand, development of efficient algorithms for Hankel transforms is now an active area of research.[4-6]

Method B exhibits three attractive features. The first is that through a series of one-dimensional transforms the angular spectrum domain can be built in a format which closely approximates a rectangular raster.[10] The second feature is that the data collected by this method lies midway between the angular spectrum domain and the time-space domain. Evaluation of the pressure field at the plane of measurement, therefore, requires only a series of inverse, one-dimensional transforms. A third feature is that acoustic pressure can be computed along a line transverse to the direction of sound propagation by a single inverse one-dimensional transform.

2.0 THEORY OF METHOD A

Consider a harmonic sound field being produced by a planar transducer. Let the pressure at a distance z from the transducer be expressed as

$$p(x,y,z,t) = \tilde{p}(x,y,z)\exp(-j\omega t) \qquad (1)$$

In the following discussion the time variance will be dropped for convenience.

Line integrals across the pressure field at $z+z_o$ can be written

$$\tilde{p}(x,z_o) = \int \tilde{p}(x,y,z_o)dy \qquad (2)$$

where the limits of this integral and others are taken from minus infinity to plus infinity. $\tilde{p}(x,z_o)$ can be seen as a "projection" of the pressure field $\tilde{p}(x,y,z_o)$.

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In the design of this experiment, $\tilde{p}(x,z_o)$ is obtained from measurement of the Raman-Nath parameter V defined as

$$V(x,z_o) = [2\pi k/\lambda]\ \tilde{p}(x,z_o) \qquad (3)$$

where λ is the optical wavelength in vacuum and k is the medium piezo-optic coefficient which relates the index of refraction to changes in acoustic pressure. This parameter V is a measure of the optical phase retardation induced by the sound field. It is a common paramter used in most theories and can be inferred from acousto-optic measurements. This parameter, in our case, is to be measured as a phasor. Various techniques for acquiring the necessary phase and amplitude information can be found in the literature[7-8] [redacted]

We will show the relation between the projected pressure and the Fourier domain assuming $p(x,z_o)$ to be a measurable quantity. Substitution of a two-dimensional transform expression into the integral of Equation (2) gives [redacted]

$$\tilde{p}(x,z_o) = \iiint \tilde{p}(k_x,k_y;z_o)\exp[j(k_x x+k_y y)]\,dy\,dk_x\,dk_y \qquad (4)$$

where k_x and k_y are components of the acoustic wave vector k . The integration of the y-variable can be completed yielding the Dirac-δ function $2\pi\delta$ (k). The sifting properties of the δ function upon integration over k produce the desired result

$$\tilde{p}(x,z_o) = 1/2\pi \int p(k_x,0;z_o)\exp(jk_x x)\,dk_x\ . \qquad (5)$$

This result, sometimes referred to as the "Fourier projection-slice theorem," states that the Fourier transform of a projection is a slice of the Fourier transform of the projected function. In other words, the one-dimensional transform of $\tilde{p}(x,z_o)$ produces a single line in the Fourier domain. This line lies perpendicular to the direction of the light paths, that is the line of interrogation. Consequently, if one rotates the transducer as specified for Method A in equal angular increments (essentially around the sound field axis), the projections obtained are related to values in the Fourier domain located along radial lines. In other words, taking a discrete one-dimensional transform of the projections $\tilde{p}\ (x,z_o)$ yields values shown as circles in Figure 2. This result is not restricted to sound fields and is a general result of projection theory.

