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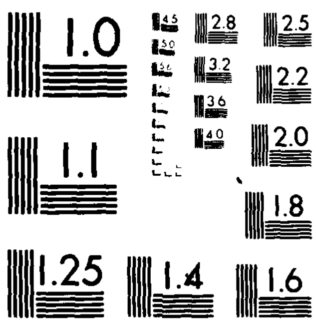
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LOCALIZED NECKING: AN INCLINED IMPERFECTION MODEL

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The Marciniak and Kuczynski (M-K) analysis, which associates localized necking in sheet metal with pre-existing imperfections, has been modified such that the imperfection or groove is inclined (rather than normal) to the major principal strain axes. The analysis, which is closely related to the calculations of Hutchinson and Neale, takes into account the requirement for localized necking that a direction of zero-extension is inclined for strain states between uniaxial tension and plane strain.

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20. Abstract (cont'd)

Based on the inclined imperfection model, calculations are presented for the negative minor strain regime of a forming limit diagram showing the dependence of localized necking on work hardening, strain rate sensitivity, plastic anisotropy, and imperfection factor. Theoretical calculations of forming limit curves indicate good agreement between the model and previously obtained experimental data for several alloys. Such agreement is obtained for realistically small imperfections, although the limit strains predicted by the model itself are relatively insensitive to the magnitude of the imperfection.

LOCALIZED NECKING: AN INCLINED IMPERFECTION MODEL

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Summary - The Marciniak and Kuczynski (M-K) analysis, which associates localized necking in sheet metal with pre-existing imperfections, has been modified such that the imperfection or groove is inclined (rather than normal) to the major principal strain axes. The analysis, which is closely related to the calculations of Hutchinson and Neale, takes into account the requirement for localized necking that a direction of zero-extension is inclined for strain states between uniaxial tension and plane strain. Based on the inclined imperfection model, calculations are presented for the negative minor strain regime of a forming limit diagram showing the dependence of localized necking on work hardening, strain rate sensitivity, plastic anisotropy, and imperfection factor. Theoretical calculations of forming limit curves indicate good agreement between the model and previously obtained experimental data for several alloys. Such agreement is obtained for realistically small imperfections, although the limit strains predicted by the model itself are relatively insensitive to the magnitude of the imperfection.

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NOTATION

n	work hardening exponent in the Swift equation
m	strain rate sensitivity
R	plastic anisotropy parameter
f	imperfection factor
$\bar{\varepsilon}_o$	constant in the Swift equation
K	strength constant in the Swift equation
t_A, t_B	thickness of sheet in reagion A and B respectively
ε_1^*, ε_2^*	major and minor limit strain
ϕ	the angle between the normal of the imperfection and the σ_1 axis
ϕ^*	Hill's angle of direction of zero-extension
α	stress ratio = σ_2/σ_1
ρ	strain ratio = $\varepsilon_2/\varepsilon_1$
$\bar{\sigma}$, $\bar{\varepsilon}$	effective stress and strain
σ_1, σ_2	principal stresses
ε_1, ε_2	principal strains
σ_x, σ_y, τ_{xy}	stress components in the x-y coordinate system
ε_x, ε_y, ε_{xy}	strain components in the x-y coordinate system

Introduction

Sheet materials deforming under multiaxial states of stress, as in sheet metal forming operations, usually fail by localized necking. The current interest in understanding sheet metal formability has led to several theoretical analyses of localized necking based on different criteria. These localized necking criteria include: a localized shear zone along a direction of zero-extension [1], material imperfections [2], or the presence of a vertex on the yield surface [3], and void growth [4].

Localized necking along a direction of zero-extension was originally proposed by Hill [1]. Hill's theory predicts that the maximum principal strain ε_1^* prior to localized necking (i.e., the limit strain) has a magnitude of $\varepsilon_1^* = n$ at plane strain and increases to $\varepsilon_1^* = (1+R)n$ for the uniaxial tension deformation of sheet exhibiting normal anisotropy with a plastic anisotropy parameter R, which is defined as the ratio of the width strain to thickness strain of sheet specimens deformed under uniaxial tension. For plastically isotropic material (R=1), the limit strain at uniaxial tension thus reduces to the well-known $\varepsilon_1^* = 2n$ expression. Hill's theory, however, cannot explain the phenomenon of localized necking under biaxial states of stress. A direction of zero-extension does not exist when both ε_1 and ε_2 are greater than zero and thus Hill's theory is limited to the negative, minor strain regime of a forming limit diagram ($\rho \leq 0$).

Strain localization developed by local weakness of material was first proposed by Marciniak and Kuczynski (M-K) [2] and extended by Sowerby and Duncan [5]. In the M-K analysis, a material imperfection in the form of a groove or trough is postulated to occur perpendicular to the direction of major principal strain (ε_1). Imposing the same ε_2 inside and outside the groove while proportional straining is maintained outside the groove, M-K have shown that deformation within the groove occurs at a faster rate than the rest of the sheet. The concentration of strain (ε_1) within the groove eventually leads to the plane strain condition ($d\varepsilon_2 = 0$) within the groove and to localized necking. The M-K model is thus able to explain localized necking under biaxial stresses. Recently, Ghosh [6] used the M-K model to calculate the forming limit diagrams for A-K steel and 70-30 brass. He found reasonable agreement between the calculated and experimental forming limit diagrams of these alloys. Thus, he concluded that Marciniak and Kuczynski's model of imperfection growth can be used to predict sheet metal forming limits, provided that an accurate constitutive equation is used and imperfections of metallurgical origin are accounted for.

An attractive feature of the M-K analysis is that the concept of the imperfection allows one to interpret both the negative and positive sides of a forming limit diagram ($-\frac{1}{1+R} \leq \rho \leq 1$) in terms of a single criteria (as opposed to relying on Hill's theory for $-\frac{1}{1+R} \leq \rho \leq 0$ and another criteria for biaxial tension). In the M-K analysis, localized necking is developed from the strain concentration within a material imperfection in the form of a groove which is assumed to align perpendicular to the major principal strain axis (ε_1). In biaxial straining of sheet metal ($\varepsilon_1 > \varepsilon_2 > 0$), localized necking does occur normal to the major principal strain axis, as assumed in the M-K analysis. However, localized necking usually occurs at an angle <u>inclined</u> to the major principal strain axis when the loading places the strain path in the negative minor strain regime of the forming limit diagram (i.e., $\rho < 0$). The angle of inclination depends on the state of stress as well as plastic anisotropy and thus the R-value of the material. For a plastically isotropic material in uniaxial tension, localized necking occurs inclined to the stress axis at an angle of about 55°, which differs considerably from the 90° assumed by M-K.

Localized necking involving an inclined groove when $\rho < 0$ has previously been undertaken by Hutchinson and Neale [7]. In their analysis, a groove is is initially inclined at an angle to the σ_1 (or ε_1) axis. During straining, the groove is rotated toward the σ_1 axis. Their calculations show that the limit strain is influenced by the orientation of the groove with respect to the σ_1 axis. Specifically, the limit strain depends on the initial as well as the final groove orientation, and the limit strain has thus to be minimized with respect to the groove orientation. The minimum limit strain occurs when the final orientation of the groove coincides with the Hill angle [1] provided that the von Mises flow theory of plasticity is assumed [7].

