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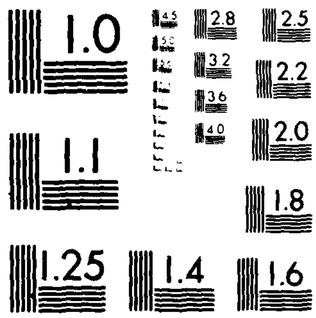

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Director of Public Affairs
Air Force Institute of Technology (ATC)
Wright-Patterson AFB, OH 45433

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The height anomaly contribution of the frequencies is calculated and compared to observed values. The portion of the variance of the 700 hPa height field explained by the frequencies is shown to be statistically significant to a very high degree of confidence in several regions.

The author concludes that an atmospheric tide develops in response to the Chandler wobble and is a significant part of the 700 hPa height field topography.

In addition, the relationship between the distribution of explained variance and the zones of maximum height field gradient is used to demonstrate the non-equilibrium nature of the tide.

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Chandler Period Atmospheric Oscillations

at the 700 HectoPascal Level

Over the Northern Hemisphere

By

Eugene Joseph Penuzzi

A thesis submitted to the Graduate School of the

University of Wisconsin-Madison in partial fulfillment

of the requirement for the degree of Master of Science

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Abstract

The response of the atmosphere to the Chandler wobble of the earth's axis of rotation is studied in a 26 year record of 700 hectoPascal (hPa) mean monthly heights at 469 Northern Hemisphere grid points. The author verifies the applicability of a four frequency spectrum of oscillation to the 700 hPa level and extracts phase and amplitude information for these frequencies plus harmonics from the data set.

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In addition, the relationship between the distribution of explained variance and the zones of maximum height field gradient is used to demonstrate the non-equilibrium nature of the tide.

List of Symbols

A_i	Fourier coefficient of sine terms of the i^{th} harmonic
B_i	Fourier coefficient of cosine terms of the i^{th} harmonic
C_i	Amplitude of the i^{th} harmonic
df_c	degrees of freedom relative to the Chandler harmonics
df_R	degrees of freedom relative to the residual harmonics
F	ratio of unbiased estimates of variance
ϕ_i	phase angle of the i^{th} harmonic
N_c	number of Chandler harmonics
N_R	number of residual harmonics
S_A^2	variance explained by the annual cycle
S_c^2	variance explained by the Chandler harmonics
S_E^2	expected variance explained by chance for the residual harmonics
S_i^2	variance explained by the i^{th} harmonic
S_R^2	variance explained by all residual harmonics
S_x^2	total variance of the sample
t	time, in months
ω_i	frequency, in radians per month, of the i^{th} harmonic
$\bar{x}$	mean of the sample
X_j	sample observation for the j^{th} time period
Z(t)	total height anomaly for time, t

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I. The Chandler Tide as a Possible Element of Atmospheric Dynamics

The atmosphere functions as a great heat engine fueled by the conversion of solar radiation to kinetic and potential energy. In the broadest sense hydrodynamicists and thermodynamicists can describe these processes. The juxtaposition of irregularly shaped, aerodynamically rough land masses and vast mobile water bodies with great heat capacities add to the complexity of the descriptive task. Superimposing a somewhat idealized thermal circulation on a rotating globe produces the familiar wave pattern of troughs and ridges distributed about both hemispheres.

In a coordinate system that rotates with the globe the resulting Coriolis acceleration makes no real contribution to the energetics of the atmosphere but does have a significant role in the formation of weather systems. Since this is so, would not other motions related to the earth's rotation produce a response in the atmosphere?

The existence of a free nutation of the axis of rotation referred to as the Chandler motion or Chandler wobble has been observed and studied for some 80 years (Rochester, 1973). Analyses of astronomical data have produced a variety of estimates of the period of the wobble. A survey of the literature on the subject by Bryson and Starr (1977) indicates that the period of the wobble falls into a fairly distinct four frequency distribution. The multiple frequency premise of Bryson and Starr (op. cit.) is used in this study and is discussed in greater detail in Section III.

Despite the lack of precise knowledge of the period or periods of the Chandler wobble, the possible response of the atmosphere-hydrosphere-lithosphere system has been studied. The response of the atmosphere, in particular, has been investigated by Maksimov (1958), Maksimov, et al (1967) and most recently by Bryson and Starr (1977). These investigators conducted their studies on time series of sea level pressures. In each case a "nutational contribution" or an "atmospheric compensation" was noted in the sea level records studied; however the "noise" in the sea level data due to local temporal or spatial variations tended to obscure the tidal contribution. A similar atmospheric tide signature should be evident in the upper air data acquired by radiosonde and should be less disturbed by surface interference. Such data would be more likely to yield clearer evidence of the atmospheric oscillations at the period of the Chandler wobble.