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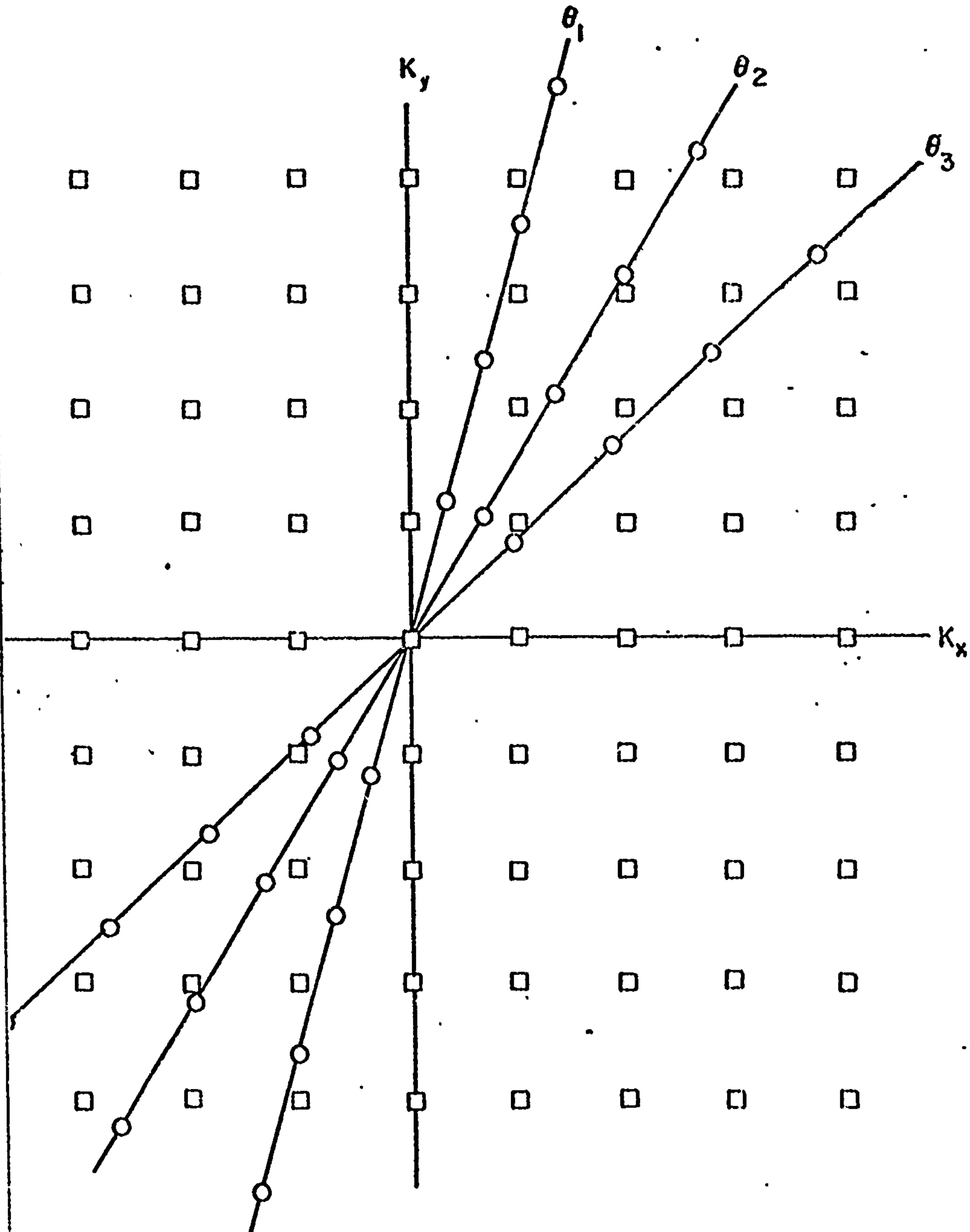

Fig. 2. Fourier domain, Method A.

[redacted]

[redacted]

The previously stated difficulty is in the inversion of data in the Fourier domain to the time-space domain from the polar format. In addition to interpolation to a rectangular format, Mersereau and Oppenheim suggest other schemes to circumvent this problem.[9]

3.0 THEORY OF METHOD B

Consider the origin of the (x,y,z) coordinate axes to be the point of intersection of the acoustic axis and the plane of the light slice. Again, z is the acoustic axis. The light path slice will now be at an angle with the y-axis. We can write the projection of the pressure field with the notation changed to account for the angle as

[redacted]

[redacted] $$\tilde{p}(x,\phi,z) = \int \tilde{p}(x,y,z)\,dy' \qquad (6)$$

where dy' is along the light paths at an angle ϕ with the y-axis.

The Fourier domain pressure can be expressed as

$$\tilde{p}(k_x,k_y,z) = \tilde{p}_o(k_x,k_y)\exp(jk_z z) \qquad (7)$$

where $\tilde{p}_o(k_x,k_y)$ is the Fourier component at $z = 0$ and k_z is the z-component of the acoustic wave vector. We again substitute the Fourier description of the pressure field in Equation (6) and incorporate Equation (7) to give

$$\tilde{p}(x,\phi,z) = \iiint \tilde{p}_o(k_x,k_y)\exp[j(k_x x+k_y y+k_z z)]\,dy'\,dk_x\,dk_y \qquad (8)$$

The light path slice and the x-axis defines a new coordinate system (x,y',z'). This is related to the (x,y,z) coordinate system by the transformation

$$\begin{aligned} y &= y'\cos\phi + z'\sin\phi \\ z &= -y'\sin\phi + z'\cos\phi \end{aligned} \qquad (9)$$