This paper suggests an alternative and efficient method of predicting the limit strain of sheet metal failed by localized necking inclined to the σ_1 axis for strain paths $\rho < 0$. The formulation of the present analysis is similar to that of Hutchinson and Neale [7] for the case of J flow theory of plasticity and therefore a smooth yield surface. In this anlaysis the orientation of the inclined groove is, however, rigidly fixed along Hill's angle of zero-extension. This simplifying assumption allows the calculation of the minimum forming limit strain without minimizing the limit strain as a function of the groove orientation. This permits a straightforward examination of the dependence of localized necking (for $\rho < 0$) on such factors as work hardening, strain rate sensitivity, plastic anisotropy and the size of the imperfection. Comparison of analysis with experimental data can also be readily determined.

In reality, material imperfections are not likely to be conveniently aligned along a direction of zero-extension. However, we suggest that localized necking is initiated at some arbitrary inclined angle and quickly rotates into the zero-extension configuration during subsequent straining. This analysis is thus an approximate solution, but it does give predictions of limit strains virtually identical to those calculated by Hutchinson and Neale [7] who allow the imperfection to rotate.

The present analysis relates to the original M-K analysis [2,5] in that it converges with the M-K predictions at plane strain. For deformation under biaxial tension ($\varepsilon_1 > \varepsilon_2 > 0$), the orientation of the localized neck is not inclined but normal to the ε_1 axis. Thus previous calculations [2,5] and comparison [6] with the positive side of experimental forming limit diagrams apply.

Analysis

The present approach follows the procedure used by Marciniak, Kuczynski and Pokora [8]. The analysis is within the framework of generalized plane stress and is a "long-wavelength" approximation as discussed by Hutchinson, Neale and Needleman [9].* As shown in Fig. 1, a material imperfection is postulated to occur on an angle inclined to the principal stress axis (σ_1) of a sheet metal under biaxial stresses of σ_1 and σ_2. The inclined imperfection is designated as region B and its thickness is t_B. The region outside the imperfection is considered to be uniform and is referred as region A; its thickness is designated as t_A. The angle between the normal of the imperfection and the σ_1 axis is noted as ϕ.

Following Marciniak et al.'s approach, [8] the equilibrium requirement between regions A and B gives:

$$t_B \, \sigma_{x_B} = t_A \, \sigma_{x_A}$$

and

$$t_B \, \sigma_{xy_B} = t_A \, \sigma_{xy_A}, \qquad \text{Eqn. (1)}$$

where σ_{x_A}, σ_{x_B}, σ_{xy_A} and σ_{xy_B} are the respective stress components in regions A and B. The stresses σ_{x_A}, σ_{y_A} and σ_{xy_A} are obtained by stress transformation of σ_1 and σ_2 into the x-y coordinate system.

*The basis for a long wavelength approximation is that the ratio of the width of the localized neck to sheet thickness is large (>4) such that no stress or strain gradient exists across the thickness of the sheet [9].

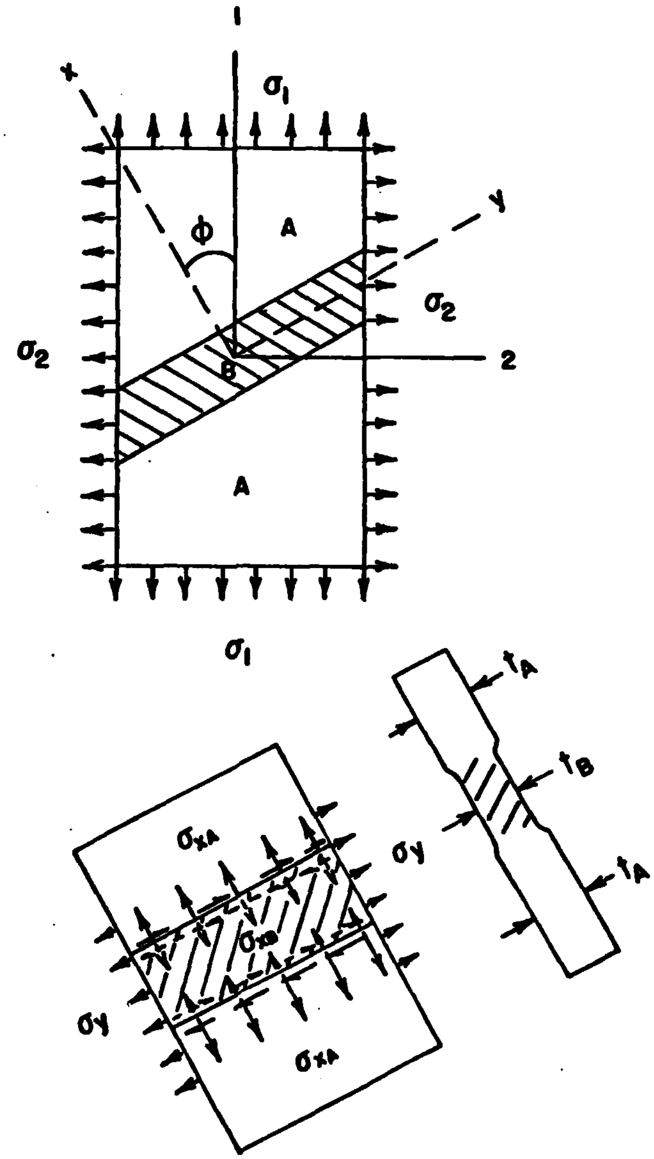


Fig. 1. A schematic view of a sheet subjected to the biaxial stresses σ_1 and σ_2 and containing an imperfection inclined to the σ_1 axis.

$$\sigma_{x_A} = \sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi$$
$$\sigma_{y_A} = \sigma_1 \sin^2\phi + \sigma_2 \cos^2\phi \qquad \text{Eqn. (2)}$$
$$\sigma_{xy_A} = (\sigma_1 - \sigma_2)\ \sin\phi\ \cos\phi$$

To relate stresses to strains, a power law type effective stress-effective strain constitutive equation is used similar to that proposed by Swift [10]:

$$\bar{\sigma} = K\ (\bar{\varepsilon}_o + \bar{\varepsilon})^n\ \dot{\bar{\varepsilon}}^{\,m} \qquad \text{Eqn. (3)}$$

where $\bar{\sigma}$ = effective stress, $\bar{\varepsilon}$ = effective strain, $\bar{\varepsilon}_o$ = a prestrain term, n = strain-hardening exponent, K = strength constant, $\dot{\bar{\varepsilon}}$ = effective strain rate and m = strain-rate sensitivity exponent. The imperfection is assumed to be a variation in K x thickness. The imperfection factor, f, is thus defined as $(K_B t_B)/(K_A t_A)$, where K_A and K_B are the strength constants in regions A and B respectively. Combining Eqns. (1) and (3), w obtain:

$$\frac{(\sigma_{x_A}/\bar{\sigma}_A)(d\bar{\varepsilon}_A/d\bar{\varepsilon}_B)^m}{(\sigma_{x_B}/\bar{\sigma}_B)} = f\left[\frac{\bar{\varepsilon}_o + \bar{\varepsilon}_B}{\bar{\varepsilon}_o + \bar{\varepsilon}_A}\right]^n \exp[\varepsilon_{z_B} - \varepsilon_{z_A}] \qquad \text{Eqn. (4)}$$

where ε_{z_A} and ε_{z_B} are the thickness strains in regions A and B respectively.

