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TABULAR AIDS FOR FITTING WEIBULL MOMENT ESTIMATES.(U)
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It is well known that moment estimates for the Weibull distribution with shape parameters β and scale parameter δ can be found by solving for $\hat{\beta}$ the equation

$$s^2/\bar{x}^2 = [\Gamma(1 + 2/\hat{\beta})/\Gamma^2(1 + 1/\hat{\beta})] - 1; \quad (1)$$

and having found $\hat{\beta}$, obtaining the estimate for δ as

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$$\hat{\delta} = x/\Gamma(1 + 1/\hat{\beta}), \quad (2)$$

where $\bar{x}$ and s^2 are the sample mean and variance.

To facilitate the solutions for β and δ , we provide a table of β vs. the right-hand side of equation (1) and, in parallel, of $\Gamma(1 + 1/\beta)$.

The method-of-moments is known to be inefficient with respect to a number of other estimation procedures and should be used only when the nature of the data available does not permit the use of one of these better methods, or if the sample size is "large". Examples of the former situation, and of the use of the tables, are provided.

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Tabular Aids for Fitting Weibull Moment Estimates¹

W.R. Blischke,² L. Guin,² E.M. Scheuer³

1. Introduction and Summary

It is well-known that moment estimates for the Weibull distribution with shape parameter β and scale parameter δ ,

$$(1) \quad f(t) = \frac{\beta}{\delta} \left(\frac{t}{\delta} \right)^{\beta-1} \exp \left[- (t/\delta)^\beta \right], \quad \begin{matrix} t > 0 \\ \beta > 0, \delta > 0 \end{matrix}$$

can be found by solving for $\hat{\beta}$ the equation

$$(2) \quad \hat{\sigma}^2 / \hat{\mu}^2 = \left[\Gamma(1 + 2/\hat{\beta}) / \Gamma^2(1 + 1/\hat{\beta}) \right] - 1$$

and, having found $\hat{\beta}$, obtaining the estimate for δ as

$$(3) \quad \hat{\delta} = \hat{\mu} / \Gamma(1 + 1/\hat{\beta}).$$

The quantities $\hat{\mu}$ and $\hat{\sigma}^2$ are sample estimates of population mean and variance.

To facilitate the solution for $\hat{\beta}$ and $\hat{\delta}$ we provide herein a table of β vs. the right-hand side of equation (2) and, in parallel, of $\Gamma(1 + 1/\beta)$.

2. Why use the method-of-moments?

It is well-known that the method-of-moments is inefficient with respect to a number of other estimation procedures and that it should be used only when the nature of the data available does not permit the use of one of these better methods, or if the sample size is "large." Here is an instance of the former situation.

The data available for analysis come from k independent samples, all presumably from the same Weibull population. The sample sizes are

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$n_1, n_2, \dots, n_k$. The individual values T_{ij} ($j = 1, 2, \dots, n_i; i=1, \dots, k$)

are not available; only the sums $T_i = \sum_{j=1}^{n_i} T_{ij}$ -- representing the total

time-on-test in the i -th sample, and the sample sizes, n_i ($i=1, 2, \dots, k$) are known.

Let $T = \sum_{i=1}^k T_i$ and $n = \sum_{i=1}^k n_i$. It is easy to verify that $\hat{\mu}_i = T_i/n_i$

is an unbiased estimate of μ , the population mean, based on data from the i -th sample alone, that $\hat{\mu} = T/n$ is an unbiased estimate of μ , and that

$$(4) \quad \hat{\sigma}^2 = \frac{1}{k-1} \sum_{i=1}^k n_i (\hat{\mu}_i - \hat{\mu})^2$$

is an unbiased estimate of σ^2 , the population variance.*

An example follows:

3. Example

We generated random variates from a Weibull distribution with $\delta = 1000$ and $\beta = 3$. We took $k = 4$, $n_1 = 5$, $n_2 = 8$, $n_3 = 9$, $n_4 = 5$. We found $T_1 = 5525.22$, $T_2 = 5321.26$, $T_3 = 10,783.55$ and $T_4 = 5806.61$. Thus $\hat{\mu}_1 = 1105.04$, $\hat{\mu}_2 = 665.16$, $\hat{\mu}_3 = 1198.17$, $\hat{\mu}_4 = 1161.32$, $\hat{\mu} = 1016.17$, $\hat{\sigma}^2 = 476,213.29$ and $\hat{\sigma}^2/\hat{\mu}^2 = .461177$. Using our table, we obtain $\hat{\beta} \approx 1.50$ and $\hat{\delta} = \frac{1016.17}{.90275} = 1125.64$.

