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FINITE ELEMENT TECHNIQUES FOR STATIC STRESS AND FREE
VIBRATION ANALYSIS OF SANDWICH COMPOSITES

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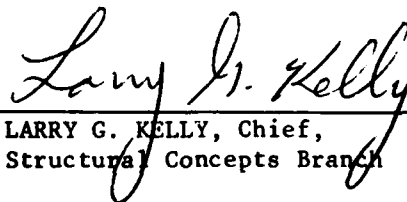
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operation at an interactive terminal, and includes provisions for automatic mesh generation and tabular output. Generated input and output data are in a form which may be accessed by existing computer graphics programs for plotting geometry and results. Sample analyses are presented to demonstrate the developed techniques.

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FOREWORD

This report describes work performed by the University of Dayton Research Institute (UDRI) under Air Force Contract F33615-77-C-3075, Structural Sandwich Composites. The effort was conducted for the Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, under the administration and technical direction of the Air Force Project Engineer, Mr. Harold C. Croop (AFWAL/FIBCB).

Administrative project supervision at the UDRI was provided by Mr. Dale H. Whitford (Supervisor, Aerospace Mechanics Division), and technical supervision was provided by Dr. Fred K. Bogner (Group Leader, Analytical Mechanics Group).

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SECTION 1 INTRODUCTION

Structural sandwich composites are commonly employed in the design of structural components requiring high stiffness-to-weight ratio characteristics. Such materials are particularly attractive for aerospace applications in which high performance and maximum payload are typically sought.

While sandwich materials offer attractive structural characteristics for many applications, effective design of sandwich construction is generally difficult due to the complexity of response and the possibility of failure in modes which are unique to sandwich materials. In order to make effective use of these advanced material concepts, improved analytical techniques are needed for use in design applications. The scope of analysis must be broad, including static and dynamic response for most applications, buckling analysis for sandwich panels and shells, and thermal stress calculations for heat-shielding and similar components.

The development presented in this report focuses upon such problems as static response, thermal stress determination and natural frequency calculations, all of which may be repeated a number of times during preliminary design and are thus most useful when analyzed in an interactive mode. Buckling, large deflection response and plastic analysis of sandwich often require a rather detailed and time-consuming analysis; such nonlinear problems are discussed in a companion report¹. Similar interactive techniques for buckling analysis and design optimization are also available^{2,3}.

1.1 GENERAL APPROACH

The analytical techniques reported here are oriented toward layered sandwich panels, including stiffening elements and edge

members, and are limited to linear elastic responses. Due to the practical requirements of considering many different types of boundary conditions and stiffener arrangements, and possibly of considering non-rectangular or curved panel geometries, the finite element method is chosen as the analytical basis throughout. With the finite element approach, only the task of data generation need be specialized to particular classes of geometries for interactive analysis.

The particular finite elements employed are derived from the same principles as the nonlinear elements developed in Reference 1. A typical sandwich panel is discretized using thin shell-type elements for the face sheet layers, and three-dimensional, shear-flexible elements in the core layer. The shell (face sheet) elements are formulated using a penalty function approach which eliminates the need for slope compatibility between elements and permits the joining of shell (face sheet) and solid (core) elements directly in a straightforward manner. Stiffening members derived from an approach similar to the shell elements can be used to model either face sheet stiffeners or full-depth spars and edge closeouts. In each case, displacement compatibility is achieved with the more complex elements used to represent the layers of the sandwich panel itself. Consistent static and thermal loadings are provided for each of these classes of elements in static analysis, as well as consistent mass formulations for natural frequency calculation.

1.2 SCOPE OF ANALYSIS

In the present development, the linear (small displacement, elastic) behavior of sandwich materials is considered for the cases of static loading and free vibration. For the case of static analysis, both mechanical and thermally-induced forces may be treated. Provisions are made for performing analyses of models of moderate size; when larger analyses are necessary, the data generated in the interactive mode may be used in

conjunction with the larger finite element program described in Reference 1.

Due to limitations in the data generation phase of the computer program, the class of sandwich panels considered is flat and rectangular, with arbitrary boundary conditions at each edge. However, the data generation scheme is organized so that geometries for non-rectangular or curved panels may be generated through minor modification of the existing code (see Section 4). Furthermore, the input information accepted by the analysis program is in the formats described in Reference 1, so that data also may be generated manually and the use of existing interactive plotting software⁴ is possible.

Each layer of a sandwich panel is modeled explicitly in the present analysis, so that localized deformations and thermal stresses due to non-uniform heating can be represented with high accuracy. The use of a layered model also facilitates the representation of face sheet and full-depth stiffeners or panel edge members. These stiffening elements may be located arbitrarily within the sandwich panel.

SECTION 2 THEORETICAL FORMULATION

The theoretical basis of each class of finite element (face sheet, core, stiffener) used in the present development is described in the following paragraphs. Further details of the element formulations may be found in References 2, 5 and 6. In each case the starting point is the element potential energy Π_p ; for static problems, the true state of equilibrium is defined simply by the condition

$$\delta \Pi_p = 0. \quad (1)$$

In dynamic (natural frequency) analysis Hamilton's principle

$$\delta \int_{t_1}^{t_2} (T - \Pi_p) dV = 0 \quad (2)$$

provides the governing equations, in which T is the kinetic energy.

2.1 THIN SHELL/FACE SHEET ELEMENTS

The development of shell-type finite elements suitable for the representation of sandwich face sheets is complicated by the compatibility requirements for elements derived from classical thin plate or shell theories. The rotational degrees of freedom dictated by the condition of slope continuity preclude the direct enforcement of compatibility with standard isoparametric solid elements. If the layers of the sandwich are to be modeled explicitly, it is necessary either to derive plate/shell elements using only displacement degrees of freedom at the bond lines or to construct higher-order solid elements to represent the core layers. In the present work, the former approach is adopted. The thin plate or shell approximation is obtained herein by

appropriate specialization of three-dimensional continuum theory, as described in References 5 and 6.

The undeformed geometry of a typical element of a shell is pictured in Figure 1. Local coordinates x, y are imbedded in the base plane of the element, and the height of the middle surface above the base plane is denoted by $Z(x, y)$. Distances away from the midsurface are measured by a third independent coordinate, ζ . The position of a generic point in the shell is then

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} + \zeta\hat{n} \quad (3)$$

where, if the shell is locally shallow, the unit normal vector \hat{n} is

$$\hat{n} = -z_{,x}\hat{i} - z_{,y}\hat{j} + \hat{k}. \quad (4)$$

The assumption of local shallowness also implies that the coordinate system (x, y, ζ) is approximately Cartesian.

If the same element of shell material is subject to a displacement field

$$\vec{u} = u\hat{i} + v\hat{j} + w\hat{k}, \quad (5)$$

the displaced position of an arbitrary point is

$$\begin{aligned} \vec{R} = & (x - \zeta z_{,x} + u)\hat{i} + (y - \zeta z_{,y} + v)\hat{j} \\ & + (z + \zeta + w)\hat{k}. \end{aligned} \quad (6)$$

By comparing Equations 3 and 6, the three dimensional strains can be identified, since

$$2e_{ij}dx_i dx_j = \vec{dR} \cdot \vec{dR} - \vec{dr} \cdot \vec{dr}. \quad (7)$$

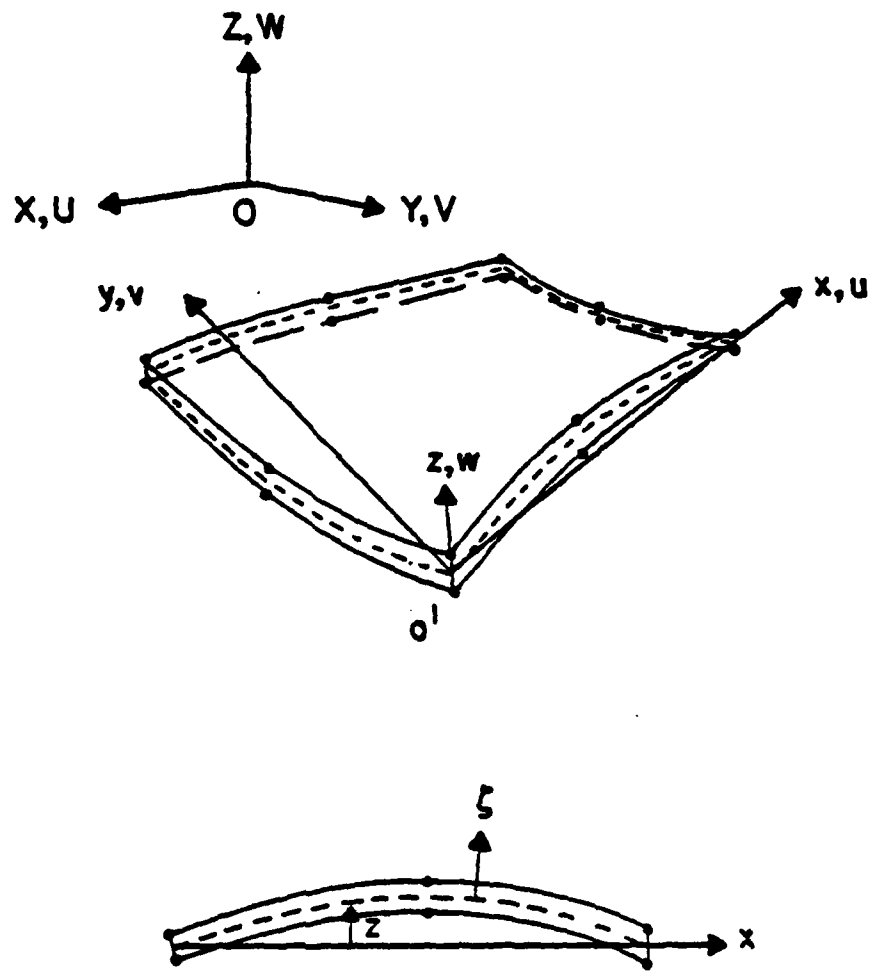


Figure 1. Undeformed Geometry of a Shell Element.