The sheet is assumed to have normal anisotropy and we assume that the effective stress is adequately described by Hill's original yield function for anisotropic material:*

$$\bar{\sigma} = \sqrt{\frac{3}{2}}\sqrt{\frac{R+1}{R+2}}\sqrt{\sigma_x^2 - \frac{2R}{R+1}\sigma_x\sigma_y + \sigma_y^2 + \frac{2(1+2R)}{1+R}\sigma_{xy}^2} \qquad \text{Eqn. (5)}$$

where R is the plastic anisotropy parameter. The effective strain thus becomes

*In order to study the influence of plastic anisotropy on localized necking and thus the forming limit strain, an anisotropic yield function is required. Hill [11] has proposed a new anisotropic yield function which consists of a stress exponent parameter, M. The value of the parameter, M, has to be determined by biaxial experiments and is not available for most materials. The earlier function is adequate for $R \gtrsim 1$.

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}}\sqrt{\frac{(2+R(1+R)}{1+2R}}\sqrt{d\varepsilon_x^{\,2} + \frac{2R}{1+R}\,d\varepsilon_x d\varepsilon_y + d\varepsilon_y^{\,2} + \frac{2}{1+R}\,d\varepsilon_{xy}^{\,2}} \qquad \text{Eqn. (6)}$$

and the associated flow rules are

$$\frac{d\varepsilon_x}{(\frac{1+R}{R})\ \sigma_x-\sigma_y} = \frac{d\varepsilon_y}{(\frac{1+R}{R})\ \sigma_y-\sigma_x} = \frac{d\varepsilon_z}{-(\frac{1+R}{R}\ -1)\,(\sigma_x+\sigma_y)} =$$

$$\frac{d\varepsilon_{xy}}{(\frac{1+2R}{R})\ \sigma_{xy}} = \frac{3R}{2(2+R)}\ \frac{d\bar{\varepsilon}}{\bar{\sigma}}\ . \qquad \text{Eqn. (7)}$$

From the theory of Marciniak et al [8], the constraint condition between regions A and B is

$$d\varepsilon_{y_A} = d\varepsilon_{y_B} \qquad \text{Eqn. (8)}$$

If the imperfection orientation ϕ is taken along a direction of zero-extension such that plane strain condition is satisfied as $d\varepsilon_{y_A} = d\varepsilon_{y_B} = 0$. The angle ϕ is designated as ϕ^* and is given by (1):

$$\phi^* = 90° - \tan^{-1}\sqrt{\frac{\alpha - (\frac{1+R}{R})}{(\frac{1+R}{R})\ \alpha - 1}} \qquad \text{Eqn. (9)}$$

After a lengthy mathematical manipulation, the following expression for describing the deformation of sheet metal exhibiting localized necking inclined to the σ_1 axis is obtained:

$$d\bar{\varepsilon}_A^{\ m}\,[\bar{\varepsilon}_o+\bar{\varepsilon}_A]^{\ n}\ \exp[-C\bar{\varepsilon}_A] = f\,[\bar{\varepsilon}_o+\bar{\varepsilon}_B]^{\ n}\ \exp[-F\cdot\bar{\varepsilon}_B]\,d\bar{\varepsilon}_B^{\ m} \qquad \text{Eqn. (10)}$$

where $$F = \sqrt{\frac{3}{2}}\sqrt{\frac{1+2R}{(R+2)(R+1)}}\ \left[1+2(R+1)\ \alpha_2^{\,2}\right]^{-\frac{1}{2}}$$

$$C = \sqrt{\frac{3}{2}}\sqrt{\frac{1+2R}{(R+2)(R+1)}}\ \left[1+2(R+1)\ \rho_2^{\,2}\right]^{-\frac{1}{2}}$$

and $$\alpha_2 = \frac{(1-\alpha)\ \sin\phi^*\ \cos\phi^*}{\cos^2\phi^* + \alpha\ \sin^2\phi^*}\quad ;\quad \alpha = \frac{\sigma_2}{\sigma_1}$$

$$\rho_2 = \frac{(1-\rho)\ \sin\phi^*\ \cos\phi^*}{\cos^2\phi^* + \rho\ \sin^2\phi^*}\quad ;\quad \rho = \frac{\varepsilon_2}{\varepsilon_1}$$

Results

As shown in Fig. 2, the limit strains calculated from Eqn. (1), which are solved numerically by a simple incremental method, are almost identical with those previously obtained in a more complex and rigorous method by Hutchinson and Neale [7] for the case of an imperfection which is allowed to rotate. The results are shown in Fig. 2 for the negative portion of the FLD, recalling that inclined localized necking occurs only when $\rho < 0$.

Comparisons of calculated limit strains between M-K analysis with perpendicular groove (ϕ=0) and the inclined groove model have also been made and an example of their differences is depicted in Fig. 3 for the case of A-K steel. When $\rho = \varepsilon_2/\varepsilon_1 = 0$ as in plane strain condition, Eqn. (9) gives $\phi^* = 0$ and therefore the inclined imperfection model is equivalent to the M-K analysis and both models given the same effective limit strain for plane strain loading. The M-K model, however, predicts a much greater effective limit strain as the strain ratio ρ becomes more negative for the small imperfection (f = .998) used. For example, in uniaxial tension ($\rho = -0.60$), M-K predicts a limit strain of $\varepsilon_1^* = 2.8$ while the present analysis predicts $\varepsilon_1^* = 0.74$. Experimentally, the limit strain for localized necking of A-K steel is $\varepsilon_1^* = 0.79$ [12]. The M-K analysis can be brought into better agreement with the experimental value by using a larger value of f (f < .98), but this seems unrealistically large. In addition, localized necking in A-K steel is observed to be inclined at an angle of $\phi = 27°$ which compares favorably to $\phi = 37°$ from Eqn. (9).

The distribution of the strain components of the effective strain attained outside the imperfection site at the onset of plastic instability is depicted in Fig. 4. The limit effective strain $\bar{\varepsilon}_A^*$, computed by the inclined imperfection model, is seen to increase as the strain state (ρ) becomes more negative. This trend, however, is not always observed as far as individual strain components are concerned. The strain components ε_x^* and ε_y^* are both constant and independent of ρ ($\varepsilon_x^* = 0.30$ and $\varepsilon_y^* = 0.0$). The shear strain component ε_{xy}^*, on the other hand,