In this thesis I will describe the Chandler wobble contribution, hereafter referred to as the Chandler tide, as expressed in the 700 hectoPascal (hPa) height field record, and show that this contribution is statistically significant.

II. The Data Set

In any study of the Chandler tide in the atmosphere using sea level data, the magnitude of the contribution could be masked by errors resulting from instrument error, operator error or the effects of local variability. It is this latter consideration that is most effectively reduced by using upper level data.

For this reason, a 700 hPa record of mean monthly heights was used. The data covers the period from January 1951 through January 1976. A portion of this record had been used by Wahl (1972) in calculations of zonal and meridional indices at the 700 hPa level.

The values are arranged in a diamond grid between 80°N and 20°N latitude. On even latitudes the data points are at the intersections with the even-numbered meridians while on the odd latitudes (75°N, 65°N, etc.) the points are at the intersections with the odd meridians. A single value is included at the pole to produce a total of 469 data points. The mean monthly height values at each point are computed from twice daily (0000 GMT and 1200 GMT) maps and are entered in meters to an accuracy of better than ±2 meters.

Several assumptions were made with regard to the data. These were that (a) the monthly means are independent with the exception of the long term periodicities being studied; (b) the observations are approximately normally distributed and (c) the individual samples are stationary time series whose statistics are representative of the population parameters. The questioning of the independence of meteorological data is reasonable since persistence or "red noise" (Laurmann and Gates, 1977) is a well-known phenomenon. The use of mean monthly values reduces the temporal correlations, while using the largest available sample and limiting the application of derived sample statistics to a restricted population should suffice to ensure the validity of the assumptions.

In subsequent calculations, the first 312 monthly values, 26 full years, were used.

III. The Computations

1. The Period of the Chandler Tide at 700 hPa

As stated in Section I, the period of the Chandler wobble is complex. The initial investigation of the Chandler tide at 700 hPa must deal with the period of oscillation since all following studies are predicated on knowledge of this period.

Bryson and Starr (1977) have studied the subject in detail, therefore this investigation was limited to one of determining the applicability of their findings at sea level to the 700 hPa response. An analytical technique developed by Schickendanz and Bowen (1977) was used.

The analytical method uses a non-integer value in lieu of the harmonic number. The sine and cosine terms fitted through the data set need not begin or terminate at the ends of the record. The lack of orthogonality of the sines and cosines is taken into account and a correction for the correlation of the terms is made. The more familiar Fourier analysis method is shown by the authors to be a special case of this more general method. Additionally, spectral estimates can be produced for intermediate values at steps of 0.1

units, for example, thereby increasing the number of such estimates tenfold. This permits better resolution of the periodic components evident in a relatively short record but does not permit drawing conclusions about the relative amplitude or variance contribution of these periodic elements. The amplitude, phase and variance of each frequency will therefore be studied independently.

This technique was first applied to the raw data at a mid-latitude grid point in a data rich area. The resulting periodogram clearly showed that the leakage of the annual cycle overwhelmed all other frequencies in a broad band about the annual maximum. To overcome this problem, the data was normalized.

The ensemble means for each calendar month were calculated for the 26 year period as were the standard deviations. The departure of each successive monthly height value from the applicable mean was computed and divided by the respective month's standard deviation. This normalizing procedure suppressed the annual peak. A periodogram prepared using the normalized data for the same mid-latitude station used in the initial attempt was far better behaved in showing apparent periodicities in the record.

Thirty-three points of the 469 were selected for the frequency study. Selection criteria were that the grid point falls within plus or minus one degree of longitude and latitude of an actual observing site (Crutcher and Meserve, 1970) and that the period of operation for the station coincide with or exceed the 26 years of record (WMO Publication #217, 1973). Periodograms were prepared for each of the points as well as a composite periodogram of all 33 stations.

Each periodogram showed one or more peaks in the frequency band between .8500 to .9346 cycles per year with .8577 being the most common (8 cases) and .8646 the second most frequently observed (5 cases). The composite or mean periodogram showed a .8615 cycle per year frequency with a period of 13.93 months as the dominant value in the Chandler frequency range.

The bimodal distribution implied by the 33 cases studied supports the multiple frequency findings of Bryson and Starr (1977). Their analysis of pole position data for the period 1900-1975 showed four frequencies of .8075, .8352, .8543, and .8912 cycles per year with .8352 and .8543 being the more pronounced. An analysis using a long record (1899-1970) of the Northern Hemisphere sea level pressures produced similar results.