The resulting expressions become, after linearization,

$$\begin{aligned}
 e_{xx} &= u_{,x}(1 - \zeta Z_{,xx}) - \zeta Z_{,xy} v_{,x} + Z_{,x} w_{,x} \\
 e_{yy} &= v_{,y}(1 - \zeta Z_{,yy}) - \zeta Z_{,xy} u_{,y} + Z_{,y} w_{,y} \\
 e_{\zeta\zeta} &= w_{,\zeta} - Z_{,x} u_{,\zeta} - Z_{,y} v_{,\zeta} \\
 2e_{y\zeta} &= v_{,\zeta}(1 - \zeta Z_{,yy}) + w_{,y} \\
 &\quad - Z_{,x} u_{,y} + Z_{,y}(w_{,\zeta} - v_{,y}) - \zeta Z_{,xy} u_{,\zeta} \\
 2e_{x\zeta} &= u_{,\zeta}(1 - \zeta Z_{,xx}) + w_{,x} \\
 &\quad - Z_{,y} v_{,x} + Z_{,x}(w_{,\zeta} - u_{,x}) - \zeta Z_{,xy} v_{,\zeta} \\
 2e_{xy} &= u_{,y}(1 - \zeta Z_{,xx}) + v_{,x}(1 - \zeta Z_{,yy}) \\
 &\quad + Z_{,x} w_{,y} + Z_{,y} w_{,x} - \zeta Z_{,xy}(u_{,x} + v_{,y}). \tag{8}
 \end{aligned}$$

The behavioral constraints leading to thin plate or shell response are expressed directly in the form

$$e_{x\zeta} = e_{y\zeta} = e_{\zeta\zeta} = 0. \tag{9}$$

Furthermore, the condition of vanishing normal stress through the thickness is imposed exactly, and the remaining stress-strain relation is one of plane stress,

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}. \tag{10}$$

Equation 10 is written symbolically in the form

$$\underline{\underline{\sigma}} = \underline{\underline{D}} \underline{\underline{e}} \quad (11)$$

in which case the potential energy of a shell is simply

$$\Pi_{\rho} = \frac{1}{2} \int_V \underline{\underline{e}}^T \underline{\underline{D}} \underline{\underline{e}} \, dV - \int_{\partial V} \underline{\underline{u}}^T \underline{\underline{t}} \, dA. \quad (12)$$

Here ∂V represents the shell boundary, and $\underline{\underline{t}}$ the components of the applied mechanical forces.

To complete the theoretical formulation of the element, the constraints summarized in Equation 9 must be introduced. This is accomplished by introducing the penalty functional,

$$F = \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^3 W_{ij} e_{jz}^2(\vec{r}_{ij}). \quad (13)$$

In Equation 13, \vec{r}_{ij} and W_{ij} represent collocation points and weights, which may be chosen independently for each of the transverse strains. The weights W_{ij} play the role of penalty factors, and as $W_{ij} \rightarrow \infty$ the transverse strain constraints are enforced exactly at each of the points \vec{r}_{ij} . In practice, the magnitudes of the W_{ij} are chosen to provide adequate satisfaction of the constraints and good numerical conditioning simultaneously for the practical range of element length-to-thickness ratios⁶. The element basis is simply the augmented functional

$$\begin{aligned} \Pi'_{\rho} = & \frac{1}{2} \int_V \underline{\underline{e}}^T \underline{\underline{D}} \underline{\underline{e}} \, dV - \int_{\partial V} \underline{\underline{u}}^T \underline{\underline{t}} \, dA \\ & + \frac{1}{2} \sum_{i=1}^M \underline{\underline{\gamma}}_i^T \underline{\underline{W}} \underline{\underline{\gamma}}_i \end{aligned} \quad (14)$$

in which

$$\underline{\gamma}_i^T = \left[e_{x\zeta}(\vec{r}_{ix}) \ e_{y\zeta}(\vec{r}_{iy}) \ e_{z\zeta}(\vec{r}_{iz}) \right] \quad (15)$$

and \underline{W} is the matrix of weighting factors W_{ij} .

In thermal stress analysis, the constitutive relation (Equation 11) is replaced by

$$\underline{\sigma} = \underline{D}(\underline{e} - \underline{\alpha}T) \quad (16)$$

which leads to the energy

$$\begin{aligned} \Pi'_\rho = & \frac{1}{2} \int_V \underline{e}^T \underline{D} \underline{e} \, dV - \int_V \underline{e}^T \underline{D} \underline{\alpha} T \, dV \\ & - \int_{\partial V} \underline{u}^T \underline{t} \, dA + \frac{1}{2} \sum_{i=1}^M \underline{\gamma}_i^T \underline{W} \underline{\gamma}_i \quad (17) \end{aligned}$$

The second term of Equation 17 leads to the consistent definition of the thermal "loading" induced by a temperature change $T(x,y,\zeta)$ from the temperature in the reference state.

For dynamic calculations, the kinetic energy is formed directly in three-dimensional form,

$$T = \frac{1}{2} \int_V \rho (\dot{\underline{R}} - \dot{\underline{r}}) \cdot (\dot{\underline{R}} - \dot{\underline{r}}) \, dV. \quad (18)$$

From Equations 3 and 6,

$$T = \frac{1}{2} \int_V \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) \, dV. \quad (19)$$

Since the potential and kinetic energies formulated above involve derivatives no higher than first order, the finite element approximation must satisfy the continuity of displacements (but not bending slopes) between adjacent elements. In the

present study, the simplest permissible form of approximation, based upon linear polynomial shape functions, is used. The resulting element is shown in Figure 2. Despite the simplicity of the approximation, the element exhibits high accuracy in both displacement and stress computations, and is particularly straightforward to use in comparison with many other shell finite elements. Linear and nonlinear analyses which demonstrate the accuracy and economy of the present shell element formulation can be found in Reference 6.

2.2 SANDWICH CORE ELEMENTS

A fully three-dimensional formulation is used herein to obtain sandwich core elements compatible with the face sheet elements described in Paragraph 2.1. Define the strain and stress vectors

$$\underline{e}^T = \left[e_{xx} \ e_{yy} \ e_{zz} \ 2e_{yz} \ 2e_{xz} \ 2e_{xy} \right] \quad (20)$$

and

$$\underline{\sigma}^T = \left[\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{yz} \ \sigma_{xz} \ \sigma_{xy} \right]. \quad (21)$$

Here the strains are defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (22)$$

The stress-strain relation is (see Reference 1, Paragraph 2.2)

$$\underline{\sigma} = \underline{D}(\underline{e} - \underline{\alpha}T) \quad (23)$$

and the potential energy can be expressed as

$$\begin{aligned} \Pi_p = \frac{1}{2} \int_V \underline{e}^T \underline{D} \underline{e} \, dV - \int_V \underline{e}^T \underline{D} \underline{\alpha} T \, dV \\ - \int_{\partial V} \underline{u}^T \underline{t} \, dA. \end{aligned} \quad (24)$$

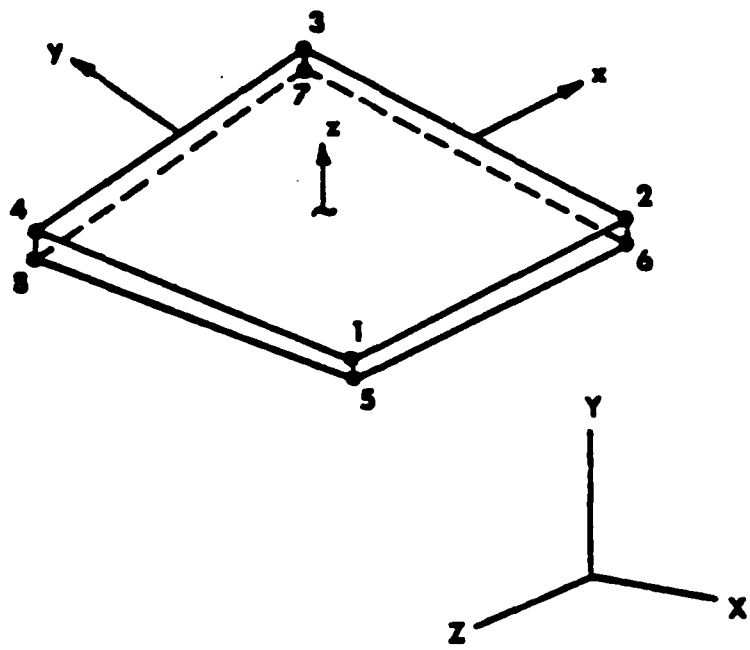


Figure 2. Trilinear Shell Finite Element.

Similarly, the kinetic energy is

$$T = \frac{1}{2} \int_V \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dV. \quad (25)$$

The above energy expressions are discretized using a linear polynomial displacement field in isoparametric coordinates, for compatibility with the face sheet elements. The resulting finite element has eight nodes (Figure 3) and 24 degrees of freedom.

In the approximation of bending response, the above eight-node element is notoriously poor when the potential energy (Equation 24) is evaluated exactly⁶. However, the stabilizing effect of surrounding face sheet and stiffener elements or closeouts permits the element energy (i.e., the stiffness matrix) to be computed using a reduced integration scheme (one point Gaussian quadrature). This single-point integration, though inexact, is sufficient for convergence⁷ and permits a much-improved simulation of the panel bending response over the conventional eight-node element.

2.3 FACE SHEET AND FULL-DEPTH STIFFENERS

Discrete stiffener elements for use with the present face sheet and core layer elements are obtained using a penalty function approach similar to that employed in Paragraph 2.1, in two dimensions rather than three. The resulting element is displacement-compatible with the remaining elements, represents axial and primary bending effects well, and may be arbitrarily oriented in three-dimensional space.

Both types of stiffeners are shown in Figure 4. In each case, the stiffener is considered to be relatively small in width, so that all stresses are approximately uniform in the width direction. This assumption causes inplane bending effects to be ignored, which is consistent with the resolution provided by the face sheet discretization. Normal stresses in the width direction

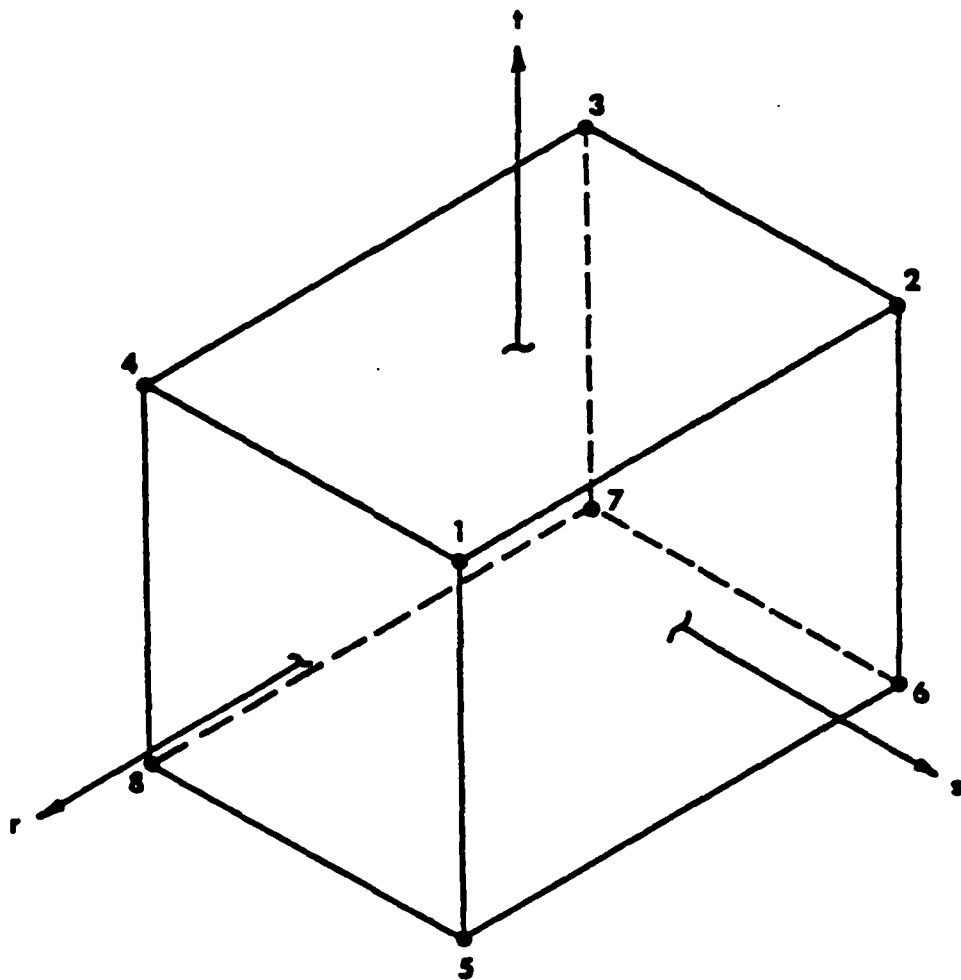


Figure 3. Sandwich Core Finite Element.