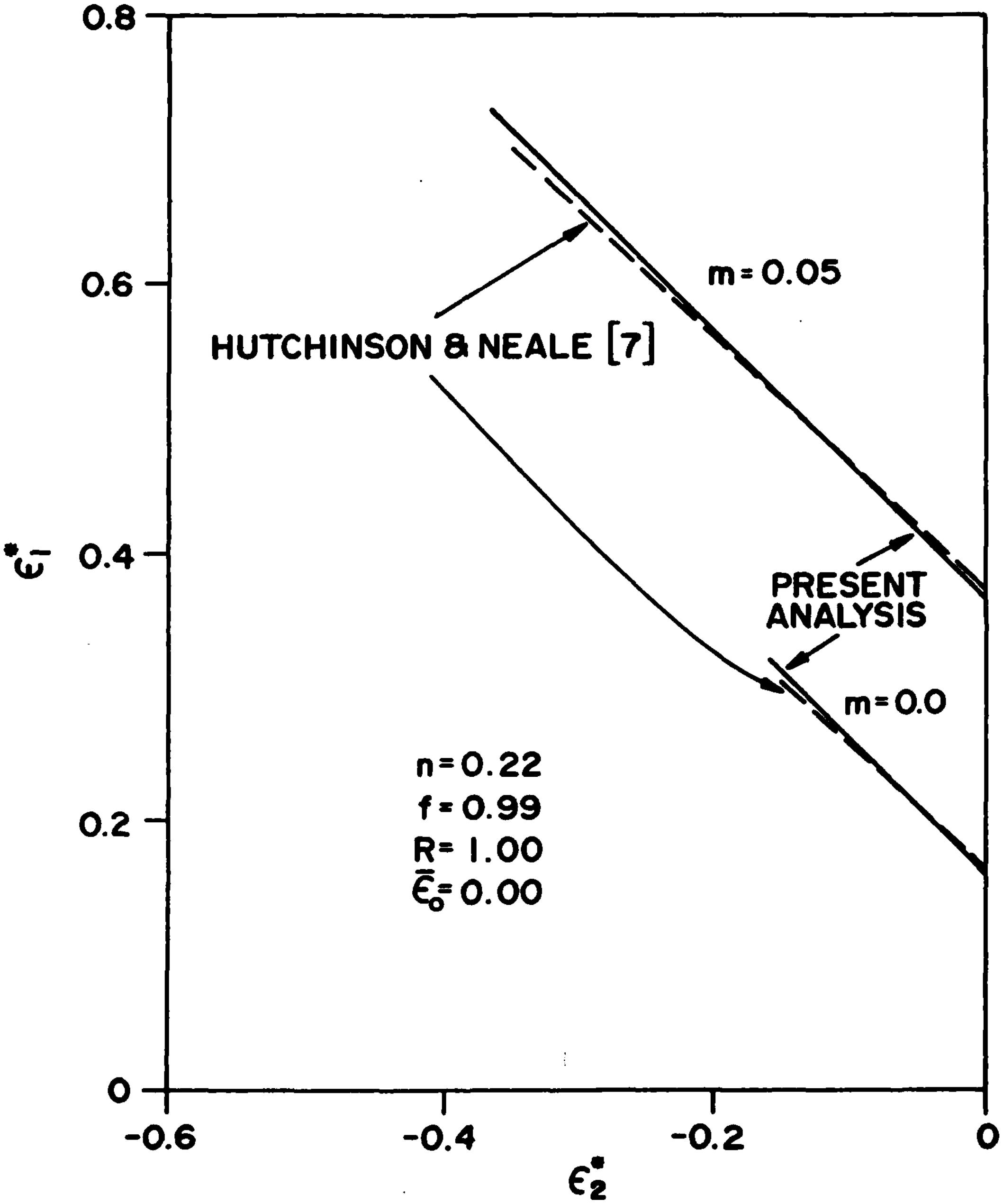


Fig. 2. A comparison of the calculated limit strains based on the present analysis and that of Hutchinson and Neale [7].

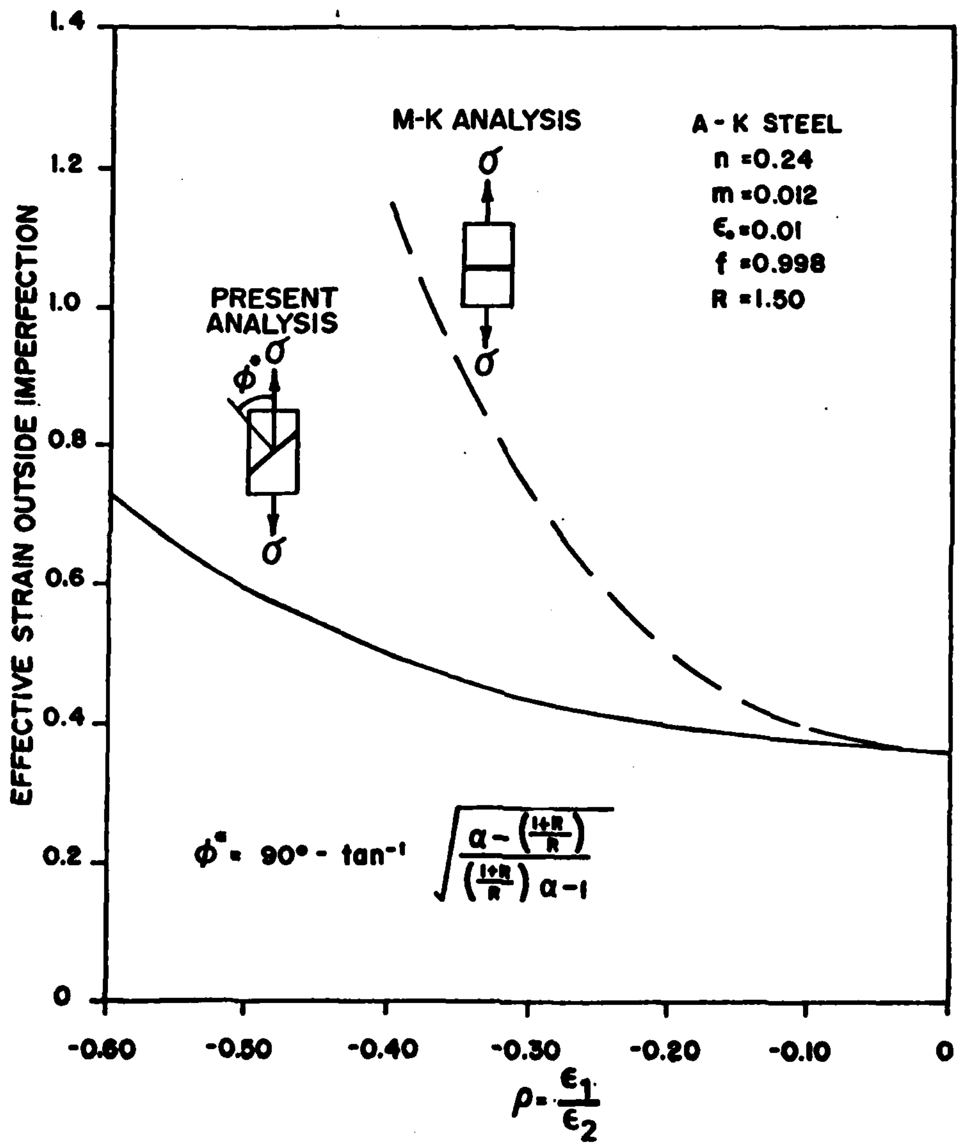


Fig. 3. A comparison between the M-K analysis and the inclined imperfection model based on the dependence of the calculated limit strains on the strain ratio for the case of A-K steel.