The poor resolution obtained from analysis of the 26 year long 700 hPa record did not provide a sufficiently strong reason to adopt a frequency profile other than that found by Bryson and Starr (op. cit.). The following studies of the phase and amplitude of the Chandler tide at 700 hPa will therefore use the four frequencies determined by Bryson and Starr (op. cit.).

2. The Phase of the Chandler Tide

The phase of the atmospheric tide was calculated for the two primary frequencies, .8352 and .5843 cycles per year, by the non-integer analytical technique described above, at each of the 469 data points. Figures 1 and 2 are phase diagrams of the Chandler tide at the respective frequencies. Both diagrams depict a far more complex structure than the smooth, uniform co-tidal pattern described by Maksimov (1958) and shown here in Figure 3. Neither phase diagram has a singular polar amphidromic point but rather tends toward a mid to low latitude distribution of nodes. We note in comparing the Chandler phase diagrams for the two main frequencies that certain of these nodes appear to coincide geographically; for example those along the eastern seaboard of the United States, the West coast of Baja California, North Africa and Central Russia. The complexity of these patterns and the existence of amphidromic points unique to one or the other of the frequencies implies an interaction or beating of the generated tides which will modulate their amplitude contribution to the 700 hPa height field.

Applying this reasoning to a four frequency structure means that any study of the amplitude contribution of the atmospheric tide must consider the phase relation of the waves generated at each frequency. The study of the amplitude will therefore include all four frequencies plus three harmonics of each. No weighting is applied since it is not known which mode of oscillation is dominant at any time.

3. Amplitude Contribution of the Four Frequency Chandler Tide

We used the normalized data described in Section III, Subsection 1, to calculate the amplitude, in units of standard deviation, for each frequency and its harmonics at each grid point. The total amplitude contribution at each point was calculated for the first 14 months of record, January 1951 through February 1952, by the following relation:

$$Z\ (t) = \sum_{j}\sum_{i} [c_i \cos (\omega_i t - \phi_i)]_j \tag{1}$$

where we first summed the contributions of each frequency and its harmonics to produce a group value and then summed over all groups. This assumes that each frequency is equally likely to occur which, as noted above, is not necessarily true. The amplitude contribution of the Chandler tide is in essence a height anomaly and is referred to as such in the subsequent discussion.

The height anomaly at each point was transformed from units of standard deviation to meters by multiplying the calculated anomaly by the point value of the standard deviation for the applicable month. These values were then plotted to yield 14 individual height anomaly charts for the prescribed period. The charts were analyzed by drawing the contour field at intervals of plus or minus 10 meters about the zero meter mean isopleth. Figure 4 shows the chart for January 1951, as an example. Several points can be made regarding the distribution and magnitude of the height anomaly centers based on the chart series.

First, the anomaly pattern is far from the uniform pattern produced by Maksimov (1958) [see Figure 3], as was anticipated, based on the comparison of the differences in the phase diagrams. Second, the height anomalies of greatest absolute magnitude are concentrated in the higher latitude bands as observed by Maksimov (op. cit.); however no uniform distribution around the $45^{o}N$ latitude band is observed as predicted by an equilibrium tide theory. A review of the data provides a simple explanation for this distribution. The magnitudes of the standard deviations are far greater in the higher latitudes, therefore even small tidal contributions, percentagewise, produce large anomalies. The waxing and waning of the anomaly magnitudes with the seasons, also noted by Bryson and Starr (1977), is evident in this chart series.

Figure 5 shows the placement and sense (positive or negative) of the anomaly centers produced by the four frequency model for January 1951 and compares them to the observed 700 hPa height anomaly field for the same period as published in Monthly Weather Review (Klein, 1951). The agreement of the two with respect to both location and sense is good and implies the reality of an atmospheric Chandler tide despite the poorer fit observed over Northern Asia. This is the result of data smoothing where the monthly values are based on only a limited number of observations. A more rigorous statistical discussion in the following section will give greater credence to this contention.

The final observation relates to the translation of the height anomalies during the 14 month period. The tracks of a major positive height anomaly shown in Figure 6 is far less coherent than either of the single frequency tracks of surface pressure anomalies shown by Bryson and Starr (op. cit.). The interaction of the multiple frequencies are responsible for this behavior. The previously noted agreement between calculated and observed anomalies supports both the multiple frequency hypothesis and the essential correctness of the frequencies used; however, the use of such a short record does not permit an extrapolation of the anomaly pattern beyond the period of record with any degree of confidence.