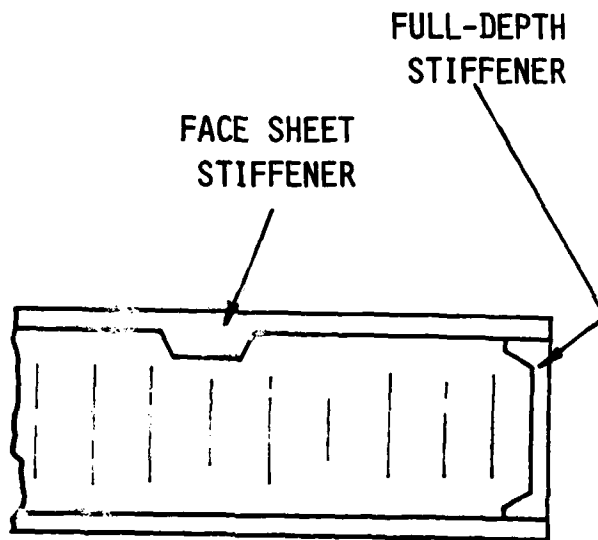


Figure 4. Face Sheet and Full-Depth Stiffeners.

are also neglected; thus, the stiffener is subject to a plane state of stress in the remaining dimensions, although arbitrary warping and twisting of the element is permitted.

The strain energy of a typical element involves only the longitudinal strains, which give rise to both axial and bending forces in the deformed state. Remaining components of strain (transverse shear and normal strain in the depth direction) are suppressed through the use of penalty function constraints (see Paragraph 2.1). Denoting the longitudinal direction by x , the potential energy is simply⁶

$$\begin{aligned} \Pi_{\rho} = & \frac{1}{2} \int_V E e_{xx}^2 dV - \int_V E \alpha^T e_{xx} dV \\ & - \int_{\partial V} \tilde{u}^T \tilde{t} dA \end{aligned} \quad (26)$$

where E is the elastic modulus, and the kinetic energy is

$$T = \frac{1}{2} \int_V \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dV. \quad (27)$$

To enforce the remaining constraints

$$e_{zz} = e_{xz} = 0, \quad (28)$$

introduce the penalty functional

$$F = \frac{1}{2} W_x e_{xz}^2(\vec{r}_x) + \frac{1}{2} W_z e_{zz}^2(\vec{r}_z) \quad (29)$$

in which W_x , W_z are the constraint weights and \vec{r}_x , \vec{r}_z are the corresponding collocation points. Analogous to the procedure of Paragraph 2.1, the functional F is added to the potential energy (Equation 26) to form the constrained energy on which the element stiffness matrix is based:

$$\begin{aligned}
\Pi'_\rho = & \frac{1}{2} \int_V E e_{xx}^2 dV - \int_V E \alpha T e_{xx} dV \\
& + \frac{1}{2} W_x e_{xz}^2(\vec{r}_x) + \frac{1}{2} W_z e_{zz}^2(\vec{r}_z) \\
& - \int_{\partial V} \underline{u}^T \underline{t} dV.
\end{aligned} \tag{30}$$

The above potential and kinetic energies are cast in discrete form through a linear approximation of the displacements \underline{u} within individual elements. The resulting finite elements, shown in Figure 5, have four nodes and twelve degrees of freedom. For this order of discretization, the element strain energy may be evaluated exactly using a four-point (2 by 2 Gauss) integration rule. The transverse strain constraints (Equation 28) are each applied at a single point corresponding to the element centroid location.

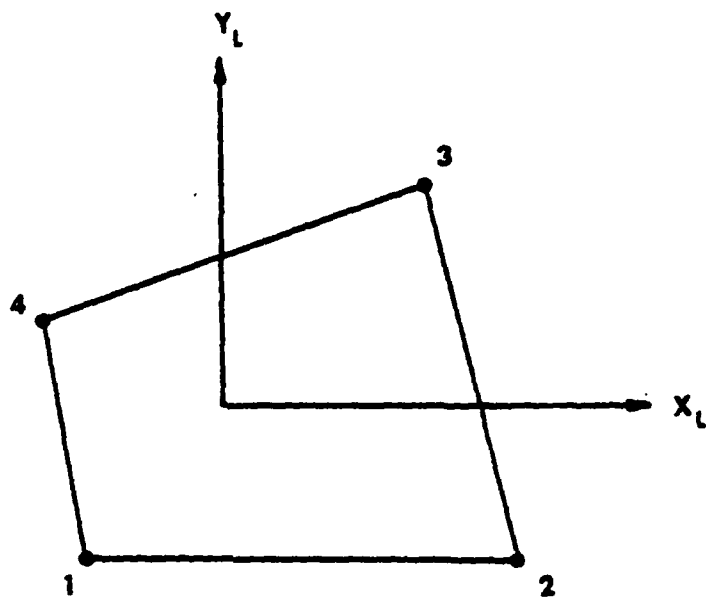


Figure 5. Stiffener Finite Element in Local Coordinates.

SECTION 3
NUMERICAL SOLUTION TECHNIQUES

The methods used in solving the governing matrix equations for a complete finite element model are described in this Section. Two general types of solutions are presented: linear simultaneous equations, for static analysis with mechanical and/or thermal loading; and generalized eigenvalue systems, for use in natural vibration analysis.

3.1 LINEAR STATIC ANALYSIS

In static analysis, minimization of the discrete potential energy for an assemblage of elements leads to the matrix equation

$$\underline{\underline{K}} \underline{U} = \underline{T} \quad (31)$$

in which $\underline{\underline{K}}$ is the stiffness matrix, \underline{U} are the (unknown) nodal displacements, and \underline{T} is the load vector including both mechanical and thermal generalized forces. A direct solution is obtained by a Gauss-Doolittle factorization of $\underline{\underline{K}}$, followed by forward and backward substitution. The decomposition of $\underline{\underline{K}}$ is of the form

$$\underline{\underline{K}} = \underline{\underline{L}} \underline{\underline{D}} \underline{\underline{L}}^T. \quad (32)$$

Here $\underline{\underline{L}}$ is a unit lower triangular matrix,

$$l_{ij} = \begin{cases} (\text{nonzero}), & i > j \\ 1, & i = j \\ 0, & i < j \end{cases} \quad (33)$$

and $\underline{\underline{D}}$ is a strictly diagonal matrix. Next let

$$\underline{Z} = \underline{\underline{D}} \underline{\underline{L}}^T \underline{U} \quad (34)$$

and the relation

$$\underline{L}\underline{Z} = \underline{T} \quad (35)$$

can be solved for \underline{Z} by a forward substitution. From Equation 34,

$$\underline{L}^T \underline{U} = \underline{D}^{-1} \underline{Z} \quad (36)$$

which can then be solved for \underline{U} in a backward substitution step.

Having solved for the nodal displacements \underline{U} over the entire finite element model, a complete vector of displacements can be extracted for each individual element. The element displacement vector, along with the shape functions for an element, completely describe the state of displacement (and, therefore, strain and stress) within each element of the model.

3.2 NATURAL FREQUENCY SOLUTION

In natural vibration analysis, the discretization of both the potential and kinetic energies and the assumption of harmonic motions yield the governing system of equations

$$\underline{K}\underline{U} = \omega^2 \underline{M}\underline{U} \quad (37)$$

Here \underline{K} and \underline{M} are the stiffness and mass matrices, respectively, \underline{U} is the nodal displacement vector, and ω is the circular frequency of vibration.

It should be noted in Equation 37 that the mode shape \underline{U} describes only the relative displacements of the mesh points ($a\underline{U}$, where a is a constant, is a solution as well). If matrices \underline{K} and \underline{M} are of order N , then there exist N -solutions of Equation 37: $\omega_i, \underline{U}_i; i=1,2,\dots,N$. Usually only the lowest several frequencies and mode shapes are of interest, and thus a vector iteration procedure is most appropriate for solving the system. An approach based upon simultaneous vector iteration⁸ is used herein.

Factoring the stiffness matrix by Choleski's method gives

$$\underline{K} = \underline{L}\underline{L}^T. \quad (38)$$

Equation 37 can now be restated as the standard eigenvalue problem

$$\underline{A}\underline{V} = \lambda\underline{V} \quad (39)$$

in which

$$\underline{A} = \underline{L}^{-1}\underline{M}\underline{L}^{-T} \quad (40)$$

$$\underline{V} = \underline{L}^T\underline{U} \quad (41)$$

and

$$\lambda = \frac{1}{\omega^2}. \quad (42)$$

The eigenvalue solution is performed using a set of m trial vectors, where m is greater than the number of frequencies desired but is generally much less than N . These trial vectors, which are arranged in the partial modal matrix

$$\underline{\phi} = [\underline{V}_1 \ \underline{V}_2 \ \dots \ \underline{V}_m] \quad (43)$$

gradually converge to the first m eigenvectors (mode shapes) as the iteration proceeds.

The central step in the simultaneous iteration is the function of the m by m interaction matrix

$$\underline{B} = \underline{\phi}^T \underline{A} \underline{\phi}. \quad (44)$$

When the iteration has converged, \underline{B} is a diagonal matrix since all of the vectors \underline{V}_i are orthogonal with respect to \underline{A} . Before

the solution has converged, an approximate eigenvalue analysis of \underline{B} is performed at each iteration to generate improved estimates of the eigenvalues and to modify the iteration matrix $\underline{\phi}$. Details of the implementation of this procedure may be found in Reference 8.

SECTION 4 COMPUTER PROGRAM DESCRIPTION

The finite element discretization and numerical solution procedures summarized in Sections 2 and 3 are implemented in a computer program which permits the interactive solution of static and natural frequency problems for sandwich composites. Auxiliary programs are used in conjunction with the primary analysis program to perform the tasks of data input and generation, and of presentation of results.