4. The Tables

As stated at the outset, we tabulate β vs $\left[\Gamma(1 + 2/\beta) / \Gamma^2(1 + 1/\beta) \right] - 1$ vs $\Gamma(1 + 1/\beta)$ for β over the range 0.200 to 10.000, this chosen because

*It should be noted that the foregoing doesn't depend at all upon the Weibull assumption. It is true generally under the assumptions of independence and homogeneity - provided, of course, that the common distribution has a variance.

it covers what we believe to be the set of values likely to be encountered in practice. The increments for β are not uniform, but selected to provide for "smooth" coverage of the functions being tabulated. The tabulation is for $\beta = 0.2(.001)0.5(.01)1.0(.10)10.0$.

The tables are appended.

5. Other Treatments of This Problem

Marquina (1979) has suggested a root-finding procedure to estimate β from eq. (2) -- or rather its reciprocal. Shooman (1968) has suggested an iterative approach. The latter seems less tractable than our tabular approach. If more accuracy is needed, our approach could be used to obtain a good starting value for a root-finding procedure, such as the one proposed by Marquina.

REFERENCES

Marquina, Nelson (1979), "A Simple Approach to Estimating Weibull Parameters," presented at the Annual Meeting of the American Statistical Association, Washington, D.C., 13-16 August 1979.

Shooman, Martin L. (1968), Probabilistic Reliability: An Engineering Approach, McGraw-Hill.

β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$
0.200	251.00010	120.00006
0.201	243.03555	115.02019
0.202	235.40051	110.30550
0.203	228.07858	105.83927
0.204	221.05433	101.60598
0.205	214.31318	97.59121
0.206	207.84140	93.78157
0.207	201.62602	90.16461
0.208	195.65479	86.72872
0.209	189.91613	83.46312
0.210	184.39912	80.35776
0.211	179.09340	77.40325
0.212	173.98920	74.59085
0.213	169.07725	71.91239
0.214	164.34880	69.36025
0.215	159.79554	66.92729
0.216	155.40960	64.60685
0.217	151.18353	62.39270
0.218	147.11026	60.27899
0.219	143.18307	58.26025
0.220	139.39561	56.33135
0.221	135.74183	54.48749
0.222	132.21599	52.72415
0.223	128.81265	51.03710
0.224	125.52660	49.42235
0.225	122.35294	47.87617
0.226	119.28697	46.39505
0.227	116.32424	44.97567
0.228	113.46050	43.61492
0.229	110.69171	42.30987
0.230	108.01403	41.05775
0.231	105.42379	39.85598
0.232	102.91749	38.70207
0.233	100.49180	37.59374
0.234	98.14356	36.52877
0.235	95.86972	35.50512
0.236	93.66740	34.52084
0.237	91.53383	33.57406
0.238	89.46639	32.66307
0.239	87.46255	31.78619
0.240	85.51991	30.94189
0.241	83.63616	30.12867
0.242	81.80913	29.34513
0.243	80.03669	28.58996
0.244	78.31684	27.86189
0.245	76.64766	27.15974
0.246	75.02731	26.48236
0.247	73.45402	25.82870
0.248	71.92611	25.19772
0.249	70.44196	24.58846
0.250	69.00002	24.00001
0.251	67.59881	23.43148
0.252	66.23690	22.88205
0.253	64.91293	22.35092
0.254	63.62559	21.83735
0.255	62.37362	21.34061
0.256	61.15582	20.86003
0.257	59.97104	20.39496
0.258	58.81816	19.94477
0.259	57.69611	19.50888

