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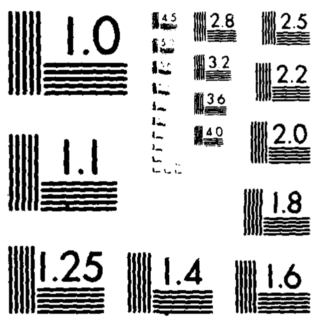


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# FOREIGN TECHNOLOGY DIVISION

CALCULATING INPUT RESISTANCE OF AN INSULATED VIBRATOR IN A SEMICONDUCTING MEDIUM

by

Yu. N. Sokolov

# EDITED TRANSLATION

FTD-ID(RS)T-1133-81 15 Oct[redacted]81

MICROFICHE NR: FTD-81-C-000942

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By: Yu. N. Sokolov

~~English pages:~~

~~Source:~~ Antenny ~~Nov 7, 1970, pp. 82-88~~

Country of origin: USSR
Translated by: Victor Mesenzeff.
Requester: USAMICOM
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## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\text{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\text{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

CALCULATING INPUT RESISTANCE OF AN INSULATED VIBRATOR IN A SEMICONDUCTING MEDIUM

Yu. N. Sokolov

This paper presents a generalization of the Bechmann formula for the case of a designated arbitrary current distribution. The obtained expressions make it possible to calculate the radiation resistance of the vibrator, taking into account the effect of a dielectric coating.

## Preliminary remarks and initial relations

When placing an electric vibrator in a semiconducting medium, most often, it becomes advantageous to use a dielectric coating, which isolates the vibrator from the ambience. When the conductivity of the ambience is sufficient, the presence of such a coating leads to a significant increase in the field excited by the vibrator.

We will not devote too much attention to this question in this work but deal only with the field of the vibrator in its immediate vicinity to determine its input resistance, taking into account the effect of the isolating dielectric. The presents of the latter leads to the fact that the wave number of electromagnetic vibrations along the vibrator ($\gamma$) is different from the wave number of the ambience ($k$) and, in conducting media, this difference can be very significant.

The methods used to determine the value of $\gamma$ for a wire with a dielectric insulation are known and are described, in particular, in work [3] for any number of concentric sheaths surrounding the wire. Thus, hereafter, we will assume that this value is known and that the current distribution along the vibrator follows the law

$$I(\zeta)=I_0\sin\gamma(l-\zeta). \tag{1}$$

The value of z, the electric-field component of the vibrator, we determine with the aid of the expression

$$E_z=\kappa^2\Pi_z+\frac{\partial^2\Pi_z}{\partial z^2}, \tag{2}$$

where

$$\Pi_z=\frac{1}{i4\pi\omega\varepsilon'}\int_0^l I(\zeta)\left[\frac{e^{-i\kappa\sqrt{r^2+(z-\zeta)^2}}}{\sqrt{r^2+(z-\zeta)^2}}+\frac{e^{-i\kappa\sqrt{r^2+(z+\zeta)^2}}}{\sqrt{r^2+(z+\zeta)^2}}\right]d\zeta, \tag{3}$$

k is the wave nummber of the ambience.

The quantity r will represent the external radius of the dielectric insulation.

Making use of the widely known method of induced electromotive forces, we determine the radiation resistance of the vibrator by the relation

$$Z_\Sigma=-\frac{2}{|I_0|^2}\int_0^l E_z I^*(z)\,dz, \tag{4}$$

where

$$I^*(z)=I_0\sin\gamma^*(l-z), \tag{5}$$

$\gamma^*$ is the value of conjugation with $\gamma$.

The solution of this problem, i.e., determination of expression (4), can be obtained using the Bechmann method [2], which, however, requires special consideration in this case.