To this point, all evidence of the existence of an atmospheric tide forced by the Chandler wobble has been drawn by inference. We will now support the contention of the physical reality of the tide by means of the analysis of variance and a statistical significance test. We will also be able to draw conclusions regarding the non-equilibrium nature of the atmospheric tide.

IV. Evidence of the Chandler Tide in the Free Atmosphere

1. Variance Explained by the Chandler Tide

We have assumed, following Bryson and Starr (1977), that the Chandler wobble of the earth's rotation axis forces a response in the atmosphere in the form of a redistribution of mass in order to maintain a constant angular momentum budget. If this is so, then a portion of the variability in the height of the 700 hPa surface will be attributable to this phenomenon.

This assumption is based on the premise that the total variance of the data can be separated into components of experimental interest and expressed as a linear combination of these component variances (Hoel, 1966, p. 299). According to Panofsky and Brier (1968, p. 66) the component variances are viewed as being either one of two types, random variance or systematic variance. Random variations or "white noise" (Laurmann and Gates, 1977) are unpredictable and physically unexplainable. Systematic variance, on the other hand, is a response to a physical process and is essentially predictable.

We must first calculate the sample variance of the total 26 year record of mean monthly 700 hPa heights at each grid point by the following formula taken from Panofsky and Brier (1968, p. 26):

$$S_x^2 = \sum_j (x_j - \bar{x})^2 \qquad j = 1,\ldots,312 \tag{2}$$

Here S_x^2 is the total variance of the sample record of 312 months and $\bar{x}$ is the sample mean for the same period.

Applying the concept of component variance, we separate the total sample variance into a portion attributable to, and explainable by, the annual cycle and an unexplained remainder or residual variance, such that,

$$S_x^2 = S_A^2 + S_R^2 \tag{3}$$

where S_A^2 and S_R^2 are the aforementioned variances, respectively. The values assigned to S_A^2 are calculated by the method of Fourier analysis. The variance of the fundamental frequency (one cycle per year) and each of five harmonics of the fundamental is found by solving for the coefficients, A_i and B_i, using a discrete Fourier analysis method (Panofsky and Brier, 1968, p. 130). The variance accounted for by each of the six harmonics is given by the relation (Panofsky and Brier, 1968, p. 133):

$$S_i^2 = C_i^2/2 \tag{4}$$

where C_i is the amplitude of the i^{th} harmonic. The variance of the last harmonic is given by $S_{N/2}^2 = C_{N/2}^2$. The sum of the variances of the annual plus the five harmonics is the variance attributable to the year

$$S_A^2 = \sum_i S_i^2 \tag{5}$$

Substituting the values for S_x^2 and S_A^2 into (2) and solving yields the values for S_R^2, the remaining unexplained or residual variance.

The variance attributable to the four Chandler frequencies and three harmonics of each is calculated by the non-integer technique of Schickedanz and Bowen (1977) since the periods related to the Chandler frequencies are not evenly divisible into the 312 month record. Once the A and B coefficients have been determined, the amplitude and variance may be calculated by standard formulae. The total variance accounted for by all Chandler frequencies and their harmonics is:

$$S_c^2 = \sum_j \sum_i (S_i^2)_j \tag{6}$$

where we sum the variance attributable to each frequency with that attributable to its harmonics to form a group variance and then sum over all groups. The total Chandler variance is therefore composed of 16 component variances.

The percentage of the residual variance explained (PRE) is the ratio of the Chandler variance to the residual variance times 100:

$$PRE = S_C^2/S_R^2 \times 100 \tag{7}$$

The distribution of PRE is shown in Figure 7. This figure will be discussed as an adjunct to the statistical significance test in Section IV, Subsection 3 below.

2. Analysis of Variance

The primary tool for testing the validity of the assumption that the total variance can be expressed as a summation of systematic and random variances is the "analysis of variance" method.

If we attribute all the systematic variations to the annual variance, then all else is purely random and therefore unpredictable (Ramage, 1976), but we have already implied that there is an observable redistribution of the mass of the atmosphere in response to the Chandler wobble of the earth's axis of rotation. We will now show that the atmospheric oscillation in response to the Chandler wobble explains a sizeable portion of the residual variance in several areas around the Northern Hemisphere. We will show that the percent variance explained, as depicted in Figure 7, is in many areas far too large to be attributed to chance. In addition we will explain why the distribution of explained variance supports the concept of a non-equilibrium atmospheric tide.