β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$
0.260	56.60387	19.08673
0.261	55.54045	18.67777
0.262	54.50492	18.28150
0.263	53.49636	17.89743
0.264	52.51389	17.52509
0.265	51.55667	17.16403
0.266	50.62391	16.81384
0.267	49.71480	16.47410
0.268	48.82862	16.14443
0.269	47.96464	15.82445
0.270	47.12215	15.51381
0.271	46.30050	15.21217
0.272	45.49905	14.91921
0.273	44.71716	14.63460
0.274	43.95424	14.35806
0.275	43.20972	14.08930
0.276	42.48304	13.82803
0.277	41.77366	13.57402
0.278	41.08107	13.32699
0.279	40.40477	13.08671
0.280	39.74428	12.85294
0.281	39.09914	12.62547
0.282	38.46889	12.40409
0.283	37.85311	12.18858
0.284	37.25138	11.97875
0.285	36.66329	11.77442
0.286	36.08845	11.57540
0.287	35.52650	11.38152
0.288	34.97706	11.19261
0.289	34.43979	11.00850
0.290	33.91434	10.82906
0.291	33.40038	10.65412
0.292	32.89761	10.48354
0.293	32.40571	10.31719
0.294	31.92449	10.15473
0.295	31.45335	9.99664
0.296	30.99233	9.84310
0.297	30.54104	9.69447
0.298	30.09924	9.54436
0.299	29.66666	9.40075
0.300	29.24307	9.26053
0.301	28.82823	9.12361
0.302	28.42191	8.98988
0.303	28.02389	8.85926
0.304	27.63394	8.73164
0.305	27.25188	8.60694
0.306	26.87748	8.48508
0.307	26.51056	8.36598
0.308	26.15093	8.24954
0.309	25.79841	8.13571
0.310	25.45280	8.02439
0.311	25.11395	7.91553
0.312	24.78168	7.80906
0.313	24.45584	7.70490
0.314	24.13625	7.60299
0.315	23.82278	7.50327
0.316	23.51527	7.40568
0.317	23.21357	7.31016
0.318	22.91755	7.21666
0.319	22.62706	7.12512

$$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1 \quad \Gamma(1+\frac{1}{\beta})$$

0.320	22.34199	7.03548
0.321	22.06219	6.94770
0.322	21.78755	6.86173
0.323	21.51794	6.77752
0.324	21.25324	6.69502
0.325	20.99334	6.61419
0.326	20.73814	6.53499
0.327	20.48751	6.45737
0.328	20.24135	6.38129
0.329	19.99957	6.30671
0.330	19.76206	6.23360
0.331	19.52873	6.16191
0.332	19.29948	6.09162
0.333	19.07422	6.02268
0.334	18.85286	5.95507
0.335	18.63532	5.88875
0.336	18.42150	5.82368
0.337	18.21133	5.75984
0.338	18.00473	5.69720
0.339	17.80162	5.63573
0.340	17.60192	5.57540
0.341	17.40557	5.51618
0.342	17.21248	5.45805
0.343	17.02259	5.40098
0.344	16.83582	5.34494
0.345	16.65213	5.28991
0.346	16.47143	5.23588
0.347	16.29366	5.18280
0.348	16.11877	5.13067
0.349	15.94669	5.07946
0.350	15.77737	5.02915
0.351	15.61075	4.97971
0.352	15.44677	4.93114
0.353	15.28537	4.88340
0.354	15.12652	4.83648
0.355	14.97014	4.79037
0.356	14.81621	4.74504
0.357	14.66465	4.70048
0.358	14.51543	4.65667
0.359	14.36851	4.61359
0.360	14.22383	4.57123
0.361	14.08134	4.52957
0.362	13.94102	4.48859
0.363	13.80281	4.44829
0.364	13.66667	4.40865
0.365	13.53257	4.36965
0.366	13.40045	4.33128
0.367	13.27030	4.29352
0.368	13.14206	4.25637
0.369	13.01570	4.21981
0.370	12.89119	4.18383
0.371	12.76848	4.14842
0.372	12.64755	4.11356
0.373	12.52837	4.07925
0.374	12.41089	4.04546
0.375	12.29509	4.01220
0.376	12.18094	3.97945
0.377	12.06840	3.94720
0.378	11.95744	3.91544
0.379	11.84805	3.88417

$$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1 \quad \Gamma(1+\frac{1}{\beta})$$

