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A CLOSED FORM LOCAL SOLUTION OF A
NONLINEAR STRUCTURAL DESIGN PROBLEM IN
TERMS OF THE DESIGN PARAMETERS

by

Anthony V. Fiacco
Abolfazl Ghaemi

Serial T-449
10 August 1981

The George Washington University
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20. ABSTRACT (cont'd)

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THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
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Program in Logistics

Abstract
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10 August 1981

A CLOSED FORM LOCAL SOLUTION OF A
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Using the results of our previous analysis of the corrugated bulkhead model, a closed form of the optimal solution of this model is easily derived as a function of the many design parameters. The analytic solution is valid over large changes in the parameters. This demonstrates the possibility of making very accurate (in this instance, exact) local estimates of a markedly nonlinear parametric solution, once one solution is found, even though the original problem statement might appear to make this prohibitive. It is shown that the local parametric solution obtained can be extended to include large changes in the parameters, particularly when the parametric perturbations are restricted and highly structured, as is typical in practical applications. Aside from the practical value of such a result, it is reported because this problem is well publicized and has been utilized for some time as a test problem. Knowledge of the precise solution for a range of parameter values provides ideal information for further studies.

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1. THE PROBLEM

The purpose of this study is to further characterize the optimal solution behavior of a nonlinear programming model for the optimal sizing of a vertically corrugated transverse bulkhead of an oil tanker, presented in [1,Ch. 6] and subsequently analyzed by the authors [2]. The objective of the model is to determine the dimensions of the bulkhead such that certain engineering constraints are met, with minimum possible bulkhead weight.

Figures 1 and 2 depict the configuration of the bulkhead and design variables, respectively. Figure 3, together with Table 1, defines the model parameters and their values.