Substituting this expression into Equation (8) and collecting terms of integration of dy', we find the integral

$$\int \exp[j(k_y\cos\phi - k_z\sin\phi)y']\,dy' = 2\pi\delta(k_y\cos\phi - k_z\sin\phi) \qquad (10)$$

[redacted]

If we define [redacted]

$$k_y' = k_z \tan\phi \qquad (11)$$

we can write the Dirac-δ function of Equation (10) as $2\pi\delta(k_y - k_y')$. Equation (8) can now be written as

[redacted]

$$\tilde{p}(x,\phi,z) = 1/2\pi \int \tilde{p}_o(k_x,k_y)\,\delta(k_y-k_y')\ X$$

[redacted]

$$\exp[j(k_x x + k_y z'\sin\phi + k_z z'\cos\phi)]\,dk_x dk_y \qquad (12)$$

[redacted]

The integral over dy' can be evaluated using the sifting properties of the δ function,

AND ADDRESS

$$\tilde{p}(x,\phi,z) = 1/2\pi \int \tilde{p}_o(k_x,k_y')\ X$$

[redacted]

$$\exp[j(k_x x + k_y' z'\sin\phi + k_z z'\cos\phi)]\,dk_x \qquad (13)$$

Since the origin of the coordinate system is in the plane of the light path slice, we have $z' = z_o = 0$. Equation (13) now becomes

$$\tilde{p}(x,\phi,z_o) = 1/2\pi \int \tilde{p}_o(k_x,k_y')\exp(jk_x x)\,dk_x \qquad (14)$$

Equation (11) can be restated as

$$k_y' = (k^2 - k_x^2)\sin\phi. \qquad (15a)$$

If k_x is small compared to k, then the following approximation holds.

$$k_y' = k\sin\phi \qquad (15b)$$

When this approximation is substituted into Equation (14), $\tilde{p}(x,\phi,z_o)$ becomes $\tilde{p}(x,k_y',z_o)$

$$\tilde{p}(x,k_y',z_o) = 1/2\pi \int \tilde{p}_o(k_x,k_y')\exp(-jk_x x)\,dk_x \qquad (16)$$

[redacted]

which is the main result of Method [redacted]

It is important to note that $\tilde{p}(x,k_y',z_o)$ lies midway between the Fourier domain and the time-space domain. If we take the inverse Fourier transform of $\tilde{p}(x,k_y',z_o)$ with respect to the variable k_y', we obtain

$$1/2\pi \int \tilde{p}(x,k_y',z_o)\exp(jk_y'y)dk_y' =$$

[redacted] $$(1/2\pi)^2 \iint \tilde{p}_o(k_x,k_y',z_o)\exp[j(k_x x+k_y'y)]dk_x dk_y' \qquad (17)$$

The term on the right-hand side can be recognized as $p(x,y,z_o)$. Thus from a set of measurement taken at a given elevation (x fixed) and varying angle, we can apply a one-dimensional Fourier transform to obtain the pressure along the y-axis for that value of x. The pressure over a specified x-y plane can also be obtained by a series of such transforms taken at equally spaced values of x.

[redacted])

If we take the Fourier transform of $\tilde{p}(x,k_y',z_o)$ with respect to the variable x, we obtain

$$\tilde{p}(k_x,k_y',z_o) = \int \tilde{p}(x,k_y',z_o)[\exp(-jk_x x)]dk_x \qquad (18)$$

which is the pressure field in the Fourier domain.

Implementation of Method B using DFFT algorithms requires the approximation that k_y' does not depend on k_x. This approximation is valid when the ultrasound is confined to a narrow beam as with sound field produced by most medical and NDE transducers.

To illustrate this approximation we show in Figure 3 the nearly rectangular format for data obtained from Equation (16) using an DFFT. The error incurred by assuming this format to be rectangular will be small if most of the radiated energy is near the origin in the Fourier domain. The illustration in Figure 3 is for a sound field produced by a circular transducer of radius $a = 10\lambda$ where λ is the acoustic wavelength. Each of the larger concentric circles has associated with it a percentage of total radiated energy contained within the circle. The percentages were calculated using an Airy pattern to approximate the sound field. Figure 3 shows the format to be essentially rectangular for 98% of the radiated energy in this case. Figure 4 shows the general curve for the fraction of total energy radiated as a function of $ka(\sin\phi)$ for the Airy function.