## Generalization of the Bechmann formula for the case of an arbitrary distribution of current along the vibrator

The quantity $Z_\Sigma$ is determined easily with the aid of (2) and (4) and leads, as is known, to the Bechmann's formula if the distribution of current along the vibrator obeys the relation $\frac{\partial^2 I(\zeta)}{\partial\zeta^2}=-\kappa^2 I(\zeta)$. We will show that the expressions analogous to the Bechmann formula have a place also in the case when

$$\frac{\partial^2 I(\zeta)}{\partial\zeta^2}=-\gamma^2 I(\zeta). \tag{6}$$

In this section, no additional conditions are imposed on the quantity $\gamma$, generally speaking.

Having substituted in (4) the value of $E_z$ from expression (2), we obtained the following expression after a double integration by parts:

$$Z_2 = -\frac{2}{|I_0|^2}\left\{\left[I^*(z)\frac{\partial \Pi_z}{\partial z}\right]_{z=0}^{l} - \left[\frac{\partial I^*(z)}{\partial z}\Pi_z\right]_{z=0}^{l} + (\kappa^2-\gamma^{*2})\int_0^l I^*(z)\Pi_z dz\right\}. \quad (7)$$

Then, having substituted expressions (2) and (3) in (4) and using the same procedure of double integration by parts, in this case performed with respect to $\zeta$, we obtain

$$Z_2 = -\frac{2}{i4\pi\omega\varepsilon|I_0|^2}\left\{\int_0^l I^*(z)\left[I(\zeta)\frac{\partial f(r)}{\partial \zeta}\right]_{\zeta=0}^{l} dz - \right.$$
$$\left. - \int_0^{l} I^*(z)\left[\frac{\partial I(\zeta)}{\partial \zeta}f(r)\right]_{\zeta=0}^{l} dz + (\kappa^2-\gamma^2)\int_0^l I^*(z)\Pi_z dz\right\}, \quad (8)$$

in which the following is introduced:

$$f(r) = \left[\frac{e^{-i\kappa\sqrt{r^2+(z-\zeta)^2}}}{\sqrt{r^2+(z-\zeta)^2}} + \frac{e^{-i\kappa\sqrt{r^2+(z+\zeta)^2}}}{\sqrt{r^2+(z+\zeta)^2}}\right].$$

In deriving expression (8), we made use of the fact that
After introducing the designation $\frac{\partial^2 f(r)}{\partial z^2} = \frac{\partial^2 f(r)}{\partial \xi^2}$.

$$\Pi_\zeta = \frac{1}{i4\pi\omega\varepsilon}\int_0^l I^*(z) f(r)\, dz \quad (9)$$

and excluding the last integral from expressions (7) and (8), we find

$$Z_2 = -\frac{2}{I_0^2}\left\{\frac{\gamma^2-\kappa^2}{\gamma^{*2}-\gamma^2}\left[\Pi_z\frac{\partial I^*(z)}{\partial z}\right]_{z=0}^{l} - \frac{\gamma^{*2}-\kappa^2}{\gamma^{*2}-\gamma^2}\left[\Pi_\zeta\frac{\partial I(\zeta)}{\partial \zeta}\right]_{\zeta=0}^{l}\right\}. \quad (10)$$

Expression (10) is obtained with the consideration of the conditions:
$\frac{\partial \Pi_z}{\partial z} = \frac{\partial \Pi_\zeta}{\partial \zeta} = 0$ with $z=\zeta=0$, $I(\zeta)=I(z)=0$ with $z=\zeta=l$.

For the generalization of these results, we note that in determining the value of mutual energy of two vibrators located at the axes $\zeta$ and $z$ with different constants of distribution of the electromagnetic waves along them, which are designated by $\gamma_1$ and $\gamma_2$, respectively, we have the relation which is analogous to expression (10):

$$W = \frac{\kappa^2 - \gamma_1^2}{\gamma_2^{*2} - \gamma_1^2}\left\{\left[I^*(z)\frac{\partial \Pi_z}{\partial z}\right]_{z_1}^{z_2} - \left[\frac{\partial I^*(z)}{\partial z}\Pi_z\right]_{z_1}^{z_2}\right\} - \frac{\kappa^2 - \gamma_2^{*2}}{\gamma_2^{*2} - \gamma^2} \times$$

$$\times\left\{\left[I(\zeta)\frac{\partial \Pi_\zeta}{\partial \zeta}\right] - \left[\frac{\partial I(\zeta)}{\partial \zeta}\Pi_\zeta\right]_{\zeta_1}^{\zeta_2}\right\}, \qquad (11)$$

where

$$\Pi_z = \int_{\zeta_1}^{\zeta_2} I(\zeta)\frac{e^{-i\kappa R}}{R}d\zeta; \quad \Pi_\zeta = \int_{z_1}^{z_2} I^*(z)\frac{e^{-i\kappa R}}{R}dz;$$