4.1 PROGRAM FUNCTIONS AND ORGANIZATION

The system of computer programs which are used for the interactive analysis of sandwich panels are pictured in Figure 6. A data generation program is used to prepare data interactively for use in the main analysis program. The sandwich analysis input data (file INPDAT) conforms to the input data formats of Reference 1, which in turn are patterned after those of the MAGNA general purpose finite element program⁴. Thus, the geometry plotting capabilities described in Reference 9 may be used to obtain line drawings of the generated mesh for documentation purposes when required. Once the requested analysis has been performed, a summary of results is stored on disc (file POSTFL) for deformed geometry plotting⁹, listing by separate output formatting program, or listing on the line printer.

4.2 INTERACTIVE OPERATION

The interactive programs described here are of three types: data input and generation; static or natural frequency analysis; and tabular output. Operating instructions for compatible pre- and post-processing graphics programs are given in Reference 9.

Data generation is performed using a minimal amount of input from the user concerning the panel geometry, properties, supports, loading, temperature distribution and mesh divisions. In particular, the following data are requested:

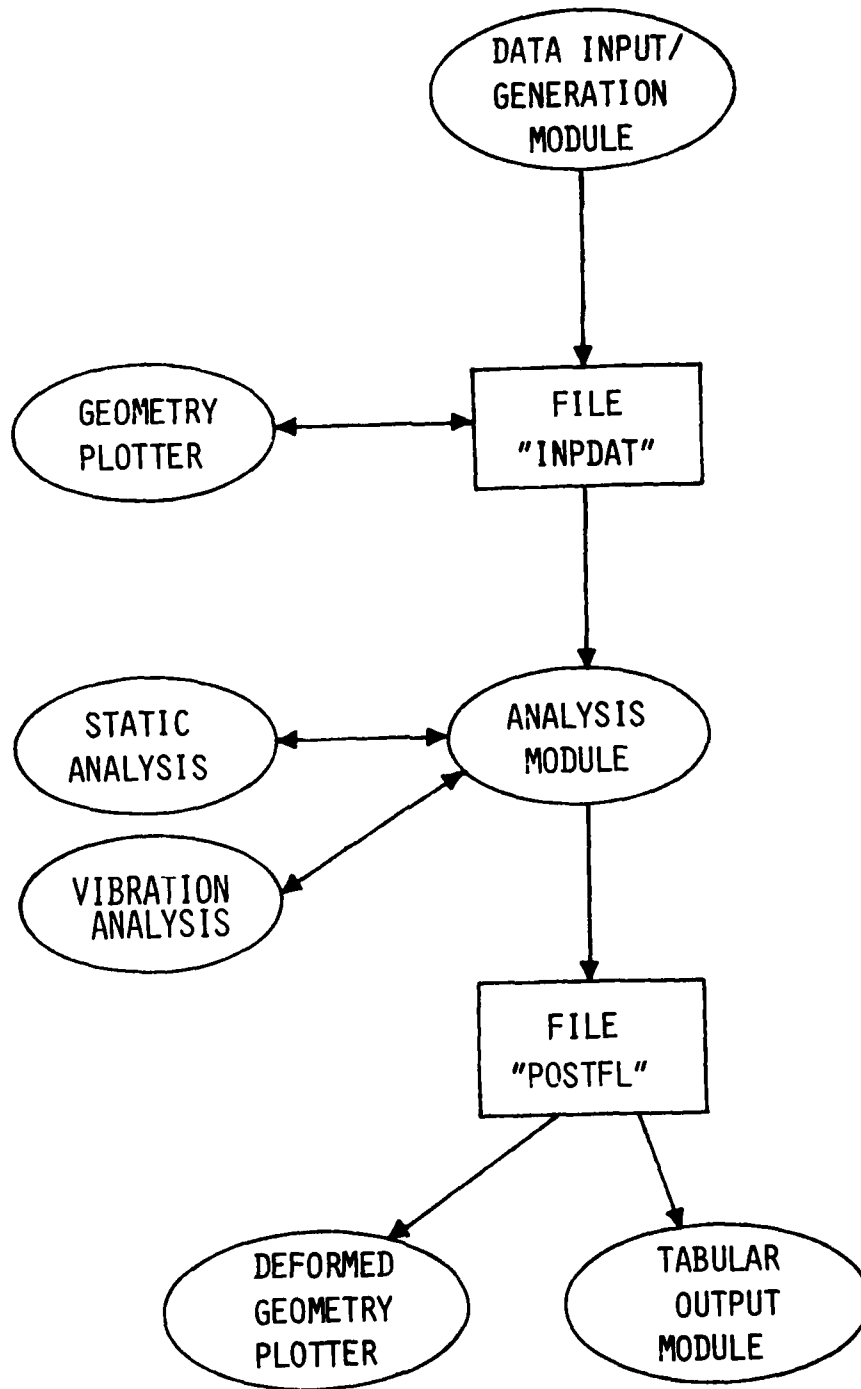


Figure 6. Computer Program Organization.

1. ANALYSIS TYPE: static or natural frequency solution.
2. NUMBER OF MODES: number of natural frequencies and mode shapes to be solved in vibration analysis.
3. TYPE OF LOADING: mechanical or thermal loads (or both) in static analysis.
4. PANEL DIMENSIONS A,B: planform dimensions of the sandwich panel in the x,y directions. At present, the data generation program is restricted to flat, rectangular panels.
5. LAYER THICKNESS TL,TC,TU: thicknesses of lower face sheet, core layer, and upper face sheet.
6. FACE SHEET PROPERTIES E,NU,ALFA,RHO: Young's modulus, Poisson's ratio, coefficient of thermal expansion and density (requested for each face sheet in turn).
7. CORE PROPERTIES GX,GY,ALFAX,ALFAY,RHO: shear moduli, coefficients of thermal expansion and density for the core layer.
8. NUMBER OF STIFFENERS NSX,NSY: number of stiffening members in the x and y directions, respectively. Both face sheet and full depth stiffeners are included.
9. STIFFENER LOCATIONS X(OR Y), ITYPE: for each stiffening member, the constant value of x or y at which the member is located is requested; valid x-values are between 0 and A, and valid y-values are between 0 and B. For each member, the type is also requested (lower face, upper face, full depth).
10. STIFFENER PROPERTIES E, ALFA, T, RHO: for each stiffener, these are the values of the Young's modulus, the coefficient of thermal expansion, the stiffener width, and the mass density.
11. SUPPORT CONDITIONS ISO,ISA,JSO,JSB: for each of the edges of the panel ($x = 0, a$; $y = 0, b$), a boundary condition must be specified. Support conditions which can be chosen independently at each panel edge include: free, simple support, clamped, symmetric and antisymmetric.
12. PRESSURE LOADINGS PL,PU: in linear static analysis, pressures may be specified on the lower or upper face sheet surfaces, or both. Note that all pressure values are positive upward.

13. POINT LOADINGS XP,YP,FORCE: location and magnitude of point loadings for static analysis.
14. NUMBER OF ELEMENTS PER EDGE NEX,NEY: this data defines the degree of mesh refinement in the model by dividing the sandwich panel into a rectangular grid of NEX by NEY finite elements. Usually a moderate number of elements (4-6 per edge) provides a good balance of accuracy and solution cost.
15. GRID POINT LOCATIONS XG (OR YG): along each side of the panel, the locations of grid lines may be specified to generate nonuniform (graded) meshes. NEX+1 locations must be specified along the x-axis, and NEY+1 locations on the y-axis. Note that each of the stiffener locations specified in Item 9 must correspond to a grid line location.

In addition to geometric parameters and other data defined in the interactive mode, temperature data must be supplied if thermal stress analysis is to be performed. By default, the data generator will create temperature data for a uniformly heated panel ($T = 100^\circ$, in consistent units). To specify a different temperature distribution, the user must supply a separate subroutine to generate the required data $T = T(x,y,z)$. This user-written subroutine has the form shown in the following example, for which the temperature distribution is

$$T(x,y,z) = 1.2Ze^{-0.028x} \quad (45)$$

The appropriate user subroutine (which always has the name UTEMPS) is

```

SUBROUTINE UTEMPS (X,Y,Z,T)
T = 1.2*Z*EXP(-0.028*X)
RETURN
END .

```

User subroutine UTEMPS is called once for every generated point in the finite element model.

The analysis is next performed by simply loading and executing the analysis module. While the final results of an analysis are written to the file POSTFL for later processing,

certain information, including diagnostic and warning messages, is output directly at the user's terminal. In the event that a problem becomes too large for the interactive program, the analysis can be performed in the batch mode using the program of Reference 1, since the input data are fully compatible.

Following the analysis step, tabular results may be printed by simply executing the output module. Analysis results provided in table form include deflection data on selected grid lines, and strain and stress data at individual element centroids. Alternatively, the file POSTFL may be used as input to graphics programs⁹ which produce deformed geometry plots for each visualization of the problem results.

SECTION 5
SAMPLE ANALYSES

Typical analyses performed using the solution techniques described in Sections 2 and 3 are presented in this section. Static problems involving mechanical and thermal loading, as well as natural frequency solutions, are discussed. The operation of the analysis program is demonstrated in greater detail in the sample terminal session included in the Appendix.

5.1 STATIC ANALYSIS WITH MECHANICAL AND THERMAL LOADS

To demonstrate the static analysis option, a rectangular panel previously studied by Monforton¹⁰ and Kan and Huang¹¹ is considered. The flat sandwich plate has the following geometry and properties:

Planform:	$a=b = 50$ inches
Thicknesses:	$t_f = 0.015$ inches
	$t_c = 1.000$ inches
Face Material:	$E_f = 10.5 \times 10^6$ lb/in ²
	$\nu_f = 0.30$
	$\alpha_f = 12.4 \times 10^{-6}$ in/in°F
Core Material:	$G_c = 50000.$ lb/in ²
	$\alpha_c = 12.4 \times 10^{-6}$ in/in°F .

Using these parameters, a static solution is performed for the case of a fully clamped boundary and uniform lateral pressure (Figure 7). Due to symmetry of the problem only one quarter of the panel is modeled. The mesh consists of a 5 by 5 grid of elements in each layer, for a total of 260 unconstrained degrees of freedom. For a unit lateral pressure, the computed central displacement is $w_c = 0.113$ inches. This value is in good agreement with previously published results; for example, linearization of the results of Reference 11 gives $w_c = 0.09497$ inches.