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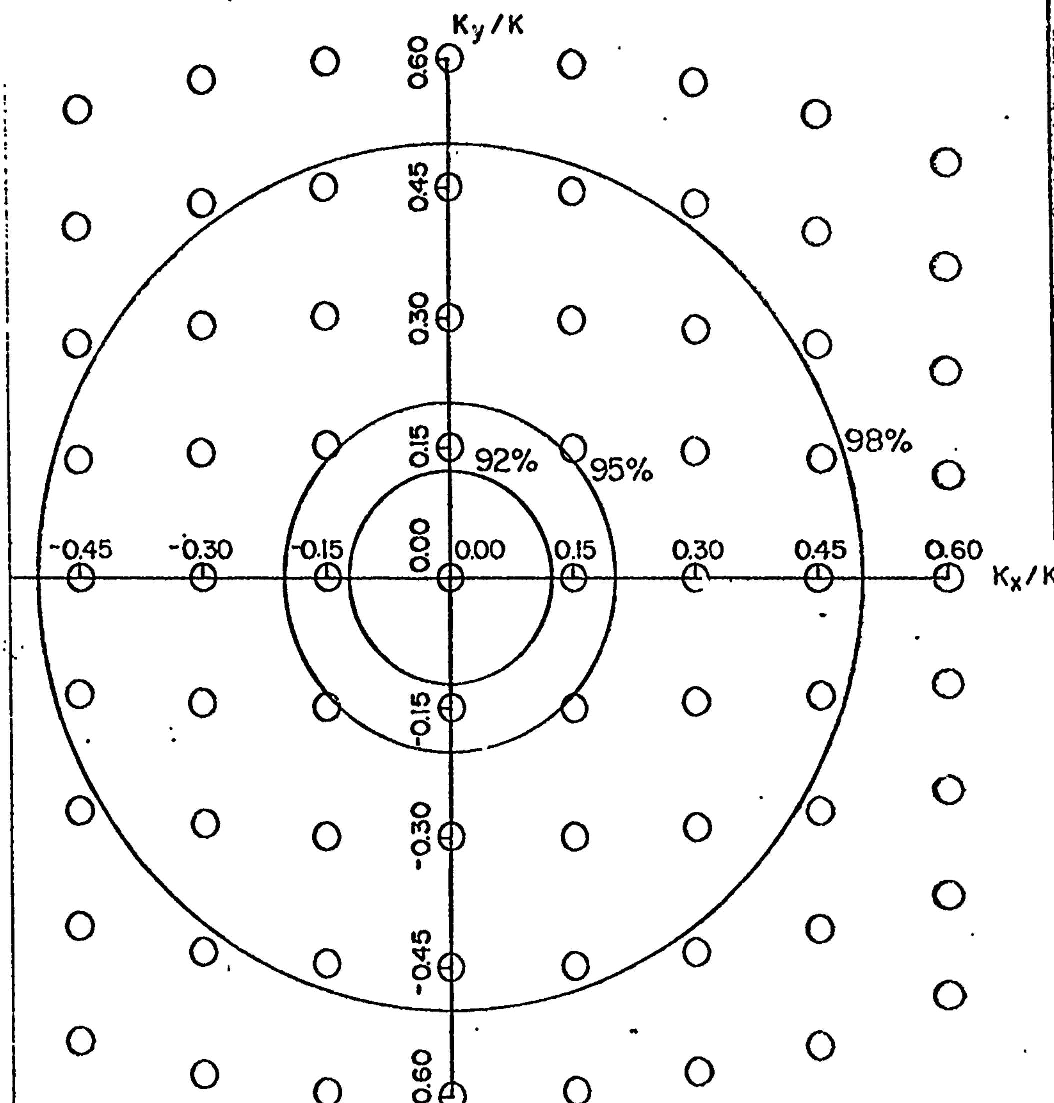

Fig. 3. Fourier domain, Method B.