3. Test of Significance of Variance Explained

As a hypothesis, let us assume that $S_c^2 = 0$ and use an F test to either accept or reject this hypothesis at some specified confidence level. The F test (Hoel, 1966, p. 285) uses a value, F, formed by a ratio of two unbiased estimates of variance. Tabulations of F for various confidence levels are published in a number of statistical texts and in the Chemical Rubber Company Standard Math Tables, which are used for the following calculations. This test is particularly good in that it is independent of population parameters and is one in which we are not likely to accept the tested hypothesis if it is actually false.

A time series of 312 observations can be decomposed into 156 terms which when recombined will reproduce that time series. The generally acknowledged systematic variance of the time series is explained by six of these terms, the annual or fundamental, and five harmonics. By our hypothesis, the residual variance is explained by the remaining 150 harmonics. The expected value of the contribution of any of these harmonics must be 1/150th of this variance since the white noise hypothesis is one of equal probability.

The contribution of any 16 harmonics, the number of Chandler components used in the calculation of S_C^2, will be (16) x (1/150) or 10.7% of S_R^2. We note that most values plotted in Figure 7 exceed this. Could these higher percentage values have occurred by chance? Applying the F test will answer this question.

The F ratio is defined by Hoel (1966, p. 285) as:

$$F = \frac{\frac{N_C}{N_C-1} \cdot S_C^2}{\frac{N_R}{N_R-1} \cdot S_E^2} \tag{8}$$

where N_C = the number of Chandler harmonics,

N_R = the number of residual harmonics,

$N_C-1 = df_C$ = number of degrees of freedom related to the Chandler harmonics,

$N_R-1 = df_R$ = number of degrees of freedom related to the residual harmonics,

S_C^2 = the portion of the residual variance explained by the Chandler harmonics,

and S_E^2 = the expected variance explained by the residual harmonics.

Rather than calculate an F value for each of the 469 point values of S_C^2, we find the F value for the appropriate degrees of freedom and a specified confidence level and solve for the S_c^2 required to equal or exceed that F value.

Table 1 lists the threshold values of explained variance, in percent, that meet the criteria for the respective confidence levels. The degrees of freedom employed are $df_c = 16-1 = 15$ and $df_R = 150-16-1 = 133$ for the F ratios shown.

Table 1

Confidence Limit	.90	.95	.975	.99	.995	.999
S_c^2 Threshold Value	15.6	17.6	19.5	22.0	24.0	28.0
F Ratio Value	1.54	1.74	1.93	2.18	2.37	2.77

Figure 8 is an analysis of the areas which have statistical significance level of 90% or greater and is isoplethed to delineated areas where the 95, 99 and 99.9% levels are attained or exceeded.

Several features of the analysis are immediately evident, primarily the continentality of the maxima and their distribution around the 45°N latitude circle. The placement of the maxima over the land masses is probably the result of the data density. Few observations exist over the vast reaches of the oceans and therefore only greatly smoothed estimates of the height field are available for these areas. The resultant reduction in variance is so great that it limits the ability to resolve the Chandler tide contribution from this data with any degree of accuracy. The pattern of maximum explained variance about the 45°N latitude band shown in Figures 7 and 8 bears an undeniable similarity to the pattern observed in the 700 hPa wind field (Figure 9) with respect to placement and shape of the wind maxima. The significance of this similarity rests in the consideration of the mode of oscillation of the atmospheric tide.

The zones of steepest height gradients are the zones of maximum winds. A displacement of the gradient would produce a sizeable change in the height field of the region into which such a displacement was made. Equilibrium tide theory cannot account for the required lateral displacements and therefore could not explain the observed relationship between the maxima of the wind field and the maxima of explained variance of the Chandler tide. The atmospheric tide oscillation must be a non-equilibrium phenomenon.

V. Conclusions

The statistical significance test permits rejecting the no-variance-explained hypothesis with a very high degree of confidence. This significance is further enhanced by the fact that the frequencies used in the investigation were selected *a priori*. The probability of being in error is less than one chance in twenty in many regions and less than one in one thousand over the heartland of North America.

With so small a likelihood that random chance produced the observed distribution, it is now possible to state that a portion of the residual variance must be systematic variance. Since systematic variance, by definition, is based in physics it follows that a physical process must explain this variance.

The Chandler wobble of the earth's axis of rotation must be the forcing function. To argue that the converse is true, that the redistribution of atmospheric mass drives the Chandler wobble, neglects the observed fact that the frequencies of oscillation have been shown to be relatively discrete. Even if the atmosphere does contribute to the excitation of the wobble, the significance of the explained variance in the atmosphere at those particular frequencies dictates that a feedback mechanism must be present.

We can now state the following conclusions:

(1) An oscillation of the atmosphere related to the observed wobble of the earth's axis is to a very high degree of certainty a physical reality.

(2) The atmospheric tide associated with the wobble must be a non-equilibrium tide.