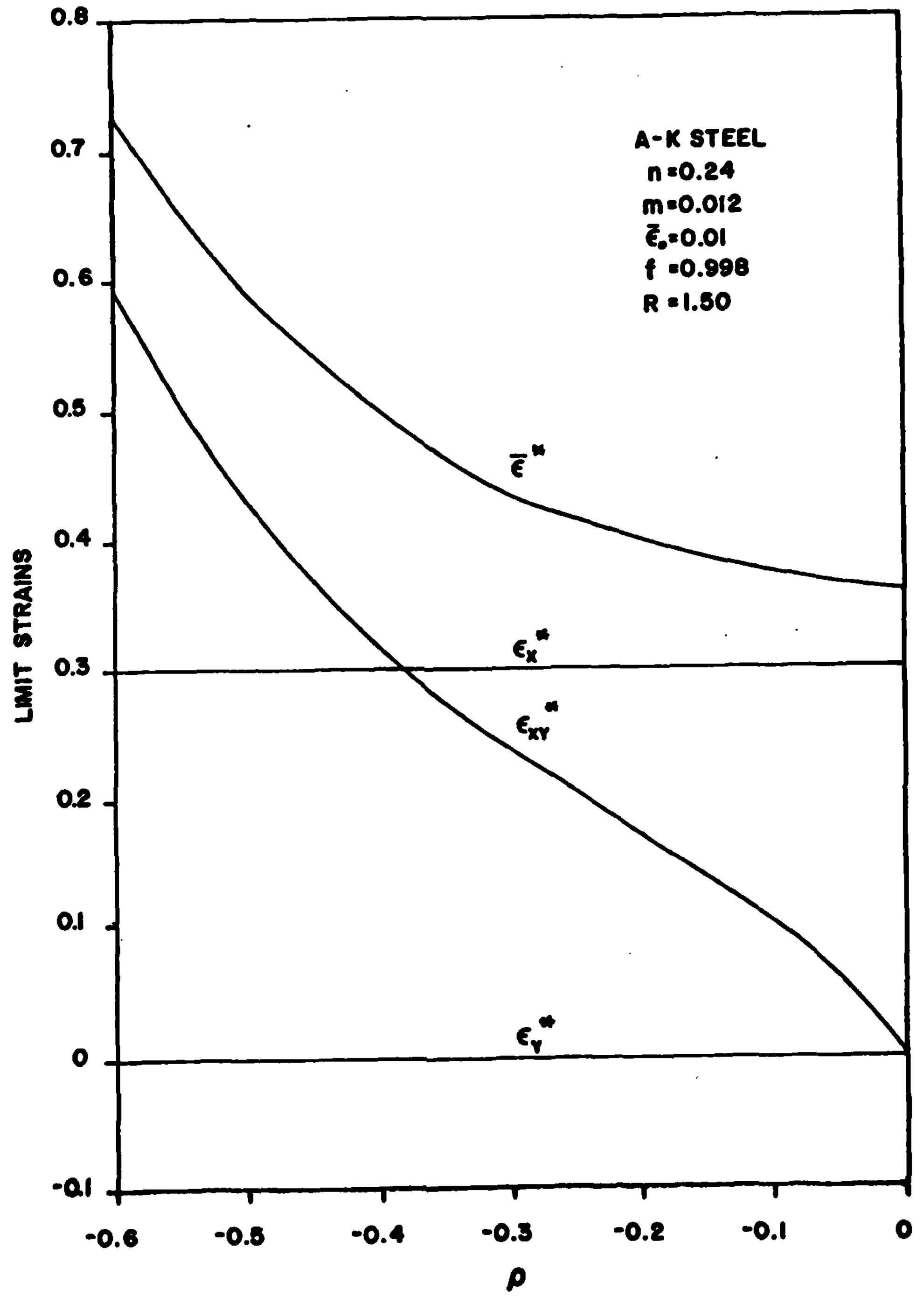


Fig. 4. The distribution of the strain components of the effective limit strain as a function of strain ratio.

increases very rapidly as ρ becomes more negative. In fact, the increase in $\bar{\varepsilon}_A^*$ is largely due to the increase in ε_{xy}^* as the stress state turns toward the uniaxial tension (Fig. 4).

On the Influence of Materials Properties

The influence of material properties such as n, m and R on localized necking can be readily demonstrated using the inclined imperfection model. The strain ratio has been chosen at uniaxial tension ($\rho = -\frac{R}{1+R}$) and a very small imperfection is used (f=0.9999). Figure 5 depicts the influence of the strain hardening exponent, n, on the flow localization process, while maintaining the other parameters constant (m=0, $\bar{\varepsilon}_o$=0, R=1). The results indicate that the strain within the imperfection site is always larger than that outside the imperfection, and when flow instability occurs, strain is localized within the imperfection while the region outside ceases to deform at the limit strain. Figure 5a clearly shows that a higher strain hardening exponent, n, can delay the onset of instability to higher strain value and thus enhance the limit strain. Interestingly, the limit strains calculated for each of the n values is just slightly smaller than 2n which is the prediction of Hill's theory for isotropic material with R=1.

The influence of strain rate hardening exponent, m, on the imperfection growth (n=0.20 in this case) is shown in Fig. 5b. The strain localization process occurs quite abruptly at m=0. However, strain rate hardening appears to have a stabilizing effect on strain localization as it is delayed and occurs at a much more gradual rate as m increases. The consequence of this stabilizing effect is the enhancement of the limit strain, which increases from slightly less than 2n (ε_1^*=0.38) at m = 0.0 to much greater than 2n (ε_1^*=0.84) at m = 0.02.

The plastic anisotropy parameter, R, can also exert a beneficial influence by delaying localized neck formation for $\rho < 0$. Figure 5c shows that the limit

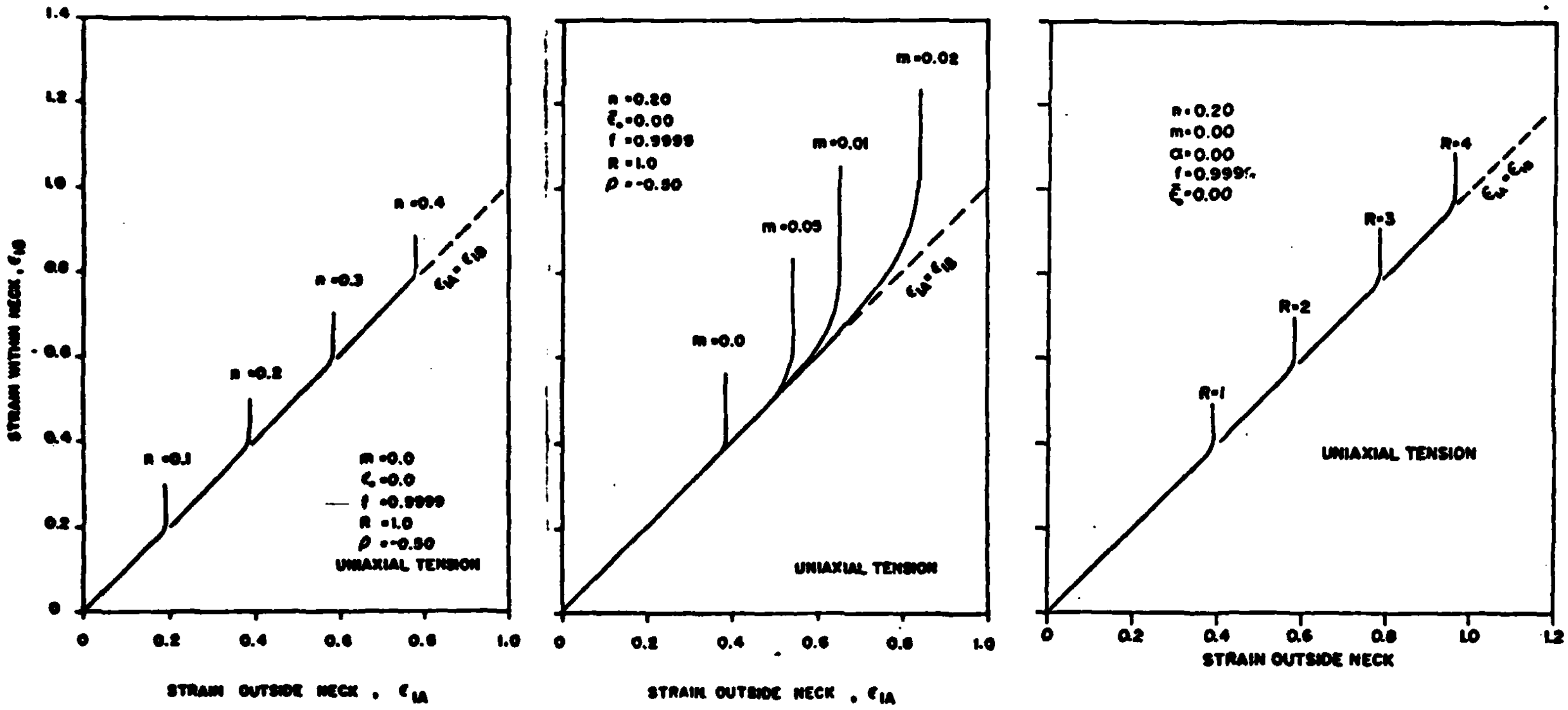


Fig. 5. Strain localization for sheet metal deformed in uniaxial tension as influenced by: (a) the work hardening exponent n, (b) the strain rate sensitivity exponent m, and (c) the plastic anisotropy parameter R. The calculations are based on the inclined imperfection model.