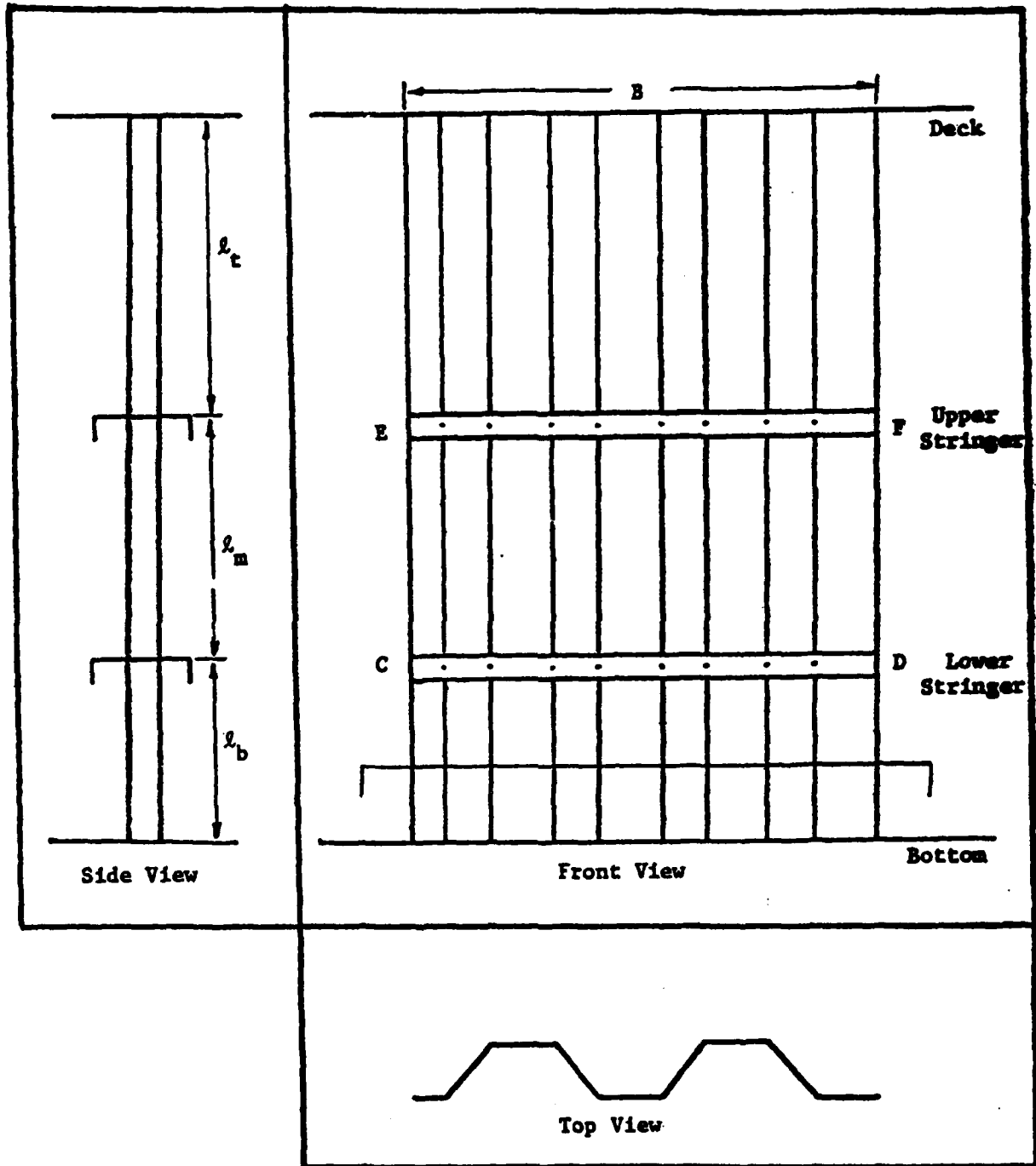


Figure 1. Vertical corrugated transverse bulkhead.

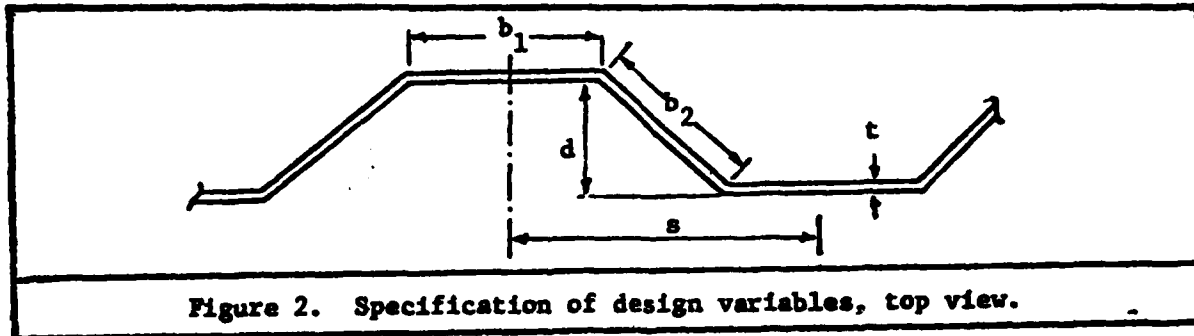


Figure 2. Specification of design variables, top view.

$x_1 = b_1 =$ width of flange

$x_2 = b_2 =$ length of web

$x_3 = d =$ depth of corrugation

$x_4 = t_t =$ thickness of top panel

$x_5 = t_m =$ thickness of middle panel

$x_6 = t_b =$ thickness of bottom panel.