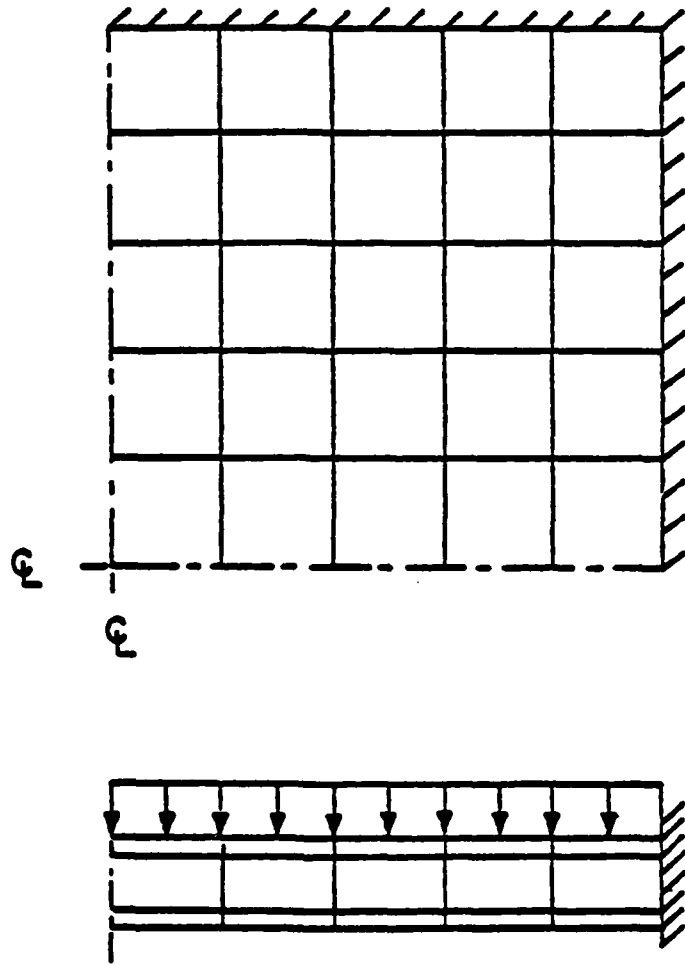


Figure 7. Clamped Panel Under Uniform Pressure.

A static analysis has also been performed for the above panel considering thermal loading only. In this case, the panel is assumed to be simply supported. The temperature distribution is such that the two face sheets are at opposite uniform temperatures, and the core experiences a linear thermal gradient. The following user subroutine is used to define the thermal field:

```

SUBROUTINE UTEMPS (X,Y,Z,T)
T = 200.*Z
IF (ABS(Z).GT.0.5)T = 100.*T/ABS(T)
RETURN
END .

```

This antisymmetric temperature field produces a pure moment type of loading, and thus a bending response. The resulting deformed shape of the panel is shown in Figure 8. Typical Stress results (in this case the core shear stress T_{xz}) are depicted in graphical form in Figure 9.

5.2 NATURAL FREQUENCY ANALYSIS

A simply supported sandwich panel is considered to determine its free vibration response. The following geometric and material parameters are used:

Planform:	$a = 62.25$ inches
	$b = 43.50$ inches
Thicknesses:	$t_f = 0.072$ inches
	$t_c = 1.856$ inches
Face Sheets:	$E_f = 10. \times 10^6$ lb/in ²
	$\nu_f = 0.33$
	$\rho_f = 0.000244$ lb-sec ² /in ⁴
Core:	$G_c = 30000.$ lb/in ²
	$\rho_c = 0.0000123$ lb-sec ² /in ⁴ .

The finite element model of one quarter of the panel is shown in Figure 10. Thirty-six elements are used in each layer of the model.

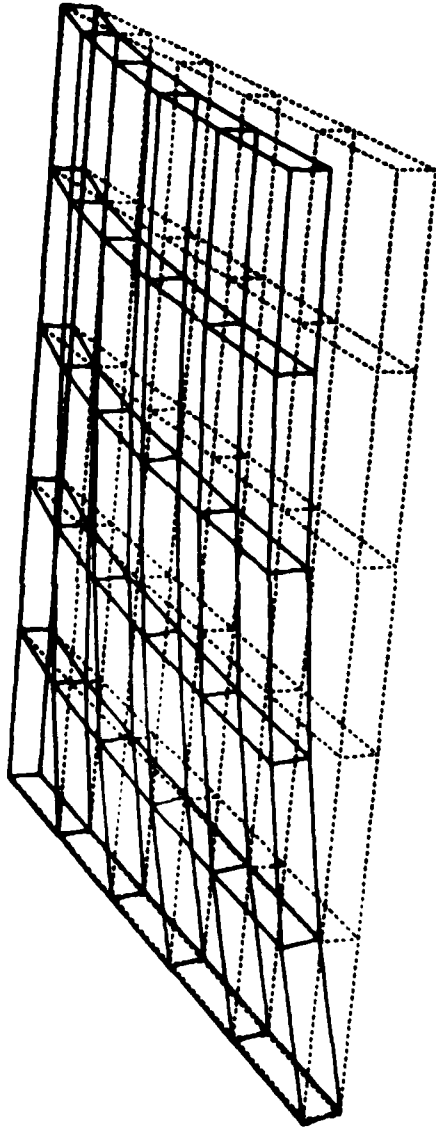


Figure 8. Deformed Geometry of Square Panel Under Antisymmetric Thermal Loading.

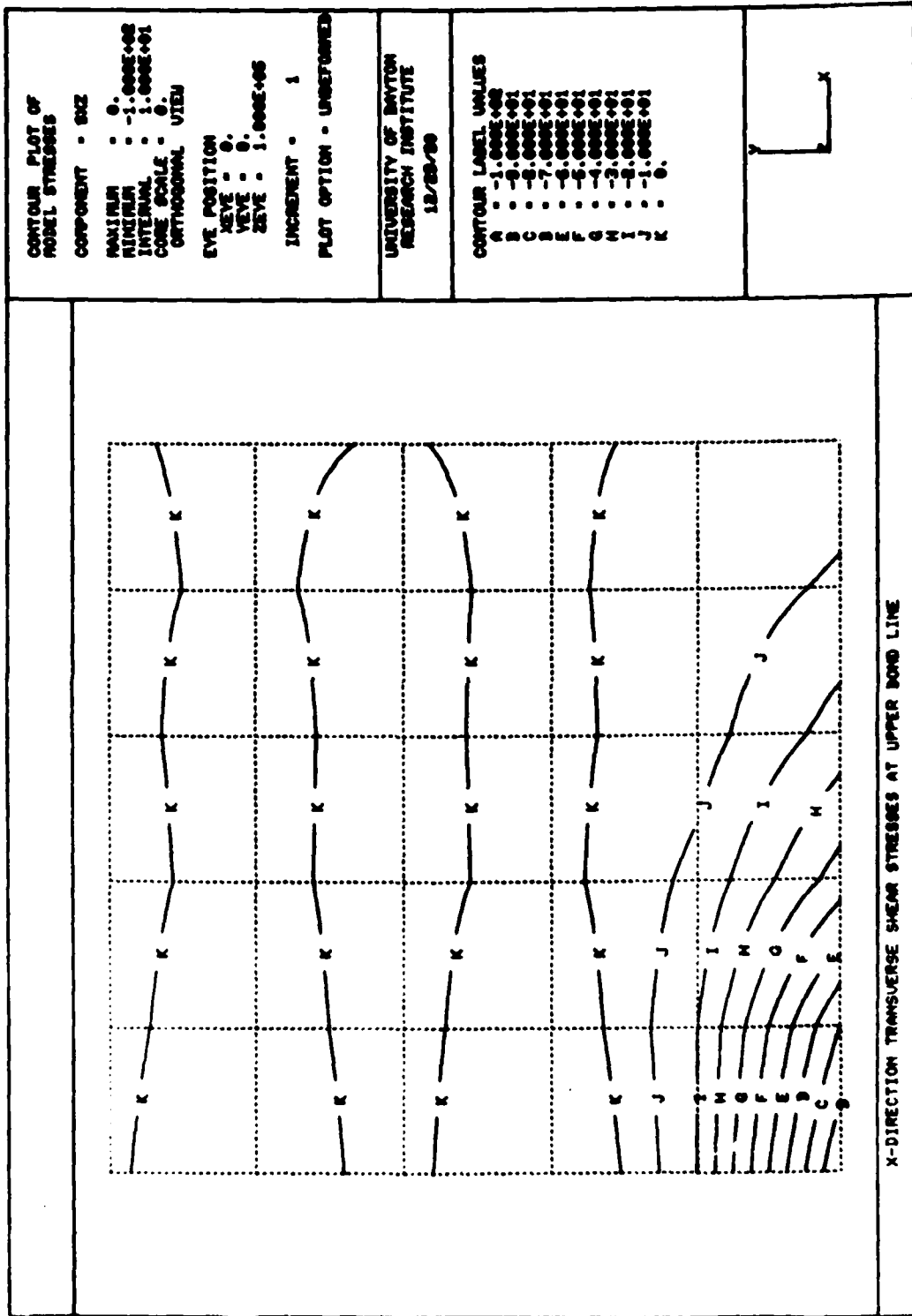


Figure 9. Core Shear Stresses for Sandwich Panel with Antisymmetric Temperature Distribution.

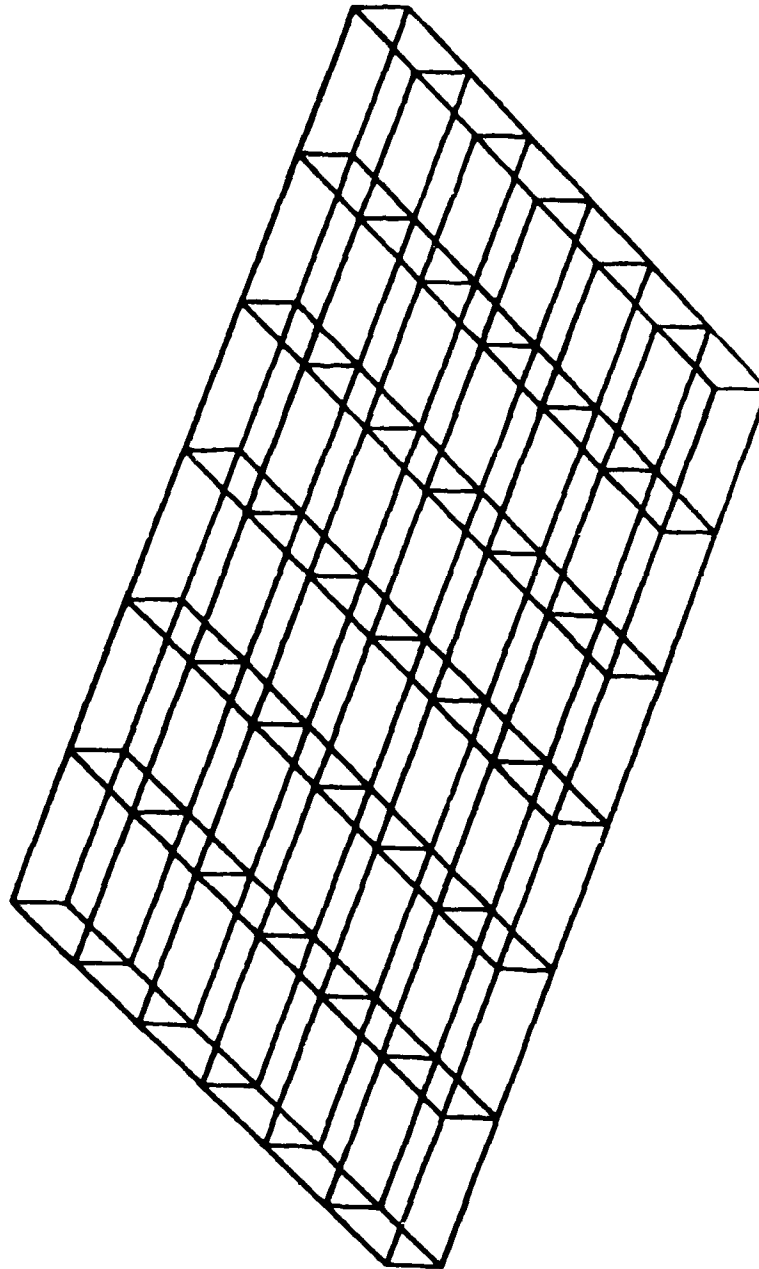


Figure 10. Sandwich Panel for Natural Frequency Analysis.