strain is enhanced significantly as the R value increases. The dependence of the limit strain on the R-value is also predicted by Hill's theory. The difference between the predicted effect of R according to Hill's theory vs. the inclined imperfection model is small when m = 0.0 but the difference becomes greater as m increases. Thus, for moderate m (.01) values, the present analysis predicts a limit strain in uniaxial tension which is about 60% greater than that predicted by Hill's theory.

The onset of localized necking is obviously influenced by the severity of the imperfection. Figure 6 depicts the dependence of the limit strain on the imperfection factor f.* Similar to the M-K analysis, the inclined imperfection model is affected by the imperfection size. The limit strain ε_1^* is reduced as the imperfection size gets larger (f becomes smaller), indicating that imperfection growth is accelerated by an increasing imperfection size. However, unlike the M-K analysis, the inclined imperfection model is much less sensitive to the imperfection factor f. At a constant f, the M-K analysis, again, predicts a limit strain larger than that obtained by the inclined imperfection model. The discrepancy becomes larger when the imperfection size becomes small, i.e., when f approaches to unity.

Limit Strain at Uniaxial Tension

Incorporating material parameters of n, m and R into the inclined imperfection model, the uniaxial tensile limit strains (ε_1^* at α = 0) of various sheet metals have been calculated. The materials investigated include several steels, aluminum alloys, brass, and commercially pure (CP) titanium. In all computations, a small imperfection of f = 0.998 is used and $\bar{\varepsilon}_o$ is set equal to zero, which zero is equivalent to assuming that the material obeys a power law hardening behavior: $\bar{\sigma} = k\bar{\varepsilon}^{n}\dot{\bar{\varepsilon}}^{m}$. The orientation of the imperfection is chosen to lie along a direction of zero-extension and ϕ^* is given by Eqn. (9). The

*Recall that $f \equiv k_B t_B / k_A t_A$. The term f can be a geometric groove (in which case $f \simeq t_B/t_A$) or a material imperfection (in which case $f = k_B/k_A$).

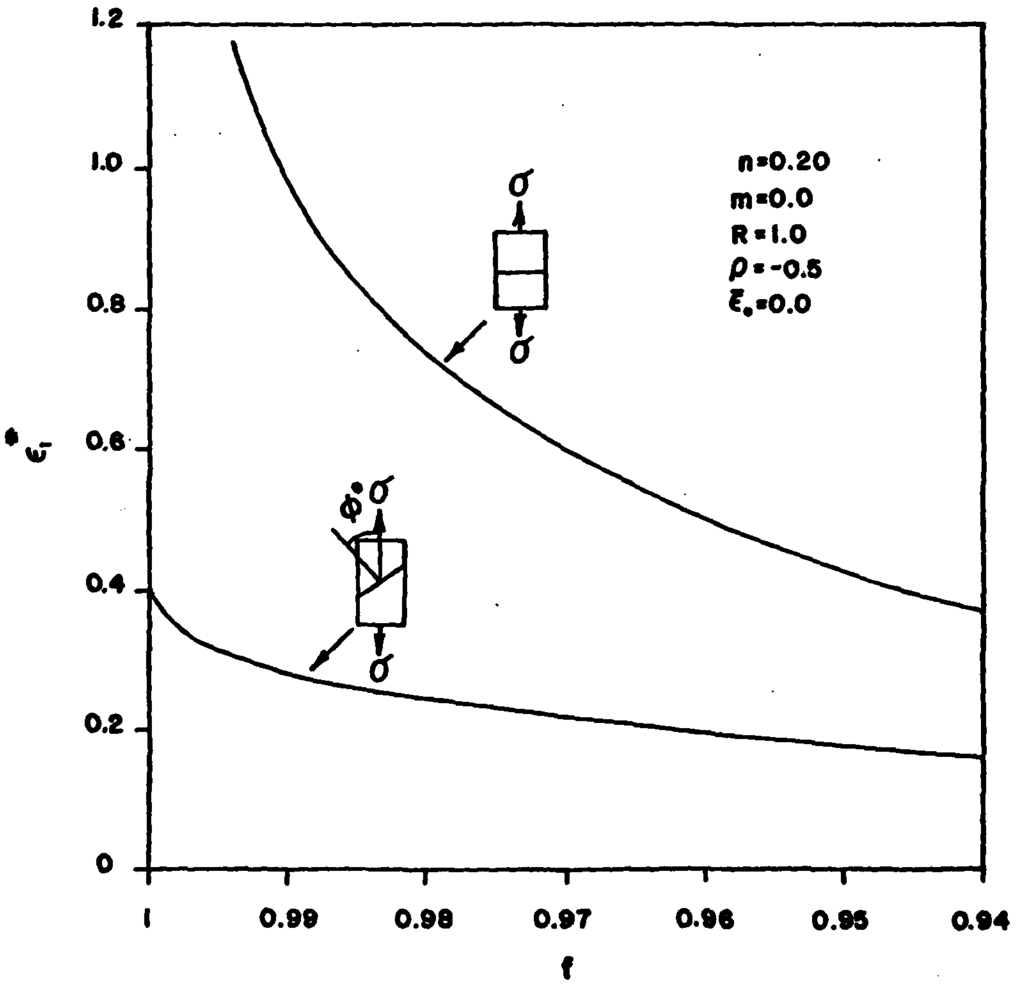
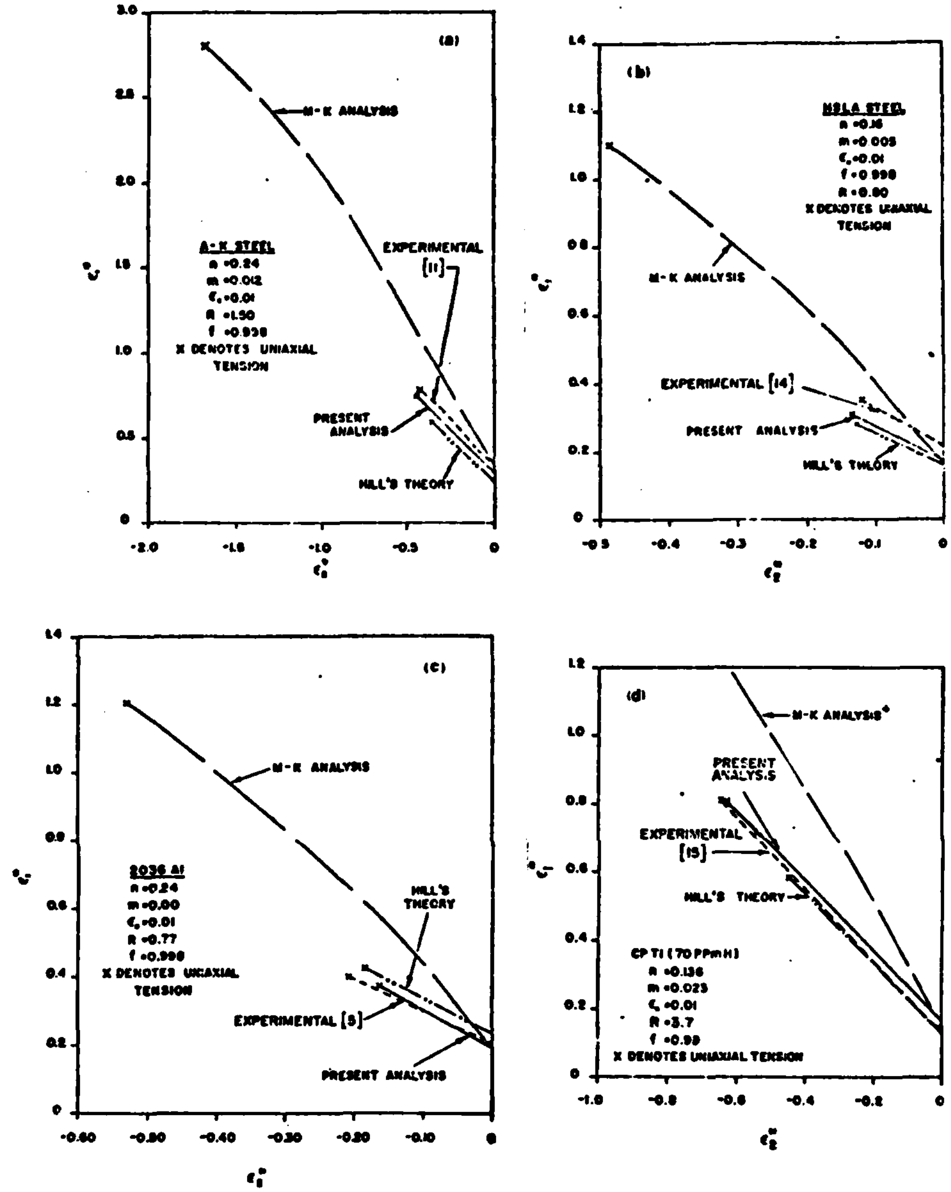