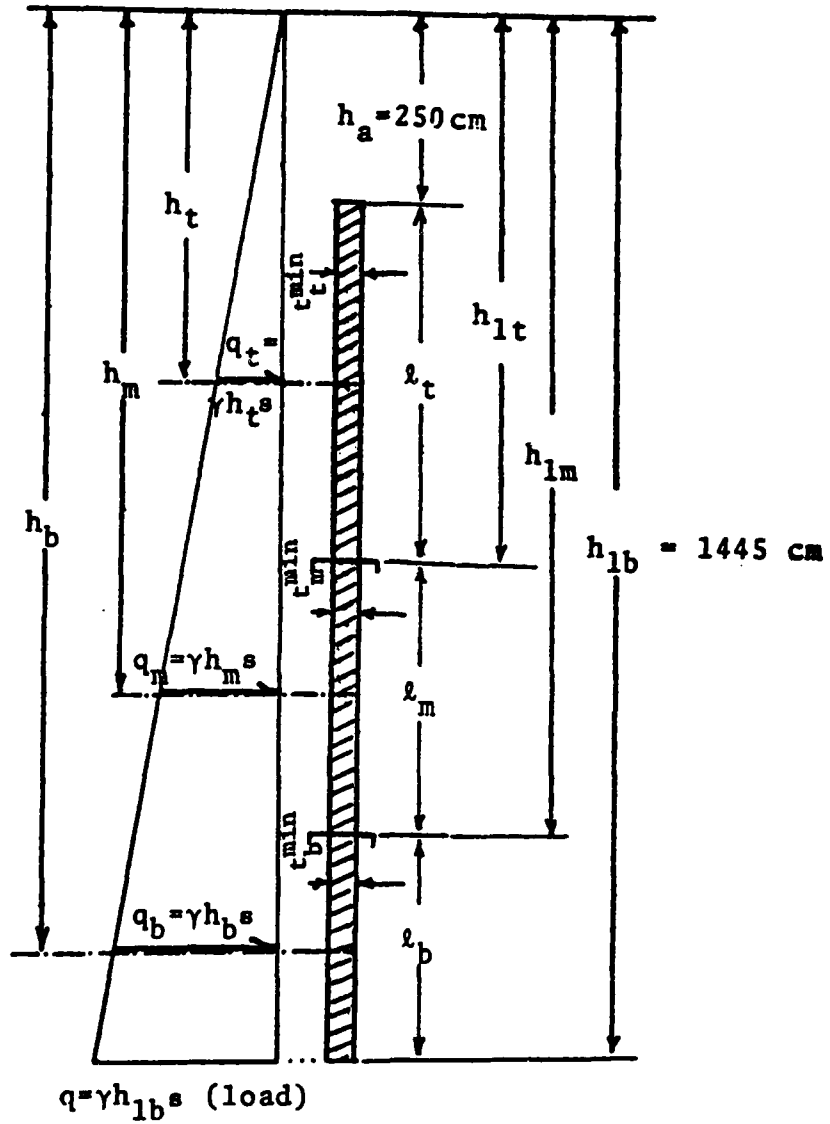


Figure 3. Specification of some of the design parameters and indication of load levels, side view.

TABLE 1
 PROBLEM DESIGN PARAMETERS
 (MODEL INPUT DATA)

| No. | Parameter | Meaning | Value |
|-----|--------------------|---|------------------------|
| 1 | Γ | Weight per unit volume of the material | 7.85 gm/cm^3 |
| 2 | B | Width of the panel | 476 cm |
| 3 | l_t | Length of the top panel | 495 cm |
| 4 | l_m | Length of the middle panel | 385 cm |
| 5 | l_b | Length of the bottom panel | 315 cm |
| 6 | h_a | Distance between free liquid level and top of structure | 250 cm |
| 7 | h_t | Distance between free liquid level and middle of top panel | 498 cm |
| 8 | h_m | Distance between free liquid level and middle of middle panel | 938 cm |
| 9 | h_b | Distance between free liquid level and middle of bottom panel | 1288 cm |
| 10 | h_{lt} | Distance between free liquid level and base of top panel | 745 cm |
| 11 | h_{lm} | Distance between free liquid level and base of middle panel | 1130 cm |
| 12 | h_{lb} | Distance between free liquid level and base of bottom panel | 1445 cm |
| 13 | t_t^{min} | Minimum allowable thickness of top panel | 1.05 cm |
| 14 | t_m^{min} | Minimum allowable thickness of middle panel | 1.05 cm |

Table 1 -- Continued

| No. | Parameter | Meaning | Value |
|-----|--------------|--|---------------------------------------|
| 15 | t_b^{\min} | Minimum allowable thickness of bottom panel | 1.05 cm |
| 16 | e | Effectiveness of the flange (dimensionless) | 0.8 |
| 17 | k_1 | A derived coefficient (function of maximum allowable bending stress) | $6.94 \times 10^{-8} \text{ cm}^{-1}$ |
| 18 | k_2 | Corrosion coefficient | 0.15 cm |

The resulting nonlinear programming problem obtained in [1] takes the following form.

$$\min_x f \equiv [B(x_1 + x_2)(l_t x_4 + l_m x_5 + l_b x_6)[x_1 + (x_2^2 - x_3^2)^{1/2}]^{-1} \text{ (weight)}$$

subject to

(i) Geometrical constraint:

$$g_1 \equiv x_2 - x_3 \geq 0 ;$$

(ii) Bending stress constraints:

$$g_2 \equiv x_2 x_3 x_4 + 3e x_1 x_3 x_4 - 6k_1 h_t \ell_t^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

$$g_3 \equiv x_2 x_3 x_5 + 3e x_1 x_3 x_5 - 6k_1 h_m \ell_m^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

$$g_4 \equiv x_2 x_3 x_6 + 3e x_1 x_3 x_6 - 6k_1 h_b \ell_b^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