The solution for the fundamental frequency of the panel using the model shown yields $\omega_1 = 174$ Hz. This value compares well with the classical analytical solution¹² of $\omega_1 = 170$ Hz.

A second solution has also been performed for a similar panel of larger aspect ratio. Panel data for this example is identical to the problem above, with the following exceptions:

$$b = 34.00 \text{ inches}$$

$$t_f = 0.063 \text{ inches}$$

$$t_c = 1.874 \text{ inches.}$$

In this case, the computed frequency of 240 Hz is also in good agreement with a closed-form solution¹² of $\omega_1 = 250$ Hz.

SECTION 6
SUMMARY AND CONCLUSIONS

A finite element approach for the static stress analysis of sandwich panels under mechanical and thermal loadings, and for the natural frequency analysis of sandwich, has been described. Layers of a sandwich panel are modeled explicitly, so that unequal face sheets, stiffening members, and other design details are represented with relatively high accuracy. Due to the finite element basis of the present methodology, rather arbitrary geometries and variations in mechanical properties are easily considered.

The methods described are suitable for interactive use, to facilitate parametric and design studies. Data generation utilities, which permit the definition of simple panel geometries from a minimal amount of input, are provided to support this mode of usage. Generated input is compatible with that of a comparison program¹ with larger capability. Results are output in a form suitable for plotting using existing computer graphics programs.⁹

The developments described herein can be improved to provide more detailed and comprehensive analytical capabilities (e.g., curved panels, arbitrary stiffener shapes, linear buckling analysis, etc.). However, in their present form, the methods presented represent a valuable capability for practical design analysis of sandwich composite structures.

APPENDIX
SAMPLE INTERACTIVE TERMINAL SESSION

A typical interactive analysis session using the sandwich analysis program described herein is reproduced in this Appendix. This sample session demonstrates the procedures for accessing and executing the data generation, analysis and plotting programs, and illustrates the types of output which can be obtained from the analysis.

The analysis procedure is initiated by accessing a stored control language procedure file (denoted by "PROCFIL" in the example), and issuing a command to start the procedure:

BEGIN,SWICH,PROFIL.

This command automatically invokes the data generation, analysis and tabular output modules. By appending the keyword "PLOT" to the BEGIN command, a geometry plotting utility is also executed before and after the analysis module. The "PLOT" option is used in the sample session to clarify the problem geometry and results.

COMMAND- ATTACH_PROCEIL_PROCEIL_ID=DR18114.MB-1.
AT CV=001 SH=APFD
COMMAND- BEGIN_SHIGH_PROCEIL_PLOT.

BEGIN DATA GENERATION PHASE

ANALYSIS TYPE CODES

- 1 - STATIC SOLUTION
- 2 - NATL. FREQUENCY

ENTER ANALYSIS TYPE..... 1

LOADING TYPE CODES

- 1 - MECHANICAL LOADS
- 2 - THERMAL LOADS
- 3 - BOTH

ENTER LOADING TYPE..... 1

ENTER PANEL DIMENSIONS

(A,B)..... 30.24

ENTER FACE SHEET AND CORE

THICKNESSES (TL,TC,TU)... 0.050, 0.875, 0.035

PROPERTIES FOR FACE SHEET 1

ENTER E, PR, ALFA, RHO... 10.3E6, 0.3, 0.0

PROPERTIES FOR FACE SHEET 2

ENTER E, PR, ALFA, RHO... 10.3E6, 0.3, 0.0

CORE LAYER PROPERTIES

ENTER GX, GY, ALFA.

ALFA, RHO..... 30000, 50000, 0.0, 0.0

STIFFENING MEMBER DATA

ENTER THE NUMBER OF

STIFFENERS WITH X=CONST

AND Y=CONST (NSX,MSY).... 2, 2

ENTER THE X-STATIONS AT

WHICH STIFFENERS ARE

TO BE LOCATED..... 0, 20

ENTER THE TYPE OF EACH STIFFENER

1 - LOWER FACE

2 - UPPER FACE

3 - FULL DEPTH..... 3, 2

ENTER THE Y-STATIONS AT

WHICH STIFFENERS ARE

TO BE LOCATED..... 0, 24

ENTER THE TYPE OF EACH STIFFENER

1 - LOWER FACE

2 - UPPER FACE

3 - FULL DEPTH..... 3, 3

ARE ALL STIFFENERS TO

BE ASSIGNED IDENTICAL

PROPERTIES (Y/N)..... Y

ENTER PROPERTIES

E, T, RHO, ALFA..... 10.0E6, .1, 0.0

BOUNDARY CONDITION CODES

0 - FREE EDGE

1 - SIMPLE SUPPORT

2 - CLAMPED EDGE

3 - SYMMETRIC EDGE

ENTER BOUNDARY CODES FOR

X=0, X=A, Y=0, Y=B..... 2, 3, 0, 0

ENTER LOWER AND UPPER

SURFACE PRESSURES PL,PU.. 1, 0

ENTER THE NUMBER

OF POINT LOADS..... 0

ENTER THE NUMBER OF

ELEMENTS IN EACH DI-

RECTION (NEX,NEY)..... 5, 4

ENTER X-AXIS NODE

LOCATIONS (NEX+1 VALUES): 0, 5, 10, 15, 20, 25, 30

ENTER Y-AXIS NODE

LOCATIONS (NEY+1 VALUES): 0, 6, 12, 18, 24

ENTER THE CHARACTERS PER SECOND

STOP

023300 MAXIMUM EXECUTION FL.

.228 CP SECONDS EXECUTION TIME.

TEKTRONIX OR HP PLOTTER(T,H)

MODEL INPUT FILE TYPES:

1 - INPUT DATA FILE

2 - POSTPROCESSOR (MPOST) FILE

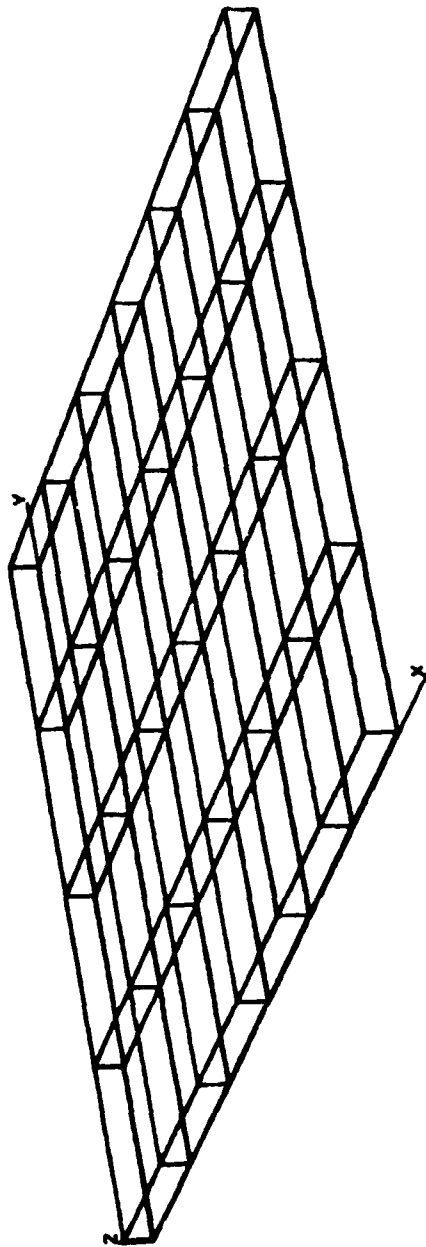
ENTER FILE TYPE(I,2)

ENTER FILE TYPE(I,2)

```

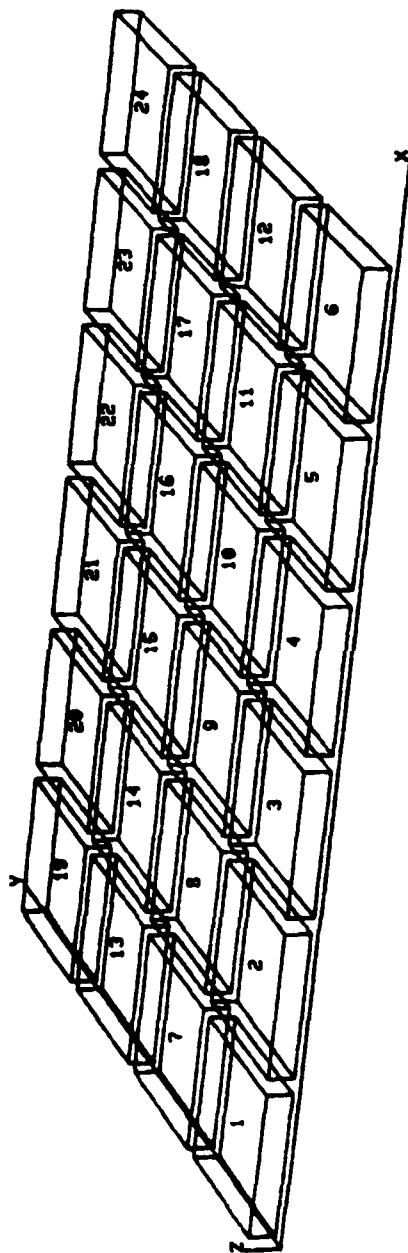
ENTER THE STRUCTURE NUMBER .....: 1
HELP
COMMAND
SERIES
AXES
CLIP
CUBE
DEFAULT
DEFORN
DRAW
ELEMENTS
EYE
HELP
LABELS
NEW
PROJECTION
REFLECT
ROTATE
SCALE
SWINK
STOP
SUMMARY
TIME
TRANSLATE
VERTICAL
ZOOM
SITE
90DEG
PLOT AND LABEL THE AXES(P,L,N) .....: Y
ENTER THE EYE POSITION.
XEYE, YEYE, ZEYE .....: 200, -100, 60
SERIES