Fig. 6. The influence of the imperfection factor (f) on the calculated limit strains based on either the inclined imperfection model or the M-K model.

calculated limit strain is compared with experimental data from literature [5,13,14,15,16] and Hill's theory which predicts $\varepsilon_1^* = (1+R)n$. All results are summarized and shown in Table I together with the appropriate material parameters. The uniaxial tensile limit strains calculated from the inclined imperfection in most cases are in good agreement with the experimental values (with the exception of 1/4 hard Al). Hill's theory, on the other hand, is only in fair agreement with the experimental data. The results indicate that Hill's theory tends to overestimate the limit strain when m=0 and underestimates the limit strain when m > 0. The M-K analysis has also been used to calculate the tensile limit strain for A-K steel, HSLA steel, 2036-Al, 70-30 brass and CP Ti. The results indicate that the M-K analysis greatly overestimates the tensile limit strains of these materials (see Table [illegible]. A large imperfection (for example, $f \geq .98$) usually can bring the M-K prediction into better agreement with experimental results, but such a large imperfection is unrealistic.

Forming Limit Diagram

The negative side of the forming limit diagram has been calculated for A-K steel, 2036-Al, HSLA steel, 70-30 steel, and CP Ti and are shown in Fig. 7a, b, c, d and e respectively. The imperfection sizes are realistically small, ranging from 0.998 to 0.99, depending on the materials. The results are compared with their counterparts using the M-K model, Hill's theory and experimental forming limit curves. As illustrated in Fig. 7a, b, c, d, and e, the forming limit curve calculated from the inclined imperfection is in good agreement with the experimental forming limit curve in all cases (A-K steel, HSLA steel, 2036-Al, 70-30 brass and CP Ti). The agreement between Hill's theory and observed behavior is also good and compares favorably to that between the present analysis and the data. M-K analysis with a perpendicular groove shows poor agreement with the experimental results in all cases. As before, the M-K analysis could be forced into a much better fit with a larger imperfection factor, which in most cases would be unrealistically large.



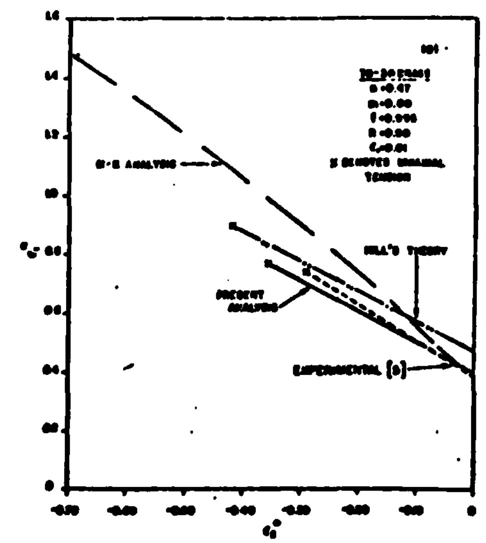


Fig. 7. Comparisons of the negative side of the FLD between Hill's theory [1], the inclined imperfection model, the M-K analysis and experimental results for:
(a) A-K steel [12],
(b) HSLA steel [14],
(c) 2036 Al [5],
(d) CP Ti [16], and
(e) 70-30 brass [5].

Table I. Theoretical predictions of uniaxial tensile limit strain by the inclined imperfection model and comparisons of the calculated values with previously reported experimental results, Hill's theory and the M-K analysis.

Material		n	m	R	Limit Strain at Uniaxial Tension, ε_1^*			
					Experiment	Inclined Imperfection Model	Hill's Theory: $\varepsilon_1^* = (1+R)n$	M-K Analysis
A-K Steel	[11]	0.24	0.012	1.50	0.79	0.74	0.60	2.8
R Steel	[14]	0.21	0.011	1.11	0.67	0.57	0.44	-
HSLA Steel	[15]	0.16	0.005	0.80	0.35	0.31	0.29	1.1
Brass	[5]	0.47	0.00	0.90	0.74	0.81	0.89	1.47
1/4-hard Al	[14]	0.04	0.003	0.76	0.28	0.07	0.07	-
2036-Al	[5]	0.24	0.00	0.77	0.40	0.37	0.42	1.21
3003-0	[14]	0.20	0.006	0.76	0.42	0.39	0.35	-
5154-Hill	[14]	0.20	0.00	0.76	0.22	0.30	0.35	-
6061-T4	[14]	0.20	0.00	0.64	0.29	0.28	0.33	-
CP Ti	[16]	0.136	0.023	3.70	0.81	1.07	0.58	4.6

Discussion

In this analysis localized necking for $-\frac{1}{1+R} < \rho < 0$ is assumed to occur at Hill's angle of direction of zero-extension. By rigidly maintaining an imperfection under such configuration during straining, a simple equation (Eqn. 10) has been obtained for predicting the limit strains and thus FLDs involving negative ε_2 strains. Good agreement is obtained by the present analysis both with previously obtained experimental results and with the calculations of Hutchinson and Neale [7]. This suggests that localized necking, when induced by material imperfection, is quickly controlled by the behavior of an imperfection near the orientation of zero-extension, i.e., the bifurcation orientation.