(iii) Moment of inertia constraints:

$$g_5 \equiv x_2 x_3^2 x_4 + 3e x_1 x_3^2 x_4 - 26.4(k_1 h_t \ell_t^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

$$g_6 \equiv x_2 x_3^2 x_5 + 3e x_1 x_3^2 x_5 - 26.4(k_1 h_m \ell_m^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

$$g_7 \equiv x_2 x_3^2 x_6 + 3e x_1 x_3^2 x_6 - 26.4(k_1 h_b \ell_b^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

(iv) Thickness requirement constraints:

$$g_8 \equiv x_4 - t_t^{\min} \geq 0 ;$$

$$g_9 \equiv x_4 - [.39 \cdot 1.05(.01h_{1t})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{10} \equiv x_4 - [.39 \cdot 1.05(.01h_{1t})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

$$g_{11} \equiv x_5 - t_m^{\min} \geq 0 ;$$

$$g_{12} \equiv x_5 - [.39 \cdot 1.05(.01h_{1m})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{13} \equiv x_5 - [.39 \cdot 1.05(.01h_{1m})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

$$g_{14} \equiv x_6 - t_b^{\min} \geq 0 ;$$

$$g_{15} \equiv x_6 - [.39 \cdot 1.05(.01h_{1b})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{16} \equiv x_6 - [.39 \cdot 1.05(.01h_{1b})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

(v) Natural constraints:

$$g_{17}, g_{18}: x_i \geq 0 \quad i = 1, 3$$

Table 2 gives the optimal solution and Table 3 shows the constraint values and Lagrange multipliers corresponding to this solution. These results were obtained in our previous study [1].

TABLE 2

OPTIMAL SOLUTIONS OF THE MODEL
WITH PARAMETER VALUES GIVEN IN TABLE 1

$$x_1 = b_1 = 57.82 \text{ cm}$$

$$x_2 = b_2 = 57.82 \text{ "}$$

$$x_3 = d = 35.69 \text{ "}$$

$$x_4 = t_t = 1.05 \text{ "}$$

$$x_5 = t_m = 1.05 \text{ "}$$

$$x_6 = t_b = 1.05 \text{ "}$$

$$f = w = 5.25 \text{ tons}$$

TABLE 3
LAGRANGE MULTIPLIERS AND CONSTRAINT VALUES AT THE OPTIMAL SOLUTION
(excluding nonnegativity constraints)

| Constraint s_i | Value of Constraint s_i | Value of Lagrange Multiplier u_i |
|------------------|---------------------------|------------------------------------|
| s_1 | 22.13 | $.4518784 \times 10^{-2}$ |
| s_2 | 2117.22 | $.4723479 \times 10^{-4}$ |
| s_3 | 1385.41 | $.7218857 \times 10^{-4}$ |
| s_4 | 1868.92 | $.5351946 \times 10^{-4}$ |
| s_5 | 41993.15 | $.2381633 \times 10^{-5}$ |
| † s_6 | .043 | $.2300013 \times 10^1$ |
| s_7 | 27945.57 | $.3580375 \times 10^{-5}$ |
| † s_8 | $.48 \times 10^{-5}$ | $.2078305 \times 10^7$ |
| s_9 | .25 | .3940568 |
| s_{10} | .25 | .3940570 |
| † s_{11} | $.96 \times 10^{-5}$ | $.1040634 \times 10^7$ |
| s_{12} | .10 | .9604345 |
| s_{13} | .10 | .9604360 |
| † s_{14} | $.49 \times 10^{-4}$ | $.2029215 \times 10^6$ |
| † s_{15} | $.31 \times 10^{-4}$ | $.3221582 \times 10^6$ |
| † s_{16} | $.13 \times 10^{-4}$ | $.7974950 \times 10^6$ |

†Binding constraints.