Since there is no reason to believe that the data used in the analysis is not representative of the 700 hPa height field, we contend that the evidence of the Chandler tide in the atmosphere is not restricted to the time period studied.

Having shown with a high degree of probability that the Chandler tide at the 700 hPa level does exist, all the inferential evidence previously put forth regarding the phase, amplitude and distribution of height anomalies is now on a firm base. It is obvious and implicit in the physics that the surface pressure anomaly of the Chandler tide described by Bryson and Starr (1977) must also be present.

VI. Areas of Further Investigation

A goal of any ongoing study of the Chandler tide is to use the anomaly pattern as a predictor of climatic events 1 to 10 years ahead through the production of analog anomaly charts and as an input to numerical models. Further studies to increase our knowledge of the sequence in which the frequencies occur or the dominance of one or more of the frequencies at any given time would permit a weighting of their respective contributions and thereby enhance the accuracy of the forecast product.

The low level of statistical significance of the atmospheric tide over much of the world ocean should act as an incentive to further research into the behavior of the pole tide in the seas. The coincidence of seven year cycles described in folk lore in such apparently diverse oceanic phenomena as fluctuations of the blue-claw crab population to the onset of El Niño (Scherdtfeger, 1978) and the beating of the Chandler period with the annual cycle at seven year intervals warrants investigation. The formation of sea surface temperature anomalies that are believed to cause modifications in the general circulation patterns (Namias, 1969) might be related to the

Chandler tides in the sea and the atmosphere.

The role of the Chandler wobble in the dynamics of the atmosphere is as yet undetermined. Does the variation of latitude affect the β term? Could the Chandler wobble possibly produce small ageostrophic components that either sustain or modulate the intensities of the climatological pressure centers?

All of these questions may now be looked upon as more than idle suppositions since the initial step necessary to pose them in a serious vein has been taken.