R is the distance from the ζ axis to the surface of the vibrator at the z axis.

Expression (11) can be used, for example, for calulating asymmetrical vibrators whose arms are under different conditions.

These expressions can be easily generalized for the case of any designated current distribution which permits expansion in harmonic components.

Derivation of calculation relations for radiation resistance

Having expanded the first term in expression (10), we obtain

$$\left[\Pi_z\frac{\partial I^*}{\partial z}\right]_{z=0} = \frac{i I_0^2\gamma^*}{4\pi\omega\varepsilon}\left\{\int_0^l \sin\gamma(l-\zeta)\frac{e^{-i\kappa\sqrt{r^2+(l-\zeta)^2}}}{\sqrt{r^2+(l-\zeta)^2}}d\zeta - \right.$$

$$\left. + \int_0^l \sin\gamma(l-\zeta)\frac{e^{-i\kappa\sqrt{r^2+(l+\zeta)^2}}}{\sqrt{r^2+(l+\zeta)^2}}d\zeta - 2\cos\gamma^* l\int_0^l \sin\gamma(l-\zeta)\frac{e^{-i\kappa\sqrt{r^2+\zeta^2}}}{\sqrt{r^2+\zeta^2}}d\zeta\right\}. \qquad (12)$$

Let us consider the integrals entering this expression. We will conduct the analysis for the case of relatively long vibrators, when the condition $l \gg r$ is satisfied.

After transforming the first integral to the form

$$I_1 = \frac{1}{2i}\left\{\int_0^l \frac{e^{i\gamma(l-\zeta)-i\kappa\sqrt{r^2+(l-\zeta)^2}}}{\sqrt{r^2+(l-\zeta)^2}}d\zeta - \int \frac{e^{-i\gamma(l-\zeta)-i\kappa\sqrt{r^2+(l-\zeta)^2}}}{\sqrt{r^2+(l-\zeta)^2}}d\zeta\right\}. \qquad (13)$$

and then after multiplying and dividing each term by $e^{i\kappa(l-\zeta)}$, and also after introducing the substitution $x=(l-\zeta)+\sqrt{r^2+(l-\zeta)^2}$, we obtain

$$I_1 = \frac{1}{2i}\left\{\int_r^{2l}\frac{e^{-i\frac{\kappa-\gamma}{2}x}}{x}e^{-i\frac{\kappa+\gamma}{2}\frac{r^2}{x}}dx - \int_r^{2l}\frac{e^{-i\frac{\kappa+\gamma}{2}x}}{x}e^{-i\frac{\kappa-\gamma}{2}\frac{r^2}{x}}dx\right\}. \qquad (14)$$

In the last expression the change in the factors $e^{-i\frac{\kappa+\gamma}{2}\frac{r^2}{\zeta}}$ for the case $l \gg r$ can be disregarded, then, with an accuracy to the terms of the order of kr, we obtain

$$I_1 = \frac{1}{2i}\left\{\mathrm{Ei}\,[-il(\kappa-\gamma)] - \mathrm{Ei}\,[-il(\kappa+\gamma)] + \ln\frac{\kappa-\gamma}{\kappa-\gamma}\right\}. \qquad (15)$$

where Ei(z) is the integral exponential function.

On the basis of (14), when necessary, it easy to obtain an expression for $I_1$ with the consideration of the following expansion terms of the factors $e^{-i\frac{\kappa+\gamma}{2}\frac{r^2}{\zeta}}$.