As indicated in Table 3, constraints g_6 , g_8 , g_{11} and $g_{14} - g_{16}$ are binding. It was shown in [2] that the corresponding gradients of these functions are linearly independent. The optimal solution of the model, with the given data base (Table 1), thus corresponds to the locally unique solution of the following nondegenerate system of nonlinear equations:

$$\begin{aligned}
 g_6 &= x_2 x_3^2 x_5 + 3e x_1 x_3^2 x_5 - 26.4(k_1 h_m \ell^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} = 0 \\
 g_8 &= x_4 - t_t^{\min} = 0 \\
 g_{11} &= x_5 - t_m^{\min} = 0 \\
 g_{14} &= x_6 - t_b^{\min} = 0 \\
 g_{15} &= x_6 - [(.39)(1.05)(.01h_{1b})^{1/2}(.01x_1) + k_2] = 0 \\
 g_{16} &= x_6 - [(.39)(1.05)(.01h_{1b})^{1/2}(.01x_2) + k_2] = 0
 \end{aligned} \tag{1}$$

In [2] we determined that, with parameter perturbations as large as two percent of the parameter base values, the set of bending constraints remains unchanged. This means that, for relatively large changes of data in a neighborhood of the data base given in Table 1, the optimal parametric solution of the nonlinear program is unique and must satisfy the system of equation (1).

In the next section, we obtain a closed form solution for this system of nonlinear equations.

2. DERIVATION OF CLOSED FORM SOLUTION

For notational simplicity, we denote the coefficients of Equations (1) as follows:

$$c_1 = 3e$$

$$c_2 = 26.4(k_1 h_m \ell_m^2)^{4/3}$$

$$c_3 = k_2$$

$$c_4 = t_t^{\min}$$

$$c_5 = t_m^{\min}$$

$$c_6 = t_b^{\min}$$

$$c_7 = (.39)(1.05)(.01h_{1b})^{1/2}(.01)$$

Thus, the system of equation (1) becomes

$$g_6 : x_2 x_3^2 x_5 + c_1 x_1 x_3^2 x_5 - c_2 [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} = 0$$

$$g_8 : x_4 - c_4 = 0$$

$$g_{11} : x_5 - c_5 = 0$$

$$g_{14} : x_6 - c_6 = 0$$

$$g_{15} : x_6 - c_7 x_1 - c_3 = 0$$

$$g_{16} : x_6 - c_7 x_2 - c_3 = 0$$

From g_8 , g_{11} and g_{14} we obtain

$$x_4 = c_4, \quad x_5 = c_5 \quad \text{and} \quad x_6 = c_6 \quad (3)$$

From g_{15} and g_{16} and (3),

$$x_1 = x_2 = (c_6 - c_3)/c_7 \equiv c_8 \quad (4)$$

Substituting x_1 and x_2 from (4) and x_5 from (3) in g_6 , we obtain

$$(c_5 c_8 + c_1 c_8 c_5) x_3^2 - c_2 [c_8 + (c_8^2 - x_3^2)^{1/2}]^{4/3} = 0$$

or

$$(c_5 c_8 (1 + c_1)/c_2)^{3/4} x_3^{3/2} = [c_8 + (c_8^2 - x_3^2)^{1/2}]$$

Letting $c_9 = \left(\frac{c_5 c_8 (1 + c_1)}{c_2} \right)^{3/4}$, (5)

then subtracting c_8 from both sides of the equation and squaring, yields

$$c_9^2 x_3^3 + x_3^2 - 2c_8 c_9 x_3^{3/2} = 0$$

or

$$x_3^{3/2} [c_9^2 x_3^{3/2} + x_3^{1/2} - 2c_8 c_9] = 0$$

Since we must have $x_3 > 0$ near the given base value solution and since we may assume $c_1, c_5, c_2 > 0$ and $c_6 > c_3$, then $c_8 > 0$

and $c_9 \neq 0$, so

$$(\sqrt{x_3})^3 + \frac{1}{c_9} \sqrt{x_3} - \frac{2c_8}{c_9} = 0 \quad (6)$$

a reduced cubic equation in $\sqrt{x_3}$.

According to a classical result discovered by Ferro and first published by Cardan [3], a reduced cubic equation of the form

$$y^3 + ay + b = 0 \quad (7)$$

with $(\frac{b}{2})^2 + (\frac{a}{3})^3 > 0$ has only one real root.