0.380	11.74018	3.85336
0.381	11.63381	3.82302
0.382	11.52892	3.79312
0.383	11.42547	3.76368
0.384	11.32345	3.73467
0.385	11.22281	3.70608
0.386	11.12355	3.67791
0.387	11.02563	3.65016
0.388	10.92904	3.62281
0.389	10.83374	3.59585
0.390	10.73972	3.56928
0.391	10.64695	3.54309
0.392	10.55542	3.51727
0.393	10.46509	3.49182
0.394	10.37595	3.46673
0.395	10.28799	3.44199
0.396	10.20116	3.41759
0.397	10.11547	3.39354
0.398	10.03089	3.36982
0.399	9.94740	3.34642
0.400	9.86498	3.32335
0.401	9.78361	3.30059
0.402	9.70329	3.27815
0.403	9.62398	3.25600
0.404	9.54567	3.23416
0.405	9.46834	3.21261
0.406	9.39199	3.19135
0.407	9.31659	3.17037
0.408	9.24213	3.14967
0.409	9.16859	3.12924
0.410	9.09595	3.10908
0.411	9.02421	3.08918
0.412	8.95335	3.06955
0.413	8.88335	3.05016
0.414	8.81420	3.03103
0.415	8.74589	3.01215
0.416	8.67840	2.99350
0.417	8.61172	2.97510
0.418	8.54583	2.95693
0.419	8.48073	2.93898
0.420	8.41641	2.92127
0.421	8.35284	2.90377
0.422	8.29002	2.88650
0.423	8.22794	2.86944
0.424	8.16658	2.85259
0.425	8.10594	2.83595
0.426	8.04600	2.81951
0.427	7.98675	2.80327
0.428	7.92819	2.78724
0.429	7.87029	2.77139
0.430	7.81306	2.75574
0.431	7.75647	2.74038
0.432	7.70053	2.72500
0.433	7.64522	2.70990
0.434	7.59053	2.69499
0.435	7.53646	2.68025
0.436	7.48299	2.66568
0.437	7.43011	2.65129
0.438	7.37782	2.63706
0.439	7.32611	2.62300

β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$	β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$
0.440	7.27497	2.60911	0.500	5.00000	2.00000
0.441	7.22438	2.59537	0.510	4.73210	1.92951
0.442	7.17435	2.58179	0.520	4.48658	1.86516
0.443	7.12487	2.56837	0.530	4.26097	1.80624
0.444	7.07592	2.55511	0.540	4.05316	1.75218
0.445	7.02750	2.54199	0.550	3.86129	1.70243
0.446	6.97960	2.52902	0.560	3.68373	1.65655
0.447	6.93221	2.51620	0.570	3.51908	1.61415
0.448	6.88534	2.50352	0.580	3.36608	1.57489
0.449	6.83896	2.49099	0.590	3.22365	1.53845
0.450	6.79307	2.47859	0.600	3.09080	1.50458
0.451	6.74767	2.46634	0.610	2.96667	1.47303
0.452	6.70274	2.45422	0.620	2.85050	1.44360
0.453	6.65829	2.44223	0.630	2.74160	1.41611
0.454	6.61431	2.43038	0.640	2.63937	1.39038
0.455	6.57078	2.41865	0.650	2.54324	1.36627
0.456	6.52770	2.40706	0.660	2.45274	1.34365
0.457	6.48507	2.39559	0.670	2.36741	1.32240
0.458	6.44289	2.38424	0.680	2.28687	1.30241
0.459	6.40113	2.37302	0.690	2.21073	1.28358
0.460	6.35980	2.36192	0.700	2.13869	1.26587
0.461	6.31890	2.35093	0.710	2.07043	1.24906
0.462	6.27841	2.34007	0.720	2.00569	1.23323
0.463	6.23833	2.32932	0.730	1.94423	1.21825
0.464	6.19866	2.31868	0.740	1.88581	1.20407
0.465	6.15938	2.30816	0.750	1.83023	1.19064
0.466	6.12050	2.29775	0.760	1.77731	1.17790
0.467	6.08201	2.28745	0.770	1.72687	1.16580
0.468	6.04390	2.27725	0.780	1.67875	1.15432
0.469	6.00618	2.26716	0.790	1.63280	1.14339
0.470	5.96882	2.25718	0.800	1.58889	1.13300
0.471	5.93183	2.24730	0.810	1.54690	1.12311
0.472	5.89521	2.23752	0.820	1.50672	1.11369
0.473	5.85895	2.22784	0.830	1.46822	1.10470
0.474	5.82304	2.21826	0.840	1.43133	1.09613
0.475	5.78748	2.20878	0.850	1.39594	1.08796
0.476	5.75227	2.19940	0.860	1.36197	1.08014
0.477	5.71739	2.19011	0.870	1.32935	1.07268
0.478	5.68286	2.18091	0.880	1.29800	1.06554
0.479	5.64865	2.17181	0.890	1.26785	1.05872
0.480	5.61477	2.16280	0.900	1.23884	1.05219
0.481	5.58122	2.15388	0.910	1.21090	1.04593
0.482	5.54798	2.14504	0.920	1.18399	1.03994
0.483	5.51506	2.13630	0.930	1.15806	1.03419
0.484	5.48246	2.12764	0.940	1.13304	1.02869
0.485	5.45015	2.11907	0.950	1.10891	1.02341
0.486	5.41816	2.11058	0.960	1.08561	1.01834
0.487	5.38646	2.10217	0.970	1.06310	1.01347
0.488	5.35505	2.09385	0.980	1.04136	1.00880
0.489	5.32394	2.08561	0.990	1.02033	1.00431
0.490	5.29312	2.07745	1.000	1.00000	1.00000
0.491	5.26258	2.06936	0.82849	0.82849	0.96491
0.492	5.23233	2.06136	1.200	0.70041	0.94066
0.493	5.20235	2.05343	1.300	0.60174	0.92358
0.494	5.17265	2.04558	1.400	0.52382	0.91142
0.495	5.14321	2.03780	1.500	0.46100	0.90275
0.496	5.11405	2.03009	1.600	0.40948	0.89657
0.497	5.08515	2.02246	1.700	0.36661	0.89224
0.498	5.05651	2.01490	1.800	0.33048	0.88929
0.499	5.02813	2.00742	1.900	0.29970	0.88736