```



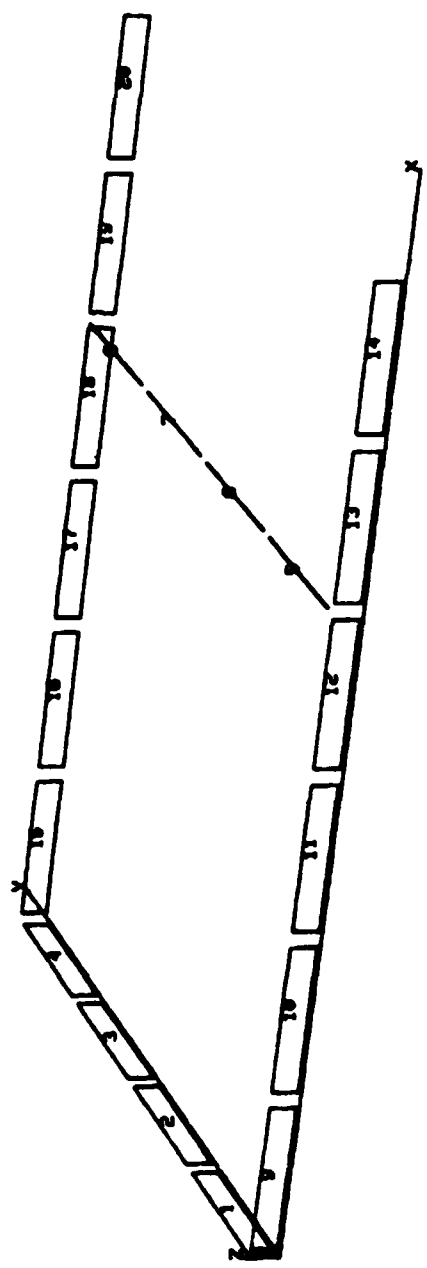
SEVE ENTER THE EYE POSITION,: 100, -300, 75
 WEVE, VEVE, ZEVE
 SELE
 PLOT ALL THE ELEMENTS(V,N): N
 ENTER THE NUMBER OF ELEMENT TYPES TO PLOT: 1
 ELEMENT INPUT OPTIONS
 OPTION 1: RANDOM ELEMENTS(4,13,84,....)
 OPTION 2: RANGE OF ELEMENTS(1,0.2 FOR 1-9, INC OF 2)
 OPTION 3: PLOT ALL ELEMENTS
 ENTER THE ELEMENT TYPE: 1
 ENTER THE INPUT OPTION(1,2,3): 1
 SEWZIK
 ENTER THE SHRINK SCALE(0.-1.): .1
 SLANK
 LABEL THE ELEMENTS(V,N): Y
 LABEL THE MODES(V,N): N
 SEBOM

CORE ELEMENTS ONLY FOR 30 X 24 SYMMETRIC PLATE



3) PLOT ALL THE ELEMENTS (V, N) 4
ENTER THE NUMBER OF ELEMENT TYPES TO PLOT ... 1
ELEMENT INPUT OPTIONS
OPTION 1: RANDOM ELEMENTS (4, 12, 64, ...) 1
 WHERE FIRST DIGIT - NO. ELEMENTS
OPTION 2: RANGE OF ELEMENTS (1, 8 FOR 1-8, INC OF 2)
ENTER THE ELEMENT TYPE 2
ENTER THE INPUT OPTION (1, 2, 3) 1
3) END

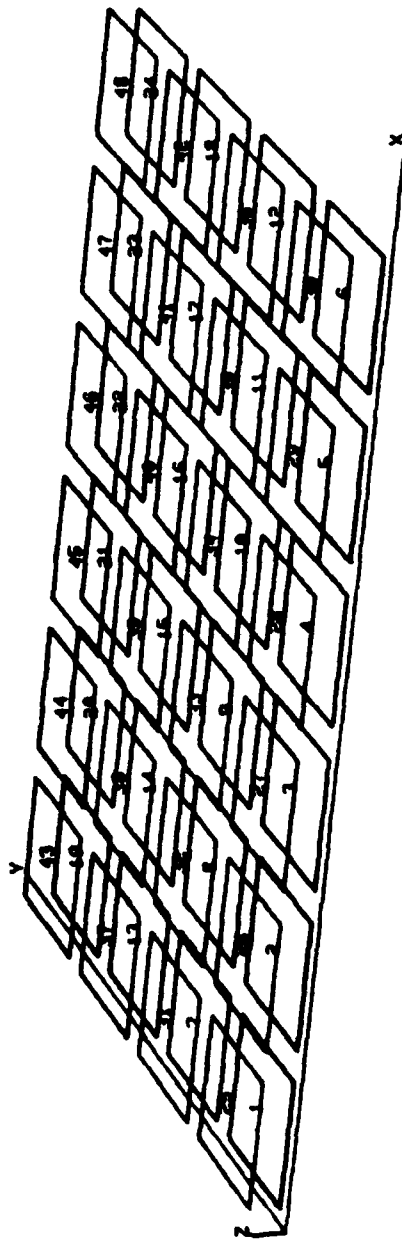
STIFFENER ELEMENTS ONLY FOR 30 X 84 SYMMETRIC PLATE



```

SELECT
PLOT ALL THE ELEMENTS(Y,N) .....: N
ENTER THE NUMBER OF ELEMENT TYPES TO PLOT ...: 1
ELEMENT INPUT OPTIONS
OPTION 1: RANGE ELEMENTS(4,13,64,...)
        MAKE FIRST DIGIT NO. ELEMENTS
OPTION 2: RANGE OF ELEMENTS(1,9,8 FOR 1-9,INC OF 2)
OPTION 3: PLOT ALL ELEMENTS
ENTER THE ELEMENT TYPE .....: 1
ENTER THE INPUT OPTION(1,2,3) .....: 1
SCALE
ENTER THE SIZING SCALE(0.-1.) .....: .2
SCREEN

```



STOP

STOP
047000 MAXIMUM EXECUTION FL.
8.573 CP SECONDS EXECUTION TIME.

.....
BEGIN POSTPROCESSING PHASE

OUTPUT OPTIONS FOR ANALYSIS DATA -

- 0 . STOP
- 1 . LIST LOWER SURFACE DISPL.
- 2 . LIST UPPER SURFACE DISPL.
- 3 . LIST LOWER FACE STRESSES
- 4 . LIST UPPER FACE STRESSES
- 5 . LIST LOWER FACE STRAINS
- 6 . LIST UPPER FACE STRAINS
- 7 . LIST CORE LAYER STRESSES
- 8 . LIST CORE LAYER STRAINS
- 9 . LIST TIME/FREQUENCY VALS.

ENTER OUTPUT OPTION NUMBER.....
STOP
051500 MAXIMUM EXECUTION FL.
13.505 CP SECONDS EXECUTION TIME.
E01 ENCOUNTERED AFTER COPY OF FILE
0, RECORD 1

ENTER INCREMENT, LOAD CASE NO.
OR NODE NO. FOR OUTPUT ---1

NODE NUMBER	ORIGINAL COORDINATES			DISPLACEMENTS		
	X	Y	Z	U	V	W
1	0.	0.	0.	0.00000	0.00000	0.00000
5	0.	.600E+01	0.	0.00000	0.00000	0.00000
9	0.	.120E+02	0.	0.00000	0.00000	0.00000
13	0.	.180E+02	0.	0.00000	0.00000	0.00000
17	0.	.240E+02	0.	0.00000	0.00000	0.00000
21	.500E+01	0.	0.	.00230	-.00523	-.01640
25	.500E+01	.600E+01	0.	.00354	-.00021	-.01970
29	.500E+01	.120E+02	0.	.00210	-.00000	-.01970
33	.500E+01	.180E+02	0.	.00250	-.00021	-.01970
37	.500E+01	.240E+02	0.	.00320	-.00028	-.01640
41	.100E+02	0.	0.	.00450	-.00198	-.05500
45	.100E+02	.600E+01	0.	.00370	-.00033	-.05870
49	.100E+02	.120E+02	0.	.00360	-.00000	-.05000
53	.100E+02	.180E+02	0.	.00370	-.00033	-.05870
57	.100E+02	.240E+02	0.	.00450	-.00198	-.05000
61	.150E+02	0.	0.	.00360	-.00033	-.05870
65	.150E+02	.600E+01	0.	.00370	-.00061	-.10400
69	.150E+02	.120E+02	0.	.00370	-.00000	-.10300
73	.150E+02	.180E+02	0.	.00370	-.00051	-.10400
77	.150E+02	.240E+02	0.	.00360	-.00000	-.10000
81	.200E+02	0.	0.	.00280	-.00047	-.14000
85	.200E+02	.600E+01	0.	.00280	-.00173	-.14000
89	.200E+02	.120E+02	0.	.00280	-.00000	-.14000
93	.200E+02	.180E+02	0.	.00280	-.00173	-.14000
97	.200E+02	.240E+02	0.	.00280	-.00000	-.14000
101	.250E+02	0.	0.	.00160	-.00000	-.17000
105	.250E+02	.600E+01	0.	.00160	-.00000	-.17000
109	.250E+02	.120E+02	0.	.00160	-.00000	-.16000
113	.250E+02	.180E+02	0.	.00160	-.00000	-.17000
117	.250E+02	.240E+02	0.	.00160	-.00000	-.17000
121	.300E+02	0.	0.	0.00000	-.00000	-.18000
125	.300E+02	.600E+01	0.	0.00000	-.00000	-.18000
129	.300E+02	.120E+02	0.	0.00000	-.00000	-.17000
133	.300E+02	.180E+02	0.	0.00000	-.00000	-.18000
137	.300E+02	.240E+02	0.	0.00000	-.00000	-.18000

OUTPUT OPTIONS FOR ANALYSIS DATA

- 0 - STOP
- 1 - LIST LOWER SURFACE DISPL.
- 2 - LIST UPPER SURFACE DISPL.
- 3 - LIST UPPER FACE STRESSES
- 4 - LIST LOWER FACE STRESSES
- 5 - LIST UPPER FACE STRAINS
- 6 - LIST LOWER FACE STRAINS
- 7 - LIST CORE LAYER STRESSES
- 8 - LIST CORE LAYER STRAINS
- 9 - LIST TIME/FREQUENCY UALS.