Since in (6), $a = \frac{1}{c_9}$ is always positive, $(\frac{b}{2})^2 + (\frac{a}{3})^3$ is

always positive for our problem for all changes of these coefficients that result from any (feasible) model parameter perturbations.

The unique real root of (7) is given by Cardan's formula as follows:

$$y = \left[\frac{-b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3} \right]^{1/3} + \left[\frac{-b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3} \right]^{1/3}$$

thus, the solution for x_3 from (6) is

$$x_3 = \left\{ \left[\frac{c_8}{c_9} + \left(\frac{c_8^2}{c_9^2} + \frac{1}{27c_9^6} \right)^{1/2} \right]^{1/3} + \left[\frac{c_8}{c_9} - \left(\frac{c_8^2}{c_9^2} + \frac{1}{27c_9^6} \right)^{1/2} \right]^{1/3} \right\}^2 \quad (8)$$

We have thus obtained a closed form parametric solution for the optimal problem variables near the given initial parameter values. Substituting $x_1 - x_6$ from (3), (4) and (8) in the objective function f of the model will of course, yield the closed form formula for the optimal value function.

3. NUMERICAL CORROBORATION

The closed formulas we have derived provide an opportunity to check the computer solution. In this section, we use the data base given in Table 1 and the solution formulas we have derived to calculate the optimal solution components and objective function value, and compare these with those obtained by the computer solution and listed in Table 2.

Using the data in Table 1, we obtain the following values for $c_1 - c_7$:

$$c_1 = 2.4 , c_2 = 542.311667 , c_3 = .15 , c_4 = c_5 = c_6 = 1.05$$

and $c_7 = .015566 .$

$$\text{From (4) , } c_8 = \frac{c_6 - c_3}{c_7} = 57.818322 ,$$

and
from (5) , $c_9 = \left(\frac{c_5 c_8 (1+c_1)}{c_2} \right)^{3/4} = .484578 .$

Substitution of c_8 and c_9 in (8) yields.

$$x_3 = (5.973894)^2 = 35.687409 .$$

Thus, the analytically calculated solution vector is, to two decimal points of accuracy,

$$x = (57.82 , 57.82 , 35.69 , 1.05 , 1.05 , 1.05) ,$$

which corresponds precisely to the computer solution given in Table 2. Substantiation of these values of x in the objective function f and rounding to two decimal points yields a value of 5.25 , again in precise correspondence with the computer solution.

As an example of a verification of previously derived sensitivity results, we calculate the sensitivity of the optimal value function to the parameter t_{\min}^t , the right-hand side of the constraint g_8 which we denoted by c_4 .

Denoting x_3 by c_{10} and substituting the closed form solutions for x_i $i=1,6$ in the objective function f , we obtain the formula for the optimal value function,

$$f^*(\cdot) = 2\Gamma B c_8 (\lambda_t c_4 + \lambda_n c_5 + \lambda_b c_6) / \sqrt{(c_8^2 - c_{10}^2)^{1/2} + c_8}. \quad (9)$$

Thus,

$$\frac{\partial f^*(\cdot)}{\partial c_4} = 2\Gamma B c_8 \lambda_t \left[(c_8^2 - c_{10}^2)^{1/2} + c_8 \right]^{-1/2}.$$

Substituting in (9) the numerical values of Γ , B and λ_t given in Table 1 and the calculated values of c_8 and c_{10} given in the above formulas, we obtain

$$\frac{\partial f^*(\cdot)}{\partial c_4} = 2070337.126.$$

The computer solution obtained in [1] was 2078305. This corresponds with the calculated Lagrange multipliers value associated with constraint g_8 , which is given in Table 3. These calculations are in close agreement, in terms of relative error.

4. DEVELOPMENT OF A COMPLETE PARAMETRIC SOLUTION

A careful study of the local solution that has been obtained and the constraint structure indicates that a great deal of progress can be made towards obtaining a global parametric solution, particularly if the parameter perturbations are restricted to those that typically arise in practice. For example, perturbations of considerable interest are based on the assumption that some or all of the parameters change "proportionately." e.g., if a given parameter vector is denoted by p , then a perturbed value of p would be given by $p + \alpha p = (1 + \alpha)p = \beta p$, where α is the proportionality factor and $\beta = 1 + \alpha$, a scalar. With this restriction, all the parameters that change are functions of the single scalar valued parameter β and the perturbation analysis is enormously simplified. In the present example, given this type of perturbation, we could readily track the course of the closed form parametric solution that we have obtained as a function of the single parameter β , as β increases or decreases. As long as the given binding constraints remain binding, the given system of equations hold and the given parametric solution would apply. When a new constraint becomes binding at a new value of β , we would have to introduce a new equation and when a binding constraint becomes nonbinding, the corresponding equation would be removed from the binding constraint equations. Given the relative simplicity of our closed form solution and the exploitable structure of the constraints, as we shall indicate, a complete closed form parametric solution could probably be developed over the domain of parameter values of interest.