β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$
2.000	0.27324	0.88623
2.100	0.25029	0.88569
2.200	0.23024	0.88562
2.300	0.21260	0.88591
2.400	0.19699	0.88648
2.500	0.18310	0.88726
2.600	0.17069	0.88821
2.700	0.15954	0.88928
2.800	0.14948	0.89045
2.900	0.14037	0.89169
3.000	0.13209	0.89298
3.100	0.12455	0.89431
3.200	0.11765	0.89565
3.300	0.11132	0.89702
3.400	0.10551	0.89838
3.500	0.10015	0.89975
3.600	0.09519	0.90111
3.700	0.09061	0.90245
3.800	0.08635	0.90379
3.900	0.08239	0.90510
4.000	0.07871	0.90640
4.100	0.07527	0.90768
4.200	0.07205	0.90894
4.300	0.06904	0.91017
4.400	0.06622	0.91138
4.500	0.06357	0.91257
4.600	0.06108	0.91374
4.700	0.05873	0.91488
4.800	0.05652	0.91600
4.900	0.05444	0.91710
5.000	0.05247	0.91817
5.100	0.05060	0.91922
5.200	0.04883	0.92025
5.300	0.04716	0.92125
5.400	0.04557	0.92224
5.500	0.04406	0.92320
5.600	0.04263	0.92414
5.700	0.04127	0.92507
5.800	0.03997	0.92597
5.900	0.03873	0.92685
6.000	0.03755	0.92772
6.100	0.03642	0.92857
6.200	0.03534	0.92940
6.300	0.03432	0.93021
6.400	0.03333	0.93100
6.500	0.03239	0.93178
6.600	0.03149	0.93254
6.700	0.03062	0.93329
6.800	0.02979	0.93402
6.900	0.02900	0.93474
7.000	0.02823	0.93544
7.100	0.02750	0.93613
7.200	0.02679	0.93680
7.300	0.02611	0.93746
7.400	0.02546	0.93811
7.500	0.02483	0.93874
7.600	0.02423	0.93937
7.700	0.02364	0.93998
7.800	0.02308	0.94058
7.900	0.02254	0.94117

β	$\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1$	$\Gamma(1+\frac{1}{\beta})$
8.000	0.02201	0.94174
8.100	0.02151	0.94231
8.200	0.02102	0.94286
8.300	0.02055	0.94341
8.400	0.02009	0.94395
8.500	0.01965	0.94447
8.600	0.01923	0.94499
8.700	0.01881	0.94550
8.800	0.01841	0.94599
8.900	0.01803	0.94648
9.000	0.01765	0.94697
9.100	0.01729	0.94744
9.200	0.01694	0.94790
9.300	0.01660	0.94836
9.400	0.01627	0.94881
9.500	0.01594	0.94925
9.600	0.01563	0.94968
9.700	0.01533	0.95011
9.800	0.01504	0.95053
9.900	0.01475	0.95094
10.000	0.01447	0.95135

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