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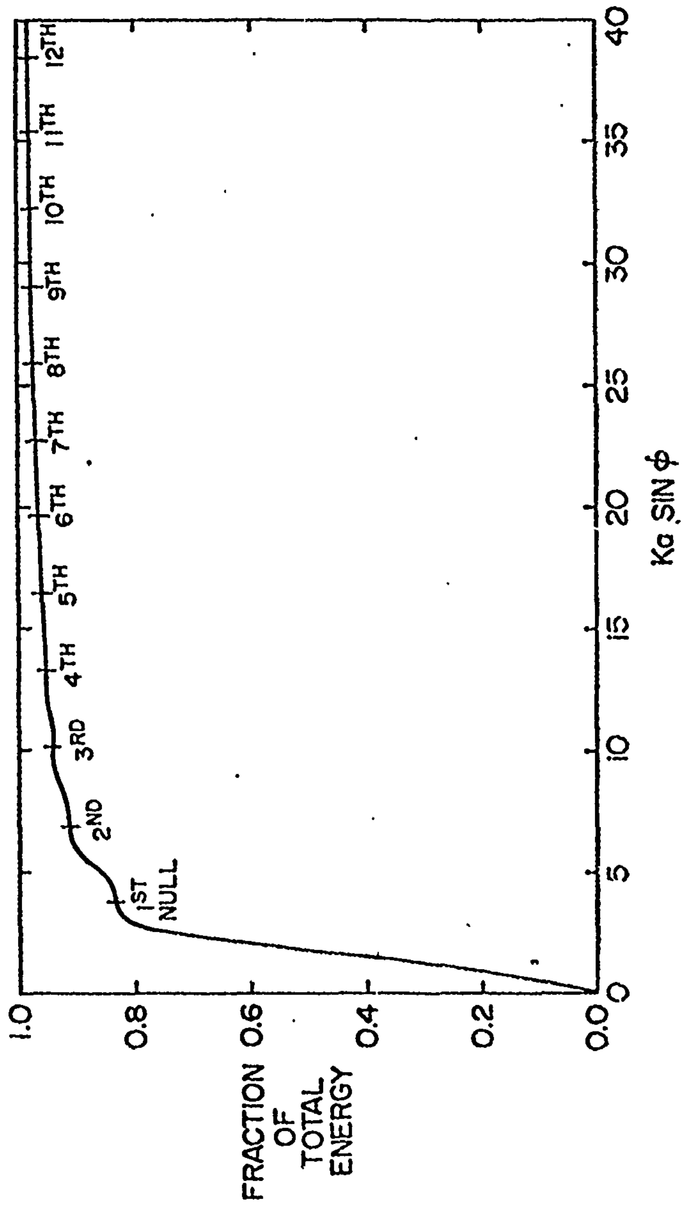

Fig. 4. Distribution of radiated energy.

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4.0 DATA ACQUISITION USING METHOD [redacted])

Equations (15a) and (15b) indicate that the angle ϕ should change by equal increments of $\sin\phi$, or that

$$\sin\phi = +/- n\alpha \tag{19}$$

[redacted]

where $n = 0,1,2,\ldots,N$ with $2N+1$ = number of experimental points and α = specified increment for $\sin\phi$.

The maximum value of k_y observed in the data is

[redacted]

[redacted] $$(k_y)_{max} = N k\alpha \tag{20}$$

[redacted]

From Equation (15b), we also know

[redacted]

$$(k_y)_{max} = k(\sin\phi_{max}) \tag{21}$$

Combining Equations (19), (20) and (21) yields

$$\Delta k_y = k\alpha \tag{22}$$

The increments of y in the transformed data returned by the DFFT are $y = m\Delta y$ where $m = 0,1,2,\ldots,M$ and M is the number of points used in the DFFT. From the periodicity of the DFFT, we find

$$M\Delta k_y\Delta y = 2\pi \tag{23}$$

or

$$\Delta y = c/(M\alpha f) \tag{24}$$

where c is the sound speed, and the acoustic frequency $f = c/\lambda$. The total range on the y-axis is then

$$M\Delta y = c/(\alpha f) . \tag{25}$$

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5.0 EXPERIMENTAL RESULTS [redacted]

Cook and Berlinghieri have experimentally demonstrated the validity of Method A.[2] They generated a rectangular array of points in the Fourier domain by rotating the transducer through the proper angles and sampling the linear scan at proper intervals. They then used a two-dimensional DFFT to reconstruct the acoustic field in the plane of interrogation and in nearby planes. [redacted]

[redacted] Method B was used to calculate the pressure distribution over a transverse plane for a 1.25 cm. diameter PZT transducer submersed in water. The transducer was operating at 2.2 Mhz (a = 9.3 λ). A total of 41 x 41 data points using a value of ϕ = sin(1 degree) were used to measure the pressure in a plane 5 cm. from the transducer face. This data was used to construct the pressure field over an area 3.9 x 3.9 cm. The results are shown in Figure 5. The effects of the approximation made in Equation (15) are noticeable in the reconstruction. The reconstructed field is slightly elliptical rather than circular due to the outward displacement of the K wave-vector components in the reconstruction.