At a general level, note that constraints g_1 and $g_8 - g_{16}$ are simple linear constraints. Each of these constraints that are binding will either determine the value of a variable or determine one variable in terms of another, thus, each removing one degree of freedom in the solution. If any of the constraints $g_2 - g_4$ are binding, they can each be solved for any variable in terms of the others involved in the particular constraint. Finally, if either of $g_5 - g_7$ are binding

then, x_1 , x_4 , x_5 , or x_6 , can be expressed in terms of the other variables, and if $x_1 = x_2$, then the remaining variables, x_2 and x_3 , as we have shown, can be solved in terms of the other variables. Thus, the preponderance of the various possible combinations of equations that can arise from the constraints that are binding at a solution corresponding to a given parameter value can be resolved in closed form.

We conclude with an extension of the local closed solution that we have obtained to illustrate our observations. Consider changing the parameter t_t^{\min} from its base value 1.05 cm, leaving the other parameters fixed as given in Table 1. Denote the parameter vector by ϵ , so that ϵ_i denotes the i th parameter value, and the closed form solution vector that we have obtained by $x(\epsilon)$. If $\epsilon_{13} = t_t^{\min}$ increases from its base value to any larger value \bar{t}_t^{\min} , with the other ϵ_i fixed at their initial values ϵ_i^* , then the optimal parametric solution vector is given by $x_4(\epsilon) = \bar{t}_t^{\min}$, with the other components $x_i(\epsilon)$ of the solution unchanged. On the other hand, if t_t^{\min} decreases to \underline{t}_t^{\min} , with the other parameter values fixed at their given initial values, then we find that the optimal parametric solution is given by $x_4(\epsilon) = \underline{t}_t^{\min}$, with no change in all the other components, providing $\underline{t}_t^{\min} - \bar{t}_t^{\min}$ is small. However, the nonbinding constraints g_2 , g_5 , g_9 and g_{10} evaluated at this solution decrease so the solution remains optimal providing

$$t_t^{\min} \geq \max(m_2, m_5, m_9, m_{10})$$

where m_i is the quantity involved in the i th constraint when the i th constraint is equivalently expressed in the form $x_4 \geq m_i[x(\epsilon), \epsilon]$, $i = 2, 5, 9, 10$, and the m_i are evaluated at the given solution $x(\epsilon)$

for the given parameter data base $\bar{\epsilon}$. Hence, the given closed form parametric solution is valid for $t_t^{\min} \geq \underline{t}_t^{\min}$, where

$$\underline{t}_t^{\min} = \max(m_2, m_5, m_9, m_{10}) \quad (10)$$

Clearly, $g_6, g_8, g_{11}, g_{14}, g_{15}$ and g_{16} will remain binding, while g_2, g_5, g_9 or g_{10} will become binding when $x_4 = \underline{t}_t^{\min}$.

We do not pursue other possible extensions of the parametric solution. A moral of the foregoing analysis is that closed form parametric solutions are not the exclusive province of "toy" problems. Closed form solutions provide precise and complete information, a goal that may be perfunctorily relinquished for "practical problems." Even if a complete parametric solution cannot be obtained, various components may be possible to derive in closed form over parameter perturbations of interest, thus greatly simplifying further analysis. In this problem, for example, even if $x_3(\epsilon)$ could not have been obtained, the other components of the local parametric solution are immediately available in closed form.

Obviously, the existence of a complete closed form parametric solution is a fortunate happenstance. Components of nonlinear programming parametric solutions will inevitably have to be estimated, perhaps most aptly by predictor-corrector techniques and their contemporary refinements, e.g., continuation methods [4], [5]. Nonetheless, experience suggests that some closed form functional relationships characterizing a solution will inevitably be available and are worth exploiting once a solution has been determined.

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