After presenting the third integral in expression (12) in the form

$$I_3 = \frac{1}{2i}\left\{e^{i\gamma l}\int_0^l \frac{e^{-i\gamma\zeta - i\kappa\sqrt{r^2+\zeta^2}}}{\sqrt{r^2+\zeta^2}}\,d\zeta - e^{-i\gamma l}\int_0^l \frac{e^{-i\gamma\zeta - i\kappa\sqrt{r^2+\zeta^2}}}{\sqrt{r^2+\zeta^2}}\,d\zeta\right\} \qquad (16)$$

and after multiplying and dividing each term by $e^{i\kappa\zeta}$, after the substitution $x=\zeta+\sqrt{r^2+\zeta^2}$ under the integrals, we obtain expressions which are totally analogous to the integrands in (14). Calculation of the integral yields

$$I_3 = \frac{1}{2i}\left\{e^{i\gamma l}\left[\mathrm{Ei}\,[-il(\kappa+\gamma)] + \ln\frac{2}{i(\kappa+\gamma)r} - E\right] - e^{-i\gamma l}\left[\mathrm{Ei}\,[-il(\kappa-\gamma)] + \ln\frac{2}{i(\kappa-\gamma)r} - E\right]\right\}. \qquad (17)$$

In the second integral entering expression (12), we can disregard the value of r immediately, which leads to the expression

$$I_2 = \frac{1}{2i}\{e^{i2\gamma l}[[\mathrm{Ei}\,[-i2l(\kappa+\gamma)] - \mathrm{Ei}\,[-il(\kappa+\gamma)]] - e^{-i2\gamma l}[\mathrm{Ei}\,[-i2l(\kappa-\gamma) - \mathrm{Ei}\,[-il(\kappa-\gamma)]]\}. \qquad (18)$$

The second terms entering expression (10) is calculated exactly the same way, and the final formulas differ only in the substitution of $\gamma^*$ for $\gamma$.

Thus, the final expression for the radiation resistance of this system we obtain in the form

$$Z_2 = -\frac{30}{\sqrt{\epsilon'}}\frac{1}{\kappa}[nA + n'A'], \qquad (19)$$

where

$$n = \frac{\gamma^2-\kappa^2}{\gamma^{*2}-\gamma^2}\gamma_1^*,\quad n' = \frac{\kappa^2-\gamma^{*2}}{\gamma^{*2}-\gamma^2}\gamma.$$

According to (12), (15), (17), and (19), after all the transformations, the expression for A has the form:

$$
\begin{aligned}
A = \mathrm{Ei}(\lambda_1 l) - \mathrm{Ei}(\lambda_2 l) + \ln\frac{\lambda_2}{\lambda_1} + e^{i2\beta l}\Big\{ e^{2\alpha l}[\mathrm{Ei}(2\lambda_2 l) - \\
- \mathrm{Ei}(\lambda_2 l)] - \mathrm{Ei}(\lambda_2 l) - \ln\frac{2}{-\lambda_2 r} + E\Big\} - e^{-2\alpha l}[\mathrm{Ei}(2\lambda_1 l) - \mathrm{Ei}(\lambda_1 l)] - \\
- \mathrm{Ei}(\lambda_1 l) - \ln\frac{2}{-\lambda_1 l} + E\Big\} - e^{2\alpha l}\left[\mathrm{Ei}(\lambda_2 l + \ln\frac{2}{-\lambda_2 r} - E\right] + \\
+ e^{-2\alpha l}\left[\mathrm{Ei}(\lambda_1 l) + \ln\frac{2}{-\lambda_1 r} - E\right],
\end{aligned} \tag{20}
$$

where E is the Euler constant,

$$\lambda_1 = -i(\kappa - \gamma), \quad \lambda_2 = -i(\kappa + \gamma), \tag{21}$$

$\alpha$ and $\beta$ are determined by the relation $\gamma = \beta - i\alpha$.

The expression for A' is obtained from expression (20) by substituting $-\alpha$ for $\alpha$, i.e., $\gamma^*$ for $\gamma$.