6.0 REFERENCES

1. J. Berlinghieri and B. Cook, J. Acoust. Soc. Am. 58, 823-827 (1975).

2. B. Cook and J. Berlinghieri, Proc. IEEE Ultrason. Symp. 75, CHO 944-SU, 133-135 (1975).

3. B. Cook and J. Berlinghieri, J. Acoust. Soc. Am. 61, 147-1480 (1977).

4. J. P. Clero and C. H. Durney, Proc. IEEE 67, 1463 (1979).

5. Alan V. Oppenheim, G. V. Frisk, and D. R. Martinez, Proc. IEEE 66, 264-265 (1978).

6. A. J. Jerri, Applicable Analysis 7, 97-109 (1978).

7. Ward A. Riley, J. Acoust. Soc. Am. 65, 82-85 (1979).

8. B. Cook, J. Acoust. Soc. Am. 60, 95-99 (1976).

9. R. Mersereau and A. Oppenheim, Proc. IEEE 62, 1319-1338 (1974). [redacted]

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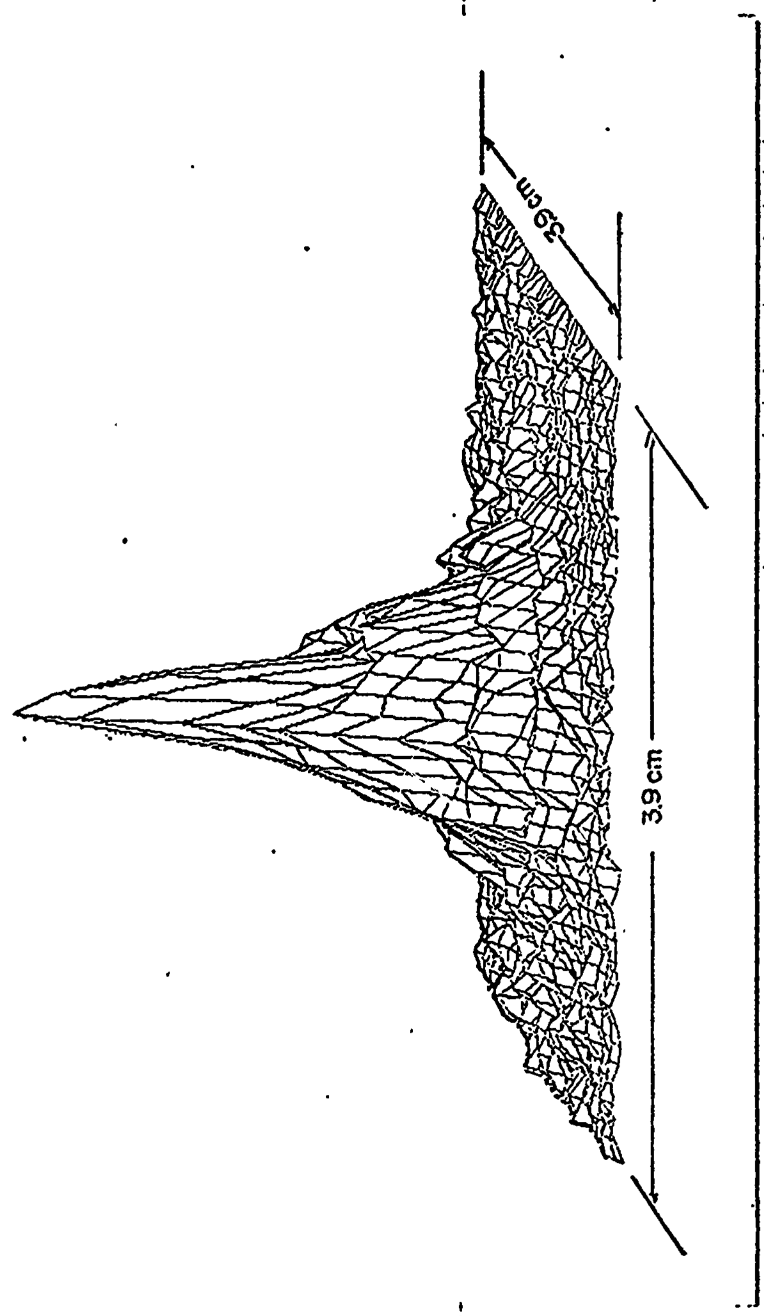

Fig. 5. Reconstructed image.

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