ENTER OUTPUT OPTION NUMBER....7

ENTER INCR. OR LOAD CASE NO. -1

STRESS RESULTS FOR CORE LAYER

ELEMENT	X2	Y2
1	.210E+02	-.254E+01
2	.211E+02	-.273E+01
3	.166E+02	-.278E+00
4	.134E+02	-.930E+00
5	.794E+01	-.147E+01
6	.332E+01	-.161E+01
7	.210E+02	-.254E+01
8	.250E+02	-.251E+01
9	.150E+02	-.112E+00
10	.123E+02	-.252E+00
11	.519E+01	-.765E+00
12	.327E+01	-.777E+00
13	.221E+02	-.250E+01
14	.250E+02	-.261E+01
15	.150E+02	-.112E+00
16	.123E+02	-.252E+00
17	.519E+01	-.765E+00
18	.327E+01	-.777E+00
19	.218E+02	-.264E+01
20	.211E+02	-.272E+01
21	.166E+02	-.272E+00
22	.134E+02	-.930E+00
23	.794E+01	-.147E+01
24	.332E+01	-.161E+01

OUTPUT OPTIONS FOR ANALYSIS DATA -

- 0 - STOP
- 1 - LIST LOWER SURFACE DISPL.
- 2 - LIST UPPER SURFACE DISPL.
- 3 - LIST LOWER FACE STRESSES
- 4 - LIST UPPER FACE STRESSES
- 5 - LIST LOWER FACE STRAINS
- 6 - LIST UPPER FACE STRAINS
- 7 - LIST CORE LAYER STRESSES
- 8 - LIST CORE LAYER STRAINS
- 9 - LIST TIME/FREQUENCY UNLS.

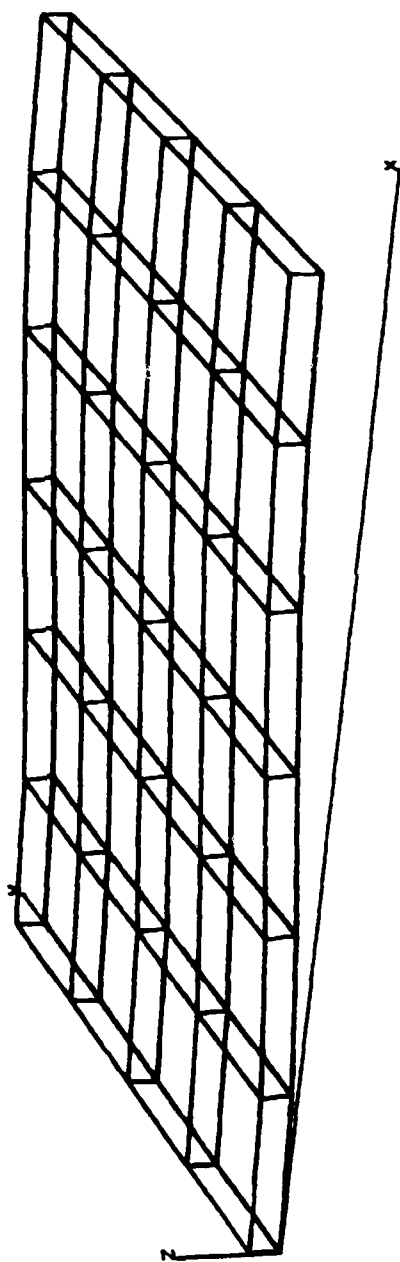
ENTER OUTPUT OPTION NUMBER.....0
ENTER THE CHARACTERS PER SECOND)

STOP
23500 MAXIMUM EXECUTION FL.
.400 CP SECONDS EXECUTION TIME.
E01 ENCOUNTERED AFTER COPY OF FILE
0 - RECORD 1
128
TEKTRONIX OR HP PLOTTER(T,H) 1
MODEL INPUT FILE TYPES:
1 - INPUT DATA FILE
2 - POSTPROCESSOR (MPOST) FILE
ENTER FILE TYPE(1,2) 2

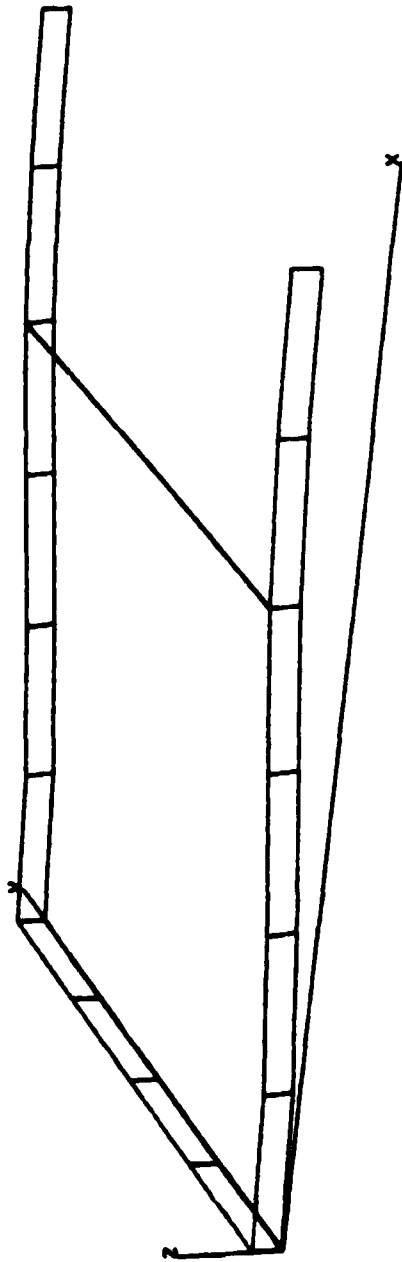
```

ENTER THE STRUCTURE NUMBER .....: 1
HELP
COMMAND
-----
AXES
CLIP
CUBE
DEFAULT
DEFORM
DRAW
ELEMENTS
EYE
HELP
LABELS
MENU
PROJECTION
REFLECT
ROTATE
SCALE
SHRINK
STOP
SUMMARY
TIME
TRANSLATE
VERTICAL
ZOOM
SITE
AXES
PLOT AND LABEL THE AXES(Y,N) .....: 1
EYE
ENTER THE EYE POSITION.
XE,YE,ZE .....: 100,-200,75
DEFORM
STRUCTURE PLOT OPTIONS.
OPTION 1: UNDEFORMED.
OPTION 2: DEFORMED.
OPTION 3: BOTH 1 AND 2.
ENTER THE PLOT OPTION(1,2,3) .....: 2
UNDEFORMED MINIMA AND MAXIMA
MINIMUM X 0.
MINIMUM Y 0.
MINIMUM Z 0.
ENTER THE TIME/LOAD INCREMENT .....: 1
DEFORMED MINIMA AND MAXIMA
MINIMUM X 0.
MINIMUM Y -7.71000E-04
MINIMUM Z 0.
MAXIMUM DISPLACEMENT OCCURS AT NODE 124
ENTER THE SCALE FACTOR .....: 10
DRAW

```



SELE
PLOT ALL THE ELEMENTS(V,N) I
ENTER THE NUMBER OF ELEMENT TYPES TO PLOT I
ELEMENT INPUT OPTIONS
OPTION 1: RANDOM ELEMENTS(4,13,64,....)
 MASK FIRST DIGIT - NO. ELEMENTS
OPTION 2: RANGE OF ELEMENTS(1,9,2 FOR 1-9, INC OF 2)
OPTION 3: PLOT ALL ELEMENTS
ENTER THE ELEMENT TYPE I
ENTER THE INPUT OPTION(1,2,3) I
\$END

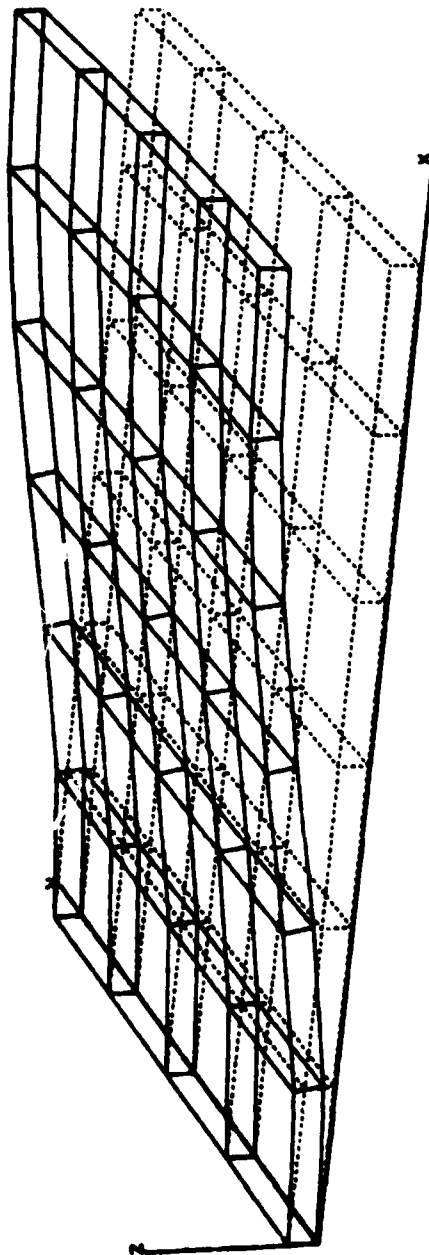


```

SELECT ALL THE ELEMENTS(V,N) .....: 1
ENTER THE NUMBER OF ELEMENT TYPES TO PLOT ...: 1
ELEMENT INPUT OPTIONS
OPTION 1: RANGE ELEMENTS(4,13,64,...)
          WHERE FIRST DIGIT = NO. ELEMENTS
OPTION 2: RANGE OF ELEMENTS(1,8,8 FOR 1-9, INC OF 8)
OPTION 3: PLOT ALL ELEMENTS
ENTER THE ELEMENT TYPE .....: 1
ENTER THE INPUT OPTION(1,2,3) .....: 1

STRUCTURE PLOT OPTIONS.
OPTION 1: UNFORMED.
OPTION 2: REFORMED.
OPTION 3: BOTH 1 AND 2.
ENTER THE PLOT OPTION(1,2,3) .....: 1
UNFORMED MINIMA AND MAXIMA
MINIMUM X 0. MAXIMUM X 3.0000E+01
MINIMUM Y 0. MAXIMUM Y 2.4000E+01
MINIMUM Z 0. MAXIMUM Z 9.6000E-01
ENTER THE TIME/LOAD INCREMENT .....: 1
REFORMED MINIMA AND MAXIMA
MINIMUM X 0. MAXIMUM X 3.0000E+01
MINIMUM Y -7.7100E-04 MAXIMUM Y 2.4000E+01
MINIMUM Z 0. MAXIMUM Z 1.1450E-00
MAXIMUM DISPLACEMENT OCCURS AT NODE 124
ENTER THE SCALE FACTOR .....: 20

```



STOP

047880 MAXIMUM EXECUTION FL. TIME.
3.698 CP SECONDS EXECUTION TIME.
38828 WORDS (237180 MAX USED)
CPA 21.717 SEC. 17.698 ADJ.
IO 22.554 SEC. 24.316 ADJ.
CR 228.466 KWD. 27.489 ADJ.
CPLS 182.475
COST 8.53
PP 110.871 SEC. 8 DATE 04/07/81
COMMAND FILES
--LOCAL FILES--
POSTFL SPROCFIL SINPUT SOUTPUT
INPRT
COMMAND BATCH,ALLIUT,PRINT,INER
COMMAND RE,USER,IN,ENTL,POSTFL,PROCFIL.

STOP

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