The arguments of the functions entering the expression for A' have the form:

$$\lambda_1' = -i(\kappa - \gamma^*), \quad \lambda_2' = -i(\kappa + \gamma^*).$$

For further analysis, it is convenient to transform formula (19) to the form

$$Z_2 = \frac{30}{\sqrt{\varepsilon'}}\frac{1}{4\kappa}\left[\frac{c_2}{\beta}(A + A') + i\frac{c_1}{\alpha}(A - A')\right], \tag{22}$$

where

$$c_1 = \alpha^2 + \beta^2 - \kappa^2, \qquad c_2 = \alpha^2 + \beta^2 + \kappa^2.$$

Let us consider some of the particular cases of this solution. The simplest relations are obtained when $\beta l \ll 1$. Then we find the following with an accuracy to the terms of the order of $(\kappa l)^4$ and $(\gamma l)^4$ :

$$A = -4(i\beta l + \alpha l)\left(\ln\frac{l}{r} - 1\right), \quad A' = -4(i\beta l - \alpha l)\left(\ln\frac{l}{r} - 1\right) \tag{23}$$

and according to (22)

$$Z_2 = -i\frac{120}{\sqrt{\varepsilon'}}\frac{\alpha^2 + \beta^2}{\kappa}\left(\ln\frac{l}{r} - 1\right). \tag{24}$$

Expression (22) permits a limited transition $\alpha \to 0$ when $\gamma \neq k$, which becomes possible if we make use of the relations:

$$
\left.
\begin{aligned}
\mathrm{Ei}(\lambda_1 l) \simeq \mathrm{Ei}(\lambda_1' l) + i\frac{2\alpha}{\lambda_1'}e^{\lambda_1' l} \\
\mathrm{Ei}(\lambda_2' l) \simeq \mathrm{Ei}(\lambda_2 l) + i\frac{2\alpha}{\lambda_2}e^{\lambda_2 l}
\end{aligned}
\right| \tag{25}
$$

and, in the expressions for A and $A'$, proceed to the functions only of the arguments $\lambda_1'$ and $\lambda_2$ (the remaining functions of $\lambda_1$ and $\lambda_2'$ can be obtained from (25) by replacing $l$ by $2l$ or r).

Finally, in the case where $\gamma \to k$, from (19) we have

$$Z_z = \frac{30}{\sqrt{\varepsilon}} A'.$$

The arguments of functions Ei(z) assume the form: $\lambda_1' = -2\alpha,\ \lambda_2' = i2\beta$, after which, it is possible to proceed to the functions Si(x) and Ci(x) of the actual arguments and, ultimately, evolve the actual and imaginary parts. We will not write out the expressions obtained in this case, since they are congruent, with respect to accuracy, with the expressions obtained by Yu. K. Murav'yev, who used the method of induced electromotive forces for the case $\gamma=k$.

In conclusion, it is necessary to note that this analysis does not fully exhausts the solution of the problem of input resistance of the isolated vibrator, since introduction into the calculation of the quantity $\gamma$, which is different from the wave number of the ambience, makes it possible to determine with certainty only $q$ - the magnetic-field component. As regards the quantity z - the electric-field component, which enters the expression of energy connected with the vibrator as a second cofactor, then for a precise solution of the problem this value must be determined not on the surface of the insulation but on the surface of the wire, taking into account the reflections at the insulation-medium boundary. In the absence of losses in the wire insulation, the magnitude of the active component, however, does not vary as compared with the actual part of expression (22). With regard to the active component yielded by this expression, in the case of thick insulations and strong reflections at the insulation-medium interface, this solution needs further refinement, which can be obtained on the basis of this solution by introducing a reflected wave when considering the electrical field.

In the course of this work the author was fortunate to receive valuable suggestions and constant consultation from the Dr. of Tech. Sciences G. A. Lavrov to whom he is deeply indebted.

The author also expresses his appreciation to the Dr. of Tech. Sciences Yu. K. Murav'yev, who made a number of valuable remarks while re-

viewing this work.

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