A consequence of restricting the imperfection along a direction of zero-extension is that the strain along the imperfection, ε_y, must always be maintained at zero (plane strain), regardless of the strain ratio (ρ). The state of stress of the imperfection can thus be viewed as that of plane strain superimposed with a shear stress τ whose magnitude depends on ρ (see Fig. 1). Specifically, τ increases as ρ becomes more negative, i.e., as the imperfection is inclined at smaller angles to the σ_1 axis. The principal strain normal to the imperfection is, however, limited to a constant value ($\varepsilon_x^* = 0.3$ in Fig. 4), independent of ρ. On the other hand, the shear strain term ε_{xy}^* becomes more important as the strain ratio becomes more negative. As evidenced in Fig. 4, it is the dominant shear strain term ε_{xy}^* which is responsible for the increase of the effective limit strain as ρ is decreased from zero to more negative values.

In the negative minor strain region ($\rho < 0$), the M-K model predicts an $\bar{\varepsilon}_A$ larger than that of the inclined imperfection (Fig. 3). According to Marciniak [17], this is due to the fact that a change of stress ratio occurs in the M-K analysis as the strain path within the imperfection tends toward plane strain. The result is an increase in the apparent strength of the imperfection when Hill's anisotropic yield function is used. Thus, the change of stress ratio within the imperfection can stabilize the process of straining, delay flow localization and cause an increase of the effective limit strain $\bar{\varepsilon}_A$. When the orientation of the imperfection is at Hill's angle of a direction of zero-extension, an initial strain path of plane strain is specified within the imperfection and no change in the stress ratio or increase in apparent strength occurs. As a result, the inclined imperfection predicts an effective limit strain considerably lower than that of the M-K analysis. The difference is largest at uniaxial tension and diminishes to zero at plane strain when $\rho = 0$.

Influence of Materials Properties on the Forming Limit

The influence of work hardening in rate and strain rate sensitivity m on forming limit strain is well known. Materials with high n and/or m values tend to have superior sheet formability because strain is distributed more uniformly [5]. Figure 5a clearly depicts the beneficial effect of n on the limit strain while Fig. 5b indicates that even a small m tends to stabilize the localized neck and enhance the limit strain by allowing the material to deform in a quasi-stable manner. The stabilizing effect of m on localized necking has also been discussed by Ghosh [5]. An interesting observation of Figs. 5a and 5b is that the inclined imperfection model gives a limit strain close to that predicted by Hill's theory when the strain rate effect is not taken into account (m=0). Large discrepancies, however, occur between the two analyses when the strain rate effect is also considered. The inclined imperfection predicts a limit strain considerably higher than that of Hill's theory when $m > 0$. Close agreement between the inclined imperfection model and Hill's theory also exist in predicting that an increase in R will increase limit strains, as shown in Fig. 5c. It should be noted that Fig. 5c assumes m=0 for $m > 0$, the inclined imperfection model predicts larger limit strain than does Hill's theory.

The result shown in Fig. 6 is indicative of of the detrimental effect of the imperfection size on the forming limit strain. Compared to the M-K model, the inclined imperfection is found to be much less sensitive to the imperfection size. This is probably due to the fact that in the M-K analysis a large imperfection size is more effective in changing the strain path toward plane strain. Since an inclined groove is always maintained at plane strain, it is therefore less sensitive to the imperfection size, as illustrated in Fig. 8.

Uniaxial Tensile Behavior and Forming Limit Diagrams

The limit strain of various sheet metals under uniaxial tension loading can be reasonably predicted by the inclined imperfection method when proper material parameters of n, m, and R are accounted for an a realistic characteristic imperfection level (f=0.998) is used (see Table I). The good agreement between theoretical calculations and experimental results suggests that localized necking can indeed originate from material imperfection. Strictly speaking, according to Hill's theory, an imperfection is not required for localized necking to occur on sheet metal under uniaxial tension. However, Hill's theory is not able to predict accurately the limit strain in cases where the flow stabilizing effect of strain rate hardening is important (see Fig. 5b). The M-K model with a perpendicular groove, on the other hand, predicts an unrealistically large limit strain at uniaxial tension as shown in Table I. By restricting the groove to align along a direction of zero-extension, the inclined imperfection can predict a reasonable uniaxial tensile limit strain which compares favorably with the experimental one. The analysis is also consistent with the fact that most sheet metals under uniaxial tensile loading show a localized neck inclined to the stress axis.

In addition to the fairly good prediction of localized necking in uniaxial tension, the inclined imperfection model predicts with reasonable accuracy the negative minor strain regime of the forming limit diagram for A-K steel, HSLA steel, 2031-Al, 70-30 brass and CP Ti. These materials exhibit a fairly wide range of material parameters. The good agreement between the present analysis and the experimental result suggests that flow localization within material imperfection can indeed result in localized necking. Hill's theory also provides a fair prediction of the forming limit strains, but it tends to give an underestimation when $m > 0$ but an overestimation when $m = 0$; this is principally due to the beneficial effect of strain rate hardening in delaying flow localization. The M-K analysis, on the other hand, takes a strain path different than that of the inclined imperfection model by using a perpendicular groove and predicts an unrealistically large forming limit strain.

With regard to the entire forming diagram including the biaxial side, the use of a single criterion, a small imperfection, is appealing. However, while analyses based on an inclined imperfection ($f \simeq .998$) yield good predictions of localized necking for $\rho < 0$, the same small imperfection factor predicts limit strains which are much greater than those experimentally observed in biaxial tension when $\rho = 1$ [6]. Possible reasons for the discrepancies between experiment and the M-K predictions at $\rho = 1$ have been discussed elsewhere [6], and include the intervention of fracture and strain-associated changes in biaxial work hardening behavior. Analysis of behavior under balanced biaxial tension thus remains a problem.

Conclusion

The Marciniak-Kuczynski (M-K) analysis of localized necking, which is based on an imperfection normal to the major principal strain ε_1 axis, has been modified for $\rho < 0$ to take into account a groove inclined to the ε_1 axis. The orientation of the inclined imperfection is assumed to lie along a direction of zero-extension. Such an imperfection orientation coincides with the minimum value of the limit strain obtained by Hutchinson and Neale by minimizing the limit strain with respect to the orientation of the imperfection. Calculations of forming limits predicted by the inclined groove model indicate that, for a given strain state, the limit strain increases with increasing work hardening rate n, the strain rate hardening exponent m, the plastic anisotropy parameter R, but decreased with increasing imperfection size (i.e., as f becomes smaller). The inclined imperfections model has been found to be much less sensitive to the imperfection factor, f, than that of the M-K analysis. When applied to the calculation of the uniaxial and forming limit strains of various steels, aluminum alloys, 70-30 brass and CP Ti in the negative minor strain regime, the inclined imperfection model gives better agreement with the experimental results than Hill's theory or the M-K analysis. Especially for the case of uniaxial tension, Hill's theory tends to overestimate the limit strain when $m = 0$ but underestimates it when $m > 0$. Because it involves a change in stress ratio within the imperfection which strengthen it during straining even for $\rho < 0$, the M-K analysis, on the other hand, predicts a limit strain which is either too large or requires an imperfection factor that is unrealistically large.

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