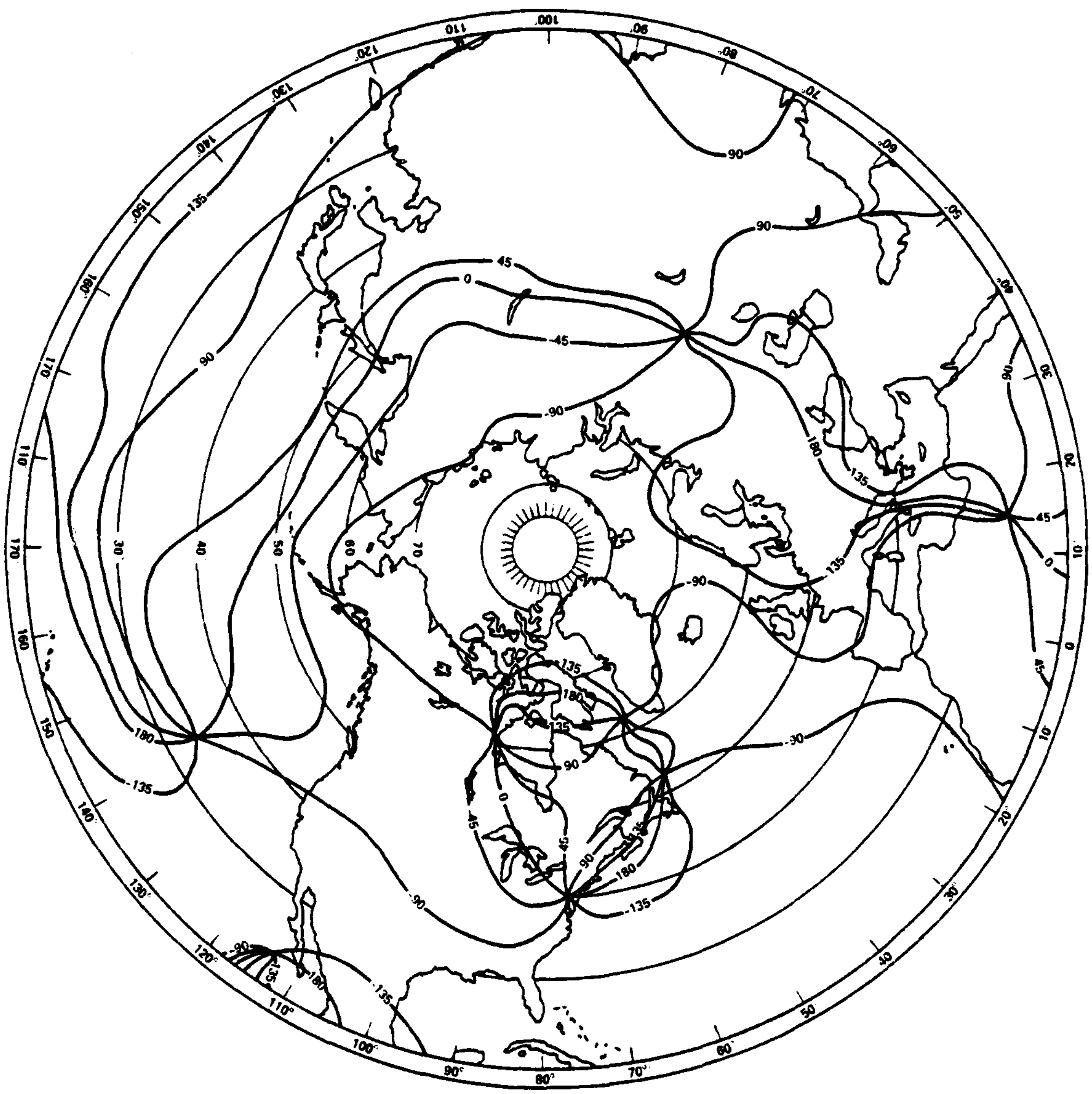
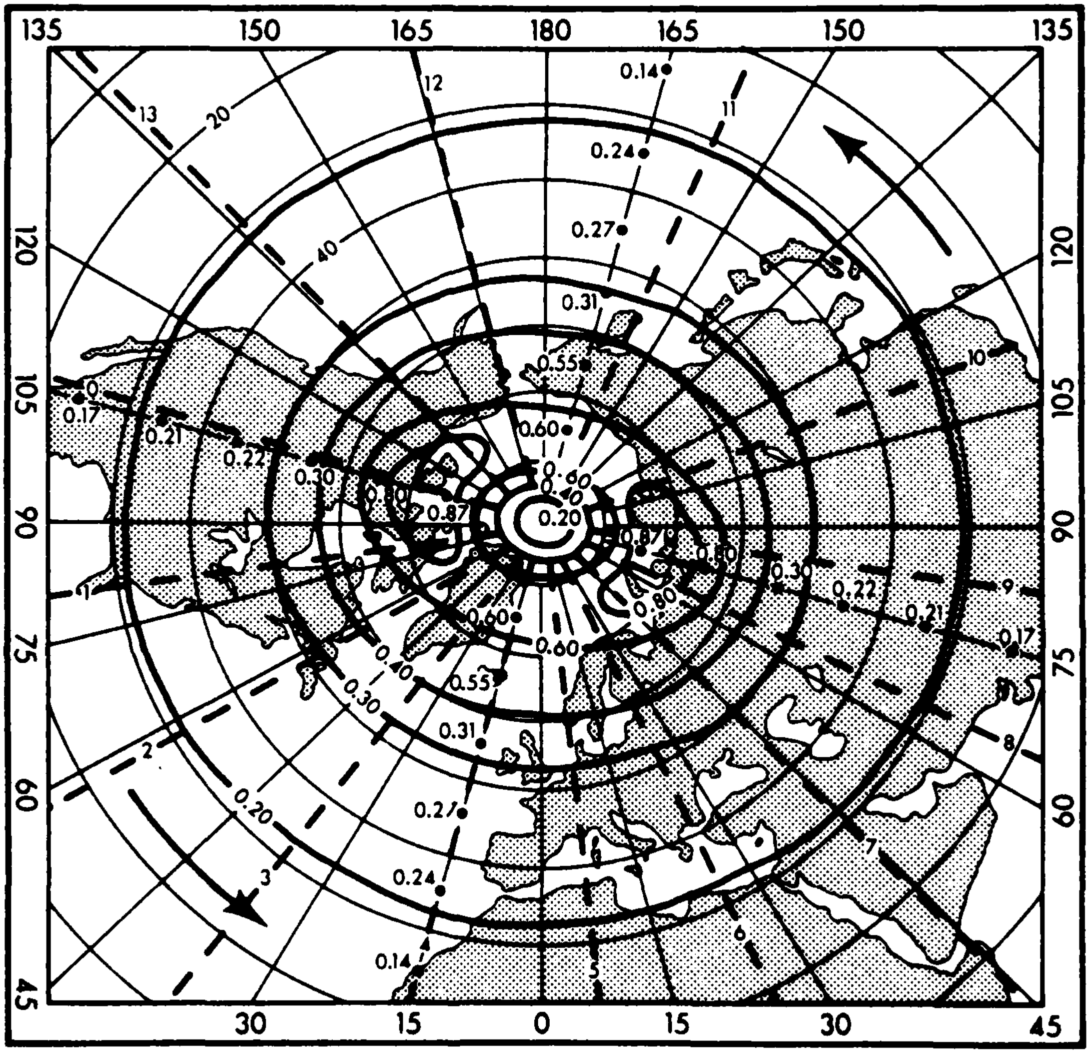


Figure 1

Figure 2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150
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0.27
0.31
0.55
0.60
0.20
0.87
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Figure 3

Figure 4

Figure 5

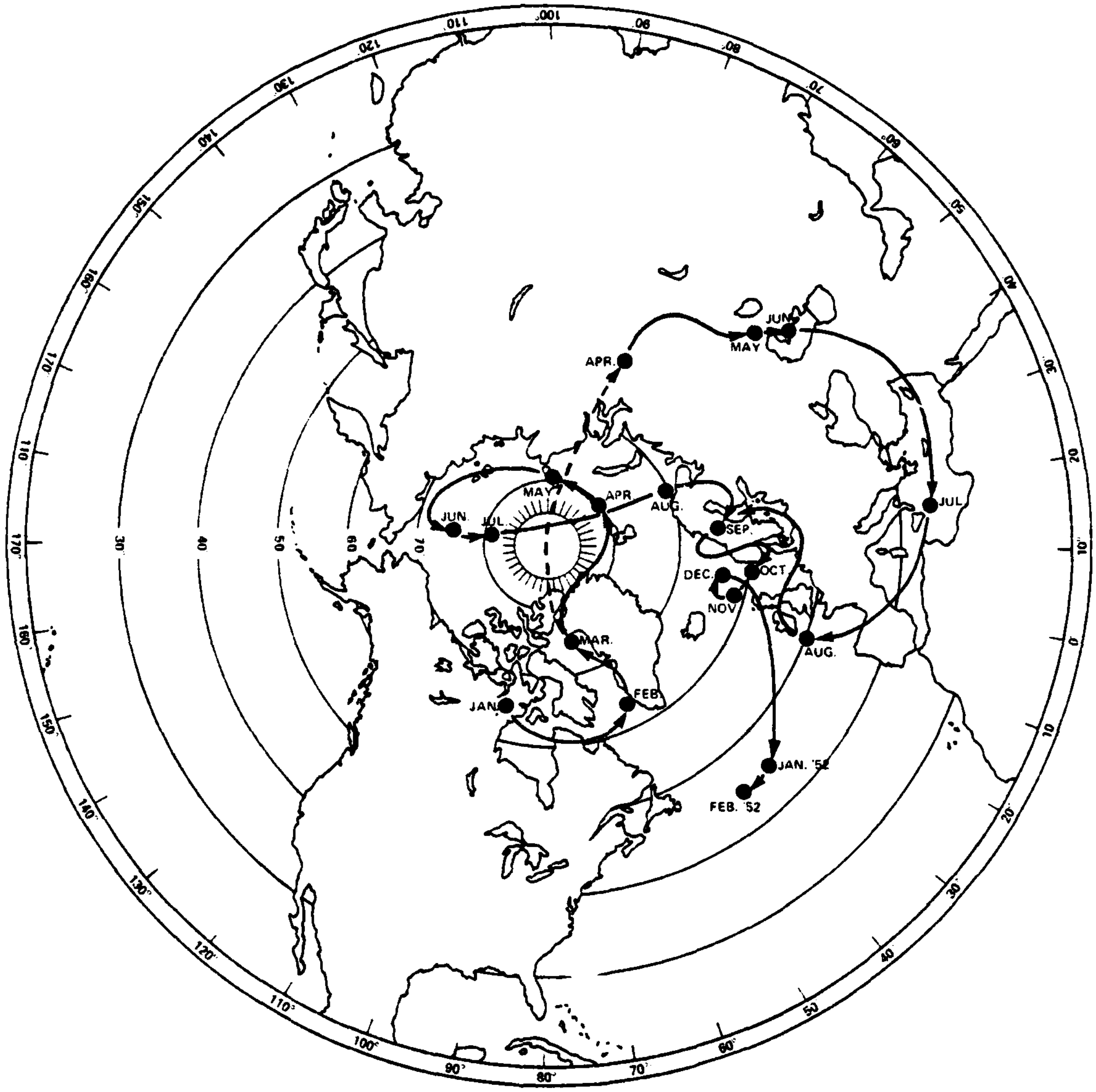
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Figure 6

Figure 7

Figure 8

110
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80°
90°
100°
110°
120°
130°
140°
150
160
170
170
160
150
140
130
120
1
4
9
16
9
4
1
16
9
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Figure 3

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